# Use of Constrained Optimization in the Conceptual Design of a Medium-Range Subsonic Transport

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NASA

NASA Technical Paper 1762

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## SUMMARY

Constrained parameter optimization was used to perform the optimal conceptual design of a medium-range transport configuration. The impact of choosing a given performance index was studied, and the required income for a 15-percent return-on-investment was proposed as a figure-of-merit. A number of design constants and constraint functions were systematically varied to document the sensitivities of the optimal design to a variety of economic and technological assumptions. A comparison was made for each of the parameter variations between the baseline configuration and the optimally redesigned configuration.

## INTRODUCTION

As new technologies are developed for subsonic transport aircraft, they are sometimes evaluated through rudimentary trade studies upon frozen configurations. The proposed improvements in technology are sometimes added with only minor alterations based upon the airplane designer's intuition. Performance gains may be obtained without redesigning the airplane to take advantage of the new technologies. However, it is obvious that such an approach is suboptimal and may either exaggerate improvements if operational constraints are not properly accounted for or may not yield the maximum potential of the proposed technology.

Generally, in studies in which geometric change is allowed to enhance the airplane performance, it too is done in a suboptimal fashion. The geometry is modified to improve subgoals based upon engineering judgment. Some of these modifications might even adversely affect the economic goals of the airline industry which uses the product. Supercritical airfoil technology, for example, could be used to increase cruise Mach number, decrease drag, or decrease structural weight, depending upon how the designer chooses to reconfigure the transport design. These design changes, which are used to take advantage of supercritical aerodynamics, could be beneficial with respect to some measures of performance but are harmful with respect to others.

In addition to needing a systematic approach for evaluating the adoption of new technologies or the impact of economic factors upon airplane design, an automated procedure is required to obtain reasonable turnaround time coupled with improved accuracy. The approach used in this report is that of constrained parameter optimization. A performance index is minimized in the presence of operational, performance, regulatory, and flying-qualities constraints. This procedure optimizes the aircraft configuration for a given set of independent design parameters, provided the aircraft operation has been properly modeled and is consistent with the level of accuracy desired in preliminary design.

Direct optimization techniques have been used for airplane design with varying degrees of success in previous studies (for example, refs. 1 to 3). However, many of these uses suffered from some of the following: inappropriate choice of performance indices; inadequate set of independent design variables; inaccurate model of the airplane and its environment; and exclusion of important operational constraints. Furthermore, despite the growing computational capabilities in industry, there has been reluctance to adopt and expand such direct optimization procedures as a result of numerical convergence problems. If the airplane design problem is properly posed, numerical optimization techniques could potentially be an efficient tool for performing conceptual design studies, for example, evaluation of the application of new technologies.

A number of new technologies to augment the performance of transport airplanes have been proposed and are being developed for potential use by industry. In order to assess the potential benefits of utilizing these new technologies, a computer program was developed as a preliminary design tool for transports (ref. 4). The computer program has been used to perform a sensitivity analysis of relaxed-static-stability augmentation systems, and to perform an analysis of the impact of choosing unaugmented longitudinal-flying-qualities design criteria upon the performance and configuration of a medium-range transport (refs. 5 and 6). The results from using this computer program, Optimal Preliminary Design of Transports (OPDOT), to study the impact of selecting performance indices and imposing constraints upon the design of a transport are reported in this paper.

The choice of performance index, the parameter about which the configuration is optimized, has a significant bearing upon the effectiveness of the aircraft to accomplish its mission. Typically, however, the aircraft is much more sensitive to the constraints that are imposed upon the performance, configuration, and operation. It is important to ascertain these sensitivities because many of the mission-related constraints and constants have traditionally been chosen by the designer or the airline in a heuristic fashion. For instance, stage length and passenger capacity are examples of marketing decisions that the manufacturer or the airlines must make. Valuable information about the relative trade-offs of these decisions could be obtained in an efficient way from constrained numerical optimization. In the present study, a computer program was used to perform such a study for a medium-range transport with a fuel-efficient mission profile similar to the one to which the next generation of jet transports is expected to adhere.

## SYMBOLS AND ABBREVIATIONS

$AR_t$      horizontal-tail aspect ratio, $(\text{Tail span})^2/S_t$

$AR_w$      wing-trapezoidal aspect ratio, $(\text{Wing span})^2/S_w$

$a$      speed of sound, km/hr

$B$      Breguet range factor, km, $B = \dfrac{M(L/D)a}{\text{TSFC}}$

| | |
|---|---|
| $C_{A\$}$ | aircraft purchase price, 1980 U.S. dollars |
| $C_D$ | drag coefficient, $D/qS$ |
| $C_L$ | lift coefficient, $L/qS$ |
| $C_{m,o}$ | pitching moment coefficient at zero lift |
| $C_{M\$}$ | aircraft maintenance cost per block hour, 1980 U.S. dollars |
| c.g. | center of gravity |
| D | drag, N |
| $d_f$ | fuselage diameter, m |
| DOC | direct operating cost per block hour, 1980 U.S. dollars |
| DOC' | direct operating cost per flight, 1980 U.S. dollars |
| $F_\$$ | fuel price, 1980 dollars/liter (dollars/gallon) |
| g | acceleration due to gravity, 9.8 m/sec$^2$ |
| $h_{cr}$ | cruise altitude, m |
| I | income per block hour, 1980 U.S. dollars |
| $I_{req}$ | income required per flight for an annual return-on-investment of 15 percent, 1980 U.S. dollars |
| $I'_{req}$ | income required per kilometer for annual return-on-investment of 15 percent, 1980 U.S. dollars |
| $I''_{req}$ | income required per seat-flight for annual return-on-investment of 15 percent, 1980 U.S. dollars |
| IOC | indirect operating cost per block hour, 1980 U.S. dollars |
| J | unaugmented performance index |
| k | conversion factor from annual income to per flight income, k = 1/Number of flights per year |
| L | lift, N |
| $\ell_f$ | fuselage length, m |
| L/D | airplane aerodynamic efficiency, $C_L/C_D$ |
| LFL | landing field length, m |

| M | cruise Mach number |
|---|---|
| MAC | mean aerodynamic chord, m |
| MLA | maneuver load alleviation |
| n | seating capacity |
| $\Delta NG$ | margin in center-of-gravity between aft center-of-gravity limit and landing gear position to insure sufficient nose gear steering, percent MAC |
| OPDOT | computer program, Optimal Preliminary Design of Transports |
| p | parameter held constant during design optimization |
| q | free-stream dynamic pressure, $N/m^2$ |
| R | design range, km |
| ROI | annual return-on-investment, percent |
| S | lifting surface area, $m^2$ |
| $S_t$ | horizontal-tail area, $m^2$ |
| $S_w$ | wing area, $m^2$ |
| SM | static margin, percent MAC |
| T | installed thrust, N |
| t/c | wing thickness ratio, Maximum wing-section thickness/Wing-section chord |
| TOFL | balanced take-off field length, m |
| TSFC | thrust specific fuel consumption, Fuel weight per unit time/Thrust force |
| tx | annual tax rate |
| U | annual utilization, block hours per year |
| VOL | volume for passenger seating, $m^3$ |
| $\Delta Wt$ | weight overrun in a given component, N |
| $Wt_{to}$ | gross take-off weight for design mission, N |
| $X_{cg}$ | aftmost center-of-gravity position, percent MAC |
| $X_{lg}$ | landing gear position, percent MAC |

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$\eta_F$         fuel efficiency, seat-kilometers per liter

$\Lambda$         wing sweep, deg

$\lambda$         wing taper ratio

Subscripts:

a         approach

av        available

cg        center-of-gravity

cr        cruise

f         fuselage

F         fuel

lg        landing gear

max       maximum

req       required

t         tail

to        take-off

w         wing

A bar over a symbol denotes a normalized value. A superscript asterisk denotes the optimum value of a quantity.


METHOD OF CALCULATION

The general optimization scheme for OPDOT (ref. 4) is shown in figure 1. Nominal values for a set of independent design variables are used as input along with the required design constants for specifying nonvarying geometries, mission economic factors, mission profile data, and the nonlinear aerodynamic terms. The 12 independent design variables chosen for this study are shown in table I along with the allowable ranges which act as side constraints that are applied directly to the design state. The major wing planform parameters - wing area, wing aspect ratio, wing taper ratio, wing thickness ratio, Mach number, and wing sweep angle - were chosen to be degrees of freedom and were expected to have the most impact upon the design. Tail sizing was accomplished by including tail area, tail aspect ratio, and center-of-gravity position as independent design variables, which were expected to have only a small impact upon the transport

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sizing. Lastly, fuselage length, fuselage diameter, and installed thrust were kept as independent design variables to match the airplane size to the mission and wing planform.

The set of independent design variables are incremented by the optimizer logic in an attempt to improve the design. The performance index (parameter to be optimized) is determined from the independent design variables and information from the data base. This index is selected from a list of possible performance indices. The performance indices which were considered are listed in table I.

The constraint functions, involving inequality relationships, represent operational, flying-qualities, and performance constraints and are based upon certification regulations, mission definition, or common sense. Constraints are integrated into the optimization process by adding a penalty to the performance index for each constraint violation. Each penalty term is proportional to the square of the violation times a weighting factor. The performance index plus these penalty terms form what is called an augmented performance function. If the weighting factor is sufficiently large, minimizing the augmented performance function is equivalent to finding the minimum performance index while satisfying all the constraints. When it is desirable to maximize the performance index $\eta_F$, ROI, or $(L/D)_{max}$, the optimization problem is converted to a minimization problem by changing the sign. Constraints that are nearly violated at a solution point are identified as active.

The numerical optimization logic which iterates the independent design variables to minimize the augmented function is a subject of intense research in nearly all fields of engineering. Surveys including a variety of gradient methods that may be applicable to airplane design are included in references 1 and 7. A description of a feasible direction/search method coupled with a gradient method for the final stage is contained in reference 8. The previous studies and the author's experience indicate that these methods suffered from numerical difficulties when analytical equations were not available to provide the gradients and also from initialization problems when the number of active constraints was large with respect to the number of independent design variables. When aircraft design is posed as a numerical optimization problem, it is common to lack analytical gradients and an initially feasible solution.

A direct sequential search simplex algorithm, which is explained and illustrated in references 9 and 10, was utilized to overcome these difficulties. It is extremely reliable and robust in terms of convergence, albeit it suffers from slow convergence in regions of the independent design variables with low gradients of the performance index. It can be argued that computer resources are much cheaper than the manpower required to supervise other more efficient algorithms which need frequent adjustments to insure proper convergence. Therefore, the robustness and reliability of the simplex algorithm make it a highly desirable one to use.

During the iteration, the optimizer routine which contains the sequential simplex algorithm sends the values of the independent design variables and the design constants to the performance function evaluation routines. A schematic

representation of the calling sequence for the performance index evaluation routines is shown in figure 2.

Airplane weight was estimated by simulating the design mission and repeating it until the hypothesized gross take-off weight at the beginning of a weight iteration was within ±0.22 N of the sum of the individual airplane component weights, the payload, and the fuel weight. Industry statistics for the airplane component weights came from references 11 to 14 and were functions of all the independent design variables, the gross take-off weight, and about 20 of the design constants input through the data base. The fuel weight was calculated by summing estimates of the following mission segments: (1) taxi; (2) take-off and climb; (3) cruise; (4) descent; (5) taxi; and (6) reserve.

The mission profile as modeled is shown in figure 3. It consists primarily of a multiple-step cruise/climb approximation to an optimal fuel profile. The cruise portion is broken into 10 equally spaced segments, and Breguet-type relationships are used for calculating the amount of fuel burned during each segment (ref. 11). Comparisons with optimum, continuous-flight profiles (ref. 15) show differences of less than 5 percent.

