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An Analytical Study of the Longitudinal Response of Airplanes to Positive Wind Shear

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SUMMARY

Wind shear, the variation of horizontal atmospheric winds with altitude, has been identified as a causative factor in several airplane accidents and may have been a contributing factor in others. Consequently, wind shear and its effect on aircraft have become the subjects of research. This study extends past work and concentrates on longitudinal motion. The airplane is represented by the three-degree-of-freedom equations for longitudinal motion. Because stability and control problems occur only if the wind shear parameter σ_u exceeds 1, all wind shears used in this study produced a value of σ_u greater than 1. Previous investigations have shown that wind shear has little effect on the short-period mode for the type of airplanes used in this study. The u stability derivatives (derivatives with respect to perturbation velocity) were varied to determine the effect of changes in their magnitudes on the stability of the long-period mode in wind shear. It was found that increases in the pitching-moment derivative M_u and decreases in the vertical-force derivative Z_u made the airplane more stable in wind shear. During the study of the u derivatives, a wind shear tolerance factor was developed, which is a function of the basic stability derivatives of the airplane. If this factor is greater than 1, the airplane is stable in positive wind shear.

INTRODUCTION

Wind shear, the vertical (altitude) variation of horizontal wind, has been a causative factor in many airplane accidents (refs. 1 and 2). Wind shears can occur at any altitude. During midcourse flight, when the airplane is at high altitude, an encounter with wind shear poses little danger to the airplane. However, during take-off and landing operations, when the airplane is close to the ground, an accident may occur before the airplane can recover from a wind shear encounter. Because low-altitude encounters represent a hazardous condition, this investigation was restricted to the study of the problems of wind shear encounters on landing approach below an altitude of 130 meters.

References 3, 4, and 5 give the results of general studies of wind shear for descending and climbing flight near the ground. The wind shear parameter σ_u, introduced in references 3 and 4 and also used in this study, is a measurement of the severity of the wind shear. Since a wind shear parameter greater than 1 produces unstable conditions, gradients for the study were selected to give $\sigma_u > 1$.

The principal thrust of this study was to examine the role the speed stability derivatives play in the interaction of the airplane and wind shear. In addition, the use of airspeed control systems to control an airplane during a wind shear encounter was further investigated.

The equations of motion are derived in appendix A. A program written for the Hewlett Packard HP-67 programmable pocket calculator, which represents an improvement on program 1 of reference 6, is given in appendix B. The use of this calculator does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

SYMBOLS

A aspect ratio

$a_1, a_2, \ldots, a_{15}$ coefficients of 3 × 3 matrix (eqs. (A8)) representing linearized equations of motion for stability; defined in equations (A9)

$b_1, b_2, \ldots, b_6$ coefficients used in equations (A8) and defined in equations (A9)

$C_0, C_1, \ldots, C_6$ coefficients of longitudinal characteristic equation with $\sigma_w = 0$ (eq. (A10)); defined in equations (A12)

$C_{\bar{D}}$ drag coefficient, $\dfrac{2\bar{D}}{\rho S U_0^2}$

$C_{\bar{D},o}$ drag coefficient at $C_L = 0$

$C_{\bar{D}\alpha}$ $= \dfrac{\partial C_{\bar{D}}}{\partial \alpha}$, rad^{-1}

$C_{\bar{D}\delta e}$ $= \dfrac{\partial C_{\bar{D}}}{\partial \delta_e}$, rad^{-1}

C_L lift coefficient, $\dfrac{2L}{\rho S U_0^2}$

$C_{L,o}$ lift coefficient at $\alpha = 0$

C_{L_q} $= \dfrac{\partial C_L}{\partial q}$, rad^{-1}-sec

C_{L_u} $= \dfrac{\partial C_L}{\partial u}$, m^{-1}-sec

$C_{L\alpha}$ $= \dfrac{\partial C_L}{\partial \alpha}$, rad^{-1}

$C_{L_{\dot{\alpha}}}$ $= \dfrac{\partial C_L}{\partial \dot{\alpha}}$, $\text{rad}^{-1}\text{-sec}$

$C_{L_{\delta_e}}$ $= \dfrac{\partial C_L}{\partial \delta_e}$, rad^{-1}

$C_{L_{\delta_T}}$ $= \dfrac{\partial C_L}{\partial \delta_T}$, rad^{-1}

$C_{L_{\dot{\theta}}}$ $= \dfrac{\partial C_L}{\partial \dot{\theta}}$, $\text{rad}^{-1}\text{-sec}$

