# GENERATION OF C-TYPE CASCADE GRIDS

FOR VISCOUS FLOW COMPUTATION

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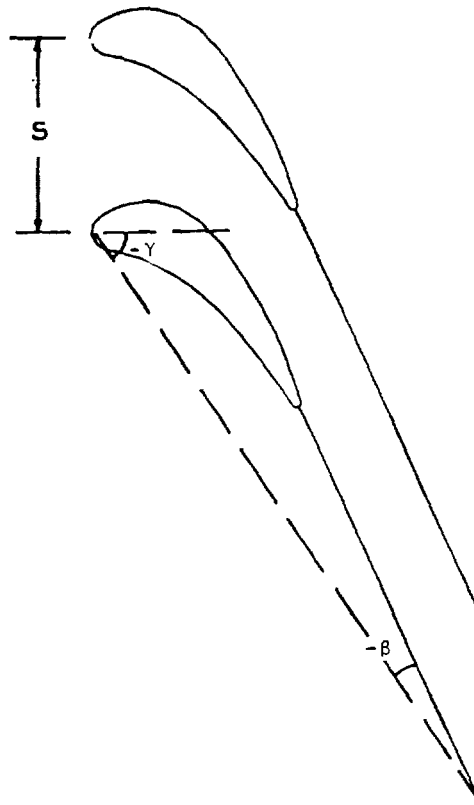NASA Lewis Research Center
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ABSTRACT

This paper presents a rapid procedure for generating C-type cascade grids suitable for viscous flow computations in turbomachinery blade rows. The resulting mesh is periodic from one blade passage to the next, nearly orthogonal, and continuous across the wake downstream of a blade. The procedure employs a pair of conformal mappings that take the exterior of the cascade into the interior of an infinite strip with curved boundaries. The final transformation to a rectangular computational domain is accomplished numerically. The boundary values are obtained from a panel solution of an integral equation and the interior values by a rapid ADI solution of Laplace's equation. Examples of C-type grids are presented for both compressor and turbine blades and the extension of the procedure to three dimensions is briefly outlined.

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Most of the coordinate systems in current use for turbomachinery flow computations are of one of three types. The channel grid has one family of lines starting upstream, passing through the blade rows, and continuing on downstream. The O-type grid has one family of lines that form closed loops around the blades. Finally, the C-type grid has one family of lines that wrap around the blade leading edge and continue on downstream. While the channel grid can be aligned with the flow and is fairly easy to generate, the resolution around the leading edge is usually poor and a choice usually has to be made between periodicity or near orthogonality for highly staggered cascades. Although the O-type grid provides excellent resolution around leading and trailing edges and may be both periodic and orthogonal, in general there is no mesh line aligned with the downstream flow and, hence, it is unsuitable for viscous computation. The C-type grid, on the other hand, appears to be a good choice for viscous flows. It provides good leading edge resolution, it can be both periodic and orthogonal, and it can be aligned with the downstream flow. This paper presents a rapid procedure for generating such C-type grids.

O-GRID

CHANNEL GRID

C-GRID
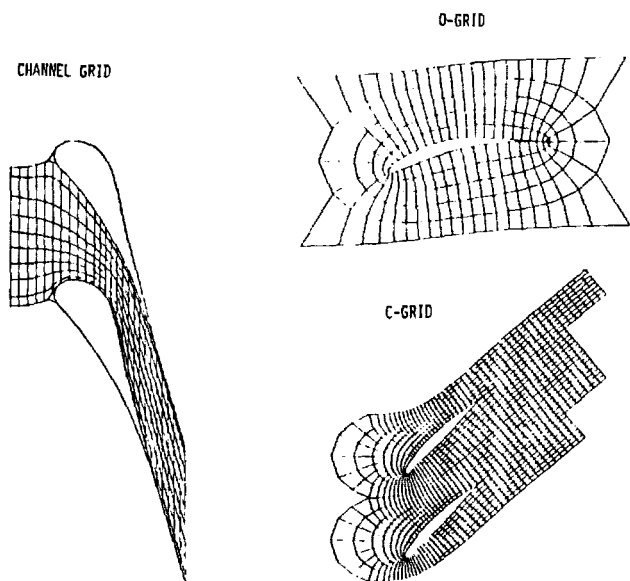
The procedure starts with a conformal mapping which takes the exterior of a cascade of semi-infinite flat plates in the Z-plane into the interior of the unit circle in the W-plane. Upstream infinity maps to the origin and downstream infinity to +1 on the real axis. This mapping is a limiting form of the standard mapping for a cascade of finite-chord flat plates (1). When this mapping is applied to a real geometry, such as the turbine cascade in the figure, the semi-infinite flat plate is taken to run from a point Z_1 inside the leading edge through the downstream end of the wake.

