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USE OF INFORMATION ON THE MOTION OF SPACECRAFT AROUND THE CENTER OF BODIES IN THE UNDERTAKING OF DETECTING SOURCES OF X-RAY AND GAMMA RADIATION

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16. Abstract		

Methods for localizing sources of X-ray and gamma radiation which use information about the motion of spacecraft in relation to mass centers are examined.

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USE OF INFORMATION ON THE MOTION OF SPACECRAFT AROUND THE CENTER OF BODIES IN THE UNDERTAKING OF DETECTING SOURCES OF X-RAY AND GAMMA RADIATION

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The problem of localizing sources of X-ray and gamma surges may, as is well-known, be solved via the installation of suitable monitors on several dispersed spacecraft. In this manner determination of the coordinates of the source is possible, as a rule, only with the use of four spacecraft. In the case of the use of 3 or 2 spacecraft it is necessary to use supplementary information (1). Data on the motion of spacecraft around the center of a mass may be used in the capacity of such information. /3*

If the character of the radiation source is such that the phase of the signal registered on one spacecraft relative to the signal registered on another craft may not be measured reliably, then determination of the position of the radiation source in space is possible only on the basis of knowledge of the motion of spacecraft relative to the center of bodies. Obviously, **the use of radiation** monitors with an anistropic directional graph is a necessary prerequisite for such localization of a source.

We shall suggest that one of the spacecraft on which the experiment is conducted be a high-apogee satellite stabilized by rotation around an extended structural X axis. The other two structural axes have been designated Y and Z. Control of the motion of the spacecraft relative to the mass center is attained with the aid of engines in such a way that the craft is directed along the extended X axis to the sun and is then twisted relative to the X axis, after which the engines are turned off. The craft moves further without the action of directing torques until the direction towards the sun does not

*Numbers in the margin indicate pagination in the foreign text.

deviate from the initial direction towards the fixed angle. After this rotation the spacecraft is halted by turning on the engines. The X axis is again directed towards the sun and a new torque of the /4 spacecraft along the X axis is effected.

We suggest that the axis of the directional graph of the radiation source coincide with the X axis and that the graph represent the rotation shape so that the recorded signal w

$$w = w(\Theta)$$

where Θ is the angle between the direction towards the source and the direction of the monitor axis.

We will examine the specifics of the motion of a spacecraft stabilized by rotation. In the nominal instance, the central axis of the inertia of the spacecraft coincides with the structural axis and the vector of the momentum created by the engines is directed along the structural X axis. Therefore, after the torque the motion of the spacecraft is a uniform rotation, if the momentums of the external forces are equal to zero.

However, in the real experiment one must calculate the possibilities for error in the balancing of the spacecraft. In consequence of this, the main X axis of inertia is not directed precisely along the structural X axis. In addition, because of errors in the placement of the engines, the vector of momentum made by them in the process of torquing may deviate sligntly from the X axis. Therefore, for zero external momentums- and this hypothesis is taken up later- after the torque of the spacecraft, we have, generally speaking, an instance of Euler-Poinsot's(2) motion of a solid body relative to the center of bodies.

Knowing the law of the motion of a spacecraft relative to mass center and having used the necessary measurements, which permit the fixation of the direction relative to the system of coordinates of the satellite in absolute space, one can obtain the constants of the law of motion.

We will suggest that the spacecraft be supplied with two groups of optical monitors, one of which measures the angle between the X axis and the projection of the vector of the direction towards the sun on the plane YZ and also the angle between the X axis and the projection of the vector of the direction towards the sun on plane XZ. The other group of monitors registers the momentum of the passage of the center of the Earth's illuminated disc through the fixed plane containing the X axis. /5

The problem of processing the measurements of the optical monitors, which consists of the computation of the constant characterising the motion is solved by systems of transcendental equations with rather complex left parts through the use of the first integrals of differential equations of motion. Therefore, the question arises about the reasonable approximation of a law of motion which could permit one to construct with satisfactory precision an algorithm which secures the reliable and rapid convergence of computed inertias. Having made a series of assumptions, we will select approximating functions.

