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CURVILINEAR GRIDS FOR SINUOUS RIVER CHANNELS

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CENTERLINE INTRODUCTION

In order to effectively analyze the flow in sinuous river channels a curvilinear grid system must be developed for use in the appropriate hydrodynamic code. The CENTERLINE program has been designed to generate a two-dimensional grid for this purpose.

The Cartesian coordinates of a series of points along the boundaries of the sinuous channel represent the primary input to CENTERLINE. The program calculates the location of the river centerline, the distance downstream along the centerline, and both radius of curvature and channel width, as a function of such distance downstream. These parameters form the basis for the generation of the curvilinear grid.

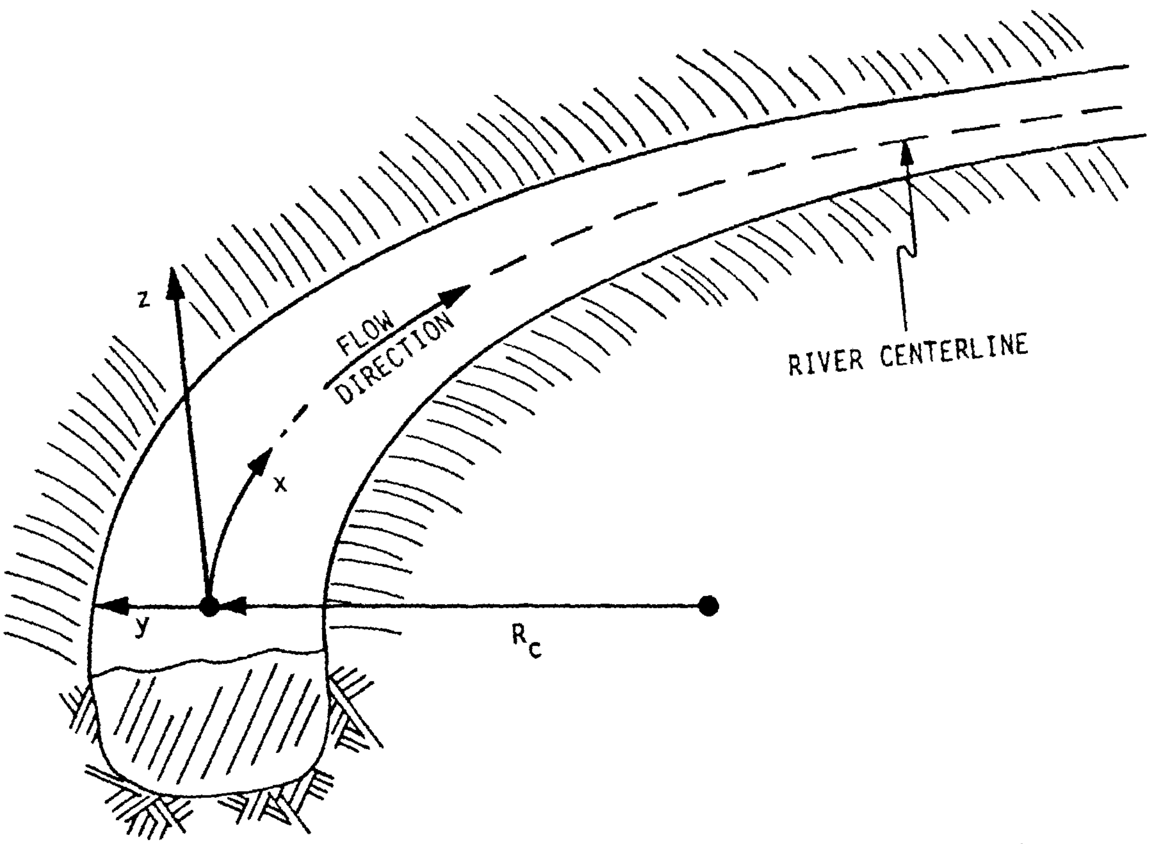
Based on input values for longitudinal and lateral grid spacing, the corresponding grid system is generated and a file is created containing the appropriate parameters for use in the associated explicit finite difference hydrodynamic programs. Because of the option for a nonuniform grid, grid spacing can be concentrated in areas containing the largest flow gradients.

For the case of sinuous channels of constant or nearly constant width the resulting curvilinear grid is orthogonal. The grid generation procedure also provides for dividing the overall flow area under consideration into a series of regions connected along common boundaries. This concept of multiple regions tends to improve computational efficiency.

For many sinuous channels the assumption of constant width is not appropriate. In such situations CENTERLINE generates a nonorthogonal grid which takes into account the nonuniform channel width.

The CENTERLINE program is currently operational and has been used successfully in conjunction with both two- and three-dimensional incompressible hydrodynamic programs. To the authors' knowledge, it is the only *curvilinear* grid program currently coupled with *operational* incompressible hydrodynamic programs for computing two- and three-dimensional river flows.

BASIC CURVILINEAR COORDINATE SYSTEM



GOVERNING EQUATIONS FOR INCOMPRESSIBLE CURVILINEAR FLOW

CONTINUITY:

$$\frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x}(h_y h_z u) + \frac{\partial}{\partial y}(h_z h_x v) + \frac{\partial}{\partial z}(h_x h_y w)\right] = 0$$

X-MOMENTUM:

$$\rho\left[\frac{\partial u}{\partial t} + \frac{u}{h_x}\frac{\partial u}{\partial x} + \frac{v}{h_y}\frac{\partial u}{\partial y} + \frac{w}{h_z}\frac{\partial u}{\partial z}\right.$$

$$\left. - v\left(\frac{v}{h_y h_x}\frac{\partial h_y}{\partial x} - \frac{u}{h_x h_y}\frac{\partial h_x}{\partial y}\right) + w\left(\frac{u}{h_x h_z}\frac{\partial h_x}{\partial z} - \frac{w}{h_z h_x}\frac{\partial h_z}{\partial x}\right)\right]$$

$$= \frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x}(h_y h_z \sigma_{xx}) + \frac{\partial}{\partial y}(h_z h_x \sigma_{yx}) + \frac{\partial}{\partial z}(h_x h_y \sigma_{zx})\right]$$

$$+ \frac{\sigma_{xy}}{h_x h_y}\frac{\partial h_x}{\partial y} + \frac{\sigma_{zx}}{h_x h_z}\frac{\partial h_x}{\partial z} - \frac{\sigma_{yy}}{h_x h_y}\frac{\partial h_y}{\partial x} - \frac{\sigma_{zz}}{h_x h_z}\frac{\partial h_z}{\partial x} + F_x$$

Y-MOMENTUM:

$$\rho\left[\frac{\partial v}{\partial t} + \frac{u}{h_x}\frac{\partial v}{\partial x} + \frac{v}{h_y}\frac{\partial v}{\partial y} + \frac{w}{h_z}\frac{\partial v}{\partial z}\right.$$

$$\left. - w\left(\frac{w}{h_z h_y}\frac{\partial h_z}{\partial y} - \frac{v}{h_y h_z}\frac{\partial h_y}{\partial z}\right) + u\left(\frac{v}{h_y h_x}\frac{\partial h_y}{\partial x} - \frac{u}{h_x h_y}\frac{\partial h_x}{\partial y}\right)\right]$$

$$= \frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x}(h_y h_z \sigma_{xy}) + \frac{\partial}{\partial y}(h_z h_x \sigma_{yy}) + \frac{\partial}{\partial z}(h_x h_y \sigma_{zy})\right]$$

$$+ \frac{\sigma_{yz}}{h_y h_z}\frac{\partial h_y}{\partial z} + \frac{\sigma_{xy}}{h_y h_x}\frac{\partial h_y}{\partial x} - \frac{\sigma_{zz}}{h_y h_z}\frac{\partial h_z}{\partial y} - \frac{\sigma_{xx}}{h_y h_x}\frac{\partial h_x}{\partial y} + F_y$$

Z-MOMENTUM :

$$\rho\left[\frac{\partial w}{\partial t} + \frac{u}{h_x}\frac{\partial w}{\partial x} + \frac{v}{h_y}\frac{\partial w}{\partial y} + \frac{w}{h_z}\frac{\partial w}{\partial z}\right.$$

$$\left. - u\left(\frac{u}{h_x h_z}\frac{\partial h_x}{\partial z} - \frac{w}{h_z h_x}\frac{\partial h_z}{\partial x}\right) + v\left(\frac{w}{h_z h_y}\frac{\partial h_z}{\partial y} - \frac{v}{h_y h_z}\frac{\partial h_y}{\partial z}\right)\right]$$

$$= \frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x}\left(h_y h_z \sigma_{xz}\right) + \frac{\partial}{\partial y}\left(h_z h_x \sigma_{yz}\right) + \frac{\partial}{\partial z}\left(h_x h_y \sigma_{zz}\right)\right]$$

$$+ \frac{\sigma_{zx}}{h_x h_z}\frac{\partial h_z}{\partial x} + \frac{\sigma_{yz}}{h_z h_y}\frac{\partial h_z}{\partial y} - \frac{\sigma_{xx}}{h_z h_x}\frac{\partial h_x}{\partial z} - \frac{\sigma_{yy}}{h_z h_y}\frac{\partial h_y}{\partial z} + F_z$$