Parasite drag was calculated from a component buildup including compressibility and Reynolds number effects using references 11 and 14 to 17. Calculations of stability and control derivatives were typical of those used in preliminary design (refs. 18 and 19) and included empirical adjustments from aerodynamic wind-tunnel and flight data (refs. 20 to 24) for compressibility, elasticity, and the use of supercritical airfoil sections. Induced drag was estimated using nonlinear corrections to parabolic drag polars for airfoil-section camber (ref. 25) and by adding terms for the tail induced drag and wing-tail interference drag (ref. 26). An iterative, nonlinear trim routine was used for determining the wing and tail loads in both cruise and approach phases of flight.

The cost data were approximated from industry statistics for manufacturing, maintenance, and the other components of direct operating costs as well as the indirect operating costs (refs. 11, 13, and 27 to 30). The direct operating cost is an augmented form of the industry standard, and it includes the following: maintenance, depreciation, delay, crew, flight attendant, control, support, spares, insurance, landing fee, and fuel. Indirect operating cost is composed of the following elements: maintenance burden, food, movies, passenger insurance, miscellaneous passenger expenses, advertising, sales commissions, reservations, passenger handling, baggage handling, cargo handling, and servicing. A simple return-on-investment is calculated in the following manner:

$$ROI = \left[ \frac{I - DOC - IOC}{0.9C_{A\$}} (1 - tx)U \right] \times 100 \qquad (1)$$

Hourly income, I, minus direct and indirect operating costs, DOC and IOC, is the profit per hour. Determining the after-taxes profit using tx as the

tax rate and multiplying by the annual utilization, $U$, and then dividing by the airplane purchase price minus the 10-percent investment tax credit, $0.9C_{A\$}$, yield the annual return-on-investment.

Additionally, another economic performance index considered in the present study is the income required per flight for a 15-percent return-on-investment $I_{req}$. It basically involves solving for $I$ in equation (1) and converting to a per flight basis as follows:

$$I_{req} = \left[ \frac{0.9C_{A\$}(ROI)(0.01)}{(1 - tx)U} + DOC + IOC \right] k \qquad (2)$$

where $k$ is a conversion factor from annual income to per flight income.

To provide a basis for performing the trade studies, a baseline mission was chosen. Table II lists the design constants chosen for the baseline mission that were used along with the indicated ranges of independent design variables and constraint functions which are listed in table I.

RESULTS AND DISCUSSION

Program Validation

The key feature of this method of airplane conceptual design is that an optimizer is used in conjunction with an analysis tool. This analysis tool accepts as input a set of independent design variables, usually airplane geometries or key mission variables, and some design constants, and then the tool returns as output a performance index in addition to a set of violated constraints. The optimizer will iterate the independent design variables until a convergence criterion is satisfied. Hence, the accuracy of a design study performed using this method is primarily a function of the accuracy of the analysis code. It is expected that sections of computer code for evaluation of the performance index and constraint function can always be enhanced to improve the model of the relationships between the design variables and the airplane operating environment. Although this report is meant to be primarily an illustration of an approach for implementing constrained optimization into airplane design and a demonstration of the potential flexibility that it affords, it may be useful to consider the accuracy of the model used in the analysis section of the computer program used herein.

The prime advantages OPDOT has over other optimizations for airplane design are that it (1) includes airplane geometric parameters (e.g., wing and tail planform) as independent design variables; (2) has a moderately extensive set of industry statistics for weight and economics; (3) contains a fairly complete representation of the drag aerodynamics and engine performance; (4) generates stability and control derivatives for flying-quality analyses; (5) includes a model for the interference effects between the wing and tail; (6) iterates non-linear force and moment equations to satisfy longitudinal trim requirements;

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and (7) contains a set of equations of motion for both performance and flying quality analysis. The key limitation in precision is probably the industry statistical relationships in (2) above which are expected to be accurate within about 10 percent. However, since the primary use of this design tool is the comparison of configurations resulting from varying the mission definitions, design constraints, or levels of technology, relative trade-offs and trends are more important than the absolute precision of predicting the performance index.

Mission and design variables for a popular medium-range trijet designed in the early 1960's were input to the analysis section of the computer code without the optimizer attached as a means of partially verifying the program accuracy. Although some levels of technology (e.g., supercritical aerodynamics) and the fuel-efficient mission profile implied in OPDOT differed significantly from the conventional trijet, good agreement was obtained for some of the key parameters that were computed. Each of the component weights was within 18 percent of the actual. The overall weight was within 12 percent, the fuel burn was within 9 percent, and ratios of fuel weight and payload to gross take-off weight were within 5 percent of actual. Additionally, a comparison of the direct operating cost components with one airline's estimates showed that OPDOT was within 4 to 18 percent of the actual values. These numbers are considered to be good for conceptual transport design.

In an effort to validate the overall optimization as being a useful model of the preliminary design process, the mission and design constraints from the baseline configuration of a study of the application of active controls to a modern transport (ref. 31) were input into OPDOT. After performing the optimization, the independent design variables agreed within 5 to 15 percent of the predictions from the preliminary design study reported in reference 31. Individual weight components were within 12 percent, the cost and fuel components were within 10 percent, and the ratios of fuel to gross take-off weight and payload to gross take-off weight were within 5 percent of the estimates from reference 31. Albeit certain areas of improvement can be identified for the models contained within OPDOT, the preceding comparisons tend to indicate that enough of the aspects of the preliminary design problem have been modeled to yield valuable insights during trade studies at the conceptual design level.

## Performance Index Sensitivity

Choosing an index for the optimization has received some attention in past preliminary design studies, for example, references 1 and 32 to 36. It is a complex issue since the function should be real and single-valued in order for the optimization process to be applicable. Trying to develop a criterion artificially that encompasses many objectives through a weighted algebraic summation adds to the complexity, and the arbitrary nature of choosing weights detracts from the general applicability of the results. Classically, weight and/or drag were minimized during design trade studies. More recently, in systems studies, direct operating cost and fuel usage have become the most important criteria. It is obvious that from the standpoint of product marketing, if a reliable economic factor could be estimated in a reasonably unambig-

uous sense with regard to actual airplane operation, it would be preferable to an intermediate airplane performance result.

Return-on-investment ROI has been generally regarded as the richest of the available economic variables (ref. 32). Direct operating cost suffers from an ambiguity in that the methods of calculation adhere to no universally accepted standard at the present time. As previously discussed, an augmented version of a standard industry model is used in the present paper. However, there remains the issue of which method represents the proper breakdown of direct operating costs and indirect operating costs. Because of the complete accounting of all costs, ROI avoids this issue and is, therefore, thought to be a more desirable index for the purposes of the present paper. Unfortunately, some problems remain unresolved even when ROI is used as a performance index.

A fundamental problem in using annual return-on-investment is trying to determine the income term for equation (1). It requires predicting the impact of price and traffic growth upon supply and demand. The assumption that trying to maximize airline ROI is equivalent to optimizing the transport manufacturer's profitability is typically used and is relied upon in this analysis. Even so, major complaints about using airline ROI are that it requires modeling income, which is different for each city-pair and each airline; and, it requires airline income statistics as a function of the important design parameters which are not readily available to the designer. Since a major portion of ROI is the income generated by the transport airplane, the simple formulas used for estimating this in the past tended to negate the accuracy of the rest of the analysis. It has been shown that relatively minor modifications to the assumptions used for developing the income result in significantly different implications during trade studies (ref. 34).

In this paper a performance index which has not previously been considered in optimizations is proposed as a means of alleviating some of the adverse effects of assuming an income model. Although this index is strictly cost-derived, it should still give to the designer an economically rich, single-valued parameter which can be used to market the probable economic success of a given configuration to the airlines. The income required per flight for a 15-percent annual return-on-investment $I_{req}$ is chosen as this figure-of-merit. It is comprised entirely of the cost elements of ROI and is equivalent to the per flight income required to exceed the predicted costs by enough margin to realize the 15 percent ROI.

Specifically, using $I_{req}$ as the performance index eliminates the need to assume a passenger load factor and an average income per seat-kilometer for the mission range, as is required for computing ROI. Additionally, using $I_{req}$ negates the necessity of assuming breakdown of income sources from the payload, i.e., first class, economy class, and cargo. Its prime advantage then is that it eliminates the designer's need to develop a comprehensive income model for each airline and yet has the same design sensitivities that would be required to optimize the airline's profitability. Since the value of $I_{req}$ can be returned for a number of design and economic ranges, it can be easily adapted to specific city-pairs, multiple stage lengths, or average missions.

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The results of performing the optimization using the baseline mission and constraints for each of the performance indices currently available in OPDOT are given in table III. A chart with the engineering units of table III converted to percent degradation with respect to the optimum value obtained for each index is shown as table IV. Optimizing to maximum aerodynamic efficiency $(L/D)_{max}$ resulted in an extremely heavy and impractical configuration when compared with optimizing to other performance indices. Optimizing to $(L/D)_{max}$ is inferior in terms of economic considerations.

Since the primary variable in the cost relations is operating weight, optimizing to a minimum purchase price and a minimum take-off weight induced nearly identical configurations with relatively low wing aspect ratios and wing sweep angles. Given the small differences in tables III and IV between optimizing to direct operating cost per block hour, direct operating cost per flight, return-on-investment, and the income required per flight for a 15-percent annual return-on-investment, it could be inferred that $I_{req}$ would be a good compromise because it is as good as the other indices while at the same time, it solves some of the previously mentioned problems. Additionally, convergence of the computer program was as good or better for $I_{req}$ than for the other performance indices, indicating that its use enhances robustness.

Although trends similar to those observed in reference 33 are shown, there exist significant differences. For example, there were unusual results in that paper for the cases in which the design was optimized to noneconomically derived performance indices. In those cases, it is probable that the regression analysis which was utilized broke down or was good only in a neighborhood of the nominal design point. Typically, when approximations to constrained optimization are used for airplane design, constraints are no longer on the boundaries and may be violated. In fact, many of the nonlinear interactions are not modeled when perturbations from the nominal become large. This would explain the large departure from the expected results exhibited by the configurations of reference 11 which were optimized for fuel weight and take-off weight. Such problems illustrate the advantage of using direct optimization to find a more accurate representation of the compromises and trade-offs required during optimal aircraft design.

Essentially all optimizations in table III, with the exception of the $(L/D)^*$ case, have the same set of active constraints. The following constraint functions were active at the solutions: sufficient thrust in cruise, sufficient friction for nose wheel steering, sufficient cabin volume for passengers, static margin of 5 percent or more in approach at aft center-of-gravity limit, and adequate elevator power to unstick the nose gear prior to lift-off. Additionally, the following side constraints upon the independent design variables were active: Mach number greater than or equal to 0.8, and wing thickness ratio less than or equal to 0.14. The only exception was the case for maximum L/D, which opted for the thinnest wing possible (t/c = 0.09) and went to the smallest diameter fuselage coupled with the longest length fuselage possible. It also converged upon the maximum allowable trapezoidal-wing aspect ratio, 14. These unreasonable values for the independent design variables illustrate the impracticality of using this intermediate performance variable as a figure-of-merit (performance index) for constrained optimization.

A set of design cases were also considered with the lower Mach number constraint reduced to 0.6. All but three design cases immediately converged to this Mach number. Direct operating cost per flight converged to a Mach number of 0.65, the speed at which wing compressibility effects became significant with the minimum allowable sweep angle ($10^O$). Return-on-investment and the income required per flight for a 15-percent annual return-on-investment tend to converge at a Mach number of nearly 0.7, which corresponds to the maximum speed without compressibility effects for a low sweep angle of $15^O$. Apparently, some compromise in structural weight was warranted to enhance productivity based upon the economic assumptions of the performance function.