We are suggesting that the duration of intervals for the processing of information is such that in each interval one can consider the direction towards the sun to be constant in absolute space. We can consider the vector of the momentary angular speed of the spacecraft $\bar{\omega}$ to be only slightly divergent from the main central axis of inertia X_r, making a small angle with the extended X axis of the craft. This is in accordance with the proposition of the smallness of projections q and r of vector $\bar{\omega}$ on the main axes Y_r and Z_r in comparison with projection p on the X axis.

We will compute the latter assumption by analyzing the expression for the coefficient $\bar{\omega}$ (2)

$$\omega^2 = \lambda_2^2 \frac{B(B-C)}{A(A-C)} + \lambda_1^2 \frac{B(A-B)}{C(A-C)} -$$

$$-\lambda_1^2\left[\frac{BC(B-C)+AB(A-B)}{AC(A-C)}-1\right]sn^2(\lambda_2\sigma t+\alpha_2), \qquad /6$$

where A,B,C are forces of the inertia of the body relative to the main axes X_r, Y_r, Z_r where $(A>B>C)$,

$$\lambda_1^2=q_0^2+\frac{C(A-C)}{B(A-B)}r_0^2,\quad \lambda_2^2=q_0^2+\frac{A(A-C)}{B(B-C)}p_0^2$$

(index "O" denotes p,q,r taken for several randomly selected fixed instants of time).

In virtue of the propositions relative to the projections ω on the main axes, the magnitude λ_1^2 is small in comparison with λ_2^2; therefore, the coefficient of $sn^2(\lambda_2\beta\tau+\alpha_2)$ is small. Consequently, the circumfrences I and 2(see drawing I) of radii $\rho_1=\sqrt{\kappa\omega^2_{min}-\beta^2}$ and $\rho_2=\sqrt{\kappa\omega^2_{max}-\beta^2}$, which the hyperbole I touches are close.

Proceeding from this fact, we will replace the hyperbole by some circle lying between circles I and 2. In the same way the stationary axoid is approximated by a spherical cone. We will consider the non-stationary axoid an elliptical cone with an axis directed along the main axis X_r of the ellipsoid of inertia. At any moment in time the cross-section of the non-stationary and stationary axoids, a plane orthagonal to the X axis, will represent two touching ellipses. We will examine the motion, i.e. examine the rotation of absolute space relative to the spacecraft. Then for various moments in time in the cross-section of the axoids in a plane, we will obtain a series of ellipses corresponding to the stationary axis. All these ellipses will touch one and the same ellipse corresponding to the stationary axoid, but they will, generally speaking, differ from one another. They will be identical in the frequent instance when the non-stationary cone but the plane of the cross-section is orthagonal to its axis. We will lable the sole vector of direction towards the sun $\bar{S}$. It is rigidly connected with the stationary axoid in the form of the sum of two components, one of which is directed along the X axis and the other of which r lies in the plane of the cross-section of the axoids. /7

In drawing 2 $\bar{r}$ is the radius-vector of the center of the cross-section of the mobile axoid $O_1, \bar{r}_1 = O_1O_2$ is the vector from O_1 to the center of the stationary axoid O_2, $\bar{r}_2 = \overline{O_2S}$ (S is the projection of the end of vector $\bar{S}$ to the plane, and O_2 is the center of the cross-section of the stationary axoid.

The vector $\bar{r}$ which interests us is equal to

$$\bar{r}_0 + \bar{r}_1 + \bar{r}_2.$$

For the above-indicated frequent instance of the projection of vectors $\bar{r}_1$ and $\bar{r}_2$ on the Y and Z axes

$$r_{1Y} = b\cos(\omega_1 t + \varphi_0),$$
$$r_{1Z} = a\sin(\omega_1 t + \varphi_0),$$
$$r_{2Y} \simeq c\cos(\omega_2 t + \psi_0),$$
$$r_{2Z} \simeq d\sin(\omega_2 t + \psi_0)$$

where 1, 2 are respectively angular and mean-angular on account of the rotation of the speed of the motion of vectors $\bar{r}_1$, $\bar{r}_2$ relative to the Y and Z axes. (The latter two approximate equalities would be exact for $/\bar{r}_2/$=constant).

