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Prediction of Composite Thermal Behavior Made Simple

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PREDICTION OF COMPOSITE THERMAL BEHAVIOR MADE SIMPLE

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ABSTRACT

A convenient procedure is described to determine the thermal behavior (thermal expansion coefficients and thermal stresses) of angleplied fiber composites using a pocket calculator. The procedure consists of equations and appropriate graphs for various ($\pm\theta$) ply combinations. These graphs present reduced stiffness and thermal expansion coefficients as functions of $\pm\theta$ in order to simplify and expedite the use of the equations. The procedure is applicable to all types of balanced, symmetric fiber composites including interply and intraply hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples.

1.0 INTRODUCTION

Thermal expansion coefficients and thermal and residual strains and stresses in angleplied laminates (figs. 1 and 2) are frequently required for the initial sizing of structural components made from fiber composites. These coefficients strains and stresses are commonly referred to as composite thermal behavior. The significance of composite thermal behavior particularly lamination residual stresses are extensively discussed in reference 1. Thermal expansion coefficients, thermal and lamination residual strains and stresses are determined using composite mechanics and laminate theory usually available in a computer code (refs. 2 and 3). A computer code was used effectively (ref. 4) to evaluate lamination residual stresses in angleplied laminates and thereby assess the effects of these stresses on the structural intergrity of composites. It is generally recognized that the use of a computer code is expedient and quite general. However, it does not provide the user with insight and instant feedback of the laminate thermal behavior and capability as he proceeds with the design/analysis of the component. Also, a computer code may not be readily accessible to the user.

A convenient procedure (method) is described in this paper which can be used to determine the thermal behavior of angleplied laminates. The procedure is suitable for hand calculations using a pocket calculator. It consists of simple equations and appropriate graphs of ($\pm\theta$) ply combinations from the most frequently used composites (figs. 3 to 18). Graphs for other composites can be generated by using uniaxial properties from table I and well-known transformation equations (ref. 4). The procedure makes use of the well-known transformation equations, and laminate theory equations. Its structure is similar to that in reference 5. The procedure can handle all types of composites includ-

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ing interply and intraply hybrids. The procedure is illustrated using selected step-by-step numerical examples. The discussion in this paper is limited to linear mechanical and thermal behavior of composites. Several additional examples and implications to design applications are described in a NASA TP report under preparation.

The paper and numerical examples are written mainly in a tutorial manner in order to illustrate the step-by-step procedure. For this reason, the various sections are self contained as much as is practical at the expense of some duplication. The notation used is defined when it first appears and also summarized under symbols for convenient reference. Customary units are used throughout the numerical examples since these serve primarily to illustrate step-by-step numerical calculations. Appropriate conversion factors are given in the symbols.

2.0 THERMAL EXPANSION COEFFICIENTS

The in-plane thermal expansion coefficients (TECs) α_{cxx} and α_{cyy} of $[0/\pm\theta]_S$ angleplied laminates are determined from the following equations

$$\alpha_{cxx} = \frac{1}{E_{cxx}} \left\{ V_{p\theta}\left[(Q_{\theta 11} - \nu_{cxy}Q_{\theta 21})\alpha_{\theta 11} + (Q_{\theta 12} - \nu_{cxy}Q_{\theta 22})\alpha_{\theta 22}\right] + V_{p0}\left[(Q_{\ell 11} - \nu_{cxy}Q_{\ell 21})\alpha_{\ell 11} + (Q_{\ell 12} - \nu_{cxy}Q_{\ell 22})\alpha_{\ell 22}\right]\right\} \tag{2.1}$$

$$\alpha_{cyy} = \frac{1}{E_{cyy}} \left\{ V_{p\theta}\left[(Q_{\theta 21} - \nu_{cyx}Q_{\theta 11})\alpha_{\theta 11} + (Q_{\theta 22} - \nu_{cyx}Q_{\theta 21})\alpha_{\theta 22}\right] + V_{p0}\left[(Q_{\ell 21} - \nu_{cyx}Q_{\ell 11})\alpha_{\ell 11} + (Q_{\ell 22} - \nu_{cyx}Q_{\ell 21})\alpha_{\ell 22}\right]\right\} \tag{2.2}$$

The composite moduli E_{cxx} and E_{cyy} and the composite Poisson's ratios ν_{cxy} and ν_{cyx} are given by

$$\left.\begin{aligned} E_{cxx} &= Q_{cxx} - \frac{Q_{cxy}^2}{Q_{cyy}}; \quad E_{cyy} = Q_{cyy} - \frac{Q_{cxy}^2}{Q_{cxx}} \\ \nu_{cxy} &= \frac{Q_{cxy}}{Q_{cyy}}; \quad \nu_{cyx} = \frac{Q_{cxy}}{Q_{cxx}} \end{aligned}\right\} \tag{2.3}$$

The "reduced stiffness" Q_c's are given by:

$$\left.\begin{aligned} Q_{cxx} &= V_{p\theta}Q_{\theta 11} + V_{p0}Q_{\ell 11} \\ Q_{cyy} &= V_{p\theta}Q_{\theta 22} + V_{p0}Q_{\ell 22} \\ Q_{cxy} &= V_{p\theta}Q_{\theta 12} + V_{p0}Q_{\ell 12} = Q_{cyx} \end{aligned}\right\} \tag{2.4}$$

The Q_θ's are obtained from the appropriate figures 3 to 18. The Q_ℓ's are equal to Q_θ's at $\theta = 0$ in figures 3 to 18. Note $Q_{\ell 21} = Q_{\ell 12}$. The α_θ's and α_ℓ's are obtained from the same figures. The parameter $V_{p\theta}$ denotes the thickness ratio of the $\pm\theta$-plies to the total laminate thickness while V_{p0} is the corresponding ratio for the 0-plies. $V_{p\theta}$ and V_{p0} satisfy the identity

$$V_{p\theta} + V_{p0} = 1 \tag{2.5}$$

The following procedure is convenient to calculate numerical values for α_{cxx} and α_{cyy} for a given $[0/\pm\theta]_S$ APL using equations (2.1) and (2.2):

1. Obtain values for $Q_{\theta 11}$, $Q_{\theta 22}$, $Q_{\theta 12} = Q_{\theta 21}$, $Q_{\ell 11}$, $Q_{\ell 22}$, $Q_{\ell 12} = Q_{\ell 21}$ $\alpha_{\theta 11}$, $\alpha_{\theta 22}$, $\alpha_{\ell 11}$ and $\alpha_{\ell 22}$ from the appropriate figures note that ten values are needed.

2. Calculate values for $V_{p\theta}$ and V_{p0}, respectively,

$$V_{p\theta} = \frac{\text{thickness of } \pm\theta\text{-plies}}{\text{thickness of APL}} \tag{2.6}$$

$$V_{p0} = \frac{\text{thickness of 0-plies}}{\text{thickness of APL}} \tag{2.7}$$

3. Calculate values for Q_{cxx}, Q_{cyy} and Q_{cxy} using equations (2.4), using the information obtained in item (1) above and that obtained from equations (2.6) and (2.7).

4. Calculate values for E_{cxx}, E_{cyy}, ν_{cxy} and ν_{cyx} using the information obtained in item (3) above. We use the well known relationship

$$\nu_{cyx} = \nu_{cxy} \frac{E_{cyy}}{E_{cxx}} \tag{2.8}$$

to check our numerical values. In summary, we need to look-up ten quantities from the graphs and calculate a minimum of seven others in order to determine the thermal expansion coefficients (TECs) α_{cxx} and α_{cyy} using equations (2.1) and (2.2).

It is worth noting in equations (2.1) and (2.2) that the APL TECs depend on the properties of the $\theta\pm$ -plies (Q_θ's), the 0-plies (Q_ℓ's) and the APL (integrated) properties E_c and ν_c. Also, the APL Poisson's ratios (ν_c) restrain the TECs of the APL since these quantities are preceded by a minus sign and, therefore, subtract from the total. Furthermore, the shear moduli contribute to the TECs through the Q_θ's. In addition, equations (2.1) and (2.2) can be easily extended for more than one set of $\pm\theta$-ply combinations. Similar terms

$V_{p\theta_1}[\;] + V_{p\theta_2}[\;]$ + etc., are added to accommodate this case as will be described with a numerical example later

Example 2.1. - Calculate the TECs of the 8-ply $[\pm 30/0_2]_S$ APL made from AS/E composite: (1) Following the procedure outlined in item (1) above we obtain the Q's from figure 7 and the α's from figure 8 both at $\theta = \pm 30$ and 0 (within curve reading accuracy):

$$Q_{\theta 11} = 11.3\mathrm{x}10^6 \text{ psi} \qquad Q_{\ell 11} = 18.7\mathrm{x}10^6 \text{ psi}$$

$$Q_{\theta 22} = 2.9\mathrm{x}10^6 \text{ psi} \qquad Q_{\ell 22} = 2.0\mathrm{x}10^6 \text{ psi}$$

$$Q_{\theta 21} = Q_{\theta 12} = 3.7\mathrm{x}10^6 \text{ psi} \qquad Q_{\ell 21} = Q_{\ell 12} = 0.6\mathrm{x}10^6 \text{ psi}$$

$$\alpha_{\theta 11} = -2.6\mathrm{x}10^{-6} \text{ in/in/}^\circ\text{F} \qquad \alpha_{\ell 11} = 0.4\mathrm{x}10^{-6} \text{ in/in/}^\circ\text{F}$$

$$\alpha_{\theta 22} = 12.8\mathrm{x}10^{-6} \text{ in/in/}^\circ\text{F} \qquad \alpha_{\ell 22} = 16.4\mathrm{x}10^{-6} \text{ in/in/}^\circ\text{F}$$

(2) Following item (2), we calculate $V_{p\theta}$ and V_{p0}

$$V_{p\theta} = 4/8 = 0.5$$

$$V_{p0} = 4/8 = 0.5$$

(3) Following item (3), we calculate the Q_c's using equations (2.4) and carrying the units selectively for convenience

$$Q_{cxx} = V_{p\theta}Q_{\theta 11} + V_{p0}Q_{\ell 11}$$

$$= 0.5 \times 11.3\mathrm{x}10^6 + 0.5 \times 18.7\mathrm{x}10^6 = 15.0\mathrm{x}10^6 \text{ psi}$$

$$Q_{cyy} = V_{p\theta}Q_{\theta 22} + V_{p0}Q_{\ell 22}$$

$$= 0.5 \times 2.9\mathrm{x}10^6 + 0.5 \times 2.0\mathrm{x}10^6 = 2.45\mathrm{x}10^6 \text{ psi}$$

$$Q_{cyx} = Q_{cxy} = V_{p\theta}Q_{\theta 12} + V_{p0}Q_{\ell 12}$$

$$= 0.5 \times 3.7\mathrm{x}10^6 + 0.5 \times 0.6\mathrm{x}10^6 = 2.15\mathrm{x}10^6 \text{ psi}$$

(4) Following item (4), we calculate E_{cxx}, E_{cyy} and ν_{cxy}

$$E_{cxx} = Q_{cxx} - \frac{Q_{cxy}^2}{Q_{cyy}} = 15.0\times10^6 - \frac{(2.15\times10^6)^2}{2.45\times10^6} = 13.1\times10^6 \text{ psi}$$

$$E_{cyy} = Q_{cyy} - \frac{Q_{cxy}^2}{Q_{cxx}} = 2.45\times10^6 - \frac{(2.15\times10^6)^2}{15.0\times10^6} = 2.14\times10^6 \text{ psi}$$

$$\nu_{cxy} = \frac{Q_{cxy}}{Q_{cyy}} = \frac{2.15\times10^6}{2.45\times10^6} = 0.878$$

$$\nu_{cyx} = \frac{Q_{cxy}}{Q_{cxx}} = \frac{2.15\times10^6}{15.0\times10^6} = 0.143$$

