A THREE-DIMENSIONAL BODY-FITTED COORDINATE SYSTEM FOR FLOW FIELD CALCULATIONS ON ASYMMETRIC NOSETIPS

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ABSTRACT

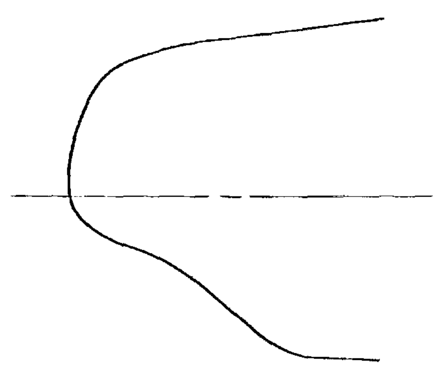
This presentation describes a three-dimensional body-fitted coordinate system developed for use in the calculation of inviscid flows over ablated, asymmetric reentry vehicle nosetips. Because of the potential geometric asymmetries, no standard coordinate system (e.g., spherical, axisymmetric reference surface-normal) is capable of being closely aligned with the nosetip surface. To generate a 3-D, body-fitted coordinate system an analytic mapping procedure is applied that is conformal within each meridional plane of the nosetip; these transformations are then coupled circumferentially to yield a three-dimensional coordinate system. The mappings used are defined in terms of "hinge points", which are points selected to approximate the body contours in each meridional plane. The selection of appropriate hinge points has been automated to facilitate the use of the resulting nosetip flow field code.

PROBLEM DEFINITION

The goal of this effort is the development of a procedure for calculating supersonic/hypersonic inviscid flows over asymmetric ablated reentry vehicle nosetips. These asymmetric shapes, such as illustrated in this figure, result from asymmetric transition on the nosetip, which occurs at the lower altitudes during reentry (i.e., below 15.24 km). Because these shapes occur in the high Reynolds number, turbulent regime, with thin boundary layers, an inviscid solution is capable of accurately predicting the pressure forces on the nosetip. The nosetip flow field solution is also required to provide the required initial data for afterbody calculations; this coupling of nosetip and afterbody codes allows accurate prediction of the effects of the nosetip shape on the afterbody flow field.

The flow field code developed is a finite-difference solution of the unsteady Euler equations in "non-conservation" form (i.e., the dependent variables are the logarithm of pressure, P, the velocity components, u,v,w, and the entropy, s). In this approach the steady flow solution is sought as the asymptotic limit of an unsteady flow, starting from an assumed initial flow field.

CALCULATION OF SUPERSONIC/HYPERSONIC INVISCID FLOWS OVER ASYMMETRIC ABLATED REENTRY VEHICLE NOSETIPS



ASYMMETRIC ABLATED NOSETIP SHAPE

APPROACH

- FINITE-DIFFERENCE SOLUTION OF UNSTEADY EULER EQUATIONS
- STEADY FLOW SOLUTION SOUGHT AS THE ASYMPTOTIC LIMIT OF UNSTEADY FLOW

COORDINATE SYSTEM REQUIREMENTS

It is well known that accurate numerical calculation of fluid flows requires the use of a coordinate system closely aligned with the principal features of the flow. For the nosetip problem this requirement would be satisfied by a coordinate system which closely follows the body shape and, hence, the streamlines of the flow. Because of the asymmetric nosetip geometries being considered, standard coordinate systems (e.g., spherical, axisymmetric reference surface-normal) are incapable of being aligned with the nosetip surface at all points. Thus, a coordinate transformation is sought that will align the coordinate system with an arbitrary nosetip geometry. By requiring the transformation to be in analytic form, the need of solving partial differential equations to define the transformation can be avoided. Finally, the transformation should be in a form that readily lends itself to automated definition, minimizing the inputs required of a user of the code.

