CONFORMAL MAPPINGS OF MULTIPLY CONNECTED REGIONS

ONTO REGIONS WITH SPECIFIED BOUNDARY SHAPES

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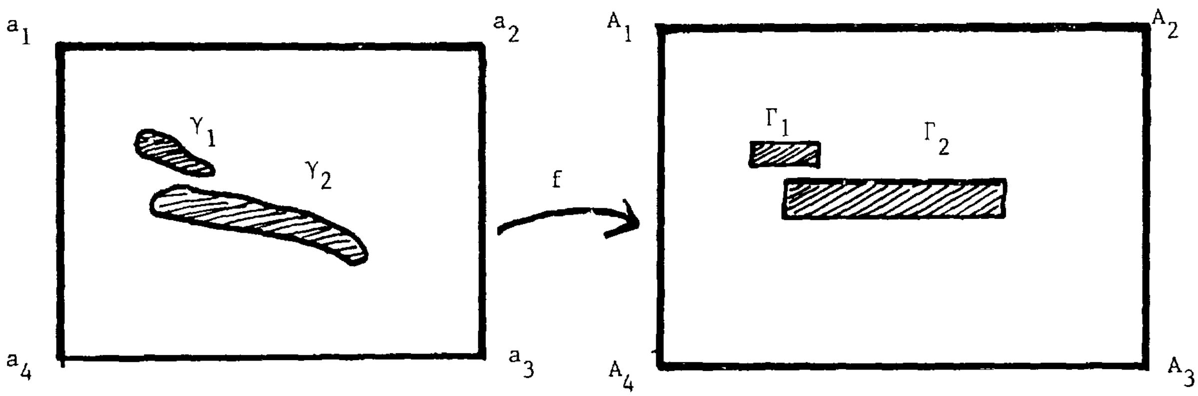
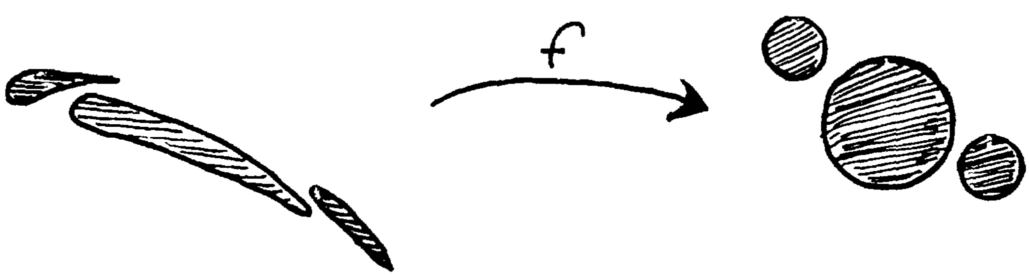


f conformal $\quad f(a_j) = A_j,\ \gamma_j \xrightarrow{f} \Gamma_j$

The author has developed and implemented a numerical procedure to compute the conformal mapping of a given n-tuply connected region onto a region with any specified boundary shapes and with several possible normalizations. If we start with a region whose outer boundary is a rectangle, we may arrange that the outer boundary of the image region is also a rectangle and the vertices map to vertices. We may choose the inner boundaries to map to rectangles or to any other shapes.



We may also consider unbounded regions and find a mapping normalized at ∞ $z + O(1/z)$. We may choose the boundaries of the image region to be circles or any other shapes.

Method

Though we may specify boundary shapes and orientations arbitrarily, the proper translation and magnification parameters must be calculated to determine the image domain and the mapping. For example, in order to find a conformal mapping between n-tuply connected regions R and S containing ∞ with $f(\infty) = \infty$, we must satisfy conditions on G_R and G_S, the analytic completions of the Green's functions for R and S with pole at ∞. We must have

$$G_R(r_j) = G_S(s_j) \qquad j = 1, 2, \ldots n-1$$

where r_j and s_j, $j = 1, 2, \ldots n-1$, are the critical points for G_S and G_R labeled in the figure. Using Symm's method to approximate Green's functions one may easily calculate the appropriate parameters. Then $G_S(f(z)) = G_R(z)$.

The dotted curves are the level curves of Re G_R and Re G_S which branch at the critical points.