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FINAL REPORT

ON

TWO-DIMENSIONAL ZONAL MEAN FLOW MODEL

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# FINAL REPORT

# ON

# TWO-DIMENSIONAL ZONAL MEAN FLOW MODEL

by

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### Final Report



### Users Guide

# SECTION 1 - INTRODUCTION

The interactions of dynamics, photochemistry and radiation in the stratosphere can be described[1] by the continuity, momentum and energy equations. A more comprehensive model for stratospheric transport theory has been developed for the purpose of aiding predictions of changes in the stratospheric ozone content as a consequence of natural and anthropogenic processes. This model is time dependent and the dependent variables are zonal means of the relevant meteorological quantities which are functions of latitude and height. The report sets out the detailed formulation of a numerical model both in physics and mathematics. A set of fundamental dynamical equations will be described in Section 2. Section 3 describes the numerical method used in the integration. The results and discussions are presented in Section 4.

## SECTION 2 - BASIC EQUATIONS

The model is an integration of an extensive set of fundamental equations which can be used to predict the future state of the atmospheric circulation from knowledge of its present state. In the log-pressure coordinate system $(x, y, \xi)$ in which x = distance eastwards, y = distance northwards, and $\xi = \log (p_o/p)$; with $p_o$ = 1000 mb and p = pressure, u, v, and w are the three corresponding velocity components respectively. In this log-pressure system the eastward component of the momentum equation is:[2]

$$\frac{\partial}{\partial t} u + u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u + w \frac{\partial}{\partial \xi} u = \frac{uv}{a} \tan\varphi + 2\Omega v \sin\varphi - \frac{\partial Z}{\partial x} \tag{1}$$

where a = radius of the earth, $\Omega$ = rate of rotation of the earth, $\varphi$ = latitude, Z is geopotential. The thermodynamic equation is:

$$\frac{\partial}{\partial t} \theta + u \frac{\partial}{\partial x} \theta + v \frac{\partial}{\partial y} \theta + w \frac{\partial}{\partial \xi} \theta = q \tag{2}$$

where $\theta = T (p_o/p)^k$, T = temperature, q is the rate of change of potential temperature, k is the ratio of gas constant for dry air R to specific heat of air at constant pressure $C_p$. The equation for conservation of matter is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{v}{a} \tan\varphi + \frac{\partial w}{\partial \xi} - w = 0 \,. \tag{3}$$

If $X_i$ is the mixing ration of a substance in the stratosphere, then the eq. for the time change of $X_i$ may be written in eq. (4) where $P_i$ is the production rate which includes the production and loss terms.

$$\frac{\partial X_i}{\partial t} + u \frac{\partial X_i}{\partial x} + v \frac{\partial X_i}{\partial y} + w \frac{\partial X_i}{\partial \xi} = P_i \qquad (4)$$

Our aim is to deal explicitly with zonal mean quantities only. Accordingly, let an over bar denote the zonal mean and a dash denote the departure therefrom: then equations (1) to (4) yield, after taking an average around a latitude circle:[3]

$$\frac{\partial}{\partial t}(\bar{\tau} e^{-\xi} \cos\varphi) + \frac{\partial}{\partial y}(\bar{V}\,\bar{\tau}) + \frac{\partial}{\partial \xi}(\bar{W}\,\bar{\tau}) = -H \qquad (5)$$

$$\frac{\partial}{\partial t}(\bar{\theta} e^{-\xi} \cos\varphi) + \frac{\partial}{\partial y}(\bar{V}\,\bar{\theta}) + \frac{\partial}{\partial \xi}(\bar{W}\,\bar{\theta}) = \bar{q}\, e^{-\xi} \cos\varphi + Q \qquad (6)$$

$$\frac{\partial}{\partial y}\bar{V} + \frac{\partial}{\partial \xi}\bar{W} = 0 \qquad (7)$$

$$\frac{\partial}{\partial t}(\bar{X}_i\, e^{-\xi} \cos\varphi) + \frac{\partial}{\partial y}(\bar{V}\,\bar{X}_i) + \frac{\partial}{\partial \xi}(\bar{W}\,\bar{X}_i) = P_i\, e^{-\xi} \cos\varphi + F \qquad (8)$$

where $\tau = (\Omega\, a \cos\varphi + u)\, a \cos\varphi$

$$H = -\frac{\partial}{\partial y}(\overline{V'\tau'}) - \frac{\partial}{\partial \xi}(\overline{W'\tau'})$$

$$Q = -\frac{\partial}{\partial y}(\overline{V'\theta'}) - \frac{\partial}{\partial \xi}(\overline{W'\theta'})$$

$$F = -\frac{\partial}{\partial y}(\overline{V'X_i'}) - \frac{\partial}{\partial \xi}(\overline{W'X_i'})$$

$$V = v\, e^{-\xi} \cos\varphi$$

$$W = w\, e^{-\xi} \cos\varphi$$

Note that the continuity equation (3) takes the simple form (6) which is the advantage of using the pressure coordinates.

Equations (5) to (8) may become a complete set by adding the y component momentum and hydrostatic equations. The hydrostatic approximation has been used in the $\xi$ direction. For our own purpose of not introducing the gravity waves we introduce an approximation to the y component momentum equation that the zonal mean wind is in geostrophic equilibrium with the zonal mean geopotential field. Combining this with the hydrostatic equation leads to the thermal wind relationship:

$$\frac{\partial}{\partial \xi} \tau \doteq - \frac{R \exp(-k\xi)}{2\Omega \tan\varphi} \frac{\partial \theta}{\partial y} \qquad (9)$$

Equations (5) to (9) can be regarded as a closed set which completely determines the relationships among the dependent variables $\tau$, $\theta$, V, W, Xi. The other variables can be written in terms of the relevant quantities and are held as functions of time, latitude and altitude.

## SECTION 3 - NUMERICAL METHOD

It is convenient to make the equations dimensionless. The scaling quantities are given a subscript s. Equations (5) to (9) become:

$$\frac{\partial}{\partial t}(\bar{\tau}\, e^{-\xi_s \xi} \cos\varphi) + \frac{\partial}{\partial \varphi}(\bar{\tau}\, \bar{V}) + \frac{\partial}{\partial \xi}(\bar{\tau}\, \bar{W}) = H \qquad (10)$$

$$\frac{\partial}{\partial t}(\bar{\theta}\, e^{-\xi_s \xi} \cos\varphi) + \frac{\partial}{\partial \varphi}(\bar{\theta}\, \bar{V}) + \frac{\partial}{\partial \xi}(\bar{\theta}\, \bar{W}) = \bar{q}\, e^{-\xi_s \xi} \cos\varphi + Q$$

$$\frac{\partial}{\partial \xi}\bar{\tau} = -\frac{e^{-k\xi\xi_s}}{\tan\varphi}\frac{\partial\theta}{\partial\varphi} \qquad (12)$$

$$\bar{V} = -\frac{\partial\Psi}{\partial\xi} \qquad W = \frac{\partial\Psi}{\partial\varphi} \qquad (13)$$

$$\frac{\partial}{\partial t}(\bar{X}\, e^{-\xi_s \xi} \cos\varphi) + \frac{\partial}{\partial \varphi}(\bar{V}\, \bar{X}) + \frac{\partial}{\partial \xi}(W\, X) = P\, e^{-\xi_s \xi} \cos\varphi + F$$

where H and Q are the divergence of eddy momentum and eddy heat flux, respectively. $\Psi$ is a stream function which is introduced to rewrite equation (7) in the form of equation (13). The method of solution is to eliminate the time derivatives from (10) and (11) using (12). Substituting for V and W from (13) gives an equation for $\Psi$.

$$-\bar{\tau}_\varphi \Psi_{\xi\xi} + 2\bar{\tau}_\xi \Psi_{\xi\varphi} + \frac{\bar{\theta}_\xi e^{-k\xi\xi_s}}{\tan\varphi}\Psi_{\varphi\varphi} - \left[\xi_s\, \bar{\tau}_\varphi + \frac{\bar{\theta}_\varphi e^{-k\xi\xi_s}(1+\sin^2\varphi)}{\sin^2\varphi}\right]\Psi_\xi +$$

$$\left[\xi_s\, \bar{\tau}_\xi + e^{-k\xi\xi_s}\bar{\theta}_\xi + k\xi_s\, e^{-k\xi\xi_s}\bar{\theta}_\varphi/\tan\varphi\right]\Psi_\varphi = H_\xi + \xi_s H +$$

$$e^{-k\xi\xi_s}\left[e^{-\xi\xi_s}\cos\varphi\, q_\varphi + Q_\varphi + Q\tan\varphi\right]/\tan\varphi \tag{15}$$

The $\xi$ and $\varphi$ used as subscripts denote differentiation with respect to that quantity. It may be solved with suitable boundary conditions for $\Psi$ which may be used in (10) and (11) to get new values of $\tau, \theta$ at the advanced time step. Equation (15) is then solved at the new time value and the process repeated.

A Crank-Nicholson implicit finite difference scheme[4] is used in the numerical calculations. The Alternate Direction Implicit method[4] is adopted so that the finite difference equation can be solved by simply inverting a tridiagonal matrix in each of the two dimensions sequentially. The nonlinear coefficients in the equations can be taken into account by using a predictor - corrector procedure. A more detailed description follows

Eq. (15) is an elliptic type, second order partial differential equation which can be solved by running a marching method for the time-dependent equation to a sufficiently large time. Equations (10), (11) and (14) are time-dependent first order partial differential equations. A general form for the set of equations can be written as:

$$S_t = A\, S_{\xi\xi} + B\, S_{\xi\phi} + C\, S_{\phi\phi} + D\, S_\xi + E\, S_\phi + F \tag{16}$$

The ADI scheme for equation (16) is the following:

$$\begin{aligned} S^{n+\frac{1}{2}} = S^n + \Delta t \Big[ & A\delta_\xi^2\,(S^{n+\frac{1}{2}}) + B\, S_{\xi\phi}^n + C\delta_\phi^2 (S^n) \\ & + D\delta_\xi\,(S^{n+\frac{1}{2}}) + E\delta_\phi\,(S^n) + F\Big] \end{aligned} \tag{17}$$

$$S^{n+1} = S^n + \Delta t \left[ A \; \delta_\xi^2 (S^{n+\frac{1}{2}}) + B \; S_{\xi\phi}^n + C \, \delta_\phi^2 \; (S^{n+1}) \right.$$

$$\left. + D \delta_\xi \, (S^{n+\frac{1}{2}}) + E \delta_\phi \, (S^{n+1}) + F \right] \qquad (18)$$

where:

$$\delta_\xi^2 \quad S_{jk} = (S_{j+1,k} - 2 \, S_{j,k} + S_{j-1,k}) / \Delta\xi^2$$

$$\delta_\phi^2 \quad S_{jk} = (S_{j,k+1} - 2 \, S_{j,k} + S_{j,k-1}) / \Delta\phi^2$$

$$\delta_\xi \quad S_{jk} = (S_{j+1,k} - S_{j-1k}) / 2\Delta\xi$$

$$\delta_\phi \quad S_{jk} = (S_{j,k+1} - S_{j,k-1}) / 2\Delta\phi$$

A simple tridiagonal matrix inversion will solve equations (17) and (18) to give the solution of equation (16).