Check: $\nu_{cyx} = \nu_{cxy} \dfrac{E_{cyy}}{E_{cxx}}$

$$0.143 = 0.878 \frac{2.14\times10^6}{13.1\times10^6} \Rightarrow 0.143 = 0.143 \quad \text{O.K.}$$

(5) Using the above information in equations (2.1) and (2.2) and cancelling 10^6 with 10^{-6} within the braces:

$$\alpha_{cxx} = \frac{1}{E_{cxx}} \left\{ V_{p\theta}\left[(Q_{\theta11} - \nu_{cxy}Q_{\theta21})\alpha_{\theta11} + (Q_{\theta12} - \nu_{cxy}Q_{\theta22})\alpha_{\theta22}\right] + V_{p0}\left[(Q_{\ell11} - \nu_{cxy}Q_{\ell21})\alpha_{\ell11} + (Q_{\ell12} - \nu_{cxy}Q_{\ell22})\alpha_{\ell22}\right]\right\}$$

$$= \frac{1}{13.1\times10^6}\left\{0.5\left[(11.3 - 0.878 \times 3.7)(-2.6) + (3.7 - 0.878\times2.9)(12.8)\right] + 0.5\left[(18.7 - 0.878 \times 0.6)(0.4) + (0.6 - 0.878\times2.0)(16.4)\right]\right\}$$

$$= \frac{1}{13.1\times10^6}\left\{0.5\left[-20.9 + 14.8\right] + 0.5\left[7.27 - 19.0\right]\right\}$$

$$\alpha_{cxx} = -0.68\times10^{-6} \text{ in/in/}^{\circ}\text{F}$$

(Cont'd.)

$$\alpha_{cyy} = \frac{1}{E_{cyy}} \left\{ V_{p\theta}\left[(Q_{\theta 21} - \nu_{cyx}Q_{\theta 11})\alpha_{\theta 11} + (Q_{\theta 22} - \nu_{cyx}Q_{\theta 21})\alpha_{\theta 22}\right]\right.$$

$$\left. + V_{p0}\left[(Q_{\ell 21} - \nu_{cyx}Q_{\ell 11})\alpha_{\ell 11} + (Q_{\ell 22} - \nu_{cyx}Q_{\ell 21})\alpha_{\ell 22}\right]\right\}$$

$$= \frac{1}{2.14\text{x}10^6} \left\{0.5\left[(3.7 - 0.143 \text{ x } 11.3)(-2.6) + (2.9 - 0.143 \text{ x } 3.7)(12.8)\right]\right.$$

$$\left. + 0.5\left[(0.6 - 0.143 \text{ x } 18.7)(0.4) + (2.0 - 0.143 \text{ x } 0.6)(16.4)\right]\right\}$$

$$= \frac{1}{2.14\text{x}10^6} \left\{0.5\left[-5.42 + 30.3\right] + 0.5\left[-0.830 + 31.4\right]\right\}$$

$$\alpha_{cyy} = 13.0\text{x}10^6 \text{ in/in/}^{\circ}\text{F}$$

Example 2.2. - Calculate the TECs of the $[0/\pm 45/90]_S$ APL made from AS/E composite. This APL is commonly called pseudo-isotropic or quasi-isotropic laminate meaning that the laminate behaves like an isotropic material with respect to its in-plane elastic and thermal properties. The meaning of this terminology will also be illustrated by a numerical example later. Again we follow the procedure outlined in items (1) through (5) and extend it to three different ply configurations (0, ±45, 90):

(1) Obtain from figures 7 and 8 (within curve-reading accuracy) and using the first subscript θ to denote all three conditions:

Q and α	$\theta = \pm 45$	$\theta = 0$	$\theta = 90$
$Q_{\theta 11}$ (10^6 psi)	6.1	18.7	2.0
$Q_{\theta 22}$ (10^6 psi)	6.1	2.0	18.7
$Q_{\theta 21} = Q_{\theta 12}$ (10^6 psi)	4.8	0.6	0.6
$\alpha_{\theta 11}$(10^{-6} in/in/$^{\circ}$F)	2.2	0.4	16.4
$\alpha_{\theta 22}$(10^{-6} in/in/$^{\circ}$F)	2.2	16.4	0.4

Note the 0-ply and 90-ply properties are complementary as expected.

(2) The respective thickness ratios for this 8-ply APL are:

θ	Number of plies	$V_{p\theta}$
±45	4	4/8 = 0.50
0	2	2/8 = 0.25
90	2	2/8 = 0.25

(3) The corresponding Q_c's are:

$$Q_{cxx} = V_{p\pm45}Q_{\pm4511} + V_{p0}Q_{011} + V_{p90}Q_{9011}$$
$$= (0.50 \times 6.1 + 0.25 \times 18.7 + 0.25 \times 2.0)\text{x}10^6 = 8.22\text{x}10^6 \text{ psi}$$

$$Q_{cyy} = V_{p\pm45}Q_{\pm4522} + V_{p0}Q_{022} + V_{p90}Q_{9022}$$
$$= (0.50 \times 6.1 + 0.25 \times 2.0 + 0.25 \times 18.7)\text{x}10^6 = 8.22\text{x}10^6 \text{ psi}$$

$$Q_{cyx} = Q_{cxy} = V_{p\pm45}Q_{\pm4512} + V_{p0}Q_{012} + V_{p90}Q_{9022}$$
$$= (0.5 \times 4.8 + 0.25 \times 0.6 + 0.25 \times 0.6)\text{x}10^6 = 2.70\text{x}10^6 \text{ psi}$$

(4) The APL moduli and Poisson's ratio are:

$$E_{cxx} = Q_{cxx} - \frac{Q^2_{cxy}}{Q_{cyy}} = \left(8.22 - \frac{2.70^2}{8.22}\right) \times 10^6 = 7.33\text{x}10^6 \text{ psi}$$

$$E_{cyy} = Q_{cyy} - \frac{Q^2_{cyx}}{Q_{cxx}} = \left(8.22 - \frac{2.70^2}{8.22}\right) \times 10^6 = 7.33\text{x}10^6 \text{ psi}$$

$$\nu_{cxy} = \frac{Q_{cxy}}{Q_{cyy}} = \frac{2.70}{8.22} = 0.328$$

$$\nu_{cyx} = \frac{Q_{cyx}}{Q_{cxx}} = \frac{2.70}{8.22} = 0.328$$

Check: $\nu_{cyx} = \nu_{cxy} \dfrac{E_{cyy}}{E_{cxx}}$

$$0.328 = 0.328 \frac{7.33}{7.33} \Rightarrow 0.328 = 0.328 \quad \text{O.K.}$$

The calculations for E_{cyy}, ν_{cyx} and for the "check" were carried out for completeness since it is obvious that for this laminate $E_{cyy} = E_{cxx}$ and $\nu_{cyx} = \nu_{cxy}$.

(5) The TECs for this APL are calculated from equations (2.1) and (2.2) but are generalized to include more than two different ply combinations. The form of the equations using the summation sign are:

$$\alpha_{cxx} = \frac{1}{E_{cxx}} \sum_{\substack{\theta = \\ \pm 45,0,90}} V_{p\theta}\left[(Q_{\theta 11} - \nu_{cxy}Q_{\theta 21})\alpha_{\theta 11} + (Q_{\theta 12} - \nu_{cxy}Q_{\theta 22})\alpha_{\theta 22}\right]$$

$$\alpha_{cyy} = \frac{1}{E_{cyy}} \sum_{\substack{\theta = \\ \pm 45,0,90}} V_{p\theta}\left[(Q_{\theta 21} - \nu_{cyx}Q_{\theta 11})\alpha_{\theta 11} + (Q_{\theta 22} - \nu_{cyx}Q_{\theta 21})\alpha_{\theta 22}\right] \qquad (2.9)$$

where the sum is taken over $\theta = \pm 45$, 0 and 90. Using the values calculated previously in equations (2.9) and cancelling the 10^6 term with the 10^{-6} within the braces we have:

$$\alpha_{cxx} = \frac{1}{7.33\text{x}10^6}\Big\{0.50\left[(6.1 - 0.328 \times 4.8)(2.2) + (4.8 - 0.328 \times 6.1)(2.2)\right]$$
$$+ 0.25\left[(18.7 - 0.328 \times 0.6)(0.4) + (0.6 - 0.328 \times 2.0)(16.4)\right]$$
$$+ 0.25\left[(2.0 - 0.328 \times 0.6)(16.4) + (0.6 - 0.328 \times 18.7)(0.4)\right]\Big\}$$

$$= \frac{1}{7.33\text{x}10^6}\Big\{0.50\left[9.96 + 6.16\right] + 0.25\left[7.40 - 0.92\right] + 0.25\left[29.6 - 2.21\right]\Big\}$$

$\alpha_{cxx} = 2.25\text{x}10^{-6}$ in/in/°F

$$\alpha_{cyy} = \frac{1}{7.33\text{x}10^6}\Big\{0.50\left[(4.8 - 0.328 \times 6.1)(2.2) + (6.1 - 0.328 \times 4.8)(2.2)\right]$$
$$+ 0.25\left[(0.6 - 0.328 \times 18.7)(0.4) + (2.0 - 0.328 \times 0.6)(16.4)\right]$$
$$+ 0.25\left[(0.6 - 0.328 \times 2.0)(16.4) + (18.7 - 0.328 \times 0.6)(0.4)\right]\Big\}$$

$$= \frac{1}{7.33\text{x}10^6}\Big\{0.50\left[6.16 + 9.96\right] + 0.25\left[-2.21 + 29.6\right] + 0.25\left[-0.92 + 7.40\right]\Big\}$$

$\alpha_{cyy} = 2.25\text{x}10^{-6}$ in/in/°F as expected. The reader may find it instructive to note that: (1) the values within the various parentheses for α_{cxy} and α_{cyy} are complementary and (2) the values for α_{cxx} and α_{cyy} are about equal to those for $\alpha_{\theta 11}$ and $\alpha_{\theta 22}$ for $\theta = \pm 45$.

The reader will obtain valuable practice and insight by using the procedure to calculate TECs of APL with ply configuration of his choice and a different composite system.

3.0 TRANSFORMATION OF THERMAL EXPANSION COEFFICIENTS

The thermal expansion coefficients (TECs) about any x'-y' coordinate axes of an orthotropic angleplied laminate APL with material symmetry about the x-y coordinate axes are given by:

$$\alpha_{cx'x'} = \alpha_{cxx}\cos^2\phi + \alpha_{cyy}\sin^2\phi$$

$$\alpha_{cy'y'} = \alpha_{cxx}\sin^2\phi + \alpha_{cyy}\cos^2\phi \tag{3.1}$$

$$\alpha_{cx'y'} = (\alpha_{cyy} - \alpha_{cxx})\sin 2\phi$$

where the notation in equation (3.1) is as follows: $\alpha_{cx'x'}$, $\alpha_{cy'y'}$ and $\alpha_{cx'y'}$ are the TECs about the new coordinate system x'-y'; α_{cxx} and α_{cyy} are the TECs about the x-y coordinate system and are calculated as described in Section 2; ϕ is the angle that the x' axis makes with the x axis. Note, $\alpha_{cx'y'}$ is a shear-type thermal deformation which is present along any coordinate system x'-y' located at some angle ϕ where $0 < \phi < 90$ since sine $2\phi \neq 0$ in this range.

