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On the Propagation of Long Waves in Acoustically Treated, Curved Ducts

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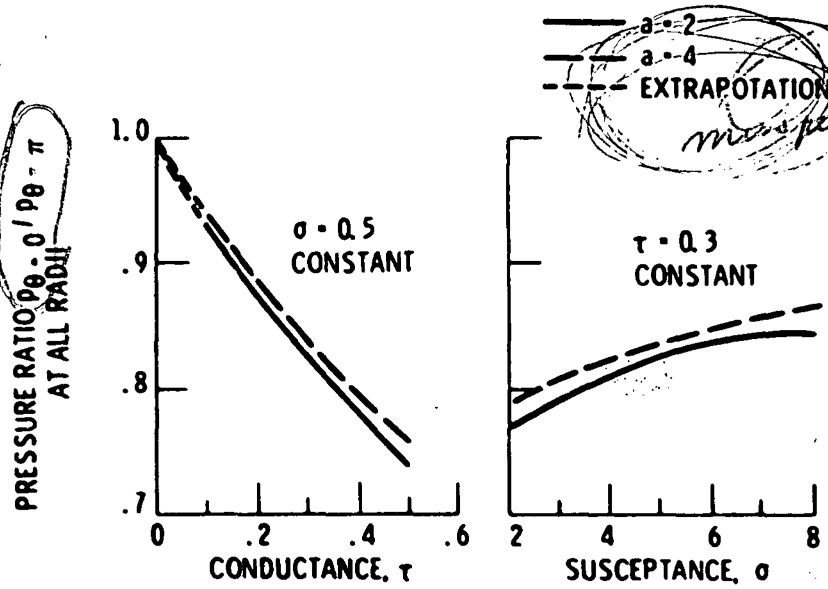
May 1981

Figure 4: In the legend, the word "timing" should be "lining."

Figure 4: The ordinate scale label should read PRESSURE RATIO $p_{\theta=\pi}/p_{\theta=0}$ AT ALL RADII.

Phase be inverted



misspelling

Fig. 4. - Acoustic pressure decay in curved ducts for various values of timing parameters.

lining

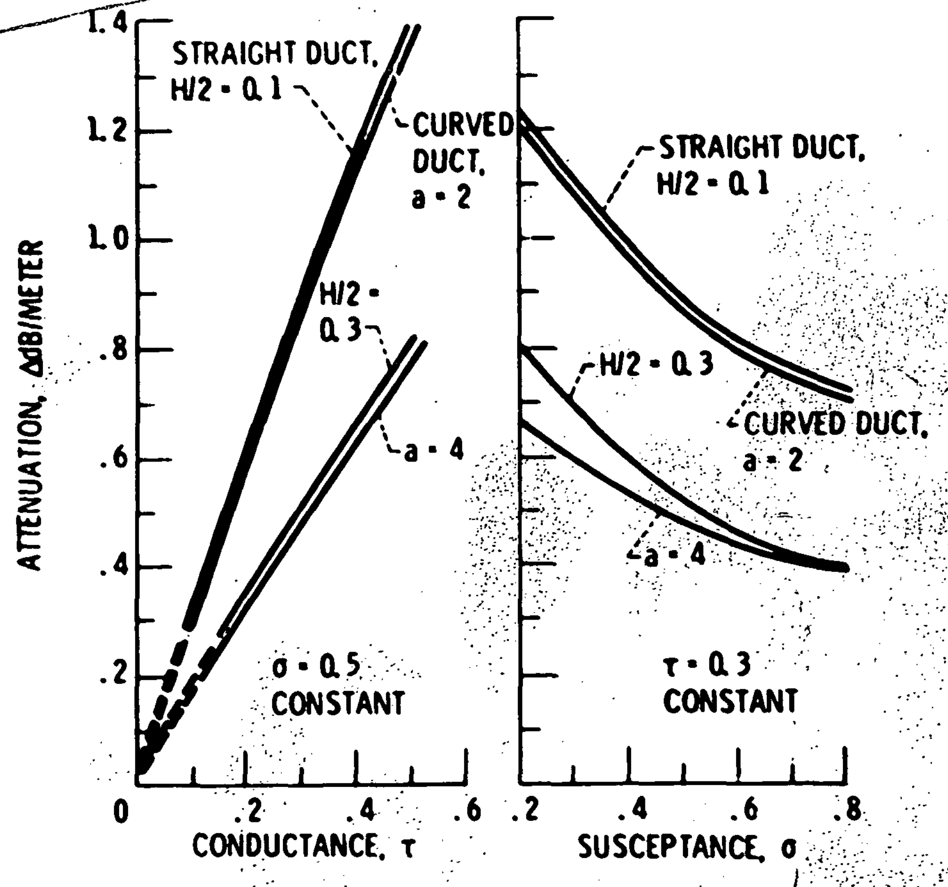


Fig. 5. - Comparison of attenuation of sound in straight and curved ducts.

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ON THE PROPAGATION OF LONG WAVES IN ACOUSTICALLY TREATED, CURVE DUCTS

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ABSTRACT

A two-dimensional, detailed study is presented on the behavior of long waves in lined, curved ducts. The analysis includes a comparison between the propagation in curved and straight lined ducts. A parametric study was conducted over a range of wall admittance and duct wall separation. The complex eigenvalues of the characteristic equation, which in the case of a curved duct are also the angular wavenumbers, have been obtained by successive approximations.

