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EQUILIBRIUM AND STABILITY OF A SATELLITE INFLUENCED BY GRAVITATIONAL AND AERODYNAMIC TORQUES

By Zachary J. Galaboff
Systems Dynamics Laboratory

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16. ABSTRACT

Equilibrium and stability of a satellite influenced by gravitational and aerodynamic torques are investigated. A circular orbit and constant atmospheric density are assumed. Presented is a computer program which determines equilibrium attitudes and the associated eigenvalues of these attitudes. Demonstration of the use of this program is made using the former Skylab satellite as an example.

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TECHNICAL MEMORANDUM

EQUILIBRIUM AND STABILITY OF A SATELLITE INFLUENCED BY GRAVITATIONAL AND AERODYNAMIC TORQUES

INTRODUCTION

The stability of satellites under the influence of only gravity gradient torques has been treated by several investigators [1-4]. The inclusion of the effects of aerodynamic torques was considered by Nurre [5] and Sperling [6]. In addition, the instability produced when a gravity-gradient stabilized body is subjected to a torque which forces it slightly away from the true gravity-gradient equilibrium position (i.e., axis of minimum moment of inertia aligned with the radius vector and the axis of maximum moment of inertia aligned with the perpendicular to the orbit plane) has been studied by Garber [7]. The purpose of this report is to document a method which can be used to determine the equilibrium conditions and investigate the stability of these conditions for satellites in circular orbits under the influence of gravity-gradient and aerodynamic torques. The procedures considered are demonstrated using the former Skylab satellite as an example.

The satellite is assumed to be rigid and the atmosphere constant for a given altitude. A linearized infinitesimal analysis is used to assess the stability of equilibrium positions found by a variation of the Newton-Raphson method.

COORDINATE SYSTEMS

Orbital Reference System

The orbital reference system has its origin at the center of mass of the satellite and rotates uniformly at the rate of one revolution per orbit (Ω). The X_o axis is positive in the direction of flight and lies in the orbit plane. The Z_o axis coincides with the orbital radius vector and is positive away from the Earth. The Y_o axis is parallel to the orbit normal and the orbital angular momentum vector.

Body-Fixed Systems

Two body fixed systems are portrayed in Figure 1. The X_B, Y_B, Z_B is a geometric system located at the center of mass. The principal axes system, X_p, Y_p, Z_p, also has its origin at the center of mass and is oriented such that the product of inertia terms vanish. It is in the principal axes system that the equation of motion is written. The rotation from the orbital system to the principal axes system is accomplished by three positive rotations ψ_1, ψ_2, and ψ_3 about the three successive axes 1, 2, 3, respectively. Thus,

$$\bar{X}_p = [\psi] \bar{X}_o$$

where over-scored symbols represent vectors

$$[\psi] = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix} ,$$

or,

$$[\psi] = \begin{bmatrix} c_2\ c_3 & c_1\ s_3 + s_1\ s_2\ c_3 & s_1\ s_3 - c_1\ s_2\ c_3 \\ -c_2\ s_3 & c_1\ c_3 - s_1\ s_2\ s_3 & s_1\ c_3 + c_1\ s_2\ s_3 \\ s_2 & -s_1\ c_2 & c_1\ c_2 \end{bmatrix} .$$

In these matrices c_1 and s_1 represent the cosine and sine of ψ_1, respectively, etc.

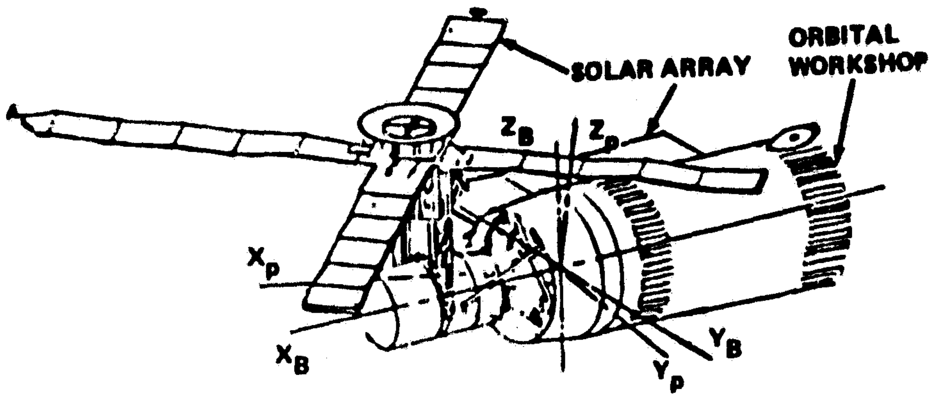

Figure 1. Skylab.

EQUATION OF MOTION

The equation of rotational motion of the satellite in the principal axes system is

$$I\dot{\overline{w}}^* + \overline{w}^* \times I\overline{w}^* = \overline{L} \quad ,$$

where

I = the constant moment of inertia matrix

$\overline{w}^*$ = angular velocity of the satellite with respect to inertial space, represented in the principal axes system

$\overline{L}$ = external torques, which for our purposes are gravitational and aerodynamic torques,

In particular, we have

$$\overline{w}^* = \overline{w} + \overline{\Omega} \quad ,$$

where

$\bar{w}$ = angular velocity of the body system with respect to the orbital system

$\bar{\Omega} = [\psi]\ \bar{\Omega}^*$; $\bar{\Omega}^*$ is the angular velocity of the orbital system with respect to inertial space. For representation in the body system it is multiplied by the transformation matrix $[\psi]$.

Now,

$$\bar{w} = [Q]\ [\dot{\bar{\psi}}] = \begin{bmatrix} c_2\, c_3 & s_3 & 0 \\ -c_2\, s_3 & c_3 & 0 \\ s_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \end{bmatrix} ,$$

and

$$\bar{\Omega} = \Omega \begin{bmatrix} c_1\, s_3 + s_1\, s_2\, c_3 \\ c_1\, c_3 - s_1\, s_2\, s_3 \\ - s_1\, c_2 \end{bmatrix} .$$

Therefore,

$$\dot{\bar{w}}^* = \dot{\bar{w}} + \dot{\bar{\Omega}} = \dot{\bar{w}} + [\dot{\psi}]\ \bar{\Omega}^* + [\psi]\ \dot{\bar{\Omega}}^* .$$

In our case $\dot{\bar{\Omega}}^* = 0$ and

$$\dot{\bar{w}} = \begin{bmatrix} c_2\, c_3 & s_3 & 0 \\ -c_2\, s_3 & c_3 & 0 \\ s_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\psi}_1 \\ \ddot{\psi}_2 \\ \ddot{\psi}_3 \end{bmatrix} + \begin{bmatrix} -s_2\, c_3 & -c_2\, s_3 & c_3 \\ s_2\, c_3 & -c_2\, c_3 & -s_3 \\ c_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi}_1\, \dot{\psi}_2 \\ \dot{\psi}_1\, \dot{\psi}_3 \\ \dot{\psi}_2\, \dot{\psi}_3 \end{bmatrix} .$$

Thus the equation of motion is

$$I\,(\dot{\overline{w}} - \overline{w} \times \overline{\Omega}) + (\overline{w} + \overline{\Omega}) \times I\,(\overline{w} + \overline{\Omega}) - \overline{L} = \overline{0} \quad . \qquad (1)$$

EXTERNAL TORQUES

The gravity gradient torque on a rigid body in circular orbit about its three principal axes is:

$$\overline{L}_g = 3\,\Omega^2\,\overline{r} \times I\overline{r} \quad ,$$

where

$$\overline{r} \times I\overline{r} = \begin{bmatrix} (I_3 - I_2)\,c_1\,c_2\,(s_1\,c_3 + c_1\,s_2\,s_3) \\ (I_1 - I_3)\,c_1\,c_2\,(s_1\,s_3 - c_1\,s_2\,c_3) \\ (I_2 - I_1)\,(s_1\,s_3 - c_1\,s_2\,c_3)\,(s_1\,c_3 + c_1\,s_2\,s_3) \end{bmatrix} \quad .$$

The aerodynamic torque is not so easily written. It is of the form

$$\overline{L}_a = K \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

where $K = 1/2\ \rho\ A\ D\ V^2$ in which ρ is the atmospheric density, A is the reference area, D is the reference length, and V is the magnitude of the velocity.

For $i = 1,\ 2,\ 3$

$$L_i = \sum_{j=1}^{6} \left\{ \sum_{k=1}^{6} (AA_{ijk} \cos (k\phi) + AB_{ijk} \sin (k\phi)) \cos (j\alpha) \right.$$

$$\left. + \sum_{k=1}^{6} (BA_{ijk} \cos (k\phi) + BB_{ijk} \sin (k\phi)) \sin (j\alpha) \right\} ,$$

in which AA, AB, BA, and BB are the coefficients of the double Fourier series representing the aerodynamic torque in the principal axes system. The angles α and ϕ are the total angle of attack and the aerodynamic roll angle, respectively. These angles are depicted in Figure 2 and are defined as follows:

$$\alpha = \cos^{-1} (v_1)$$

$$\phi = \tan^{-1} (v_2/v_3) ,$$

where v_1, v_2, v_3 are unit vectors of the velocity vector expressed in the geometric body reference frame. It is in this reference frame that the aerodynamic moment and force coefficients are rendered [8].

EQUILIBRIUM

The equilibria of the linearized equation of motion (1) are solutions of

$$\bar{\nu} = \bar{\Omega} \times I \bar{\Omega} - \bar{L} = \bar{0} ,$$

To find an equilibrium point it is necessary to find a zero of $\bar{\nu}$. However, the aerodynamic torque component of the external torque is sufficiently complicated to preclude the analytic determination of zeros of $\bar{\nu}$. Now, $\bar{\nu}$ is a function of

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} ,$$

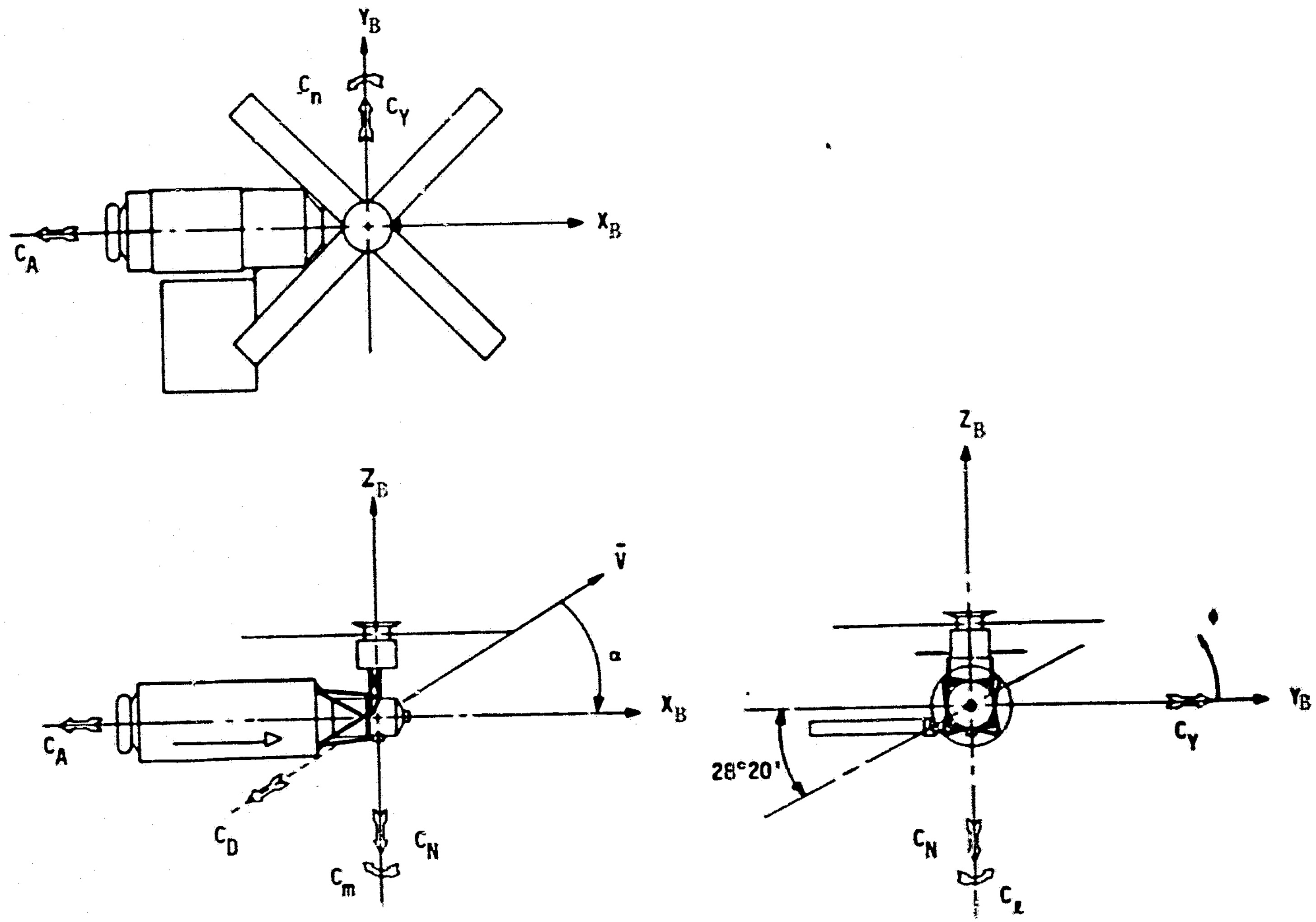

Figure 2. Geometric body reference frame with angle of attack and roll angle defined.

and an iterative procedure like that of the Newton-Raphson method can be used to solve for the values $\bar{\psi}$ which render $\bar{\nu} = 0$. We then have

$$\bar{\nu}_{i+1} = \bar{\nu}_i + \left[\frac{\partial \nu}{\partial \psi}\right] \Delta \bar{\psi}$$

where succession approximations of $\bar{\nu}$ are noted by subscripts i and i+1. Then for $\bar{\nu}_{i+1}$ to be zero, it is necessary that

$$\Delta \bar{\psi} = - \left[\frac{\partial \nu}{\partial \psi}\right]^{-1} \bar{\nu}_i \quad .$$

LINEAR STABILITY

For a linear stability analysis, we let $\bar{\psi} = \bar{\psi}^* + \bar{\delta}$ where $\bar{\psi}^*$ is a zero of $\bar{\nu}$ and

$$\bar{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \quad ,$$

is a small displacement from an equilibrium point. Note that $\dot{\bar{\psi}} = \dot{\bar{\delta}}$ and $\ddot{\bar{\psi}} = \ddot{\bar{\delta}}$. The equation of motion [equation (1)] can be expanded to

$$I\dot{\bar{w}} - I(\bar{w} \times \bar{\Omega}) + \bar{w} \times I\bar{\Omega} + \bar{\Omega} \times I\bar{w} + \bar{w} \times I\bar{w} + \bar{\nu} = \bar{0} \quad .$$

After linearizing and recalling that $\bar{w}$ was of the form [Q] $\dot{\bar{\psi}}$ and $\dot{\bar{w}}$ of of the form [Q] $\ddot{\bar{\psi}}$, it is possible to arrive at

$$[A]\, \ddot{\bar{\delta}} + [B]\, \dot{\bar{\delta}} + [C]\, \bar{\delta} = \bar{0}$$

or,

$$\ddot{\bar{\delta}} + [A]^{-1} [B] \dot{\bar{\delta}} + [A]^{-1} [C] \bar{\delta} = \bar{0} \quad ,$$

where

$$[A] = [I] [Q] \quad ,$$

$$[B] = \begin{bmatrix} 0 & (I_3 - I_1 - I_2) \Omega_3 & -(I_2 - I_3 - I_1) \Omega_2 \\ -(I_3 - I_1 - I_2) \Omega_3 & 0 & -(I_1 - I_2 - I_3) \Omega_1 \\ (I_2 - I_3 - I_1) \Omega_2 & -(I_1 - I_2 - I_3) \Omega_1 & 0 \end{bmatrix} \quad ,$$

$$C = \left[\frac{\partial \nu}{\partial \psi}\right] \quad ,$$

where I_i and Ω_i are the components of I and $\dot{\Omega}$, respectively. (The matrix A can become singular when the cosine of ψ_2 equals 0.) If we let

$$\bar{\varepsilon} = \begin{bmatrix} \bar{\delta} \\ \dot{\bar{\delta}} \end{bmatrix}$$

then

$$\dot{\bar{\varepsilon}} = \begin{bmatrix} \dot{\bar{\delta}} \\ \ddot{\bar{\delta}} \end{bmatrix} \quad .$$

It is then possible to write equation (1) in terms of $\bar{\varepsilon}$ as

$$\dot{\bar{\varepsilon}} + \begin{bmatrix} \emptyset & -U \\ A^{-1}C & A^{-1}B \end{bmatrix} \bar{\varepsilon} = \bar{0} \quad , \qquad (2)$$

where

$$\emptyset = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

To determine how unstable the equilibrium point is, it is necessary to obtain the eigenvalues of equation (2) and compute the unstable time constant or the logarithm of the amplitude over one orbit (logarithmic increment).

A computer program was developed to first determine combinations of ψ_1, ψ_2, ψ_3 which would render $\bar{\nu} = 0$ and then to determine the eigenvalues of the system for each such attitude. In the case of the Skylab, it can be demonstrated that $\bar{\nu}$ is not a unimodal function so that finding all equilibrium attitudes cannot be easily done except perhaps by exhaustive search. However, for purposes of demonstrating the operation of this program, an initial starting set of angles was chosen close to an equilibrium point and after a fixed number of iterations, the stability of the Skylab for that equilibrium point was assessed for combinations of altitude and density.

COMPUTER PROGRAM

The form of the program is relatively straight forward. Input quantities are read in first. The aerodynamic quantities, which are the Fourier coefficients for aerodynamic forces and moments in the geometric system, are transformed into the Fourier coefficients for the aerodynamic moment in the principal axes system. This is done in Subroutine ATRAN. The program then accepts combinations of density and altitude and initial angles ϕ, α, ϕ_β for starting the iterative search for combinations of ψ_1, ψ_2, ψ_3 (Euler angles) such that $\bar{\nu} = 0$. Upon reaching the iteration limit, the program outputs the values of ψ_1, ψ_2, ψ_3 in the form of ϕ, α, ϕ_β . (for ease of visualization). The magnitude and components of $\bar{\nu}$ are also displayed for visual determination of how close to equilibrium the last angles are. These quantities, as well as the associated eigenvalues, are then stored for possible future plotting.

A listing of the main program and associated subroutines is given in Appendix A. A listing of major program variables and input data are supplied in Appendices B and C, respectively. In Appendix D are presented the results of a sample case and some relevant plots.

The program was written for the Honeywell CP-V System. It should be noted that there are two machine dependent parameters; one in Subroutine BALANC and the other in Subroutine HQR2, which must be altered for use of this program on other computing systems.