ENERGY :

$$\frac{\partial T}{\partial t} + \frac{u}{h_x}\frac{\partial T}{\partial x} + \frac{v}{h_y}\frac{\partial T}{\partial y} + \frac{w}{h_z}\frac{\partial T}{\partial z}$$

$$= \frac{1}{h_x h_y h_z}\left[\frac{\partial}{\partial x}\left(\frac{\alpha_x h_y h_z}{h_x}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\alpha_y h_z h_x}{h_y}\frac{\partial T}{\partial y}\right)\right.$$

$$\left. + \frac{\partial}{\partial z}\left(\frac{\alpha_z h_x h_y}{h_z}\frac{\partial T}{\partial z}\right)\right]$$

COMPUTATION OF METRIC COEFFICIENTS

FUNDAMENTAL CONSIDERATIONS:

- APPEAR IN GOVERNING EQUATIONS
- ONLY h_x REQUIRES COMPUTATION
- EVALUATED FOR EACH GRID POINT
- DERIVATIVES ALSO REQUIRED

BASIC RELATIONS:

$$h_x = \frac{R_c + y}{R_c}$$

$$h_y = 1$$

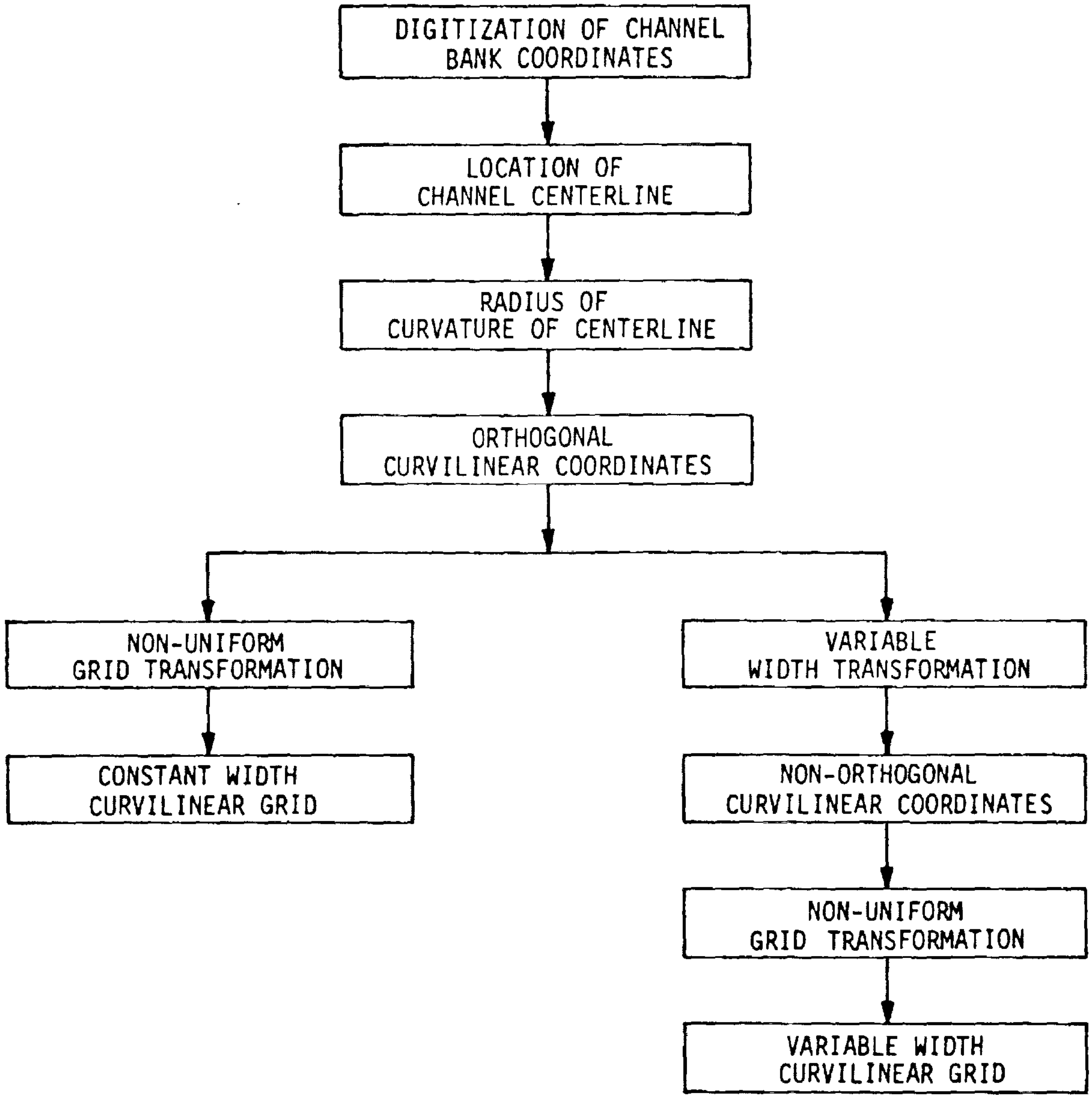
$$h_z = 1$$

DERIVATIVES:

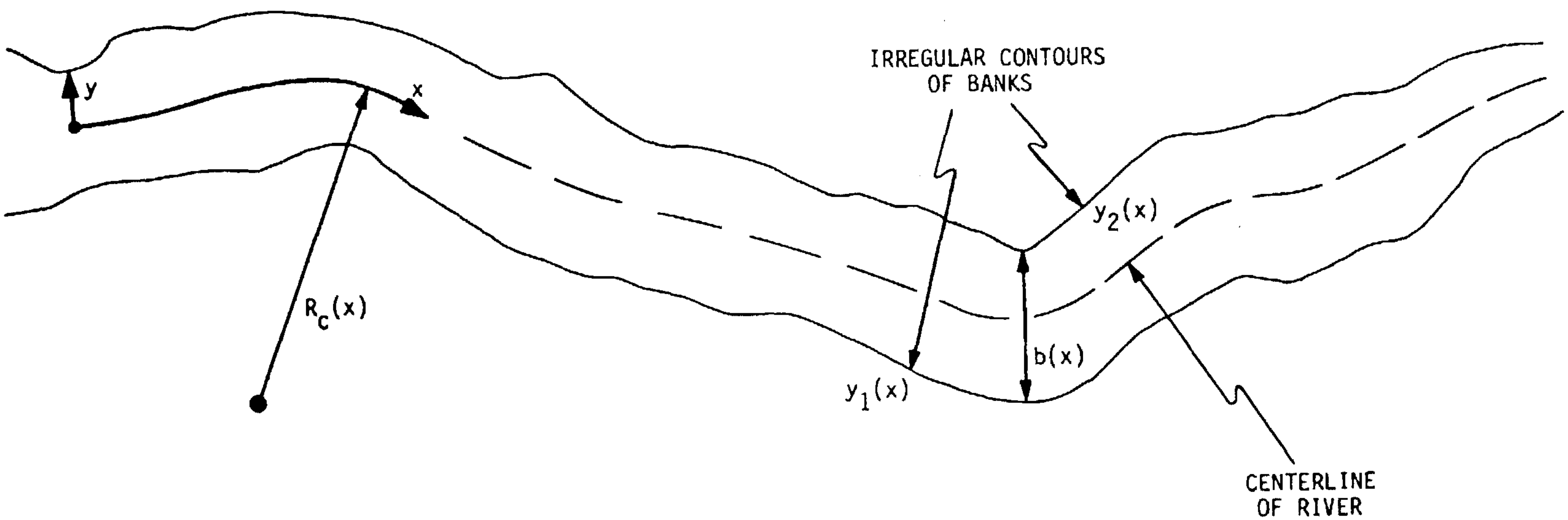
$$\frac{\partial h_x}{\partial x} = -\frac{y}{R_c^2}\frac{dR_c}{dx}$$