This experience indicates that the proposed figure-of-merit $I_{req}$, the income per flight required to generate a 15-percent ROI, has the desirable properties for use in constrained optimization. The index $I_{req}$ was robust in terms of convergence; it was sensitive to the key independent design variables; and it appeared to be an excellent economic compromise between the other economic indices. Possibly, the 15-percent standard for the ROI is open to debate as being inappropriate, especially in today's economic environment. However, at the conceptual design level, the absolute magnitude of a particular run is not of prime importance. The significant factors are the relative comparisons during trade studies. A limited number of cases using $I_{req}$ for the performance index were run with different levels of required ROI, and approximately the same values for the optimum independent design variables were obtained. For this reason and the ones mentioned previously, $I_{req}$ is assumed to be a viable and useful parameter for a performance index, no matter what level of ROI is required.

## Sensitivity to Parameter Variations

As a means of illustrating the ease with which a trade study can be performed with direct optimization and to gain some insight into the impact of choosing constraints, defining missions, and deciding upon technology levels, a parameter sensitivity study was performed for the baseline mission. Table V is a compilation of rates of change of performance index from a series of optimizations using variations in several design constants and constraint functions. Approximately 5- and 10-percent variations in a parameter or constraint were made; and rates of change were calculated, numerically normalized, and averaged for direct operating cost per block hour and income per flight required for a 15-percent ROI. The sensitivity ratios were calculated by the following equation:

$$\frac{\bar{J}}{\bar{p}} = \frac{\dfrac{J^*(p + \Delta p) - J^*(p)}{J^*(p)}}{\Delta p/p} \tag{3}$$

where $J(p)$ and $J(p + \Delta p)$ are values of a performance index $J$ at $p$ and $p + \Delta p$, respectively; $p$ is the value of a varied design constant or constraint; $\Delta p$ is the change in that parameter; and the asterisk denotes an optimum value.

The numbers in table V were generated from the optimizations with the full set of 12 independent design variables previously defined, with a weak convergence criterion, and with only one restart. Restarts are auxiliary calls to the optimizer with a new initial guess for the independent design variables and are necessary to insure that a global minimum has been found. Hence, the results are thought to be only representative of the order of magnitude of relative sensitivities. In using this table, the reader should be wary that the units for DOC are dollars per block hour, which is a popular way of reporting these results, while $I_{req}$ has units of dollars per flight, which is proposed as a more realistic way of presenting the data. In terms of percent of parameter change, the sensitivities are greatest for percent variations in Mach number, weight increase, load alleviation, and range. In contrast, the design is fairly insensitive to percent changes in maximum lift coefficient, static margin, and maintenance costs. It should be noted, however, that the ease of realizing a 1-percent change in a parameter varies considerably with the physical nature of the parameter. As expected, since fuel is such a large component of DOC and $I_{req}$, parameter changes which have the largest influence on fuel usage have the biggest impact upon the outcome.

Indications from table V are that the designs from optimizing direct operating cost per block hour were more sensitive to Mach number variations and fuel price changes than were the designs from optimizing the income required per flight for a 15-percent ROI. This is because fuel cost makes up a greater proportion of DOC, and the fixed utilization assumption, which is a large part of $I_{req}$, generally requires a higher optimum Mach number. Otherwise, the two performance indices have reasonably close sensitivities in terms of the trends to the set of parameters shown.

Accepting $I_{req}$ as a reliable and robust figure-of-merit, a more thorough analysis of the sensitivity of the optimum $I_{req}$ design to parameter and constraint variations was then performed. To facilitate interactive computing, the independent design variables were reduced from 12 to 9. The independent design variables that tended to be against side constraint boundaries were input as design constants, which is equivalent to assuming equality constraints. Specifically, Mach number (0.8), wing taper ratio (0.38), and wing thickness ratio (0.14) were input as constant design parameters for the remaining studies.

Parameter or constraint variations were performed in four categories: mission definitions, economic assumptions, production predictions, and technology improvements. Each factor was varied through a range thought to be reasonable in terms of yielding information for the transport designer or consumer. Since some nonlinear variations existed, these were then plotted showing percent savings or percent increase in $I_{req}$ as a function of the parameter variation. Where applicable, a curve is also shown depicting the $I_{req}$ variation with the independent design variables fixed at the nominal values for the optimum baseline configuration. The difference between the two curves illustrates the contribution to the $I_{req}$ variation due to optimally configuring the airplane for each parameter variation. Comparing these curves shows the benefits from optimally resizing the aircraft as well as the relative economic sensitivity to uncertainties in predicting parameter values. Additionally, table VI shows the

numerical sensitivities of the optimal design variables, the optimally resized $I_{req}$ and the baseline $I_{req}$ to the parameter variations.

Mission definition.- The landing field length, Mach number constraints, design range, and seating capacity were varied. The sensitivity of $I_{req}$ to the respective parameter variations is shown in figures 4 to 7. Reducing the field length resulted in a rapidly increasing $I_{req}$, requiring significant configuration changes to optimally satisfy the constraints. The following independent variables needed increases of 10 percent or more for a 300-meter decrease in field length: $S_w$, $\ell_f$, $S_t$, and $X_{cg}$. On the other hand, $AR_w$, $AR_t$, and $\Lambda$ required substantial decreases in magnitude.

The importance of resizing while choosing Mach number is shown in figure 5. Although the performance is highly sensitive to Mach number at cruise, selection of Mach number is one of the items needed for the marketing oriented decision determining the desirability of decreasing the block time, among other factors. An increase of 0.1 in the nominal Mach number saved 1 percent in block time but cost 0.8 percent in $I_{req}$ if the aircraft was optimally resized and 1.4 percent in $I_{req}$ for the baseline configuration. As expected for the fixed wing thickness ratio, fairly large changes in wing sweep and aspect ratio were obtained when optimally resized.

Choosing design range and seating capacity is also an important compromise decision between performance and economy, as illustrated in figures 6 and 7. The improvement in $I_{req}$ from a decrease in range is due principally to the fixed utilization assumption and to the decrease in the amount of fuel used. The income required per kilometer for a 15-percent ROI ($I'_{req}$) shows nearly 2 percent savings (when optimized to $I_{req}$) with a 15-percent decrease in range. Range changes required only moderate changes in aircraft size. In contrast, decreasing the seating capacity decreased $I_{req}$ but increased $I''_{req}$, the income required per seat to generate a 15-percent ROI. As anticipated, fairly significant size changes were obtained as the aircraft was reconfigured for different seating capacities. The baseline configuration results are not extended beyond the nominal baseline values in figures 6 and 7 because constraint violations indicate that the designs are infeasible.

Economic assumptions.- As implied previously, the economic assumptions made during vehicle design affect the optimization results. The impact of fuel price and annual utilization are two such assumptions. The augmentation in $I_{req}$ due to fuel price increases is fairly severe, since at \$0.23 per liter (\$1.00 per gallon), fuel is already a significant portion of the overall operating costs. It should be noticed (fig. 8) that optimally resizing the aircraft saves 1 percent in $I_{req}$ compared with the baseline configuration at a 50-percent fuel price increase, indicating a sizable benefit (approximately 3 percent in fuel) from properly predicting the fuel price during preliminary design studies and making the fairly small changes to optimize aircraft configuration.

The impact of varying the annual utilization $U$ (fig. 9) does not result in much variation in the configuration if the optimizer is allowed to work upon the nominal with the changes in $U$. In other words, although $U$ is a major component of equation (2) and does have a very significant impact upon the mag-

nitude of the optimum $I_{req}$, its choice for the operation model does not significantly impact the preliminary design process. Since this is an obvious result, it might be considered an indication that the optimization in OPDOT is converging to consistent results.

Production predictions.- The designer must estimate the purchase price of the airplane, its gross take-off weight, and the cost of maintaining the airframe, engines, and systems. The need to accurately predict the initial purchase price and the per hour maintenance cost is shown in figure 10 to not alter the optimum design significantly, since the differences between the optimally resized configurations and baseline configurations are small. These two costs, however, seriously affect the absolute magnitude of the income required per flight for a 15-percent annual return-on-investment.

In contrast, figure 11 shows that weight changes have a large impact upon the economy and design of the baseline configuration. An increase of 8900 N (2000 lbf) in the weight of a component costs about 1.5 percent in $I_{req}$. If the airplane could be built with a decrease of 4450 N (1000 lbf) in the empty weight, it could be resized to save a little over 1 percent in $I_{req}$. When the aircraft was not resized to take advantage of weight decrease (principally through using wing area reductions), only half of the benefits in $I_{req}$ were obtained. Calculations of the performance of the fixed baseline configuration with assumed weight overruns did not satisfy the performance constraints and are not plotted in figure 11. An additional penalty would have to be added to reflect the increases in installed thrust and wing area that would be needed to achieve the design objectives. The large differences between the optimum and baseline configurations illustrate the obvious importance of correctly predicting the weight of the production airplanes during conceptual design. Perhaps this indicates the need for refining the weight estimation techniques available at this level of preliminary design.

Technology improvements.- As a means of assessing the potential benefits of technology improvements, a variety of parameters were varied on the nominal baseline configuration. The technology changes that were considered include variations of the following: wing drag, thrust specific fuel consumption, pitching moment coefficient, empty weight structure, load alleviation, maximum lift coefficient, and static margin.

The wing drag coefficient was varied to see the improvements that could be obtained if wing aerodynamic efficiency could be enhanced. It is shown in figure 12 that a 10-percent improvement in drag at zero lift yielded slightly over 1 percent savings in the income required per flight for a 15-percent ROI. When the aircraft was not reconfigured to take advantage of the wing drag improvement, only about 65 percent of the improvement was realized. The nonlinear variation in the curve is in favor of larger changes.

If engine technology improvements could be achieved, large reductions in $I_{req}$ could be realized (fig. 13). The benefits cascaded through the design process because as less fuel was required, the empty weight could be reduced along with the size. When the nominal baseline configuration was not resized to take advantage of the synergism from improving thrust specific fuel consumption, only half of the savings were obtainable.

OPDOT was programmed with the assumption that a supercritical airfoil section was utilized to improve the drag and/or structural characteristics. A characteristic of such sections, however, is a substantial pitching moment, which could have a significant impact upon the configuration. It is shown in figure 14 that if research is able to achieve a 50-percent reduction in pitching-moment coefficient at zero lift, $C_{m,o}$, nearly a 1-percent savings in $I_{req}$ is realizable. About 75 percent of these benefits came from being able to reconfigure the nominal baseline design, while about 25 percent of the improvements came from a reduction in the tail lift used to trim the large pitching moments.

If the structural efficiency could be enhanced, for example through new materials, very large gains could be obtained. The synergism possible from the use of new materials is depicted in figure 15 as a 1-percent savings in $I_{req}$ for every 1-percent reduction in empty weight. Only half that rate of gain was achievable when the nominal configuration was not resized. Also indicated in figure 15 is the anticipated synergism in weight reduction from improving the structural efficiency.

Two active controls concepts were considered next, load alleviation and relaxed static stability. Maneuver load alleviation is shown by figure 16 to be a concept with some large potential gains. Assuming there were no added weight or costs from utilizing the concept, a 0.2g incremental reduction in the design limit load resulted in a 2-percent savings in $I_{req}$. As a precautionary statement, it should be pointed out that fatigue loads and other dynamic modes (which probably become critical when structural material is removed to reduce the design ultimate maneuver loads) were not modeled. Hence, this analysis is probably overly optimistic. Even so, if the nominal baseline configuration was not resized and just the forecast weight savings benefits were included, about half the benefits were realized.