Eq. (15) is an elliptic type, boundary value problem so that the crossing term $\psi_{\xi\varphi}$ is not a dominant term which can be approximated at the old time step as shown in eqs. (17) and (18). This enables us to use the Crank-Nicholson scheme and the ADI method to solve eq. (15) without rotating the coordinates to get rid of the crossing term. We have taken the average for each two consecutive values of $\psi$ in the process of getting fast convergence. In practice Eq. (15) has been solved with less than 50 total time steps to reach the steady state solution.

The values of $\psi$ are then used to determine V and W which are then substituted in Eqs. (10), (11) and (13) for solving $\tau$, $\theta$ and X at the advanced time step. This process has been repeated for the time dependent solution.

## SECTION 4 RESULTS AND DISCUSSIONS

The terms in the right hand side of Eq. (15) are source functions which drive the wind fields. Q is the summation of the rate of heating due to absorption of solar radiation and the rate of cooling by thermal radiation. Eddy fluxes of heat and momentum are treated by large-scale diffusion coefficients. For these fixed driving terms the time-dependent solutions of the set of equations depend strongly on the initial data available. Therefore we have to find the steady state solution first before seeking any time-dependent solutions.

For the winter northern hemisphere case the generated zonal (u), meridional (v) and vertical (w) winds are shown in Figure (1), (2) and (3) respectively. The agreement between the observed and calculated data may be considered fully satisfactory. Figure (4) shows the temperature distribution from south pole to north pole (x axis) as well as from ground to about 55 km (Y axis) approximately. As we mentioned early H, Q and q are the three source functions which are shown in Figure (5) to Figure (7) respectively. From Eq. (15) it is possible to regard $\Psi$ as the sum of separate contributions by a different term on the right hand side. Thus one may set to zero all the terms on the right-hand side of (15) except those in q and the solution then gives that part of the mean circulation induced by the net heating. The meridional and vertical winds arising from heating only are shown in Figure (8) and (9). It shows the heating plays an important role in the stratosphere but not in the troposphere. Figure (10) and (11) show the results arising from eddy heat fluxes. Finally Figure (12) and (13) are due to eddy momentum fluxes alone. The structure of the wind fields in the troposphere are heavily determined by eddy fluxes of heat and momentum.

## References


1. Holton, J.R., "The Dynamic Meteorology of the Stratosphere and Mesophere," American Meteorological Society, (1974).

2. Lorenz, E.N., "The Nature and Theory of the General Circulation of the Atmosphere," World Meteorological Organization, 1967.

3. Harwood, R.S., Pyle, J.A., "A Two-dimensional mean circulation model for the Atmosphere below 80 Km," Quart. J.R. Met. Soc. (1975), 101, pp. 723-747.

4. Richtmyer, R.D. and Morton, K.W., "Difference Methods for Initial-Value Problems," John Wiley & Sons, N.Y. (1967)

# Figure Captions

For the figures shown below, a northern hemisphere winter case (December to February) has been chosen. The x coordinate shows the latitude from south pole to north pole. The y coordinate gives the pressure and altitude in the following way.

| Grid Point | Pressure (mb) | Altitude (km) |
|---|---|---|
| 28 | .41 | 54.4 |
| 27 | .55 | 52.2 |
| 26 | .74 | 49.9 |
| 25 | .98 | 47.7 |
| 24 | 1.31 | 45.4 |
| 23 | 1.75 | 43.2 |
| 22 | 2.33 | 41.1 |
| 21 | 3.11 | 38.9 |
| 20 | 4.14 | 36.9 |
| 19 | 5.52 | 34.8 |
| 18 | 7.37 | 32.8 |
| 17 | 9.82 | 30.9 |
| 16 | 13.10 | 29.0 |
| 15 | 17.46 | 27.1 |
| 14 | 23.28 | 25.2 |
| 13 | 31.04 | 23.4 |
| 12 | 41.39 | 21.5 |
| 11 | 55.19 | 19.7 |
| 10 | 73.58 | 17.9 |
| 9 | 98.11 | 16.1 |
| 8 | 130.81 | 14.3 |
| 7 | 174.42 | 12.5 |
| 6 | 232.56 | 10.7 |
| 5 | 310.08 | 8.8 |
| 4 | 413.44 | 6.8 |
| 3 | 551.25 | 4.7 |
| 2 | 735.00 | 2.5 |
| 1 | 980.00 | .2 |

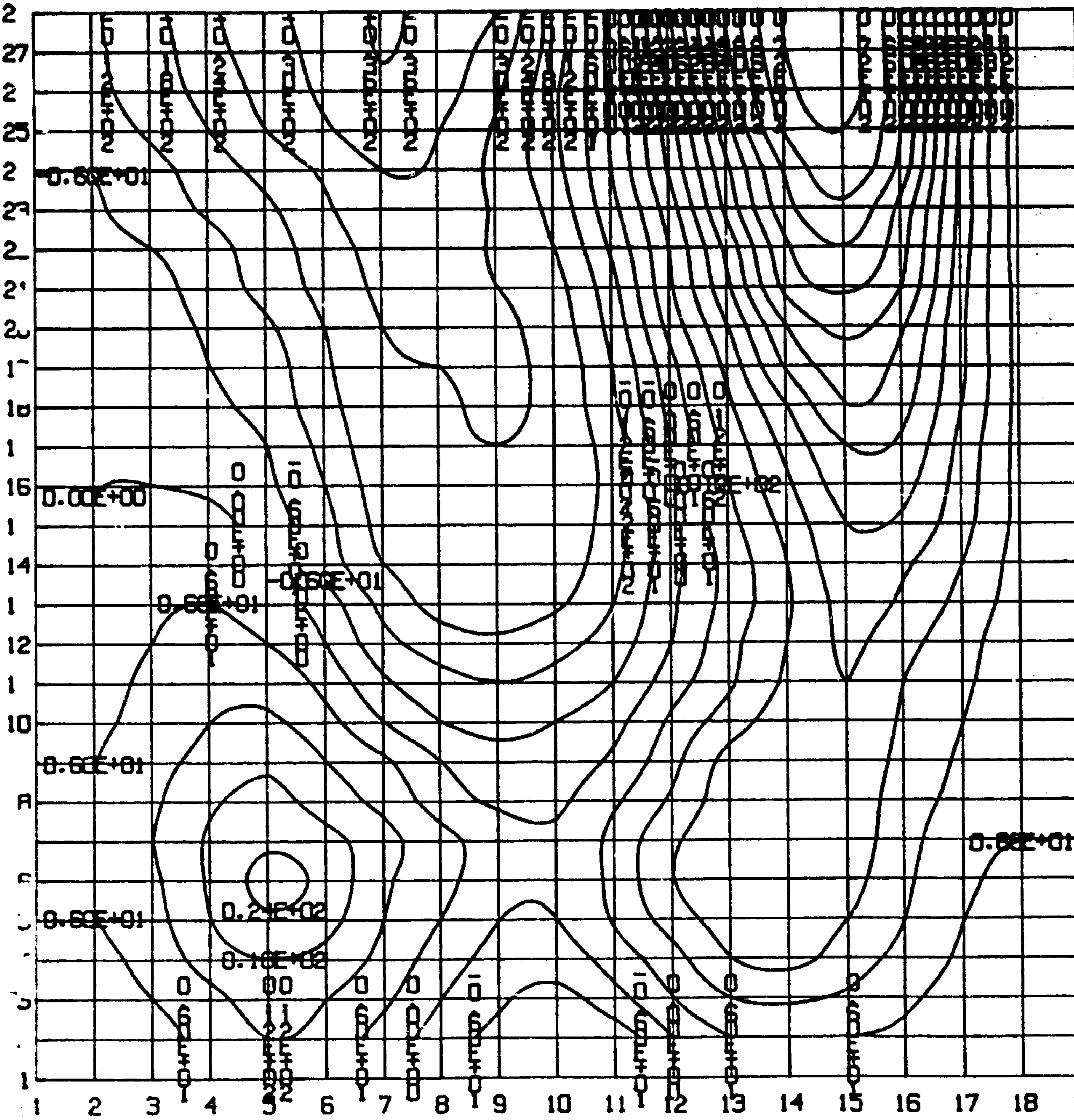
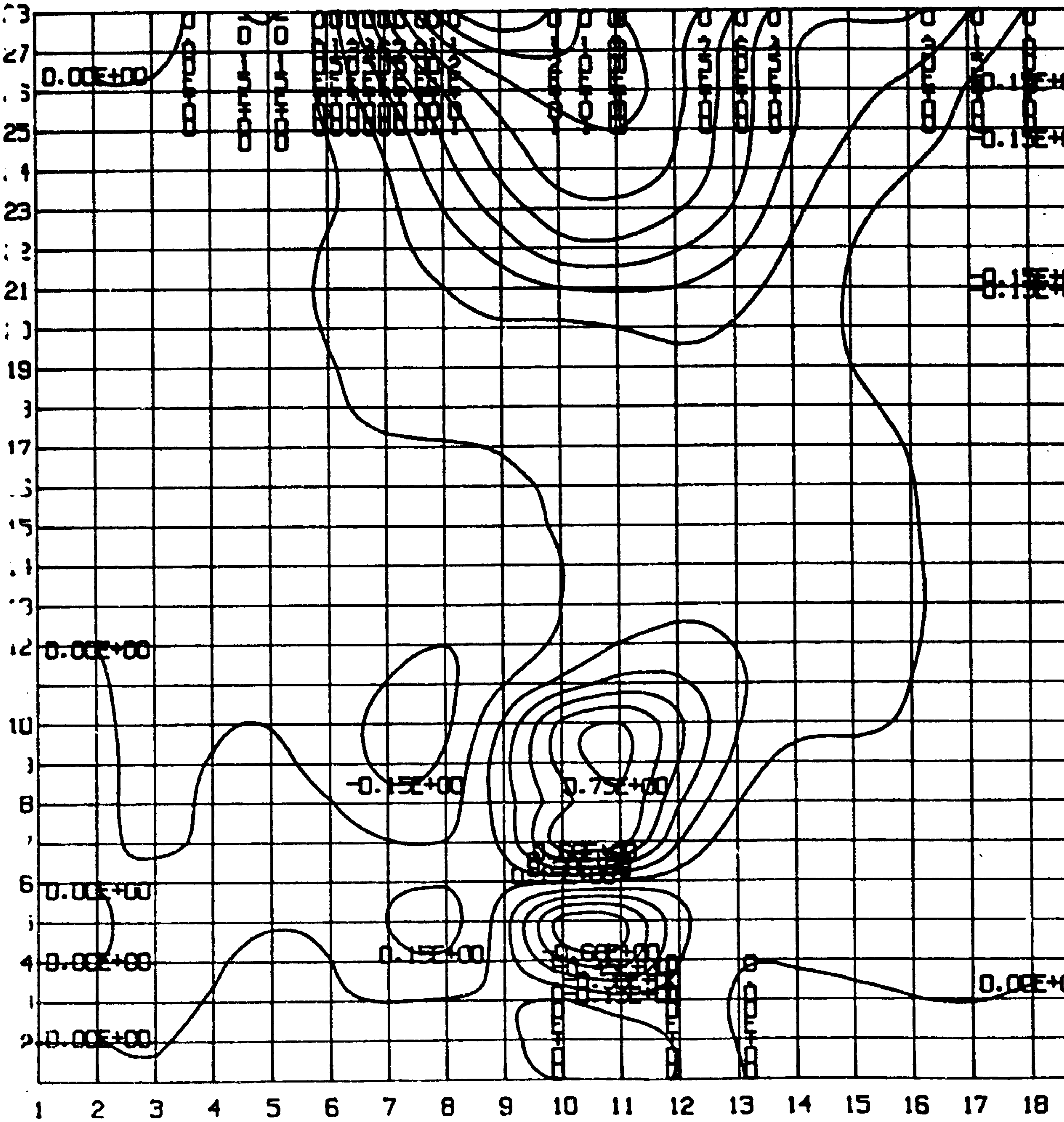