$$\frac{\partial h_x}{\partial y} = \frac{1}{R_c}$$

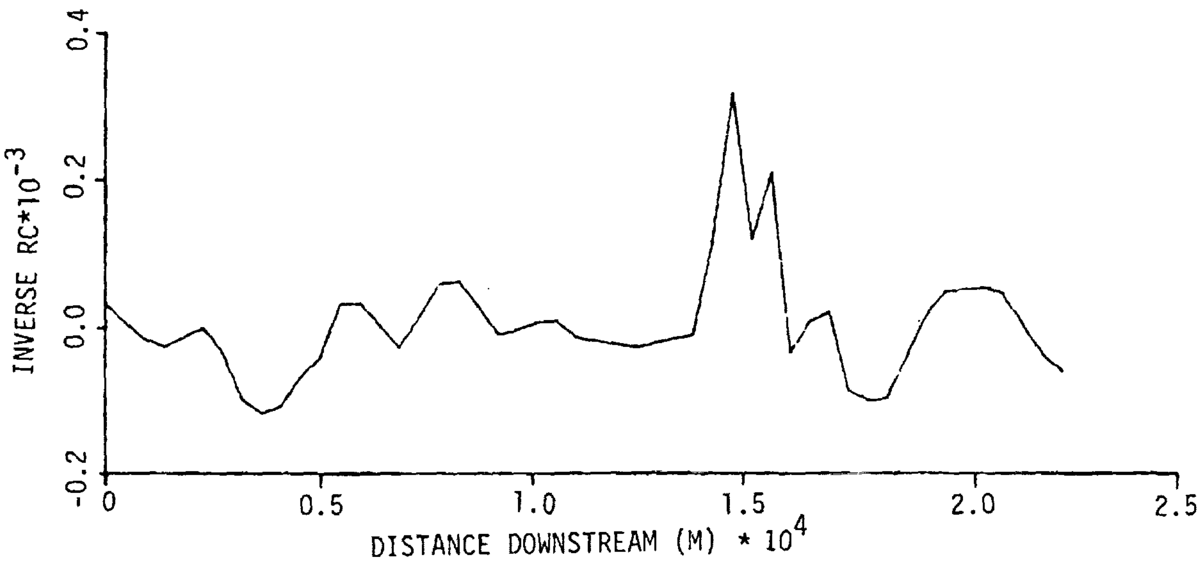
GENERATION OF CURVILINEAR GRIDS



TYPICAL RESERVOIR



INVERSE RADIUS OF CURVATURE VS DISTANCE DOWNSTREAM FOR TYPICAL RESERVOIR



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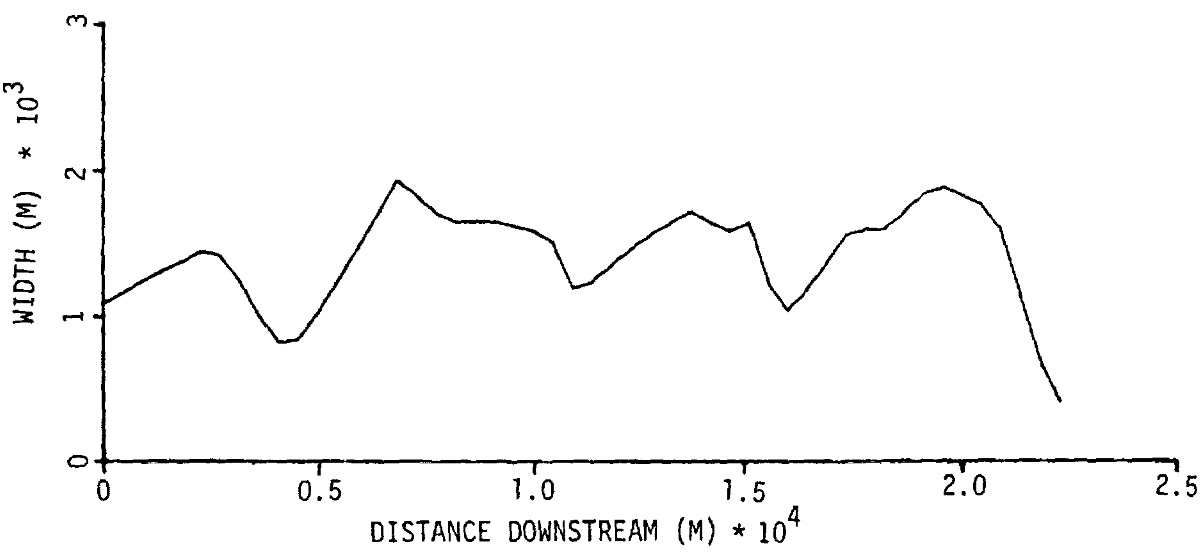
CHANNEL WIDTH VS DISTANCE DOWNSTREAM FOR TYPICAL RESERVOIR



COMPUTATION OF RADIUS OF CURVATURE AND CHANNEL WIDTH

- DIGITIZE CARTESIAN COORDINATES OF CHANNEL BANKS
- LOCATE GEOMETRIC CENTERLINE
- COMPUTE DISTANCE ALONG CENTERLINE, x
- COMPUTE RADIUS OF CURVATURE, $R_c(x)$
- COMPUTE CHANNEL WIDTH, $b(x)$

VARIABLE WIDTH TRANSFORMATION

BASIC TRANSFORMATION:

$$b = y_2(x) - y_1(x)$$

$$X = x$$

$$Y = y/b$$

TRANSFORMATION DERIVATIVES:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial X} + \frac{\partial f}{\partial Y} Y'$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial Y} \frac{1}{b}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial X^2} + 2 \frac{\partial^2 f}{\partial X \partial Y} Y' + \frac{\partial^2 f}{\partial Y^2} (Y')^2 + \frac{\partial f}{\partial Y} Y''$$

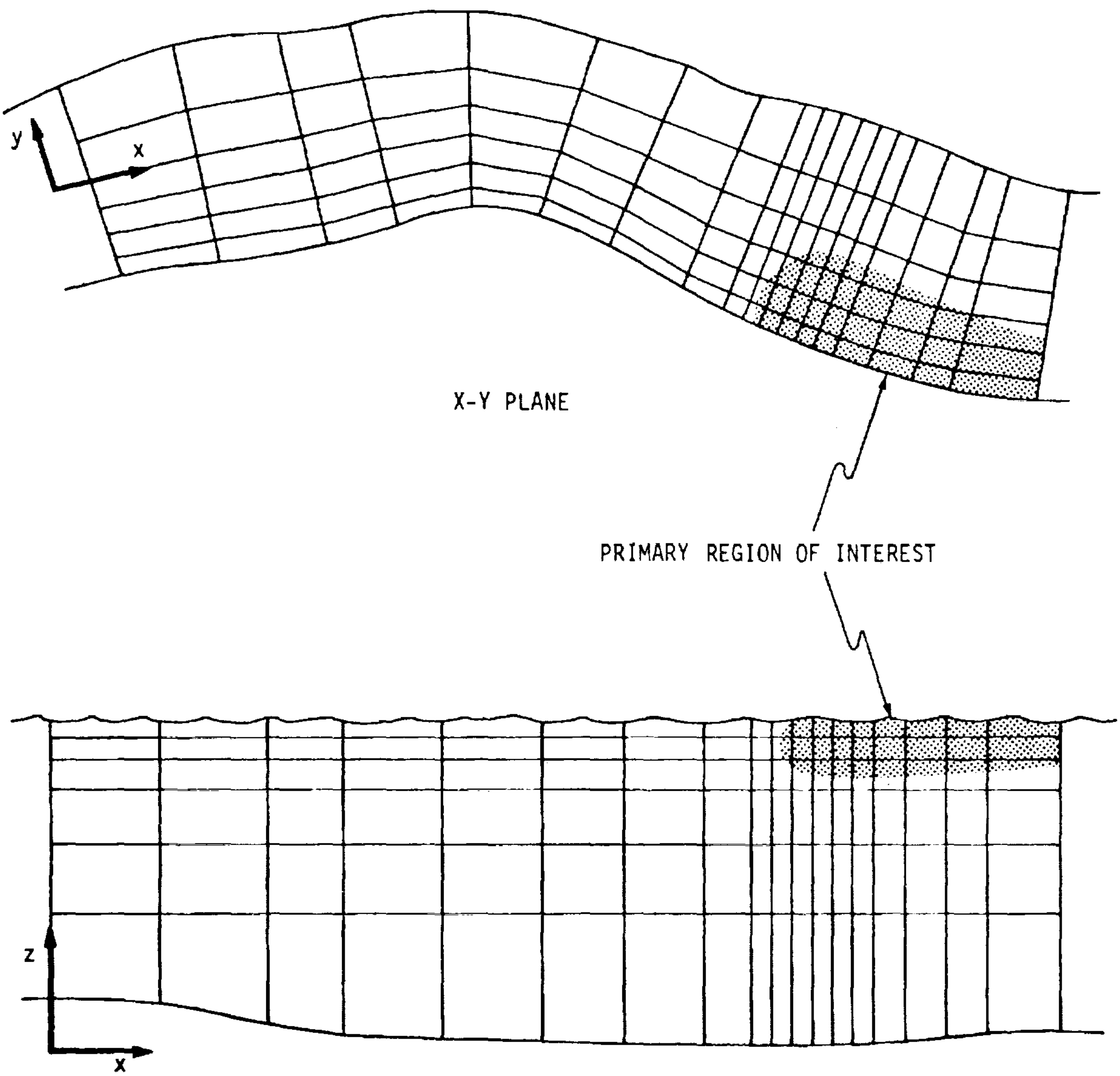
$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial Y^2} \frac{1}{b^2}$$