The nature of the baseline configuration prevented the optimizer from achieving less than a -6.8-percent static margin during static margin variations (fig. 17). Again assuming no weight or cost penalty, when the static margin was reduced from the nominal of 5 percent to the lowest achievable (-6.8 percent), 1.8 percent in $I_{req}$ was saved. A more thorough study of the benefits of designing transports with relaxed static stability using OPDOT is presented in references 5 and 6.

At the preliminary design stage, the aerodynamicist must choose the level of complexity for high-lift devices. Making the unrealistic assumption of no penalty for adding greater lifting capability (i.e., cost or weight), figure 18 shows the gains that were obtained from the optimizer. The sensitivity is relatively large and, as expected, resulted in adjustments to wing area with minor changes in wing sweep and wing aspect ratio required to satisfy the landing field length constraints.

Trade-studies.- Seeing the sensitivities about the nominal parameters for the baseline configuration, the designer can make some initial decisions about potential changes at the preliminary design level. For example, it might be proposed to add a more complex flap system at a cost of a 4450-N (1000-lbf) increase in weight and a 0.5-percent increase in purchase price to get a gain of 0.2 in $C_{L,max}$. If superposition of the curves can be assumed, then a

potential savings of 0.4 percent in $I_{req}$ could be achieved from summing the appropriate components of figures 10, 11, and 18.

Given the possibility that the proposed improvement in the airplane system could enhance the aircraft operational economy, a more complete analysis of the proposed changes could then be pursued. The data modeling the improvements should be inserted into the appropriate modules in an optimum design program like OPDOT. To yield the maximum synergism possible, each concept change needs to be implemented through optimally redesigning the configuration as a means of investigating the relative trade-offs. In fact, there are cases when significant improvements in some components yield little or even negative economic improvement in the aircraft design.

## CONCLUDING REMARKS

A constrained parameter optimization technique for the preliminary design of an optimal, medium-range transport has been performed. A result of this study is that income required per flight for a 15-percent return-on-investment $I_{req}$ was shown to be a robust, economically rich performance index for use as a figure-of-merit for numerical optimizations. This performance index had the advantage of being useful as an indication of profitability without requiring detailed income information or assumptions.

As a means of illustrating its use as a conceptual design tool, direct optimization was used to perform an interactive sensitivity study to parameter variations for a variety of design constants and constraint functions. The optimal design in terms of aircraft geometry was shown to be relatively insensitive to certain design assumptions and economic parameters although impacting the magnitude of optimized $I_{req}$. The insensitive parameters included the following: annual utilization, aircraft purchase price, and aircraft maintenance costs. In contrast, choosing landing field length, Mach number, design range, seating capacity, and fuel price were mission or economic choices that had significant impacts upon the optimal configurations as well as on the value of optimum $I_{req}$.

A series of design optimizations was made for a number of potential technology-based improvements. Sizable savings in $I_{req}$ were possible with moderate enhancements in structural efficiency, engine fuel consumption, and maneuver load alleviation. Modest gains were observed with reductions in wing drag coefficient, wing pitching moment, and static margin. In all these cases, the maximum benefits were realized only after the baseline configuration was optimally resized. It is thus concluded that the feasibility and future usefulness of constrained parameter optimization for aircraft design have been demonstrated.

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November 20, 1980

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TABLE I.- LIST OF DEFINING PARAMETERS FOR OPDOT

(a) Upper and lower limits of independent design variables

| Independent design variable | Lower limit | Upper limit |
|---|---|---|
| Wing area, $S_w$, m$^2$ . . . . . . . . . . . . . . . . . . . . . . . . | 93 | 372 |
| Wing aspect ratio, $AR_w$ . . . . . . . . . . . . . . . . . . . . . | 3 | 14 |
| Wing sweep angle, $\Lambda$, deg . . . . . . . . . . . . . . . . . . . | 10 | 45 |
| Wing thickness ratio, $t/c$ . . . . . . . . . . . . . . . . . . . | 0.10 | 0.14 |
| Wing taper ratio, $\lambda$ . . . . . . . . . . . . . . . . . . . . . | 0.25 | 0.50 |
| Horizontal-tail area, $S_t$, m$^2$ . . . . . . . . . . . . . . . . . | 9.29 | 150 |
| Horizontal-tail aspect ratio, $AR_t$ . . . . . . . . . . . . . | 2 | 12 |
| Aftmost center-of-gravity, $X_{cg}$, percent MAC . . . . . . . . | 0 | 100 |
| Installed thrust, T, kN . . . . . . . . . . . . . . . . . . . . | 178 | 667 |
| Fuselage length, $\ell_f$, m . . . . . . . . . . . . . . . . . . . . | 36.6 | 79.2 |
| Fuselage diameter, $d_f$, m . . . . . . . . . . . . . . . . . . . | 5.66 | 6.10 |
| Cruise Mach number, M . . . . . . . . . . . . . . . . . . . . . | 0.8 | 0.9 |

(b) Baseline function limits of available constraint functions

| Available constraint function | Baseline function limits |
|---|---|
| Thrust for cruise/climb . . . . . . . . . . . . . . . . . . . . . . | $\dfrac{T_{av}}{T_{req}} \geq 1$ |
| Second-segment climb gradient . . . . . . . . . . . . . . . . . . . | $\dfrac{T_{av}}{T_{req}} \geq 1$ |
| Missed-approach climb gradient . . . . . . . . . . . . . . . . . . . | $\dfrac{T_{av}}{T_{req}} \geq 1$ |
| Landing field length, m . . . . . . . . . . . . . . . . . . . . . . . | LFL $\leq$ 2130 |
| Take-off field length, m . . . . . . . . . . . . . . . . . . . . . . | TOFL $\leq$ 2430 |
| Nosewheel steering traction . . . . . . . . . . . . . . . . . . . . . | $X_{cg} \leq X_{1g} - \Delta NG$ |
| Passenger volume . . . . . . . . . . . . . . . . . . . . . . . . . . | $\dfrac{VOL_{req}}{VOL_{av}} \leq 1$ |
| Cruise altitude, m . . . . . . . . . . . . . . . . . . . . . . . . . | $h_{cr} \geq 9100$ |
| Cruise wing lift coefficient . . . . . . . . . . . . . . . . . . . . | $C_{L,w} \leq 0.7$ |
| Static margin (cruise and approach), percent MAC . . . . . . . . | SM $\geq$ 5 |
| Tail lift coefficient in approach . . . . . . . . . . . . . . . . . . | $C_{L,t} \geq -0.8$ |
| Nose gear unstick . . . . . . . . . . . . . . . . . . . . . . . . . . | $\dfrac{L_{t,av}}{L_{t,req}} \geq 1$ |

(c) Available performance indices

Direct operating cost per block hour, DOC
Direct operating cost per flight, DOC'
Return-on-investment, ROI
Fuel efficiency, $\eta_F$
Maximum glide ratio, $(L/D)_{max}$
Take-off gross weight, $Wt_{to}$
Airplane purchase price, $C_{A\$}$
Income required per flight for
   15 percent ROI, $I_{req}$

TABLE II.- KEY DESIGN CONSTANTS USED FOR DESIGN OPTIMIZATION

(a) Mission

| | |
|---|---|
| Design range, km | 5600 |
| Number of seats | 200 |
| Cargo, N | 33 400 |
| Maximum lift coefficient | 3.15 |
| Landing field requirement, m | 2130 |
| Take-off field requirement, m | 2430 |

(b) Geometry

| | |
|---|---|
| Wing incidence angle, deg | 2 |
| Wing geometric twist, deg | 5 |
| Tail thickness ratio | 0.10 |
| Tail sweep angle, deg | 30 |
| Tail taper ratio | 0.4 |
| Vertical-tail sweep, deg | 35 |
| Ratio of rudder area to vertical-tail area | 0.30 |
| Ratio of elevator chord to horizontal-tail chord | 0.25 |
| Ratio of flap span to wing span | 0.6 |
| Maximum flap deflection, deg | 45 |
| Height of aerodynamic center above c.g., percent MAC | 8 |
| Height of thrust vector above c.g., percent MAC | -12 |
| Height of horizontal tail above c.g. | 0 |
| Number of engines | 2 |

(c) Economics

| | |
|---|---|
| Fuel cost, $/L | 0.20 |
| Load factor | 0.55 |
| Passenger revenue, ¢/seat-km | 4.9 |
| Utilization,  U, hr/yr | 3200 |
| Depreciation period, yr | 14 |
| Residual value, percent | 12 |
| Tax rate,  tx | 0.48 |
| Year of study | 1980 |
| Assumed annual inflation rate | 0.07 |
| Number of prototype aircraft | 2 |
| Aircraft fleet size | 250 |
| Initial production rate, per mo | 0.5 |
| Full production rate, per mo | 5 |
| Engineering rate (1974), $/hr | 19.55 |
| Tooling rate (1974), $/hr | 14.00 |
| Labor rate (1974), $/hr | 10.90 |
| Engines for test aircraft | 3 |
| Ratio of manufacturer's airframe weight to take-off weight | 0.75 |

(d) Miscellaneous

| | |
|---|---|
| Maximum dynamic pressure, $N/m^2$ | 5.13 |
| Pressurized volume, $m^3$ | 178.2 |
| Number of pilots | 3 |
| Number of attendants | 8 |
| Air conditioning flow rate, kg/min | 200 |
| Autopilot channels (with multiplexers) | 5 |
| Generator capacity, kV-A | 750 |
| Maintenance complexity factor | 1.6 |
| Hydraulics volume flow rate, L/min | 300 |
| Number of inertial platform systems | 1 |
| Ratio of auxiliary-power-unit on-time to engine on-time | 0.1 |
| Ratio of first class to economy class seating | 0.15 |
| Maximum speed, m/sec | 248.5 |
| Airfoil design lift coefficient | 0.4 |
| Baseline engine | CF-6 |
| Elevator servo time constant, sec | 0.1 |
| Curved windshield | |
| Supercritical airfoil technology | |
| Some nonlinear aerodynamics terms | |