CONTOUR IN CASCADE PLANE

$$Z = Z_0 + \lambda \left[e^{-i\gamma}(\log W - i\pi) - 2 \cos \gamma \, \log(1 - W) \right]$$

$$\lambda = \frac{S}{2\pi} e^{i\gamma}, \quad Z_0 = Z_1 + 2\lambda \left[\gamma \sin \gamma + \cos \gamma \, \log(2 \cos \gamma) \right]$$

The mapping of the turbine cascade and wakes of the preceding figure produces the highly distorted "circle" in the adjacent figure. Note that the contour actually crosses the real axis twice between zero and one. The next mapping takes the interior of the unit circle in the W plane, with a branch cut from zero to one along the real axis, to the interior of the infinite strip between the real axis and ($-i \frac{\pi}{2}$) in the ζ -plane. The upper and lower sides of the wake at downstream infinity are mapped to plus and minus infinity, respectively, while upstream infinity maps to the origin. Since W is a function of ζ^2, reflection of ζ through the origin leaves W unchanged.

CONTOUR IN W PLANE
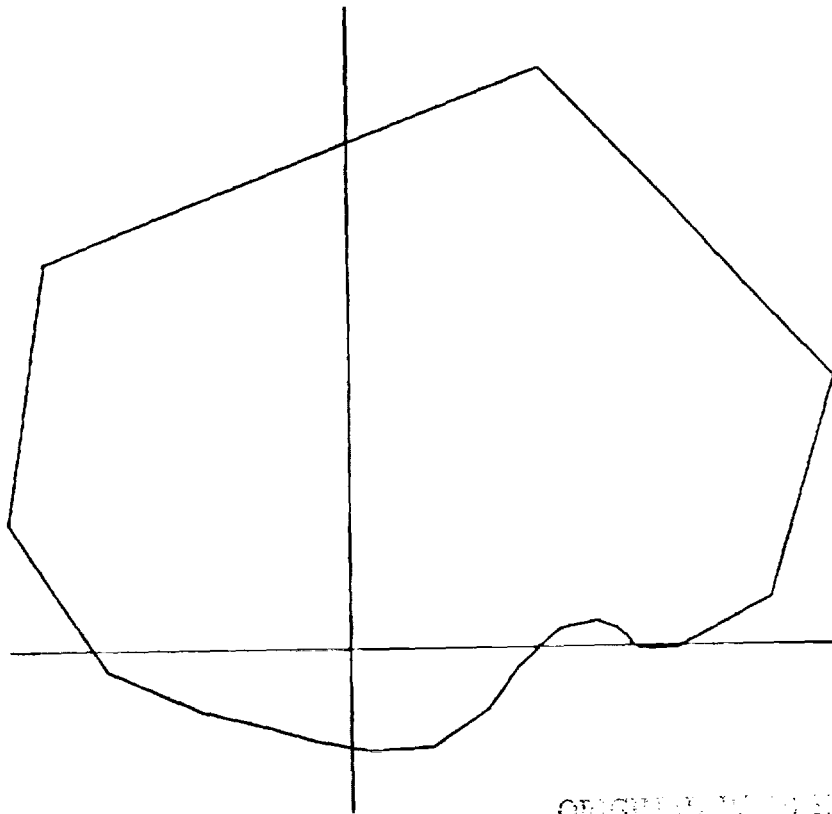
$$W = \tanh^2 \frac{\zeta}{2}$$

The image of the cascade of turbine blades and wakes in the ζ -plane is a pair of parallel straight lines connected by a roughly S-shaped curve. In actual practice W is eliminated between the two functions and the transformation from Z to ζ is obtained by Newton iteration proceeding from point to point around the contour. To insure that the branch cuts of the logarithms are never crossed, the imaginary parts of these logarithms are saved. Whenever the change in either of these quantities between adjacent points exceeds $\pm \pi$, the computed value of the logarithm at the new point is incremented by $\mp 2 \pi i$, i.e. in the opposite direction.