where

$$Y' = - \frac{Y}{b} \frac{db}{dx}$$

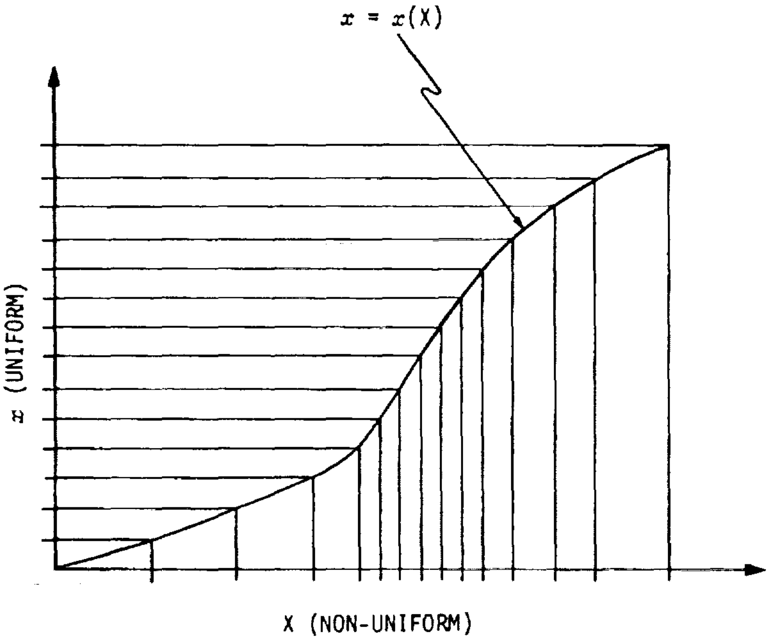
$$Y'' = \frac{2Y}{b^2} \frac{db}{dx} - \frac{Y}{b} \frac{d^2 b}{dx^2}$$

NON-UNIFORM GRID SYSTEM



X-Z PLANE

RELATIONSHIP BETWEEN NON-UNIFORM AND UNIFORM GRIDS



TRANSFORMATION FROM NON-UNIFORM TO UNIFORM GRID

PROCEDURE:

- IDENTIFY "REGIONS OF INTEREST"
- INPUT DESIRED GRID SPACING
- GENERATE TRANSFORMATION DERIVATIVES

BASIC TRANSFORMATION:

$$\left.\begin{aligned} x &= x(X) \\ y &= y(Y) \\ z &= z(Z) \end{aligned}\right\} \text{ANALYTICAL TRANSFORMATION FUNCTIONS NOT REQUIRED}$$

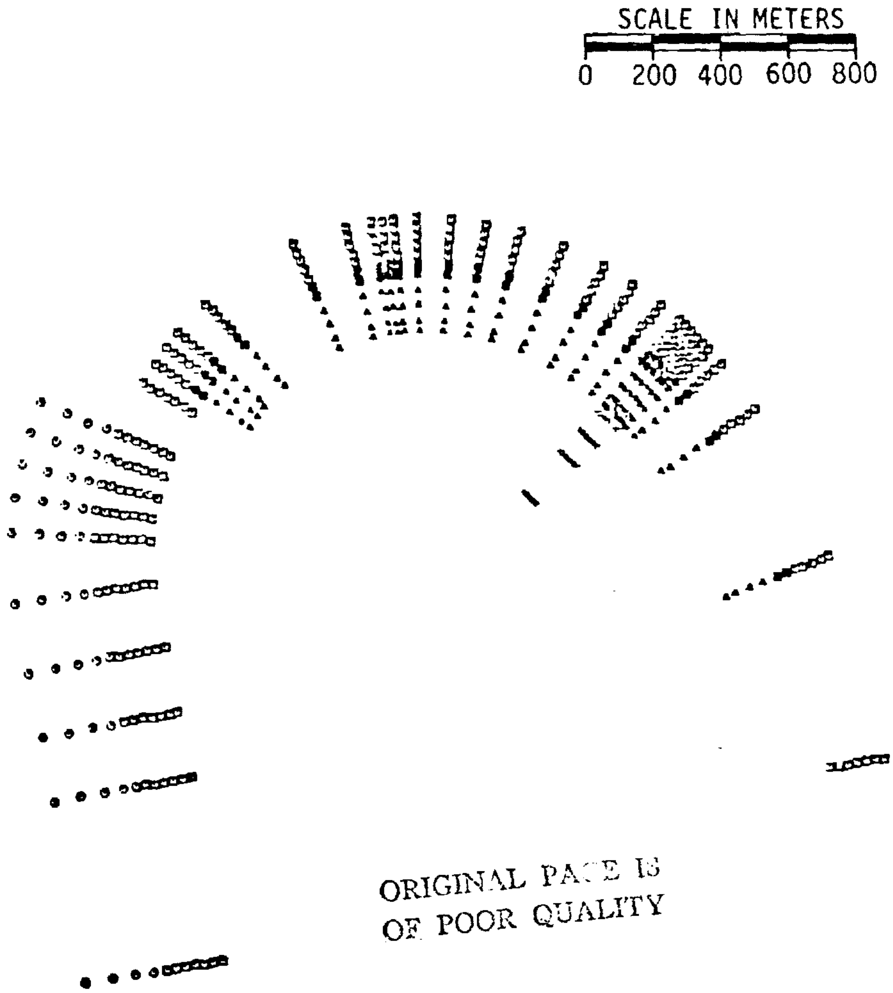
TRANSFORMATION DERIVATIVES:

$$\frac{\partial q}{\partial X} = \frac{\partial q}{\partial x}\frac{\partial x}{\partial X}$$

$$\frac{\partial^2 q}{\partial X^2} = \frac{\partial q}{\partial x}\frac{\partial^2 x}{\partial X^2} + \frac{\partial^2 q}{\partial x^2}\left(\frac{\partial x}{\partial X}\right)^2$$

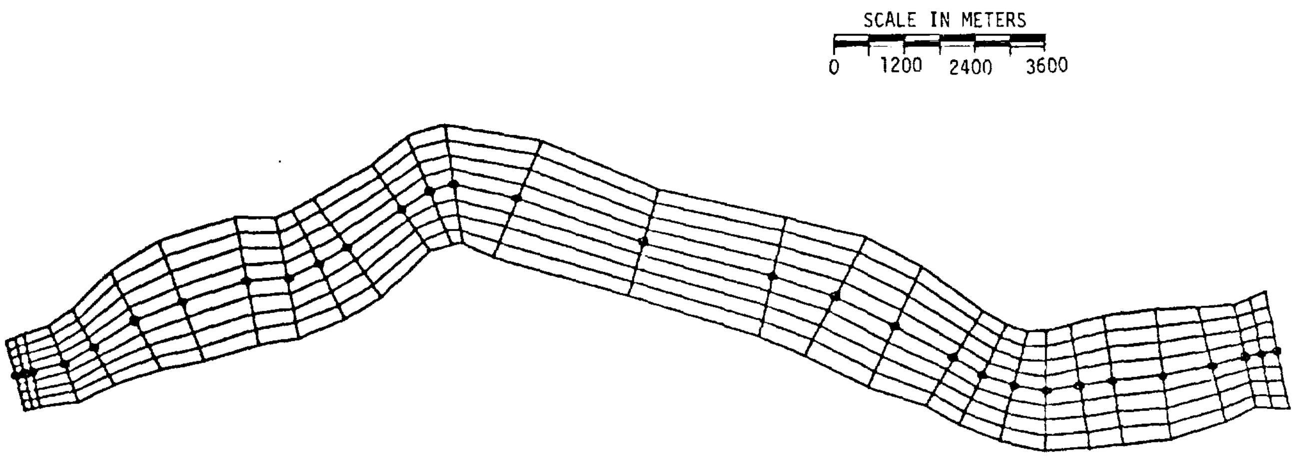
CURVILINEAR GRID FOR CUMBERLAND RIVER SEGMENT

- NEAR TVA GALATIN STEAM PLANT
- CONSTANT WIDTH CHANNEL
- NON-UNIFORM GRID (x, y, & z)
- 4 CONNECTED REGIONS
- USED IN 3-D FLOW COMPUTATIONS



