

IONOSPHERIC EFFECTS IN ACTIVE RETRODIRECTIVE ARRAY  
AND MITIGATING SYSTEM DESIGN

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Abstract

The operation of an active retrodirective array (ARA) in an ionospheric environment (that is either stationary or slowly-varying) is examined. The restrictions imposed on the pilot-signal structure as a result of such operation are analyzed. A 3-tone pilot beam system is defined which first estimates the total electron content along paths of interest and then utilizes this information to aid the phase conjugator so that correct beam pointing can be achieved.

I. INTRODUCTION

In order to make the solar power satellite system perform correctly, it is necessary to point the high power downlink beam towards a specific point on ground. The downlink beam is narrow and pointing accuracy requirements are stringent. One way of achieving this objective is to use the retrodirective array such that the down-going power beam points in the same direction from which a ground-originated pilot signal came. In this approach, the downlink wavefront is obtained by conjugating the phases of various segments of the uplink (pilot) wavefront. For operational reasons, the uplink and downlink frequencies cannot be identical. Both the uplink and downlink wavefronts are required to travel through the ionosphere. The object of this note is to examine system operation constraints imposed by the ionosphere and find possible remedies. The discussion that follows is based on the assumption that the ionosphere is stationary or slowly-varying. Also, heating effects on the medium due to the downlink power beam are not taken into account.

II. IONOSPHERIC EFFECTS ON SINGLE-TONE PILOT BEAM

It is well-known that an important feature of the retrodirective array is that the down-coming beam is phase coherent when it arrives at the source.<sup>1</sup> This statement is rigorously correct only if the propagation medium is non-dispersive, spatially homogeneous and temporally stable. In case of the ionosphere, one or more of the above conditions are violated. Under certain conditions, beam pointing error can occur and phase coherence at the source can be lost.

Consider the situation shown in Figure 1. Assume the uplink and downlink frequencies are given by  $f_u$  and  $f_D$ , respectively ( $f_u \neq f_D$ ). The (path-dependent) phase shift at  $f_u$  on one particular radio link can be written as<sup>2</sup>

$$\phi(f_u) = \frac{2\pi f_u L}{c} - \frac{b}{2\pi f_u c} \int_0^L N \, dl \quad (1)$$

where

$$b = \frac{e^2}{2 \epsilon_0 m}; \quad e = \text{electron charge, } m = \text{electron mass,}$$
$$\epsilon_0 = \text{free-space permittivity}$$
$$= 1.6 \times 10^3 \text{ mks}$$

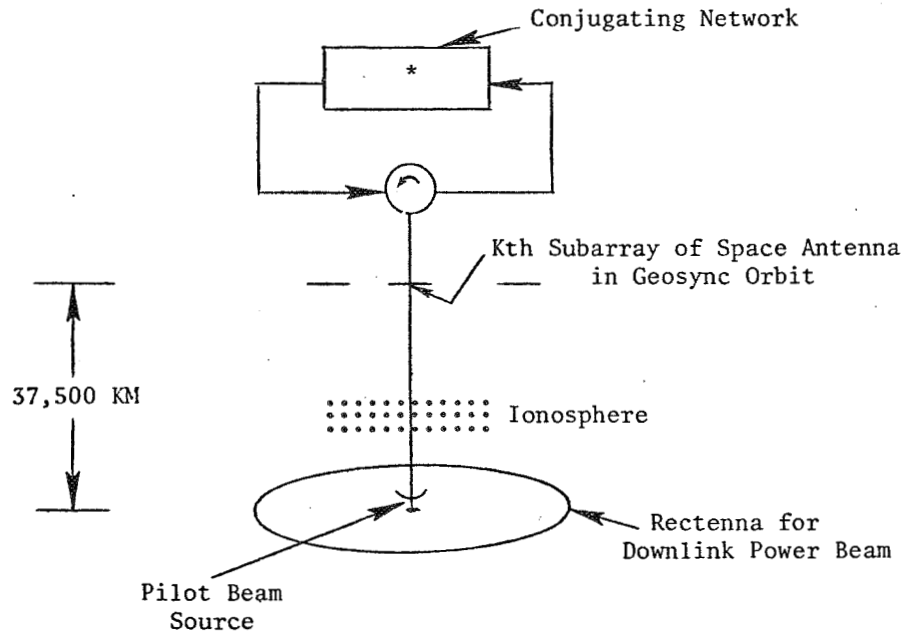


Figure 1.