INTRODUCTION

There are both scientific and practical reasons for understanding the mechanics of the propagation of acoustic waves inside acoustically lined pipes and ducts. The noise suppressors found in aircraft engine ducts are a typical application. Consequently, many papers and engineering articles have been published on the motion of waves in straight conduits with acoustically absorbent, or lined, walls.

There are some instances where extreme curvature is present in an engine duct. An example is the S-shaped inlet to the center engine of a three-engine airplane. For some time now, there has been a theoretical effort focused on the propagation of waves in lined, curved ducts. Of about 12 papers published to date on the propagation of waves in circularly curved ducts, three treat the case of soft-wall ducts of rectangular cross section. This relative scarcity of literature on this subject may be explained by the severe mathematical difficulties that exist when eigenvalue problems are encountered which involve Bessel functions and other higher transcendental functions, especially when the functions are complex. In the three papers, selected areas of the propagation process are developed.

The first paper, by Grigor'yan[1], published in 1970, considers the numerical solution of the propagation equations at two frequencies, 2 and 4 kHz. Three configurations of acoustical wall treatment are considered: both curved walls lined; only the convex wall lined; and only the concave wall lined. The radii of curvature of the bends are equal to or greater than the radial distance between the curved walls, so that $a = R_2/R_1 \leq 2$, where R_2 is the radius of the concave wall, and R_1 is the radius of the convex wall. This is a rather narrow range, considering that in the great majority of industrial applications, $a > 2$. The analysis considers two widths of the curved ducts, and two types of sound absorbing materials. A number of very interesting conclusions are obtained, among them the fact that the curvature does not always yield an increase in the attenuation over that obtainable with a straight duct.

Myers and Mungur[2], in a paper published in 1975, numerically solve the partial differential equation of motion, and numerically evaluate the propagation and attenuation of three duct modes. The zeroth, basic mode is not included. In this work, they assume one arbitrary value of wall admittance, a range of vibrational frequencies, and a single duct width characterized by $a = 2$. In their conclusions, the authors state that for locally reactive duct walls, the sound field generally attenuates more rapidly along the axial direction of a bend than does the field in a straight duct.

Finally, in 1977, Ko and Ho[3] published results of their extensive parametric study of motion in slightly bent ducts. The purpose of their work was to develop a method for predicting the attenuation of sound in acoustically lined, curved ducts. They proceed by the derivation of eigenvalues, which they list in extensive tables. Among their findings was the determination that the level of attenuation of sound is rather dependent on acoustic resistance and relatively insensitive to the sharpness of the bend. Within the limits $1.11 < a < 1.43$, only the median arc length affects the amount of sound attenuation, which is independent of the width of the duct.

The present study is intended to extend this author's two-dimensional study of the behavior of long waves in circularly bent ducts with hard walls.[4] One of the results of that work was determination of wave phase velocity, which in the case of very low frequencies is higher than in a similar straight duct and is proportional to the sharpness of the bend. This extension of that study consists of modifying the boundary conditions and evaluating the effect of the new boundaries on sound phase velocity and on the distribution and level of particle velocity and pressure. These boundary conditions assume that the curved walls of the bend will be lined with locally reactive, acoustically absorbing material. All other assumptions of the previous work are accepted without change, namely that there is a sustained, continuous, and steady propagation of long acoustic waves in a two-dimensional, infinitely long, circularly bent duct. The closest physical example of such an infinitely long bend is a tightly wound coil of which the pitch is negligible compared to the radius of curvature of the duct. The wave length is at least two orders of magnitude larger than the width of the duct. The pressure distribution within the duct satisfies the wave equation.

ANALYTICAL SOLUTION

Infinite, Lineu, Curved Duct

The basic difficulty in solving equations of motion with complex roots resides in our inability to split the final equation containing exponential and trigonometric functions into real and imaginary parts. Were this possible, it still would not guarantee yielding the answers directly, but it would help enormously, because a single term would be sufficient to satisfy the characteristic equation. With complex expressions, two real numbers must be found. So we are forced to rely on the tedious method of successive approximations.

The calculation of pressure and of components of particle velocity starts with definition of the velocity potential:

$$\phi = \sum_{\nu} [A_{\nu} J_{\nu}(kr) + B_{\nu} Y_{\nu}(kr)] \exp i(\omega t - \nu\theta) \tag{1}$$

which yields

$$p = \rho \frac{\partial\phi}{\partial t}; \qquad v_{\theta} = - \frac{1}{r} \frac{\partial\phi}{\partial\theta}; \text{ and } \qquad v_{r} = - \frac{\partial\phi}{\partial r}$$