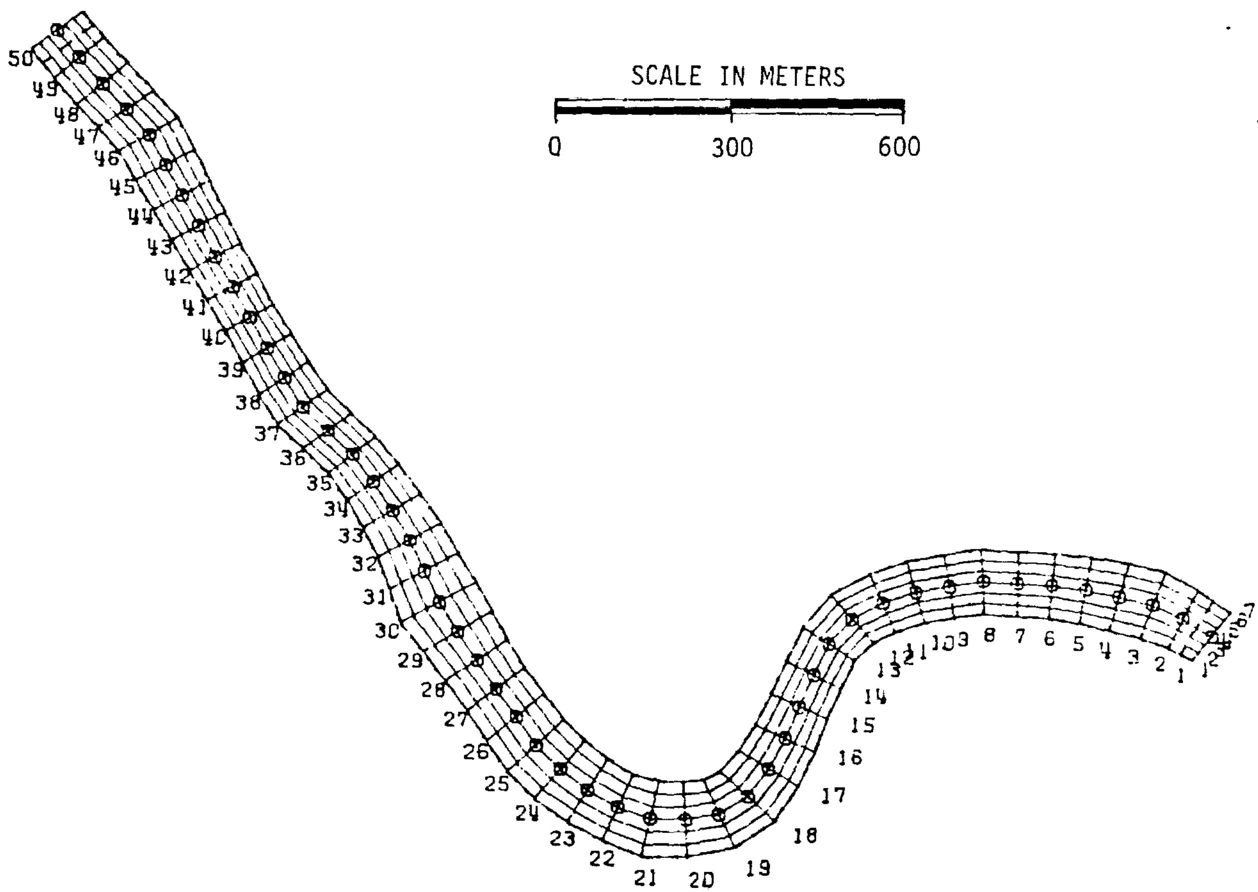
CURVILINEAR GRID FOR TENNESSEE RIVER, WILSON RESERVOIR

- BETWEEN WHEELER AND WILSON DAMS
- VARIABLE WIDTH CHANNEL
- NON-UNIFORM GRID (x only)
- USED IN 2-D DEPTH-AVERAGED FLOW COMPUTATION