CONTOUR IN ζ PLANE
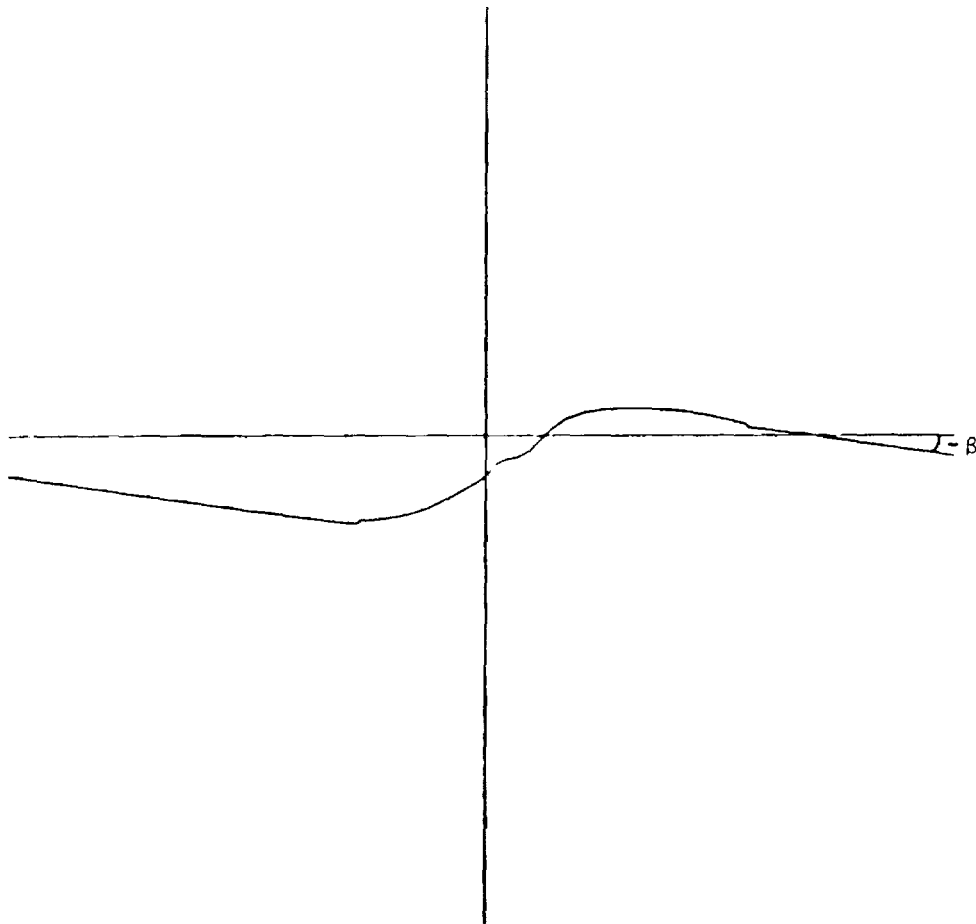
The final mapping transforms the infinite strip in the ζ -plane, bounded by the blade-wake contour and its reflection through the origin, into a rectangular domain with coordinates $F = \xi + i\eta$. If we let F be the complex potential for flow through the strip and require $F(\zeta) = -F(-\zeta)$ together with $\eta = -1$ along the contour, then we can write F as a contour integral. Here C, β, and h are, respectively, the complex velocity, flow angle, and normal channel width in the far field. The figure shows the formation of an integral equation for the unknown vortex density q_t. The source density q_n is set to cancel the normal component of the velocity C. Here s is distance along contour. A simple panel method, with flat panels and locally constant q_t and q_n, is used to solve for q_t and then to find ξ as a function of ζ along the contour.