A symbol list is found in the appendix. In the above expressions, the two Bessel functions J_{ν} and Y_{ν} are of complex order, since the angular wavenumber

$$\nu = \nu_1 + i\nu_2$$

Determination of constants B_{ν} in the definition of the velocity potential is usually obtained from the boundary condition, by assuming that at $\theta = 0$, $v_{\theta} = v_0$. However, in this paper only the lowest order mode will be considered. The attenuation of sound, in decibels (dB), is defined by $\Delta dB = 20 \log(p_{\theta}/p_{\theta=0})$. Since $p \propto \exp(-\nu_2\theta)$, $\Delta dB = 20 (-\nu_2\theta) \log e$. The angular wave phase velocity, $\dot{\theta} = \omega/\nu_1$, and the phase velocity along the median radius, $\dot{s} = \dot{\theta}R_m = c\ kR_1(a + 1)/2\nu_1$, are easily calculated. A general solution of the two-dimensional problem for both rigid and soft-wall cases has been formulated by Grigor'yan[5] and others. The equation for determination of the angular wavenumber for the case of identical lining material on both curved walls (the original equation makes it possible to stipulate different materials) of a circularly bent duct, is

$$[J_\nu'(kR_1)Y_\nu'(kR_2) - J_\nu'(kR_2)Y_\nu'(kR_1)] + i\eta[J_\nu'(kR_1)Y_\nu(kR_2) - J_\nu(kR_2)Y_\nu'(kR_1) - J_\nu(kR_1)Y_\nu'(kR_2) + J_\nu'(kR_2)Y_\nu(kR_1)] + \eta^2[J_\nu(kR_1)Y_\nu(kR_2) - J_\nu(kR_2)Y_\nu(kR_1) = 0 \qquad (2)$$

where $\eta = -\rho c\ v_r/p$ (i.e., a dimensionless wall admittance v_r/p). In general, $\eta = \tau + i\sigma$ is complex, and τ is the conductance of the walls, and σ the susceptance of the walls. It is clear that using $\eta = 0$ in Eq. (2) leaves only the expression of the cross products of the derivatives of the two Bessel functions, which corresponds to the case of the hard-walled bend.

To solve Eq. (2) for the case of long waves propagating in a soft-walled bend, the two Bessel functions are expanded in series, and in view of the smallness of the arguments, only the first terms of the series are retained. The second and following terms of the expansions are smaller than the first term at least by the order of $(kR_1)^2$. Rearranging and eliminating small terms of higher order yields

$$\tau^2 - \sigma^2 + 2i\tau\sigma + (\sigma - i\tau)\ \frac{a + 1}{akR_1}\ \frac{a^\nu + a^{-\nu}}{a^\nu - a^{-\nu}} - \frac{\nu^2}{a(kR_1)^2} = 0 \qquad (3)$$

For $\tau = \sigma = 0$, the conditions for stiff walls, Eq. (3) yields $a^\nu = a^{-\nu}$ or $a^{2\nu} = 1$, and $\nu = in\pi/\ln a$, pure imaginary roots derived before in

Ref. 4.. Eq. (3) also allows verification that for finite values of τ and σ, there are no pure imaginary or simple real roots. Hence, ν must be complex. Roots, which are complex angular wavenumbers, have been obtained by successive approximations. It helped to know that both the real and the imaginary parts of ν must be small because the real root for the case of the hard walls is small. In applying Eq. (3), calculations have been made for the following set of parameters: $k = 0.1$; $R_1 = 0.2$; $a = 2$ and 4; $\tau = 0.1, 0.2, \ldots, 0.5$; and $\sigma = 0.2, 0.3, 0.5$, and 0.8. The calculated complex roots of Eq. (3), which are also complex propagation constants, are listed in Table I and are shown in Fig. 1. It will be noted that quantities related to attenuation are one order of magnitude smaller than terms related to propagation. Attenuation increases with increasing τ, but attenuation is less pronounced with a wider duct. For small values of conductance τ, susceptance σ does not greatly influence attenuation; with higher τ, however, large changes occur with changing σ. The percentage change in attenuation at constant τ and varying σ is nearly twice as great for the lower range of τ as for the largest τ studied. On the same graph, the angular wavenumber for the rigid, curved wall case (i.e., for $\tau = \sigma = 0$, and $a = 2$) is also shown. Presence of acoustical lining on curved walls markedly increases the values of the propagation constants ν_1.

Infinite, Lined, Straight Duct

The propagation of acoustic waves in a straight, lined duct can be described in terms of the acoustic pressure using the expression given by Rice[6]:

$$p = \cos\left(\frac{2gy}{H}\right)\exp\left(i\omega t - \frac{\omega}{c}\xi x\right) \tag{4}$$

In Eq. (4), the g's are complex roots. Also, $\xi = \alpha + i\beta$ is a complex wave number with its propagation and attenuations terms. Now, $\xi = i\sqrt{1 - (2g/kH)^2}$

as derived by Rice, $H/2$ is the half-width of the duct, and x and y are the axial and transverse coordinates, with $y = 0$ at duct centerline.

The phase velocity in a straight duct will be simply $\dot{x} = c/\beta$. The transverse particle velocity may be expressed by $u_y = \eta p/\rho c$ or $u_y = i(\partial p/\partial y)/\rho\omega$. Equating these two expressions at $y = H/2$ yields

$$\tan(g) = i\left(\frac{\omega}{c}\frac{H}{2}\right)\left(\frac{\eta}{g}\right) \tag{5}$$

where $\eta = \tau + i\sigma$, as in the case of the curved duct. Solution of Eq. (5) yields the real and the imaginary components of the roots. Following this, a calculation of complex wavenumber is straightforward. Also, the expression for the axial component of particle velocity is easily obtainable by derivation of the expression for pressure with respect to x, the axial duct coordinate $[u_x = i(\partial p/\partial x)/\rho\omega]$.