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APPENDIX A

LISTING OF PROGRAM

```
C
C      PROGRAM TO FIND EQUILIBRIUM ATTITUDES FOR A SATELLITE UNDER
C              GRAVITY GRADIENT AND AERODYNAMIC TORQUES
C
C      FOURIER COEFFICIENTS FOR AERODYNAMIC EFFECTS ARE INPUT IN THE
C              FORM GIVEN BY NORTHROP MEMO M-9230-73-197 BY
C              P.A. GYORFI
C
C
       DIMENSION TGP(3,3),TPG(3,3),PNUS(3,3),QNUS(3,3),PNUSI(3,3),
      1DELPSI(3),PAS(3),PSI(3),V0(3),PSI0(3),C(3),S(3),V(3),XNU(3),
      2AM(3),PAMA(3),PAMP(3),PPS(3),PAUS(3,3),XI(3),PASI(3,3)
       DIMENSION ALA0(6,6),ALB0(6,6),AB0(6),ALA(6,6,6),ALB(6,6,6),
      1BLA(6,6,6),BLB(6,6,6),PSIG(3),LI(3),MI(3),PPHI(3),APHI(3,3)
       DIMENSION AKA0(3,2),AKB0(3,2),AKA(3,2,6),AKB(3,2,6),
      1BKA(3,2,6),BKB(3,2,6),ADUM(3,3),TMM(6,6),DELPG(3),TC(6)
       DIMENSION WR(6),WI(6),Z(6,6),FV(6),IV(6),DUM(3,3),EUM(3,3)
       DIMENSION QI(3,3),B(3,3),PGGS(3,3),FR(6),ZETA(6),U(3,6)
       REAL PI/3.14159265359/
       RAD=57.295779513
C
C      INPUTS
C              REFERENCE DIAMETER IN FEET
C              REFERENCE AREA IN FEET2
C              NUMBER OF ITERATIONS
C              TGP
C              MOMENTS OF INERTIA ABOUT PRINCIPAL AXES IN SLUG-FEET2
C              LOCATION OF CG IN GEOMETRIC COORDINATES IN FEET
C
       READ(5,700)DREF,AREF,LIMIT
       READ(5,701)((TGP(L,M),M=1,3),L=1,3),XI1,XI2,XI3,XCG,YCG,ZCG
       READ(5,702)M1,M2,M3
       READ(5,703)(AB0(I),I=1,M1)
       DO 5 I=1,M1
       READ(5,703)(ALA0(I,J),J=1,M2)
     5 READ(5,703)(ALB0(I,J),J=1,M2)
       DO 7 I=1,M1
       DO 6 J=1,M2
       READ(5,703)(ALA(I,J,K),K=1,M3)
     6 READ(5,703)(BLA(I,J,K),K=1,M3)
       DO 7 J=1,M2
       READ(5,703)(ALB(I,J,K),K=1,M3)
     7 READ(5,703)(BLB(I,J,K),K=1,M3)
   700 FORMAT(2G/I)
   701 FORMAT(3G)
   702 FORMAT(3I5)
   703 FORMAT(6F12.0)
       RE=20925721.785
       GM=1.40765391E16
       DO 8 L=1,3
       DO 8 M=1,3
     8 TPG(L,M)=TGP(M,L)
```

```
C
C     INTERIA TERMS
C
      XI(1)=XI1
      XI(2)=XI2
      XI(3)=XI3
      XM1=XI3-XI2
      XM2=XI1-XI3
      XM3=XI2-XI1
      XK1=XM1/XI1
      XK2=XM2/XI2
      XK3=XM3/XI3
      XD=XCG/DREF
      YD=YCG/DREF
      ZD=ZCG/DREF
C
C     CONVERSION TERMS FOR CHANGING AERODYNAMIC TORQUE
C     TO PRINCIPAL AXES REFERENCE FRAME
C
      DO 9 I=1,3
      U(I,1)=ZD*TPG(I,2)-YD*TPG(I,3)
      U(I,2)=YD*TPG(I,1)-XD*TPG(I,2)
      U(I,3)=ZD*TPG(I,1)-XD*TPG(I,3)
      U(I,4)=TPG(I,2)
      U(I,5)=TPG(I,3)
      U(I,6)=TPG(I,1)
    9 CONTINUE
C
C     CONVERSION OF AERODYNAMIC COEFFICIENTS FROM GEOMETRIC
C     REFERENCE FRAME TO PRINCIPAL AXES REFERNCE FRAME
C
      CALL ATRAN(ALAO,ALBO,ALA,ALB,BLA,BLB,AKAO,AKBO,AKA,AKB,
     1BKA,BKB,U)
C
C     RHOK = ATMOSPHERIC DENSITY IN KG/M3
C            (PROGRAM TERMINATED IF RHOK = 0)
C     ALT  = ALTITUDE IN NAUTICAL MILES
C
 1000 READ(5,701)RHOK,ALT
      IF(RHOK)1001,1002,1001
 1001 DO 10 I=1,3
      DO 10 J=1,6
   10 TMM(I,J)=0.
      TMM(1,4)=-1.
      TMM(2,5)=-1.
      TMM(3,6)=-1.
      RHO=RHOK/5.15379E+2
      RORB=RE+ALT*6.07611548E+3
      OMG=SQRT(GM/RORB)/RORB
      OM2=OMG*OMG
      GG=3.*OMG*OMG
      DEN=RHO*DREF*AREF
```

```
      VMAG= RORD*OMG
      CLA=.5*DEN*VMAG*VMAG
      READ(5,701)PSI0
C
C     INITIAL ANGLES:
C
C     PSI0(1) = ALPHA = TOTAL ANGLE OF ATTACK
C     PSI0(2) = PHI = ROLL ANGLE
C     PSI0(3) = PHI BETA = BANK ANGLE
C
C     CONVERSION TO EULER ANGLES PSI(1), PSI(2), PSI(3)
C
      DO 11 LJ=1,3
   11 PPHI(LJ)=PSI0(LJ)/RAD
      CP1=COS(PPHI(1))
      CP2=COS(PPHI(2))
      CP3=COS(PPHI(3))
      SP1=SIN(PPHI(1))
      SP2=SIN(PPHI(2))
      SP3=SIN(PPHI(3))
      APHI(1,1)= CP2
      APHI(1,2)=-SP2*SP3
      APHI(1,3)= SP2*CP3
      APHI(2,1)= SP2*SP1
      APHI(2,2)=-CP1*CP3+SP1*CP2*SP3
      APHI(2,3)=-CP1*SP3-SP1*CP2*CP3
      APHI(3,1)= SP2*CP1
      APHI(3,2)= SP1*CP3+CP1*CP2*SP3
      APHI(3,3)= SP1*SP3-CP1*CP2*CP3
      CALL MAT3(IPG,APHI,ADUM)
      PSI(1)=ATAN2(-ADUM(3,2),ADUM(3,3))
      PSI(3)=ATAN2(-ADUM(2,1),ADUM(1,1))
      ADU=ADUM(3,3)/COS(PSI(1))
      PSI(2)=ATAN2(ADUM(3,1),ADU)
C
C     BEGIN ITERATION LOOP
C
      DO 100 KOUNT=1,LIMIT
      C1=COS(PSI(1))
      C2=COS(PSI(2))
      C3=COS(PSI(3))
      S1=SIN(PSI(1))
      S2=SIN(PSI(2))
      S3=SIN(PSI(3))
      CS1=C1*C1-S1*S1
      CS2=C2*C2-S2*S2
      CS3=C3*C3-S3*S3
      SC31=3.*C1*C1-S1*S1
      QI(1,1)=C3/C2
      QI(1,2)=-S3/C2
      QI(1,3)=0.
      QI(2,1)=S3
```

```
      QI(2,2)=C3
      QI(2,3)=0.
      QI(3,1)=-S2*C3/C2
      QI(3,2)=S2*S3/C2
      QI(3,3)=1.
      B(1,1)=OMG*((XK1*CS2-1.)*S1*S3+(XK1+1.)*C1*S2*C3)
      B(1,2)=-OMG*(XK1-1.)*S1*C2*C3
      B(1,3)=OMG*(XK1+1.)*(C1*C3-S1*S2*S3)
      B(2,1)=-OMG*((XK2*CS2+1.)*S1*C3-(XK2-1.)*C1*S2*S3)
      B(2,2)=-OMG*(XK2+1.)*S1*C2*S3
      B(2,3)=OMG*(XK2-1.)*(C1*S3+S1*S2*C3)
      B(3,1)=OMG*((XK3*CS3-1.)*C1*C2-2.*XK3*S1*S2*C2*S3*C3)
      B(3,2)=OMG*((XK3*CS3+1.)*S1*S2+2.*XK3*C1*S3*C3)
      B(3,3)=0.
C
C     PGGS IS THE PARTIAL OF OMEGA X I*OMEGA  - GRAVITY GRADIENT
C     WITH RESPECT TO PSI AND EXPRESSED IN THE PRINCIPAL AXES
C     REFERENCE FRAME
C
      PGGS(1,1)=-4.*OM2*C2*XM1*(C3*CS1-2.*S1*C1*S2*S3)
      PGGS(1,2)=-XM1*OM2*(CS2*S3*SC31-4.*S1*C1*S2*C3)
      PGGS(1,3)=-OM2*C2*XM1*(SC31*S2*C3-4.*S1*C1*S3)
      PGGS(2,1)=-4.*OM2*C2*XM2*(S3*CS1+2.*S1*C1*S2*C3)
      PGGS(2,2)= XM2*OM2*(CS2*C3*SC31+4.*S1*C1*S2*S3)
      PGGS(2,3)=-OM2*C2*XM2*(SC31*S2*S3+4.*S1*C1*C3)
      PGGS(3,1)= 4.*OM2*XM3*(CS1*S2*CS3-2.*S1*C1*(1.+S2*S2)*S3*C3)
      PGGS(3,2)= 2.*OM2*C2*XM3*(SC31*S2*S3*C3+2.*S1*C1*CS3)
      PGGS(3,3)= OM2*XM3*(CS3*(S2*S2*SC31+C1*C1-3.*S1*S1)-
     116*S1*C1*S2*S3*C3)
C
C     UNIT VELOCITY VECTOR IN PRINCIPAL AXES REFERENCE FRAME
C
      V0(1)=C2*C3
      V0(2)=-C2*S3
      V0(3)=S2
C
C     TRANSFORMATION OF UNIT VELOCITY VECTOR INTO GEOMETRIC
C     REFERENCE FRAME
C
      CALL MATVEC(TGP,V0,V)
      V(1)=AMAX1(AMIN1(1.0,V(1)),-1.0)
C
C     ALPHA = TOTAL ANGLE OF ATTACK
C     PHI   = ROLL ANGLE
C
C
      ALPHA=ACOS(V(1))
      PHI=ATAN2(V(2),V(3))
      CONST1=1.0-V(1)*V(1)
      IF(CONST1)20,25,20
   20 CONST2=1./SQRT(CONST1)
      GO TO 30
```

```
   25 CONST2=1.5E70
   30 CONTINUE
C
C     CALCULATION OF AERODYNAMIC TORQUE AND PARTIALS OF
C     AERODYNAMIC TORQUE WITH RESPECT TO ALPHA AND PHI
C
      CALL ZAERO(AKAO,AKBO,AKA,AKB,BKA,BKB,ALPHA,PHI,CLA,
     1PAMA,PAMP,AM)
C
C     PAS = PARTIAL OF ALPHA WITH RESPECT TO PSI
C     PPS = PARTIAL OF PHI WITH RESPECT TO PSI
C
      PAS(1)=0.
      PAS(2)=(S2*(TPG(1,1)*C3-TPG(2,1)*S3)-TPG(3,1)*C2)*CONST2
      PAS(3)=C2*(TPG(1,1)*S3+TPG(2,1)*C3)*CONST2
      PPS(1)=0.
      PPS(2)=(V(2)*(S2*(TPG(1,3)*C3-TPG(2,3)*S3)-TPG(3,3)*C2)-
     1V(3)*(S2*(TPG(1,2)*C3-TPG(2,2)*S3)-TPG(3,2)*C2))/CONST1
      PPS(3)=C2*(V(2)*(TPG(1,3)*S3+TPG(2,3)*C3)-V(3)*(TPG
     1(1,2)*S3+TPG(2,2)*C3))/CONST1
C
C     CALCULATION OF PARTIAL OF AERODYNAMIC
C     TORQUE WITH RESPECT TO PSI
C
      DO 40 I=1,3
      DO 40 J=1,3
   40 PAMS(I,J)=PAMA(I)*PAS(J)+PAMP(I)*PPS(J)
C
C     CALCULATION OF PARTIAL OF EXTERNAL TORQUE
C     WITH RESPECT TO PSI
C
      DO 50 I=1,3
      DO 50 J=1,3
      PNUS(I,J)=PGGS(I,J)-PAMS(I,J)
      PASI(I,J)=PNUS(I,J)/XI(I)
   50 QNUS(I,J)=PNUS(I,J)
C
C     CALCULATION OF NU
C
      XNU(1)=-OM2*XM1*C2*(4.*S1*C1*C3+S2*S3*SC31)-AM(1)
      XNU(2)= OM2*XM2*C2*(S2*C3*SC31-4.*S1*C1*S3)-AM(2)
      XNU(3)=OM2*XM3*(4.*S1*C1*S2*CS3+S3*C3*(S2*S2*SC31+C1*C1-
     13.*S1*S1))-AM(3)
      CALL MINV(QNUS,3,DI,LI,MI)
C
C     PNUSI = INVERSE OF PARTIAL OF NU WITH RESPECT TO PSI
C
      DO 60 I=1,3
      DO 60 J=1,3
   60 PNUSI(I,J)=QNUS(I,J)
      CALL MATVEC(PNUSI,XNU,DELPSI)
      XNUMAG=0.
```

```
      DO 70 I=1,3
      XNUMAG=XNUMAG+XNU(I)*XNU(I)
      DELPG(I)=-DELPSI(I)*RAD
      PSIG(I)=PSI(I)*RAD
   70 DELPSI(I)=-DELPSI(I)
      XNUMAG=SQRT(XNUMAG)
      ALPHAG=ALPHA*RAD
      PHIG=PHI*RAD
      UXN=-(TGP(1,1)*(C1*S3+S1*S2*C3)+TGP(1,2)*(C1*C3-S1*S2*S3)-
     1TGP(1,3)*S1*C2)
      UXD=TGP(1,1)*(S1*S3-C1*S2*C3)+TGP(1,2)*
     1(S1*C3+C1*S2*S3)+TGP(1,3)*C1*C2
      PHIBG=ATAN2(UXN,UXD)*RAD
      DO 80 L=1,3
      PSI(L)=AMOD(PSI(L)+DELPSI(L),2.0*PI)
   80 IF(ABS(PSI(L)).GT.PI+.01)PSI(L)=PSI(L)-SIGN(2.0*PI,PSI(L))
  800 FORMAT(//,5X,9HALTITUDE ,F6.1,3X,8HDENSITY ,E11.5)
  801 FORMAT(2X,4HPHI ,F11.6,2X,6HALPHA ,F10.6,2X,5HPHIB ,F10.6)
  802 FORMAT(1X,3F15.8,1X,E15.8,/)
  803 FORMAT(7X,4HREAL,13X,4HIMAG,13X,4HFREQ,5X,4HZETA,6X,2HTC)
  804 FORMAT(1X,E16.8,1X,E16.8,2X,F10.5,2X,F6.4,2X,E9.2)
  100 CONTINUE
      CALL MAT3(QI,PASI,DUM)
      CALL MAT3(QI,B,EUM)
      DO 110 I=1,3
      J=I+3
      DO 110 K=1,3
      LK=K+3
      TMM(J,K)=DUM(I,K)
  110 TMM(J,LK)=EUM(I,K)
C
C     DETERMINATION OF EIGENVALUES
C
      IERR=0
      CALL EIS(TMM,WR,WI,Z,FV,IV,6,6,IERR)
C
C     DETERMINATION OF UNSTABLE TIME CONSTANTS
C     OR LOGARITHMIC INCREMENTS
C
      CALL EISORT(WR,WI,FR,TC,ZETA,OMG,PI,6)
C
C     OUTPUT TO TERMINAL SCREEN
C
      WRITE(102,800)ALT,RHOK
      WRITE(102,801)PHIG,ALPHAG,PHIBG
      WRITE(102,802)XNU,XNUMAG
      WRITE(102,803)
      WRITE(102,804)(WR(I),WI(I),FR(I),ZETA(I),TC(I),I=1,6)
C
C
C     WRITE OUTPUT FILE FOR PLOTTING
C
      WRITE(10)RHOK,ALT,WR(1),WI(1),WR(3),WI(3),WR(5),WI(5),PHIG,
     1ALPHAG,PHIBG,XNUMAG,TC(1),TC(3),TC(5)
C
C     PAUSE FOR TERMINAL SCREEN RESET
C
C     PAUSE
C
      GO TO 1000
 1002 STOP
      END
```

```
      SUBROUTINE MINV(A,N,D,L,M)                                        MIN 0001
C***********************************************************************MIN 0002
C                                                                       MIN 0003
C     MATRIX INVERSE, OF AN NXN MATRIX=A INTO THE SAME MATRIX=A         MIN 0004
C                                                                       MIN 0005
C          -1                      D = OUTPUT DETERMINANT OF ORIGINAL A MIN 0006
C        A    = A                  L, M ARE TWO WORKING INTEGER VECTORS MIN 0007
C                                    DIMENSIONED IN MAIN, OF LENGTH NMIN 0008
C                                                                       MIN 0009
C     NOTE THAT THE ORIGINAL A-MATRIX IS DESTROYED                      MIN 0010
C     ALSO NOTE THAT A CHECK ON D SHOULD BE MADE IN CASE MATRIX=A EVER  MIN 0011
C                                                  BECOMES SINGULAR    MIN 0012
C                                                                       MIN 0013
C***********************************************************************MIN 0014
      DIMENSION A(1),L(1),M(1)                                          MIN 0015
C        SEARCH FOR LARGEST ELEMENT                                     MIN 0016
      D=1.0                                                             MIN 0017
      NK=-N                                                             MIN 0018
      DO 80 K=1,N                                                       MIN 0019
      NK=NK+N                                                           MIN 0020
      L(K)=K                                                            MIN 0021
      M(K)=K                                                            MIN 0022
      KK=NK+K                                                           MIN 0023
      BIGA=A(KK)                                                        MIN 0024
      DO 20 J=K,N                                                       MIN 0025
      IZ=N*(J-1)                                                        MIN 0026
      DO 20 I=K,N                                                       MIN 0027
      IJ=IZ+I                                                           MIN 0028
      IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20                               MIN 0029
   15 BIGA=A(IJ)                                                        MIN 0030
      L(K)=I                                                            MIN 0031
      M(K)=J                                                            MIN 0032
   20 CONTINUE                                                          MIN 0033
C        INTERCHANGE ROWS                                               MIN 0034
      J=L(K)                                                            MIN 0035
      IF(J-K) 35,35,25                                                  MIN 0036
   25 KI=K-N                                                            MIN 0037
      DO 30 I=1,N                                                       MIN 0038
      KI=KI+N                                                           MIN 0039
      HOLD=-A(KI)                                                       MIN 0040
      JI=KI-K+J                                                         MIN 0041
      A(KI)=A(JI)                                                       MIN 0042
   30 A(JI) =HOLD                                                       MIN 0043
C        INTERCHANGE COLUMNS                                            MIN 0044
   35 I=M(K)                                                            MIN 0045
      IF(I-K) 45,45,38                                                  MIN 0046
   38 JP=N*(I-1)                                                        MIN 0047
      DO 40 J=1,N                                                       MIN 0048
      JK=NK+J                                                           MIN 0049
      JI=JP+J                                                           MIN 0050
      HOLD=-A(JK)                                                       MIN 0051
      A(JK)=A(JI)                                                       MIN 0052
```

```
   40 A(JI) =HOLD
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
   45 IF(ABS(BIGA)-1.E-20)46,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C        REDUCE MATRIX
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
C        DIVIDE ROW BY PIVOT
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C        PRODUCT OF PIVOTS
      D=D*BIGA
C        REPLACE PIVOT BY RECIPROCAL
      A(KK)=1.0/BIGA
   80 CONTINUE
C        FINAL ROW AND COLUMN INTERCHANGE
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
      END
```

```
      SUBROUTINE MATVEC(A,B,C)
      REAL A(3,3),B(3),C(3)
C
C     THIS ROUTINE SETS C=A      TIMES B
C               C    & B   MUST NOT BE IDENTICAL VARIABLES
C
      DO 1 I=1,3
    1 C(I)=A(I,1)*B(1)+A(I,2)*B(2)+A(I,3)*B(3)
      RETURN
      END
```

```
      SUBROUTINE MAT3(A,B,C)
      REAL A(3,3),B(3,3),C(3,3)
C
C     THIS ROUTINE SET C=A       TIMES B
C               C    MUST NOT BE IDENTICAL WITH EITHER FACTOR
C
      DO 1 I=1,3
      DO 1 J=1,3
    1 C(J,I)=A(J,1)*B(1,I)+A(J,2)*B(2,I)
     X          +A(J,3)*B(3,I)
      RETURN
      END
```

```
      SUBROUTINE MATMAT(A,B,C,L,M,N)
      REAL A(L,M),B(M,N),C(L,N)
C
C     THIS ROUTINE SETS C=A      TIMES B
C               C   MUST NOT BE IDENTICAL WITH EITHER FACTOR
C
      DO 1 I=1,L
      DO 1 J=1,N
      SAVE=0.0
      DO 2 K=1,M
    2 SAVE=SAVE+A(I,K)*B(K,J)
    1 C(I,J)=SAVE
      RETURN
      END
```

```
      SUBROUTINE ATRAN(ALAO,ALBO,ALA,ALB,BLA,BLB,AKAO,AKBO,AKA,AKB,
     1BKA,BKB,U)
      DIMENSION ALAO(6,6),ALBO(6,6),ALA(6,6,6),ALB(6,6,6),BLA(6,6,6),
     1BLB(6,6,6),AKAO(3,2),AKBO(3,2),AKA(3,2,6),AKB(3,2,6),
     2BKA(3,2,6),BKB(3,2,6),U(3,6)
      DO 300 I=1,3
      DO 300 LJ=1,2
      J=3*LJ/2
      AKAO(I,LJ)=0.
      AKBO(I,LJ)=0.
      DO 100 L=1,6
      AKAO(I,LJ)=AKAO(I,LJ)+U(I,L)*ALAO(L,J)
  100 AKBO(I,LJ)=AKBO(I,LJ)+U(I,L)*ALBO(L,J)
      DO 200 K=1,6
      AKA(I,LJ,K)=0.
      BKA(I,LJ,K)=0.
      AKB(I,LJ,K)=0.
      BKB(I,LJ,K)=0.
      DO 200 L=1,6
      AKA(I,LJ,K)=AKA(I,LJ,K)+U(I,L)*ALA(L,J,K)
      BKA(I,LJ,K)=BKA(I,LJ,K)+U(I,L)*BLA(L,J,K)
      AKB(I,LJ,K)=AKB(I,LJ,K)+U(I,L)*ALB(L,J,K)
  200 BKB(I,LJ,K)=BKB(I,LJ,K)+U(I,L)*BLB(L,J,K)
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ZAERO(AKAO,AKBO,AKA,AKB,BKA,BKB,ALPHA,PHI,CLA,
     1PAMA,PAMP,AM)
C
C     AM = AERODYNAMIC TORQUE
C     PAMA = PARTIAL OF AM WITH RESPECT TO ALPHA
C     PAMP = PARTIAL OF AM WITH RESPECT TO PHI
C
      DIMENSION AM(3),PAMA(3),PAMP(3),FXX(3,2),GXX(3,2),FXXP(3,2),
     1GXXP(3,2),AKAO(3,2),AKBO(3,2),AKA(3,2,6),AKB(3,2,6),BKA(3,2,6),
     2BKB(3,2,6)
      DO 400 I=1,3
      AM(I)=0.
      PAMA(I)=0.
      PAMP(I)=0.
      DO 300 J=1,2
      FXX(I,J)=AKAO(I,J)/2.
      GXX(I,J)=AKBO(I,J)/2.
      FXXP(I,J)=0.
      GXXP(I,J)=0.
      LJ=3*J/2
      XF=FLOAT(LJ)
      CAL=COS(XF*ALPHA)
      SAL=SIN(XF*ALPHA)
      DO 200 K=1,6
      YF=FLOAT(K)
      CPH=COS(YF*PHI)
      SPH=SIN(YF*PHI)
      FXXP(I,J)=FXXP(I,J)-YF*(AKA(I,J,K)*SPH-BKA(I,J,K)*CPH)
      GXXP(I,J)=GXXP(I,J)-YF*(AKB(I,J,K)*SPH-BKB(I,J,K)*CPH)
      FXX(I,J)=FXX(I,J)+AKA(I,J,K)*CPH+BKA(I,J,K)*SPH
  200 GXX(I,J)=GXX(I,J)+AKB(I,J,K)*CPH+BKB(I,J,K)*SPH
      PAMA(I)=PAMA(I)-XF*(FXX(I,J)*SAL-GXX(I,J)*CAL)
      PAMP(I)=PAMP(I)+FXXP(I,J)*CAL+GXXP(I,J)*SAL
  300 AM(I)=AM(I)+FXX(I,J)*CAL+GXX(I,J)*SAL
  400 CONTINUE
      DO 500 I=1,3
      PAMP(I)=CLA*PAMP(I)
      PAMA(I)=CLA*PAMA(I)
  500 AM(I)=CLA*AM(I)
      RETURN
      END
```

```
      SUBROUTINE EIS(A, WR, WI, Z, FV1, IV1, N, NM, IERR)
C     EISPACK PROCEDURE FOR ALL EIGENVALUES AND EIGENVECTORS OF A REAL
C     GENERAL MATRIX
C        A          REAL INPUT MATRIX
C        WR         REAL PART OF EIGENVALUES STORED IN A REAL VECTOR
C        WI         IMAGINARY PART OF EIGENVALUES STORED IN A REAL VECTOR
C        Z          MATRIX OF COMPLEX EIGENVECTORS STORED IN EISPACK
C                   PACKED FORM IN A REAL MATRIX
C        FV1        WORKING VECTOR (REAL)
C        IV1        WORKING VECTOR (INTEGER)
C        N          NUMBER OF ROWS AND COLS IN A
C        NM         DIMENSIONED COL LENGTH OF A AND Z
C        IERR       EISPACK ERROR FLAG .NE. 0 AN ERROR HAS BEEN DETECTED
      DIMENSION A(NM, N), Z(NM, N), WR(1), WI(1), FV1(1), IV1(1)
      CALL BALANC(NM, N, A, IS1, IS2, FV1)
      CALL ELMHES(NM, N, IS1, IS2, A, IV1)
      CALL ELTRAN(NM, N, IS1, IS2, A, IV1, Z)
      CALL HQR2(NM, N, IS1, IS2, A, WR, WI, Z, IERR)
      IF(IERR .NE. 0) GO TO 9999
      CALL BALBAK(NM, N, IS1, IS2, FV1, N, Z)
      RETURN
 9999 PRINT 12, IERR
   12 FORMAT(1H0, 45H*****ERROR IN EISPACK EIGENVALUE ROUTINE HQR2 ,
     *        2X, 6HIERR =, I6 )
      RETURN
      END
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
      SUBROUTINE BALANC(NM,N,A,LOW,IGH,SCALE)                           69215004
C                                                                       69215005
      INTEGER I,J,K,L,M,N,JJ,NM,IGH,LOW,IEXC                            69215006
      REAL A(NM,N),SCALE(N)                                             69215007
      REAL C,F,G,R,S,B2,RADIX                                           69215008
C     REAL ABS                                                          69215009
      LOGICAL NOCONV                                                    69215010
C                                                                       69215011
C     THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE BALANCE,  69215012
C     NUM. MATH. 13, 293-304(1969) BY PARLETT AND REINSCH.              69215013
C     HANDBOOK FOR AUTO. COMP., VOL.II-LINEAR ALGEBRA, 315-326(1971).   69215014
C                                                                       69215015
C     THIS SUBROUTINE BALANCES A REAL MATRIX AND ISOLATES               69215016
C     EIGENVALUES WHENEVER POSSIBLE.                                    69215017
C                                                                       69215018
C     ON INPUT-                                                         69215019
C                                                                       69215020
C        NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL         69215021
C          ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM          69215022
C          DIMENSION STATEMENT,                                         69215023
C                                                                       69215024
C        N IS THE ORDER OF THE MATRIX,                                  69215025
C                                                                       69215026
C        A CONTAINS THE INPUT MATRIX TO BE BALANCED.                    69215027
C                                                                       69215028
C     ON OUTPUT-                                                        69215029
C                                                                       69215030
C        A CONTAINS THE BALANCED MATRIX,                                69215031
C                                                                       69215032
C        LOW AND IGH ARE TWO INTEGERS SUCH THAT A(I,J)                  69215033
C          IS EQUAL TO ZERO IF                                          69215034
C           (1) I IS GREATER THAN J AND                                 69215035
C           (2) J=1,...,LOW-1 OR I=IGH+1,...,N,                         69215036
C                                                                       69215037
C        SCALE CONTAINS INFORMATION DETERMINING THE                     69215038
C           PERMUTATIONS AND SCALING FACTORS USED.                      69215039
C                                                                       69215040
C     SUPPOSE THAT THE PRINCIPAL SUBMATRIX IN ROWS LOW THROUGH IGH      69215041
C     HAS BEEN BALANCED, THAT P(J) DENOTES THE INDEX INTERCHANGED       69215042
C     WITH J DURING THE PERMUTATION STEP, AND THAT THE ELEMENTS         69215043
C     OF THE DIAGONAL MATRIX USED ARE DENOTED BY D(I,J).  THEN          69215044
C        SCALE(J) = P(J),    FOR J = 1,...,LOW-1                        69215045
C                 = D(J,J),      J = LOW,...,IGH                        69215046
C                 = P(J)         J = IGH+1,...,N.                       69215047
C     THE ORDER IN WHICH THE INTERCHANGES ARE MADE IS N TO IGH+1,       69215048
C     THEN 1 TO LOW-1.                                                  69215049
C                                                                       69215050
C     NOTE THAT 1 IS RETURNED FOR IGH IF IGH IS ZERO FORMALLY.          69215051
C                                                                       69215052
C     THE ALGOL PROCEDURE EXC CONTAINED IN BALANCE APPEARS IN           69215053
C     BALANC  IN LINE.  (NOTE THAT THE ALGOL ROLES OF IDENTIFIERS       69215054
C     K,L HAVE BEEN REVERSED.)                                          69215055
```

```
C                                                                         69215056
C     QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,          69215057
C     APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY           69215058
C                                                                         69215059
C     ********** RADIX IS A MACHINE DEPENDENT PARAMETER SPECIFYING        69215062
C                THE BASE OF THE MACHINE FLOATING POINT REPRESENTATION.   69215063
C                                                                         69215064
C                **********                                               69215065
      RADIX = 2.                                                          69215066
C                                                                         69215067
      B2 = RADIX * RADIX                                                  69215068
      K = 1                                                               69215069
      L = N                                                               69215070
      GO TO 100                                                           69215071
C     ********** IN-LINE PROCEDURE FOR ROW AND                            69215072
C                COLUMN EXCHANGE **********                               69215073
   20 SCALE(M) = J                                                        69215074
      IF (J .EQ. M) GO TO 50                                              69215075
C                                                                         69215076
      DO 30 I = 1, L                                                      69215077
         F = A(I,J)                                                       69215078
         A(I,J) = A(I,M)                                                  69215079
         A(I,M) = F                                                       69215080
   30 CONTINUE                                                            69215081
C                                                                         69215082
      DO 40 I = K, N                                                      69215083
         F = A(J,I)                                                       69215084
         A(J,I) = A(M,I)                                                  69215085
         A(M,I) = F                                                       69215086
   40 CONTINUE                                                            69215087
C                                                                         69215088
   50 GO TO (80,130), IEXC                                                69215089
C     ********** SEARCH FOR ROWS ISOLATING AN EIGENVALUE                  69215090
C                AND PUSH THEM DOWN **********                            69215091
   80 IF (L .EQ. 1) GO TO 280                                             69215092
      L = L - 1                                                           69215093
C     ********** FOR J=L STEP -1 UNTIL 1 DO -- **********                 69215094
  100 DO 120 JJ = 1, L                                                    69215095
         J = L + 1 - JJ                                                   69215096
C                                                                         69215097
         DO 110 I = 1, L                                                  69215098
            IF (I .EQ. J) GO TO 110                                       69215099
            IF (A(J,I) .NE. 0.0) GO TO 120                                69215100
  110    CONTINUE                                                         69215101
C                                                                         69215102
         M = L                                                            69215103
         IEXC = 1                                                         69215104
         GO TO 20                                                         69215105
  120 CONTINUE                                                            69215106
C                                                                         69215107
      GO TO 140                                                           69215108
C     ********** SEARCH FOR COLUMNS ISOLATING AN EIGENVALUE               69215109
```

```
C                  AND PUSH THEM LEFT **********                       69215110
      130 K = K + 1                                                     69215111
C                                                                       69215112
      140 DO 170 J = K, L                                               69215113
C                                                                       69215114
             DO 150 I = K, L                                            69215115
                IF (I .EQ. J) GO TO 150                                 69215116
                IF (A(I,J) .NE. 0.0) GO TO 170                          69215117
      150    CONTINUE                                                   69215118
C                                                                       69215119
             M = K                                                      69215120
             IEXC = 2                                                   69215121
             GO TO 20                                                   69215122
      170 CONTINUE                                                      69215123
C     ********** NOW BALANCE THE SUBMATRIX IN ROWS K TO L **********   69215124
          DO 180 I = K, L                                               69215125
      180 SCALE(I) = 1.0                                                69215126
C     ********** ITERATIVE LOOP FOR NORM REDUCTION **********           69215127
      190 NOCONV = .FALSE.                                              69215128
C                                                                       69215129
          DO 270 I = K, L                                               69215130
             C = 0.0                                                    69215131
             R = 0.0                                                    69215132
C                                                                       69215133
             DO 200 J = K, L                                            69215134
                IF (J .EQ. I) GO TO 200                                 69215135
                C = C + ABS(A(J,I))                                     69215136
                R = R + ABS(A(I,J))                                     69215137
      200    CONTINUE                                                   69215138
C                                                                       69215139
             G = R / RADIX                                              69215140
             F = 1.0                                                    69215141
             S = C + R                                                  69215142
      210    IF (C .GE. G) GO TO 220                                    69215143
             F = F * RADIX                                              69215144
             C = C * B2                                                 69215145
             GO TO 210                                                  69215146
      220    G = R * RADIX                                              69215147
      230    IF (C .LT. G) GO TO 240                                    69215148
             F = F / RADIX                                              69215149
             C = C / B2                                                 69215150
             GO TO 230                                                  69215151
C     ********** NOW BALANCE **********                                 69215152
      240    IF ((C + R) / F .GE. 0.95 * S) GO TO 270                   69215153
             G = 1.0 / F                                                69215154
             SCALE(I) = SCALE(I) * F                                    69215155
             NOCONV = .TRUE.                                            69215156
C                                                                       69215157
             DO 250 J = K, N                                            69215158
      250    A(I,J) = A(I,J) * G                                        69215159
C                                                                       69215160
             DO 260 J = 1, L                                            69215161
      260    A(J,I) = A(J,I) * F                                        69215162
C                                                                       69215163
      270 CONTINUE                                                      69215164
C                                                                       69215165
          IF (NOCONV) GO TO 190                                         69215166
C                                                                       69215167
      280 LOW = K                                                       69215168
          IGH = L                                                       69215169
          RETURN                                                        69215170
C     ********** LAST CARD OF BALANC **********                         69215171
          END                                                           69215172
```

```
      SUBROUTINE ELMHES(NM,N,LOW,IGH,A,INT)                             73215004
C                                                                       73215005
      INTEGER I,J,M,N,LA,NM,IGH,KP1,LOW,MM1,MP1                         73215006
      REAL A(NM,N)                                                      73215007
      REAL X,Y                                                          73215008
C     REAL ABS                                                          73215009
      INTEGER INT(IGH)                                                  73215010
C                                                                       73215011
C     THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE ELMHES,   73215012
C     NUM. MATH. 12, 349-368(1968) BY MARTIN AND WILKINSON.             73215013
C     HANDBOOK FOR AUTO. COMP., VOL.II-LINEAR ALGEBRA, 339-358(1971).   73215014
C                                                                       73215015
C     GIVEN A REAL GENERAL MATRIX, THIS SUBROUTINE                      73215016
C     REDUCES A SUBMATRIX SITUATED IN ROWS AND COLUMNS                  73215017
C     LOW THROUGH IGH TO UPPER HESSENBERG FORM BY                       73215018
C     STABILIZED ELEMENTARY SIMILARITY TRANSFORMATIONS.                 73215019
C                                                                       73215020
C     ON INPUT-                                                         73215021
C                                                                       73215022
C        NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL         73215023
C          ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM          73215024
C          DIMENSION STATEMENT,                                         73215025
C                                                                       73215026
C        N IS THE ORDER OF THE MATRIX,                                  73215027
C                                                                       73215028
C        LOW AND IGH ARE INTEGERS DETERMINED BY THE BALANCING           73215029
C          SUBROUTINE  BALANC.  IF  BALANC  HAS NOT BEEN USED,          73215030
C          SET LOW=1, IGH=N,                                            73215031
C                                                                       73215032
C        A CONTAINS THE INPUT MATRIX.                                   73215033
C                                                                       73215034
C     ON OUTPUT-                                                        73215035
C                                                                       73215036
C        A CONTAINS THE HESSENBERG MATRIX.  THE MULTIPLIERS             73215037
C          WHICH WERE USED IN THE REDUCTION ARE STORED IN THE           73215038
C          REMAINING TRIANGLE UNDER THE HESSENBERG MATRIX,              73215039
C                                                                       73215040
C        INT CONTAINS INFORMATION ON THE ROWS AND COLUMNS               73215041
C          INTERCHANGED IN THE REDUCTION.                               73215042
C          ONLY ELEMENTS LOW THROUGH IGH ARE USED.                      73215043
C                                                                       73215044
C     QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,        73215045
C     APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY         73215046
C                                                                       73215047
C     ------------------------------------------------------------------73215048
C                                                                       73215049
      LA = IGH - 1                                                      73215050
      KP1 = LOW + 1                                                     73215051
      IF (LA .LT. KP1) GO TO 200                                        73215052
C                                                                       73215053
      DO 180 M = KP1, LA                                                73215054
         MM1 = M - 1                                                    73215055
```