FORMATION OF INTEGRAL EQUATION

COMPLEX POTENTIAL

$$F(\zeta) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \phi(t) \, \log \frac{t + \zeta}{t - \zeta} \, dt + C\zeta$$

WITH $\quad F = \xi + i\eta, \quad C = \frac{2}{h} e^{-i\beta}$

SET $\qquad \phi(t)\frac{dt}{ds} = q_t - iq_n$

WITH $\qquad q_n(t) = \text{Im}\left[C \, \frac{dt}{ds} \right]$

ON CONTOUR $\quad \eta = -1 \qquad$ AND

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[q_t \, \text{Re}\left\{ \log \frac{t + \zeta}{t - \zeta} \right\} + q_n \, \text{Im}\left\{ \log \frac{t + \zeta}{t - \zeta} \right\} \right] ds$$

$$= 1 + \text{Im}(C\zeta)$$

$$\xi = \text{Re}(C\zeta) - \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[q_n \, \text{Re}\left\{ \log \frac{t + \zeta}{t - \zeta} \right\} \right.$$

$$\left. - q_t \, \text{Im}\left\{ \log \frac{t + \zeta}{t - \zeta} \right\} \right] ds$$

Generation of the grid in the rectangular ξ , η space proceeds in two stages. First points are located on the boundaries such that the grid in the cascade plane is periodic and continuous across the wake. Periodicity is enforced by distributing points symmetrically about the origin along the ξ -axis. Continuity across the wake, away from the trailing edge, is achieved by selecting a constant mesh size $\Delta\xi$ for this region such that the spacing in the cascade plane is an integer fraction of the staggered distance $s \sin |\beta_w|$, where s is the blade pitch and β_w is the wake angle. The values of ζ along the boundary are then found by inverse interpolation in the solution of ξ vs ζ . In order to enforce continuity near the trailing edge, a local straining is first introduced that places a point directly at the trailing edge. Then pairs of neighboring points across the wake are adjusted until their images in the cascade plane coincide. The distribution of points with η at the two ends of the region is arbitrary. Uniform spacing can be used for inviscid flows, and boundary layer stretching can be used to cluster points near the blade surface and wake for viscous flows. Once the boundary values of ζ are specified, interior values are found by solving the complex Laplace equation by a cyclic ADI relaxation scheme which has the symmetry properties of ζ built in. Estimates of the maximum and minimum eigenvalues of the matrix are used to obtain near optimum values of the acceleration parameters (2). Convergence to the round-off error limit with seven place arithmetic is typically obtained in six to twelve iterations, even for cases where the maximum and minimum eigenvalues differ by five orders of magnitude.
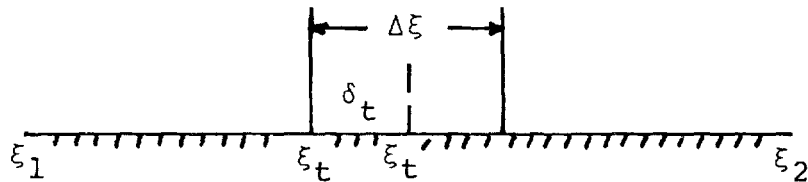
GRID GENERATION