To be able to compare the propagation in the straight duct with the propagation in the curved duct, not only the same set of values for τ and σ are used in numerical calculations as in the case of the curved duct, but also the widths of the straight ducts are selected to match the two widths used in analysis of the curved ducts. The complex wavenumber ξ for the straight, lined duct (i.e., the attenuaton and the propagation terms) are given in Table II and shown in Fig. 2. As in the case of a curved, lined duct, the wider duct has a less pronounced attenuation. The general influence of τ and σ on the complex wavenumber is similar to that in the curved duct.

RESULTS AND DISCUSSION

Phase Velocities

Using the propagation constants for both curved and straight ducts, corresponding phase velocities were calculated and ratios $\dot{s}/\dot{x}$ established.

They are shown on Fig. 3 as a function of τ. The ratios are greater than 1 for all parameters considered. They are higher for a wider duct (e.g., for $a = 4$) than for one half as wide (for $a = 2$). Generally, the phase velocities and their ratios increase with τ, but not strongly. The corresponding values of phase-velocity ratios for a hard-walled ducts are also shown. At these low frequencies, the phase-velocity ratios in lined ducts are lower than in untreated ducts.

Attenuation of Sound

Attenuation of sound in lined ducts, both curved and straight, has been calculated for all parameters shown in Tables I and II. Some of the results for curved ducts are shown in Fig. 4, in which pressure decay in a bend of 180° is plotted against parameters of the lining, both τ and σ.

There is a very significant increase in pressure loss with the increase of conductance τ. On the other hand, for a given level of conductance, higher values of susceptance σ somewhat reduce the pressure loss. Both relations are clearly nonlinear. Also, it is clear that in a wider bend the losses are somewhat less than in a narrow one. It will be noticed that on a plot of pressure ratio versus τ, the two curves can be extrapolated down to $\tau = 0$ (i.e., to the case of bends with hard walls). The extrapolation reaches a pressure ratio of unity, as it should.

Results of calculations of sound attenuation per unit length, in dB/m, and comparisons of attenuation between curved and straight ducts, are shown in Fig. 5. Attenuation has been evaluated with variable parameters, τ and σ, of the acoustical lining in ducts of two widths. Sound attenuation rapidly increases with increasing τ, but it is less pronounced in a wider duct. In general, attenuation is slightly higher in a straight duct (by 2 to 7 percent), but in this range of very long waves, it must be considered significant. All four linear functions shown in Fig. 5 extrapolate to zero attenuation at $\tau = 0$. The dependence of attenuation on σ is nonlinear, and attenuation decreases with increasing susceptance. For narrow ducts, the difference between the attenuation in a straight duct and in a curved duct remains fairly constant with increasing σ. For wide ducts, this difference in attenuation decreases with increasing σ.

Radial Distribution of Wave-Motion Parameters

The radial distribution of the particle velocity components and of the acoustic pressure has been analyzed for all cases taken into consideration previously. Samples of typical profiles in a curved, lined duct are shown in Fig. 6. The particle velocities are nondimensionalized by using v_0. Basically, very slight radial variations have been detected when τ and σ were given their full range of values. Also, when the wave was moving down the curved duct, the pressure and axial-velocity profiles remained unchanged except for a gradual decrease in their amplitudes. The profile of the radial component of the particle velocity is different in that its slope changes with distance. The radial velocity distribution exhibits a zero point near the convex wall of the bend. This point is not much displaced toward the center of the duct when a duct three times as wide is used. In Fig. 7, these profiles of particle velocity components are compared with distributions calculated for the case of unlined bends of the same geometry and for similar, very low frequencies of acoustic waves. A striking difference exists between the two radial particle velocity distributions. This was to be expected, because a lined duct has a finite value of wall impedance which does not require that the radial particle velocities vanish at the wall. The relative values of the radial velocities shown on the graph have no particular meaning, because the boundary conditions at $\theta = 0$ for the rigid and the lined bends are different. The distribution of the tangential velocity component is not affected by the presence of a lining, except for a generally lower amplitude at all radii.

The radial distribution of the acoustic pressure is markedly changed by the presence of a lining. The data are nondimensinalized by using p_0, the acoustic pressure corresponding to reference particle velocity v_0. Within a hard-walled bend, the pressure is characteristically higher at the outside wall, as was first documented by Cummings[7]. In a lined, curved duct, it is practically independent of the radial postion.

CONCLUDING REMARKS

Propagation of long waves in circularly bent ducts lined with sound-absorbing material has been analyzed for a range of acoustic parameters. A set of eigenvalues, the complex angular wavenumbers, has been obtained by a method of successive approximations. The results have been compared with the propagation of low-frequency sound in a straight, lined duct of the same width and lined with the same material as the bent duct. Furthermore, a comparison with the propagation parameters in a hard-walled, bent duct has been presented as well.