Figure 1



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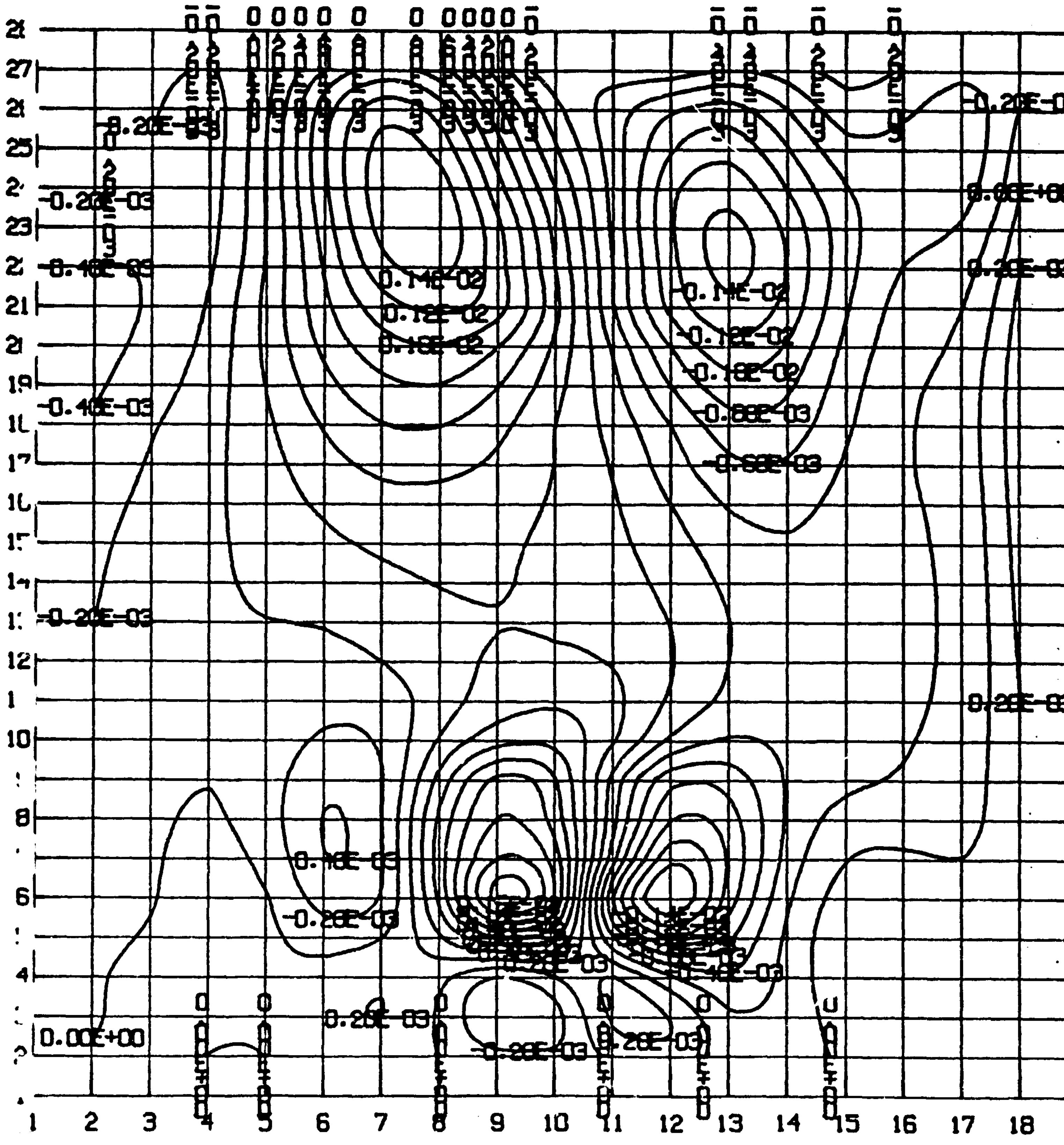

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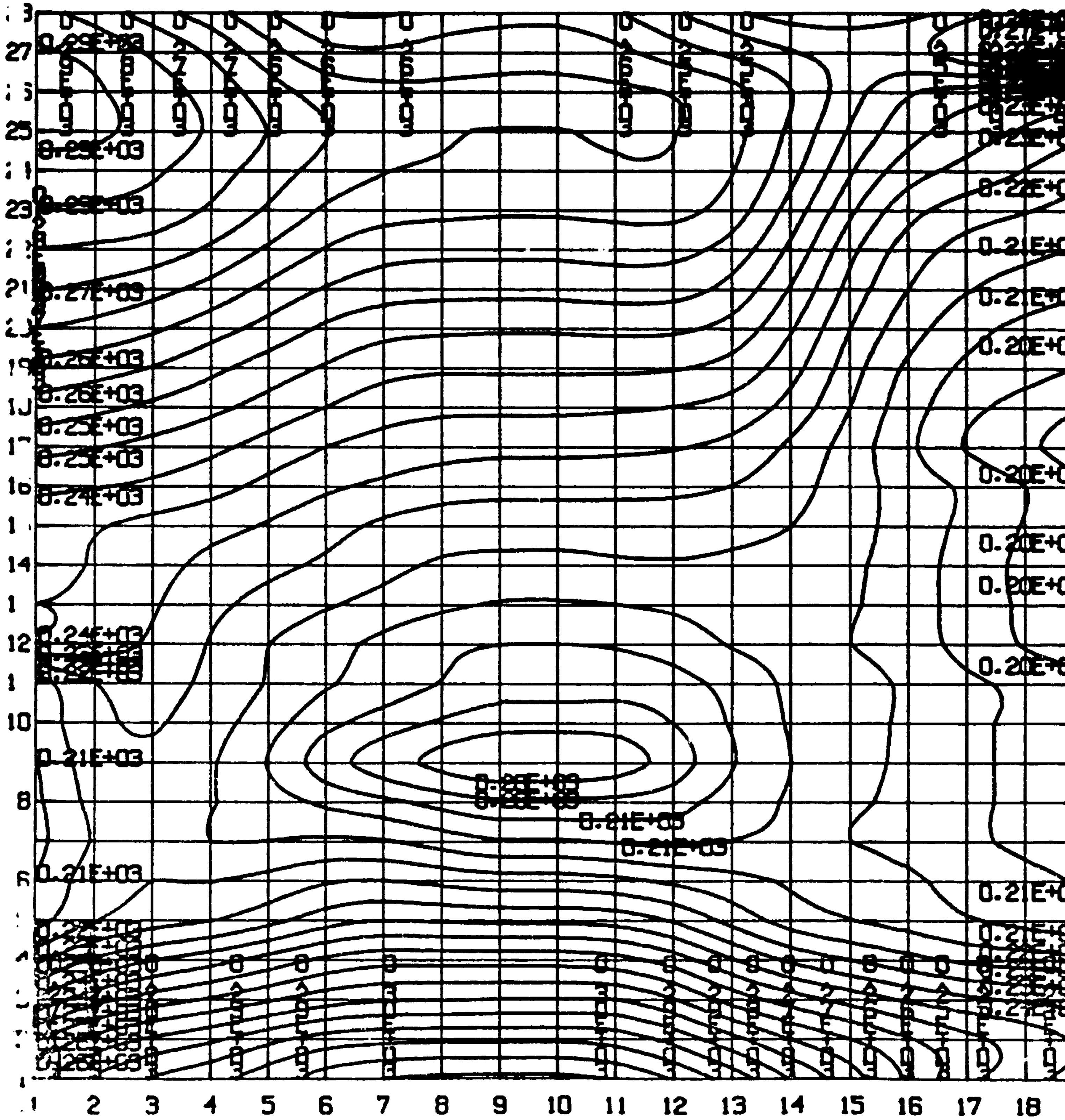

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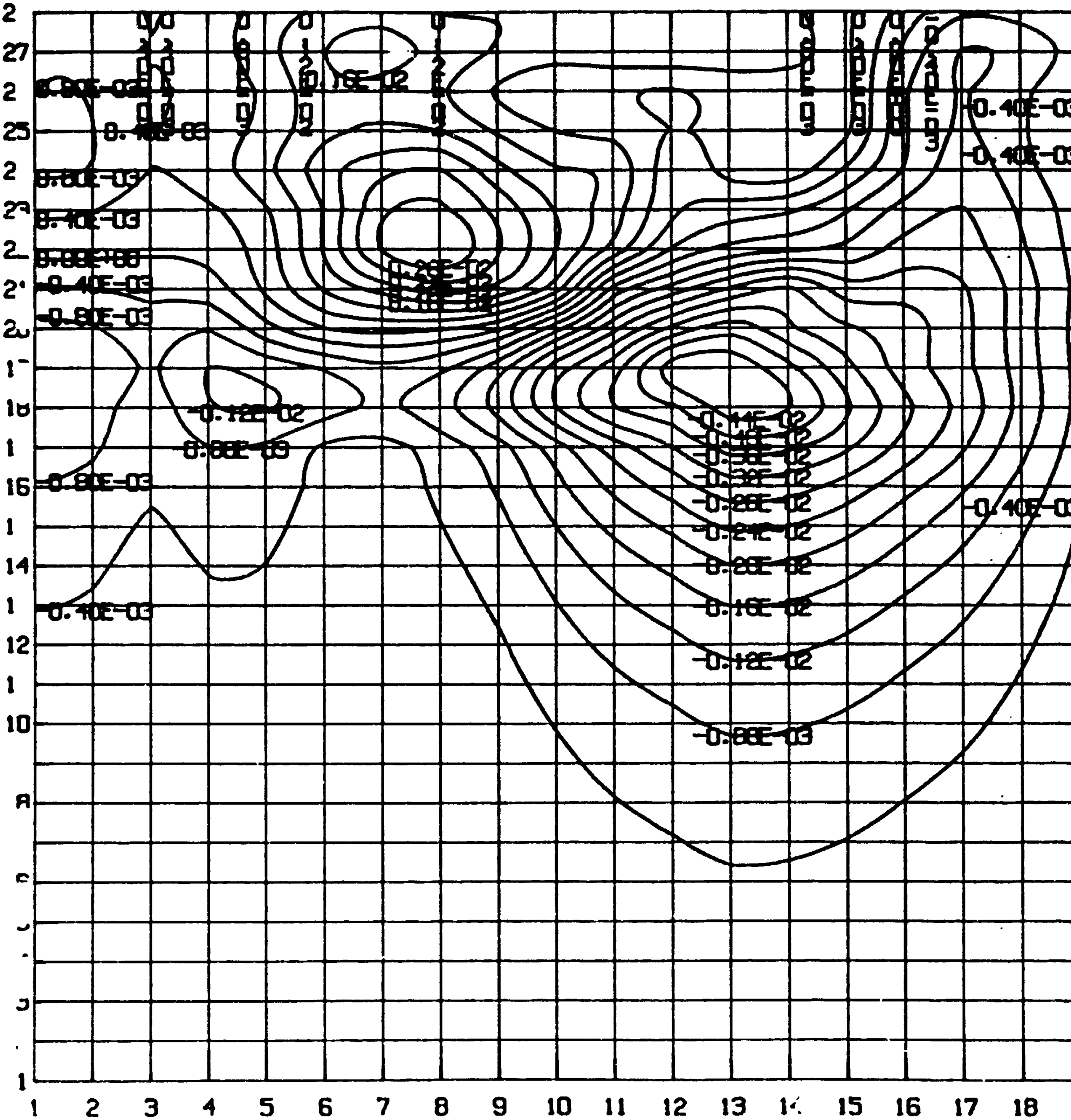

