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RAPID CORRELATION ANALYSIS IN A GROUP OF MOTIONS AND HOMOGENEOUS SCALE TRANSFORMS OF THE PLANE $(M(2)xR_+)$

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16. Abstract

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RAPID CORRELATION ANALYSIS IN A GROUP OF MOTIONS AND HOMOGENEOUS SCALE TRANSFORMS OF THE PLANE ($M(2) \times R_+$)

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A method is proposed for picking out a given image in a plane, irrespective of translations, rotations, and homogeneous scale transforms of the plane. The correlation integral is calculated, using the methods of abstract harmonic analysis. In this case, and also for groups of translations, it is possible to carry out the calculations using the rapid Fourier transform (BPF). The latter fact makes it possible to substantially reduce the number of computer operations for calculating the correlation integral, as compared with direct methods. /2*

Introduction /3

Schwartz's inequality [1] lies at the base of correlation analysis. For any functions $f(x)$ and $\phi(x)$ of L_2

$$\int_X f(x)\varphi(x)\,dx \le \left(\int_X f^2(x)\,dx \int_X \varphi^2(x)\,dx\right)^{1/2}.$$

We will assume that the functions $f(x)$ and $\Phi(x)$ are determined in a homogeneous Euclidean two-dimensional space X, with a group of motions G (G is locally-compact). Then, for any $g \in G$, Schwartz's inequality is also fulfilled:

$$\int_X f(x)\varphi(g^{-1}x)\,dx \le \left(\int_X f^2(x)\,dx \int_X \varphi^2(g^{-1}x)\,dx\right)^{1/2}. \quad (1)$$

If the measure dx is invariant relative to the group G, then the right-hand portion of (1) does not depend on g. Insofar as, in practical problems, the detection of an invariant measure is difficult, we will make use of the Euclidean measure $dx_1 dx_2$,

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relative to the invariant measure in X [2]. The problem of the identification of images amounts to a search for that element of $g \in G$ for which one of the local maxima of the normed correlation function is achieved.

$$K(g) = J^{-1/2}(g) \int_X f(x)\,\varphi(g^{-1}x)\,dx, \qquad (2)$$

where $J(g)$ is the Jacobian transforms $x \to gx$.

The methods of harmonic analysis in groups [2,3] make it possible to reduce the calculation of the correlation function to integral transforms, in the capacity of the kernel of which are matrix elements of the nonreducible unitary concepts of the corresponding groups [4]. The number of calculations is reduced substantially with numerical solution of problems of identification of the images, based on harmonic analysis. For example, for calculation of the correlation function in a group of motions of the plane M(2), with identification of images irrespective of their translations and rotations in the plane, $\sim N^3 \log_2 N$ operations are required, as opposed to $\sim N^5$ with calculation of the correlation integrals by the known methods [5] (N is the number of elements of the discrete network according to each of the parameters). Proposed in the present study is a method of rapid calculation of the correlation function in a group of motions and homogeneous scale transforms of the Euclidean plane $(M(2) \times R_+)$ for identification of the images, irrespective of their translations, rotations, and homogeneous scale transforms in the plane. Also examined are translations and scale transforms of the images in the plane. /4

1. Harmonic Expansion of Correlation Function

Let X be a homogeneous space with a locally-compact group of motions G. It is common knowledge that X may be realized as a space of left classes of contiguity, according to the stationary subgroup H of some point $a \in X$ [4]. In this case, the transforms are given by the formula $g_0 H \to g g_0 H;\ g, g_0 \in G$. The harmonic expansion of the functions in a homogeneous space X is reduced to the expansion of the functions in the group G, which are constant in the left classes of contiguity according to the subgroup H, i.e., those in which $f(g)=f(gh)$.

The correlation function in the group G has the form

$$K(g) = \int_G f(h)\varphi(g^{-1}h)\,dh, \qquad (3)$$

where $g, h \in G;\ f(h), \varphi(h)$ are the functions which are constant in the left classes of contiguity according to the subgroup H, and the measure dh is invariant in the group G. We will conduct spectral analysis of function (3). For this purpose, we will avail ourselves of the harmonic expansion of the functions $f(g)$ and $\phi(g)$, which has the form:

$$f(g) = \sum_{\alpha \in A_0} \sum_{i=1}^{d_\alpha} a_{i1}^{\alpha} \cdot t_{i1}^{\alpha}(g), \qquad (4)$$

$$\varphi(g) = \sum_{\beta \in A_0} \sum_{j=1}^{d_\beta} b_{j1}^{\beta}\ t_{j1}^{\beta}(g), \qquad (5)$$ /5

where

$$a_{i1}^{\alpha} = d_\alpha \int f(g) \cdot \overline{t_{i1}^{\alpha}(g)}\,dg, \qquad (6)$$

$$b_{j1}^{\beta} = d_\beta \int \varphi(g) \cdot \overline{t_{j1}^{\beta}}(g)\,dg \qquad (7)$$

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(the line above designates complex conjugation).