OPTIMUM COORDINATE SYSTEM FOR NUMERICAL FLOW FIELD CALCULATIONS IS BODY-ORIENTED

COORDINATE TRANSFORMATION SOUGHT THAT:

1.) ALIGNS COORDINATE SURFACES WITH BODY SURFACE

2.) IS ANALYTIC (SOLUTION OF PDE'S NOT REQUIRED TO DEFINE TRANSFORMATION)

3.) CAN BE READILY AUTOMATED (TO MINIMIZE INPUTS REQUIRED FROM USER)

COORDINATE TRANSFORMATION

The nosetip geometry is defined in an (x,y,ϕ) cylindrical coordinate system, and a mapping to a (ξ,η,θ) transformed coordinate system is sought. Since current reentry vehicle nosetips are initially axisymmetric (prior to ablative shape change), it is assumed that nosetip cross-sections retain some "axisymmetric" character during reentry. Thus, no transformation of the circumferential coordinate is required, and $\theta = \phi$ is assigned. (This transformation can readily be generalized to $\theta = f(\phi)$ if required for other applications of this approach.) Within a ϕ = constant meridional plane, the transformation reduces to the two-dimensional form $\xi = \xi(x,y)$, $\eta = \eta(x,y)$. Conformal transformations from the $z = x+iy$ to the $\zeta = \xi+i\eta$ plane are desirable, ensuring that an orthogonal (ξ,η) grid maps back onto an orthogonal grid in the (x,y) plane.

(x,y,ϕ) CYLINDRICAL COORDINATES IN PHYSICAL SPACE

(ξ,η,θ) COORDINATES IN TRANSFORMED SPACE

TRANSFORMATION OF CIRCUMFERENTIAL COORDINATE NOT REQUIRED (NOSETIPS INITIALLY AXISYMMETRIC); ASSUME TRANSFORMATION TAKES THE FORM

$$\xi = \xi(x,y,\phi)$$

$$\eta = \eta(x,y,\phi)$$

$$\theta = \phi$$

IN A MERIDIONAL PLANE (ϕ = CONSTANT), THE TRANSFORMATION REDUCES TO

$$\xi = \xi(x,y)$$

$$\eta = \eta(x,y)$$

DEFINITION OF TRANSFORMATION

The approach used to define the coordinate transformations is a modification of the "hinge point" approach of Moretti*. The mapping is defined as a sequence of conformal transformations of the form

$$z_{j+1} - 1 = [z_j - h_{j+1,j}]^{\delta_j}$$

where $z_j = x_j + iy_j$ ($j = 1$ is physical space) and $h_{i,j}$ is the i^{th} hinge point in the z_j plane. The hinge points in the physical (z_1) plane are selected to approximately model the body geometry. By mapping the hinge points sequentially onto the horizontal axis, the image of the body surface will then be a nearly horizontal contour.

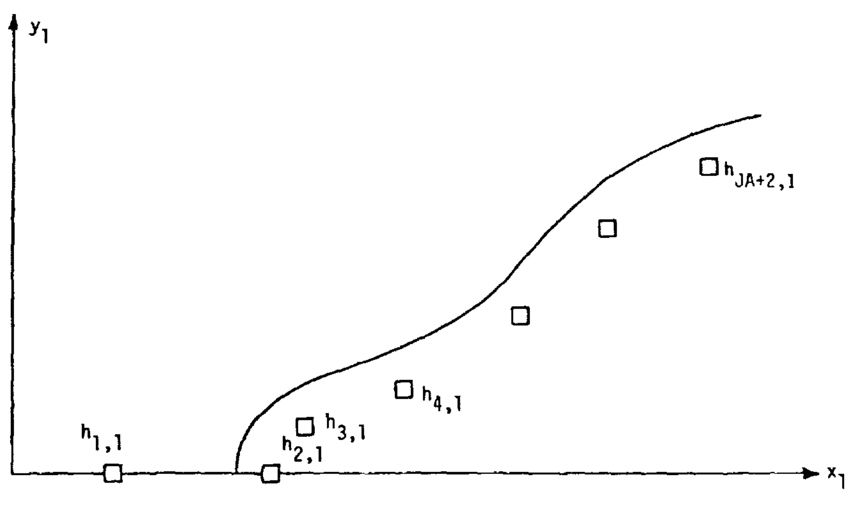
INDEPENDENTLY IN EACH MERIDIONAL PLANE, DEFINE A SEQUENCE OF CONFORMAL TRANSFORMATIONS

$$z_{j+1} - 1 = [z_j - h_{j+1,j}]^{\delta_j} \quad j = 1,2,\ldots,JA$$

$z_j = x_j + i_{yj}$ ($j = 1$ IS PHYSICAL SPACE)