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```
            X = 0.0                                                         73215056
            I = M                                                           73215057
C                                                                           73215058
            DO 100 J = M, IGH                                               73215059
               IF (ABS(A(J,MM1)) .LE. ABS(X)) GO TO 100                     73215060
               X = A(J,MM1)                                                 73215061
               I = J                                                        73215062
      100   CONTINUE                                                        73215063
C                                                                           73215064
            INT(M) = I                                                      73215065
            IF (I .EQ. M) GO TO 130                                         73215066
C     ********** INTERCHANGE ROWS AND COLUMNS OF A **********               73215067
            DO 110 J = MM1, N                                               73215068
               Y = A(I,J)                                                   73215069
               A(I,J) = A(M,J)                                              73215070
               A(M,J) = Y                                                   73215071
      110   CONTINUE                                                        73215072
C                                                                           73215073
            DO 120 J = 1, IGH                                               73215074
               Y = A(J,I)                                                   73215075
               A(J,I) = A(J,M)                                              73215076
               A(J,M) = Y                                                   73215077
      120   CONTINUE                                                        73215078
C     ********** END INTERCHANGE **********                                 73215079
      130   IF (X .EQ. 0.0) GO TO 180                                       73215080
            MP1 = M + 1                                                     73215081
C                                                                           73215082
            DO 160 I = MP1, IGH                                             73215083
               Y = A(I,MM1)                                                 73215084
               IF (Y .EQ. 0.0) GO TO 160                                    73215085
               Y = Y / X                                                    73215086
               A(I,MM1) = Y                                                 73215087
C                                                                           73215088
               DO 140 J = M, N                                              73215089
      140      A(I,J) = A(I,J) - Y * A(M,J)                                 73215090
C                                                                           73215091
               DO 150 J = 1, IGH                                            73215092
      150      A(J,M) = A(J,M) + Y * A(J,I)                                 73215093
C                                                                           73215094
      160   CONTINUE                                                        73215095
C                                                                           73215096
      180 CONTINUE                                                          73215097
C                                                                           73215098
      200 RETURN                                                            73215099
C     ********** LAST CARD OF ELMHES **********                             73215100
          END                                                               73215101
```

```
      SUBROUTINE ELTRAN(NM,N,LOW,IGH,A,INT,Z)                           20215004
C                                                                       20215005
      INTEGER I,J,N,KL,MM,MP,NM,IGH,LOW,MP1                             20215006
      REAL A(NM,IGH),Z(NM,N)                                            20215007
      INTEGER INT(IGH)                                                  20215008
C                                                                       20215009
C     THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE ELMTRANS, 20215010
C     NUM. MATH. 16, 181-204(1970) BY PETERS AND WILKINSON.             20215011
C     HANDBOOK FOR AUTO. COMP., VOL.II-LINEAR ALGEBRA, 372-395(1971).   20215012
C                                                                       20215013
C     THIS SUBROUTINE ACCUMULATES THE STABILIZED ELEMENTARY             20215014
C     SIMILARITY TRANSFORMATIONS USED IN THE REDUCTION OF A             20215015
C     REAL GENERAL MATRIX TO UPPER HESSENBERG FORM BY  ELMHES.          20215016
C                                                                       20215017
C     ON INPUT-                                                         20215018
C                                                                       20215019
C        NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL         20215020
C          ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM          20215021
C          DIMENSION STATEMENT,                                         20215022
C                                                                       20215023
C        N IS THE ORDER OF THE MATRIX,                                  20215024
C                                                                       20215025
C        LOW AND IGH ARE INTEGERS DETERMINED BY THE BALANCING           20215026
C          SUBROUTINE  BALANC.  IF  BALANC  HAS NOT BEEN USED,          20215027
C          SET LOW=1, IGH=N,                                            20215028
C                                                                       20215029
C        A CONTAINS THE MULTIPLIERS WHICH WERE USED IN THE              20215030
C          REDUCTION BY  ELMHES  IN ITS LOWER TRIANGLE                  20215031
C          BELOW THE SUBDIAGONAL,                                       20215032
C                                                                       20215033
C        INT CONTAINS INFORMATION ON THE ROWS AND COLUMNS               20215034
C          INTERCHANGED IN THE REDUCTION BY  ELMHES.                    20215035
C          ONLY ELEMENTS LOW THROUGH IGH ARE USED.                      20215036
C                                                                       20215037
C     ON OUTPUT-                                                        20215038
C                                                                       20215039
C        Z CONTAINS THE TRANSFORMATION MATRIX PRODUCED IN THE           20215040
C          REDUCTION BY  ELMHES.                                        20215041
C                                                                       20215042
C     QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,        20215043
C     APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY         20215044
C                                                                       20215045
C     ------------------------------------------------------------------20215046
C                                                                       20215047
C     ********** INITIALIZE Z TO IDENTITY MATRIX **********             20215048
      DO 80 I = 1, N                                                    20215049
C                                                                       20215050
         DO 60 J = 1, N                                                 20215051
   60    Z(I,J) = 0.0                                                   20215052
C                                                                       20215053
         Z(I,I) = 1.0                                                   20215054
   80 CONTINUE                                                          20215055
```