TABLE III.- OPTIMIZATION RESULTS FOR EACH PERFORMANCE INDEX

| Variables | Optimized performance index | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | DOC* | DOC'* | ROI* | $\eta_F$* | $(L/D)_{max}$ | $Wt^*_{to}$ | $C^*_{A\$}$ | $I^*_{req}$ |
| Independent design variables: | | | | | | | | |
| $S_w$, m$^2$ . . . . . . . . . . | 226 | 223 | 226 | 227 | 371 | 222 | 220 | 225 |
| $AR_w$ . . . . . . . . . . . . . | 10.6 | 10.0 | 10.3 | 10.5 | 14.0 | 8.84 | 9.12 | 10.1 |
| $\Lambda$, deg . . . . . . . . . . . | 22.6 | 21.6 | 20.8 | 22.6 | 31.8 | 20.6 | 19.1 | 22.3 |
| t/c . . . . . . . . . . . | 0.14 | 0.14 | 0.14 | 0.14 | 0.09 | 0.14 | 0.14 | 0.14 |
| $\lambda$ . . . . . . . . . . . . | 0.38 | 0.39 | 0.39 | 0.38 | 0.30 | 0.40 | 0.40 | 0.33 |
| $s_t$, m$^2$ . . . . . . . . . . | 71.0 | 78.7 | 70.1 | 69.6 | 73.2 | 86.0 | 99.2 | 73.2 |
| $AR_t$ . . . . . . . . . . . | 5.5 | 5.6 | 7.1 | 6.2 | 9.7 | 6.2 | 3.7 | 6.3 |
| $X_{cg}$, percent MAC . . . . . | 47 | 47 | 45 | 47 | 47 | 44 | 44 | 45 |
| T, kN . . . . . . . . . . | 344 | 345 | 346 | 344 | 497 | 370 | 360 | 348 |
| $\ell_f$, m . . . . . . . . . . | 54.5 | 51.3 | 52.7 | 53.9 | 79.2 | 50.0 | 47.2 | 52.8 |
| $d_f$, m . . . . . . . . . . | 4.97 | 5.18 | 5.09 | 5.00 | 4.27 | 5.27 | 5.49 | 5.09 |
| M . . . . . . . . . . . . | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| Performance indices: | | | | | | | | |
| DOC, $ . . . . . . . . . . | 2307 | 2309 | 2317 | 2313 | 3162 | 2345 | 2331 | 2316 |
| DOC', $ . . . . . . . . . | 16 220 | 16 190 | 16 250 | 16 240 | 22 270 | 16 410 | 16 330 | 16 240 |
| ROI, percent . . . . . . . | 8.4 | 8.6 | 8.65 | 8.3 | 0.4 | 8.3 | 8.4 | 8.5 |
| $\eta_F$, seat-km/L . . . . . . | 42.8 | 42.5 | 42.5 | 42.8 | 29.5 | 41.1 | 41.1 | 42.5 |
| $(L/D)_{max}$ . . . . . . . . | 20.0 | 19.3 | 19.8 | 20.0 | 24.5 | 18.2 | 19.6 | 18.1 |
| $Wt_{to}$, kN . . . . . . . . | 1270 | 1240 | 1260 | 1270 | 2120 | 1220 | 1260 | 1220 |
| $C_{A\$}$, millions of $ . . . . | 21.4 | 21.1 | 21.4 | 21.5 | 33.3 | 21.1 | 20.8 | 21.3 |
| $I_{req}$, $ . . . . . . . . . . | 35 080 | 35 050 | 35 070 | 35 120 | 38 210 | 35 040 | 35 040 | 35 020 |

TABLE IV.- PERCENT DEGRADATION OF A PERFORMANCE INDEX WITH RESPECT TO

OPTIMUM WHEN CONFIGURED TO ALTERNATE PERFORMANCE INDEX

| Optimized performance index | Percent degradation | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | DOC | DOC' | ROI | $\eta_F$ | $(L/D)_{max}$ | $Wt_{to}$ | $C_{A\$}$ | $I_{req}$ |
| DOC* . . . . . . . | 0 | 0.19 | 2.89 | 0.11 | 18.4 | 4.10 | 2.88 | 0.17 |
| DOC'* . . . . . . | 0.09 | 0 | .58 | 0.80 | 21.2 | 1.64 | 1.44 | 0.09 |
| ROI* . . . . . . . | 0.43 | 0.37 | 0 | 0.69 | 19.2 | 3.28 | 2.88 | 0.14 |
| $\eta_F^*$ . . . . . . . | 0.26 | 0.31 | 4.05 | 0 | 18.4 | 4.10 | 3.37 | 0.29 |
| $(L/D)_{max}^*$ . . . . | 37.1 | 37.5 | 95.38 | 31.2 | 0 | 73.77 | 60.10 | 9.11 |
| $Wt_{to}^*$ . . . . . . | 1.65 | 1.36 | 4.05 | 4.11 | 25.7 | 0 | 1.44 | 0.06 |
| $C_{A\$}^*$ . . . . . . | 1.04 | 0.86 | 2.89 | 4.11 | 20.0 | 3.28 | 0 | 0.06 |
| $I_{req}^*$ . . . . . . | 0.39 | 0.31 | 1.73 | 0.80 | 26.1 | 0 | 2.40 | 0 |

TABLE V.– SENSITIVITY OF OPTIMUM DOC AND OPTIMUM $I_{req}$

DESIGNS TO PARAMETER AND CONSTRAINT VARIATIONS

| Varied parameter, p | Symbol | $\dfrac{\overline{DOC}}{\bar{p}}$ | $\dfrac{\overline{I_{req}}}{\bar{p}}$ |
|---|---|---|---|
| **Mission:** | | | |
| Range | R | 0.37 | 1.06 |
| Landing field length | LFL | −.31 | −.33 |
| Mach number | M | .69 | .32 |
| | | | |
| **Economic:** | | | |
| Fuel price | $F_\$$ | .37 | .18 |
| Annual utilization | U | −.31 | −.47 |
| | | | |
| **Production:** | | | |
| Aircraft price | $C_{A\$}$ | .31 | .44 |
| Maintenance | $C_{M\$}$ | .028 | .058 |
| Weight overrun[a] | $\Delta Wt$ | .73 | .77 |
| | | | |
| **Technological:** | | | |
| Maximum lift coefficient | $C_{L,max}$ | −.019 | −.021 |
| Maneuver load alleviation | MLA | −.48 | −.46 |
| Static margin | SM | .0014 | .0011 |

[a]Increase in performance index per 4450-N weight overrun.

TABLE VI.- SENSITIVITY OF PERFORMANCE INDEX AND AIRPLANE GEOMETRY TO PARAMETER VARIATIONS

| Varied parameter, p | Geometry sensitivities | | | | | | | Performance sensitivities | |
|---|---|---|---|---|---|---|---|---|---|
| | $\dfrac{\bar{S_w}^*}{\bar{p}}$ | $\dfrac{\bar{AR_w}^*}{\bar{p}}$ | $\dfrac{\bar{\Lambda}^*}{\bar{p}}$ | $\dfrac{\bar{T}^*}{\bar{p}}$ | $\dfrac{\bar{S_t}^*}{\bar{p}}$ | $\dfrac{\bar{AR_t}^*}{\bar{p}}$ | $\dfrac{x_{cg}^*}{\bar{p}}$ | $\dfrac{\overline{I_{req}^*}}{\bar{p}}$ | $\dfrac{\overline{I_{req}^*}^a}{\bar{p}}$ |
| **Mission:** | | | | | | | | | |
| Landing field length | -1.05 | 0.51 | 0.32 | -0.30 | -0.61 | 1.16 | -0.07 | -0.21 | ----- |
| Mach number | .95 | -1.97 | 3.74 | 2.18 | 1.44 | -2.02 | .82 | .72 | 0.48 |
| Design range | .36 | -.08 | .24 | .45 | 1.47 | -1.69 | .38 | .96 | .84 |
| Seating capacity | .62 | .49 | -.02 | .46 | .10 | -1.12 | -.25 | .50 | .23 |
| **Economic:** | | | | | | | | | |
| Fuel price | -.03 | -.07 | .02 | .001 | .52 | -16 | .34 | .16 | .20 |
| Annual utilization | .20 | .10 | .41 | -.06 | .85 | -.33 | .11 | -.37 | -.39 |
| **Production:** | | | | | | | | | |
| Airplane price | -1.64 | .38 | 3.21 | 1.36 | 1.86 | 2.54 | .76 | .46 | .47 |
| Maintenance cost | -.12 | -.27 | .05 | .08 | .34 | .16 | .17 | .06 | .07 |
| Weight overrun[b] | 8.51 | 6.08 | 3.67 | 6.21 | 9.45 | -3.24 | -1.08 | 16.2 | 9.90 |
| **Technological:** | | | | | | | | | |
| Wing drag coefficient | -.06 | .66 | .18 | -.52 | -1.56 | .80 | -.30 | .11 | .08 |
| Engine efficiency | -.61 | .07 | -.25 | -.63 | -.97 | 1.21 | -.33 | -.38 | -.20 |
| Pitching moment | 1.70 | .96 | 2.8 | 1.35 | -.43 | .59 | -.09 | .02 | .003 |
| Structural efficiency | -1.92 | .50 | 1.57 | -1.96 | -2.1 | 1.88 | 1.22 | -1.44 | -.92 |
| Maximum lift coefficient | -1.21 | .90 | -.23 | -.69 | -10.1 | 5.9 | 1.68 | -.22 | ----- |
| Load alleviation | -.58 | .33 | -.14 | -.79 | -.30 | -.25 | -1.24 | -.36 | -.15 |
| Static margin[c] | .14 | .32 | .32 | .60 | -4.9 | 1.83 | -1.02 | .13 | ----- |

[a]Configuration same as baseline (not optimally resized).

[b]Sensitivity of unnormalized performance index or geometric variable to a 1-percent weight overrun.

[c]Unnormalized.
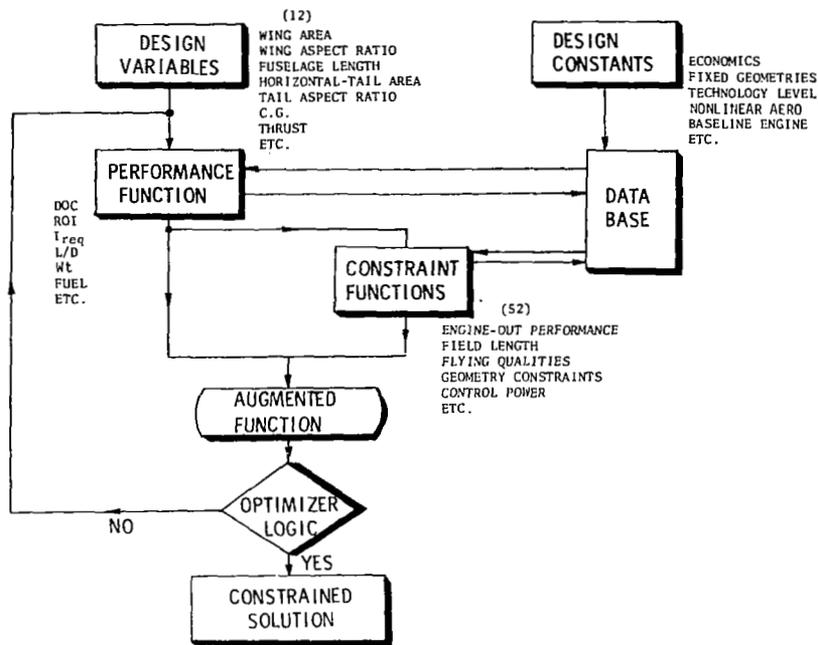
Figure 1.- Generalized flow diagram for OPDOT.