● BOUNDARY VALUES IN (ξ , η) SPACE

ξ VALUES SYMMETRIC ABOUT ORIGIN

$\Delta\xi$ CONSTANT ALONG WAKE WITH

$$|\Delta z| = \left(s \, \sin \, |\beta_w|\right)\Big/\text{integer}$$

LOCAL STRAINING NEAR TRAILING EDGE

$$\xi' = \xi + \delta_t \left[\frac{\xi - \xi_1}{\xi_t - \xi_1} \cdot \frac{\xi - \xi_2}{\xi_t - \xi_2} \right]^2$$
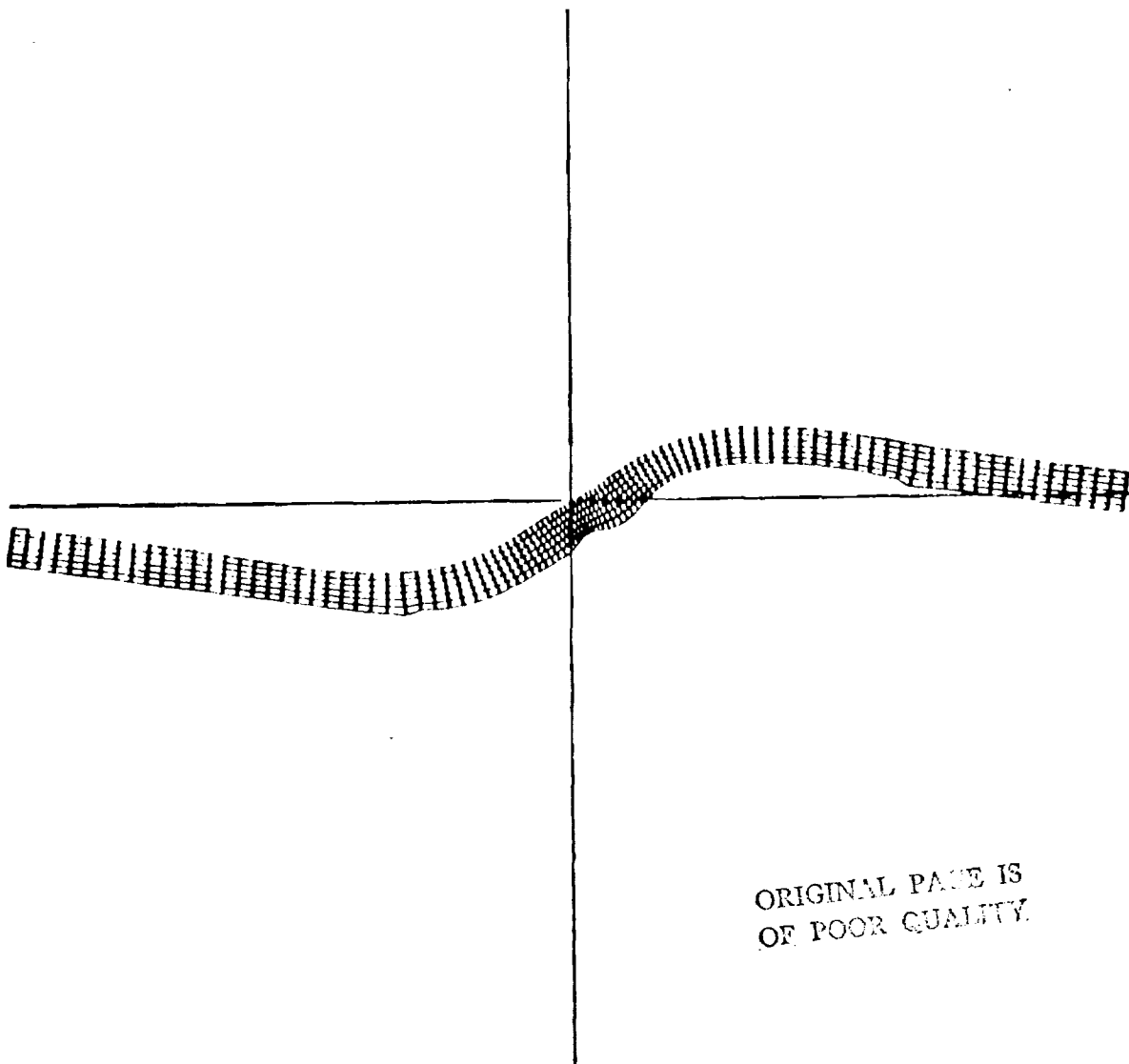

ARBITRARY DISTRIBUTION ALONG η

● INTERIOR VALUES IN (ξ , η) SPACE

$$\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \eta^2} = 0$$

SOLVED BY CYCLIC ADI RELAXATION

This figure shows the grid distribution in the ζ -plane. Note that the upper boundary in the plot, which is found by the symmetric ADI solution of Laplace's equation, maps into the upper and lower periodic lines in the cascade plane.

GRID IN ζ PLANE

The final grid in the cascade plane is obtained by conformal mapping of the solution in the ζ -plane using the two analytic functions previously introduced. This figure shows the grid distribution for the cascade of turbine blades. Note that the continuity across the wake was obtained at the expense of a small amount of nonorthogonality. The rounded cap at the upstream boundary was obtained by extrapolation from the next two inner loops. Generation of this grid (99 x 7 points) required about 1.4 sec. of CPU time on an IBM 3033 computer.