To perform the calculations using equations (3.1) we need the TECs α_{cxx}, α_{cyy} and the angle ϕ. The TECs for the APL of interest are either known or can be computed using the procedure and examples described in the previous section. The angle ϕ is known once the coordinate x'-y' axes has been selected. We can check our results using $\alpha_{cx'x'} + \alpha_{cy'y'} = \alpha_{cxx} + \alpha_{cyy}$.

<u>Example 3.1.</u> - Calculate the TECs of the $[\pm 30/0_2]_S$ APL made from AS/E composite about an x'-y' coordinate axes where the x'-axis is located by $\phi = 15^o$ from the x-axis. From example (2.1) $\alpha_{cxx} = -0.68\text{x}10^6$ in/in/^{o}F and $\alpha_{cyy} = 13.0\text{x}10^{-6}$ in/in/^{o}F. Using these values in equations (3.1) we have:

$$\begin{aligned}\alpha_{cx'x'} &= \alpha_{cxx}\cos^2\phi + \alpha_{cyy}\sin^2\phi \\ &= (-0.68\cos^2 15^o + 13.0\sin^2 15^o) \times 10^{-6}\ \text{in/in/}^o\text{F} \\ &= 0.24\text{x}10^{-6}\ \text{in/in/}^o\text{F}\end{aligned}$$

(Cont'd.)

$$\alpha_{cy'y'} = \alpha_{cxx}\sin^2\phi + \alpha_{cyy}\cos^2\phi$$

$$= (-0.68 \sin^2 15^o + 13.0 \cos 15^o) \times 10^{-6} \text{ in/in/}^oF$$

$$= 12.08\text{x}10^{-6} \text{ in/in/}^oF$$

$$\alpha_{cx'y'} = (\alpha_{cyy} - \alpha_{cxx})\sin 2\theta$$

$$= \left\{ \left[13.0 - (-0.68)\right] \sin 2(15^o) \right\} \; 10^{-6} \text{ in/in/}^oF$$

$$\alpha_{cx'y'} = 6.84\text{x}10^{-6} \text{ in/in/}^oF$$

As can be seen, the shear-type TEC $\alpha_{cx'y'}$ is substantial. Restraining this APL along the x'-y' coordinate axes will induce considerable in-plane shear stresses.

4. LAMINATE THERMAL STRAINS AND STRESSES

Along the laminate material axes. - The equations for calculating thermal strains ε_{cxy} and ε_{cyy} along the laiminate x-y coordinate axes are:

$$\begin{aligned} \varepsilon_{cxx} &= (T - T_o)\alpha_{cxx} \\ \varepsilon_{cyy} &= (T - T_o)\alpha_{cyy} \end{aligned} \qquad (4.1)$$

where T is the use temperature and T_o is the reference temperature (usually room temperature); and α_{cxy} and α_{cyy} are the TECs which are either known or can be determined as described previously. Use of equations (4.1) requires that the TECs be independent of temperature within the range $T - T_o$.

Example 4.1. - Calculate the thermal strains for the $[\pm 30/0_2]_S$ APL made from AS/E composite (Example 2.1) where $T = 270^o$ F and $T_o = 70^o$ F. The values for the TECs from Example 2.1 are:

$$\alpha_{cxx} = -0.68\text{x}10^{-6} \text{ in/in/}^oF; \; \alpha_{cyy} = 13.0\text{x}10^{-6} \text{ in/in/}^oF$$

Substituting these values in equation (4.1), we obtain:

$$\varepsilon_{cxx} = (T - T_o)\alpha_{cxx} = (270 - 70)(-0.68\text{x}10^{-6}) = -136\text{x}10^{-6} \text{ in/in}$$

$$\varepsilon_{cyy} = (T - T_o)\alpha_{cyy} = (270 - 70)(13.0\text{x}10^{-6}) = 2600\text{x}10^{-6} \text{ in/in}$$

The equations to calculate the corresponding thermal stresses, assuming that the laminate is completely restrained from thermal expansion, are:

$$\begin{aligned} \sigma_{cxx} &= -\Delta T(Q_{cxx}\alpha_{cxx} + Q_{cxy}\alpha_{cyy}) \\ \sigma_{cyy} &= -\Delta T(Q_{cyx}\alpha_{cxx} + Q_{cyy}\alpha_{cyy}) \end{aligned} \qquad (4.2)$$

where $\Delta T = (T - T_o)$; the Q_c's and α_c's are determined as described previously.

Example 4.2. - Calculate the restrained thermal stresses in Example 4.1. Referring to Examples 2.1 and 4.1, we have:

$$Q_{cxx} = 15.0\text{x}10^6 \text{ psi} \qquad \alpha_{cxx} = -0.68\text{x}10^{-6} \text{ in/in/}^o\text{F}$$

$$Q_{cyy} = 2.45\text{x}10^6 \text{ psi} \qquad \alpha_{cyy} = 13.0\text{x}10^{-6} \text{ in/in/}^o\text{F}$$

$$Q_{cyx} = Q_{cxy} = 2.15\text{x}10^6 \text{ psi} \qquad \Delta T = (270 - 70) = 200^o \text{ F}$$

Using these values in equations (4.2) we obtain (cancelling 10^6 with 10^{-6}):

$$\begin{aligned}
\sigma_{cxx} &= -\Delta T(Q_{cxx}\alpha_{cxx} + Q_{cxy}\alpha_{cyy}) \\
&= -200[15.0 \times (-0.68) + 2.15 \times 13.0] \\
&= -200(-10.2 + 28.0) \\
&= -3550 \text{ psi} \approx -3.6 \text{ ksi}
\end{aligned}$$

$$\begin{aligned}
\sigma_{cyy} &= -\Delta T(Q_{cyx}\alpha_{cxx} + Q_{cyy}\alpha_{cyy}) \\
&= -200[2.15 \times (-0.68) + 2.45 \times 13.0] \\
&= -200(-1.46 + 31.8) \\
&= -6078 \text{ psi} \approx -6.1 \text{ ksi}
\end{aligned}$$

Two points are worth noting in connection with the above values of these thermal stresses:

1. The thermal stresses σ_{cxx} and σ_{cyy} are relatively small (4 and 28 percent, respectively) compared to the corresponding compressive failure stresses of the laminate $S_{cxxC} = 83$ ksi and $S_{cyyC} = 22$ ksi, based on first ply failure (ref. 5).

2. The magnitude of these thermal stresses may be sufficiently high to cause panel buckling. For example, a 20 in. x 10 in. x 0.04 in. panel from this 8-ply APL has buckling stresses of about $\sigma_{cxx} = 540$ psi and $\sigma_{cyy} = 920$ psi (calculated using the equation in ref. 6) or approximately 15 percent of the restrained thermal stresses which are relatively low. A panel with this geometry will buckle at an increase in temperature of about 15 percent of 200^o F or 30^o F. The important conclusion from the discussion in this example is that thermal stresses need be considered carefully by the designer/analyst in situations where restraints may be present.

5.0 PLY THERMAL STRAINS AND STRESSES

It is instructive to describe the ply thermal strains and stresses in the plies of an APL by breaking them down into four "commonly thought-of" types.

These types are:

1. Restrained - APL is restrained from thermal expansion.

2. Free - APL is free to undergo thermal expansion.

3. Residual - APL laminate is cooled from cure temperature to use temperature and frequently to room temperature.

4. Combined - Combinations of free and residual.

The ply thermal strains and stresses to be described are those along the ply material axes 1, 2, 3, figure 1. For convenience, the strains along the 1-direction (fiber direction) are defined by $\varepsilon_{\ell 11}$ and the stresses $\sigma_{\ell 11}$; those along the 2-direction (transverse to the fiber direction) are defined by $\varepsilon_{\ell 22}$ and $\sigma_{\ell 22}$; and those in the 1-2 plane (intralaminar shear) are defined by $\varepsilon_{\ell 12}$ and $\sigma_{\ell 12}$.

Restrained APL. - The ply thermal strains for this case are given by

$$\varepsilon_{\ell 12} = \varepsilon_{\ell 22} = \varepsilon_{\ell 11} = 0 \tag{5.1}$$

The corresponding ply stresses are given by

$$\left.\begin{aligned} \sigma_{\ell 11} &= -\Delta T(Q_{\ell 11}\alpha_{\ell 11} + Q_{\ell 12}\alpha_{\ell 22}) \\ \sigma_{\ell 22} &= -\Delta T(Q_{\ell 21}\alpha_{\ell 11} + Q_{\ell 22}\alpha_{\ell 22}) \\ \sigma_{\ell 12} &= 0 \end{aligned}\right\} \tag{5.2}$$

where ΔT equals the use temperature minus the reference temperature. Where the Q_ℓ's are the reduced ply stiffnesses and the α_ℓ's are ply thermal expansion coefficients (ply TECs). The ply reduced stiffness Q_ℓ's and TECs can be estimated from figures 3 to 18 at $\theta = 0^\circ$. Equations (5.2) show that the ply material axes thermal stresses in an APL restrained from thermal expansion depend only on ply properties. Also, there is no intralaminar shear stress for this case.

Example 5.1. - Calculate the ply thermal stress in the plies of the $[\pm 30/0_2]_S$ APL, made from AS/E composite, where the temperature is increased from 70° F (room temperature) to 270° F and the APL is restrained from thermal expansion. The numerical values we need for these calculations are determined as follows: Figures 7 and 8 at $\theta = 0^\circ$ yield

$$Q_{\ell 11} = 18.7\text{x}10^6 \text{ psi}, \; Q_{\ell 22} = 2.0\text{x}10^6 \text{ psi}$$

$$Q_{\ell 21} = Q_{\ell 12} = 0.6\text{x}10^6 \text{ psi}$$

$$\alpha_{\ell 11} = 0.4\text{x}10^{-6} \text{ in/in/}^\circ\text{F}; \; \alpha_{\ell 22} = 16.4\text{x}10^{-6} \text{ in/in/}^\circ\text{F}$$

and $\Delta T = 270^o F - 70^o F = 200^o F$ consistent with the previous definition. Using these numerical values in equations (5.2), we calculate

$$\sigma_{\ell 11} = -\Delta T(Q_{\ell 11}\alpha_{\ell 11} + Q_{\ell 12}\alpha_{\ell 22})$$
$$= -200(18.7 \times 0.4 + 0.6 \times 16.4) = -3464 \text{ psi} \approx -3.5 \text{ ksi}$$

$$\sigma_{\ell 22} \quad -\Delta T(Q_{\ell 21}\alpha_{\ell 11} + Q_{\ell 22}\alpha_{\ell 22})$$
$$= -200(0.6 \times 0.4 + 2.0 \times 16.4) = -6608 \text{ psi} \approx -6.6 \text{ ksi}$$

Free APL. - The ply thermal strains for this case are given by:

$$\varepsilon_{\ell 11} = \Delta T(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11})$$

$$\varepsilon_{\ell 22} = \Delta T(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22}) \tag{5.3}$$

$$\varepsilon_{\ell 12} = \Delta T(\alpha_{cyy} - \alpha_{cxx})\sin^2\theta$$

The corresponding ply stresses are given by:

$$\sigma_{\ell 11} = \Delta T\left[Q_{\ell 11}(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11}) + Q_{\ell 12}(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22})\right]$$

$$\sigma_{\ell 22} = \Delta T\left[Q_{\ell 21}(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11}) + Q_{\ell 22}(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22})\right]$$

$$\sigma_{\ell 12} = \Delta T\left[Q_{\ell 33}(\alpha_{cyy} - \alpha_{cxx})\sin^2\theta\right]$$

(5.4)

Also, when the ply thermal strains are calculated using equations (5.2), the corresponding ply stresses are given by:

$$\sigma_{\ell 11} = Q_{\ell 11}\varepsilon_{\ell 11} + Q_{\ell 12}\varepsilon_{\ell 22}$$

$$\sigma_{\ell 22} = Q_{\ell 21}\varepsilon_{\ell 11} + Q_{\ell 22}\varepsilon_{\ell 22} \tag{5.5}$$

$$\sigma_{\ell 12} = Q_{\ell 33}\varepsilon_{\ell 33}$$

where the strains (ε_ℓ) are calculated from equation (5.3).