**FEATURES**

- INDUSTRY STATISTICS FOR COSTS AND WEIGHTS
- DATCOM-TYPE STABILITY AND CONTROL DERIVATIVES
- NON-LINEAR, ITERATIVE TRIM SOLUTION
- MULTIPLE-STEP, SUBOPTIMAL CRUISE/CLIMB
- GENERALIZED INTERFERENCE DRAG
- COMPUTER GRAPHICS OUTPUT OPTION
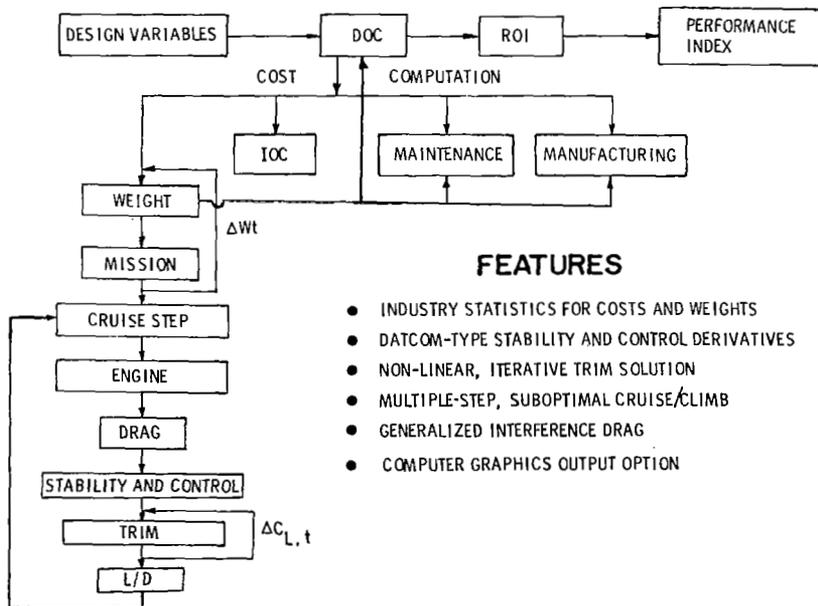
Figure 2.- Schematic representation of performance function calculation hierarchy.
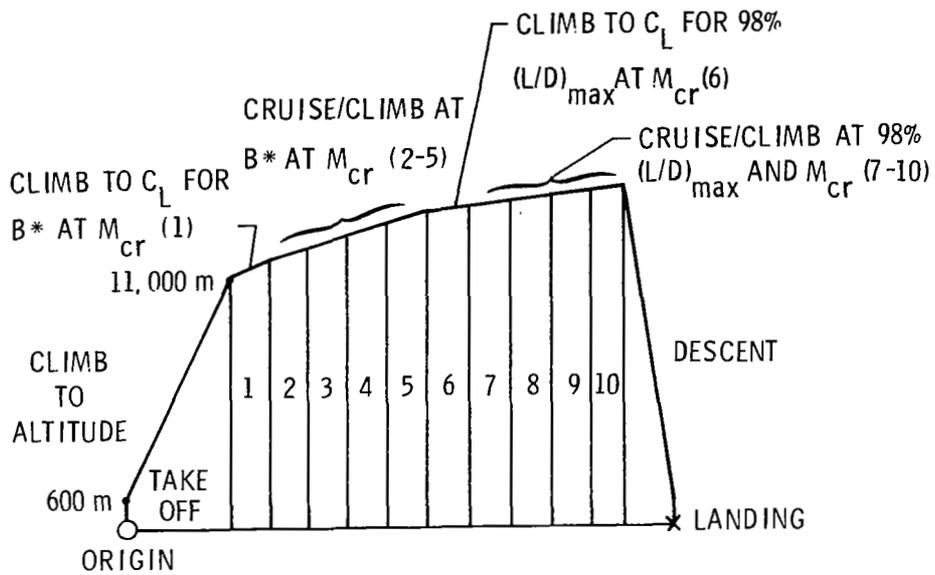
Figure 3.- Mission profile used in OPDOT.
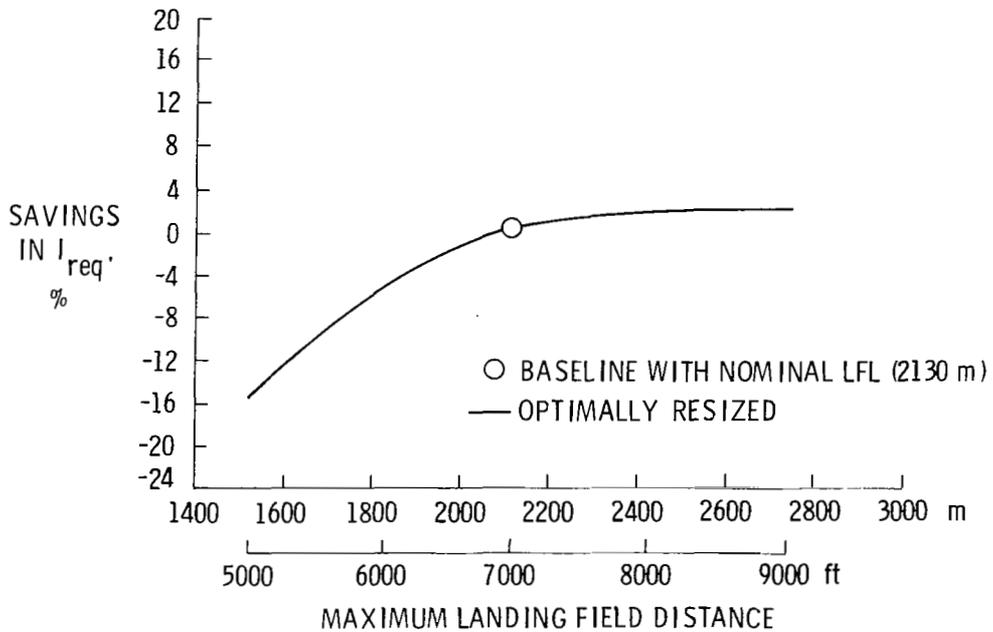


Figure 4.- Sensitivity of optimum $I_{req}$ to landing field length constraint.
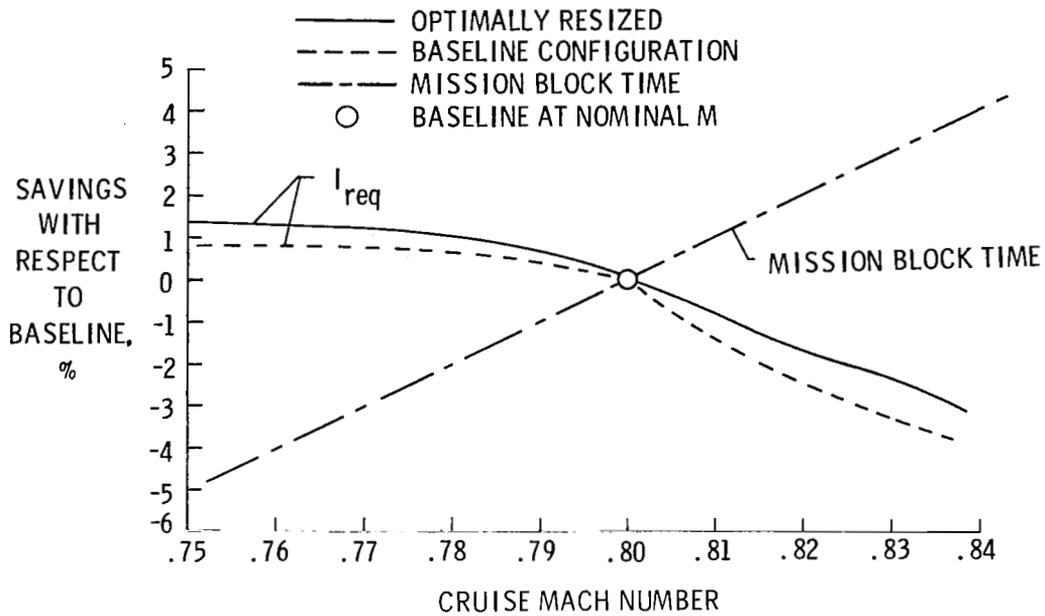
Figure 5.- Impact of cruise Mach number upon transport design.
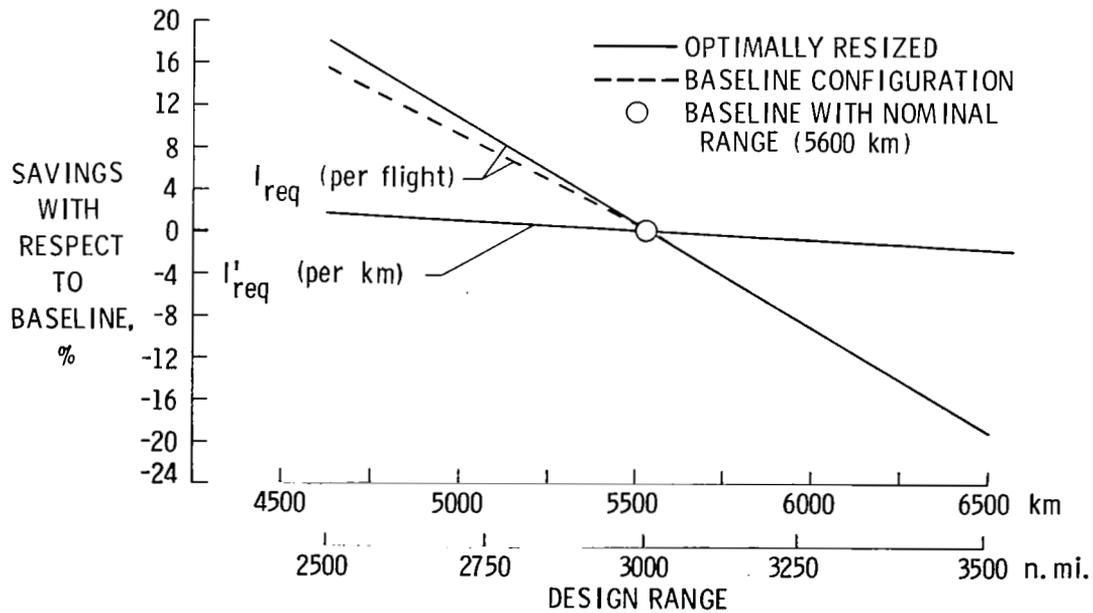


Figure 6.- Impact of design range upon transport design.
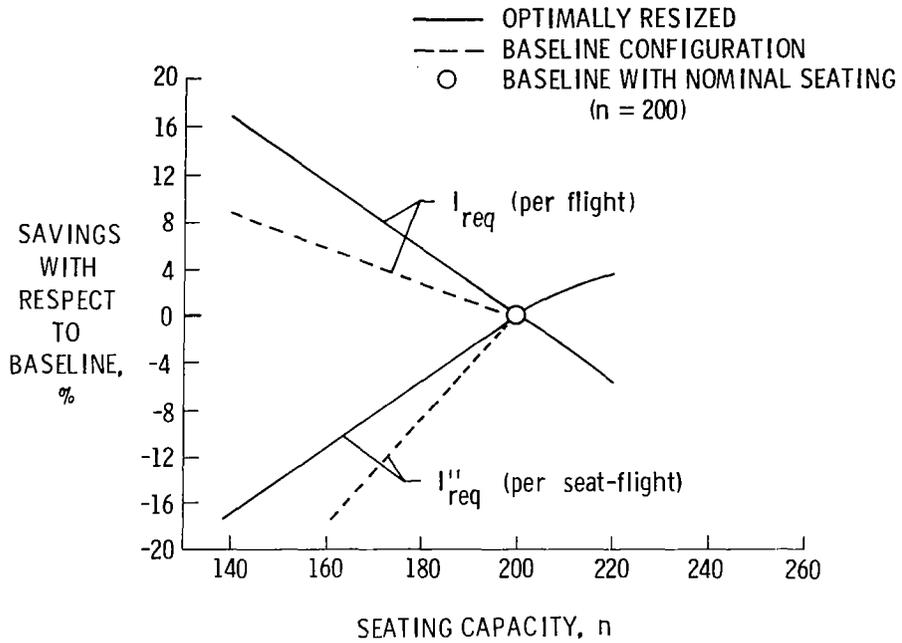
Figure 7.- Impact of varying seating capacity upon transport design.
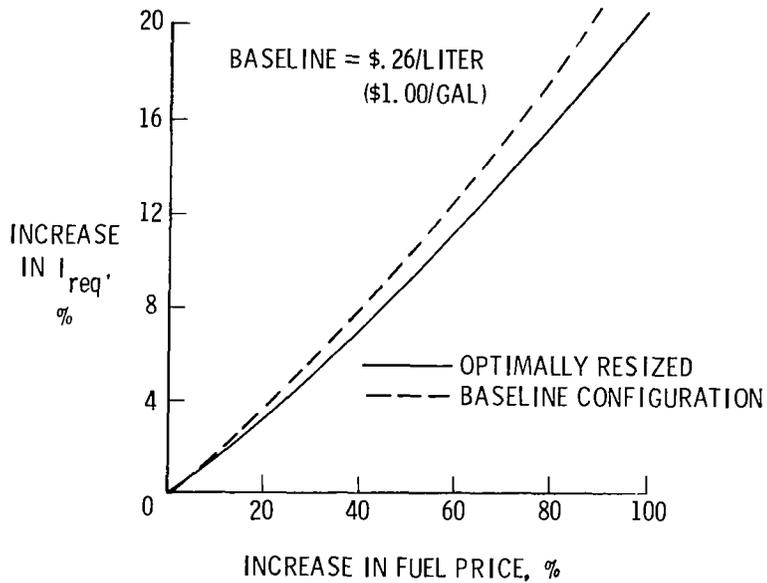