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0.80E-03
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-0.80E-03
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Figure 5

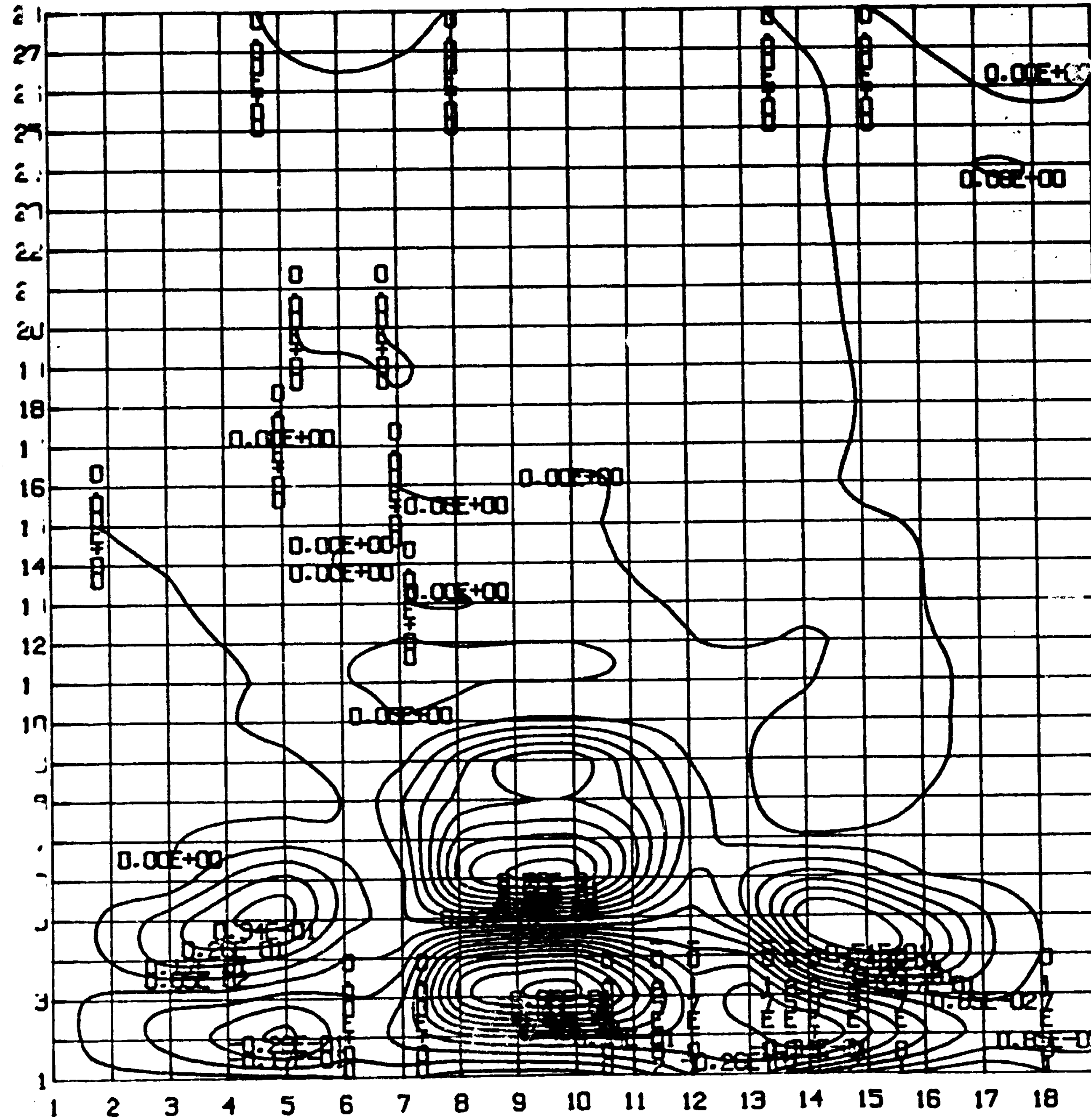

Figure 6

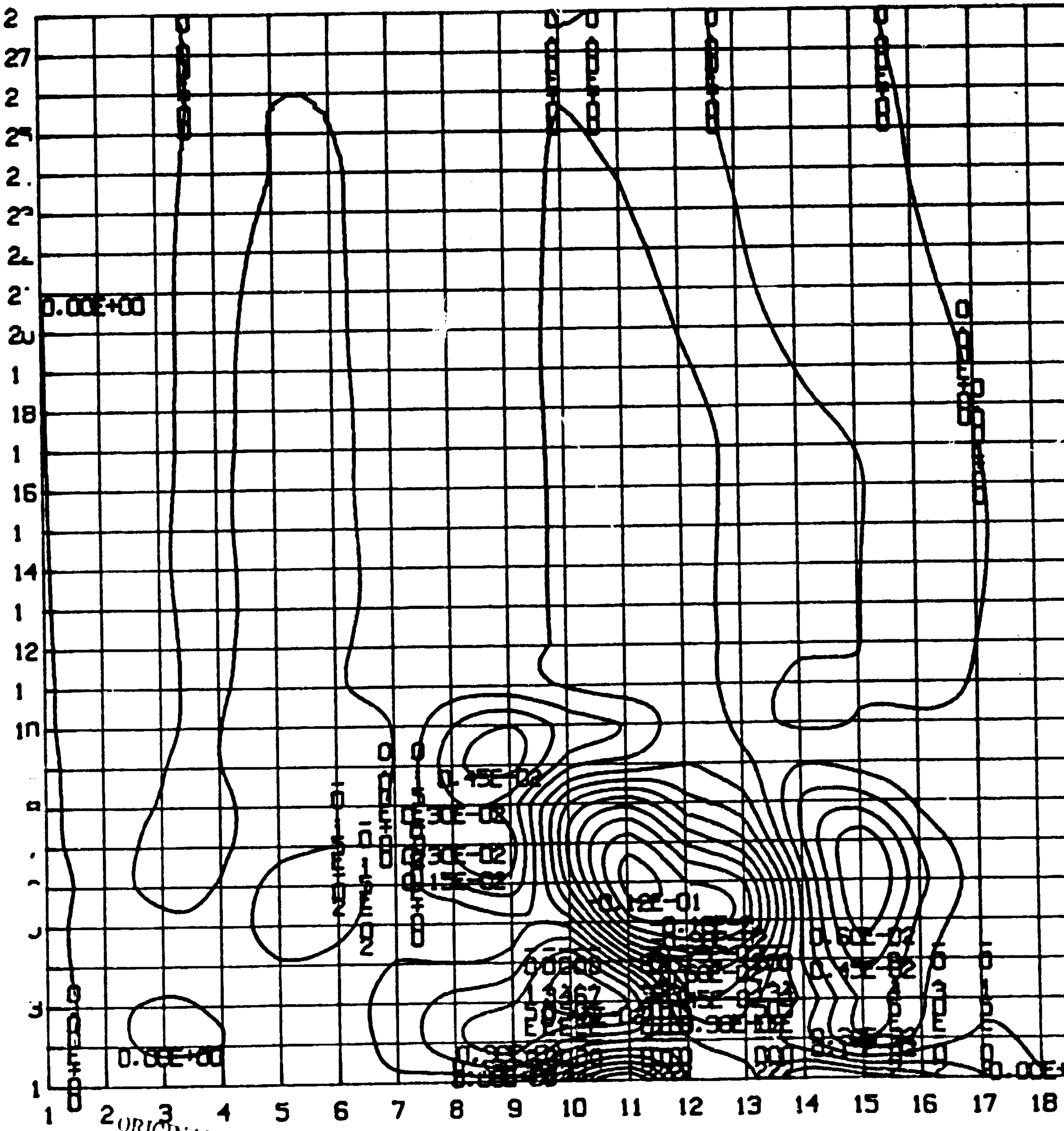


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Figure 7

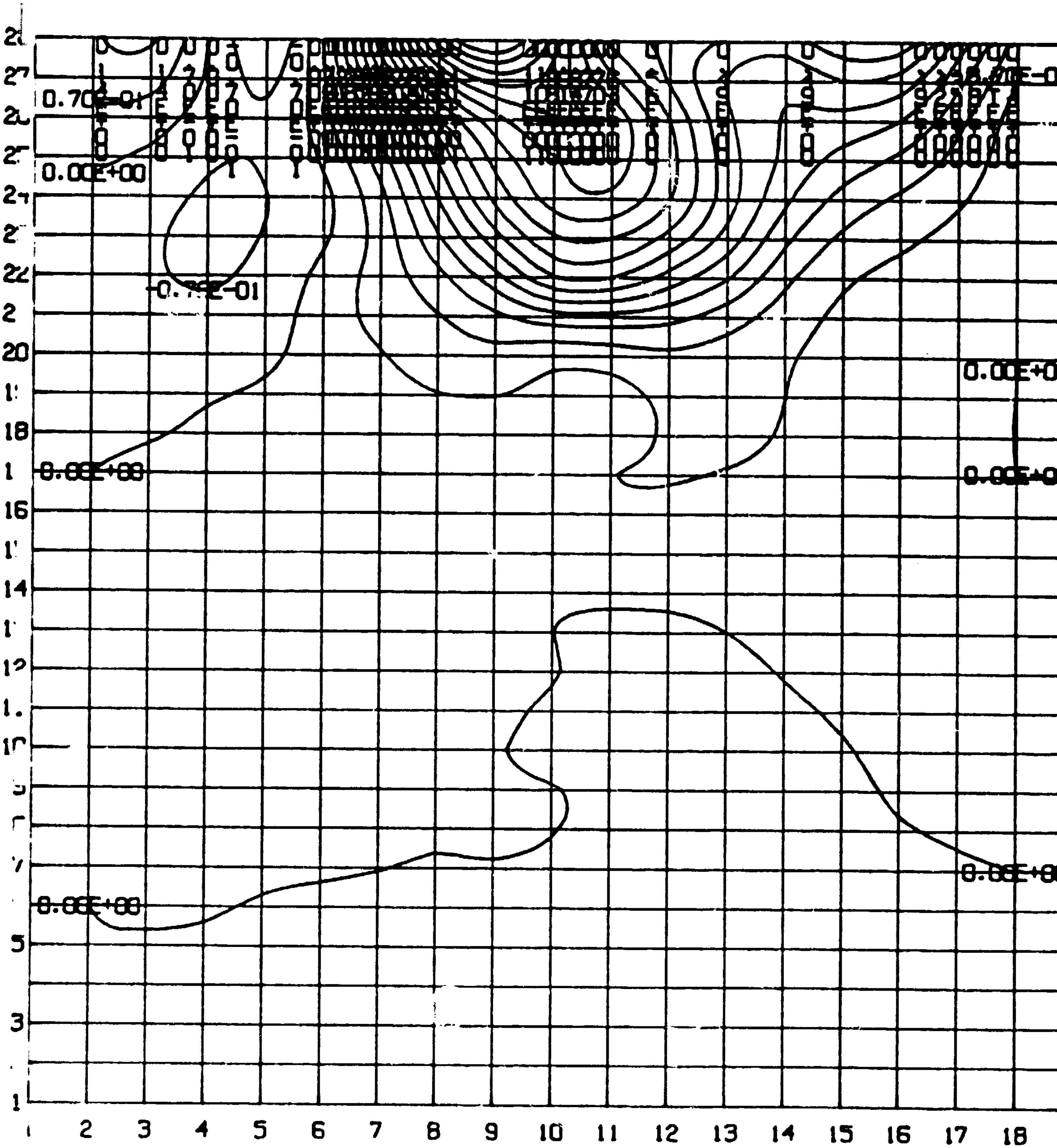
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Figure 8