```
      C                                                                 20215056
            KL = IGH - LOW - 1                                          20215057
            IF (KL .LT. 1) GO TO 200                                    20215058
      C     ********** FOR MP=IGH-1 STEP -1 UNTIL LOW+1 DO -- ********** 20215059
            DO 140 MM = 1, KL                                           20215060
               MP = IGH - MM                                            20215061
               MP1 = MP + 1                                             20215062
      C                                                                 20215063
               DO 100 I = MP1, IGH                                      20215064
        100    Z(I,MP) = A(I,MP-1)                                      20215065
      C                                                                 20215066
               I = INT(MP)                                              20215067
               IF (I .EQ. MP) GO TO 140                                 20215068
      C                                                                 20215069
               DO 130 J = MP, IGH                                       20215070
                  Z(MP,J) = Z(I,J)                                      20215071
                  Z(I,J) = 0.0                                          20215072
        130    CONTINUE                                                 20215073
      C                                                                 20215074
               Z(I,MP) = 1.0                                            20215075
        140 CONTINUE                                                    20215076
      C                                                                 20215077
        200 RETURN                                                      20215078
      C     ********** LAST CARD OF ELTRAN **********                   20215079
            END                                                         20215080
```

```
      SUBROUTINE HQR2(NM,N,LOW,IGH,H,WR,WI,Z,IERR)                      87215004
C                                                                       87215005
      INTEGER I,J,K,L,M,N,EN,II,JJ,LL,MM,NA,NM,NN,                      87215006
     X        IGH,ITS,LOW,MP2,ENM2,IERR                                 87215007
      REAL H(NM,N),WR(N),WI(N),Z(NM,N)                                  87215008
      REAL P,Q,R,S,T,W,X,Y,RA,SA,VI,VR,ZZ,NORM,MACHEP                   87215009
C     REAL SQRT,ABS,SIGN                                                87215010
C     INTEGER MIN0                                                      87215011
      LOGICAL NOTLAS                                                    87215012
      COMPLEX Z3                                                        87215013
C     COMPLEX CMPLX                                                     87215014
      REAL T3(2)                                                        87215015
      EQUIVALENCE (Z3,T3(1))                                            87215016
C                                                                       87215017
C     THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE HQR2,     87215018
C     NUM. MATH. 16, 181-204(1970) BY PETERS AND WILKINSON.             87215019
C     HANDBOOK FOR AUTO. COMP., VOL.II-LINEAR ALGEBRA, 372-395(1971).   87215020
C                                                                       87215021
C     THIS SUBROUTINE FINDS THE EIGENVALUES AND EIGENVECTORS            87215022
C     OF A REAL UPPER HESSENBERG MATRIX BY THE QR METHOD.  THE          87215023
C     EIGENVECTORS OF A REAL GENERAL MATRIX CAN ALSO BE FOUND           87215024
C     IF  ELMHES  AND  ELTRAN  OR  ORTHES  AND  ORTRAN  HAVE            87215025
C     BEEN USED TO REDUCE THIS GENERAL MATRIX TO HESSENBERG FORM        87215026
C     AND TO ACCUMULATE THE SIMILARITY TRANSFORMATIONS.                 87215027
C                                                                       87215028
C     ON INPUT-                                                         87215029
C                                                                       87215030
C        NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL         87215031
C          ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM          87215032
C          DIMENSION STATEMENT,                                         87215033
C                                                                       87215034
C        N IS THE ORDER OF THE MATRIX,                                  87215035
C                                                                       87215036
C        LOW AND IGH ARE INTEGERS DETERMINED BY THE BALANCING           87215037
C          SUBROUTINE  BALANC.  IF  BALANC  HAS NOT BEEN USED,          87215038
C          SET LOW=1, IGH=N,                                            87215039
C                                                                       87215040
C        H CONTAINS THE UPPER HESSENBERG MATRIX,                        87215041
C                                                                       87215042
C        Z CONTAINS THE TRANSFORMATION MATRIX PRODUCED BY  ELTRAN       87215043
C          AFTER THE REDUCTION BY  ELMHES, OR BY  ORTRAN  AFTER THE     87215044
C          REDUCTION BY  ORTHES, IF PERFORMED.  IF THE EIGENVECTORS     87215045
C          OF THE HESSENBERG MATRIX ARE DESIRED, Z MUST CONTAIN THE     87215046
C          IDENTITY MATRIX.                                             87215047
C                                                                       87215048
C     ON OUTPUT-                                                        87215049
C                                                                       87215050
C        H HAS BEEN DESTROYED,                                          87215051
C                                                                       87215052
C        WR AND WI CONTAIN THE REAL AND IMAGINARY PARTS,                87215053
C          RESPECTIVELY, OF THE EIGENVALUES.  THE EIGENVALUES           87215054
C          ARE UNORDERED EXCEPT THAT COMPLEX CONJUGATE PAIRS            87215055
```

```
C           OF VALUES APPEAR CONSECUTIVELY WITH THE EIGENVALUE             87215056
C           HAVING THE POSITIVE IMAGINARY PART FIRST.  IF AN               87215057
C           ERROR EXIT IS MADE, THE EIGENVALUES SHOULD BE CORRECT          87215058
C           FOR INDICES IERR+1,...,N.                                      87215059
C                                                                          87215060
C        Z CONTAINS THE REAL AND IMAGINARY PARTS OF THE EIGENVECTORS.      87215061
C          IF THE I-TH EIGENVALUE IS REAL, THE I-TH COLUMN OF Z            87215062
C          CONTAINS ITS EIGENVECTOR.  IF THE I-TH EIGENVALUE IS COMPLEX    87215063
C          WITH POSITIVE IMAGINARY PART, THE I-TH AND (I+1)-TH             87215064
C          COLUMNS OF Z CONTAIN THE REAL AND IMAGINARY PARTS OF ITS        87215065
C          EIGENVECTOR.  THE EIGENVECTORS ARE UNNORMALIZED.  IF AN         87215066
C          ERROR EXIT IS MADE, NONE OF THE EIGENVECTORS HAS BEEN FOUND.    87215067
C                                                                          87215068
C        IERR IS SET TO                                                    87215069
C          ZERO       FOR NORMAL RETURN,                                   87215070
C          J          IF THE J-TH EIGENVALUE HAS NOT BEEN                  87215071
C                     DETERMINED AFTER 30 ITERATIONS.                      87215072
C                                                                          87215073
C     ARITHMETIC IS REAL EXCEPT FOR THE REPLACEMENT OF THE ALGOL           87215074
C     PROCEDURE CDIV BY COMPLEX DIVISION.                                  87215075
C                                                                          87215076
C     QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,           87215077
C     APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY            87215078
C                                                                          87215079
C     ------------------------------------------------------------------87215080
C                                                                          87215081
C     ********** MACHEP IS A MACHINE DEPENDENT PARAMETER SPECIFYING        87215082
C                THE RELATIVE PRECISION OF FLOATING POINT ARITHMETIC.      87215083
C                                                                          87215084
C                **********                                                87215085
C     SIGMA 5 SIGMA 5 SIGMA 5 SIGMA5 SIGMA 5 SIGMA 5 SIGMA 5 SIGMA 5       * * * *
      MACHEP = 2.0**(-52)
C     SIGMA 5 SIGMA 5 SIGMA 5 SIGMA5 SIGMA 5 SIGMA 5 SIGMA 5 SIGMA 5       * * * *
C                                                                          87215087
      IERR = 0                                                             87215088
C     ********** STORE ROOTS ISOLATED BY BALANC **********                 87215089
      DO 50 I = 1, N                                                       87215090
         IF (I .GE. LOW .AND. I .LE. IGH) GO TO 50                         87215091
         WR(I) = H(I,I)                                                    87215092
         WI(I) = 0.0                                                       87215093
   50 CONTINUE                                                             87215094
C                                                                          87215095
      EN = IGH                                                             87215096
      T = 0.0                                                              87215097
C     ********** SEARCH FOR NEXT EIGENVALUES **********                    87215098
   60 IF (EN .LT. LOW) GO TO 340                                           87215099
      ITS = 0                                                              87215100
      NA = EN - 1                                                          87215101
      ENM2 = NA - 1                                                        87215102
C     ********** LOOK FOR SINGLE SMALL SUB-DIAGONAL ELEMENT                87215103
C                FOR L=EN STEP -1 UNTIL LOW DO -- **********               87215104
   70 DO 80 LL = LOW, EN                                                   87215105
```

```
            L = EN + LOW - LL
            IF (L .EQ. LOW) GO TO 100
            IF (ABS(H(L,L-1)) .LE. MACHEP * (ABS(H(L-1,L-1))
     X         + ABS(H(L,L)))) GO TO 100
   80    CONTINUE
C     ********** FORM SHIFT **********
  100    X = H(EN,EN)
         IF (L .EQ. EN) GO TO 270
         Y = H(NA,NA)
         W = H(EN,NA) * H(NA,EN)
         IF (L .EQ. NA) GO TO 280
         IF (ITS .EQ. 30) GO TO 1000
         IF (ITS .NE. 10 .AND. ITS .NE. 20) GO TO 130
C     ********** FORM EXCEPTIONAL SHIFT **********
         T = T + X
C
         DO 120 I = LOW, EN
  120    H(I,I) = H(I,I) - X
C
         S = ABS(H(EN,NA)) + ABS(H(NA,ENM2))
         X = 0.75 * S
         Y = X
         W = -0.4375 * S * S
  130    ITS = ITS + 1
C     ********** LOOK FOR TWO CONSECUTIVE SMALL
C                SUB-DIAGONAL ELEMENTS.
C                FOR M=EN-2 STEP -1 UNTIL L DO -- **********
         DO 140 MM = L, ENM2
            M = ENM2 + L - MM
            ZZ = H(M,M)
            R = X - ZZ
            S = Y - ZZ
            P = (R * S - W) / H(M+1,M) + H(M,M+1)
            Q = H(M+1,M+1) - ZZ - R - S
            R = H(M+2,M+1)
            S = ABS(P) + ABS(Q) + ABS(R)
            P = P / S
            Q = Q / S
            R = R / S
            IF (M .EQ. L) GO TO 150
            IF (ABS(H(M,M-1)) * (ABS(Q) + ABS(R)) .LE. MACHEP * ABS(P)
     X      * (ABS(H(M-1,M-1)) + ABS(ZZ) + ABS(H(M+1,M+1)))) GO TO 150
  140    CONTINUE
C
  150    MP2 = M + 2
C
         DO 160 I = MP2, EN
            H(I,I-2) = 0.0
            IF (I .EQ. MP2) GO TO 160
            H(I,I-3) = 0.0
  160    CONTINUE
C     ********** DOUBLE QR STEP INVOLVING ROWS L TO EN AND
```

```
C             COLUMNS M TO EN **********                              87215158
         DO 260 K = M, NA                                             87215159
            NOTLAS = K .NE. NA                                        87215160
            IF (K .EQ. M) GO TO 170                                   87215161
            P = H(K,K-1)                                              87215162
            Q = H(K+1,K-1)                                            87215163
            R = 0.0                                                   87215164
            IF (NOTLAS) R = H(K+2,K-1)                                87215165
            X = ABS(P) + ABS(Q) + ABS(R)                              87215166
            IF (X .EQ. 0.0) GO TO 260                                 87215167
            P = P / X                                                 87215168
            Q = Q / X                                                 87215169
            R = R / X                                                 87215170
  170       S = SIGN(SQRT(P*P+Q*Q+R*R),P)                             87215171
            IF (K .EQ. M) GO TO 180                                   87215172
            H(K,K-1) = -S * X                                         87215173
            GO TO 190                                                 87215174
  180       IF (L .NE. M) H(K,K-1) = -H(K,K-1)                        87215175
  190       P = P + S                                                 87215176
            X = P / S                                                 87215177
            Y = Q / S                                                 87215178
            ZZ = R / S                                                87215179
            Q = Q / P                                                 87215180
            R = R / P                                                 87215181
C     ********** ROW MODIFICATION **********                          87215182
            DO 210 J = K, N                                           87215183
               P = H(K,J) + Q * H(K+1,J)                              87215184
               IF (.NOT. NOTLAS) GO TO 200                            87215185
               P = P + R * H(K+2,J)                                   87215186
               H(K+2,J) = H(K+2,J) - P * ZZ                           87215187
  200          H(K+1,J) = H(K+1,J) - P * Y                            87215188
               H(K,J) = H(K,J) - P * X                                87215189
  210       CONTINUE                                                  87215190
C                                                                     87215191
            J = MIN0(EN,K+3)                                          87215192
C     ********** COLUMN MODIFICATION **********                       87215193
            DO 230 I = 1, J                                           87215194
               P = X * H(I,K) + Y * H(I,K+1)                          87215195
               IF (.NOT. NOTLAS) GO TO 220                            87215196
               P = P + ZZ * H(I,K+2)                                  87215197
               H(I,K+2) = H(I,K+2) - P * R                            87215198
  220          H(I,K+1) = H(I,K+1) - P * Q                            87215199
               H(I,K) = H(I,K) - P                                    87215200
  230       CONTINUE                                                  87215201
C     ********** ACCUMULATE TRANSFORMATIONS **********                87215202
            DO 250 I = LOW, IGH                                       87215203
               P = X * Z(I,K) + Y * Z(I,K+1)                          87215204
               IF (.NOT. NOTLAS) GO TO 240                            87215205
               P = P + ZZ * Z(I,K+2)                                  87215206
               Z(I,K+2) = Z(I,K+2) - P * R                            87215207
  240          Z(I,K+1) = Z(I,K+1) - P * Q                            87215208
               Z(I,K) = Z(I,K) - P                                    87215209
```

```
  250    CONTINUE                                                       87215210
C                                                                       87215211
  260 CONTINUE                                                          87215212
C                                                                       87215213
      GO TO 70                                                          87215214
C     ********** ONE ROOT FOUND **********                              87215215
  270 H(EN,EN) = X + T                                                  87215216
      WR(EN) = H(EN,EN)                                                 87215217
      WI(EN) = 0.0                                                      87215218
      EN = NA                                                           87215219
      GO TO 60                                                          87215220
C     ********** TWO ROOTS FOUND **********                             87215221
  280 P = (Y - X) / 2.0                                                 87215222
      Q = P * P + W                                                     87215223
      ZZ = SQRT(ABS(Q))                                                 87215224
      H(EN,EN) = X + T                                                  87215225
      X = H(EN,EN)                                                      87215226
      H(NA,NA) = Y + T                                                  87215227
      IF (Q .LT. 0.0) GO TO 320                                         87215228
C     ********** REAL PAIR **********                                   87215229
      ZZ = P + SIGN(ZZ,P)                                               87215230
      WR(NA) = X + ZZ                                                   87215231
      WR(EN) = WR(NA)                                                   87215232
      IF (ZZ .NE. 0.0) WR(EN) = X - W / ZZ                              87215233
      WI(NA) = 0.0                                                      87215234
      WI(EN) = 0.0                                                      87215235
      X = H(EN,NA)                                                      87215236
      R = SQRT(X*X+ZZ*ZZ)                                               87215237
      P = X / R                                                         87215238
      Q = ZZ / R                                                        87215239
C     ********** ROW MODIFICATION **********                            87215240
      DO 290 J = NA, N                                                  87215241
         ZZ = H(NA,J)                                                   87215242
         H(NA,J) = Q * ZZ + P * H(EN,J)                                 87215243
         H(EN,J) = Q * H(EN,J) - P * ZZ                                 87215244
  290 CONTINUE                                                          87215245
C     ********** COLUMN MODIFICATION **********                         87215246
      DO 300 I = 1, EN                                                  87215247
         ZZ = H(I,NA)                                                   87215248
         H(I,NA) = Q * ZZ + P * H(I,EN)                                 87215249
         H(I,EN) = Q * H(I,EN) - P * ZZ                                 87215250
  300 CONTINUE                                                          87215251
C     ********** ACCUMULATE TRANSFORMATIONS **********                  87215252
      DO 310 I = LOW, IGH                                               87215253
         ZZ = Z(I,NA)                                                   87215254
         Z(I,NA) = Q * ZZ + P * Z(I,EN)                                 87215255
         Z(I,EN) = Q * Z(I,EN) - P * ZZ                                 87215256
  310 CONTINUE                                                          87215257
C                                                                       87215258
      GO TO 330                                                         87215259
C     ********** COMPLEX PAIR **********                                87215260
  320 WR(NA) = X + P                                                    87215261
```

```
      WR(EN) = X + P
      WI(NA) = ZZ
      WI(EN) = -ZZ
  330 EN = ENM2
      GO TO 60
C     ********** ALL ROOTS FOUND.  BACKSUBSTITUTE TO FIND
C                VECTORS OF UPPER TRIANGULAR FORM **********
  340 NORM = 0.0
      K = 1
C
      DO 360 I = 1, N
C
         DO 350 J = K, N
  350    NORM = NORM + ABS(H(I,J))
C
         K = I
  360 CONTINUE
C
      IF (NORM .EQ. 0.0) GO TO 1001
C     ********** FOR EN=N STEP -1 UNTIL 1 DO -- **********
      DO 800 NN = 1, N
         EN = N + 1 - NN
         P = WR(EN)
         Q = WI(EN)
         NA = EN - 1
         IF (Q) 710, 600, 800
C     ********** REAL VECTOR **********
  600    M = EN
         H(EN,EN) = 1.0
         IF (NA .EQ. 0) GO TO 800
C     ********** FOR I=EN-1 STEP -1 UNTIL 1 DO -- **********
         DO 700 II = 1, NA
            I = EN - II
            W = H(I,I) - P
            R = H(I,EN)
            IF (M .GT. NA) GO TO 620
C
            DO 610 J = M, NA
  610       R = R + H(I,J) * H(J,EN)
C
  620       IF (WI(I) .GE. 0.0) GO TO 630
            ZZ = W
            S = R
            GO TO 700
  630       M = I
            IF (WI(I) .NE. 0.0) GO TO 640
            T = W
            IF (W .EQ. 0.0) T = MACHEP * NORM
            H(I,EN) = -R / T
            GO TO 700
C     ********** SOLVE REAL EQUATIONS **********
  640       X = H(I,I+1)
```

```
                Y = H(I+1,I)                                                  87215314
                Q = (WR(I) - P) * (WR(I) - P) + WI(I) * WI(I)                 87215315
                T = (X * S - ZZ * R) / Q                                      87215316
                H(I,EN) = T                                                   87215317
                IF (ABS(X) .LE. ABS(ZZ)) GO TO 650                            87215318
                H(I+1,EN) = (-R - W * T) / X                                  87215319
                GO TO 700                                                     87215320
      650       H(I+1,EN) = (-S - Y * T) / ZZ                                 87215321
      700    CONTINUE                                                         87215322
C     *********** END REAL VECTOR **********                                  87215323
             GO TO 800                                                        87215324
C     *********** COMPLEX VECTOR **********                                   87215325
  710        M = NA                                                           87215326
C     *********** LAST VECTOR COMPONENT CHOSEN IMAGINARY SO THAT              87215327
C                 EIGENVECTOR MATRIX IS TRIANGULAR **********                 87215328
             IF (ABS(H(EN,NA)) .LE. ABS(H(NA,EN))) GO TO 720                  87215329
             H(NA,NA) = Q / H(EN,NA)                                          87215330
             H(NA,EN) = -(H(EN,EN) - P) / H(EN,NA)                            87215331
             GO TO 730                                                        87215332
  720        Z3 = CMPLX(0.0,-H(NA,EN)) / CMPLX(H(NA,NA)-P,Q)                  87215333
             H(NA,NA) = T3(1)                                                 87215334
             H(NA,EN) = T3(2)                                                 87215335
  730        H(EN,NA) = 0.0                                                   87215336
             H(EN,EN) = 1.0                                                   87215337
             ENM2 = NA - 1                                                    87215338
             IF (ENM2 .EQ. 0) GO TO 800                                       87215339
C                                                                             87215340
             DO 790 II = 1, ENM2                                              87215341
                I = NA - II                                                   87215342
                W = H(I,I) - P                                                87215343
                RA = 0.0                                                      87215344
                SA = H(I,EN)                                                  87215345
C                                                                             87215346
                DO 760 J = M, NA                                              87215347
                   RA = RA + H(I,J) * H(J,NA)                                 87215348
                   SA = SA + H(I,J) * H(J,EN)                                 87215349
  760           CONTINUE                                                      87215350
C                                                                             87215351
                IF (WI(I) .GE. 0.0) GO TO 770                                 87215352
                ZZ = W                                                        87215353
                R = RA                                                        87215354
                S = SA                                                        87215355
                GO TO 790                                                     87215356
  770           M = I                                                         87215357
                IF (WI(I) .NE. 0.0) GO TO 780                                 87215358
                Z3 = CMPLX(-RA,-SA) / CMPLX(W,Q)                              87215359
                H(I,NA) = T3(1)                                               87215360
                H(I,EN) = T3(2)                                               87215361
                GO TO 790                                                     87215362
C     ********** SOLVE COMPLEX EQUATIONS **********                           87215363
  780           X = H(I,I+1)                                                  87215364
                Y = H(I+1,I)                                                  87215365
```