Figure 8.- Impact of fuel price upon transport design and optimum $I_{req}$.
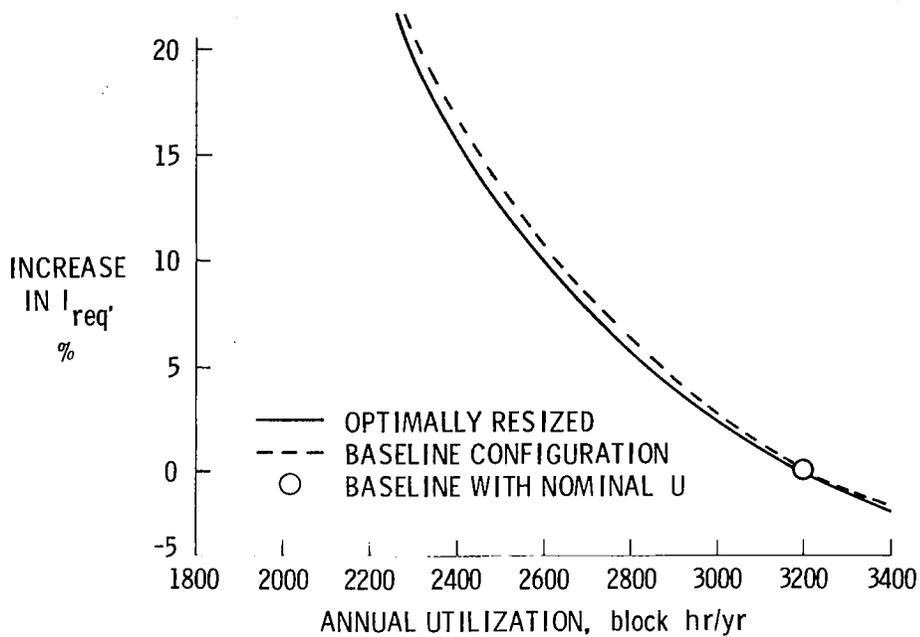
Figure 9.- Impact of annual utilization assumption upon
transport design and optimal $I_{req}$.
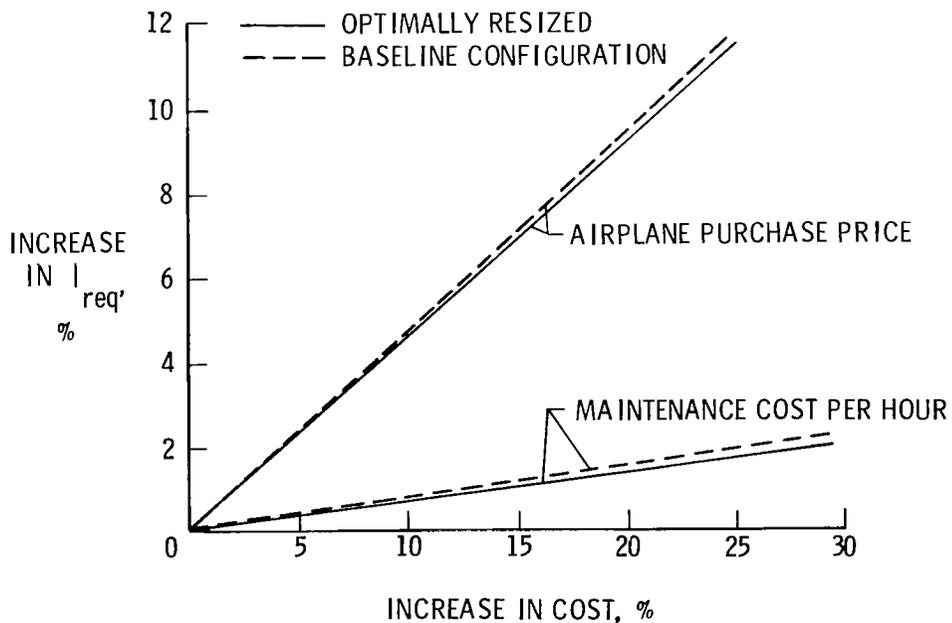


Figure 10.- Impact of increments in airplane purchase price and
per block hour maintenance cost upon transport design.
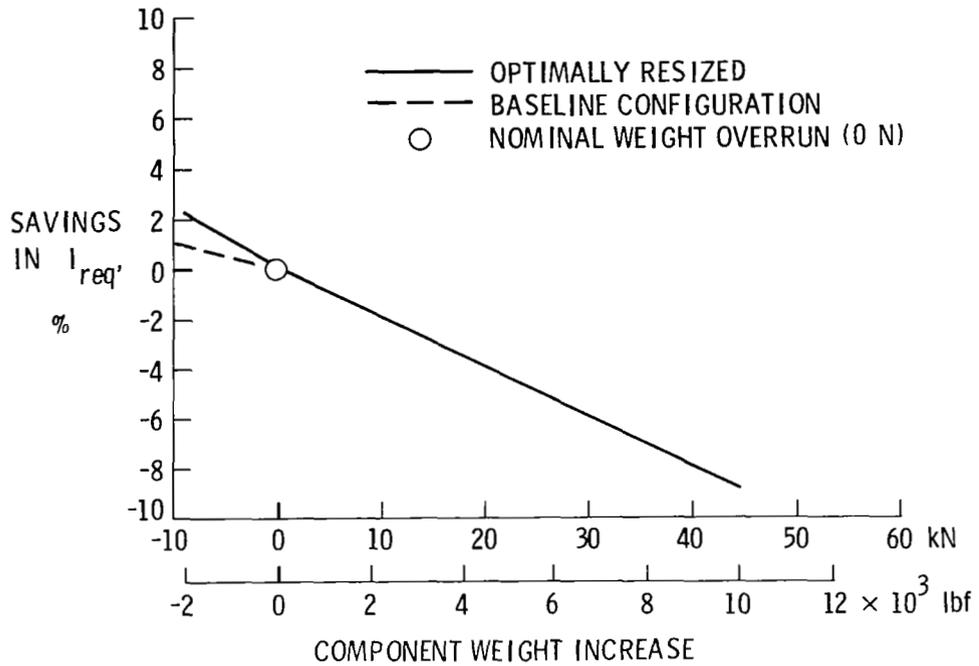
Figure 11.- Impact of weight increases upon the design and optimum $I_{req}$ of a transport.
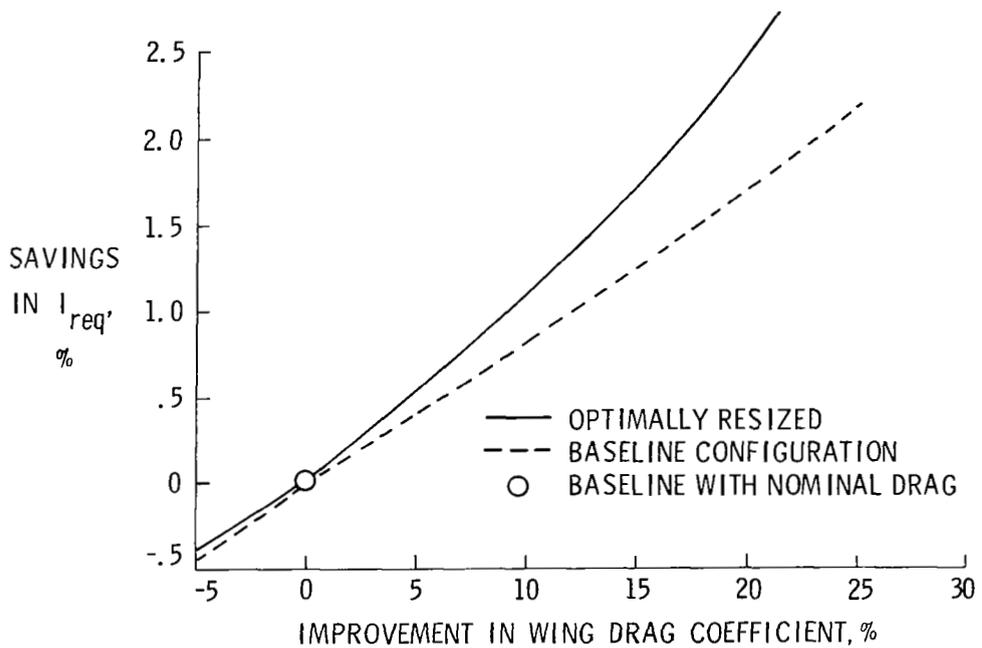


Figure 12.- Impact of reductions in wing drag coefficient upon the design and optimum $I_{req}$ of a transport.
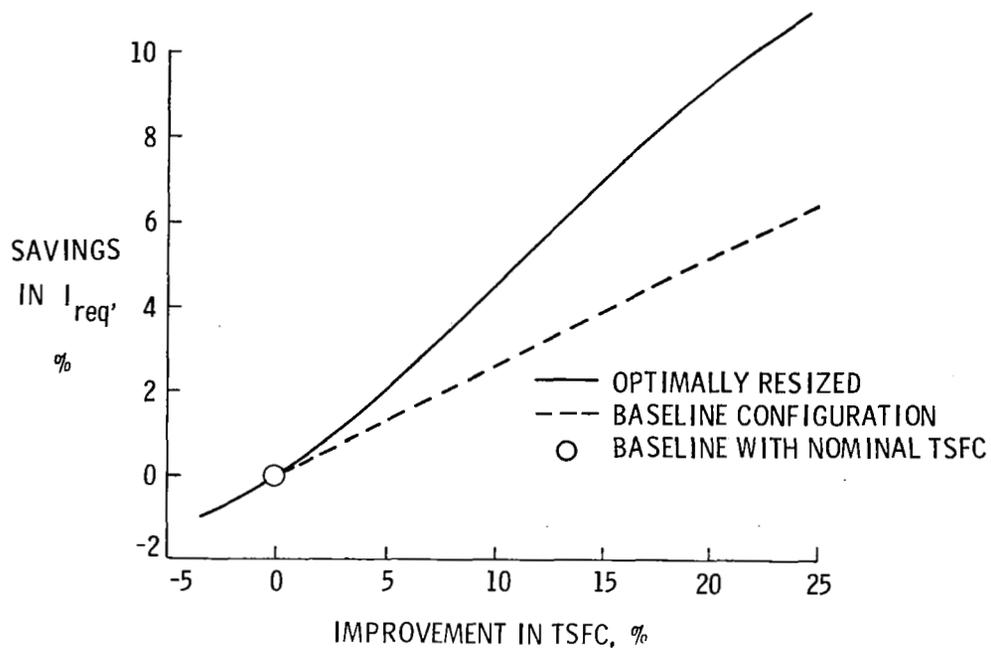
Figure 13.- Impact of technology improvements to reduce engine
fuel consumption upon design and optimum $I_{req}$ of the
transport.
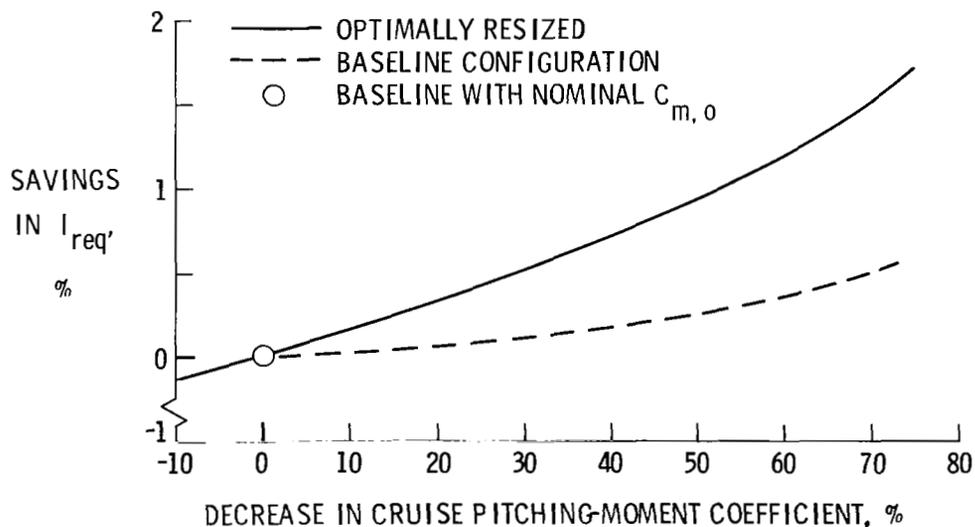