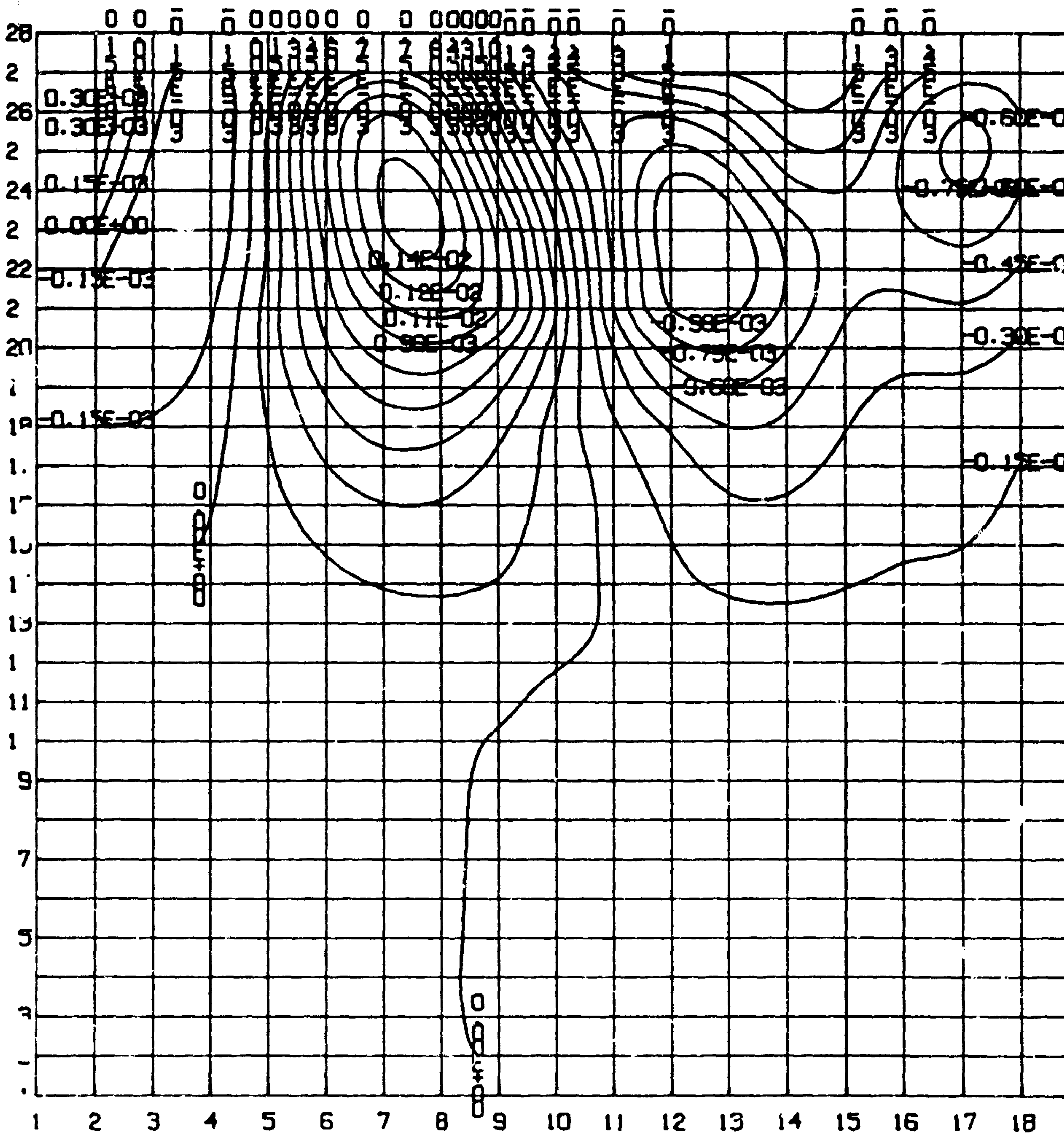

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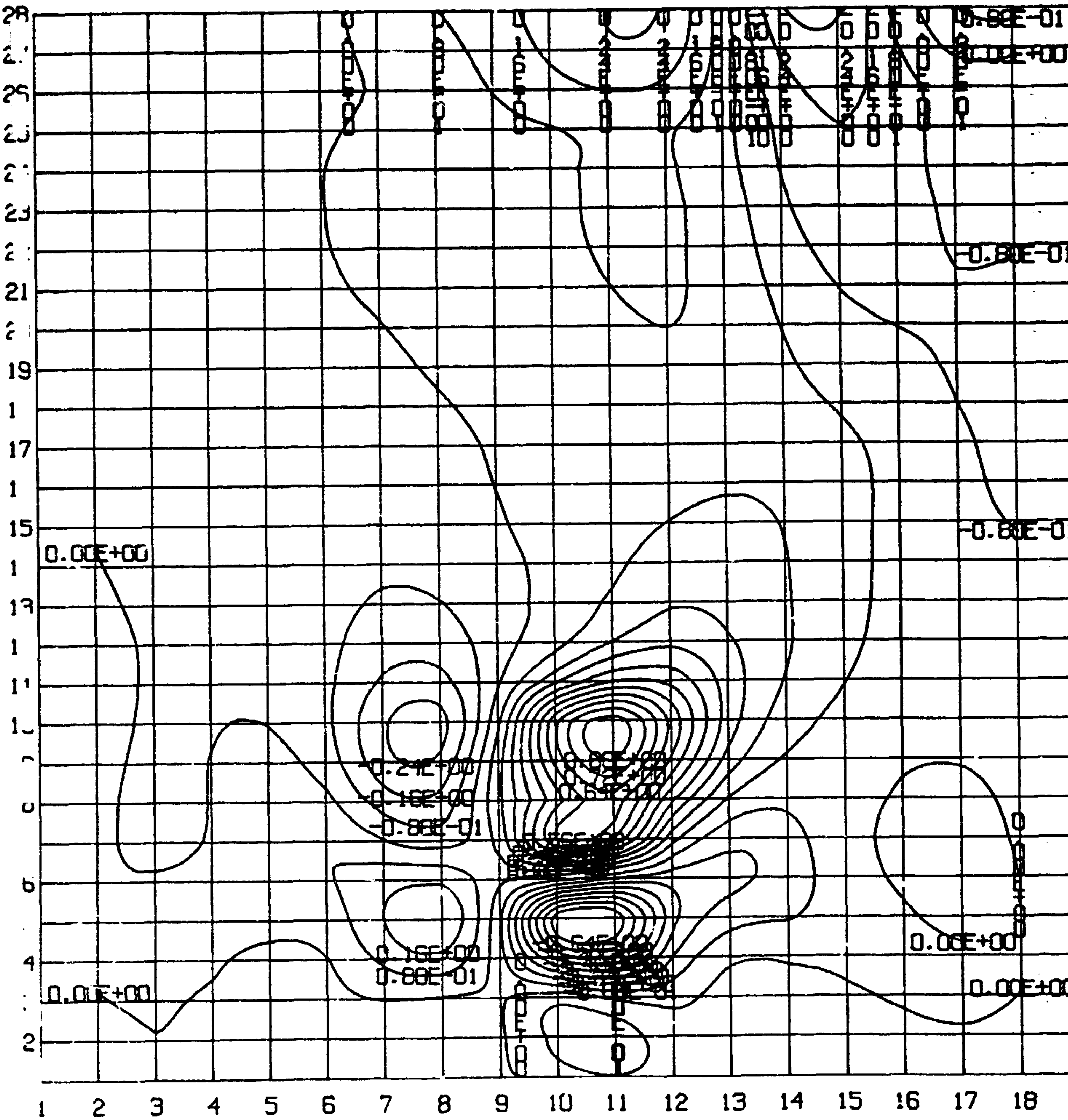
Figure 9



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0.80E-01
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Figure 10