$L$  is the physical path length involved and  $\int_0^L N \, dl$  is the integrated electron density along the path under consideration ( $\approx 10^{17} - 10^{19}$ ). Note the second quantity on the right hand side of Equation (1) accounts for ionospheric effects on a CW tone. On using appropriate constants, one can write

$$\begin{aligned} \phi(f_u) &= \frac{2\pi f_u L}{c} - 40.5 \times \frac{2\pi}{f_u c} \int_0^L N \, dl \\ &= \frac{2\pi f_u L}{c} - \frac{K_u}{f_u} \end{aligned} \quad (2)$$

Since one is interested in knowing the phase shift at  $f_D$ , a reasonable estimate of the phase can be obtained by multiplying  $\phi(f_u)$  by  $f_D/f_u$  (this estimate becomes increasingly accurate as  $f_u \rightarrow f_D$ ). Thus,

$$\begin{aligned} \tilde{\phi}(f_D) &= f_D/f_u \times \phi(f_u) \\ &= \frac{2\pi f_D L}{c} - \frac{K_u}{f_u^2} \cdot f_D \end{aligned} \quad (3)$$

On conjugating this phase, one obtains

$$\tilde{\phi}^*(f_D) = -\frac{2\pi f_D L}{c} + K_u \frac{f_D}{f_u^2} \quad (4)$$

The downlink signal at the transmitting end can be written as

$$S_{\text{down}}^T(t) = \cos \left[ \omega_D t + 2\pi f_D \frac{L}{C} - K_u \frac{f_D}{f_u^2} \right] \quad (5)$$

The downlink signal at the receiving end is given by

$$S_{\text{down}}^R(t) = \cos \left[ \omega_D t - \left( K_u \frac{f_D}{f_u^2} - \frac{K_D}{f_D} \right) \right] \quad (6)$$

For a temporally stable ionosphere and ignoring second-order effects, one can set  $K_u = K_D$  in Equation (6) and obtain

$$S_{\text{down}}^R(t) = \cos \left[ \omega_D t - K_u \left( \frac{f_D}{f_u^2} - \frac{1}{f_D} \right) \right] \quad (7)$$

If, in addition, the propagation medium is assumed non-dispersive, then the second term on the right hand side of Equation (7) involving  $K_u$  could be equated to zero. In the present situation, this kind of assumption is highly unrealistic. Note in Equation (7),  $K_u$  applies to a particular radio path and will, in general, be different on different paths because of ionospheric inhomogeneity. A consequence of this fact is that the phase coherence (at source) property of the downlink signal mentioned earlier does no longer hold good. Furthermore, if a coherent phase perturbation occurs due to some ionospheric large-scale features (such as a wedge), then even a beam pointing error is possible. The magnitude of these effects need to be evaluated for worst-case ionospheric conditions. The two tone pilot beam system which aims at alleviating some of the ionospheric problems mentioned above is discussed next.

### III. TWO-TONE PILOT BEAM SYSTEM

If two tones (symmetrically situated around the downlink frequency) are used on the uplink transmission, then under appropriate conditions an average of the phases of the uplink tones can be taken to be a good estimate of the phase at the downlink frequency. The idea here is that the phase errors caused by a stationary ionosphere can be largely eliminated by this approach. Let  $f_1$  and  $f_2$  be the two tones constituting the pilot beam and symmetrically located around the downlink frequency  $f_D$ . The choice of the offset  $\Delta f$  is based on conflicting requirements and is not discussed here.