At these very low frequencies, and in the range of parameters evaluated, the sound attenuation in a curved, lined duct is generally less pronounced than in a straight duct of the same width and same wall impedance.

Results of this study and of work of others indicate that there is need for further analysis of the propagation of sound in curved, lined ducts. The overall picture of such propagation is not yet clear.

APPENDIX - LIST OF SYMBOLS

A_ν	a constant
a	radii ratio = R_2/R_1
B_ν	a constant
c	velocity of sound
g	complex eigenvalue for the straight duct
H	depth or width of a rectangular duct
$J_\nu(kr)$	Bessel function of the first kind, of order ν
k	wavenumber
p	acoustic pressure
p_0	reference acoustic pressure
p_θ	acoustic pressure at angular location θ
R_1	curved-duct convex wall radius = 0.2 m
R_2	curved-duct concave wall radius
r	radial coordinate
s	distance along curved-duct centerline
$\dot{s}$	wave phase velocity in the curved duct
t	time coordinate
v_0	reference particle velocity
v_θ	tangential component of particle velocity
v_r	radial component of particle velocity
u_x	axial component of particle velocity
u_y	transverse particle velocity
x	axial coordinate
$\dot{x}$	wave phase velocity in a straight duct
$Y_\nu(kr)$	Bessel function of the second kind, of order ν
y	transverse coordinate

α	attenuation coefficient in a straight duct
β	propagation coefficient in a straight duct
ζ	complex wavenumber, $\zeta = \alpha + i\beta$
η	wall admittance
θ	polar coordinate
$\dot{\theta}$	angular phase velocity
ν	complex angular wavenumber, $\nu = \nu_1 + i\nu_2$
ν_1	propagation coefficient
ν_2	attenuation coefficient
ρ	density
σ	susceptance
τ	conductance
ϕ	velocity potential
ω	angular frequency = kc

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2. M.K. Myers and P. Mungur, "Sound Propagation in Curved Ducts," AIAA Paper No. 75-497 (March 1975).

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7. A. Cummings, "Sound Transmission in Curved Duct Bends," J. Sound Vib. 35(4), 451-477 (1974).

TABLE I. - COMPLEX ANGULAR WAVENUMBERS FOR CURVED LINED DUCTS FOR

$kR_1 = 0.02$ AND FOR A RANGE OF PARAMETERS OF THE LINING

τ	σ	a = 2		a = 4	
		Propagation ν_1	Attenuation ν_2	Propagation ν_1	Attenuation ν_2
0.1	0.2	0.13546	-0.03201	-------	--------
.2		.14460	-.05997	-------	--------
.3		.15577	-.08351	0.14225	-0.07680
.4		.16735	-.10364	-------	--------
.5		.17873	-.12131	-------	--------
.1	.5	.20931	-.02077	.19199	-.01925
.2		.21224	-.04097	.19462	-.03797
.3		.21672	-.06019	.19864	-.05580
.4		.22234	-.07823	.20367	-.07256
.5		.22873	-.09505	.20938	-.08823
.1	.8	.26415	-.01651	------	-------
.2		.26565	-.03283	------	-------
.3		.26805	-.04880	.24649	-.04564
.4		.27125	-.06480	------	-------
.5		.27511	-.07925	------	-------

TABLE II. - COMPLEX WAVENUMBERS FOR STRAIGHT LINED DUCTS FOR

k = 0.1 AND FOR A RANGE OF PARAMETERS OF THE LINING

τ	σ	H/2 = 0.1		H/2 = 0.3	
		Propagation k_β	Attenuation k_α	Propagation k_β	Attenuation k_α
0.3	0.2	0.52195	0.28687	0.30218	0.18561
.1	.5	.70021	.07175	.39872	.05574
.2		.71197	.13971	.40343	.08171
.3		.72886	.20486	.41296	.11986
.4		.74772	.26620	.42498	.15527
.5		.76908	.32183	.43856	.18817
.3	.8	.90287	.16522	.51268	.08939