$h_{i,j} = i^{th}$ "HINGE POINT" IN j^{th} SPACE

HINGE POINTS ARE SELECTED TO APPROXIMATE BODY GEOMETRY



HINGE POINT DEFINITION

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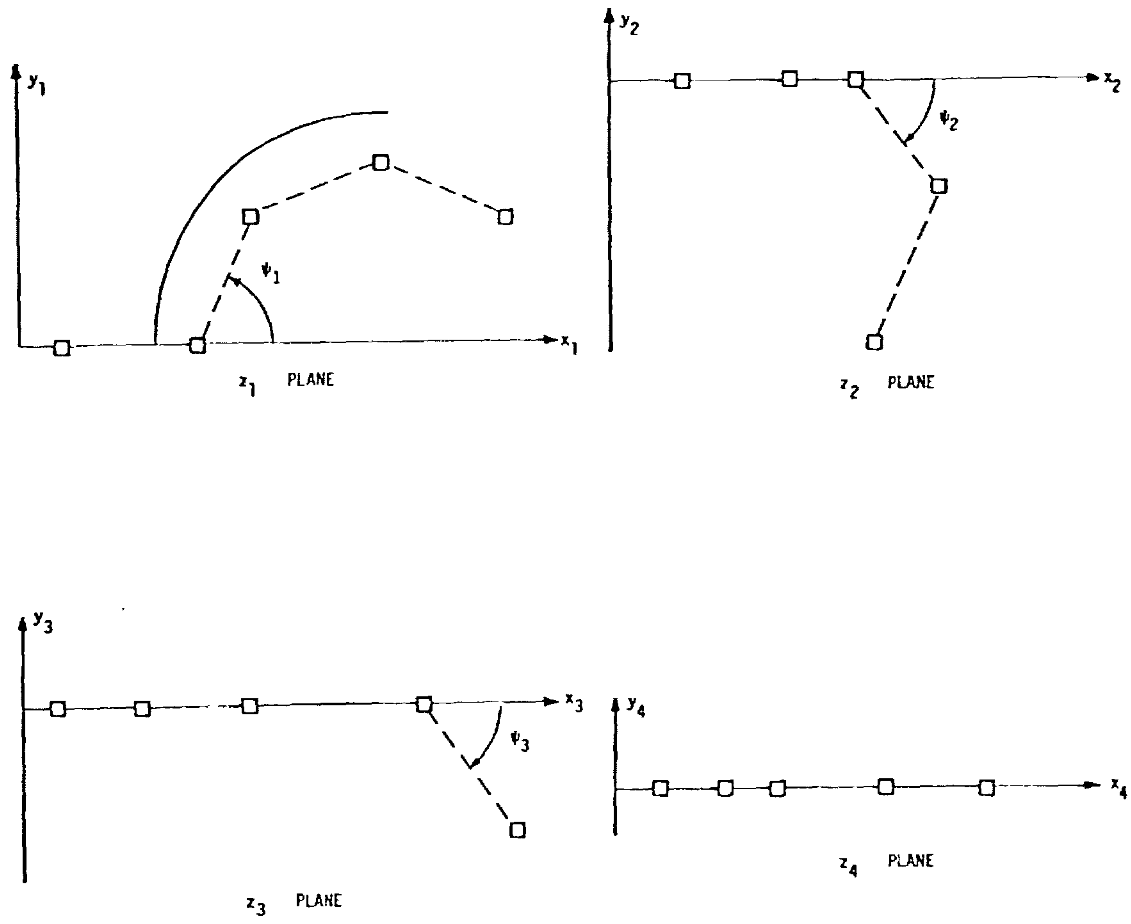
*Moretti, G., "Conformal Mappings for Computations of Steady, Three-Dimensional, Supersonic Flows," Numerical/Laboratory Computer Methods in Fluid Mechanics, ASME, 1976.