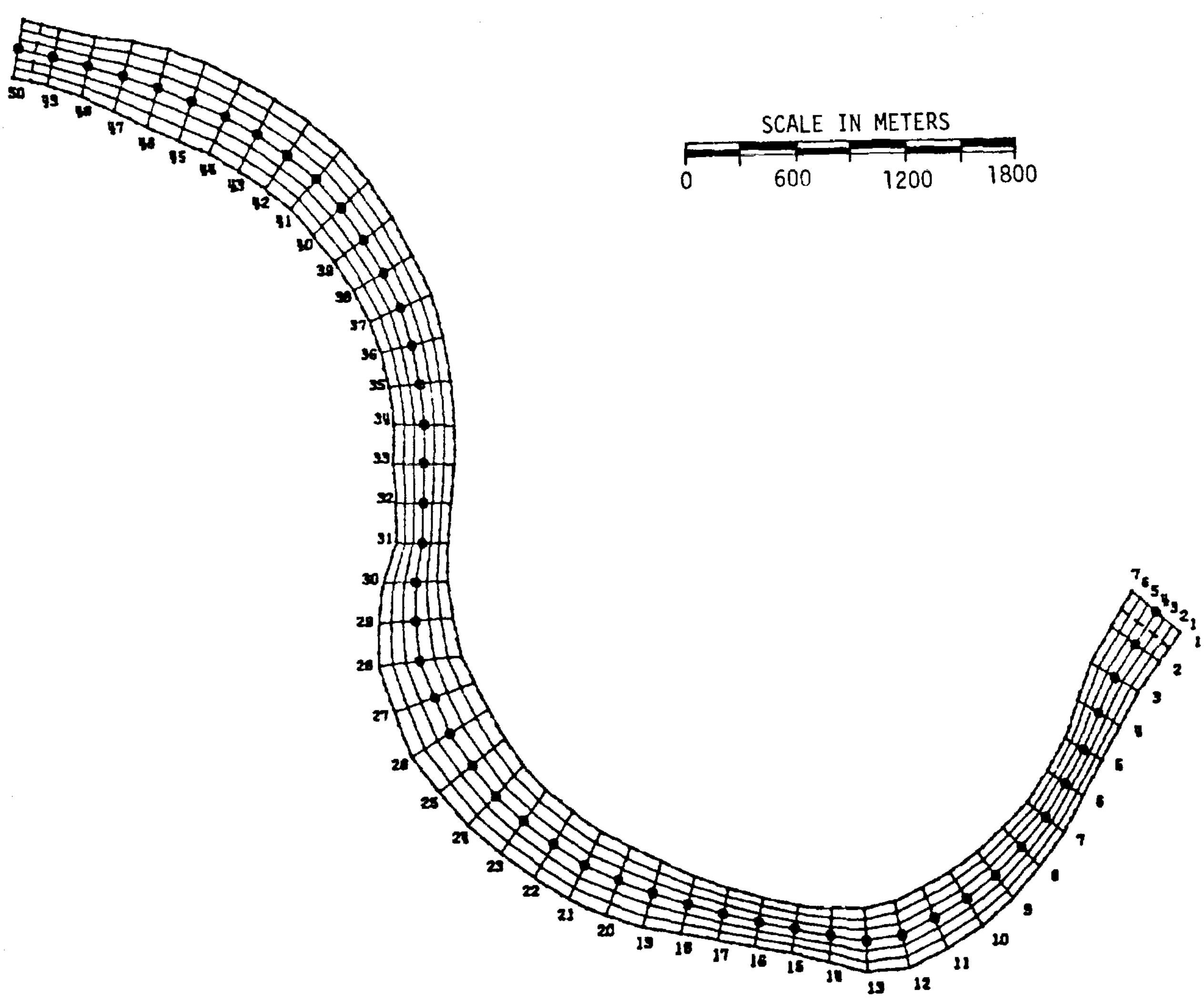
CURVILINEAR GRID FOR GREEN RIVER SEGMENT

- NEAR PARADISE STEAM PLANT
- MODERATE SINUOSITY
- VARIABLE WIDTH
- UNIFORM GRID



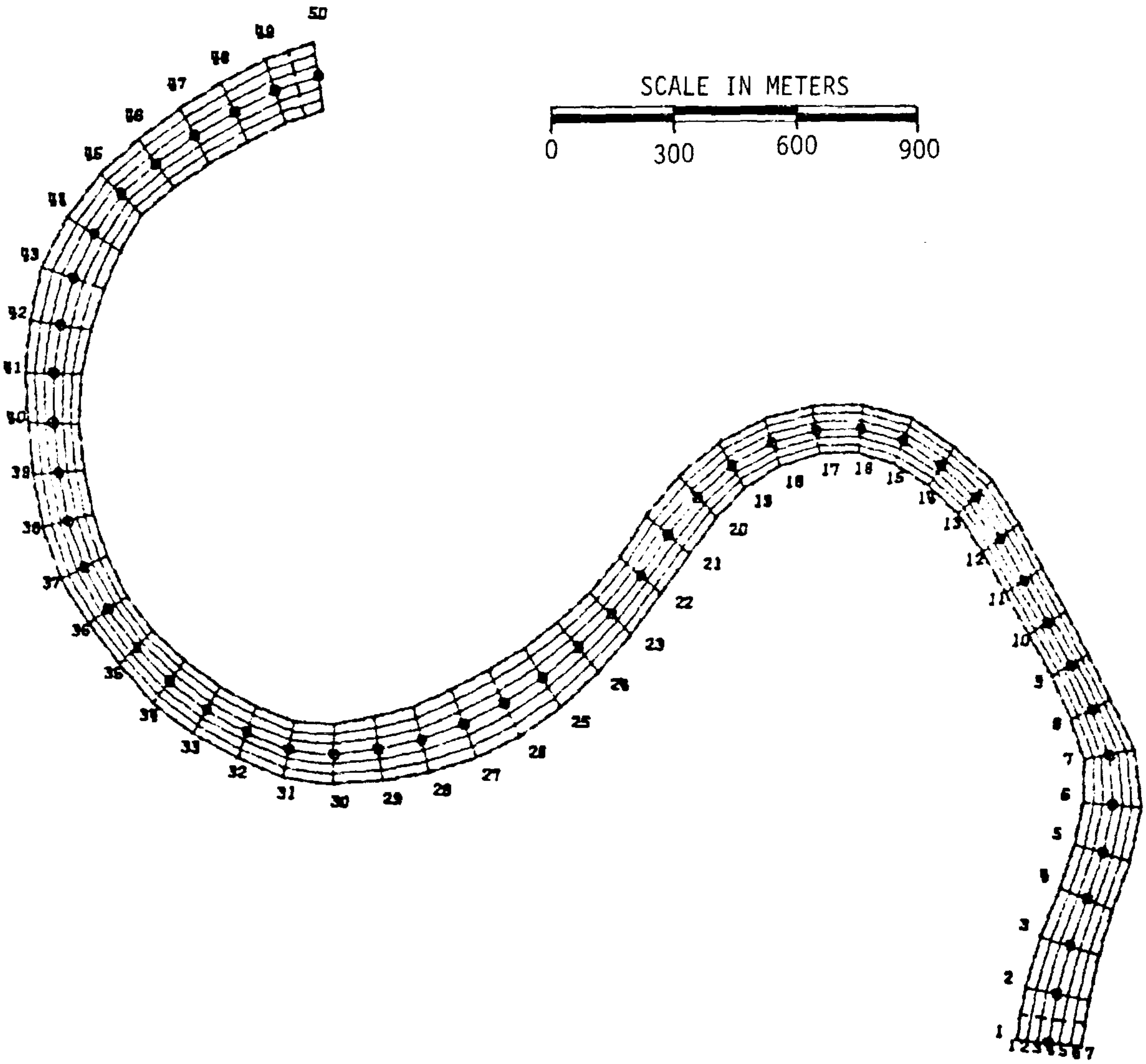
CURVILINEAR GRID FOR TENNESSEE RIVER, WHEELER RESERVOIR

- NEAR REDSTONE ARSENAL
- MODERATE SINUOSITY
- VARIABLE WIDTH
- UNIFORM GRID



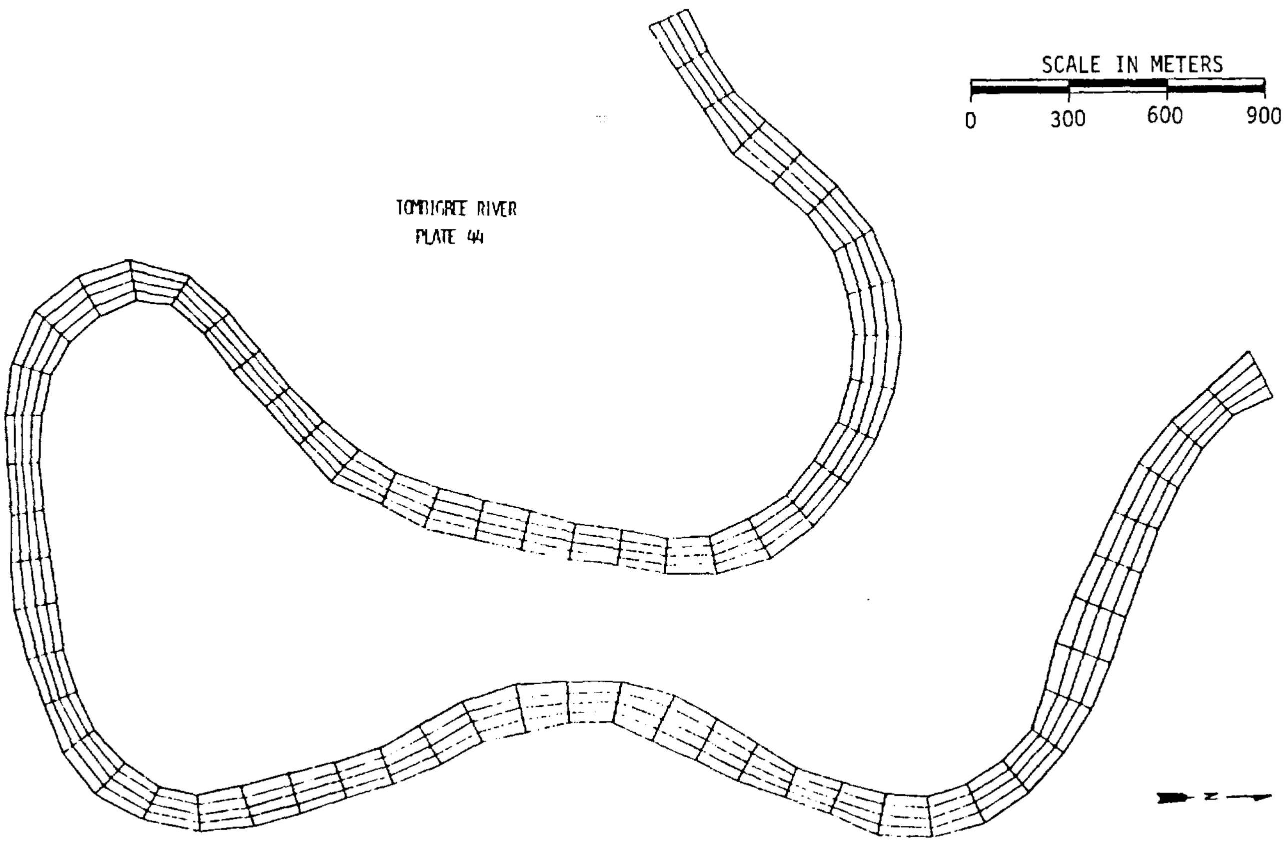
CURVILINEAR GRID FOR LITTLE TENNESSEE RIVER SEGMENT

- PART OF TELLICO LAKE
- HIGH SINUOSITY
- VARIABLE WIDTH
- UNIFORM GRID

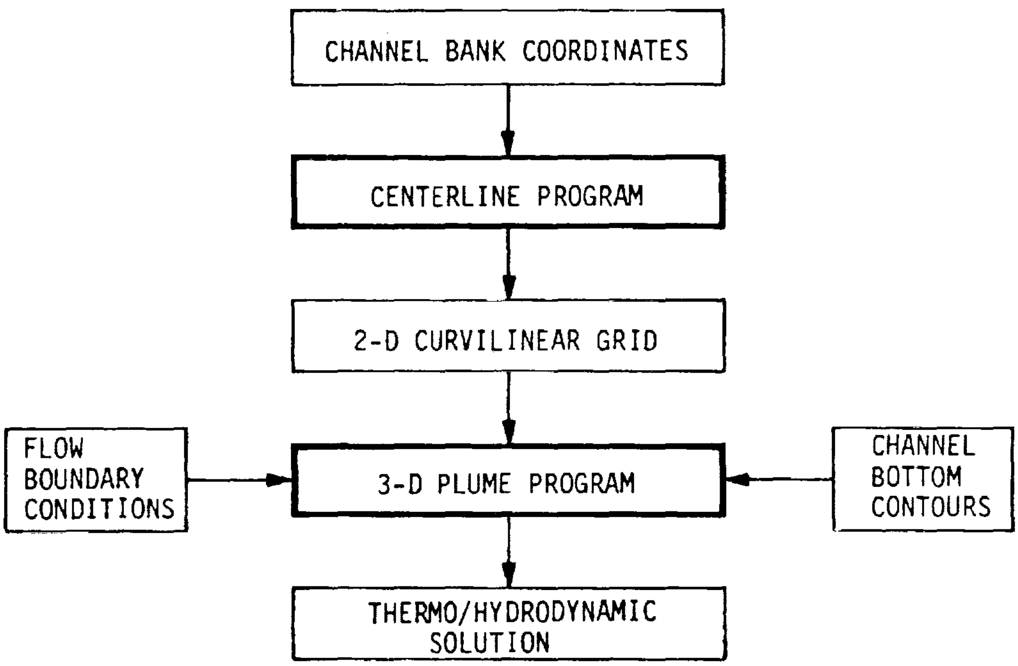