Using the notation as before, for a given link one can write

$$\phi(f_1) = 2\pi f_1 \frac{L}{C} - \frac{40.5}{f_1} \times \frac{2\pi}{C} \int_0^L N \, d\ell = \phi_1 \quad (8)$$

and

$$\phi(f_2) = 2\pi f_2 \frac{L}{C} - \frac{40.5}{f_2} \times \frac{2\pi}{C} \int_0^L N \, d\ell = \phi_2 \quad (9)$$

Then

$$\begin{aligned} \bar{\phi} &= \frac{\phi(f_1) + \phi(f_2)}{2} \\ &= 2\pi f_D \frac{L}{C} - \frac{40.5}{f_D} \times \frac{2\pi}{C} \int_0^L N \, d\ell; \quad \left| \frac{\Delta f}{f_D} \right| \ll 1 \\ &= \phi(f_D) \end{aligned} \quad (10)$$

Note  $\bar{\phi}$  is a desirable quantity as far as correct retrodirective array operation is concerned. Normally, all one needs to do is to conjugate this quantity and use it as the phase of the downlink signal leaving the space antenna. However, the arithmetic averaging indicated in Equation (10) can give wrong answers for  $\bar{\phi}$  (often called  $\Pi$  ambiguities). This can happen if

$$(i) \quad \phi(f_2) - \phi(f_1) = K (2\Pi) + \Delta; \quad |\Delta| < 2\Pi \text{ and } K \text{ is odd integer}$$

and/or

(ii) asynchronous dividers are used.

It is clear that in spite of its inherent attractiveness, the 2-tone pilot beam system cannot be used because of the  $\Pi$  ambiguities that can occur during phase averaging.

#### IV. THREE-TONE PILOT BEAM SYSTEM

Before proceeding with the main task of solving the phase conjugation problem in an ionospheric environment, it is worthwhile to find out whether  $\phi_1$  and  $\phi_2$  could indeed differ by integral multiples of  $2\Pi$  when typical SPS parameters are used. For the present problem, it is sufficient to show that ionospheric effects alone can give rise to phase differences which are multiples of  $2\Pi$ . A measure of this effect is obtained by multiplying  $\phi_1$  (Equation (8)) by  $f_2/f_1$  and subtracting  $\phi_2$  (Equation 9)). Thus

$$\begin{aligned} \Delta\phi &= \frac{f_2}{f_1} \phi_1 - \phi_2 \\ &= 2\Pi \times \left\{ \frac{40.5}{C} \times \int_0^L N \, d\ell \times \left[ \frac{1}{f_2} - \frac{f_2}{f_1^2} \right] \right\} \end{aligned} \quad (11)$$

Let

$$f_D = 2.45 \times 10^9 \quad (12a)$$

$$\text{and } \begin{cases} f_1 = f_D - \Delta f \\ f_2 = f_D + \Delta f \end{cases} \quad (12b)$$

$$(12c)$$

then, the number of  $2\Pi$  phase changes obtained for different values of  $\int N \, d\ell$  and  $\Delta f$  is shown in Table 1.

Table 1. Number of Ambiguities ( $\eta$ ) Vs.  $\Delta f$

$\Delta f$ MHz	$f_1$ GHz	$f_2$ GHz	$10^{19} \frac{el/m^2}{\eta}$	$10^{18} \frac{el/m^2}{\eta}$	$\int N d\ell$
100	2.350	2.550	92	9.2	
50	2.400	2.500	45	4.5	
10	2.440	2.460	8.9	0.89	
5	2.445	2.455	4.4	0.44	
1	2.449	2.451	0.9	0.09	

It is clear from Table 1 that in order to avoid ionospheric ambiguity for the strongest concentration under consideration,  $\Delta f$  should not exceed 1 MHz. Other operational constraints render such a choice unacceptable.

In what follows, a 3-tone approach due to Burns and Fremouw is used to resolve the ambiguity problem.<sup>3</sup> It is based on a direct measurement of  $\int N d\ell$  along the paths of interest and then using this information to estimate the path related phase shift at the downlink frequency  $f_D$ .