```
                VR = (WR(I) - P) * (WR(I) - P) + WI(I) * WI(I) - Q * Q         87215366
                VI = (WR(I) - P) * 2.0 * Q                                     87215367
                IF (VR .EQ. 0.0 .AND. VI .EQ. 0.0) VR = MACHEP * NORM          87215368
     X           * (ABS(W) + ABS(Q) + ABS(X) + ABS(Y) + ABS(ZZ))               87215369
                Z3 = CMPLX(X*R-ZZ*RA+Q*SA,X*S-ZZ*SA-Q*RA) / CMPLX(VR,VI)       87215370
                H(I,NA) = T3(1)                                                87215371
                H(I,EN) = T3(2)                                                87215372
                IF (ABS(X) .LE. ABS(ZZ) + ABS(Q)) GO TO 785                    87215373
                H(I+1,NA) = (-RA - W * H(I,NA) + Q * H(I,EN)) / X              87215374
                H(I+1,EN) = (-SA - W * H(I,EN) - Q * H(I,NA)) / X              87215375
                GO TO 790                                                      87215376
  785           Z3 = CMPLX(-R-Y*H(I,NA),-S-Y*H(I,EN)) / CMPLX(ZZ,Q)            87215377
                H(I+1,NA) = T3(1)                                              87215378
                H(I+1,EN) = T3(2)                                              87215379
  790       CONTINUE                                                           87215380
C     ********** END COMPLEX VECTOR **********                                 87215381
  800 CONTINUE                                                                 87215382
C     ********** END BACK SUBSTITUTION.                                        87215383
C                VECTORS OF ISOLATED ROOTS **********                          87215384
      DO 840 I = 1, N                                                          87215385
         IF (I .GE. LOW .AND. I .LE. IGH) GO TO 840                            87215386
C                                                                              87215387
         DO 820 J = I, N                                                       87215388
  820    Z(I,J) = H(I,J)                                                       87215389
C                                                                              87215390
  840 CONTINUE                                                                 87215391
C     ********** MULTIPLY BY TRANSFORMATION MATRIX TO GIVE                     87215392
C                VECTORS OF ORIGINAL FULL MATRIX.                              87215393
C                FOR J=N STEP -1 UNTIL LOW DO -- **********                    87215394
      DO 880 JJ = LOW, N                                                       87215395
         J = N + LOW - JJ                                                      87215396
         M = MIN0(J,IGH)                                                       87215397
C                                                                              87215398
         DO 880 I = LOW, IGH                                                   87215399
            ZZ = 0.0                                                           87215400
C                                                                              87215401
            DO 860 K = LOW, M                                                  87215402
  860       ZZ = ZZ + Z(I,K) * H(K,J)                                          87215403
C                                                                              87215404
            Z(I,J) = ZZ                                                        87215405
  880 CONTINUE                                                                 87215406
C                                                                              87215407
      GO TO 1001                                                               87215408
C     ********** SET ERROR -- NO CONVERGENCE TO AN                             87215409
C                EIGENVALUE AFTER 30 ITERATIONS **********                     87215410
 1000 IERR = EN                                                                87215411
 1001 RETURN                                                                   87215412
C     ********** LAST CARD OF HQR2 **********                                  87215413
      END                                                                      87215414
```

```
      SUBROUTINE BALBAK(NM,N,LOW,IGH,SCALE,M,Z)                         70215004
C                                                                       70215005
      INTEGER I,J,K,M,N,II,NM,IGH,LOW                                   70215006
      REAL SCALE(N),Z(NM,M)                                             70215007
      REAL S                                                            70215008
C                                                                       70215009
C     THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE BALBAK,  70215010
C     NUM. MATH. 13, 293-304(1969) BY PARLETT AND REINSCH.              70215011
C     HANDBOOK FOR AUTO. COMP., VOL.II-LINEAR ALGEBRA, 315-326(1971).   70215012
C                                                                       70215013
C     THIS SUBROUTINE FORMS THE EIGENVECTORS OF A REAL GENERAL          70215014
C     MATRIX BY BACK TRANSFORMING THOSE OF THE CORRESPONDING            70215015
C     BALANCED MATRIX DETERMINED BY  BALANC.                            70215016
C                                                                       70215017
C     ON INPUT-                                                         70215018
C                                                                       70215019
C        NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL         70215020
C          ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM          70215021
C          DIMENSION STATEMENT,                                         70215022
C                                                                       70215023
C        N IS THE ORDER OF THE MATRIX,                                  70215024
C                                                                       70215025
C        LOW AND IGH ARE INTEGERS DETERMINED BY  BALANC,                70215026
C                                                                       70215027
C        SCALE CONTAINS INFORMATION DETERMINING THE PERMUTATIONS        70215028
C          AND SCALING FACTORS USED BY  BALANC,                         70215029
C                                                                       70215030
C        M IS THE NUMBER OF COLUMNS OF Z TO BE BACK TRANSFORMED,        70215031
C                                                                       70215032
C        Z CONTAINS THE REAL AND IMAGINARY PARTS OF THE EIGEN-          70215033
C          VECTORS TO BE BACK TRANSFORMED IN ITS FIRST M COLUMNS.       70215034
C                                                                       70215035
C     ON OUTPUT-                                                        70215036
C                                                                       70215037
C        Z CONTAINS THE REAL AND IMAGINARY PARTS OF THE                 70215038
C          TRANSFORMED EIGENVECTORS IN ITS FIRST M COLUMNS.             70215039
C                                                                       70215040
C     QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,        70215041
C     APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY         70215042
C                                                                       70215043
C     ------------------------------------------------------------------70215044
C                                                                       70215045
      IF (IGH .EQ. LOW) GO TO 120                                       70215046
C                                                                       70215047
      DO 110 I = LOW, IGH                                               70215048
         S = SCALE(I)                                                   70215049
C     ********** LEFT HAND EIGENVECTORS ARE BACK TRANSFORMED            70215050
C                IF THE FOREGOING STATEMENT IS REPLACED BY              70215051
C                S=1.0/SCALE(I). **********                             70215052
         DO 100 J = 1, M                                                70215053
  100    Z(I,J) = Z(I,J) * S                                            70215054
C                                                                       70215055
```

```
  110 CONTINUE                                                          70215056
C     *********- FOR I=LOW-1 STEP -1 UNTIL 1,                           70215057
C                IGH+1 STEP 1 UNTIL N DO -- **********                  70215058
  120 DO 140 II = 1, N                                                  70215059
         I = II                                                         70215060
         IF (I .GE. LOW .AND. I .LE. IGH) GO TO 140                     70215061
         IF (I .LT. LOW) I = LOW - II                                   70215062
         K = SCALE(I)                                                   70215063
         IF (K .EQ. I) GO TO 140                                        70215064
C                                                                       70215065
         DO 130 J = 1, M                                                70215066
            S = Z(I,J)                                                  70215067
            Z(I,J) = Z(K,J)                                             70215068
            Z(K,J) = S                                                  70215069
  130    CONTINUE                                                       70215070
C                                                                       70215071
  140 CONTINUE                                                          70215072
C                                                                       70215073
      RETURN                                                            70215074
C     ********** LAST CARD OF BALBAK **********                         70215075
      END                                                               70215076
```

```
      SUBROUTINE EISORT(WR,WI,FR,TC,ZETA,OMG,PI,N)
      DIMENSION WR(1),WI(1),FR(1),TC(1),ZETA(1),SR(50),SI(50)
C
C     SR AND SI ARE TEMPORARY WORKING STORAGE
C
      JUMP=1
      NT=N
      NR=0
      NM1=N-1
      DO 100 I=1,N
      IF(JUMP)20,10,20
   10 JUMP=1
      GO TO 100
   20 IF(WI(I))60,30,60
   30 NR=NR+1
      SM=WR(I)
      DO 50 L=1,NM1
      IF(L-I)50,40,40
   40 WR(L)=WR(L+1)
      WI(L)=WI(L+1)
   50 CONTINUE
      WR(NT)=SM
      WI(NT)=0.
      NT=NT-1
      GO TO 100
   60 JUMP=0
  100 CONTINUE
      NMR=N-NR
      NMR1=NMR-1
      DO 200 I=1,NMR1,2
      WII=WI(I)
      WIR=WR(I)
      IMAX=I
      IP2=I+2
      IF(IP2-NMR)140,160,160
  140 DO 150 J=IP2,NMR,2
      IF(WII.GE.WI(J))GO TO 150
      WII=WI(J)
      WIR=WR(J)
      IMAX=J
  150 CONTINUE
  160 SR(I)=WR(IMAX)
      SR(I+1)=WR(IMAX)
      SI(I)=WI(IMAX)
      SI(I+1)=-WI(IMAX)
      WR(IMAX)=WR(I)
      WR(IMAX+1)=WR(I)
      WI(IMAX)=WI(I)
      WI(IMAX+1)=-WI(I)
      ZETA(I)=SR(I)/SQRT(SR(I)*SR(I)+SI(I)*SI(I))
      ZETA(I+1)=ZETA(I)
      TC(I)=1./(2.*PI*SR(I))
```

```
      TC(I+1)=TC(I)
      FR(I)=SI(I)/OMG
      FR(I+1)=FR(I)
  200 CONTINUE
      IF(NR)210,260,210
  210 DO 250 I=NMR,N
      SR(I)=WR(I)
      SI(I)=WI(I)
      ZETA(I)=1.
      TC(I)=1./(2.*PI*SR(I))
      FR(I)=0.
  250 CONTINUE
  260 DO 300 I=1,N
      WR(I)=SR(I)
      WI(I)=SI(I)
  300 CONTINUE
      RETURN
      END
```

APPENDIX B

DEFINITION OF SYMBOLS

SYMBOLS

ABO	Aero term
ADUM	Dummy array
AKA	transformed aero term
ALA	Aero term
AKAO	Transformed aero term
ALAO	Aero term
AKB	Transformed aero term
ALB	Aero term
AKBO	Transformed aero term
ALBO	Aero ferm
ALPHA	Angle of attack
ALT	Altitude
AM	Aerodynamic torque
APHI	Initial transformation matrix
AREF	Reference area
B	Matrix
BKA	Transformed aero term
BLA	Aero term
BKB	Transformed aero term
BLB	Aero term
CLA	Aero term
DELPSI	Delta PSI
DEN	Aero term
DI	Dummy index

SYMBOLS (Continued)

DREF	Reference diameter
DUM	Dummy array
EUM	Dummy matrix
FR	Frequency of root
FV	Working array
GG	Gravity constant, $3\Omega^2$
GM	Universal gravitational constant
IERR	Error flag
IV	Working array
LI	Working storage
LIMIT	Number of iterations
MI	Working storage
M1, M2, M3	Aero terms
OMG	Orbital angular velocity
PAMA	Partial of aerodynamic torque with respect to alpha
PAMP	Partial of aerodynamic torque with respect to PHI
PAMS	Partial of aerodynamic torque with respect to PSI
PAS	Partial of alpha with respect to PSI
PASI	Partial of aerodynamic torque with respect to PSI divided by moment of inertia
PGGS	Partial of $\Omega X \; I\Omega$ — gravity gradient torque, with respect to PSI expressed in the principal axes reference frame
PHI	Aerodynamic roll angle
PI	π
PHIB	Aerodynamic bank angle

SYMBOLS (Continued)

PNUS	Partial of NU with respect to PSI
PNUSI	Inverse of the partial of NU with respect to PSI
PPHI	Initial rotation angles
PPS	Partial of PHI with respect to PSI
PSI	Rotation angles
PSIO	Initial input of rotation angles
QI	Inverse of angular velocity transformation matrix
QNUS	Dummy matrix for PNUS
RAD	Conversion factor radians to degrees
RE	Radius of the Earth
RHO	Atmospheric density (slugs per cubic foot)
RHOK	Atmospheric density (kg per cubic meter)
RORB	Orbital radius
TC	Unstable time constant or logaritmic increment
TGP	Transformation matrix from principal to geometric system
TMM	Characteristic polynomial matrix
TPG	Transformation matrix from geometric to principal system
U	Transformation array for converting aerodynamic torques from geometric system to principal system
V	Velocity in geometric system
VMAG	Magnitude of velocity vector
VO	Velocity in principal system
WI	Array containing imaginary parts of roots
WR	Array containing real parts of roots

SYMBOLS (Concluded)

XCG	X coordinate of center of gravity
XI	Moment of inertias
XNU	NU
XNUMAG	Magnitude of NU
YCG	Y coordinate of center of gravity
ZCG	Z coordinate of center of gravity
ZETA	Ratio of the real part to the root sum square of the real and imaginary parts of the root

APPENDIX C

INPUT DATA

TABLE 1. INPUT REQUIREMENTS

Lines	Program Symbols	Variable Definition	Units	Format
1	DREF AREF	Reference Length Reference Area	Ft. Sq. Ft.	G G
2	LIMIT	Number of Iterations	Unitless	I
3 5	TGP(I,J) I 1,3 J 1,3	Transformation Matrix from Principal to Geometric Coordinates	Unitless	3G
6	XI1	X Principal Moment of Inertia	Slug Sq. Ft.	G
	XI2	Y Principal Moment of Inertia	Slug Sq. Ft.	G
	XI3	Z Principal Moment of Inertia	Slug Sq. Ft.	G
7	XCG YCG ZCG	Geometric Coordinates of Center of Gravity	Ft. Ft. Ft.	G G G
8	M1 M2 M3	Aerodynamic Table Indicies	Unitless Unitless Unitless	I5 I5 I5
9	ABO(I) I 1,M1	Coefficients of Double Fourier Series for Aerodynamic Forces and Torques	Unitless	F12.0
10 21	ALAO(I,J) ALBO(I,J) J-1,M2 I-1,M1		Unitless	F12.0
22 165	ALA(I,J,K) BLA(I,J,K) K-1,M3 J 1,M2 ALB(I,J,K) BLB(I,J,K) K-1,M3 J 1,M2 I 1,M1			
166	RHOK ALT	Atmospheric Density Altitude	Kg/cu. m. Nautical Mile	G G
167	PSIO(I) I 1,3	Angle of Attack, Roll Angle, Bank Angle	Degrees	3G
168		Blank Card		