Considering that these dependencies according to the magnitude of errors also apply in the general case, we will take the following form of the approximation functions for the projection of vector $\bar{S}$:

$$S_y = r_y = A_0 + A_1\sin\omega_1 t + A_2\cos\omega_1 t + A_3\sin\omega_2 t + A_4\cos\omega_2 t$$

$$\dot{S}_z = r_z = B_0 + B_1 \sin\omega_1 t + B_2 \cos\omega_1 t + B_3 \sin\omega_2 t + B_4 \cos\omega_2 t$$

Examining A_k, B_k, ω_1, ω_2 as parameters of congruence during the processing of the display of the optical monitors $\tilde{S}_y(t_i)$, $\tilde{S}_z(t_i)$, we find these constants by using the method of least squares, i.e. from the postulate of the minimum of the function

$$J(\omega_1, \omega_2, A_k, \ldots \quad \sum_i \{[\tilde{S}_y(t_i) - S_y(t_i)]^2 + [\tilde{S}_z(t_i) - S_z(t_i)]^2\}$$

It is easy to see that the postulate of the minimum of the function Ψ for A_k, B_k is written in the form of a system of linear equations relative to A_k, B_k. Therefore, it is easy to solve the problem by determining $\min\limits_{\omega_1,\omega_2} J_1(\omega_1, \omega_2)$ where the function $J_1(\omega_1, \omega_2) = \min\limits_{A_k, B_k} J(\omega_1, \omega_2, A_k, B_k)$ is computed as a result of the solution of this system. For the minimization $\Psi_1(\omega_1, \omega_2)$ with the use of standard procedures it is necessary to know the initial approximations ω_1 and ω_2. The problem of their selection is simplified if one allows for the assumption made earlier about the closeness of the vector of the angular speed to the main axis of inertia. It is agreed (2) that with the movement according to inertia, the projections of the momentary angular speed onto the main axis of inertia change periodically with the phase frequency

$$\nu = \sqrt{\frac{(A-B)(B-C)}{AC}} \sqrt{q_0^2 + \frac{A(A-C)}{B(B-C)} p_0^2}$$

where p_0, q_0 are values of p and q in a fixed moment in time.

Disregarding the magnitude q in comparison with p, we obtain

$$\nu \approx \sqrt{\frac{(A-B)(A-C)}{BC}} p_0$$

It is obvious that $\nu = \omega_1$ except in postulates of our proposition $p_0 = \omega_2$.

Therefore, in the capacity of an initial approximation, one may consider that

$$\omega_1 = \sqrt{\frac{(A-B)(A-C)}{BC}}\ \omega_2$$ /9

The initial approximation for ω_2 is usually known beforehand with sufficiently good precision in that the speed of the torque of the spacecraft is preset by a control system within fixed limits.

To process the measurements from the monitors registering the path of the Earth's disc through the plane containing the X axis, we will introduce the angle ψ constructed in the plane YZ from the Y axis to the projection on plane YZ of the vector of the direction to the pole of the elliptic. It is easy to obtain this angle when one has the results of the processing of the measurements from the solar monitors.

Let the angle made by the plane passing through the Earth and the X axis with the plane YX equal ψ. We will proceed from angle ψ to the projections K_x, K_y, K_z of the single vector of the direction to the Earth on the X,Y,Z axes according to the following formula:

$$K_x = \frac{1}{F^2+G^2}\left(GE - \operatorname{sgn} F \cdot \sqrt{G^2E^2+(F^2+G^2)(F^2-E^2)}\right),$$
$$K_Y = \sqrt{1-K_x^2}\cos\psi, \quad K_z = \sqrt{1-K_x^2}\sin\psi,$$

where

$$F = S_y\cos\psi + S_z\sin\psi,$$
$$G = S_x,$$
$$E = \cos\delta,$$

δ is the angle between the direction from the spacecraft to the sun and to the Earth obtained by the computation of the motion of the center of the spacecraft mass; S_x, S_y, S_z are projections to the structural axes of the spacecraft of the single vector of direction to the sun.