Example 5.2. - Calculate the ply thermal strains in the plies of the $[\pm 30/0_2]_S$ APL, made from AS/E composite, where the temperature is increased from 70^o F (room temperature) to 270^o F and the APL is not restrained from thermal expansion. We will calculate the ply thermal strains in the $+30^o$ plies, in the -30^o plies and in the 0^o-plies by using equations (5.3). The numerical values we need are:

$$\Delta T = 200^{\circ}\ F$$

$$\theta = 30^{\circ},\ -30^{\circ},\ 0^{\circ}$$

$$\left.\begin{aligned} \alpha_{cxx} &= -0.68\text{x}10^{-6}\ \text{in/in/}^{\circ}\text{F} \\ \alpha_{cyy} &= 13.0\text{x}10^{-6}\ \text{in/in/}^{\circ}\text{F} \\ \alpha_{\ell 11} &= 0.4\text{x}10^{-6}\ \text{in/in/}^{\circ}\text{F} \\ \alpha_{\ell 22} &= 16.4\text{x}10^{-6}\ \text{in/in/}^{\circ}\text{F} \end{aligned}\right\}\ \text{(These values are taken from Example 2.1)}$$

<u>30°-ply.</u> - Substituting these numerical values and $\theta = 30^{\circ}$ in equations (5.3) we calculate:

$$\epsilon_{\ell 11} = \Delta T(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11})$$

$$= 200(-0.68 \times \cos^2 30^{\circ} + 13.0 \times \sin^2 30^{\circ} - 0.4) \times 10^{-6}\ \text{in/in}$$

$$= 468\text{x}10^{-6}\ \text{in/in or} = 0.05\%$$

$$\epsilon_{\ell 22} = \Delta T(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22})$$

$$= 200(-0.68 \times \sin^2 30^{\circ} + 13.0 \times \cos^2 30^{\circ} - 16.4) \times 10^{-6}\ \text{in/in}$$

$$= -\ 1364\text{x}10^{-6}\ \text{in/in} = -0.14\%$$

$$\epsilon_{\ell 12} = \Delta T(\alpha_{cyy} - \alpha_{cxx})\sin 2\theta$$

$$= 200\left[13.0 - (-0.68)\right] \times 10^{-6}\ \sin 60^{\circ}$$

$$= 2369 \times 10^{-6}\ \text{in/in} = 0.24\%$$

<u>-30°-ply.</u> - The thermal strains in the -30° ply are the same as those in the $+30^{\circ}$ plies for $\epsilon_{\ell 11}$ and $\epsilon_{\ell 22}$ and opposite sign for $\epsilon_{\ell 12}$. The reader can readily verify that this is the case by inspection of the appropriate equations.

<u>0°-ply.</u> - Substituting the above numerical values and $\theta = 0^{\circ}$ in equations (5.3) we calculate:

$$\varepsilon_{\ell 11} = \Delta T(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11})$$

$$= 200(-0.68 \times \cos^2 0^o + 13.0 \times \sin^2 0^o - 0.4) \times 10^{-6} \text{ in/in}$$

$$= -216\text{x}10^{-6} \text{ in/in} = -0.02\%$$

$$\varepsilon_{\ell 22} = \Delta T(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22})$$

$$= 200(-0.68 \sin^2 0^o + 13.0 \times \cos^2 0^o - 16.4) \times 10^{-6}$$

$$= -680\text{x}10^{-6} \text{ in/in} = -0.07\%$$

$$\varepsilon_{\ell 12} = \Delta T(\alpha_{cyy} - \alpha_{cxx})\sin 2\theta$$

$$= 200\left[13.0 - (-0.68)\right] \times 10^{-6} \sin 2(0^o)$$

$$= 0.0$$

The reader will find it instructive to compare the corresponding thermal strains in the 30^o and 0^o plies. Both normal strains $\varepsilon_{\ell 11}$ and $\varepsilon_{\ell 22}$ in the 30^o ply are about twice those in the 0^o plies while the shear strain $\varepsilon_{\ell 12}$ has the largest magnitude in the 30^o plies and is "zero" in the 0^o plies.

Example 5.3. - Calculate the corresponding thermal stresses in the plies in the APL of Example 5.2. To calculate the corresponding thermal stresses we can use either equations (5.4) or (5.5) since we already calculated the strains in Example 5.2. We will use equations (5.5) for convenience. In order to use equations (5.5), we need numerical values for the reduced ply stiffnesses Q_ℓ and corresponding thermal strain values ε_ℓ from Example 5.2. We tabulate these numerical values for convenience.

Reduced Ply Stiffnesses in 10^6 psi (fig. 7 at $\theta = 0^o$ or from Example 5.1)

$Q_{\ell 11} = 18.7$

$Q_{\ell 22} = 2.0$

$Q_{\ell 21} = Q_{\ell 12} = 0.60$

$Q_{\ell 33} = 0.56$

Thermal Strains in 10^{-6} in/in from Example 5.1 for $\theta =$

	30^o	-30^o	0^o
$\varepsilon_{\ell 11}$	468	468	-216
$\varepsilon_{\ell 22}$	-1364	-1364	-680
$\varepsilon_{\ell 12}$	2369	-2369	0

Using corresponding values in equations (5.5) we have (cancelling 10^6 with 10^{-6} for convenience)

+30°-ply: $\sigma_{\ell 11} = Q_{\ell 11}\varepsilon_{\ell 11} + Q_{\ell 12}\varepsilon_{\ell 12}$

$= 18.9 \times 468 + 0.6 \times (-1364) = 8367$ psi

$\sigma_{\ell 22} = Q_{\ell 21}\varepsilon_{\ell 11} + Q_{\ell 22}\varepsilon_{\ell 22}$

$= 0.6 \times 468 + 2.0 \times (-1364) = -2447$ psi

$\sigma_{\ell 12} = Q_{\ell 33}\varepsilon_{\ell 33} = 0.56 \times 2369 = 1327$ psi

-30°-ply: $\sigma_{\ell 11}$ and $\sigma_{\ell 22}$ are the same as for the +30° ply:

$\sigma_{\ell 12}$ has opposite sign or $\sigma_{\ell 12} = -1327$ psi

0°-ply: $\sigma_{\ell 11} = Q_{\ell 11}\varepsilon_{\ell 11} + Q_{\ell 12}\varepsilon_{\ell 12}$

$= 18.9 \times (-216) + 0.60 \times (-680) = -4490$ psi

$\sigma_{\ell 22} = Q_{\ell 21}\varepsilon_{\ell 11} + Q_{\ell 22}\varepsilon_{\ell 22}$

$= 0.60 \times (-216) + 2.0 \times (-680) = -1490$ psi

$\sigma_{\ell 22} = Q_{\ell 33}\varepsilon_{\ell 33} = 0.56 \times (0) = 0$

The interesting point to be noted from the numerical values of the ply thermal stresses is that the transverse ply stresses $\sigma_{\ell 22}$ are compressive and are only about 22 percent of the compressive ply strength ($S_{\ell 22C}$ equals about 20 ksi at 270° F). This implies that thermal fatigue in the range $70^{\circ} \leq T \leq 270^{\circ}$ is not anticipated to cause progressive degradation to the ply transverse properties in the APL considered.

6.0 PLY LAMINATION RESIDUAL STRAINS AND STRESSES

Ply lamination residual strains and stresses are a special case of thermal strains and stresses. However, because of their importance in composite laminates they need be treated in a separate section. These strains and stresses arise from the fabrication procedure of the composite laminates: "The difference between the cure and use temperatures" as mentioned previously. They are always present in the plies of APLs (free or restrained) and their magnitude need be determined to accurately assess the thermomechanical integrity of APLs in structural applications. The laminations residual strains can be calculated using equations (5.3) and the lamination residual stresses can be calculated using equations (5.4) or equations (5.5) when the laminations residual strains are either known or they have been calculated previously. Usually, the lamination residual strains and stresses are calculated at room temperature. The temperature difference ΔT for these cases is known from the specified cure conditions. For example, the ΔT for APLs made from structural epoxies is about -300° F. The minus sign is determined from the definition of ΔT given in section 5.0:

$$\Delta T = \underset{\text{(about } 70^{\circ}\text{F)}}{\text{Room Temperature}} - \underset{\substack{\text{(about } 370^{\circ}\text{ F for} \\ \text{structural epoxies)}}}{\text{Cure Temperature}}$$

Calculations of lamination residual strains and stresses will be illustrated using the APLs considered in Example 5.2.

Example 6.1. - Calculate the lamination residual strains and stresses in the plies of the $[\pm 30/0_2]_S$ APL made from AS/E composite. First, we calculate the lamination residual strains using equations (5.3). The numerical values we need to use in these equations are obtained from Examples 5.2 and 5.3 and are summarized below for convenience.

$$\alpha_{cxx} = -0.68\times10^{-6} \text{ in/in/}^{\circ}\text{F} \qquad Q_{\ell 11} = 18.7\times10^{6} \text{ psi}$$

$$\alpha_{cyy} = 13.0\times10^{-6} \text{ in/in/}^{\circ}\text{F} \qquad Q_{\ell 22} = 2.0\times10^{6} \text{ psi}$$

$$\alpha_{\ell 11} = 0.4\times10^{-6} \text{ in/in/}^{\circ}\text{F} \qquad Q_{\ell 21} = Q_{\ell 12} = 0.60\times10^{6} \text{ psi}$$

$$\alpha_{\ell 22} = 16.4\times10^{-6} \text{ in/in/}^{\circ}\text{F} \qquad Q_{\ell 33} = 0.56\times10^{6} \text{ psi}$$

$$\theta = 30^{\circ}, -30^{\circ}, 0^{\circ}; \quad \Delta T = (70^{\circ}\text{ F} - 370^{\circ}\text{ F}) = -300^{\circ}\text{ F}$$

Using appropriate numerical values in equations (5.3) we calculate the lamination residual strain:

$$\varepsilon_{\ell 11} = \Delta T(\alpha_{cxx}\cos^2\theta + \alpha_{cyy}\sin^2\theta - \alpha_{\ell 11})$$

$\pm 30^{\circ}$ PLY: $\quad = -300\left[(-0.68)\times 0.75 + 13.0\times 0.25 - 0.4\right]\times 10^{-6} = -702\times10^{-6}$ in/in