Here, $\mathscr{A}_0$ is the set of mutually nonequivalent nonreducible unitary representations of class I [4], and $t^{\alpha}_{i1}(g)$ are the matrix elements of the columns which corresponds to the basis vectors ℓ^{α}_1, so that $T_\alpha(h)\ell_1^\alpha = \ell_1^\alpha,\ h \in H$; d_α is the dimensionality of the matrix $\|t^{\alpha}_{i1}(g)\|$.

Utilizing (4) - (7), we will calculate the coefficients of harmonic expansion of function (3). We will substitute expression (3) into the equation

$$C^{\alpha}_{mn} = \int_G K(h)\, \overline{t^{\alpha}_{mn}(h)}\, dh \tag{8}$$

in place of the function K(h). By changing the order of integrating in (8) and utilizing the expansion

$$t^{\alpha}_{mn}(g_1 g_2) = \sum_K t^{\alpha}_{mK}(g_1)\cdot t^{\alpha}_{Kn}(g_2),$$

we obtain:

$$C^{\alpha}_{mn} = \sum_K \int_G f(g)\, \overline{t^{\alpha}_{mK}(g)}\, dg \int_G \varphi(g)\cdot \overline{t^{\alpha}_{Kn}(g^{-1})}\, dg. \tag{9}$$

It follows from the unitariness of representation $T^{\alpha}(g)$, as well as from (6) and (7), that

/6

$$C^{\alpha}_{mn} = a^{\alpha}_{m1}\, \overline{b^{\alpha}_{n1}}. \tag{10}$$

Then, the expansion of the correlation function is written as:

$$K(g) = \sum_{m,n} C^{\alpha}_{mn} \, \overline{t^{\alpha}_{mn}}(g). \qquad \text{(II)}$$

Formula (11) may be written in matrix designations

$$K(g) = tr\,(C^{\alpha} T^{\alpha *}),$$

where $C^{\alpha} = \| C^{\alpha}_{mn} \|$, $T^{\alpha *} = \| \overline{t^{\alpha}_{nm}}(g) \|$. In virtue of the fact that the functions $f(g)$ and $\phi(g)$ and the matrix elements $t^{\alpha}_{i1}(g)$ are constant in the left classes of contiguity, according to the stationary subgroup H, the integrals (6) and (7) may be rewritten in the form of integrals according to the homogeneous space X=G/H [4]. Expansion (11) serves as the basis for constructing algorithms for calculating the correlation function.

2. Scale Transforms and Translations of Images in the Plane

Let $g = g(a_1, a_2, b_1, b_2) \in G,\ x \in X$ (X is the Euclidean plane). We will determine the effect of the group G on X in the following manner:

$$g(a_1, a_2, b_1, b_2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_1 x_1 + b_1 \\ a_2 x_2 + b_2 \end{pmatrix}.$$

The normed correlation function in the group G has the form:

$$K(a_1, a_2, b_1, b_2) = \frac{1}{\sqrt{a_1 a_2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(g^{-1}x)\, f(x)\, dx. \qquad (12)$$

The group G, acting on the plane, is the direct sum of the two groups of linear transforms of the line; therefore, without limiting the continuity, one can study the expansion of the correlation function in the group acting on the line. The designation for this group, as before, will be G. /7

Group G is the hybridized product of the additive group of real number (R) and the multiplicative group (R_+) [4]; utilized, therefore, for harmonic expansion are the transforms of Fourier

$$\Phi_{a,b} f(\beta) = \int_{-\infty}^{\infty} f(b)\, e^{iab}\, db \tag{13}$$

and Mellin

$$M_{a,b} f(b) = \int_{0}^{\infty} b^{a-1} f(b)\, db, \tag{14}$$

where $\mathrm{Re}\,\sigma=0$. Thus, let

$$K(a,b) = \int_{-\infty}^{\infty} \varphi(g^{-1}x)\, f(x)\, dx, \tag{15}$$

where $g(a,b)\cdot x = ax+b,\ a>0$. The Fourier transform reduces (15) to the form

$$\hat{K}(a,b) = Q(y)\ \bar{F}(ay). \tag{16}$$

where

$$Q(y) = \Phi_{y,x}\, \varphi(x),\quad \bar{F}(ay) = \Phi_{-ay,x}\, f(x).$$

In the given multiplicative form, $\hat{k}(a,y)$ is easily calculated, and, having done the inverse Fourier transform, one can obtain (15) from its expansion. However, the presence of a scale factor in the right-hand part of expansion (16) leads to the necessity of interpolation of the function $\bar{F}(ay)$, with its determination in a discrete network. This difficulty is eliminated if one applies the

Mellin transform to (16).