Thus, for the moment in time registered by the monitors we have in the system of satellite coordinates two vectors which are known in absolute space: the single vector of direction towards /10 the sun and the sole vector of direction towards the center of the Earth $\bar{K}$. If these vectors are determined simultaneously in the absolute and satellite coordinate systems, one can obtain the matrix of the transfer from the absolute system to the satellite system. According to these vectors, one can, with sufficient precision, assume that the direction from the spacecraft to the sun coincides with the direction from the center of the Earth to the sun. In the system of satellite coordinates we find the vectors

$$\bar{P}_A = \bar{S}_A \times \bar{K}_A$$

and

$$\bar{R}_A = \bar{S}_A \times \bar{P}_A$$

These same vectors have been computed analagously from the absolute coordinate system

$$\bar{P} = \bar{S} \times \bar{K},$$
$$\bar{R} = \bar{S} \times \bar{P}$$

We will form the two matrices

$$Q = (\bar{S}_A, \bar{P}_A, \bar{R}_A),$$
$$D = (\bar{S}, \bar{P}, \bar{R}).$$

The matrix of transfer

$$T = Q \cdot D^T$$

(D^t is transformed matrix D) permits one to obtain in the system of coordinates connected with the satellite XYZ any vector fixed in the absolute coordinate system including the vector directed to the pole of the elliptic. This vector is close to the direction orthagonal to the kinetic momentum of the spacecraft. Therefore, in the case when the momentary axis of the spacecraft rotation does not deviate sharply from the vector of the kinetic momentum(which is assumed in our example) the angular motion of the projection of this vector on the plane YZ will be close to isometric. Consequently, one may approximate the change in angle γ by the linear law /11

$$\gamma = C_0 + C_1 t$$

Errors in approximation for empirically possible deviations of momentum from rotation of the axis from the structural axis do not exceed several minutes(for deviations of 6° the maximum approximation error is ~7').

As a result of the processing of the measurements of the optic monitors we obtain a set of constants $A_k, B_k (k=1,2,...5)$, ω_1, ω_2, C_0, $C_1 = \omega_2$. Having these constants, it is possible to obtain the position of the vector of direction to the sun relative to the spacecraft and projections one the plane YZ of the vector of the direction to the elliptic pole for each moment in time. This is sufficient for a computation of the matrix of the transfer from the system of spacecraft coordinates to that of the absolute coordinates, i.e. for the complete determination of the position of the satellite relative to the center of the bodies.

We will evaluate the possibilities for locating "constant" X-ray sources according to data about the orientation of the satellite. We propose from the beginning that measurements of the radiation source are centered according to the center of the bodies at each interval of time between reorientations of the spacecraft. Then, for every such interval we obtain, according to one point, the position which is generally unknown on the directional graph. For convenience, we imagine that in the reorientation process the position of the axis on the radiation monitor graph is stationary and that the turn in space occurs around the pole of an elliptic. The angles of the turn from one position to another are then known. /12

If we have at least three intervals of measurements, separated by reorientations, then it is possible to determine the direction to the radiation sources. For this it is sufficient to solve the system of equations

$$\begin{aligned} w f(\theta(y, z)) &= C_1, \\ w f(\theta(y, z+\Delta z_1)) &= C_2, \\ w f(\theta(y, z+\Delta z_2)) &= C_3 \end{aligned} \qquad (1)$$

relative to w,y,z.