Consider a frequency-amplitude pattern as shown in Figure 2 where the three uplink tones  $f_1$ ,  $f_2$  and  $f_3$  are coherent at ground. Indeed, the three tones can be generated by a low-deviation phase-modulated transmitter. Thus, using equations similar to Equation (8) for three frequencies  $f_1$ ,  $f_2$  and  $f_3$ , one can write

$$\begin{aligned} \delta\phi_A &= \phi_2 - \phi_1 \\ &= \frac{2\pi}{C} \left\{ (f_2 - f_1) L - 40.5 \times \int N d\ell \times \left( \frac{1}{f_2} - \frac{1}{f_1} \right) \right\} \end{aligned} \quad (13)$$

and

$$\begin{aligned} \delta\phi_B &= \phi_1 - \phi_3 \\ &= \frac{2\pi}{C} \left\{ (f_1 - f_3) L - 40.5 \times \int N d\ell \times \left( \frac{1}{f_1} - \frac{1}{f_3} \right) \right\} \end{aligned} \quad (14)$$

The second difference of phase shift is given by

$$\begin{aligned} \delta_2\phi &= \delta\phi_A - \delta\phi_B \\ &= \frac{2\pi}{C} \times 40.5 \times \int N d\ell \times \left[ \frac{2}{f_1} - \frac{1}{f_3} - \frac{1}{f_2} \right] \end{aligned} \quad (15)$$

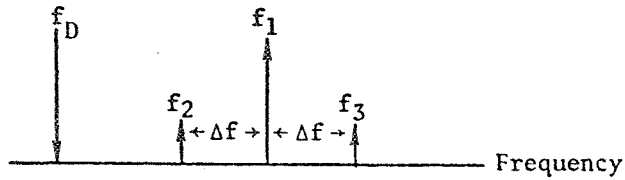


Figure 2.

For suitably chosen  $\Delta f$ , one obtains

$$\delta_2\phi \approx -\frac{2\pi}{C} \times 40.5 \times \int N d\ell \times \frac{2 \Delta f^2}{f_1^3} \quad (16)$$

Suppose one needs to avoid a  $360^\circ$  ambiguity in  $\delta_2\phi$  for values of  $\int N d\ell$  less than  $10^{19}$ . From Equation (16), one easily finds

$$\Delta f^2 \approx -\delta_2\phi \times f_1^3 / \left( \frac{2\pi}{C} \times 40.5 \times 2 \times \int N d\ell \right) \quad (17)$$

Let

$$\begin{aligned} f_1 &= 2.45 + 0.153125 \text{ (this choice will be justified later)} \\ &= 2.603125 \text{ GHz} \end{aligned} \quad (18)$$

Then

$$\begin{aligned} \Delta f^2 &= (2\pi) \times (2.6 \times 10^9)^3 \times C / (2\pi \times 81 \times 10^{19}) \\ &= 0.651 \times 10^{16} \end{aligned}$$

or

$$\Delta f \approx 80.6 \text{ MHz} \quad (19)$$

Thus, with  $\Delta f \leq 80.6 \text{ MHz}$  and assuming that  $\delta_2\phi$  can be measured, then  $\int N d\ell$  can be calculated rather easily from Equation (16). An implementation that measures  $\delta_2\phi$  with relative ease is shown in Figure 3.

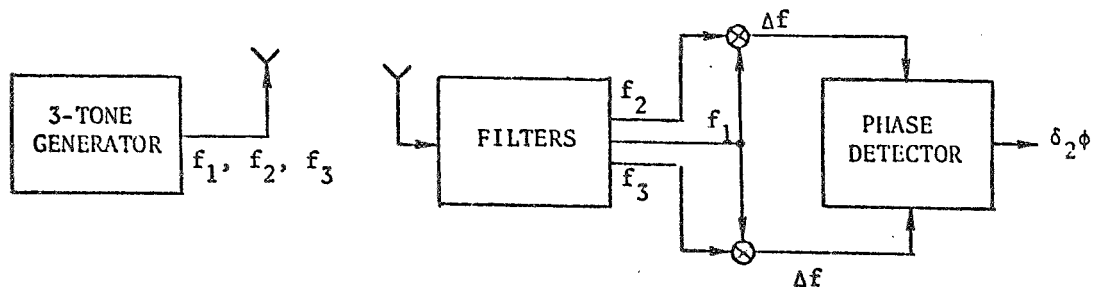


Figure 3. Measurement of  $\delta_2\phi$

Reordering Equation (16), one easily obtains

$$\begin{aligned}\hat{N} &= \text{computed value of } \int N d\ell \\ &= \frac{f_1^3}{2 \Delta f^2} \times \frac{C}{2\pi} \times \frac{1}{40.5} \times (-\delta_2 \phi)_{\text{measured}} \\ &= \alpha \cdot (-\delta_2 \phi)_{\text{measured}}\end{aligned}\tag{20}$$