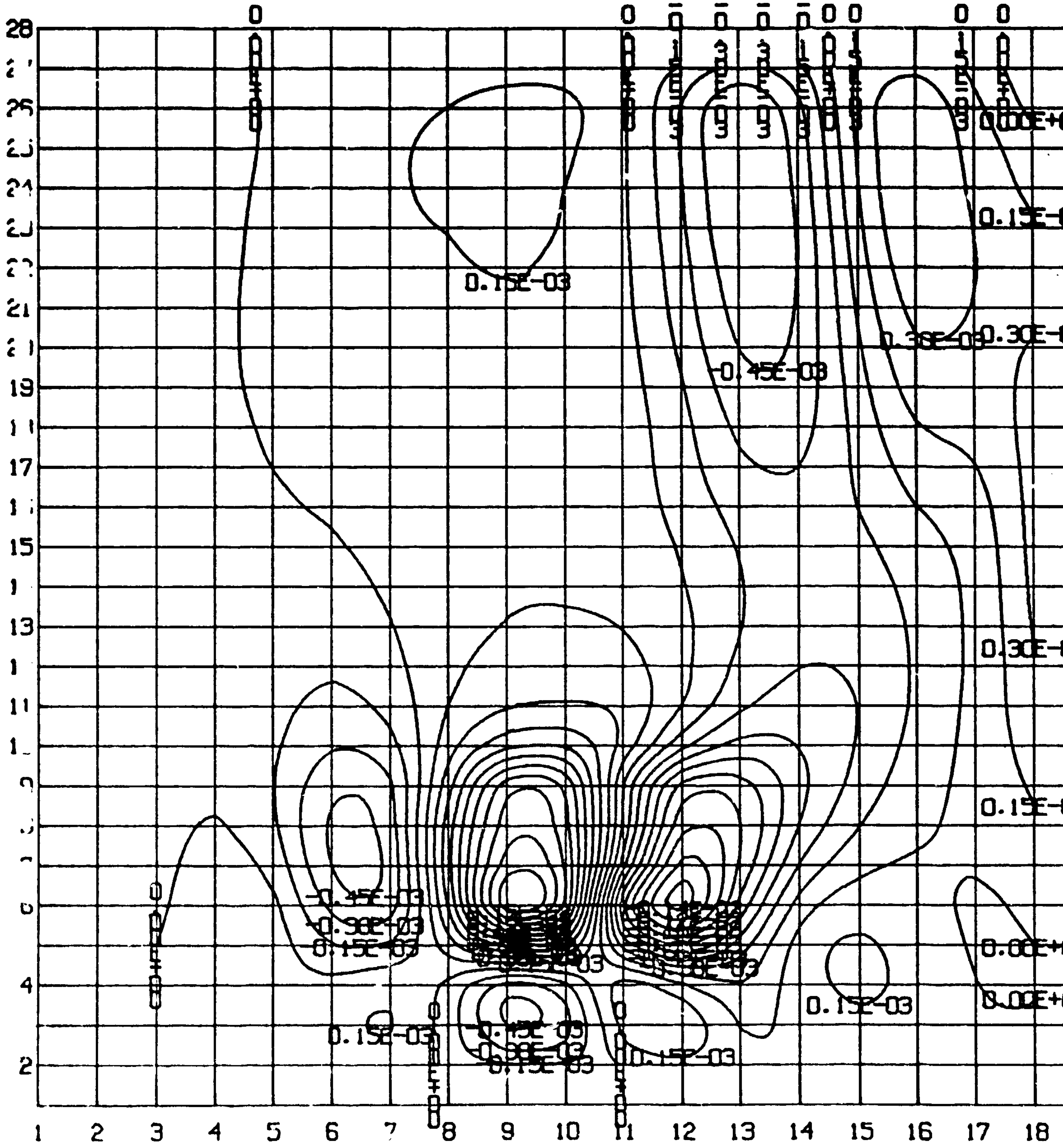


Figure 11

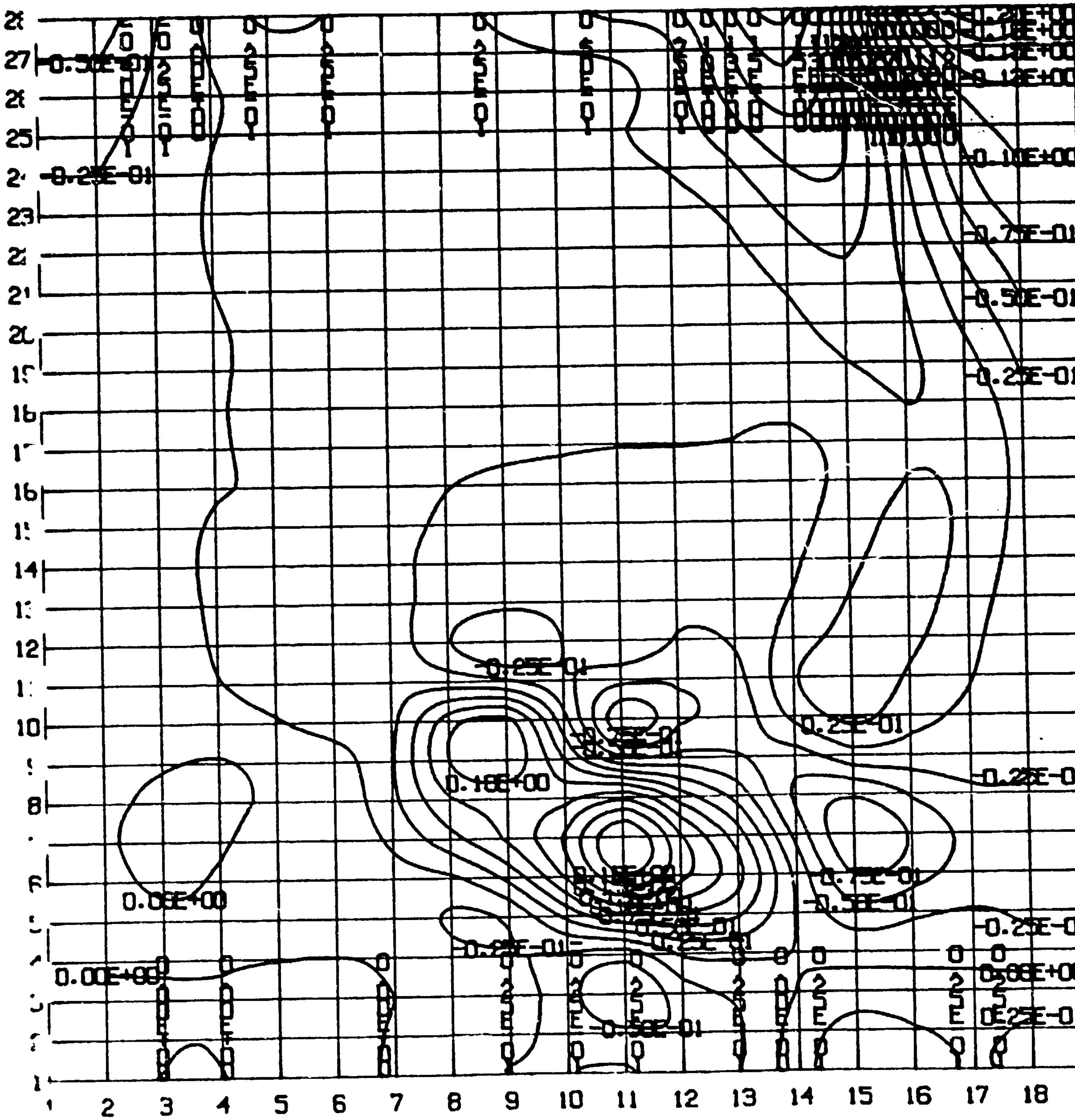

Figure 12

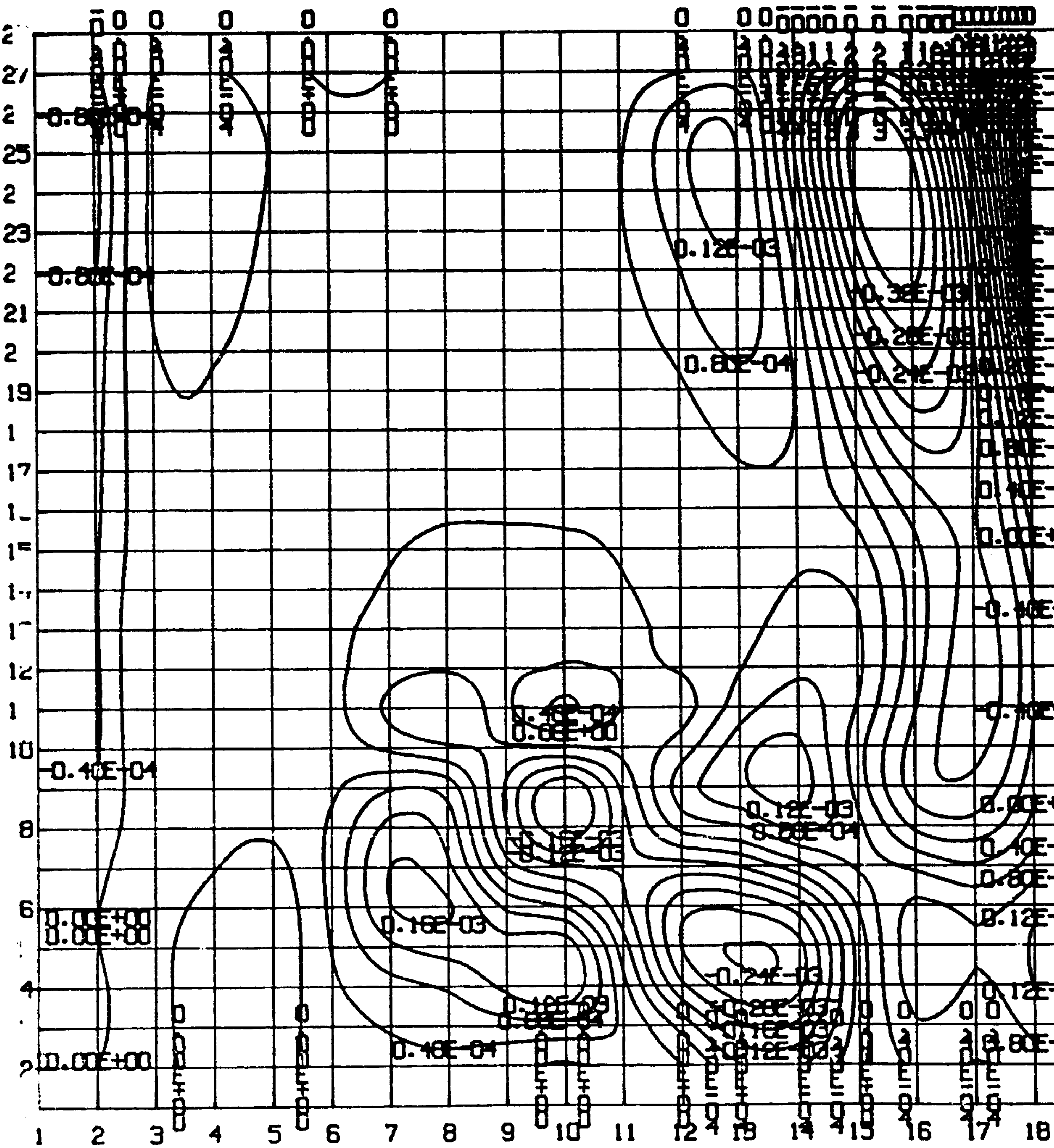

Figure 13

# Two-Dimensional Transport Model for

# Stratospheric Chemistry

## A Users Guide

HSING-CHENG CHEN

## SECTION 1 PHYSICS AND MATHEMATICS

The interactions of dynamics, photochemistry and radiation in the stratosphere can be described[1] by the continuity, momentum and energy equations. In the log-pressure coordinate system the non-dimensional forms of these equations become

$$\frac{\partial}{\partial t}\left(\tau\, e^{-\xi^*}\cos\phi\right) + \frac{\partial}{\partial\phi}(\tau v) + \frac{\partial}{\partial\xi}(\tau W) = H \qquad (1\text{-}1)$$

$$\frac{\partial}{\partial t}\left(\theta\, e^{-\xi^*}\cos\phi\right) + \frac{\partial}{\partial\phi}(\theta V) + \frac{\partial}{\partial\xi}(\theta W) = q\, e^{-\xi^*}\cos\phi + Q \qquad (1\text{-}2)$$

$$\frac{\partial}{\partial\xi}\tau = -\frac{e^{-k\xi^*}}{\tan\phi}\frac{\partial\theta}{\partial\phi} \qquad (1\text{-}3)$$

$$\frac{\partial}{\partial\phi} + \frac{\partial}{\partial\xi} = o \qquad (1\text{-}4)$$

$$\frac{\partial}{\partial t}\left(X\, e^{-\xi^*}\cos\phi\right) + \frac{\partial}{\partial\phi}(VX) + \frac{\partial}{\partial\xi}(WX) = P\, e^{-\xi^*}\cos\phi + F \qquad (1\text{-}5)$$

where the notation of Harwood and Pyle[2] has been used. This is a complete set of equations for the unknowns $\tau, \theta$, v, w and x. Equation (1-4) enables us to introduce a stream function $\psi$ such that

$$V = -\frac{\partial\psi}{\partial\xi} \qquad W = \frac{\partial\psi}{\partial\phi} \qquad (1\text{-}6)$$

The method of solution is to eliminate the time derivatives from (1-1) and (1-2) using (1-3). Substituting V and W from (1-6) gives a equation for $\psi$

$$-\tau_\phi\ \psi_{\xi\xi} + 2\,\tau_\xi\ \psi_{\xi\phi} + \left[e^{-k\xi^*}\ (\tan\phi)^{-1}\ \theta_\xi\right]\psi_{\phi\phi}$$

$$- \left[\xi_s \tau_\phi + e^{-k\xi^*} (1 + \sin^2\phi) (\sin\phi)^{-2} \theta\phi\right] \psi\phi\xi$$

$$+ \left[\xi_s \tau_\xi + e^{-k\xi^*} \theta_\xi + k\xi_s e^{-k\xi^*} (\tan\phi)^{-1} \theta\phi\right] \psi\phi$$

$$= H_\xi + \xi_s H + e^{-k\xi^*} (\tan\phi)^{-1} \left[e^{-\xi^*} \cos\phi \, q_\phi + Q\phi + Q \tan\phi\right] \tag{1-7}$$

For suitable boundary conditions Eq. (1-7) can be solved for $\psi$ which gives us V and W. Then (1-1), (1-2) and (1-5) are solved for $\tau, \frac{\theta}{N}$ and x at the advanced time step. (1-7) is again solved at the new time value and the process repeated until the solution become stationary.