0° PLY: $\quad = -300\left[(-0.68)(1.0) + 13.0\times 0.0 - 0.4\right]\times 10^{-6} = 324\times10^{-6}$ in/in

$$\varepsilon_{\ell 22} = \Delta T(\alpha_{cxx}\sin^2\theta + \alpha_{cyy}\cos^2\theta - \alpha_{\ell 22})$$

$\pm 30^{\circ}$ PLY: $= -300[(-0.68) \times 0.25 + 13.0 \times 0.75 - 16.4] \times 10^{-6} = 2046\times 10^{-6}$ in/in

0° PLY: $= -300[(-0.68) \times 0.0 + 13.0 \times 1.0 - 16.4] \times 10^{-6} = 1020\times 10^{-6}$ in/in

$$\varepsilon_{\ell 12} = \Delta T(\alpha_{cyy} - \alpha_{cxx}) \sin 2\theta$$

$\pm 30^{\circ}$ PLY: $= -300[13.0 - (-0.68)](\pm \sin 60^{\circ}) \times 10^{-6} = \mp 3554\times 10^{-6}$ in/in

0° PLY: $= -300[13.0 - (-0.68)](0.0) \times 10^{-6} = 0$

Using appropriate Q_{ℓ} values and ε_{ℓ} values in equations (5.5) we calculate the ply lamination residual stresses (cancelling 10^6 with 10^{-6}):

$$\sigma_{\ell 11} = Q_{\ell 11}\varepsilon_{\ell 11} + Q_{\ell 12}\varepsilon_{\ell 22}$$

$\pm 30^{\circ}$ PLY: $= 18.7 \times (-702) + 0.60 \times 2046 = -11\,900$ psi $= -11.9$ ksi

0° PLY: $= 18.7 \times 324 + 0.60 \times 1020 = 6671$ psi $= 6.7$ ksi

$$\sigma_{\ell 22} = Q_{\ell 21}\varepsilon_{\ell 11} + Q_{\ell 22}\varepsilon_{\ell 22}$$

$\pm 30^{\circ}$ PLY: $= 0.60 \times (-702) + 2.0 \times 2046 = 3671$ psi $= 3.7$ ksi

0° PLY: $= 0.60 \times 324 + 2.0 \times 1020 = 2234$ psi $= 2.2$ ksi

$$\sigma_{\ell 12} = Q_{\ell 33}\varepsilon_{\ell 33}$$

$\pm 30^{\circ}$ PLY: $= 0.56 \times (\mp 4104) = \mp 2298$ psi $= \mp 2.3$ ksi

0° PLY: $= 0.56 \times 0.0 = 0.0$

The transverse lamination residual stresses in the ± 30 plies are about 50 percent of the corresponding ply strength (6 to 8 ksi).

7.0 CONCLUDING REMARKS

A convenient procedure is described to determine the thermal behavior (thermal expansion coefficients, thermal and residual stresses) of angleplied fiber composites. The procedure consists of equations and appropriate graphs for various ($\pm\theta$) ply combinations. These graphs consist of reduced stiffness and thermal expansion coefficients for frequently used composites and hybrids as functions of $\pm\theta$ in order to simplify and expedite the use of the equations. The procedure is applicable to all types of balanced, symmetric fiber composites including interply and intraply hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples. The step-by-step numerical examples are set up so that the calculations can be made using a pocket calculator. Some of the numerical examples were selected to illustrate significant implications of composite thermal behavior in design applications.

8.0 SYMBOLS

APL	angleplied laminate
AS/E	AS-graphite-fiber/epoxy-matrix composite
B/E	boron-fiber/epoxy-matrix composite
E_c	laminate modulus - subscripts x,y denote structural axes directions
E_ℓ	ply modulus - subscripts 1,2 denote ply material axes directions
HMS/E	high modulus graphite-fiber/epoxy-matrix composite
K/E	Kevlar-fiber/epoxy matrix composite
Q_c	reduced laminate stiffness - subscripts x,y denote structural axes directions
Q_ℓ	reduced ply stiffness - subscripts x,y denote ply material axes directions
Q_θ	reduced stiffness for $\pm\theta$ symmetric laminate - subscripts 1,2 denote material axes directions
S_ℓ	ply strength - subscripts 1,2 denote ply material axes directions; - subscripts T, C, S denote type
S-G/E	S-glass-fiber/epoxy-matrix composite
T	temperature
T_o	reference temperature
ΔT	temperature difference between use and reference temperature
TEC	thermal expansion coefficient

V_p	ply thickness ratio - subscripts θ, 0, 90 denote ply designation to which the ratio applies
x,y,z	structural axes coordinate directions
1,2,3	material axes coordinate directions - 1 taken along the fiber direction
$[-/-/-]_S$	laminate configuration designation - numbers in the blanks denote ply stacking sequence and orientation - subscript S denotes symmetry about ply in last blank space
α_c	laminate TEC - subscripts x,y denote laminate structural axes directions
α_ℓ	ply TEC - subscripts 1,2 denote ply material axes directions
α_θ	$\pm\theta$ laminate TEC - subscripts 1,2 denote material axes directions
ϵ_c	laminate strain - subscripts x,y denote structural axes directions
ϵ_ℓ	ply strain - subscripts 1,2 denote material axes directions
θ	ply orientation angle measured from the x-laminate structural axes to the 1-ply material axes and taken positive
ν_c	laminate Poisson's ratio - subscripts x,y denote structural axes directions
ν_ℓ	ply Poisson's ratio - subscripts 1,2 denote ply material axes directions
σ_c	laminate stress - subscripts x,y denote structural axes directions
σ_ℓ	ply stress - subscripts 1,2 denote material axes directions
ϕ	laminate coordinate axes x', y', z' orientation other than the structural axes x, y, z measured from the x axis to the x'-axis and taken positive.

Conversion factors:

$^\circ C = \frac{5}{9}(^\circ F - 32^\circ)$

$\Delta ^\circ C = \frac{5}{9} \Delta ^\circ F$

$cm/cm/^\circ C = \frac{9}{5} in/in/^\circ F$

$MPa = 6.89 ksi$

REFERENCES

1. C. C. Chamis, "Lamination Residual Stresses in Cross-plied Fiber Composites," Proceedings of the 26th Annual Technical Conference, Reinforced Plastic-Composites Division, Society of the Plastics Industry, Inc., 1971, Section 17D, pp. 1-12.

2. C. C. Chamis, "Computer Code for the Analysis of Multilayered Fiber Composites - User's Manual," NASA TN D-7013, 1971.

3. C. C. Chamis, "Computerized Multilevel Analysis for Multilayered Fiber Composites," Comput. Struct., vol. 3, no. 3, 1973, pp. 467-482.

4. C. C. Chamis, "Residual Stresses in Angleplied Laminates and Their Effects on Laminate Behavior," NASA TM-78835, 1978.

5. C. C. Chamis, "Prediction of Fiber Composite Mechanical Behavior Made Simple," Rising to the Challenge of the '80's; Proceedings of the Thirty-Fifth Annual Conference, Reinforced Plastics/Composites Institute, Society of the Plastic Industry, Inc., 1980, Section 12A, pp. 1-10.

TABLE I. - TYPICAL PROPERTIES OF UNIDIRECTIONAL FIBER COMPOSITES AT ROOM TEMPERATURE

Properties	Units	Boron/ epoxy	Boron/ polyimide	Scotchply/ epoxy	Modmor I/ epoxy	Modmor I/ polyimide	Thornel 300/ epoxy	Kevlar 49/ epoxy	Graphite As/epoxy
1. Fiber volume ratio	-------	0.50	0.49	0.72	0.45	0.45	0.70	0.54	0.60
2. Density	lb/in^3	0.073	0.072	0.077	0.056	0.056	0.058	0.049	0.057
3. Longitudinal thermal coefficient	$10^{-6}in/in/^{\circ}F$	3.4	2.7	2.1	-------	0.0	0.01	-1.60	0.40
4. Transverse thermal coefficient	$10^{-6}in/in/^{\circ}F$	16.9	15.8	9.3	18.5	14.1	12.5	31.3	16.4
5. Longitudinal modulus	10^6 psi	29.2	32.1	8.8	27.5	31.3	26.3	12.2	16.0
6. Transverse modulus	10^6 psi	3.15	2.1	3.6	1.03	0.72	1.5	0.70	2.2
7. Shear modulus	10^6 psi	0.78	1.11	1.74	0.9	0.65	1.0	0.41	0.72
8. Major Poisson's ratio	-------	0.17	0.16	0.23	0.10	0.25	0.28	0.32	0.25
9. Minor Poisson's ratio	-------	0.02	0.02	0.09	-------	0.02	0.01	0.02	0.34
10. Longitudinal tensile strength	psi	199 000	151 000	187 000	122 000	117 000	218 000	172 000	220 000
11. Longitudinal compressive strength	psi	232 000	158 000	119 000	128 000	94 500	247 000	42 000	180 000
12. Transverse tensile strength	psi	8100	1600	6670	6070	2150	5850	1600	8000
13. Transverse compressive strength	psi	17 900	9100	25 300	28 500	10 200	35 700	9400	36 000
14. Intralaminar shear strength	psi	9100	3750	6500	8900	3150	9800	4000	10 000

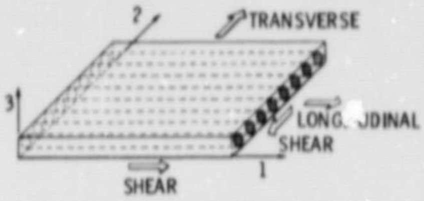

Figure 1. - Schematic of single ply.

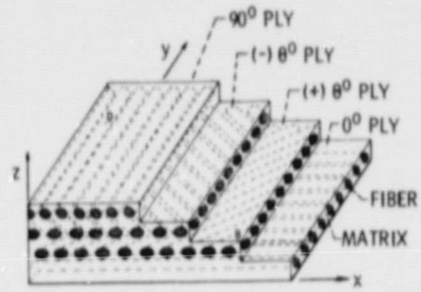

Figure 2. - Schematic of angleplied laminate.

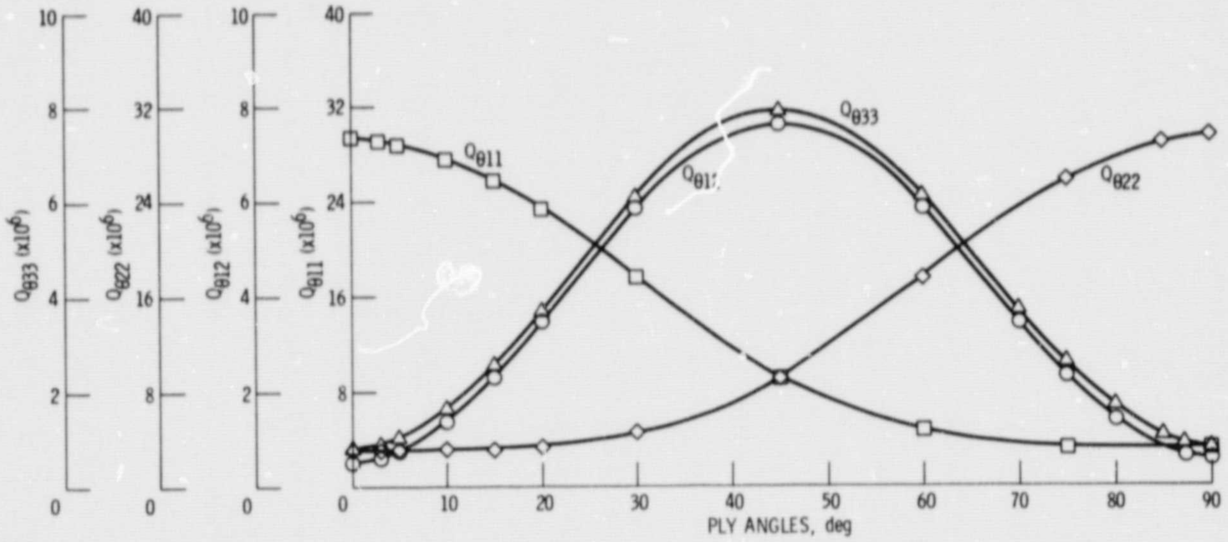

Figure 3. - Reduced stiffnesses of boron-fiber/epoxy (B/E) ±θ laminates.