We will divide the line into orbits, which are homogeneous relative to R_+. We will determine the correlation function in each of the orbits in the following manner: /8

$$\hat{K}(a,y) = \begin{cases} \hat{F}(a,y), & y>0 \\ \hat{R}(a,-|y|), & y<0 \\ K(a,0), & y=0 \end{cases} \qquad (17)$$

The spectral expansion of function (17) on the half lines y>0 and y<0 is calculated using transform (14), and has the form:

$$S(\omega,\nu) = M_{-(\nu+\omega),y}\, Q(y) \cdot M_{\omega,y} \cdot \bar{F}(y), \qquad (18)$$

for y>0

$$S(\omega,\nu) = M_{(\nu+\omega),y}\, Q(-|y|) \cdot M_{\omega,|y|} \cdot \bar{F}(-|y|) \qquad (19)$$

for y<0. Now, it is simple to recreate function (15) from expansion (18) - (19), if one uses the inverse transforms of Mellin and Fourier, and if one also notes that $\hat{K}(a,0)=Q(0)\cdot\bar{F}(0)$. The lines utilized here, and the inverse Fourier and Mellin transforms, are implemented quickly (in the sense of BPF) [5], which leads to a substantial decrease in the number of calculations. This is associated with the fact that Mellin's transform, after substitution into (14) of $b=\ell^t$, is reduced to the Fourier transform of the function $f(\ell^t)$. If one designates the function $f(b)$ in a finite set of points N, in the interval $(-\ell^T, \ell^T)$, then, selecting the points of the readings in the following manner $x_i = e^{-T+(i-1)\Delta T}$ ($i=\overline{1,N-1}$, $\Delta T = \frac{2T}{N-1}$), one can implement transform (14), using BPF.

3. Correlation Analysis in a Group of Motions and Homogeneous Scale Transforms of the Plane $(M(2)xR_+)$ /9

According to (2), the normed correlation function in the group of motions and homogeneous scale transforms of the plane $(M(2)xR_+)$ has the form:

$$K(R, a_1, a_2, \alpha) = \frac{1}{R}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(g^{-1}x)\, f(x)\, dx, \qquad (20)$$

where $g^{-1}x = (\frac{1}{R}x - a)_{-\alpha},\ g(R, a, \alpha) \in G$, x and a are the vectors with coordinates x_1, x_2 and a_1, a_2, respectively; x_α designates the vector rotated at an angle α, R is the coefficient of scale transform, and a_1 and a_2 are the translation parameters.

We will first carry out spectral expansion of function (20) in the group M(2), which is done using the Fourier-Bessel transform:

$$B^n_{\gamma,\rho} f(\rho) = \int_0^\infty f(\rho)\, J_n(\gamma\rho)\, \rho\, d\rho \qquad (21)$$

and Fourier's discrete transform:

$$\Phi_{m\alpha} f(\alpha) = \frac{1}{2\pi}\int_0^{2\pi} e^{im\alpha} f(\alpha)\, d\alpha \qquad (22)$$

As shown in study [5], the Fourier form of function (20) has the form:

$$S(R, \gamma, m, n) = 2\pi\frac{(-1)^n}{R}\left\{\Phi_{m+n,\alpha} B^n_{\frac{\gamma}{R},\rho} f(\rho,\alpha)\right\}\left\{\Phi_{-n,\alpha} B^n_{\gamma,\rho}\varphi(\rho,\alpha)\right\}, \qquad (23)$$

where

$$\Phi_{m,\psi}\,\Phi_{n\alpha}\,B^{m+n}_{\tau,\rho}\,K(R,\rho,\psi,\alpha) = S(R,\tau,m,n),$$
$$a_1 = \rho\cos\psi,\quad a_2 = \rho\sin\psi.$$

After the application of Mellin's transform to (23), the final expansion of the correlation function (20) is written in the following manner:

/10

$$S(\omega,\nu,m,n) = 2^{n+\nu}\pi(-1)^n \frac{\Gamma\left(\frac{n+1+\omega}{2}\right)\Gamma\left(\frac{n+1+\omega+\nu}{2}\right)}{\Gamma\left(\frac{n+1+\omega}{2}\right)\Gamma\left(\frac{n-\omega-\nu}{2}+1\right)}F(\omega,n+m)\,Q(\omega+\nu,n),$$

where

$$F(\omega,n+m) = M_{\omega+1,\rho}\,\Phi_{m+n,\psi}\,f(\rho,\psi)$$

$$Q(\omega+\nu,n) = M_{-(\omega+\nu)+1,\rho}\,\Phi_{-n,\psi}\,\varphi(\rho,\psi)$$

The integral transforms in (25), which are used to carry out expansion (24), are reduced to BPF through discrete realization (see paragraph 2). If the initial images are given in a discrete network N x N in size, and if N points are also taken for each of the parameters of the group, then it requires $\sim N^4\log_2 N$ operations for calculation of the correlation function (20), as opposed to $\sim N^5$, if the integrals in (20) are calculated for each g by the direct methods.

Conclusion

The use of the methods of abstract harmonic analysis for the identification of images on a computer makes it possible to considerably reduce the number of calculations. The methods indicated in the present study make it possible to rapidly identify an image, irrespective of its rotations, translations, and homogeneous scale transforms in the plane, which may prove useful, for example, during the reading of print texts on a computer.

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