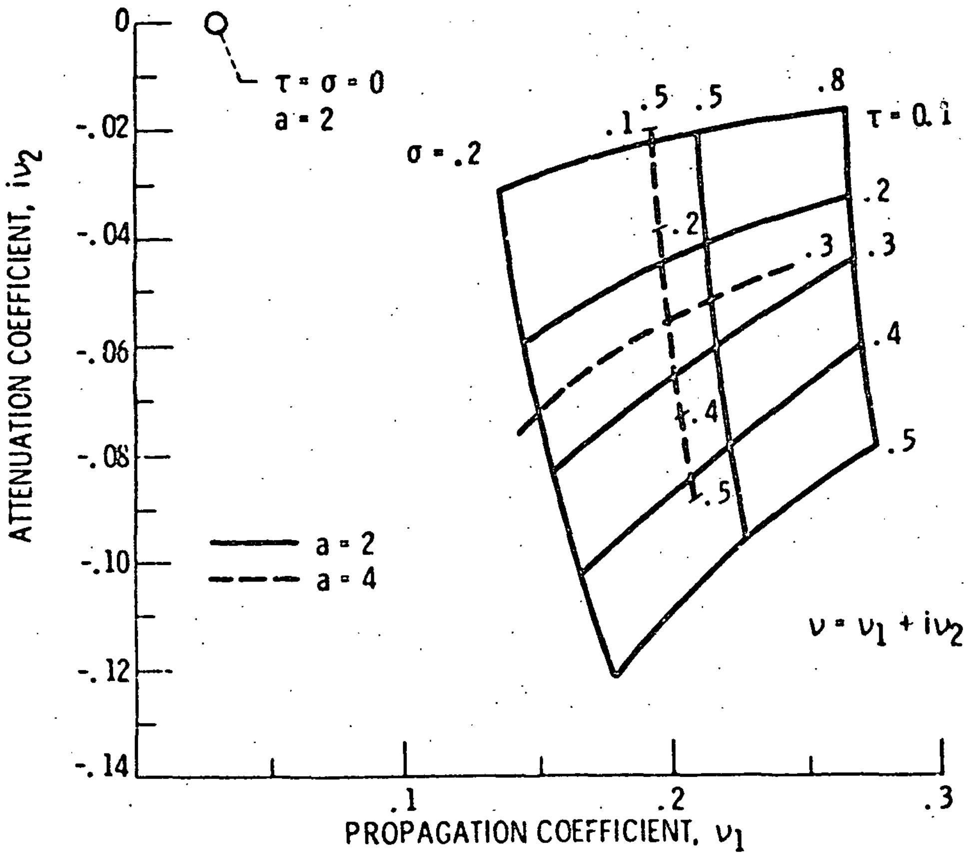


Fig. 1. - Real and imaginary parts of complex angular wavenumbers for a curved duct.

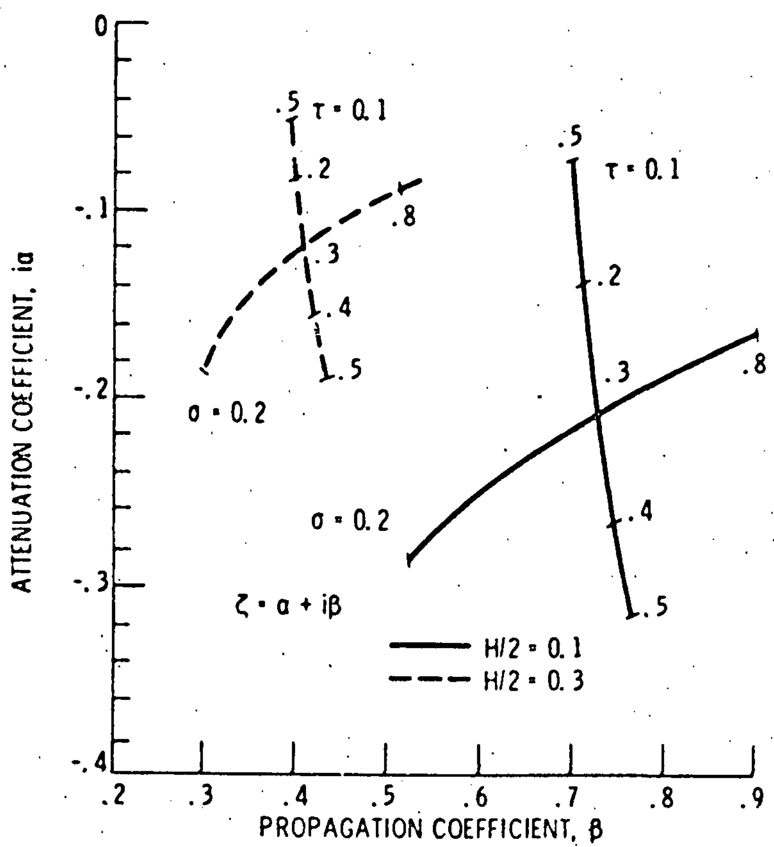


Fig. 2. - Real and imaginary parts of complex wavenumbers for a straight duct.

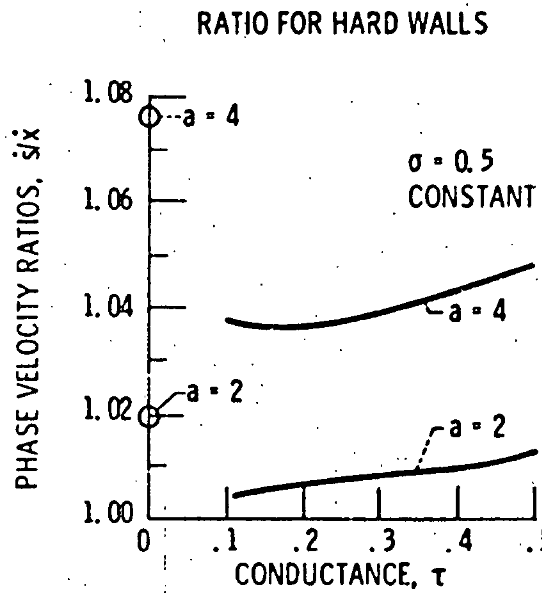


Fig. 3. - Phase velocity ratios for curved and straight ducts.