Figure 14.- Sensitivity of optimum design to reductions in the
high pitching moments assumed in the supercritical aerodynamic
model.
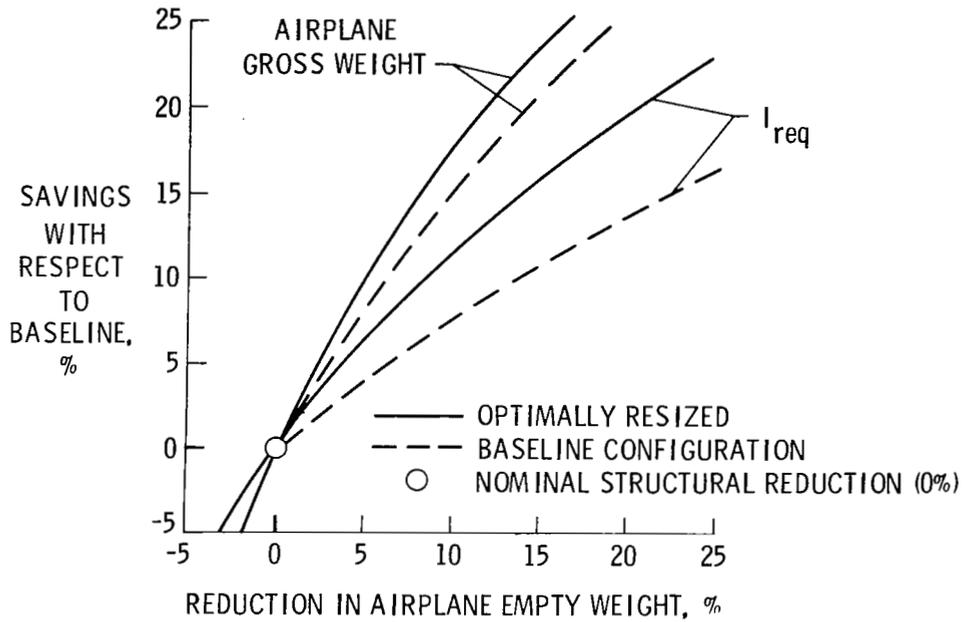
Figure 15.- Impact of structural efficiency of basic construction
materials upon optimum transport design.
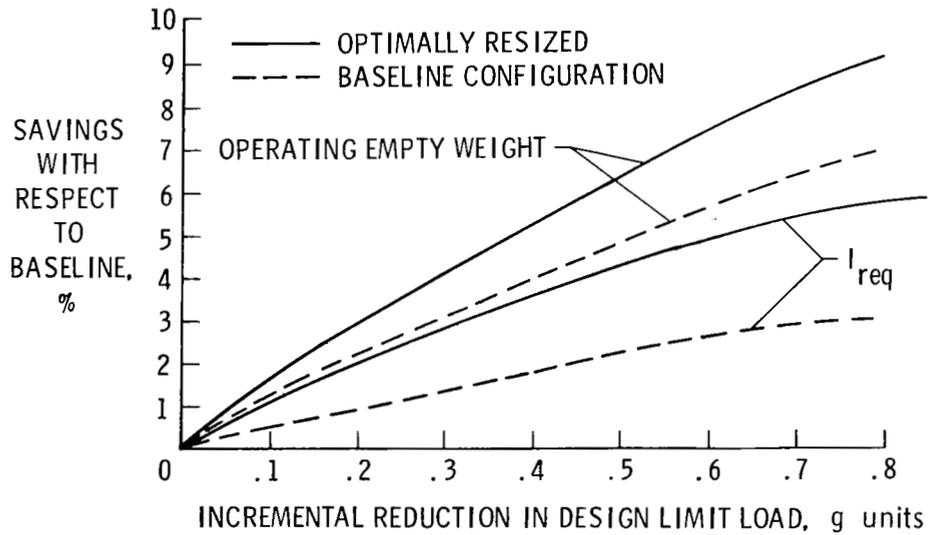


Figure 16.- Impact of reducing design limit load factors
with active controls.
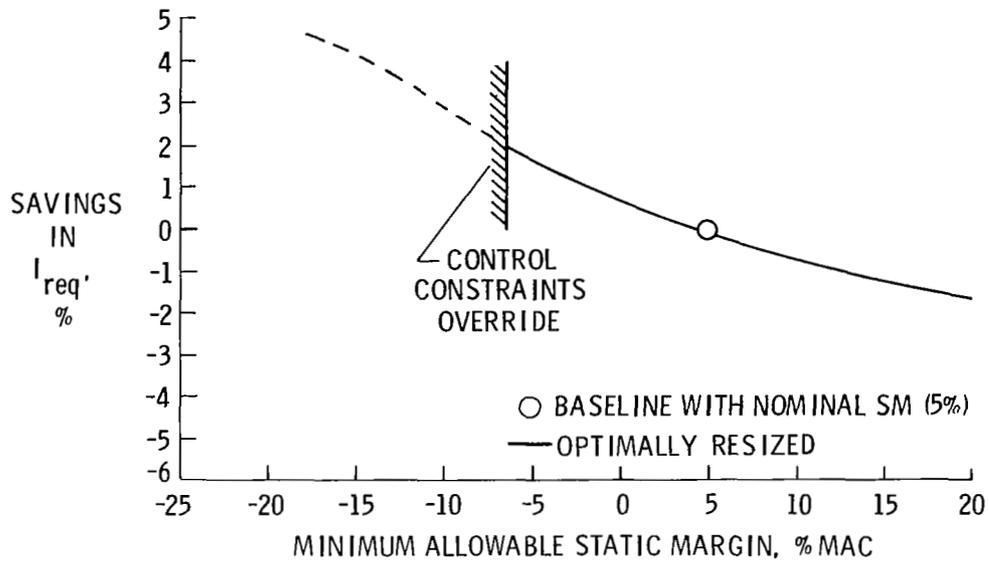
Figure 17.- Impact of reductions in static margin constraints upon $I_{req}$ of an optimum transport.
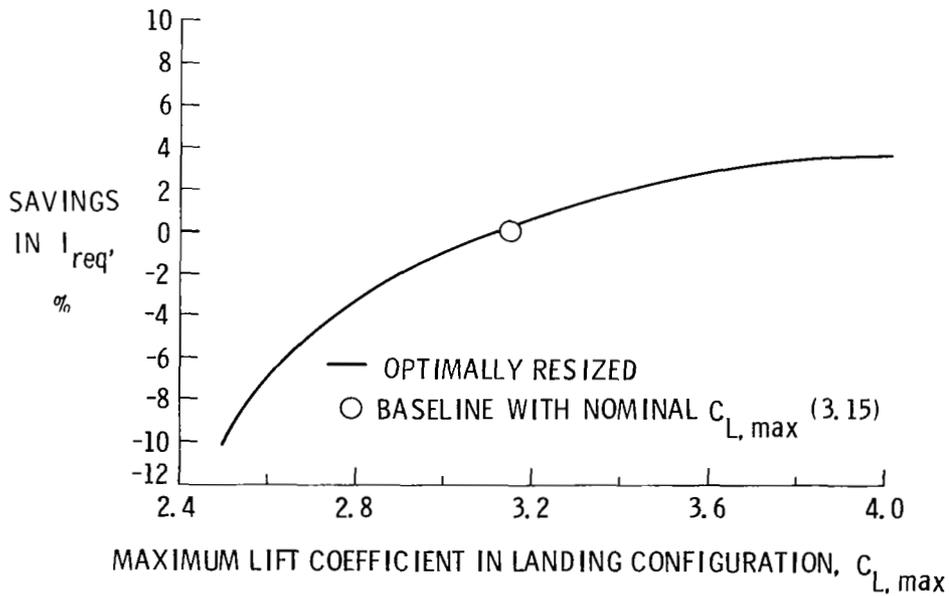


Figure 18.- Impact of changes in maximum lift coefficient on the optimum $I_{req}$ of a transport.

| 1. Report No.<br>NASA TP-1762 | 2. Government Accession No. | 3. Recipient's Catalog No. |
|---|---|---|
| 4. Title and Subtitle<br>USE OF CONSTRAINED OPTIMIZATION IN THE CONCEPTUAL<br>DESIGN OF A MEDIUM-RANGE SUBSONIC TRANSPORT | | 5. Report Date<br>December 1980 |
| | | 6. Performing Organization Code<br>505-34-33-05 |
| 7. Author(s)<br>Steven M. Sliwa | | 8. Performing Organization Report No.<br>L-13946 |
| | | 10. Work Unit No. |
| 9. Performing Organization Name and Address<br>NASA Langley Research Center<br>Hampton, VA 23665 | | |
| | | 11. Contract or Grant No. |
| | | 13. Type of Report and Period Covered<br>Technical Paper |
| 12. Sponsoring Agency Name and Address<br>National Aeronautics and Space Administration<br>Washington, DC 20546 | | |
| | | 14. Sponsoring Agency Code |

15. Supplementary Notes

16. Abstract

Constrained parameter optimization was used to perform the optimal conceptual
design of a medium-range transport configuration. The impact of choosing a given
performance index was studied, and the required income for a 15-percent return-
on-investment was proposed as a figure-of-merit. A number of design constants
and constraint functions were systematically varied to document the sensitivities
of the optimal design to a variety of economic and technological assumptions.
A comparison was made for each of the parameter variations between the baseline
configuration and the optimally redesigned configuration.

| 17. Key Words (Suggested by Author(s))<br>Conceptual design of aircraft<br>Constrained optimization<br>Nonlinear programming<br>Economics<br>Sensitivity to design parameters | 18. Distribution Statement<br>Unclassified – Unlimited<br><br>Subject Category 05 | | |
|---|---|---|---|
| 19. Security Classif. (of this report)<br>Unclassified | 20. Security Classif. (of this page)<br>Unclassified | 21. No. of Pages<br>35 | 22. Price<br>A03 |

National Aeronautics and
Space Administration

Washington, D.C.
20546

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