Here C_1, C_2, C_3 are measurements, w is the measured intensity of radiation in the case where the axis of the graph is directed at the radiation source and y,z are right angle coordinates of the single vector of direction to the source relative to the satellite in which the directional graph is constructed; the Y axis is directed to the pole of the elliptic, the Z axis is orthagonal to it and lies in the plane of the elliptic; ΔZ_1, ΔZ_2 are the shift of direction to the source relative to the initial direction during the spacecraft reorientations.

However, the recorded series of equations does not permit one to determine the sign of the magnitude Y, i.e. to establish the location of the source either above the plane or below it.
In virtue of the fact that the axis of the diagram of the direction of the apparatus does not preserve a constant direction in space but completes a scan of several limited sections of space in accordance with the motion of the spacecraft relative to the center

of the mass, the angle θ between the direction to the source and the axis of the apparatus graph changes. Following the change in the angle θ, the intensity measured by the apparatus changes with the maximum being attained at the minimal angles θ. It is obvious that for the minimum angle between the vector of direction towards the source and the axis of the diagram of direction, it is necessary that the source be located in a plane orthagonal to the described axis of the graph of the conic surface and that it pass through the generatrix of this surface. /13

The surface on which the source is located may be determined in as much as the moments at which the measured intensity attains maximums are known and the surface formed by the movement of the axis of the graph is constructed according to data about the measurement of the spacecraft.

Moreover, the described procedure for determining the plane on which the source is located permits one to compute the coordinates of the sources without the incorporation of a method using a system of equations (1). It is sufficient to adapt this procedure for two intervals of motion between which the reorientations of the spacecraft took place.

We will turn to the problem of determining the direction to the source of a surge of X-ray or gamma-radiation according to data about the orientation of a satellite. It is obvious that according to the measurements of the monitors whose axes are identically directed in absolute space, it is impossible to establish the direction to the source by using only the directional graphs of the monitors and their orientation. Location of the source is possible having, as a minimum, three monitors whose axes are directed along three non-coplanar vectors. In this case the direction towards the source is determined from the solution of a system of equations, three of which connect the angle between the axis of the monitor and the direction to the radiation source with the measured magnitude of radiation intensity and the fourth of which is a /14 purely geometric correlation between the three angles made by the direction to the source and the axis of the monitors.