```
 1 - 33.0,855.3                                   REFERENCE DIAMETER AND AREA IN FT,FT2
 2 - 20                                           NUMBER OF ITERATIONS
 3 - .984160967,.0162084438,-.176537454           TRANSFORMATION FROM
 4 - .021006465,.97813368,.20691207               PRINCIPAL TO GEOMETRIC
 5 - .176030993,-.207343161,.962299825            COORDINATES
 6 - .562022876E6,.28737495E7,.28195765E7         PRINCIPAL INERTIAS IN SLUG-FT2
 7 - -27.283,-.325,2.642                          LOCATION OF CG IN FT
 8 -      6     6     6
```

9 -	.000	.000	.000	.000	.000	.000
10 -	7.842	.000	-2.326	.000	.000	.000
11 -	-.125	.000	-.171	.000	.000	.000
12 -	-.281	.000	.287	.000	.000	.000
13 -	.059	.000	.020	.000	.000	.000
14 -	.013	.000	-.027	.000	.000	.000
15 -	-.064	.000	-.019	.000	.000	.000
16 -	-1.289	.000	.442	.000	.000	.000
17 -	-.037	.000	-.040	.000	.000	.000
18 -	-.958	.000	.269	.000	.000	.000
19 -	.166	.000	.155	.000	.000	.000
20 -	-.056	.000	.078	.000	.000	.000
21 -	.053	.000	.025	.000	.000	.000
22 -	.056	.727	-.038	-.076	.014	.107
23 -	.034	-.120	.013	.032	-.012	-.006
24 -	.000	.000	.000	.000	.000	.000
25 -	.000	.000	.000	.000	.000	.000
26 -	-.006	-.532	.026	.071	-.002	-.023
27 -	-.024	.115	-.008	-.032	.019	.005
28 -	.000	.000	.000	.000	.000	.000
29 -	.000	.000	.000	.000	.000	.000
30 -	.000	.000	.000	.000	.000	.000
31 -	.000	.000	.000	.000	.000	.000
32 -	.000	.000	.000	.000	.000	.000
33 -	.000	.000	.000	.000	.000	.000
34 -	-.260	-.014	-.064	.008	.013	-.016
35 -	-.013	.021	-.028	.007	-.005	.001
36 -	.000	.000	.000	.000	.000	.000
37 -	.000	.000	.000	.000	.000	.000
38 -	-.240	-.009	-.037	.015	.011	-.016
39 -	.001	.011	-.016	.007	.000	.000
40 -	.000	.000	.000	.000	.000	.000
41 -	.000	.000	.000	.000	.000	.000
42 -	.000	.000	.000	.000	.000	.000
43 -	.000	.000	.000	.000	.000	.000
44 -	.000	.000	.000	.000	.000	.000
45 -	.000	.000	.000	.000	.000	.000
46 -	-.072	-.173	.003	-.023	-.007	.030
47 -	.111	-.022	.016	-.018	.005	-.010
48 -	.000	.000	.000	.000	.000	.000
49 -	.000	.000	.000	.000	.000	.000
50 -	.047	.182	.005	.038	.011	-.039

51 -	-.016	.036	-.020	.027	-.005	.016
52 -	.000	.000	.000	.000	.000	.000
53 -	.000	.000	.000	.000	.000	.000
54 -	.000	.000	.000	.000	.000	.000
55 -	.000	.000	.000	.000	.000	.000
56 -	.000	.000	.000	.000	.000	.000
57 -	.000	.000	.000	.000	.000	.000
58 -	7.296	.005	.679	-.016	-.002	.005
59 -	-.162	.027	-.102	-.002	.049	-.007
60 -	.000	.000	.000	.000	.000	.000
61 -	.000	.000	.000	.000	.000	.000
62 -	-1.079	.001	-.156	-.008	.023	.000
63 -	.132	.013	.011	.003	-.022	.004
64 -	.000	.000	.000	.000	.000	.000
65 -	.000	.000	.000	.000	.000	.000
66 -	.000	.000	.000	.000	.000	.000
67 -	.000	.000	.000	.000	.000	.000
68 -	.000	.000	.000	.000	.000	.000
69 -	.000	.000	.000	.000	.000	.000
70 -	-.011	.008	.007	-.012	.003	.004
71 -	.049	.097	.011	.039	-.011	.017
72 -	.000	.000	.000	.000	.000	.000
73 -	.000	.000	.000	.000	.000	.000
74 -	.016	-.006	-.012	.015	-.002	-.005
75 -	-.032	-.094	-.010	-.053	.008	-.023
76 -	.000	.000	.000	.000	.000	.000
77 -	.000	.000	.000	.000	.000	.000
78 -	.000	.000	.000	.000	.000	.000
79 -	.000	.000	.000	.000	.000	.000
80 -	.000	.000	.000	.000	.000	.000
81 -	.000	.000	.000	.000	.000	.000
82 -	.154	.023	-.208	.027	.058	-.031
83 -	-5.278	-.046	-.925	.044	.196	-.004
84 -	.000	.000	.000	.000	.000	.000
85 -	.000	.000	.000	.000	.000	.000
86 -	-.029	.007	.050	.000	-.019	.001
87 -	.718	-.018	.225	.004	-.058	.000
88 -	.000	.000	.000	.000	.000	.000
89 -	.000	.000	.000	.000	.000	.000
90 -	.000	.000	.000	.000	.000	.000
91 -	.000	.000	.000	.000	.000	.000
92 -	.000	.000	.000	.000	.000	.000
93 -	.000	.000	.000	.000	.000	.000
94 -	.135	-.195	-.009	.080	.007	-.037
95 -	-.009	.009	-.011	.022	-.005	.011
96 -	.000	.000	.000	.000	.000	.000
97 -	.000	.000	.000	.000	.000	.000
98 -	-.073	.084	.000	-.076	-.011	.033
99 -	.009	-.022	.015	-.036	.007	-.020
100 -	.003	.000	.000	.000	.000	.000
101 -	.000	.000	.000	.000	.000	.000
102 -	.000	.000	.000	.000	.000	.000

103 -	.000	.000	.000	.000	.000	.000
104 -	.000	.000	.010	.000	.000	.000
105 -	.000	.000	.000	.000	.000	.000
106 -	-5.146	.011	-.350	.014	-.010	-.007
107 -	.199	-.017	.120	.002	-.065	.009
108 -	.000	.000	.000	.000	.000	.000
109 -	.000	.000	.000	.000	.000	.000
110 -	.779	-.007	.123	.005	-.032	-.001
111 -	-.049	-.012	-.021	.001	.032	-.001
112 -	.000	.000	.000	.000	.000	.000
113 -	.000	.000	.000	.000	.000	.000
114 -	.000	.000	.000	.000	.000	.000
115 -	.000	.000	.000	.000	.000	.000
116 -	.000	.000	.000	.000	.000	.000
117 -	.000	.000	.000	.000	.000	.000
118 -	-.067	-.266	.019	.016	-.006	.012
119 -	-.096	-.055	-.026	-.054	.018	-.009
120 -	.000	.000	.000	.000	.000	.000
121 -	.000	.000	.000	.000	.000	.000
122 -	.116	.207	-.007	-.032	.006	-.014
123 -	.155	.031	.014	.068	-.027	.012
124 -	.000	.000	.000	.000	.000	.000
125 -	.000	.000	.000	.000	.000	.000
126 -	.000	.000	.000	.000	.000	.000
127 -	.000	.000	.000	.000	.000	.000
128 -	.000	.000	.000	.000	.000	.000
129 -	.000	.000	.000	.000	.000	.000
130 -	-.140	-.014	.280	-.042	-.074	.046
131 -	1.210	.022	.454	-.052	-.055	.003
132 -	.000	.000	.000	.000	.000	.000
133 -	.000	.000	.000	.000	.000	.000
134 -	.072	.003	-.069	-.015	.032	.004
135 -	-.467	.004	-.134	.010	.003	-.001
136 -	.000	.000	.000	.000	.000	.000
137 -	.011	.000	.000	.000	.000	.000
138 -	.000	.000	.000	.000	.000	.000
139 -	.000	.000	.000	.000	.000	.000
140 -	.000	.000	.000	.000	.000	.000
141 -	.000	.000	.000	.000	.000	.000
142 -	-.071	-.038	.009	-.012	-.001	.006
143 -	.018	-.035	.013	-.017	.002	-.011
144 -	.000	.000	.000	.000	.000	.000
145 -	.000	.000	.000	.000	.000	.000
146 -	.047	.058	-.009	.018	.005	-.016
147 -	-.021	.002	-.016	.028	.000	.018
148 -	.000	.000	.000	.000	.000	.000
149 -	.000	.000	.000	.000	.000	.000
150 -	.000	.000	.000	.000	.000	.000
151 -	.000	.000	.000	.000	.000	.000
152 -	.000	.000	.000	.000	.000	.000
153 -	.000	.000	.000	.000	.000	.000
154 -	1.136	.024	.250	-.005	-.039	-.007

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155 -        .943      -.016       .430       .005      -.052      -.005
156 -        .000       .000       .000       .000       .000       .000
157 -        .000       .000       .000       .000       .000       .000
158 -       -.210       .009      -.055      -.005       .019      -.003
159 -       -.162       .005      -.099       .000       .007      -.002
160 -        .000       .000       .000       .000       .000       .000
161 -        .000       .000       .000       .000       .000       .000
162 -        .000       .000       .000       .000       .000       .000
163 -        .000       .000       .000       .000       .000       .000
164 -        .000       .000       .000       .000       .000       .000
165 -        .000       .000       .000       .000       .000       .000
166 - .2295E-11,235.
167 - 159.221,81.266,4.686
168 - .2666E-11,230.
169 - 156,910,81.501,5.045
170 - .3095E-11,225.
171 - 154.618,81.755,5.390
172 - .361E-11,220.
173 - 152.4027,82.0207,5.72385
174 - .4215E-11,215.
175 - 137.,80.,8.
176 - .493E-11,210.
177 - 148.378,82.567,6.303
178 - .5795E-11,205.
179 - 140.,80.,7.6
180 - .6805E-11,200.
181 - 141.0,83.0,7.24
182 - .8030E-11,195.
183 - 140.,84.,7.5
184 - .9075E-11,190.
185 - 139.,84.,7.7
186 - .11285E-10,185.
187 - 140.992,83.983,7.291
188 - .13385E-10,180.
189 - 138.2,85.0,7.64
190 - .1600E-10,175.
191 - 138.4,85.,7.7
192 - .1925E-10,170.
193 - 138.5,85.,7.7
194 - .2315E-10,165.
195 - 137.629,85.339,7.713
196 - .2795E-10,160.
197 - 136.2,86.0,7.9
198 - .3005E-10,155.
199 - 136.,87.,7.9
200 - .4160E-10,150.
201 - 136.,88.,7.9
202 - .512E-10,145.
203 - 135.701,87.595,7.989
204 - .634E-10,140.
205 - 135.5,89.0,8.1
206 - .7915E-10,135.
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134.8,90.5,8.2
.9945E-10,130.
134.6,91.8,8.4
.1260E-9,125.
134.4,92.9,8.5
.162E-9,120.
134.2,94.1,8.7
.210E-9,115.
134.127,94.573,8.8457
.276E-9,110.
133.8,97.5,9.5
.3685E-9,105.
133.78024,98.1895,9.7594
.5015E-9,100.
133.638,100.197,10.5303
.699E-9,95.
133.519,101.975,11.658
.10045E-8,90.
133.4,104.,13.8
.1505E-8,85.
133.353,105.352,16.141
.240E-8,80.
133.300,106.765,21.176
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APPENDIX D

PROGRAM OUTPUT AND PLOTS

ALTITUDE 235.0 DENSITY .22950E-11
PHI 159.220905 ALPHA 81.265698 PHIB 4.686589
.12143064E-16 -.57766292E-15 -.52041704E-17 .57781397E-15