For  $f_1 = 2.603$  GHz and  $\Delta f = 80.0$  MHz, one can compute

$$\alpha = 1.6 \times 10^{18}\tag{21}$$

Based on S/N ratio considerations, the accuracy of the  $\hat{N}$  computation in Equation (20) is determined by the accuracy of  $\delta_2 \phi$  measurement and is given by

$$\alpha \hat{N} = \alpha \cdot \sigma_{\delta_2 \phi}\tag{22}$$

Once an estimate of  $\int N d\ell$  for a given link is found, one needs to perform several steps of signal processing starting with the phase at  $f_1$  and finishing with the conjugated phase at  $f_D$ . These steps are shown in Figure 4.

It is fair to point out that the conjugator used is a modified version of the one in Reference 1. With the additional boxes, the new conjugator clearly takes into account steady-state ionospheric effects.

For the present configuration, the uplink and downlink frequencies are related by the equation\*

$$\frac{n}{n+2} \cdot f_1 = f_D$$

or

$$f_1 = \frac{n+2}{n} f_D\tag{23}$$

For  $f_D = 2.45$  GHz and  $n = 32$ , one obtains

$$f_1 = 2.603125 \text{ GHz (see Equation (18)).}$$

It is interesting to examine the output  $\phi^*(f_D)$  of the conjugator in Figure 4. On taking differentials, one obtains

$$\Delta \phi^*(f_D) \approx \frac{40.5}{f_D} \times \frac{2\pi}{C} (1 - f_D^2/f_1^2) \Delta \hat{N}\tag{24}$$

One using  $f_D = 2.45$  GHz and  $f_1 = 2.603$  GHz, the above equation simplifies to

$$\Delta \phi^*(f_D) \approx 3.95 \times 10^{-17} \Delta \hat{N}\tag{25}$$

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\*Note the mode of operation indicated here is different from that in Ref. 1.