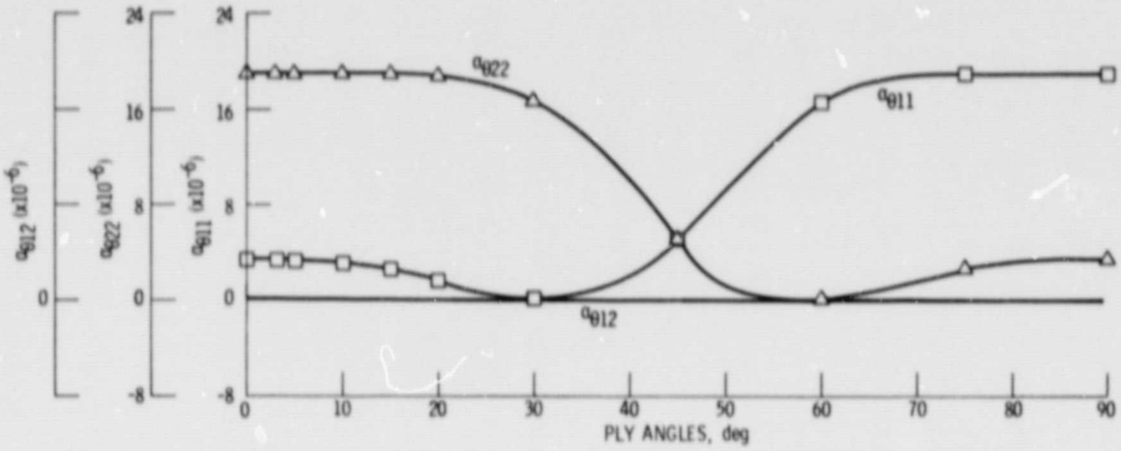

Figure 4. - Thermal expansion coefficients of boron-fiber/epoxy (B/E) ±θ laminates.

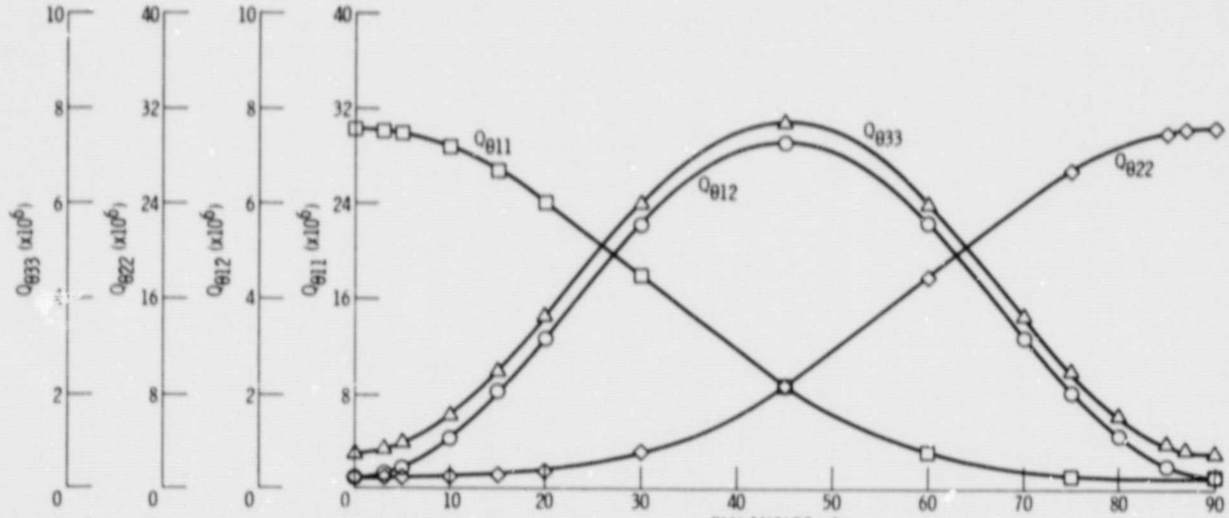

Figure 5. - Reduced stiffnesses of high modulus graphite-fiber/epoxy (HMG/E) ±θ laminates.

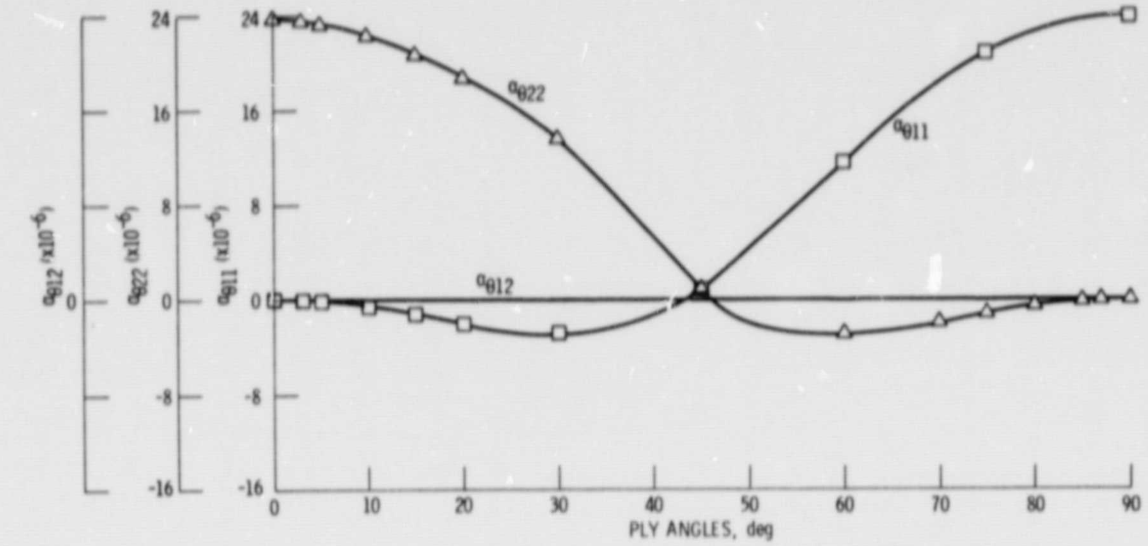

Figure 6. - Thermal expansion coefficients of high modulus graphite-fiber/epoxy (HMG/E) ±θ laminates.

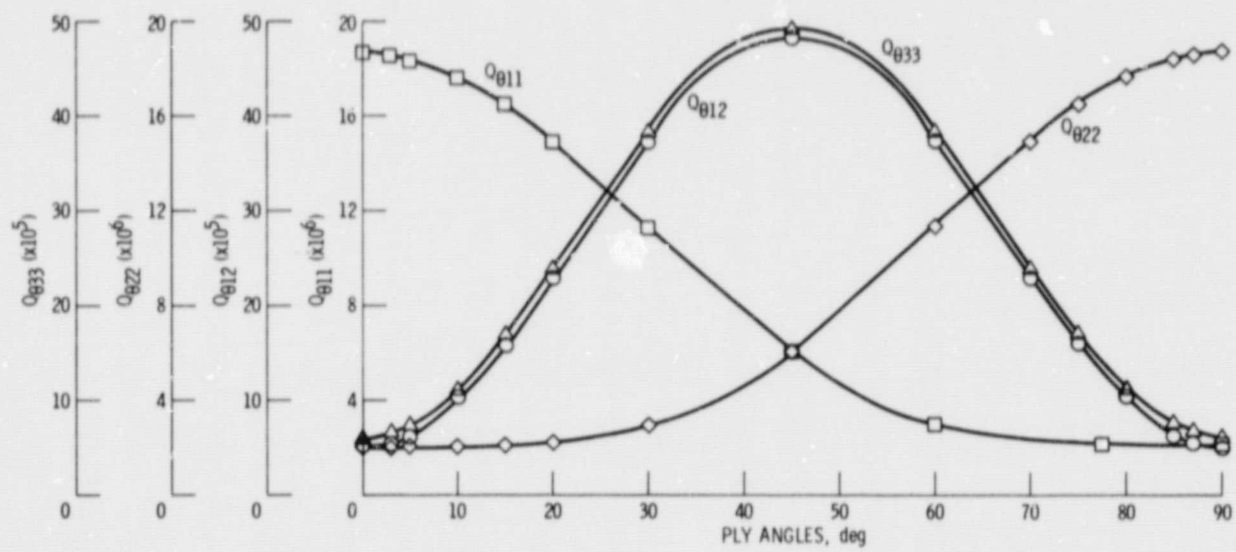

Figure 7. - Reduced stiffnesses of AS graphite-fiber/epoxy (AS/E) ±θ laminates.

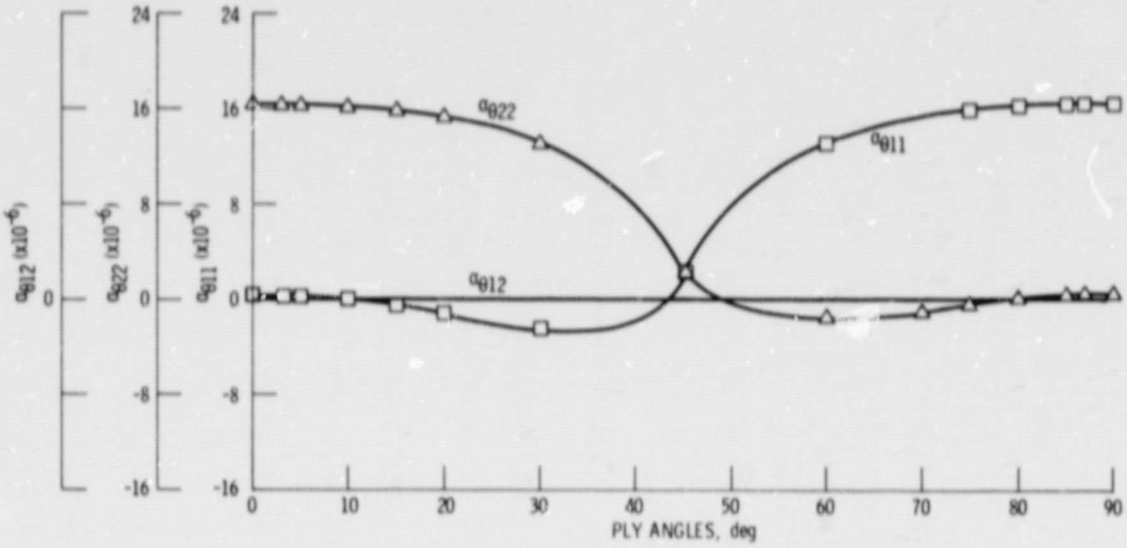

Figure 8. - Thermal expansion coefficients of AS graphite-fiber/epoxy (AS/E) ±θ laminates.