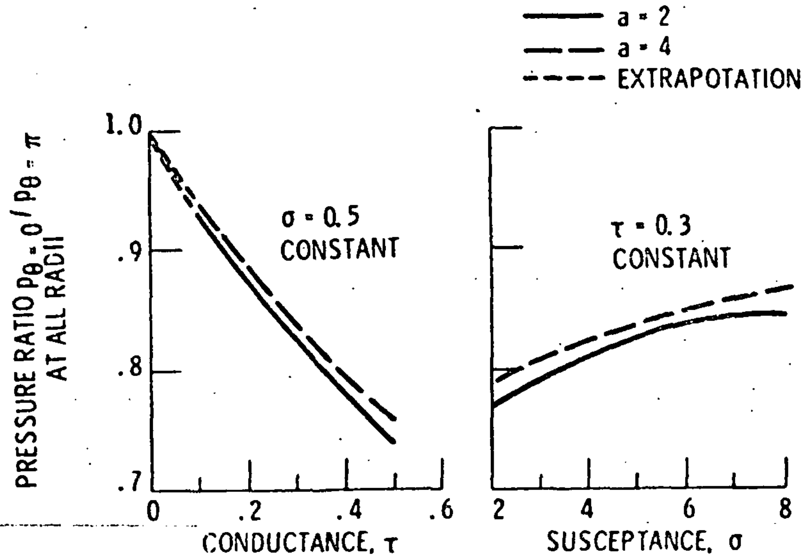


Fig. 4. - Acoustic pressure decay in curved ducts for various values of timing parameters.

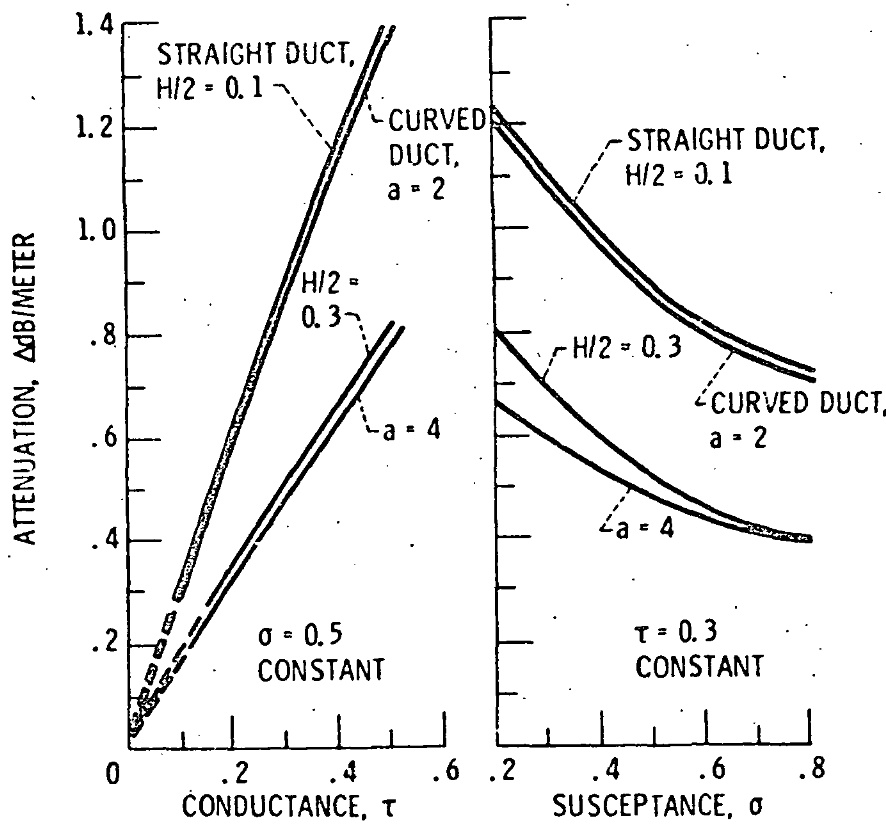
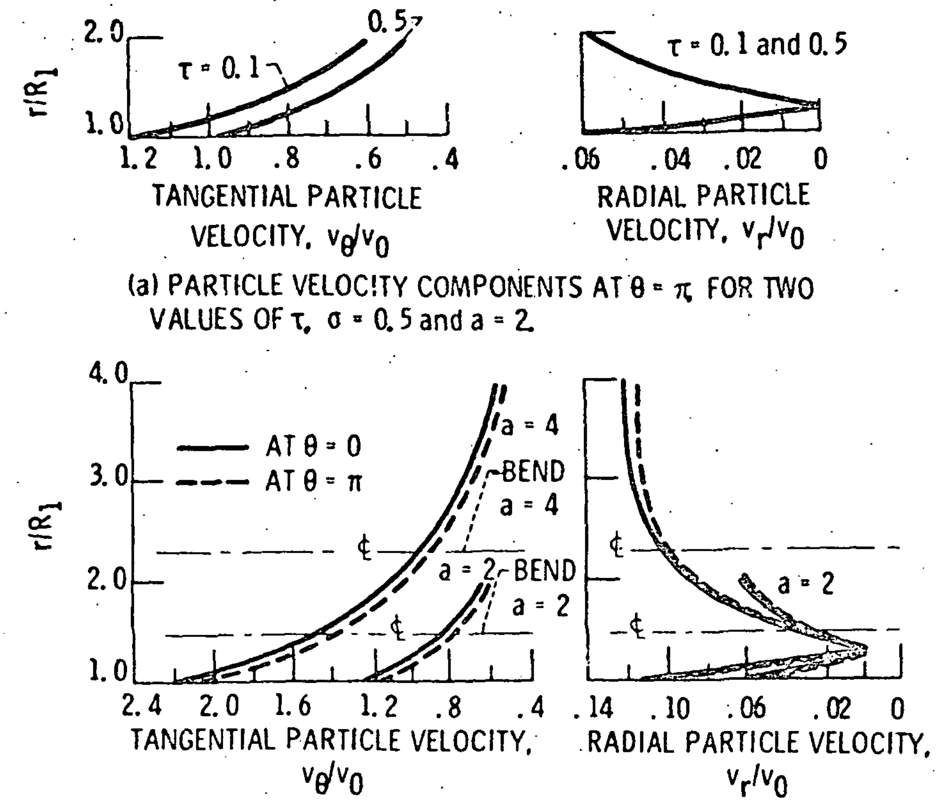