SEQUENCE OF TRANSFORMATIONS

In the j^{th} mapping of the sequence, the transformation is centered around the hinge point $h_{j+1,j}$. The mappings have the property of keeping the hinge points, $h_{i,j}$ ($i \leq j+1$) on the horizontal axis, while mapping the hinge point $h_{j+2,j}$ onto the horizontal axis. Thus, after JA transformations, all JA+2 hinge points in the JA+1 space will lie on the horizontal axis. (Each mapping in this sequence may be considered a "point-wise Schwarz-Christoffel" transformation.) This figure illustrates the sequence of transformations for JA = 3.



EXPONENTS OF TRANSFORMATIONS: $\delta_j = \frac{\pi}{\pi - \psi_j}$

In order to establish a grid suitable for flow field calculations when the image of the body contour is a nearly horizontal surface, it is desirable to have the image of the centerline external to the body lie along the vertical axis. This is achieved using an additional conformal transformation, centered around the second hinge point, of the form

$$z_{JA+2} = (z_{JA+1} - h_{2,JA+1})^{1/2}.$$

The last transformation is a simple stretching (which is also conformal):

$$\zeta = \xi + i\eta = az_{JA+2} \quad .$$

(This stretching is used in the calculation procedure along the centerline.) This figure illustrates the body contour resulting in the ζ-plane for the case of a sphere with JA = 3, where the body surface is defined as $\eta = b(\xi)$.

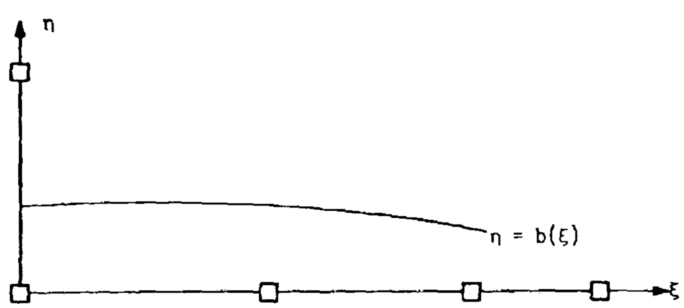
MAP CENTERLINE ONTO VERTICAL AXIS WITH

$$z_{JA+2} = (z_{JA+1} - h_{2,JA+1})^{1/2}$$

ALLOW FOR SIMPLE STRETCHING (REQUIRED FOR CENTERLINE TREATMENT) WITH

$$\zeta = \xi + i\eta = az_{JA+2}$$

RESULTING BODY CONTOUR:

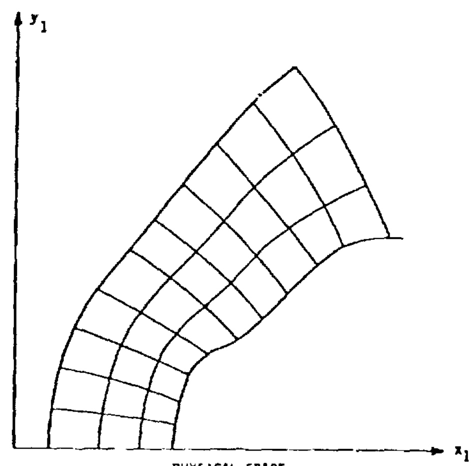


COMPUTATIONAL TRANSFORMATION

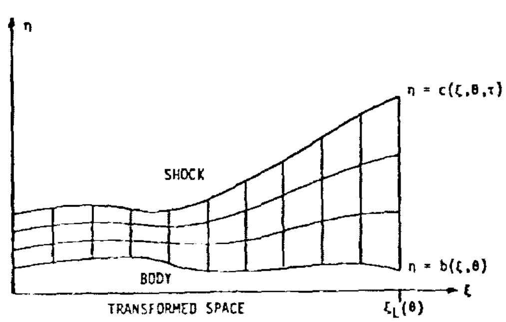
For the flow field calculation it is desirable to have equally spaced grid points. Thus, a transformation to a computational coordinate system (X,Y,Z) is used, in which grid points are equally spaced circumferentially in θ, longitudinally in ξ within each meridional plane, and in η between the body and the shock. It is important to note that the (X,Y,Z) system is not orthogonal, and that the computational transformation varies with time as the bow shock position varies during the time-dependent calculation. These sketches illustrate the computational grids resulting in a meridional plane in both physical ($z = x+iy$) and transformed ($\zeta = \xi+i\eta$) space for a typical ablated nosetip contour (with the shock layer thickness exaggerated for clarity).

DESIRE GRID POINTS EQUALLY SPACED IN ξ ALONG BODY, IN η BETWEEN BODY AND SHOCK, AND IN θ CIRCUMFERENTIALLY

$$X = \frac{\theta}{2\pi} \qquad Y = \frac{\xi}{\xi_L(\theta)} \qquad Z = \frac{\eta - b(\xi,\theta)}{c(\xi,\theta,\tau) - b(\xi,\theta)}$$



PHYSICAL SPACE



TRANSFORMED SPACE

PARAMETERS OF THE TRANSFORMATION

In transforming the governing equations from physical to the (X,Y,Z) computational coordinates, certain derivatives of the transformation are required. Because the transformation has been defined in analytic form, these derivatives can readily be evaluated analytically and are functions only of the hinge point locations. Within a meridional plane (ϕ = constant), the required derivatives are $g = \partial\zeta/\partial z$ and $\Phi = \partial(\log g)/\partial\zeta$. Circumferentially, the independent transformations in each meridional plane can be coupled to produce a three-dimensional transformation by assuming that hinge point locations can be expressed as $h_{i,j}(\phi)$. The required circumferential parameters of the transformation, ζ_ϕ and g_ϕ, can be evaluated analytically if each meridional plane has the same number of hinge points and assuming the form of interpolating functions for $h_{i,j}(\phi)$. Alternatively, it has been found to be sufficient to evaluate ζ_ϕ and g_ϕ from Taylor series expansions using data at computational (X,Y,Z) mesh points, with the forms of the resulting expressions shown in the figure.

REQUIRED IN WRITING GOVERNING EQUATIONS IN TRANSFORMED COORDINATES

$$g = \frac{\partial\zeta}{\partial z} = Ge^{i\omega} = \xi_x + i\eta_x = -i\xi_y + \eta_y$$

$$\Phi = \frac{\partial(\log g)}{\partial\zeta}$$

CAN BE EVALUATED ANALYTICALLY

CIRCUMFERENTIAL PARAMETERS OF THE TRANSFORMATION

ζ_ϕ, g_ϕ CAN BE EVALUATED ANALYTICALLY IF EACH MERIDIONAL PLANE HAS THE SAME NUMBER OF HINGE POINTS, ASSUMING INTERPOLATING FUNCTIONS FOR $h_{i,j}(\phi)$

ALTERNATIVELY, EVALUATE FROM TAYLOR SERIES EXPANSIONS:

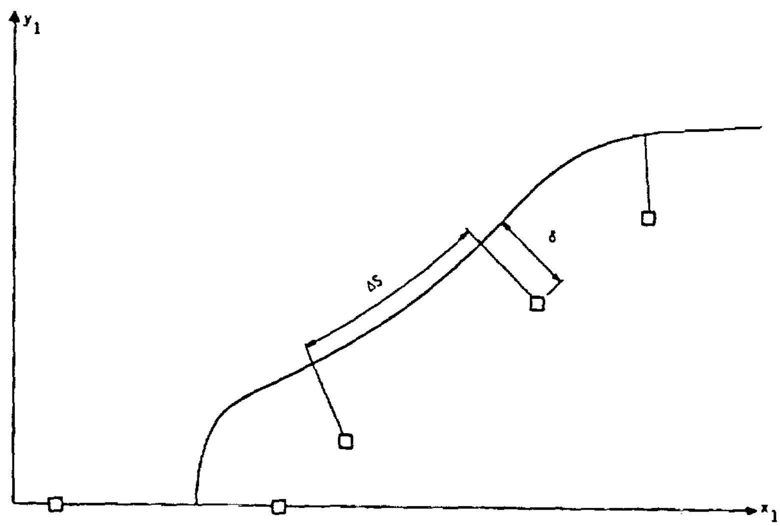
$$\zeta_\phi = \xi_\phi + i\eta_\phi = \frac{\zeta_2-\zeta_1-g(z_2-z_1)}{\phi_2-\phi_1}$$

$$g_\phi = \frac{g_2-g_1-g^2\,\Phi(z_2-z_1)}{\phi_2-\phi_1}$$

$(\ )_1 \rightarrow (X-\Delta X,Y,Z)$, $\quad (\ )_2 \rightarrow (X+\Delta X,Y,Z)$ IN COMPUTATIONAL MESH

AUTOMATIC GENERATION OF HINGE POINTS

To simplify the application of this coordinate transformation to the asymmetric nosetip flow field problem, the selection of hinge points that define the transformations has been automated. Within each meridional plane to be computed, body normals are constructed at points equally spaced in wetted length along the body profile. The hinge points are then selected to lie a distance δ inside the body along these normals. By relating δ to any convenient scale factor for a nosetip geometry, the only input required of the user of the code is the number of hinge points to be used. The locations of the first two hinge points (i.e., those that lie on the x axis) are the same in each meridional plane, in order to simplify the treatment of the centerline. Typically, no more than nine hinge points per meridional plane (JA = 7) are necessary for the nosetip flow field problem.



HINGE POINTS LOCATED DISTANCE δ ALONG INWARD BODY NORMALS, FROM BODY POINTS EQUALLY SPACED IN WETTED LENGTH

ONLY INPUT REQUIRED OF USER IS NUMBER OF HINGE POINTS TO BE USED IN EACH MERIDIONAL PLANE

TREATMENT OF CENTERLINE

The greatest complication encountered in the use of this 3-D coordinate transformation is the extra care that must be taken in treating the grid points on the centerline. Since the transformations in each meridional plane are independent, the scale factors $g = \partial\zeta/\partial z$ along the centerline will not be the same in each meridional plane. Thus, one computational grid point at the centerline will represent different physical points for each value of ϕ . To minimize these discrepancies, the stretching transformation $\zeta = az_{JA+2}$ is used to ensure that the images of the first hinge point are coincident in all meridional planes. The remaining discrepancies are small enough that simple linear interpolations can be used to account for differences in the scale factors.

In addition to the mapping complications along the centerline, the governing equations in cylindrical coordinates are singular along $y = 0$. This difficulty has been avoided by using a Cartesian (x_1,x_2,x_3) coordinate system for the centerline analysis. The required Cartesian derivatives can be expressed in terms of the radial derivative $\partial/\partial y$ in cylindrical coordinates for certain values of ϕ, as shown in this figure. The only restriction resulting from this analysis is that computational planes must be located at $\phi = 0, \pi/2, \pi$, and $3\pi/2$.