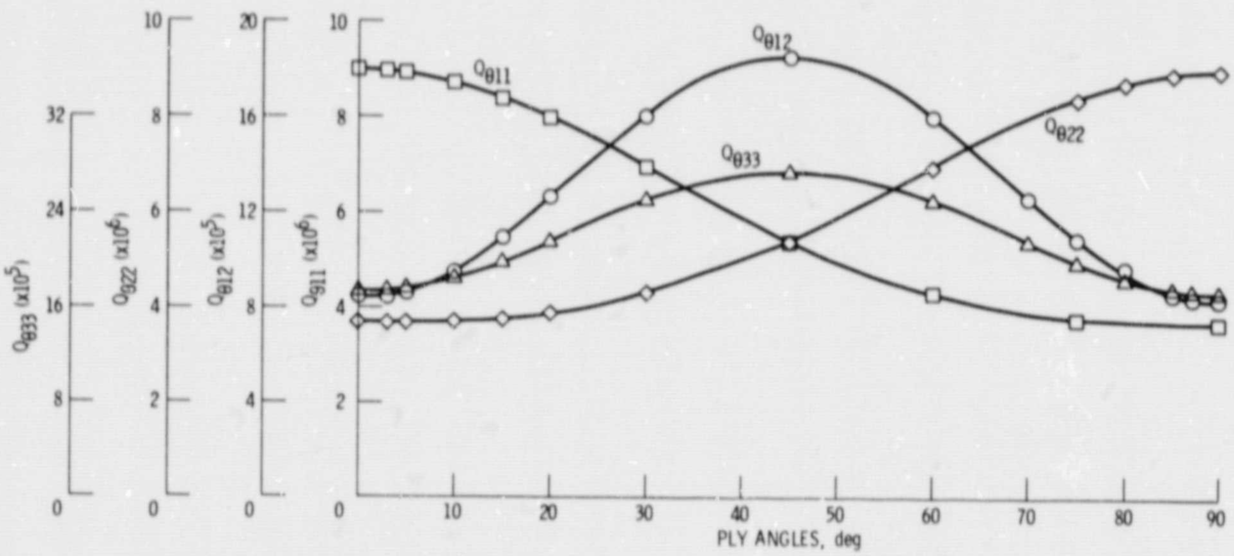

Figure 9. - Reduced stiffnesses of S-Glass-fiber/epoxy (S-G/E) ±θ laminates.

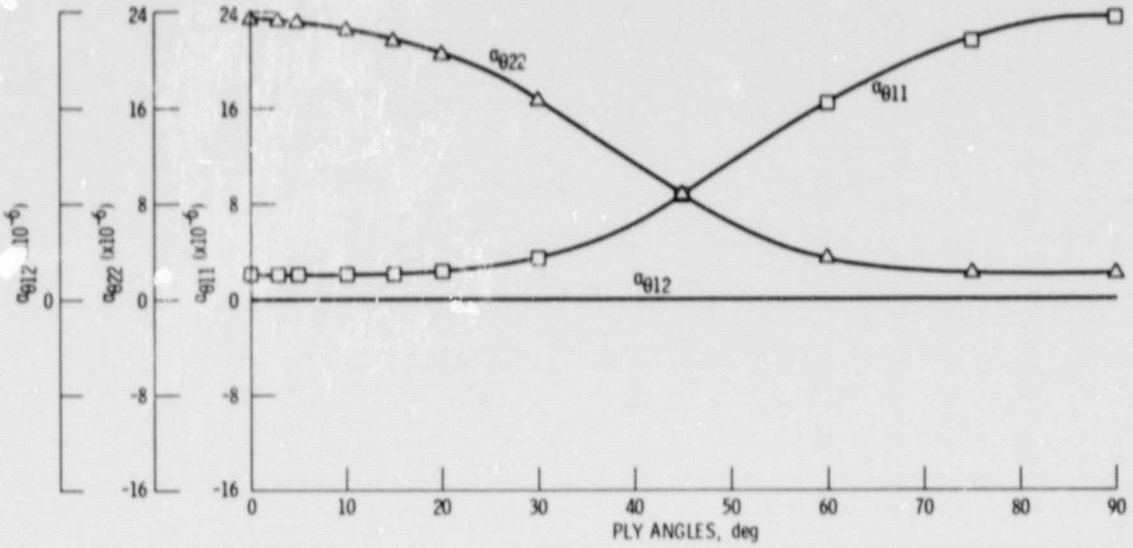

Figure 10. - Thermal expansion coefficients of S-Glass-fiber/epoxy (S-G/E)±θ laminates.

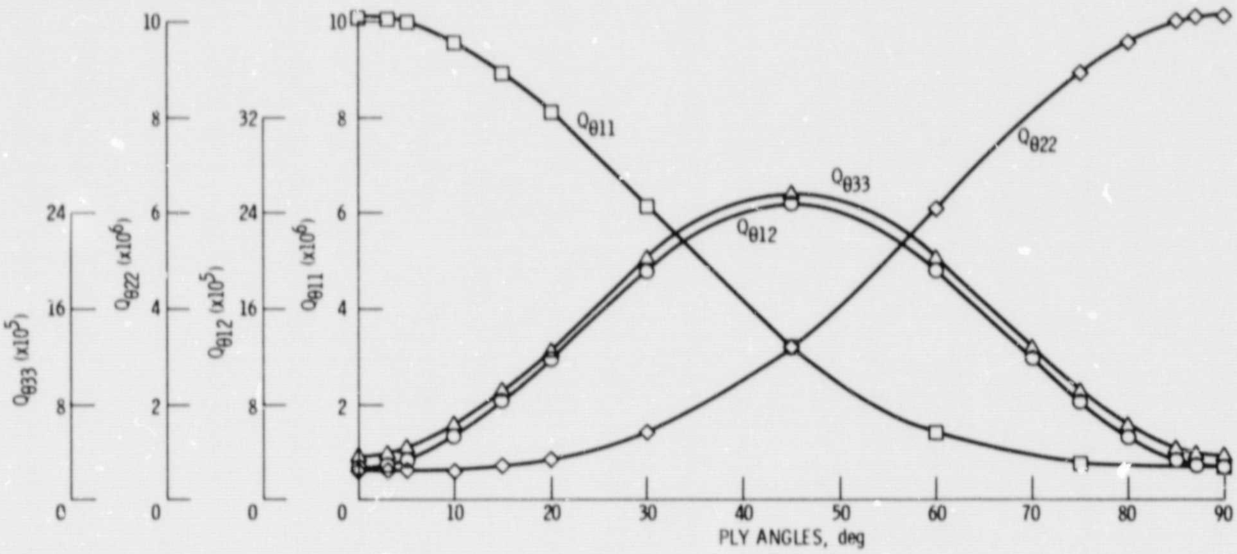

Figure 11. - Reduced stiffnesses of Kevlar 49-fiber/epoxy (K/E) ±θ laminates.

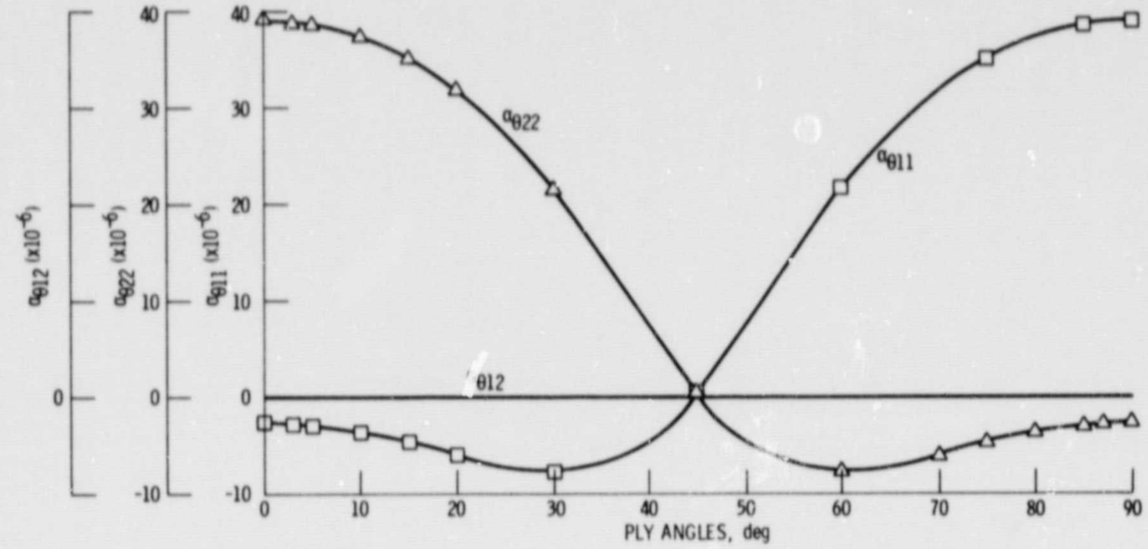

Figure 12. - Thermal expansion coefficients of Kevlar 49-fiber/epoxy (K/E) ±θ laminates.

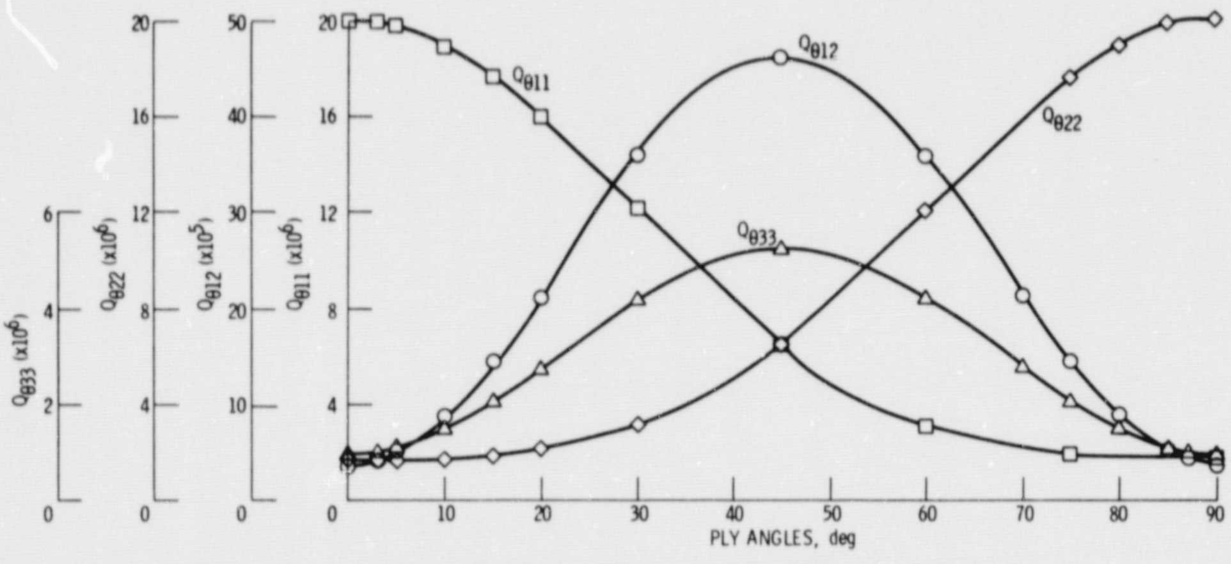

Figure 13. - Reduced stiffnesses of intraply hybrid (80% HMG/E//20% S-G/E) ±θ laminates.