A general form for the set of equations can be written as

$$S_t = A \, S_{\xi\xi} + B \, S_{\xi\phi} + C \, S_{\phi\phi} + D \, S_\xi + E \, S_\phi + F \tag{1-8}$$

A Crank-Nicholson implicit finite difference scheme[3] is used in the numerical calculations. The Alternate Direction Implicit method[3] is adopted for this two dimensional problem so that the finite difference equation can be solved by inverting a tridiagonal matrix in each of the two dimensions sequentially. The ADI scheme for (1-8) is the following

$$S^{n+\frac{1}{2}} = S^n + \Delta t \Big[ A \, \delta_\xi^2 (S^{n+\frac{1}{2}}) + B \, S^n_{\xi\phi} + C \, \delta_\phi^2 (S^n) + D \, \delta_\xi (S^{n+\frac{1}{2}}) + E \, \delta_\phi (S^n) + F \Big]$$

$$S^{n+1} = S^n + \Delta t \Big[ A \, \delta_\xi^2 (S^{n+\frac{1}{2}}) + B \, S^n_{\xi\phi} + C \, {}^2 (S^{n+1}) + D \, \delta_\xi (S^{n+\frac{1}{2}}) + E \, \delta_\phi (S^{n+1}) + F \Big]$$

where the following notations are used

$$\delta_{\xi}^{2} S_{j,k} = (S_{j+1,k} - 2\ S_{j,k} + S_{j-1,k}) / \Delta\xi^{2}$$

$$\delta_{\xi}^{2}\ S_{j,k} = (S_{j,k+1} - 2\ S_{j,k} + S_{j,k-1}) / \Delta\xi^{2}$$

$$\delta_{\xi}\ S_{j,k} = (S_{j+1,k} - S_{j-1,k}) / 2\Delta\xi$$

$$\delta_{\phi}\ S_{j,k} = (S_{j,k+1} - S_{j,k-1}) / 2\Delta\phi$$

# SECTION 2 PROGRAM OUTLINES

## 2.1 Main

The developed GSFC 2D model (latitude, altitude) introduces a better understanding of the coupling between stratospheric chemistry and dynamics. The program is constructed in American National Standard FORTRAN on a large IBM S/360-91 computer with a region less than 400K eight-bit bytes. It consists of main program, seven (7) subroutines and data. In the main program, the physical constants are defined and the mesh sizes in each of the two dimensions are chosen. The eddy diffusion coefficients, the initial profiles for zonal wind, temperature and ozone distribution are read in from the punched - card data. The five major subroutines accomplish the following tasks: (1) Subroutine HEAT deals with the calculation of heating and cooling rates of the atmosphere (2) Subroutine EDDY calculates the eddy diffusion terms due to heat fluxes and momentum fluxes. (3) Subroutine VWS computes the solutions for meridional and vertical winds. (4) Subroutine TVS solves the time-dependent solutions for zonal wind and temperature and finally, (5) Subroutine ANS prints out the solutions in two-dimensional format. A schematic diagram including the JCL cards is shown in the following

```
///ZMHCCJ28 JOB (L40042195A,T,AA0001,001001),620,MSGLEVEL=1
//*B11
// EXEC FORTRANH,PARM='OPT=2,XREF',REGION=400K
```

```
     Main Program
Subroutine EDDY
Subroutine VWS
Subroutine HEAT
Subroutine TUS
     BLOCK DATA
Subroutine TRIDJ
Subroutine ANS
// EXEC LOADER,REGION=400K,PARM='SIZE=400K'
//FT05F001 DD DSN=K3,L6CJM.ZMHCC.S1,DISP=SHR
//      DD *,DCB=BLKSIZE=2000
 /*
```

Every subroutine will be described in great detail in the later sections.

The M.K.S. System has been adopted in the calculations, Some of the important physical constants and parameters are listed and defined below.

| | | |
|---|---|---|
| PI | : | $\pi$ |
| RA | : | radius of earth (m) |
| GRAV | : | acceleration of gravity (m/sec$^2$) |
| XM | : | mass of air (Kg) |
| BOLTZ | : | Boltzmann constant. (k) ($^{\text{dyne}}$/deg (k)) |
| XC | : | speed of light (c) (m/sec) |
| XH | : | Planck constant (h) (dyne-sec) |
| XK | : | ratio of gas constant to specific heat at constant pressure. (k) |
| JJ | : | number of grid levels in vertical direction |

R : angular velocity of earth

CS : $\xi_s$

VS : $V_s$

WS : $W_s$

KK : total number of intervals in latitude

KT : total grid points in 12 hour average

DZ : grid size in latitude

DY : grid size in altitude

Y(J) : $\xi$

EX(J) : exp $(-\xi\xi s)$

EXK(J) : exp $(-k\xi\xi s)$

P(J) : pressure (mb)

Z(J) : latitude (degree)

SI : sine $(\phi)$

CO : cosine $(\phi)$

TA : tangent $(\phi)$

DT : time step (sec)

The following parameters are read in from the machine-readable data stored on a CRBE file on disk:

YYK : horizontal eddy diffusion coefficients

YZK : off-diagonal eddy diffusion coefficients

ZZK : vertical eddy diffusion coefficients

TEMP : temperature (°k)

ZA : wavelength (m)

ZB : solar flux $(m^{-2} sec^{-1})$

ZC : absorption cross section of $O_3$ $(m^2)$

ZY : absorption cross section of $O_2$ $(m^2)$

PO3 : ozone density $(m^{-3})$

UU : zonal wind velocity (m/sec)

It is then ready to call subroutines Heat, Eddy, Vws, TUS sequentially to solve the problem.

2.2 Subroutine HEAT: Heating and cooling rates are calculated mainly from the 15 $\mu$m $CO_2$ band and the 9.6 $\mu$m $O_3$ band. The parameters used are defined below:

RD : unit from radians to degrees

C2 : CH * XK *86400 *1000/$_{1.013 \times 10^5}$

AL : g m/ k t

AM : density of air $(m^{-3})$

PO2 : density of $O_2$ $(m^{-3})$

O : mixing ratio of $O_3$

QQ : heating rate $(^\circ k/day)$

XJ2 : J coefficient for $O_2$

XJ3 : J coefficient for $O_3$

Q1 : heating rate at different hour of the day

Q2 : XJ2 at different hour of the day

Q3 : XJ3 at different hour of the day

XB : column density for $O_2$

XA : column density for $O_3$

XP : distance from the center of the earth

KT : total number of points in 12 hours

S4 : Ch (X)

TAU : $\sum_i \alpha_i(\lambda) \int_z^{\infty} N_i \, (z) \, CH(x)$

The rate of cooling is written

$$\frac{dT}{dt} = XK2CO2 - XK1CO2 * B_{CO2}(T)$$
$$XK2O3 - XK1O3 * B_{O_3}(T)$$

where $B(T) = \frac{2\pi k^3 T^3}{ch^2} \quad \frac{x^3}{e^x - 1}$

and $x = {}^{hc}/_{\lambda k T}$

S3 : cooling rate due to $CO_2$

S6 : cooling rate due to $O_3$

SQ : net heating rate

2.3 Subroutine EDDY: H and Q are the divergence of eddy momentum and eddy heat fluxes respectively. They are defined as

$$H = \frac{\partial}{\partial\phi}\left[K_{yy}\frac{1}{a\upsilon_s}\frac{\partial\tau}{\partial\phi} + K_{yz}\frac{1}{\xi_s\upsilon_s}\frac{\partial\xi^*}{\partial z}\frac{\partial\tau}{\partial\xi}\right] e^{-\xi^*}\cos\phi$$

$$+ \frac{\partial}{\partial\xi}\left[K_{yz}\frac{1}{a\omega_s}\frac{\partial\xi^*}{\partial z}\frac{\partial\tau}{\partial\phi} + K_{zz}\frac{1}{\xi_s\omega_s}\left(\frac{\partial\xi^*}{\partial z}\right)^2\frac{\partial\tau}{\partial\xi}\right] e^{-\xi^*}\cos$$

$$= \frac{\partial}{\partial\phi}\left[YYK\frac{\partial\tau}{\partial\phi} + YZK\frac{\partial\tau}{\partial\xi}\right] e^{-\xi^*}\cos\phi$$

$$+ \frac{\partial}{\partial\xi}\left[ZYK\frac{\partial\tau}{\partial\phi} + ZZK\frac{\partial\tau}{\partial\xi}\right] e^{-\xi^*}\cos\phi$$

$$Q = \frac{\partial}{\partial\phi}\left|YYK\frac{\partial\theta}{\partial\phi} + YZK\frac{\partial\theta}{\partial\xi}\right| e^{-\xi^*}\cos$$

$$+ \frac{\partial}{\partial\xi}\left|ZYK\frac{\partial\theta}{\partial\phi} + ZZK\frac{\partial\theta}{\partial\xi}\right| e^{-\xi^*}\cos$$

where $\tau$ is represented in the program by TH which is the average between old value T and new value TT. Similarly $\theta$ is represented by UH. The eddy flux convergence of $O_3$ mixing ratio is represented by PF and PL is the production rate. XK2 and XK4 are rate constants for the basic reactions

$$O + O_2 + M \longrightarrow O_3 + M$$

and $O + O_3 \longrightarrow 2\,O_2$ respectively. EN is the enhancement for XK4 in order to take into account the ozone sink due to other species in the atmosphere.