AT THE CENTERLINE $(y = 0)$, SCALE FACTORS $(g = \partial\zeta/\partial z)$ VARY WITH ϕ

STRETCHING TRANSFORMATION USED TO MINIMIZE DISCREPANCIES, WITH

$$a(\phi_k) = \frac{h_{1,JA+2}(\phi = 0)}{h_{1,JA+2}(\phi = \phi_k)}$$

CARTESIAN COORDINATES (x_1,x_2,x_3) USED IN CENTERLINE ANALYSIS

$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial x_2} = \cos\phi \frac{\partial}{\partial y} - \frac{\sin\phi}{y}\frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial x_3} = \sin\phi \frac{\partial}{\partial y} + \frac{\cos\phi}{y}\frac{\partial}{\partial \phi}$$

WITH $$\lim_{y\to 0} \frac{1}{y}\frac{\partial}{\partial\phi} = \frac{\partial^2}{\partial y\partial\phi} \text{ FINITE,}$$

$$\frac{\partial}{\partial x_2} = \cos\phi \frac{\partial}{\partial y}, \; \phi = 0, \pi$$

$$\frac{\partial}{\partial x_3} = \sin\phi \frac{\partial}{\partial y}, \; \phi = \frac{\pi}{2}, \frac{3\pi}{2}$$

RESULTING FLOW FIELD CODE

The 3-D, time-dependent, inviscid nosetip flow field code that was developed using the 3-D coordinate transformation described here is called CM3DT (Conformal Mapping 3-D Transonic). This code can treat ideal or equilibrium real gas thermodynamics, has both pitch and yaw capability, and is able to treat weak embedded shocks on indented nosetips using the λ-differencing scheme*. To provide total body inviscid flow field capability, the CM3DT code has been coupled to the BMO/3IS**, NSWC/D3CSS+, and STEIN++ afterbody codes. Complete details on the CM3DT analysis and results obtained with this code may be found in the following references:

Hall, D. W., "Inviscid Aerodynamic Predictions for Ballistic Reentry Vehicles with Ablated Nosetips," Ph.D. Dissertation University of Pennsylvania, 1979.

Hall, D. W., "Calculation of Inviscid Supersonic Flow over Ablated Nosetips," AIAA Paper 79-0342, January 1979.

CM3DT (CONFORMAL MAPPING 3-D TRANSONIC)
NOSETIP FLOW FIELD CODE

- IDEAL OR EQUILIBRIUM REAL GAS THERMODYNAMICS
- PITCH AND YAW CAPABILITY
- λ-DIFFERENCING SCHEME USED TO TREAT WEAK EMBEDDED SHOCKS ON INDENTED NOSETIPS
- COUPLED TO AFTERBODY CODES FOR TOTAL INVISCID FLOW FIELD CAPABILITY
    - BMO/3IS
    - NSWC/D3CSS
    - STEIN
- $81{,}000_{10}$ CORE STORAGE REQUIRED

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*Moretti, G., "An Old Integration Scheme for Compressible Flow Revisited, Refurbished, and Put to Work," Polytechnic Institute of New York, POLY-M/AE Report 78-22, September 1978.

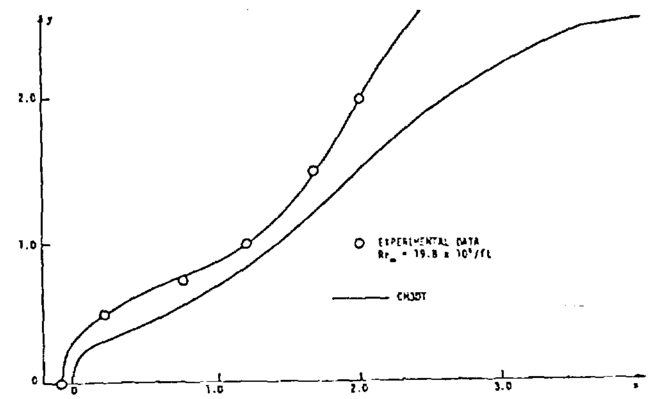
**Kyriss, C. L. and Harris, T. B., "A Three-Dimensional Flow Field Computer Program for Maneuvering and Ballistic Reentry Vehicles," 10th U.S. Navy Symposium on Aeroballistics, July 1975; also, Daywitt, J., Brant, D., and Bosworth, F., "Computational Technique for Three-Dimensional Inviscid Flow Fields about Reentry Vehicles, Volume I: Numerical Analysis," SAMSO TR-79-5, April 1978.