GRID IN CASCADE PLANE
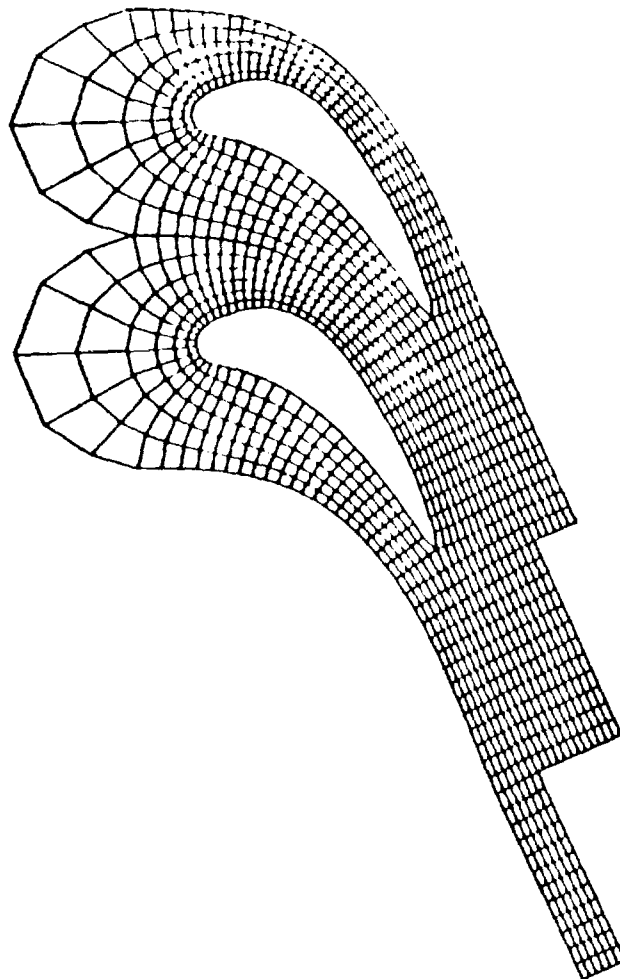
The last figure presents C grids for a compressor stator and a turbine rotor.
The stator was designed to turn the flow to the axial direction, hence, there
is zero stagger in the downstream boundary. The turbine rotor is a
particularly difficult case as it was designed to produce 126 degrees of
turning. In this case the imposition of continuity across the wake resulted
in a significant change in slope.

The extension of this procedure to the generation of three-dimensional
turbomachinery grids should be relatively straightforward. First the spanwise
direction is discretized by a number of coaxial, axisymmetric surfaces. Next,
and most difficult, the intersection of the blade with each of these surfaces
is obtained in meridional (m) and tangential (θ) coordinates. Since the
geometry is periodic in θ , these (m , θ) coordinates can be fed into the
current program to generate a C-grid on each of the axi-symmetric surfaces.
For O-grids this has already been done by Dulikravich (3).

COMPRESSOR STATOR

TURBINE ROTOR

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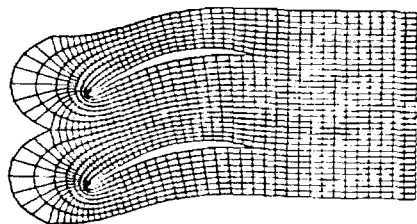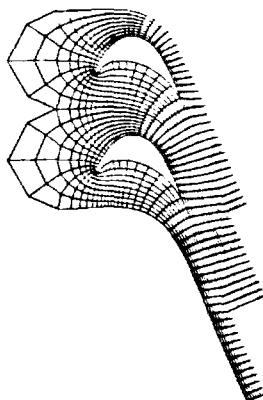
REFERENCES

1. Kober, H.: Dictionary of Conformal Representations, Dover Publications, Inc., Second ed. with corrections, 1957, p. 131.

2. Varga, R. S.: Matrix Iterative Analysis, Prentice-Hall, Inc., 1962.

3. Dulikravich, D. S.: Fast Generation of Body Conforming Grids for 3-D Axial Turbomachinery Flow Calculations. Numerical Grid Generation Techniques, NASA CP-2166, 1980. (Paper 16 of this compilation.)