2.4 Subroutine VWS: This subroutine solves a second order partial differential equation for stream function $\psi$ in the form

$$A\,\psi_{\xi\xi} + B\,\psi_{\xi\phi} + C\,\psi_{\phi\phi} + D\,\psi_{\xi} + E\,\psi_{\phi} + F = o$$

where $A = -\tau_{\phi}\tan\phi$

$B = 2\,\tau_{\xi}\tan\phi$

$C = e^{-k\xi^*}\,\theta_{\xi}$

$D = -\left\{\xi_s\tau\phi + e^{-k\xi^*}\left(\frac{1}{\sin^2\phi} + 1\right)\theta_{\phi}\right\}\tan\phi$

$E = (\xi_s\,\tau_{\xi} + e^{-k\xi^*}\,\theta_{\xi})\tan\phi + k\,\xi_s\,e^{-k\xi^*}\,\theta_{\phi}$

$F = -(H_{\xi} + \xi_s H)\tan\phi - e^{-k\xi^*}(e^{-\xi^*}\cos\phi\, q_{\phi}$

$+ Q_{\phi} + Q\tan\phi)$

The alternate-direction-implicit method has been used to solve this two-dimensional problem. The finite difference equation for the first step is

$$AA_j{*}SS_{j-1,k} + BB_j{*}SS_{j,k} + CC_j{*}SS_{j+1,k} = YY_j$$

where $AA_j = -A + D$

$BB_j = \frac{1}{\Delta t} + 2\,A$

$CC_j = -A - D$

$YY_j = S_{j-1,k}\left[A - D\right] + S_{j,k}\left[\frac{1}{\Delta t} - 2A - 2C\right] + S_{j+1,k}\left[A + D\right]$

$+ S_{j,k+1}\left[C + E\right] + S_{j,k-1}\left[C - E\right] + F + g$

where

$$g = \frac{B}{4\Delta\xi\,\Delta\phi}\left(S_{j+1,k-1} - S_{j+1,k-1} - S_{j-1,k+1} + S_{j-1,k-1}\right)$$

The second step is

$$AA_k * ST_{j,k-1} + BB_k * ST_{j,k} + CC_k * ST_{j,k+1} = YY_k$$

where $AA_k = (-C + E)/2$

$$BB_k = \frac{1}{\Delta t} + C$$

$$CC_k = (-C - E)/2$$

$$YY_k = SS_{j-1,k} \left|A - D\right| + SS_{j,k} \left|-2A\right| + SS_{j+1,k} \left|A + D\right| + S_{j-1,k} \left|A - D\right| + S_{j,k} \left|\frac{1}{\Delta t} - 2A - C\right| + S_{j+1,k} \left|A + D\right| + S_{j,k-1} \left[\frac{C}{2} - \frac{E}{2}\right] + S_{j,k+1} \left[\frac{C}{2} + \frac{E}{2}\right] + F + g$$

Subroutine TRIDJ is called to invert the tridiagonal matrix. II is the total number of time step required to satisfy the criterion (S (J,K) - ST(J,K) ) / S (J,K) $\leq$ .005 for steady state solution. Integer I is designed to average the solutions for every two time steps which speeds up the rate of convergence.

2.5 Subroutine TVS: Equations (1-1), (1-2) and (1-5) are solved at the advanced time step. They have the general form

$$S_t\, e^{-\xi^*}\cos\phi = V\,\frac{\partial S}{\partial \phi} + W\,\frac{\partial S}{\partial \xi} + RHS$$

The finite difference equation for the first step in ADI scheme is

$$AA_j * SS_{j-1,k} + BB_j * SS_{j,k} + CC_j * SS_{j+1,k} = YY_j$$

where $AA_j = -\,W/4\Delta\xi$

$BB_j = e^{-\xi^*}\cos\phi\,/\,\Delta t$

$CC_j = W/4\Delta\xi$

$$YY_j = S^n_{j-1,k}\,(W/4\Delta\xi) + S^n_{j,k}\,(e^{-\xi^*}\cos\phi/\Delta t) - S^n_{j+1,k}\,(W/4\Delta\xi) - \frac{V}{2\Delta\phi}\left(S^n_{j,k+1} - S^n_{j,k-1}\right) + RHS$$

The second step is

$$AA_k * ST_{j,k-1} + BB_k * ST_{j,k} + CC_k * ST_{j,k+1} = YY_k$$

where $AA_k = -\,V/4\Delta\phi$

$BB_k = e^{-\xi^*}\cos\phi\,/\,\Delta t$

$CC_k = V/4\Delta\phi$

$$YY_k = \frac{V}{4\Delta\phi}\left(S^n_{j,k+1} - S^n_{j,k-1}\right) - \frac{W}{4\Delta\xi}\left(SS_{j+1,k} + S_{j+1,k} - SS_{j-1,k} - S_{j-1,k}\right) + S_{j,k}\,e^{-\xi^*}\cos\phi/\Delta t + RHS$$

# SECTION 3 COMMON

In the program a double precision has been chosen for all real variables.

A. Common /TU/

| FORTRAN Name | Description and Units |
|---|---|
| T(28,19) | $\tau_t$ (absolute angular momentum) |
| TS(28,19) | $\tau_s$ (intermediate values beteeen $\tau_t$ and $\tau_{t+1}$) |
| TT(28,19) | $\tau t+1$ |
| US(28,19) | $\theta_s$ (intermediate values between $\theta_t$ and $\theta_{t+1}$ |
| UT(28,19) | $\theta t+1$ |
| Z(19) | latitude (degree) |
| Z1(28) | YY for $O_3$ |
| Y(28) | $\xi*$ $\left( - \log (p*/1000 \text{ mb}) \right)$ |
| DZ4 | 4 * DZ |
| DT | real time step (sec) |
| DTO | 1/DT |

B. Common/HEA/

| | |
|---|---|
| OS(28,19) | $O_s$ (intermediate values between $O_t$ and $O_{t+\Delta t}$ |
| P(28) | pressure in mb |
| AB(19) | not used |
| PP(19) | not used |
| QQ(28,19) | heating rate ($^{\circ}$k/day) |

| | |
|---|---|
| ZA(162) | wavelength (m) |
| ZB(162) | flux of solar photons ($m^{-2}sec^{-1}$) |
| AC(162) | absorption cross section of $O_3$ ($m^2$) |
| ZY(162) | absorption cross section of $O_2$ ($m^2$) |
| XO3 | CH/k/9.6 X $10^{-6}$(0.6 $\mu$m $O_3$ band) |
| XD | $2\pi k^3$ / (c * 100. * $h^2$) |
| XCO2 | CH/k/1.5 x $10^{-5}$ (15 $\mu$m $CO_2$ band) |
| CH | C * h |
| BOLTZ | k (Boltzmann constant) (dyne/°k) |
| RA | radius of earth (m) |
| SD | sine of declination angle |
| CD | cosine of declination angle |
| PI | $\pi$ |
| GMK | g m / k |
| KT | total points for 12-hour average |
| KTT | 2 (KT-1) |
| KTM1 | (KT-1) |

C. Common/VW/

| | |
|---|---|
| q(28,19) | defined in (2.4) |
| ZZ(28) | YY for $\tau$ |
| SQ(28,19) | q (net heating rate) |
| EXK(28) | exp (- k $\xi \xi_s$) |
| SI(19) | sine (latitude) |
| TA(19) | tangent (latitude) |

| | |
|---|---|
| AA(162) | $AA_j$ or $AA_k$ |
| BB(162) | $BB_j$ or $BB_k$ |
| CC(162) | $CC_j$ or $CC_k$ |
| YY(162) | $YY_j$ or $YY_k$ |
| W(28,19) | vertical wind velocity (m/sec) |
| XX(28) | solution for subroutine TRIDJ |
| ST(28,19) | solution for $\psi$ at t + $\Delta$t |
| SH(28,19) | average of $\psi$ between t and t + $\Delta$t |
| U(28,19) | potential temperature $\theta$ |
| S(28,19) | solution of $\psi$ at t |
| SS(28,19) | $\psi$s (intermediate values between $\psi$+ and $\psi$ t+$\Delta$t) |
| 111 | loop for integration |
| 111T | end point for loop 111 |
| 1111 | loop for integration |
| 1111T | end point for loop 1111 |
| DY4 | 4 * DY |
| DZDZ | DZ * DZ |
| DYDZ4 | 4 * DY * DZ |
| DYDY2 | 2 * DY * DY |
| XK | 1/Cp |
| CS | $\xi_s$ |

D. Common/EDD/

| | |
|---|---|
| A(28,19) | coefficient in eq. (1-8) |
| B(28,19) | coefficient in eq. (1-8) |
| C(28,19) | coefficient in eq. (1-8) |

| | |
|---|---|
| D(28,19) | coefficient in eq. (1-8) |
| E(28,19) | coefficient in eq. (1-8) |
| YYK(28,19) | eddy diffusion coefficient $K_{yy}$ |
| YZK(28,19) | eddy diffusion coefficient $K_{yz}$ |
| ZZK(28,19) | eddy diffusion coefficient $K_{zz}$ |
| PL(28,19) | production rate |
| ZYK(28,19) | eddy diffusion coefficient $K_{zy}$ |
| AM(28,19) | density of air $(m^{-3})$ |
| PO2(28,19) | density of $O_2$ $(m^{-3})$ |
| PO3(28,19) | density of $O_3$ $(m^{-3})$ |
| TEMP(28,19) | temperature ( $^\circ$K) |
| AL(28,19) | 1/H (scale height) |
| XJ2(28,19) | J coefficient for $O_2$ |
| XJ3(28,19) | J coefficient for $O_3$ |
| UU(28,19) | zonal wind velocity (m/sec) |
| F(28,19) | coefficient in eq. (1-8) |
| Q(28,19) | Q in (H & P) (dimensionless) |
| H(28,19) | H in (H & P) (dimensionless) |
| UH(28,19) | average of $\theta$ between t and t + $\Delta$t |
| TH(28,19) | average of $\tau$ between t and t + $\Delta$t |
| O(28,19) | mixing ratio of $O_3$ |
| PF(28,19) | eddy diffusion term for $O_3$ |
| CO(19) | cosine (latitude) |
| EX(28) | exp $(-\xi^*)$ |
| KKP1 | KK + 1 |
| JJP1 | JJ + 1 |

| | |
|---|---|
| DZ2 | 2 * DZ |
| DY2 | 2 * DZ |
| DY2 | 2 * DY |
| DZ | mesh size for latitude |
| DY | mesh size for altitude |
| KK | total intervals in latitude |
| JJ | total intervals in altitude |
| AR | $\Omega$ * radius of earth |

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## List of Symbols

| | |
|---|---|
| $\tau$ | absolute angular momentum |
| | $= (\Omega a \cos\phi + u)\, a \cos\phi$ |
| $\theta$ | potential temperature |
| H | $= -\frac{\partial}{\partial\phi}(\overline{V'\tau'}) - \frac{\partial}{\partial\xi}(\overline{W'\tau'})$ |
| Q | $= -\frac{\partial}{\partial\phi}(\overline{V'\theta'}) - \frac{\partial}{\partial\xi}(\overline{W'\theta'})$ |
| J | Photodissociation rate |
| V | $v\cos\phi \exp(-\xi)$ |
| W | $w\cos\phi \exp(-\xi)$ |
| a | radius of the earth |
| p | pressure |
| q | rate of change of potential temperature |
| t | time |
| R | gas contstant for dry air |
| u | eastward veolcity component |
| v | northward velocity component |
| w | vertical velocity |
| $\Omega$ | rate of rotation of earth |
| $\phi$ | latitude |
| $\psi$ | stream function for mean meridional circulation |