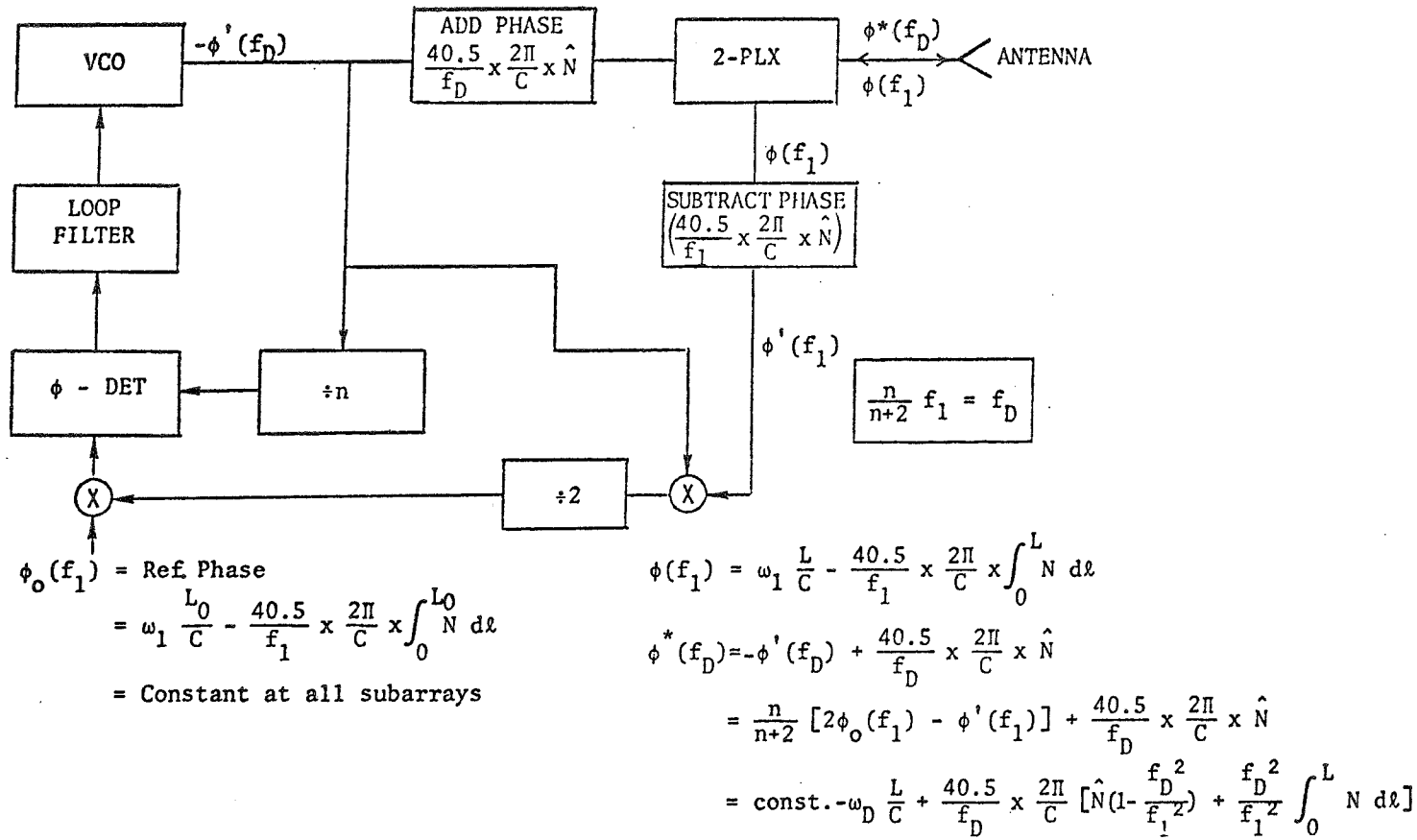


Figure 4. Modified Chernoff Conjugator



so that

$$\hat{\Delta N} = 2.53 \times 10^{16} \times \Delta\phi^*(f_D) \quad (26)$$

Suppose one requires an rms accuracy of  $10^\circ$  ( $= .174$  rad) on  $\phi^*(f_D)$ . Then the required accuracy on  $N$  is given by

$$\begin{aligned} \sigma_{\hat{N}} &= 2.53 \times 10^{16} \times .174 \\ &= 4.41 \times 10^{15} \end{aligned} \quad (27)$$

On going back to Equation (22), one finds

$$\begin{aligned} \sigma_{\delta_2} &= \sigma_{\hat{N}}/\alpha, \quad \alpha = 1.6 \times 10^{18} \\ &= 2.76 \times 10^{-3} \end{aligned} \quad (28)$$

Squaring the quantity on the right hand side of Equation (28) and on using some results in Reference 3, one obtains a value for  $(P_R/\sigma^2)$ .\*\* Thus,

$$\begin{aligned} P_R/\sigma^2 &= (S/N) \text{ ratio at the receiver sketched in Figure 3} \\ &= \frac{8}{\text{Var}(\delta_2\phi)_{\text{opt}}} \\ &= \frac{8}{7.62 \times 10^{-6}} \\ &= 1.05 \times 10^6; \text{ i.e., } 60 \text{ dB} \end{aligned}$$

As far as Figure 4 is concerned, several comments are in order. Firstly, the use of the same  $\hat{N}$  for both uplink and downlink phase compensations need justification. Secondly, the conjugator suffers from divider ambiguity problems. This makes it necessary to phase conjugate at IF and then suitably multiply the conjugator output frequency to 2.45 GHz. Preliminary design of a 3-tone conjugator operating at IF has been completed and will be reported elsewhere.

## V. CONCLUSION

An attempt has been made above to incorporate the role of the ionosphere in ARA system design. A conjugator has been sketched that compensates for steady-state ionospheric effects. Work is currently in progress to evaluate the magnitudes of ionospheric wedge effects. Based on (limited) available data<sup>2</sup> and because of geometry considerations (the proximity of the ionosphere to the rectenna), it appears unlikely that any compensation towards ionospheric effects would be necessary. However, in order to make a definite conclusion, more data on wedge structure are desirable. In addition, this problem needs examination in the light of ionospheric heating effects due to the downlink power beam.

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\*\* $P_R$  is the total 3-tone signal power received and  $\sigma^2$  is the noise power out of any one of the tone filters that have identical bandwidths.

#### REFERENCES

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