REAL	IMAG	FREQ	ZETA	TC
-.18690156E-06	.20798799E-02	1.85273	-.0001	-.85E 06
-.18690156E-06	-.20798799E-02	1.85273	-.0001	-.85E 06
-.42023415E-06	.17415523E-02	1.55135	-.0002	-.38E 06
-.42023415E-06	-.17415523E-02	1.55135	-.0002	-.38E 06
.60713570E-06	.43146157E-03	.38434	.0014	.26E 06
.60713570E-06	-.43146157E-03	.38434	.0014	.26E 06

ALTITUDE 230.0 DENSITY .26600E-11
PHI 156.910476 ALPHA 81.501266 PHIB 5.045400
.12143064E-16 .93414859E-15 -.22100377E-14 .23993849E-14

REAL	IMAG	FREQ	ZETA	TC
-.37955882E-06	.20836125E-02	1.85227	-.0002	-.42E 06
-.37955882E-06	-.20836125E-02	1.85227	-.0002	-.42E 06
-.52643271E-06	.17476387E-02	1.55360	-.0003	-.30E 06
-.52643271E-06	-.17476387E-02	1.55360	-.0003	-.30E 06
.90599153E-06	.44486910E-03	.39548	.0020	.18E 06
.90599153E-06	-.44486910E-03	.39548	.0020	.18E 06

ALTITUDE 225.0 DENSITY .30950E-11
PHI 154.617514 ALPHA 81.754979 PHIB 5.394232
-.23418767E-16 .99052710E-15 -.34000580E-15 .10475191E-14

REAL	IMAG	FREQ	ZETA	TC
-.58517083E-06	.20873855E-02	1.85184	-.0003	-.27E 06
-.58517083E-06	-.20873855E-02	1.85184	-.0003	-.27E 06
-.65069462E-06	.17539366E-02	1.55602	-.0004	-.24E 06
-.65069462E-06	-.17539366E-02	1.55602	-.0004	-.24E 06
.12358655E-05	.46087953E-03	.40887	.0027	.13E 06
.12358655E-05	-.46087953E-03	.40887	.0027	.13E 06

ALTITUDE 220.0 DENSITY .36100E-11
PHI 152.402695 ALPHA 82.020764 PHIB 5.723857
-.19949320E-16 -.64705186E-15 -.10460383E-14 .12301504E-14

REAL	IMAG	FREQ	ZETA	TC
-.78753724E-06	.20912331E-02	1.85146	-.0004	-.20E 06
-.78753724E-06	-.20912331E-02	1.85146	-.0004	-.20E 06
-.79444193E-06	.17604019E-02	1.55856	-.0005	-.20E 06
-.79444193E-06	-.17604019E-02	1.55856	-.0005	-.20E 06
.15819792E-05	.47979842E-03	.42479	.0033	.10E 06
.15819792E-05	-.47979842E-03	.42479	.0033	.10E 06

ALTITUDE 215.0 DENSITY .42150E-11
PHI 150.318658 ALPHA 82.292089 PHIB 6.027047
.52041704E-17 .97144515E-16 .12195106E-14 .12233848E-14

REAL	IMAG	FREQ	ZETA	TC
-.97080251E-06	.20951881E-02	1.85117	-.0005	-.16E 06
-.97080251E-06	-.20951881E-02	1.85117	-.0005	-.16E 06
-.95960476E-06	.17669946E-02	1.56120	-.0005	-.17E 06
-.95960476E-06	-.17669946E-02	1.56120	-.0005	-.17E 06
.19304073E-05	.50179953E-03	.44336	.0038	.82E 05
.19304073E-05	-.50179953E-03	.44336	.0038	.82E 05

ALTITUDE 210.0 DENSITY .49300E-11
PHI 148.378478 ALPHA 82.566771 PHIB 6.302888
-.95409791E-17 -.98532293E-15 -.66439909E-15 .11884353E-14

REAL	IMAG	FREQ	ZETA	TC
-.11245645E-05	.20992759E-02	1.85098	-.0005	-.14E 06
-.11245645E-05	-.20992759E-02	1.85098	-.0005	-.14E 06
-.11512996E-05	.17737296E-02	1.56394	-.0006	-.14E 06
-.11512996E-05	-.17737296E-02	1.56394	-.0006	-.14E 06
.22758641E-05	.52726675E-03	.46490	.0043	.70E 05
.22758641E-05	-.52726675E-03	.46490	.0043	.70E 05

ALTITUDE 205.0 DENSITY .57850E-11
PHI 146.581409 ALPHA 82.844995 PHIB 6.552577
.00000000E 00 .11796120E-14 -.14988011E-14 .19073251E-14

REAL	IMAG	FREQ	ZETA	TC
-.12418217E-05	.21035236E-02	1.85092	-.0006	-.13E 06
-.12418217E-05	-.21035236E-02	1.85092	-.0006	-.13E 06
-.13774609E-05	.17806583E-02	1.56682	-.0008	-.12E 06
-.13774609E-05	-.17806583E-02	1.56682	-.0008	-.12E 06
.26192826E-05	.55676693E-03	.48991	.0047	.61E 05
.26192826E-05	-.55676693E-03	.48991	.0047	.61E 05

ALTITUDE 200.0 DENSITY .68050E-11
PHI 144.947384 ALPHA 83.124143 PHIB 6.774560
-.16479873E-16 -.11518564E-14 -.86042284E-15 .14378359E-14

REAL	IMAG	FREQ	ZETA	TC
-.13166784E-05	.21079621E-02	1.85101	-.0006	-.12E 06
-.13166784E-05	-.21079621E-02	1.85101	-.0006	-.12E 06
-.16446367E-05	.17878071E-02	1.56988	-.0009	-.97E 05
-.16446367E-05	-.17878071E-02	1.56988	-.0009	-.97E 05
.29613151E-05	.59051743E-03	.51854	.0050	.54E 05
.29613151E-05	-.59051743E-03	.51854	.0050	.54E 05

ALTITUDE 195.0 DENSITY .80300E-11
PHI 143.472836 ALPHA 83.405599 PHIB 6.970618
-.32959746E-16 -.62450045E-15 .13322676E-14 .14717419E-14

REAL	IMAG	FREQ	ZETA	TC
-.13469783E-05	.21126252E-02	1.85129	-.0006	-.12E 06
-.13469783E-05	-.21126252E-02	1.85129	-.0006	-.12E 06
-.19631492E-05	.17952558E-02	1.57318	-.0011	-.81E 05
-.19631492E-05	-.17952558E-02	1.57318	-.0011	-.81E 05
.33101275E-05	.62903990E-03	.55123	.0053	.48E 05
.33101275E-05	-.62903990E-03	.55123	.0053	.48E 05

ALTITUDE 190.0 DENSITY .94750E-11
PHI 142.179816 ALPHA 83.685547 PHIB 7.139171
.52041704E-17 -.19428903E-15 -.66613381E-15 .69390891E-15

REAL	IMAG	FREQ	ZETA	TC
-.13327127E-05	.21175401E-02	1.85177	-.0006	-.12E 06
-.13327127E-05	-.21175401E-02	1.85177	-.0006	-.12E 06
-.23365150E-05	.18030114E-02	1.57672	-.0013	-.68E 05
-.23365150E-05	-.18030114E-02	1.57672	-.0013	-.68E 05
.36692277E-05	.67192999E-03	.58760	.0055	.43E 05
.36692277E-05	-.67192999E-03	.58760	.0055	.43E 05

ALTITUDE 185.0 DENSITY .11285E-10
PHI 140.992149 ALPHA 83.983350 PHIB 7.291270
-.50306981E-16 .97144515E-15 -.23037128E-14 .25006657E-14

REAL	IMAG	FREQ	ZETA	TC
-.12721114E-05	.21227970E-02	1.85254	-.0006	-.13E 06
-.12721114E-05	-.21227970E-02	1.85254	-.0006	-.13E 06
-.28014529E-05	.18114317E-02	1.58082	-.0015	-.57E 05
-.28014529E-05	-.18114317E-02	1.58082	-.0015	-.57E 05
.40735643E-05	.72222940E-03	.63028	.0056	.39E 05
.40735643E-05	-.72222940E-03	.63028	.0056	.39E 05

ALTITUDE 180.0 DENSITY .13385E-10
PHI 139.992363 ALPHA 84.280081 PHIB 7.417452
.31694470E-17 -.16653345E-15 -.12351231E-14 .12463044E-14

REAL	IMAG	FREQ	ZETA	TC
-.11716432E-05	.21283895E-02	1.85358	-.0006	-.14E 06
-.11716432E-05	-.21283895E-02	1.85358	-.0006	-.14E 06
-.33372428E-05	.18202814E-02	1.58526	-.0018	-.48E 05
-.33372428E-05	-.18202814E-02	1.58526	-.0018	-.48E 05
.45088860E-05	.77662786E-03	.67635	.0058	.35E 05
.45088860E-05	-.77662786E-03	.67635	.0058	.35E 05

ALTITUDE 175.0 DENSITY .16000E-10
PHI 139.094055 ALPHA 84.602675 PHIB 7.529699
-.34694470E-17 .15404344E-14 .12490009E-15 .15454936E-14

REAL	IMAG	FREQ	ZETA	TC
-.10270586E-05	.21344853E-02	1.85505	-.0005	-.15E 06
-.10270586E-05	-.21344853E-02	1.85505	-.0005	-.15E 06
-.39992176E-05	.18300783E-02	1.59049	-.0022	-.40E 05
-.39992176E-05	-.18300783E-02	1.59049	-.0022	-.40E 05
.50262762E-05	.83941761E-03	.72952	.0060	.32E 05
.50262762E-05	-.83941761E-03	.72952	.0060	.32E 05

ALTITUDE 170.0 DENSITY .19250E-10
PHI 138.300910 ALPHA 84.957136 PHIB 7.628477
-.81532003E-16 .31918912E-15 -.74940054E-15 .81861489E-15

REAL	IMAG	FREQ	ZETA	TC
-.84126813E-06	.21412331E-02	1.85706	-.0004	-.19E 06
-.84126813E-06	-.21412331E-02	1.85706	-.0004	-.19E 06
-.48131548E-05	.18410300E-02	1.59669	-.0026	-.33E 05
-.48131548E-05	-.18410300E-02	1.59669	-.0026	-.33E 05
.56544229E-05	.91128946E-03	.79035	.0062	.28E 05
.56544229E-05	-.91128946E-03	.79035	.0062	.28E 05

ALTITUDE 165.0 DENSITY .23150E-10
PHI 137.623370 ALPHA 85.338833 PHIB 7.712728
-.11275703E-16 .20261570E-14 -.24980018E-14 .32164347E-14

REAL	IMAG	FREQ	ZETA	TC
-.63109515E-06	.21487309E-02	1.85969	-.0003	-.25E 06
-.63109515E-06	-.21487309E-02	1.85969	-.0003	-.25E 06
-.57747166E-05	.18531003E-02	1.60383	-.0031	-.28E 05
-.57747166E-05	-.18531003E-02	1.60383	-.0031	-.28E 05
.64058117E-05	.99039545E-03	.85717	.0065	.25E 05
.64058117E-05	-.99039545E-03	.85717	.0065	.25E 05

ALTITUDE 160.0 DENSITY .27950E-10
PHI 137.047448 ALPHA 85.766419 PHIB 7.787501
.45970172E-16 -.65225603E-15 -.43021142E-15 .78270879E-15

REAL	IMAG	FREQ	ZETA	TC
-.41327615E-06	.21573097E-02	1.86324	-.0002	-.39E 06
-.41327615E-06	-.21573097E-02	1.86324	-.0002	-.39E 06
-.69301434E-05	.18667295E-02	1.61227	-.0037	-.23E 05
-.69301434E-05	-.18667295E-02	1.61227	-.0037	-.23E 05
.73434195E-05	.10793185E-02	.93219	.0068	.22E 05
.73434195E-05	-.10793185E-02	.93219	.0068	.22E 05

ALTITUDE 155.0 DENSITY .34050E-10
PHI 136.531646 ALPHA 86.265387 PHIB 7.857194
-.48572257E-16 -.81878948E-15 -.92981178E-15 .12398893E-14

REAL	IMAG	FREQ	ZETA	TC
-.23090688E-06	.21675716E-02	1.86821	-.0001	-.69E 06
-.23090688E-06	-.21675716E-02	1.86821	-.0001	-.69E 06
-.83402117E-05	.18825628E-02	1.62256	-.0044	-.19E 05
-.83402117E-05	-.18825628E-02	1.62256	-.0044	-.19E 05
.85711185E-05	.11818305E-02	1.01861	.0073	.19E 05
.85711185E-05	-.11818305E-02	1.01861	.0073	.19E 05

ALTITUDE 150.0 DENSITY .41600E-10
PHI 136.089558 ALPHA 86.835240 PHIB 7.922544
-.17347235E-17 .62450045E-15 -.10408341E-14 .12138119E-14

REAL	IMAG	FREQ	ZETA	TC
-.20036129E-16	.21800972E-02	1.87509	-.0001	-.79E 06
-.20036129E-06	-.21800972E-02	1.87509	-.0001	-.79E 06