Fig. 5. - Comparison of attenuation of sound in straight and curved ducts.



(a) PARTICLE VELOCITY COMPONENTS AT $\theta = \pi$, FOR TWO VALUES OF τ, $\sigma = 0.5$ and $a = 2$.

(b) PARTICLE VELOCITY DISTRIBUTION CHANGES WITH DISTANCE, FOR $\tau = 0.1$, $\sigma = 0.5$, AND FOR TWO VALUES OF a.

Fig. 6. - Particle velocity components in a lined curved duct.

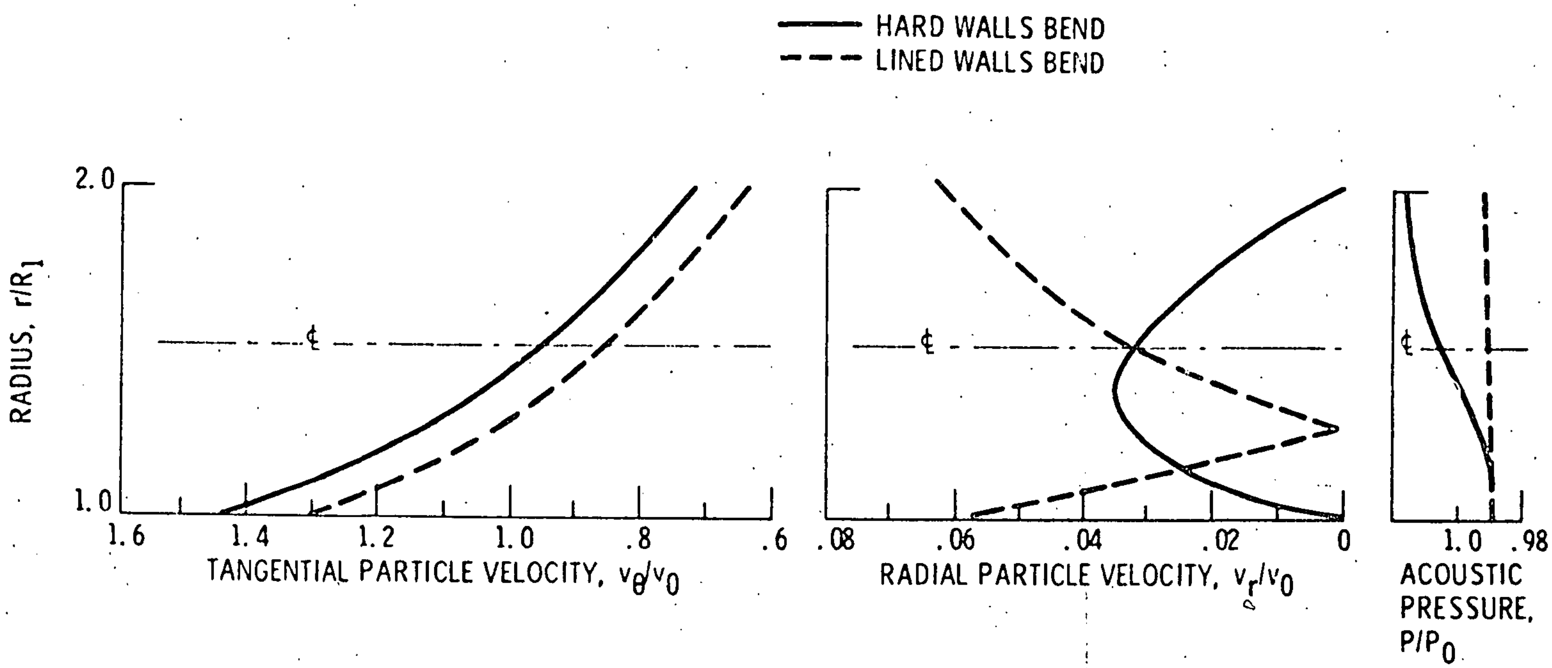


Fig. 7. - Comparison of particle velocities and pressure distribution in curved, lined, and unlined ducts, $a = 2$, $\tau = 0.1$, $\sigma = 0.5$.

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