+Solomon, J. M., Ciment, M., Ferguson, R. E., Bell, J. B., and Wardlaw, A. B., Jr., "A Program for Computing Steady Inviscid Three-Dimensional Supersonic Flow on Reentry Vehicles, Volume I: Analysis and Programming," Naval Surface Weapons Center, NSWC/WOL/TR 77-28, February 1977.

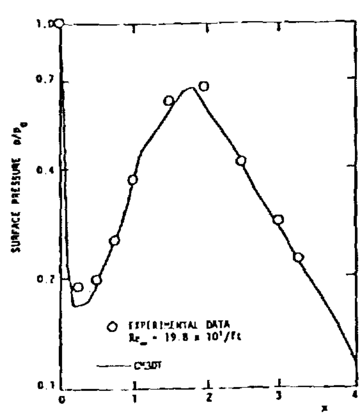
++Marconi, F., Salas, M., and Yaeger, L., "Development of a Computer Code for Calculating the Steady Super/Hypersonic Inviscid Flow around Real Configurations, Volume I. Computational Technique," NASA CR-2675, April 1976.

CM3DT RESULTS

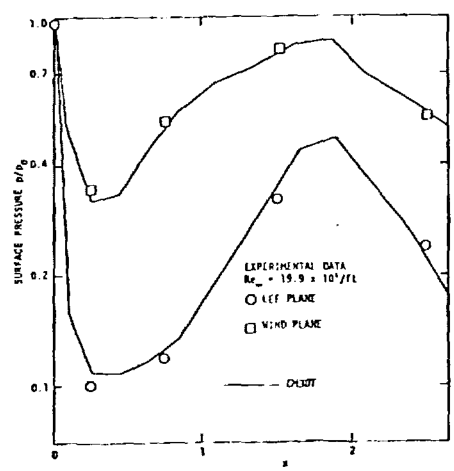
This figure presents some typical results obtained with the CM3DT inviscid nosetip flow field code. Shown are comparisons of predictions to data obtained for the PANT Triconic shape* at $M_\infty = 5$. It is significant that attempts to compute the flow over this slender shape using a time-dependent code formulated in a spherical coordinate system were unsuccessful. CM3DT, with its body-oriented coordinate system, was able to obtain converged solutions for this shape, with the predictions agreeing well with the data, as seen in this figure.



CM3DT Shock Shape Prediction for the PANT Triconic at $M_\infty = 5$, $\alpha = 0°$



Surface Pressure Predictions for the PANT Triconic at $M_\infty = 5$, $\alpha = 0°$



Surface Pressure Predictions for the PANT Triconic at $M_\infty = 5$, $\alpha = 10°$

(Note: 1 ft = 0.3048 m)

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*Abbett, M. J. and Davis, J. E., "Interim Report, Passive Nosetip Technology (PANT) Program, Volume IV. Heat Transfer and Pressure Distribution on Ablated Shapes, Part II. Data Correlation and Analysis," Space and Missile Systems Organization, TR-74-86, January 1974.

CM3DT - RUN TIMES

On a CDC Cyber 176 computer, the CM3DT inviscid nosetip code with λ-differencing requires approximately 0.00045 CP seconds per grid point per time step (iteration). Typically, 400-500 time steps are required to obtain a converged solution. It is estimated that the computer time required for a solution has been increased by approximately 20% by using the 3-D coordinate transformation described here, when the parameters of the transformation on the moving grid are updated every ten time steps. When compared to the standard MacCormack differencing scheme, the use of λ-differencing scheme increases the run time requirements approximately 50% for this code.

ON A CDC CYBER 176, CM3DT REQUIRES 0.00045 CP SECS/POINT/STEP FOR IDEAL GAS CALCULATIONS WITH λ-DIFFERENCING

- 20% PENALTY INCURRED FOR COORDINATE TRANSFORMATION (PARAMETERS ON MOVING GRID UPDATED EVERY 10 TIME STEPS)

- 50% PENALTY INCURRED FOR λ-DIFFERENCING (RELATIVE TO MAC CORMACK DIFFERENCING)