CURVILINEAR GRID FOR TOMBIGBEE RIVER SEGMENT

- PORTION OF TENNESSEE - TOMBIGBEE WATERWAY
- EXTREME SINUOSITY
- VARIABLE WIDTH
- UNIFORM GRID

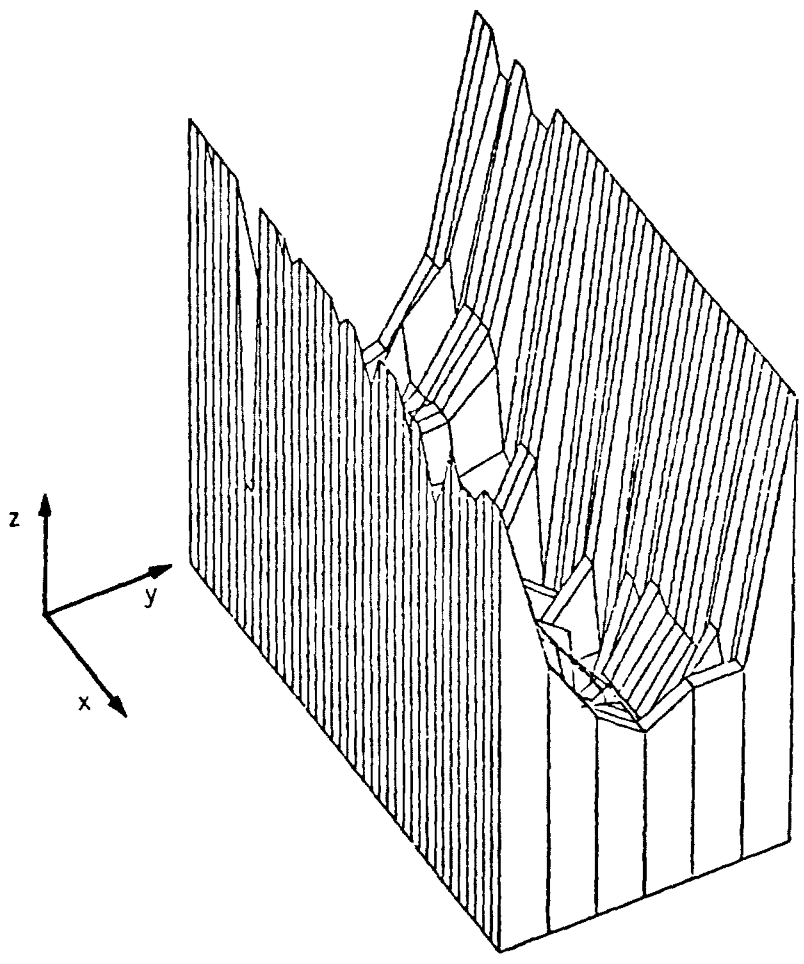


INTEGRATION OF CENTERLINE PROGRAM WITH 3-D PLUME PROGRAM

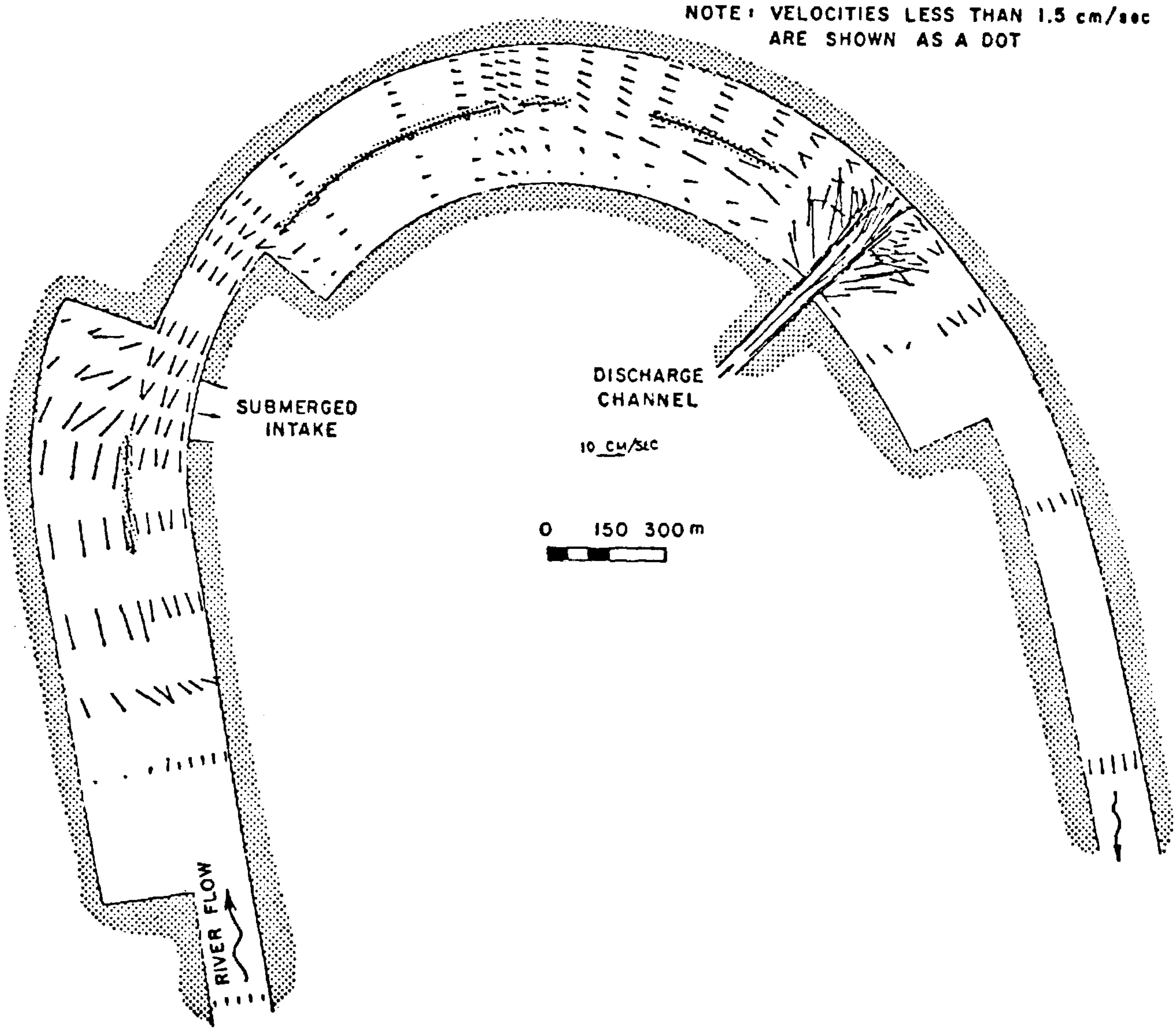


NON-UNIFORM BOTTOM CONSIDERATIONS OF CUMBERLAND RIVER SEGMENT

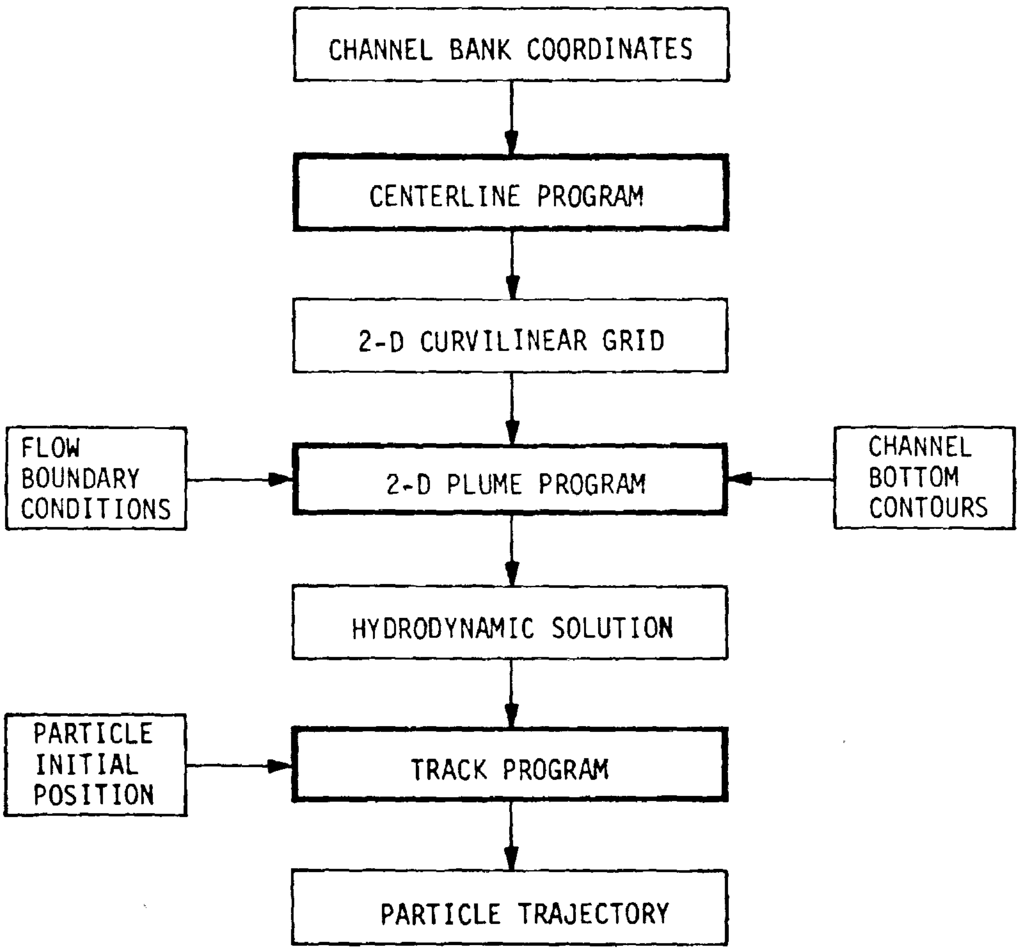
- BOTTOM PROFILES BASED ON SOUNDINGS
- LONGITUDINAL AND TRANSVERSE VARIATIONS ACCEPTED
- GRID SPACING LIMITS RESOLUTION OF BOTTOM SHAPE
- BOTTOM PROFILES NOT USED FOR TRANSFORMATION



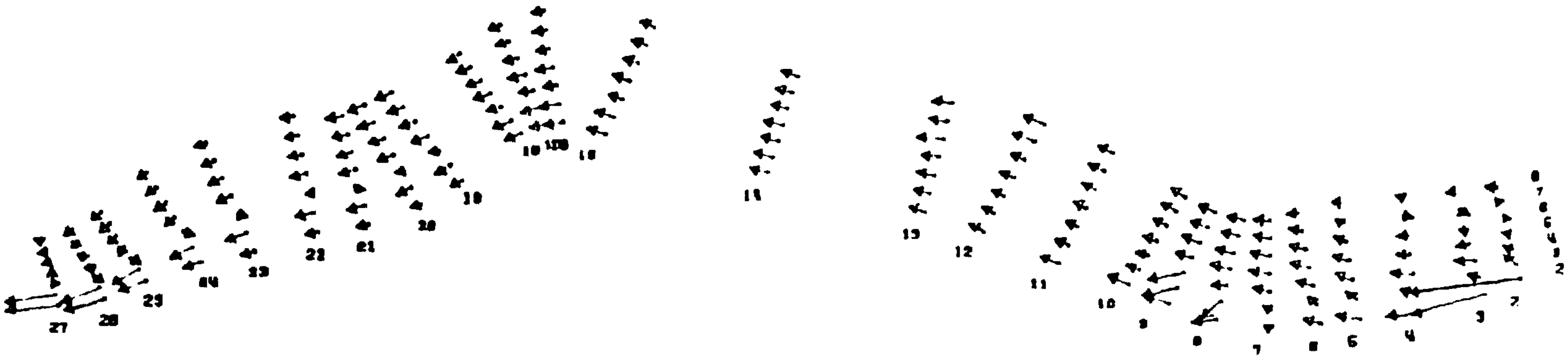