-.99615010E-05	.19006179E-02	1.63471	-.0052	-.16E 05
-.99615010E-05	-.19006179E-02	1.63471	-.0052	-.16E 05
.10161862E-04	.12461691E-02	1.11483	.0078	.16E 05
.10161862E-04	-.12961691E-02	1.11483	.0078	.16E 05

ALTITUDE 145.0 DENSITY .51200E-10
PHI 135.701305 ALPHA 87.505004 PHIB 7.988816
.16826818E-15 .15959456E-14 -.11102230E-15 .16086275E-14

REAL	IMAG	FREQ	ZETA	TC
-.57007263E-06	.21963269E-02	1.88511	-.0003	-.28E 06
-.57007263E-06	-.21963269E-02	1.88511	-.0003	-.28E 06
-.11738428E-04	.19215897E-02	1.64930	-.0061	-.14E 05
-.11738428E-04	-.19215897E-02	1.64930	-.0061	-.14E 05
.12308500E-04	.14260938E-02	1.22402	.0086	.13E 05
.12308500E-04	-.14260938E-02	1.22402	.0086	.13E 05

ALTITUDE 140.0 DENSITY .63400E-10
PHI 135.361642 ALPHA 88.288887 PHIB 8.060363
.25240227E-15 .45796700E-15 -.70776718E-15 .87998583E-15

REAL	IMAG	FREQ	ZETA	TC
-.19040264E-05	.22185041E-02	1.90016	-.0009	-.84E 05
-.19040264E-05	-.22185041E-02	1.90016	-.0009	-.84E 05
-.13285579E-04	.19457924E-02	1.66658	-.0068	-.12E 05
-.13285579E-04	-.19457924E-02	1.66658	-.0068	-.12E 05
.15189606E-04	.15718217E-02	1.34628	.0097	.10E 05
.15189606E-04	-.15718217E-02	1.34628	.0097	.10E 05

ALTITUDE 135.0 DENSITY .79150E-10
PHI 135.060111 ALPHA 89.213072 PHIR 8.144366
.21684043E-16 -.15681900E-14 .88817842E-15 .18023737E-14

REAL	IMAG	FREQ	ZETA	TC
-.54641688E-05	.22513289E-02	1.92424	-.0024	-.29E 05
-.54641688E-05	-.22513289E-02	1.92424	-.0024	-.29E 05
-.13092114E-04	.19741637E-02	1.68735	-.0066	-.12E 05
-.13092114E-04	-.19741637E-02	1.68735	-.0066	-.12E 05
.18556283E-04	.17340669E-02	1.48213	.0107	.86E 04
.18556283E-04	-.17340669E-02	1.48213	.0107	.86E 04

ALTITUDE 130.0 DENSITY .99450E-10
PHI 134.792731 ALPHA 90.286037 PHIR 8.248909
.45536491E-15 .21649349E-14 -.38857806E-15 .22461730E-14

REAL	IMAG	FREQ	ZETA	TC
-.13458942E-04	.23044598E-02	1.96553	-.0058	-.12E 05
-.13458942E-04	-.23044598E-02	1.96553	-.0058	-.12E 05
-.80829459E-06	.20106277E-02	1.71491	-.0004	-.20E 06
-.80829459E-06	-.20106277E-02	1.71491	-.0004	-.20E 06
.14267237E-04	.19030605E-02	1.62317	.0075	.11E 05
.14267237E-04	-.19030605E-02	1.62317	.0075	.11E 05

ALTITUDE 125.0 DENSITY .12600E-09
PHI 134.551868 ALPHA 91.526200 PHIR 8.386678
.37123082E-15 -.52735594E-15 -.44408921E-15 .78302735E-15

REAL	IMAG	FREQ	ZETA	TC
-.25600924E-04	.24017281E-02	2.04419	-.0107	-.62E 04
-.25600924E-04	-.24017281E-02	2.04419	-.0107	-.62E 04
.44711118E-04	.21237009E-02	1.80756	.0210	.36E 04
.44711118E-04	-.21237009E-02	1.80756	.0210	.36E 04
-.19110194E-04	.19950893E-02	1.69809	-.0096	-.83E 04
-.19110194E-04	-.19950893E-02	1.69809	-.0096	-.83E 04

ALTITUDE 120.0 DENSITY .16200E-09
PHI 134.328505 ALPHA 92.972078 PHIR 8.579343
.58546917E-15 .92981178E-15 -.44408921E-15 .11851326E-14

REAL	IMAG	FREQ	ZETA	TC
-.28315927E-04	.25828062E-02	2.19370	-.0110	-.56E 04
-.28315927E-04	-.25828062E-02	2.19370	-.0110	-.56E 04
.50433580E-04	.22735113E-02	1.93100	.0222	.32E 04
.50433580E-04	-.22735113E-02	1.93100	.0222	.32E 04
-.22117653E-04	.20282672E-02	1.72270	-.0109	-.72E 04
-.22117653E-04	-.20282672E-02	1.72270	-.0109	-.72E 04

ALTITUDE 115.0 DENSITY .21000E-09
PHI 134.127320 ALPHA 94.573132 PHIB 8.845697
.32092384E-15 .15820678E-14 .15543122E-14 .22409411E-14

REAL	IMAG	FREQ	ZETA	TC
-.20060273E-04	.28519983E-02	2.41724	-.0070	-.79E 04
-.20060273E-04	-.28519983E-02	2.41724	-.0070	-.79E 04
.40495124E-04	.24120169E-02	2.04433	.0168	.39E 04
.40495124E-04	-.24120169E-02	2.04433	.0168	.39E 04
-.20434851E-04	.20495099E-02	1.73708	-.0100	-.78E 04
-.20434851E-04	-.20495099E-02	1.73708	-.0100	-.78E 04

ALTITUDE 110.0 DENSITY .27600E-09
PHI 133.944113 ALPHA 96.327729 PHIB 9.222803
-.10148132E-15 .12628787E-14 .00000000E 00 .12669495E-14

REAL	IMAG	FREQ	ZETA	TC
-.12352042E-04	.31998528E-02	2.70635	-.0039	-.13E 05
-.12352042E-04	-.31998528E-02	2.70635	-.0039	-.13E 05
.30327220E-04	.25677453E-02	2.17173	.0118	.52E 04
.30327220E-04	-.25677453E-02	2.17173	.0118	.52E 04
-.17975178E-04	.20641223E-02	1.74578	-.0087	-.89E 04
-.17975178E-04	-.20641223E-02	1.74578	-.0087	-.89E 04

ALTITUDE 105.0 DENSITY .36850E-09
PHI 133.780241 ALPHA 98.189546 PHIB 9.759408
.19342167E-14 .23453461E-14 -.22204460E-14 .37646013E-14

REAL	IMAG	FREQ	ZETA	TC
-.68890548E-05	.36274889E-02	3.06157	-.0019	-.23E 05
-.68890548E-05	-.36274889E-02	3.06157	-.0019	-.23E 05
.22455697E-04	.27646852E-02	2.33337	.0081	.71E 04
.22455697E-04	-.27646852E-02	2.33337	.0081	.71E 04
-.15566642E-04	.20736369E-02	1.75013	-.0075	-.10E 05
-.15566642E-04	-.20736369E-02	1.75013	-.0075	-.10E 05

ALTITUDE 100.0 DENSITY .50150E-09
PHI 133.637742 ALPHA 100.096579 PHIB 10.530307
.10937432E-14 .36082248E-15 -.22204460E-15 .11729325E-14

REAL	IMAG	FREQ	ZETA	TC
-.28553036E-05	.41538151E-02	3.49837	-.0007	-.56E 05
-.28553036E-05	-.41538151E-02	3.49837	-.0007	-.56E 05
.16355717E-04	.30237654E-02	2.54664	.0054	.97E 04
.16355717E-04	-.30237654E-02	2.54664	.0054	.97E 04
-.13500413E-04	.20787831E-02	1.75077	-.0065	-.12E 05
-.13500413E-04	-.20787831E-02	1.75077	-.0065	-.12E 05

ALTITUDE 95.0 DENSITY .69900E-09
PHI 133.518696 ALPHA 101.974562 PHIB 11.659259
.39577716E-14 .22204460E-14 -.88817842E-15 .46241970E-14

REAL	IMAG	FREQ	ZETA	TC
.36844345E-06	.48148699E-02	4.04654	.0001	.43E 06
.36844345E-06	-.48148699E-02	4.04654	.0001	.43E 06
.11524984E-04	.33713794E-02	2.83339	.0034	.14E 05
.11524984E-04	-.33713794E-02	2.83339	.0034	.14E 05
-.11893428E-04	.20793348E-02	1.74753	-.0057	-.13E 05
-.11893428E-04	-.20793348E-02	1.74753	-.0057	-.13E 05

ALTITUDE 90.0 DENSITY .10045E-08
PHI 133.424090 ALPHA 103.746584 PHIB 13.371772
.65529179E-14 .56621374E-14 -.22204460E-15 .86631309E-14

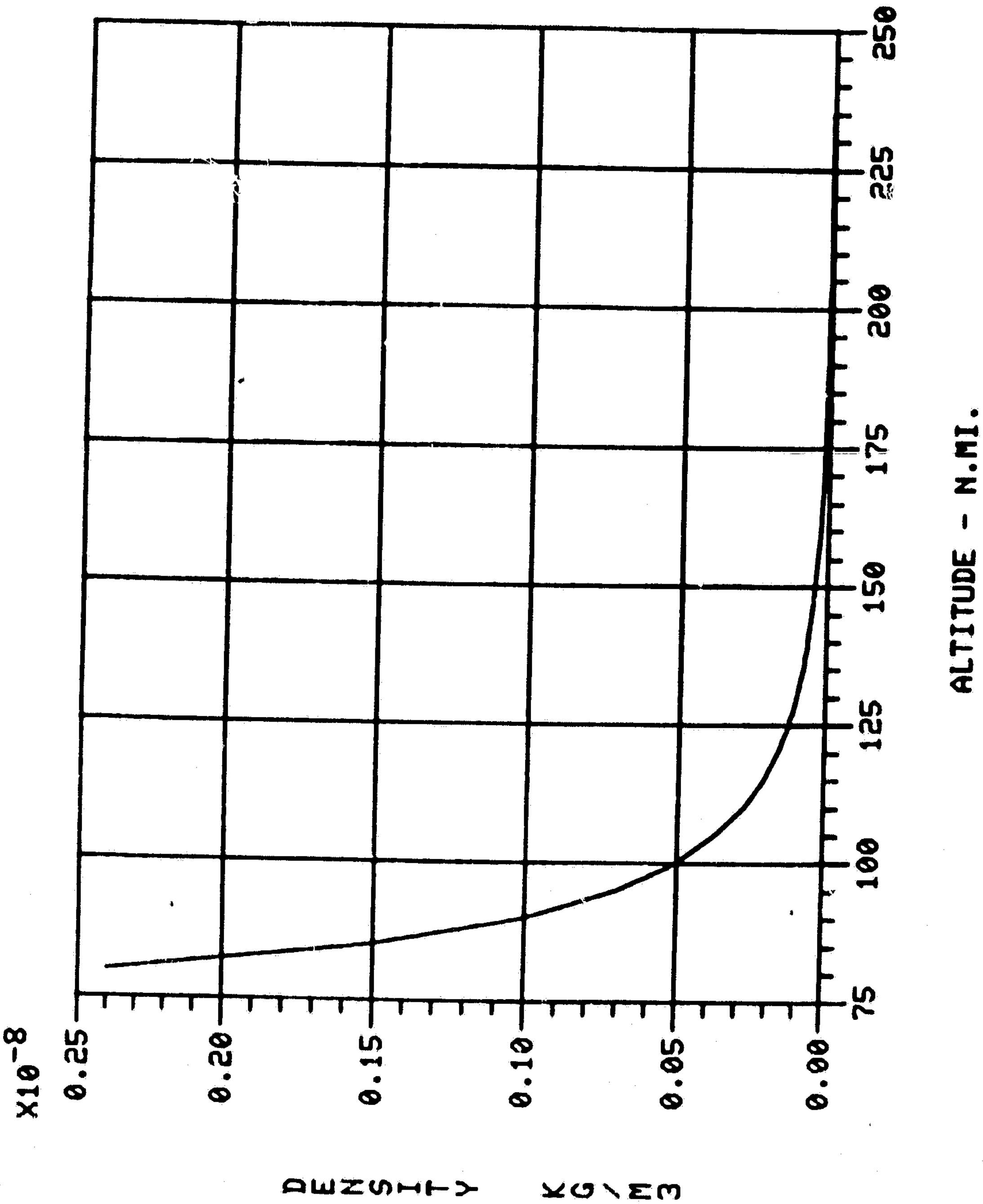
REAL	IMAG	FREQ	ZETA	TC
.32144518E-05	.56710305E-02	4.75598	.0006	.50E 05
.32144518E-05	-.56710305E-02	4.75598	.0006	.50E 05
.76304357E-05	.38471940E-02	3.22643	.0020	.21E 05
.76304357E-05	-.38471940E-02	3.22643	.0020	.21E 05
-.10844888E-04	.20731122E-02	1.73861	-.0052	-.15E 05
-.10844888E-04	-.20731122E-02	1.73861	-.0052	-.15E 05

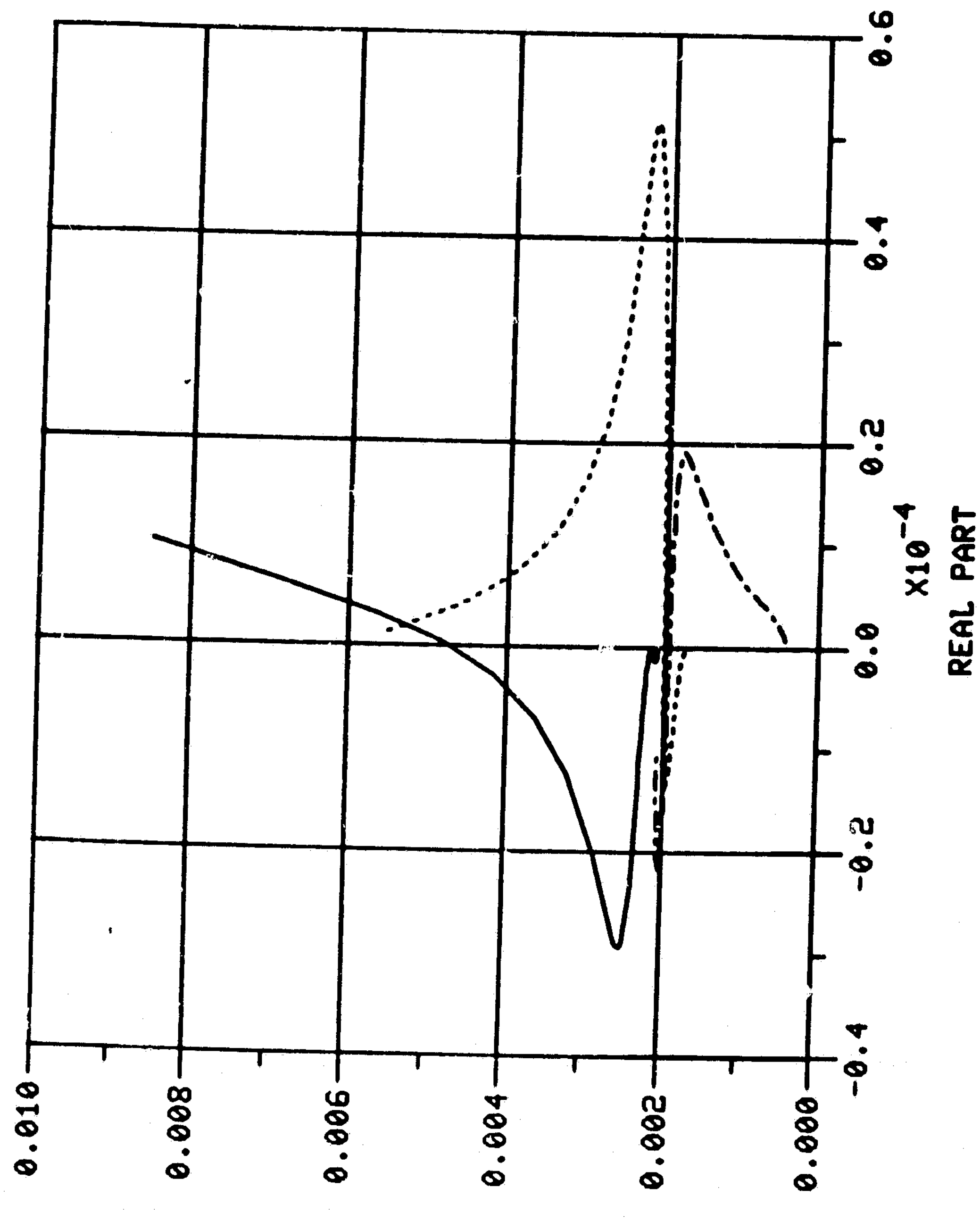
ALTITUDE 85.0 DENSITY .15050E-08
PHI 133.352620 ALPHA 105.352034 PHIB 16.140770
.17347235E-15 .23869795E-14 .22204460E-15 .24035531E-14

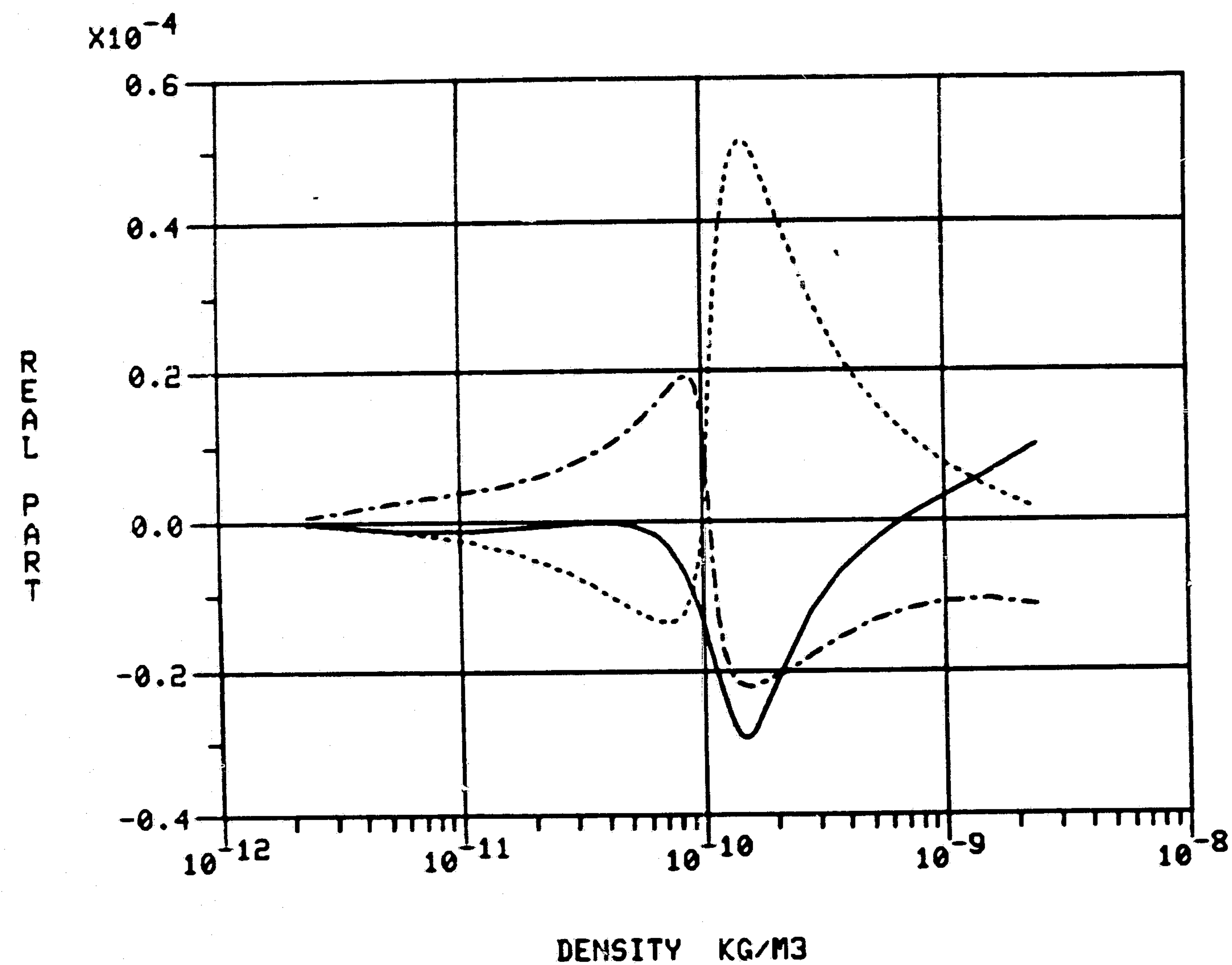
REAL	IMAG	FREQ	ZETA	TC
.61567239E-05	.68291380E-02	5.71507	.0009	.26E 05
.61567239E-05	-.68291380E-02	5.71507	.0009	.26E 05
.43806889E-05	.45189914E-02	3.78179	.0010	.36E 05
.43806889E-05	-.45189914E-02	3.78179	.0010	.36E 05
-.10537413E-04	.20522974E-02	1.71750	-.0051	-.15E 05
-.10537413E-04	-.20522974E-02	1.71750	-.0051	-.15E 05

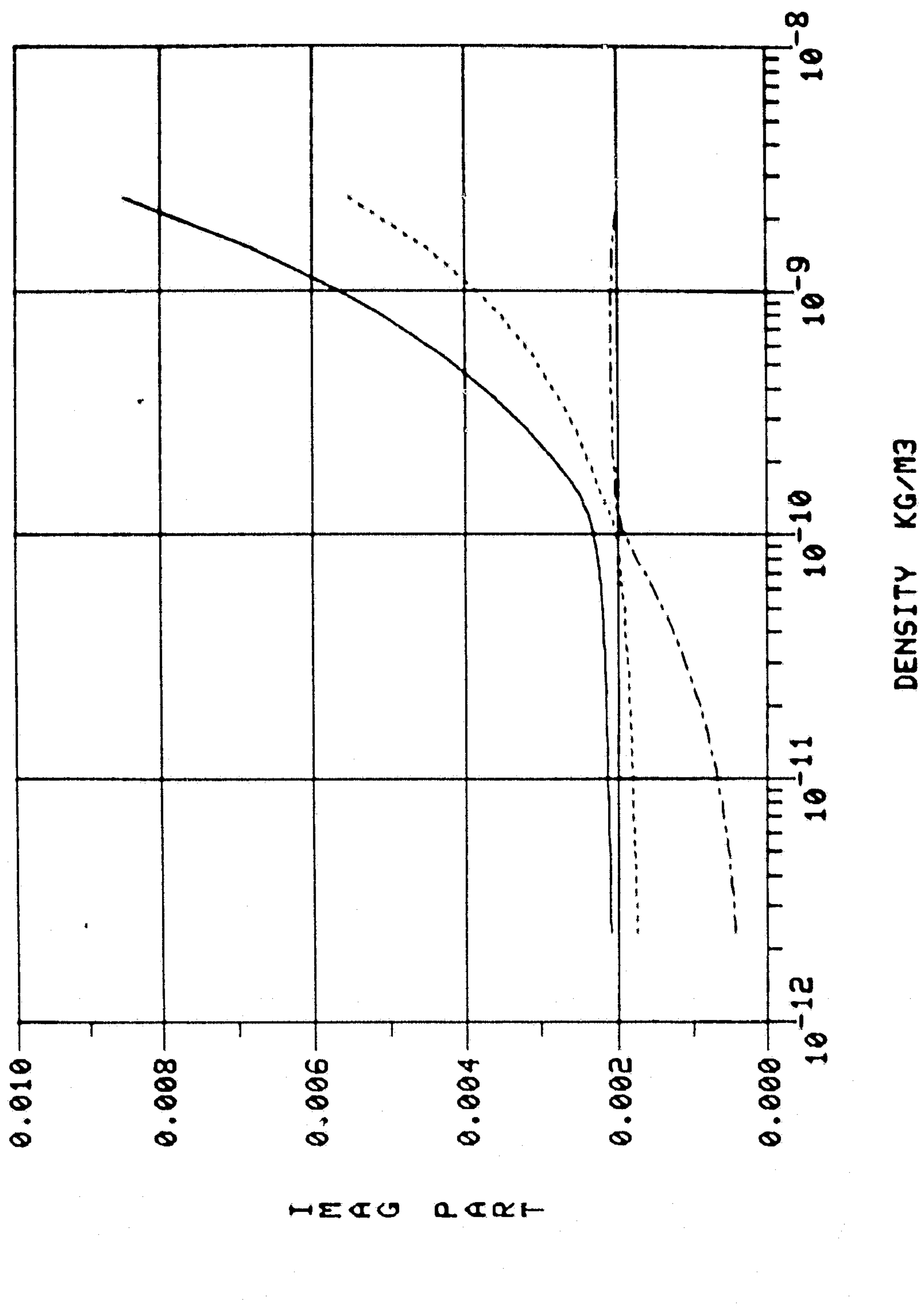
ALTITUDE 80.0 DENSITY .24000E-08
PHI 133.300324 ALPHA 106.764801 PHIB 21.176511
.27235159E-14 -.20261570E-14 -.17763568E-14 .38312262E-14

REAL	IMAG	FREQ	ZETA	TC
.10054228E-04	.84985513E-02	7.09704	.0012	.16E 05
.10054228E-04	-.84985513E-02	7.09704	.0012	.16E 05
.13877365E-05	.55188136E-02	4.60869	.0003	.11E 06
.13877365E-05	-.55188136E-02	4.60869	.0003	.11E 06
-.11441965E-04	.19866856E-02	1.65906	-.0058	-.14E 05
-.11441965E-04	-.19866856E-02	1.65906	-.0058	-.14E 05

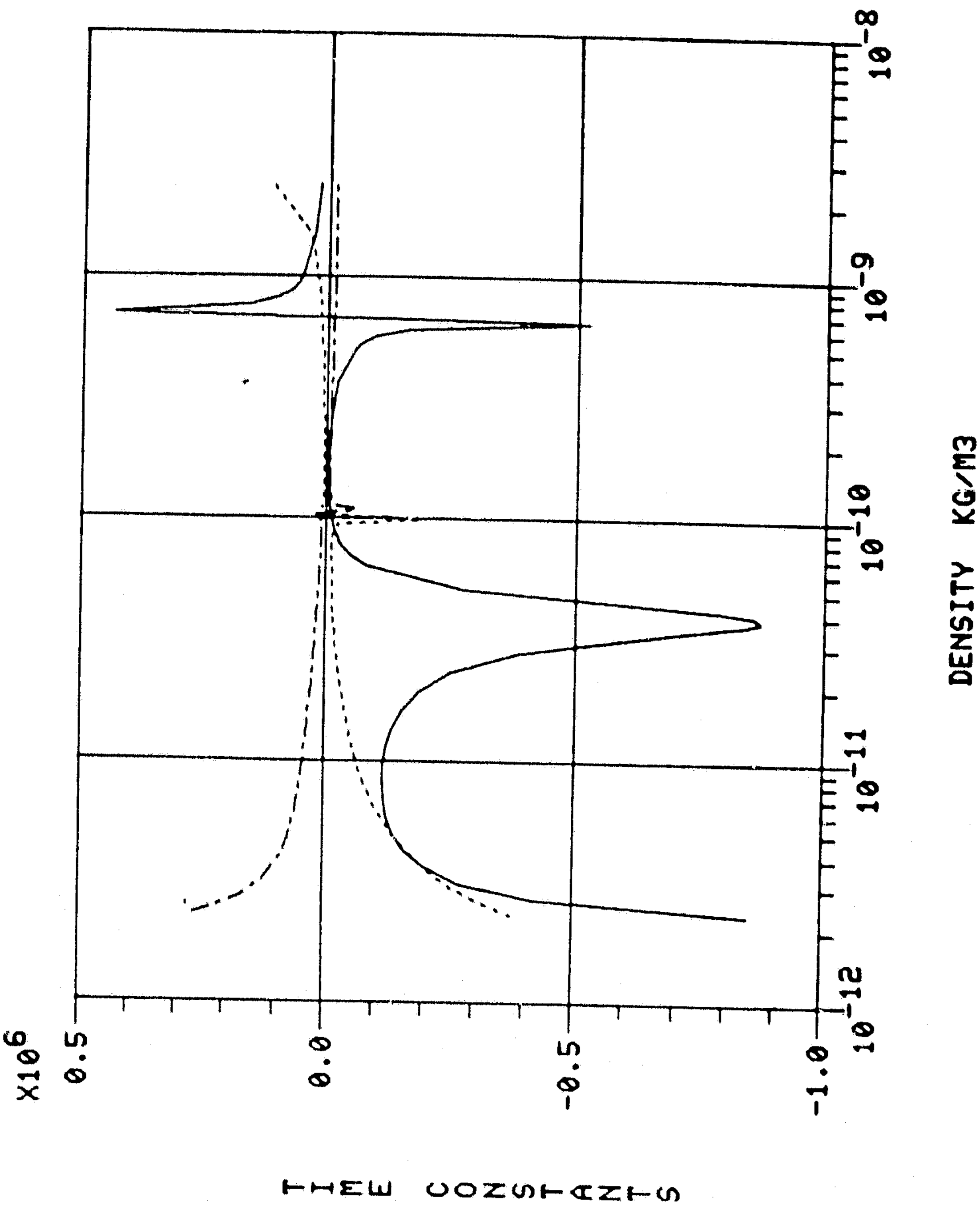

X10^-8
0.25
0.20
0.15
0.10
0.05
0.00
DENSITY KG/M3
75
100
125
150
175
200
225
250
ALTITUDE - N.MI.


REAL PART
X10^-4
-0.4
-0.2
0.0
0.2
0.4
0.6
IMAG PART
0.010
0.008
0.006
0.004
0.002
0.000

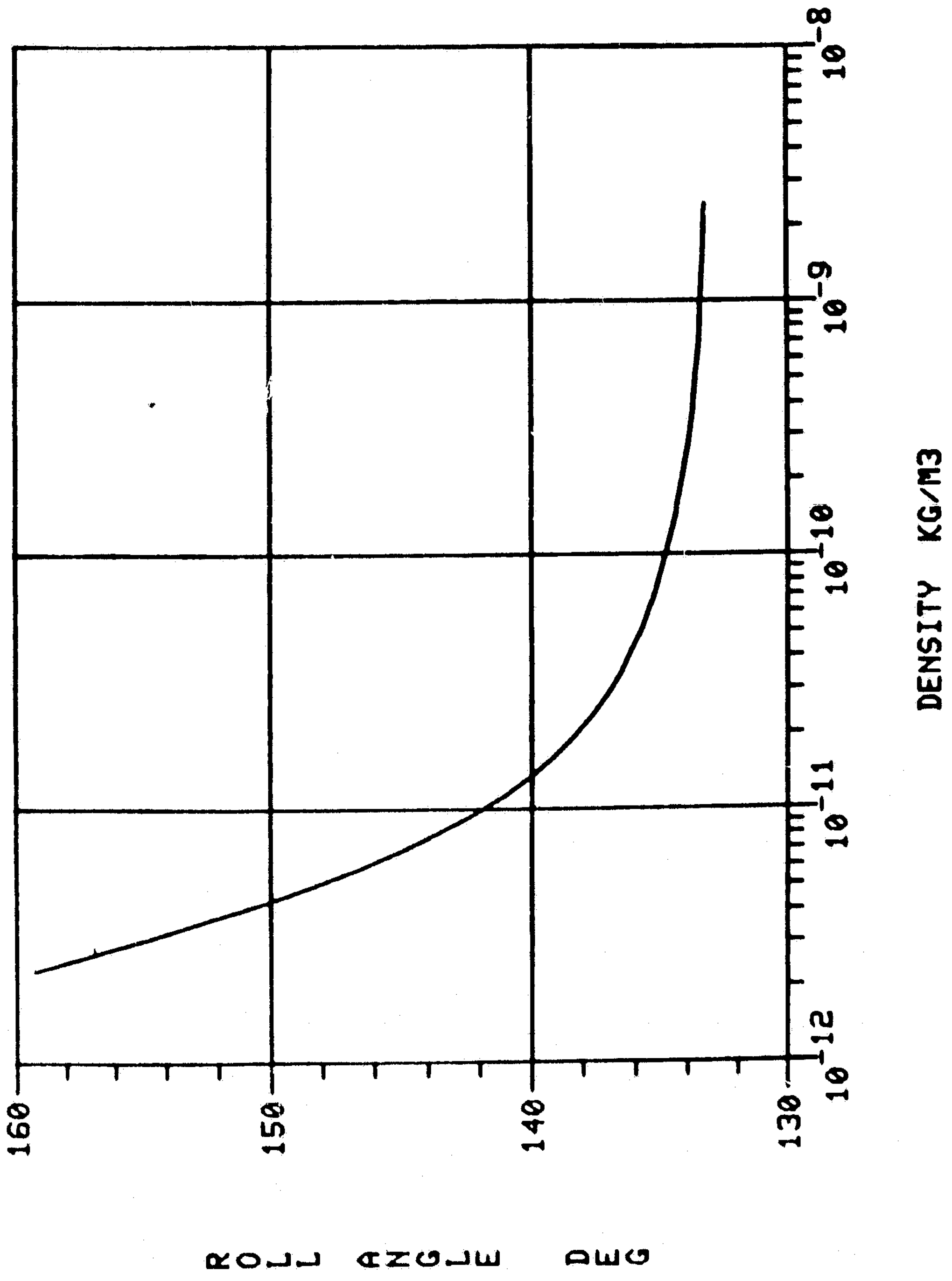

X10^-4
0.6
0.4
0.2
0.0
-0.2
-0.4
REAL PART
10^-12
10^-11
10^-10
10^-9
10^-8
DENSITY KG/M3



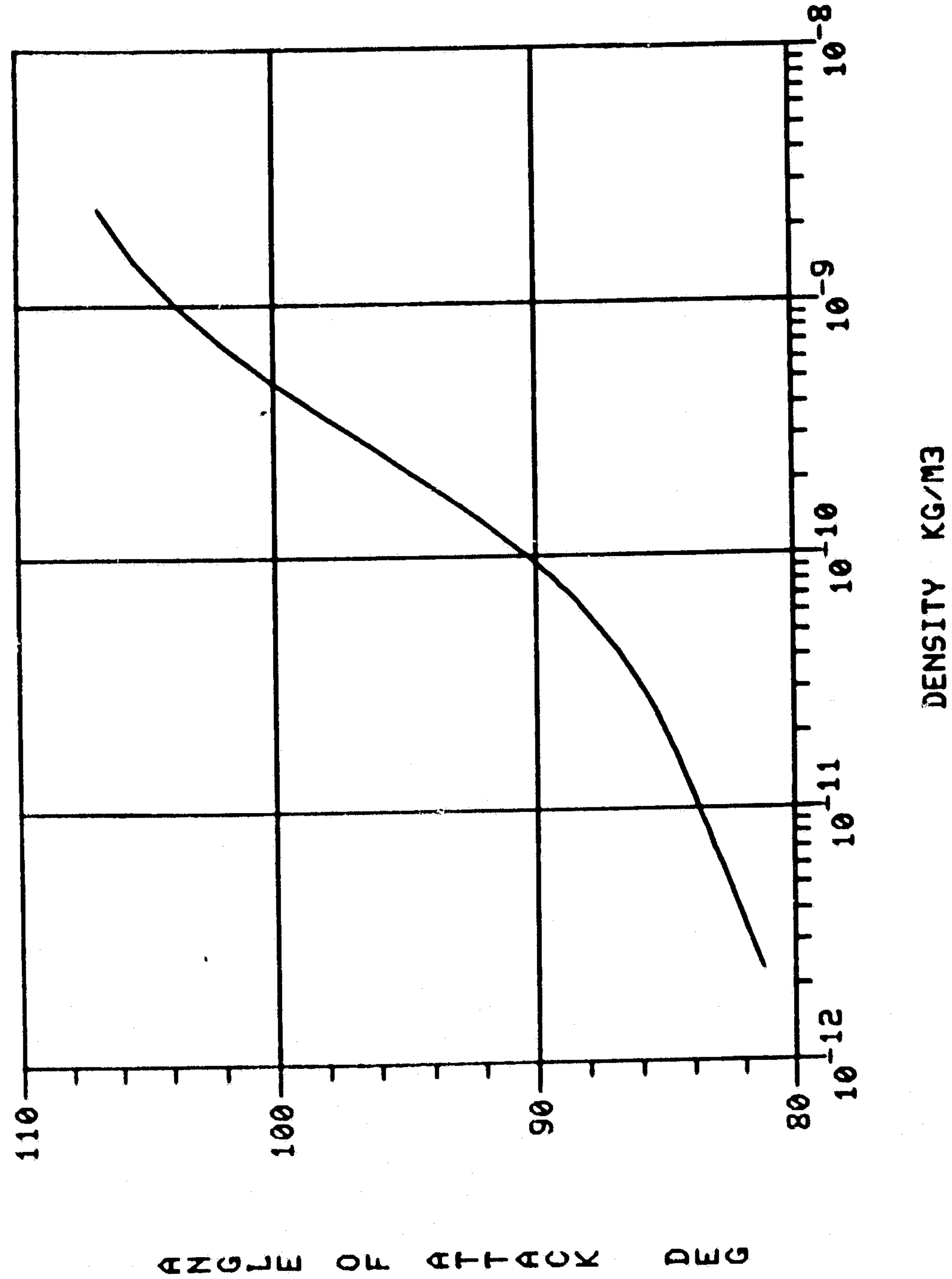
IMAG PART
0.010
0.008
0.006
0.004
0.002
0.000
DENSITY KG/M3
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}



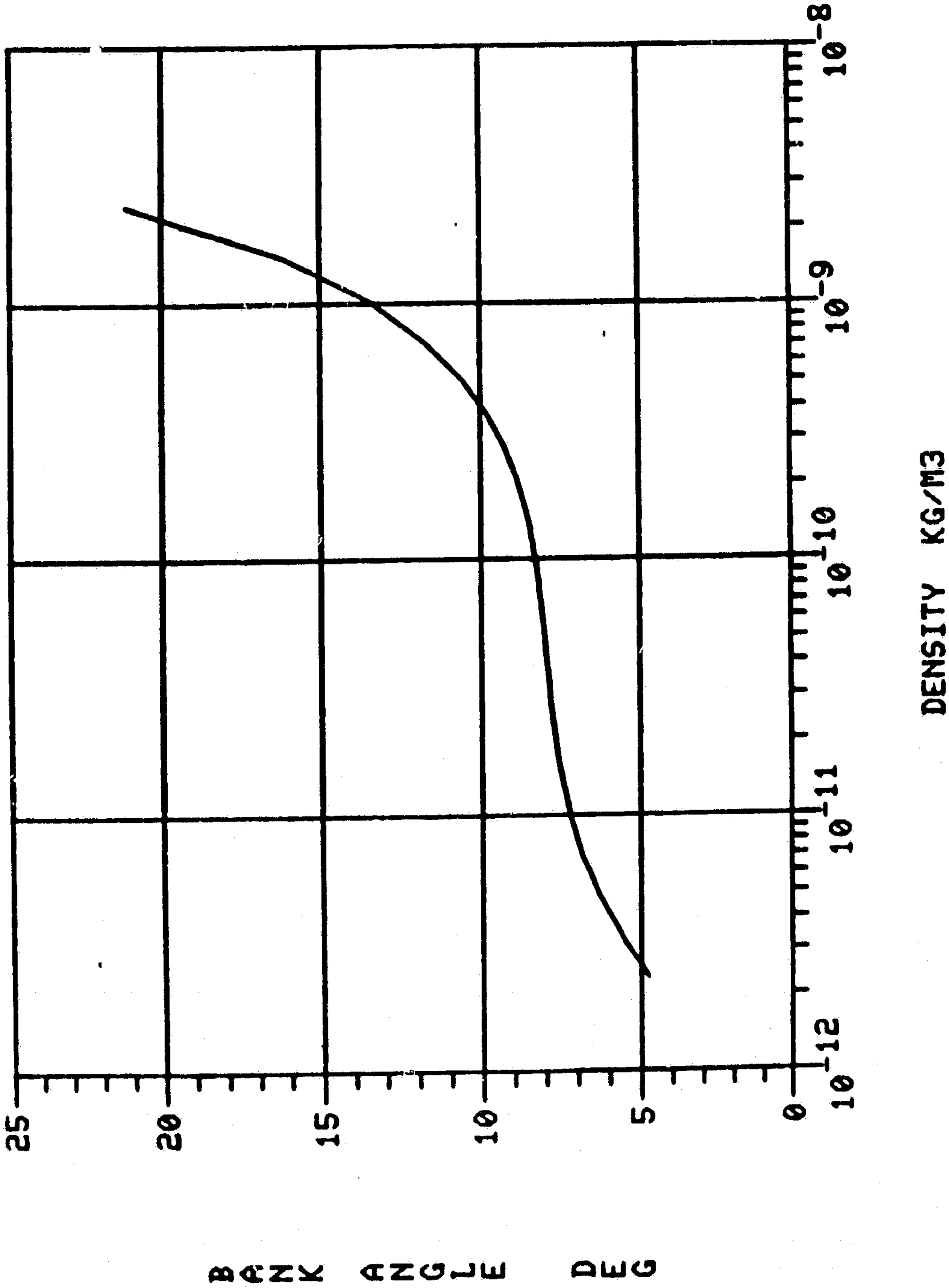
X10^6
0.5
0.0
-0.5
-1.0
TIME CONSTANTS
10^-12
10^-11
10^-10
10^-9
10^-8
DENSITY KG/M3



ROLL ANGLE DEG
160
150
140
130
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
DENSITY KG/M3



ANGLE OF ATTACK DEG
110
100
90
80
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
DENSITY KG/M3



BANK ANGLE DEG
0
5
10
15
20
25
DENSITY KG/M3
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}

APPROVAL

EQUILIBRIUM AND STABILITY OF A SATELLITE INFLUENCED BY GRAVITATIONAL AND AERODYNAMIC TORQUES

By Zachary J. Galaboff

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

G. D. HOPSON
Director, Systems Dynamics Laboratory

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