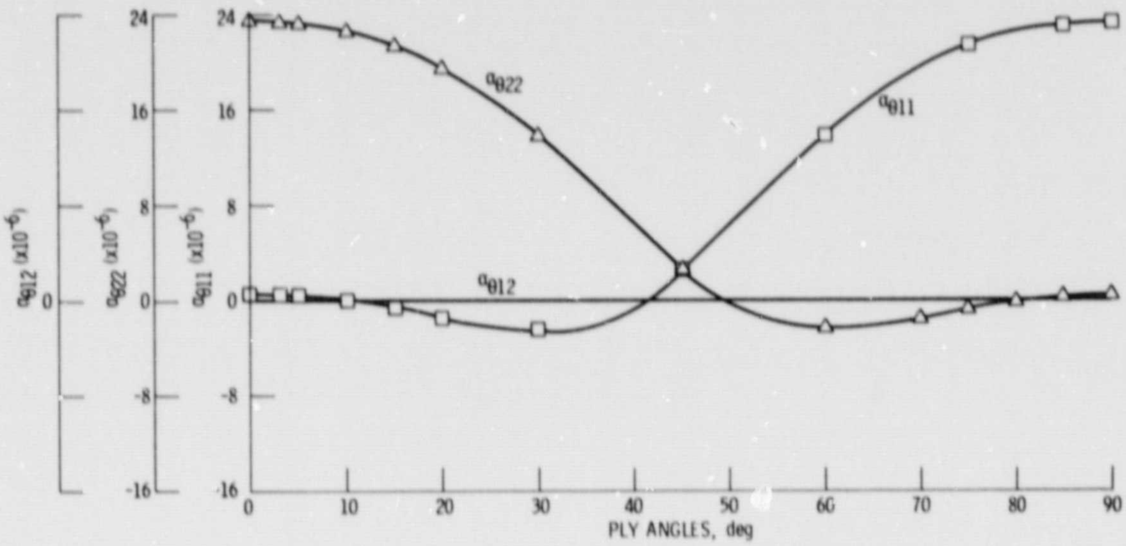

Figure 14. - Thermal expansion coefficients of intraply hybrid (80% HGM/E//20% S-G/E) ±θ laminates.

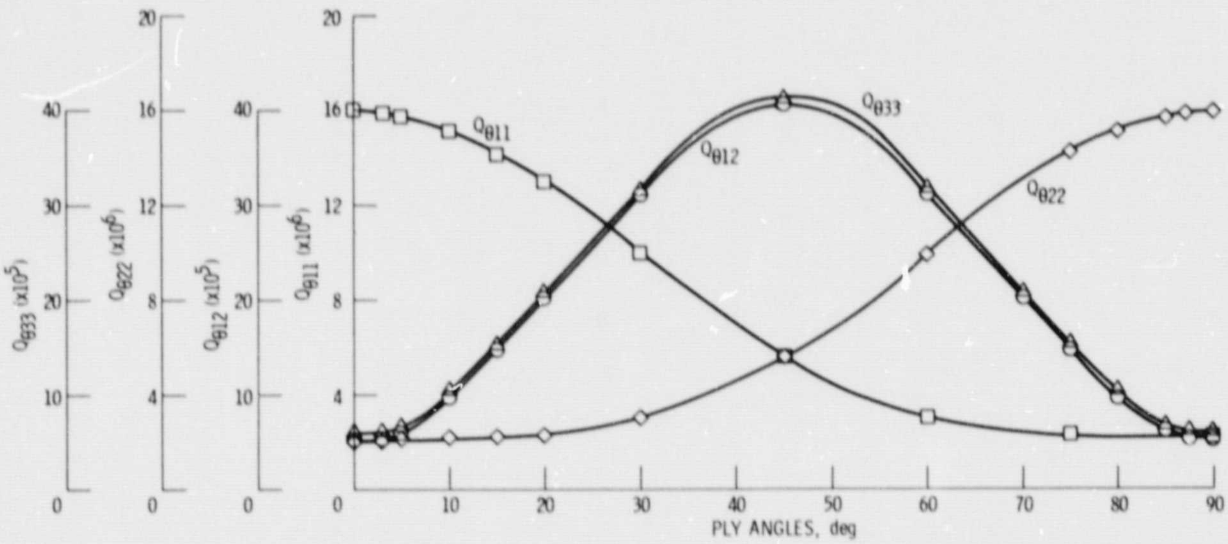

Figure 15. - Reduced stiffnesses of intraply hybrid (80% AS/E//20% S-G/E) ±θ laminates.

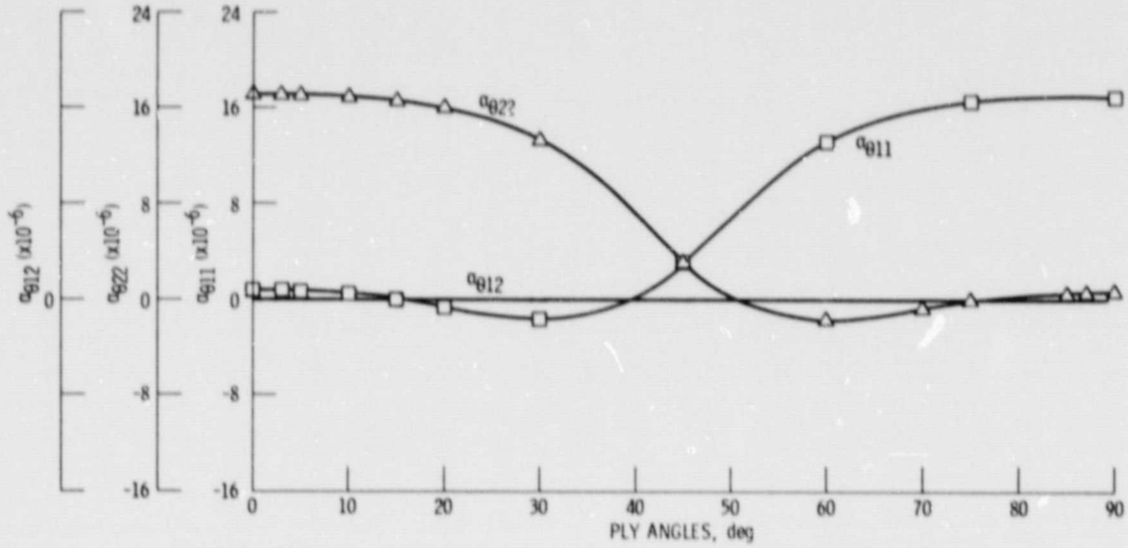

Figure 16. - Thermal expansion coefficients of intraply hybrid (80% AS/E//20% S-G/E) ±θ laminates.

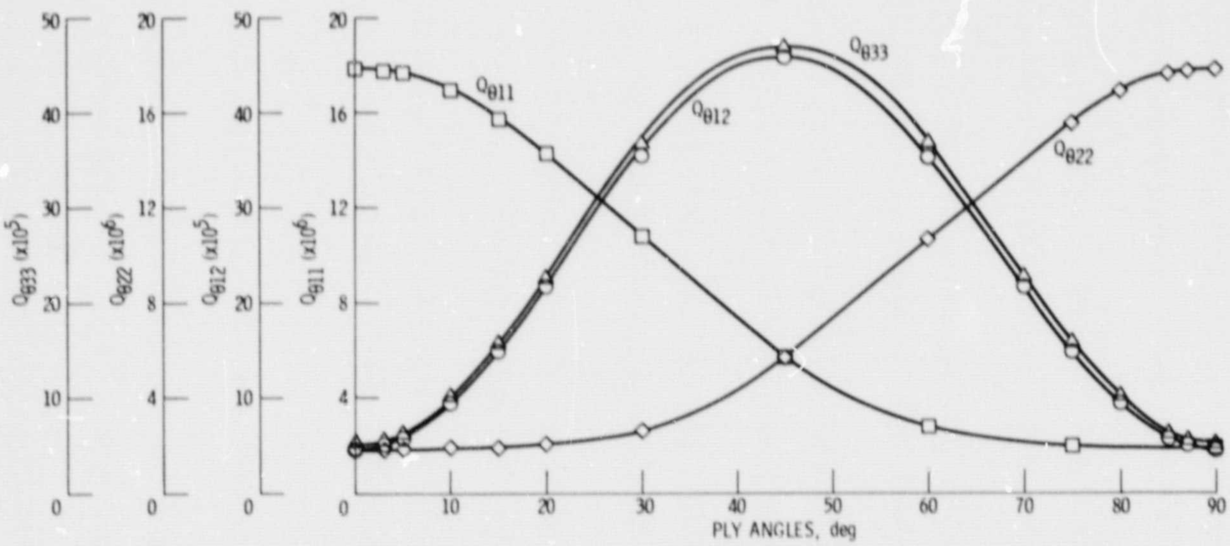

Figure 17. - Reduced stiffnesses of intraply hybrid (80% AS/E//20% K/E) ±θ laminates.

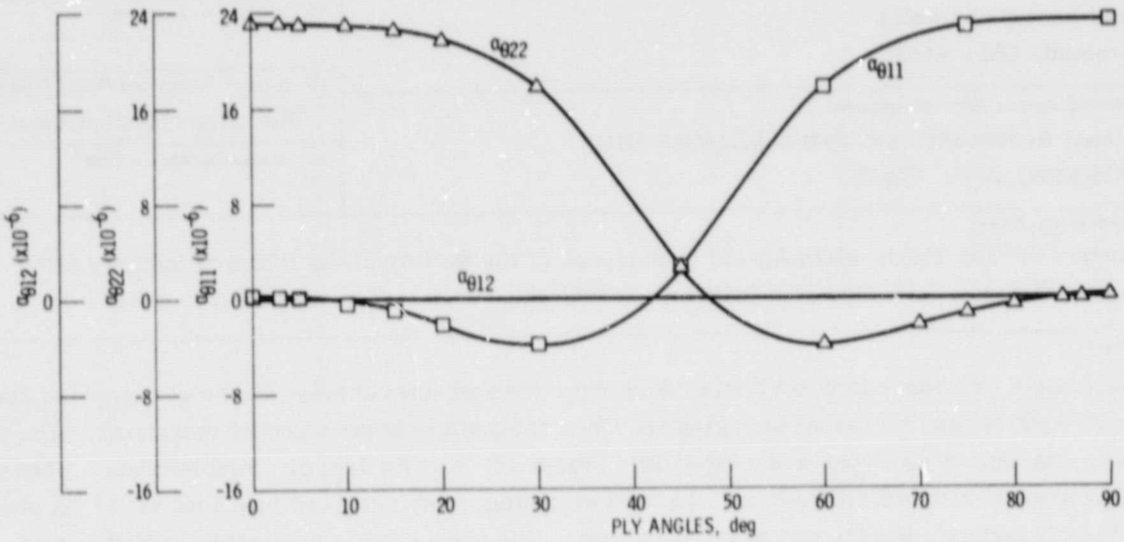

Figure 18. - Thermal expansion coefficients of intraply hybrid (80% AS/E//20%) ±θ laminates.