VELOCITY VECTOR PLOT FOR CUMBERLAND RIVER SEGMENT



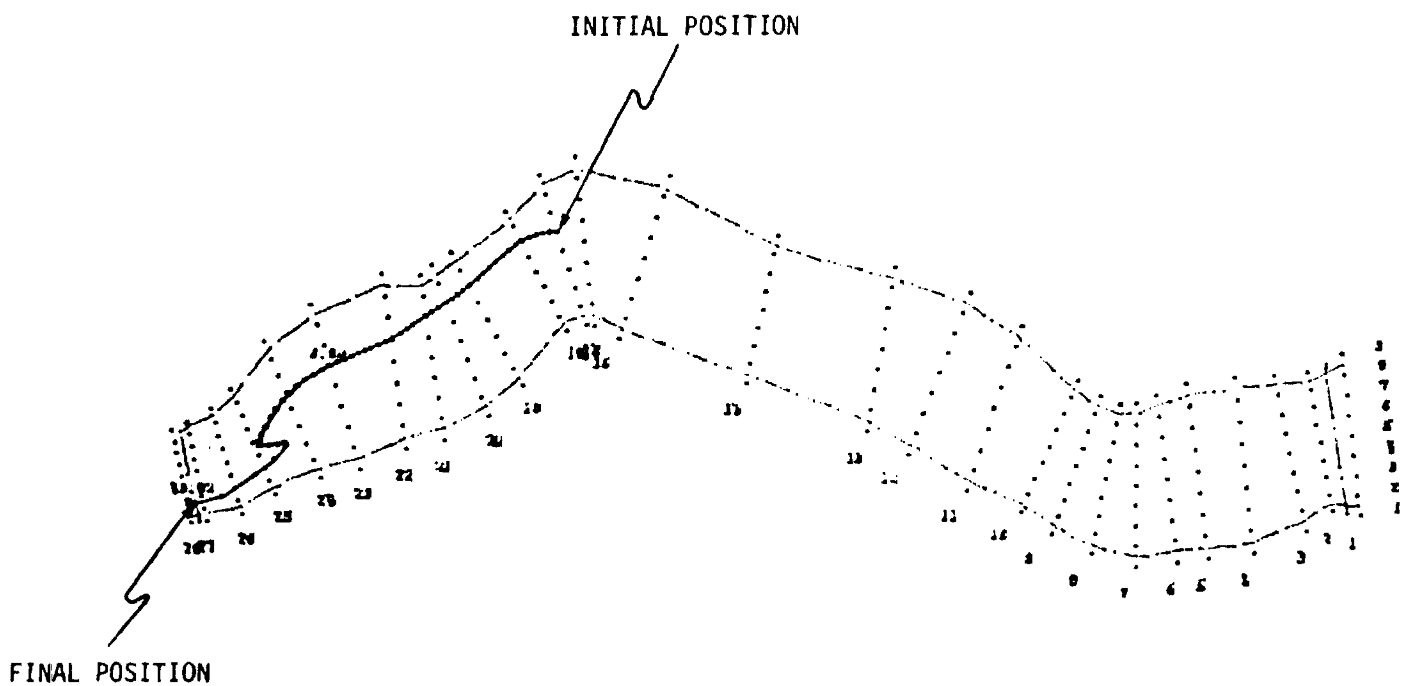
INTEGRATION OF CENTERLINE PROGRAM WITH 2-D PLUME AND TRACK PROGRAMS



VELOCITY VECTOR PLOT FOR THE TENNESSEE RIVER, WILSON RESERVOIR



PARTICLE TRAJECTORY PLOT FROM TRACK IN TENNESSEE RIVER, WILSON RESERVOIR



CENTERLINE SUMMARY

- APPLICABLE TO SINUOUS RIVER CHANNELS
- CURRENTLY OPERATIONAL
- DIGITIZATION OF CHANNEL COORDINATES
- CONSTANT/VARIABLE CHANNEL WIDTH OPTIONS
- UNIFORM/NON-UNIFORM GRID OPTIONS
- PRESENTLY COUPLED WITH 2-D AND 3-D HYDRODYNAMIC MODELS