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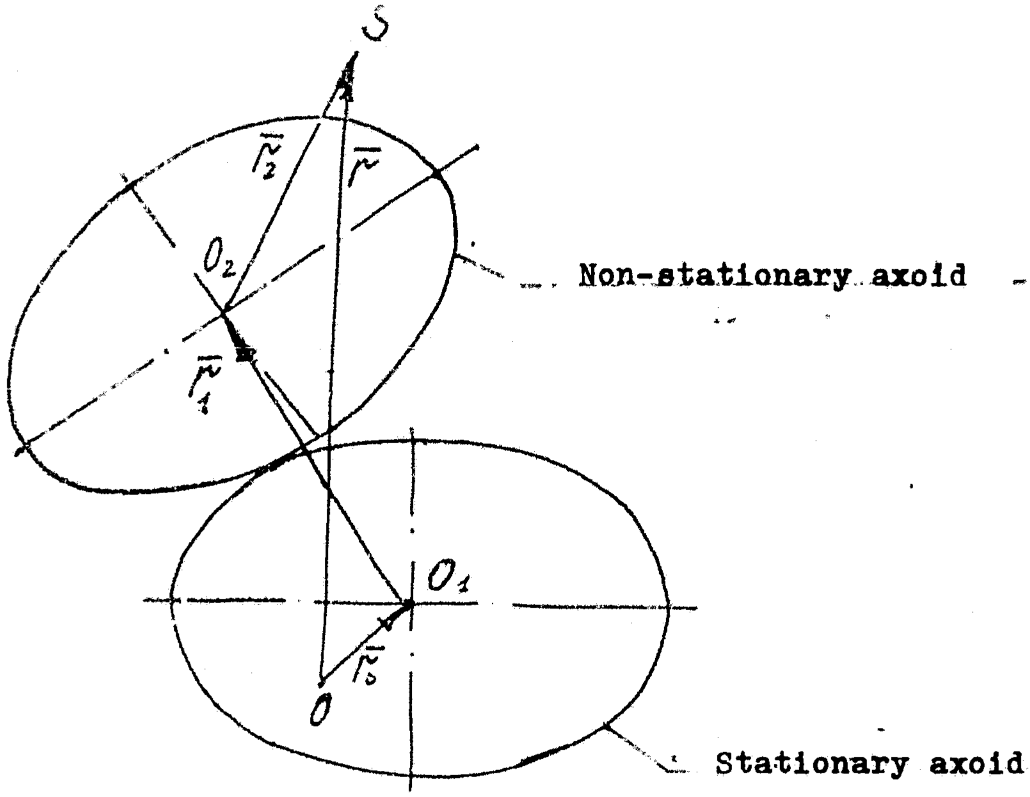
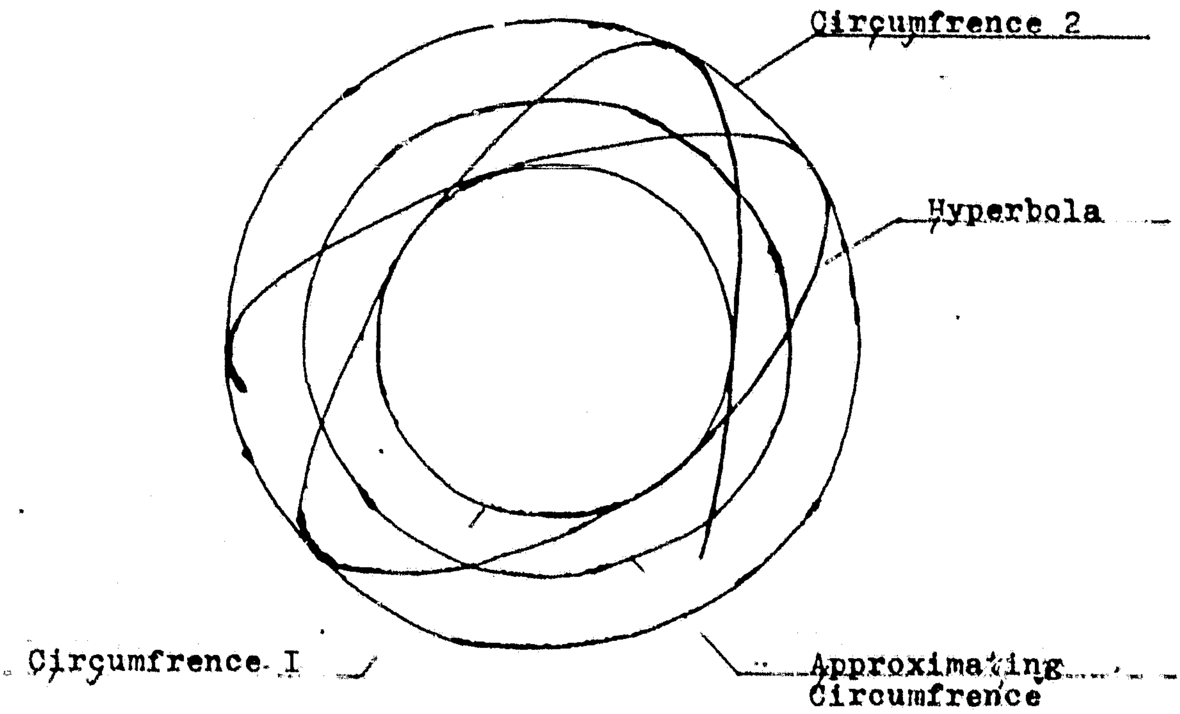

S
$\bar{r}_2$
$\bar{r}$
O_2
$\bar{r}_1$
O_1
$\bar{r}_0$
O
Non-stationary axoid
Stationary axoid

Drawing 2


Circumfrence 2
Hyperbola
Circumfrence I
Approximating
Circumfrence

Drawing I