C_m pitching-moment coefficient, $\dfrac{2M}{\rho S \bar{c} U_0{}^2}$

$C_{m,o}$ pitching-moment coefficient at $\alpha = 0$

C_{m_q} $= \dfrac{\partial C_m}{\partial q}$, $\text{rad}^{-1}\text{-sec}$

C_{m_u} $= \dfrac{\partial C_m}{\partial u}$, $\text{m}^{-1}\text{-sec}$

C_{m_α} $= \dfrac{\partial C_m}{\partial \alpha}$, rad^{-1}

$C_{m_{\dot{\alpha}}}$ $= \dfrac{\partial C_m}{\partial \dot{\alpha}}$, $\text{rad}^{-1}\text{-sec}$

$C_{m_{\delta_e}}$ $= \dfrac{\partial C_m}{\partial \delta_e}$, rad^{-1}

$C_{m_{\delta_T}}$ $= \dfrac{\partial C_m}{\partial \delta_T}$, rad^{-1}

$C_{m_{\dot{\theta}}}$ $= \dfrac{\partial C_m}{\partial \dot{\theta}}$, $\text{rad}^{-1}\text{-sec}$

C_T	thrust coefficient, $\dfrac{2T}{\rho S U_0{}^2}$
C_{T_u}	$= \dfrac{\partial C_T}{\partial u}$, m^{-1}-sec
$\bar{c}$	mean aerodynamic chord, m
D	$= \dfrac{d}{dt}$
$\bar{D}$	drag, N
$\bar{D}_u$	$= \dfrac{1}{m} \dfrac{\partial \bar{D}}{\partial u}$, sec^{-1}
F_X	force in x-direction, N
F_Z	force in z-direction, N
g	acceleration due to gravity, 9.80665 m-sec^{-2}
h	altitude, m
I_Y	moment of inertia about y-stability axis, kg-m^2
$\vec{i},\vec{j},\vec{k}$	unit vectors
i,j,k	indices
$K_4,\ldots,K_7$	airspeed control system gains
k_Y	radius of gyration, m
L	lift, N
M	pitching moment, N-m
M_q	$= \dfrac{1}{I_Y} \dfrac{\partial M}{\partial q}$, rad^{-1}-sec^{-1}
M_u	$= \dfrac{1}{I_Y} \dfrac{\partial M}{\partial u}$, m^{-1}-sec^{-1}

M_α $= \frac{1}{I_Y} \frac{\partial M}{\partial \alpha}$, rad^{-1}-sec^{-2}

$M_{\dot{\alpha}}$ $= \frac{1}{I_Y} \frac{\partial M}{\partial \dot{\alpha}}$, rad^{-1}-sec^{-1}

$M_{\delta e}$ $= \frac{1}{I_Y} \frac{\partial M}{\partial \delta_e}$, rad^{-1}-sec^{-2}

m mass, kg

P period, sec

q pitching velocity, rad-sec^{-1}

S wing area, m^2

S_{TF} shear tolerance factor

T thrust, N

T_u $= \frac{1}{m} \frac{\partial T}{\partial u}$, sec^{-1}

t_D time to double amplitude, sec

$t_{1/2}$ time to damp to half-amplitude, sec

U forward velocity, m-sec^{-1}

U_0 steady-state velocity, m-sec^{-1}

u perturbation velocity, m-sec^{-1}

u_w wind speed in x-direction in inertial coordinates, m-sec^{-1}

u_w' wind shear gradient, $\frac{du_w}{dh}$, sec^{-1}

V airspeed, m-sec^{-1}

$\vec{V}_A, V_A$ airplane velocity vector and magnitude, m-sec^{-1}

$\vec{V}_B, V_B$ airplane velocity vector in body axes, m-sec^{-1}

$\dot{\vec{V}}_B$ airplane acceleration vector in body axes, m-sec^{-2}

V_C command velocity, m-sec^{-1}

V_R resultant velocity, m-sec^{-1}

$\vec{V}_W$ wind velocity vector, m-sec^{-1}

w component of $\vec{V}_A$ in the z-direction (body axes), m-sec^{-1}

w_W speed of updrafts or downdrafts, m-sec^{-1}

w_W' updraft-downdraft gradient, $\frac{dw_W}{dx}$, sec^{-1}

x,y,z inertial axis system and coordinates

X_u $= \frac{1}{m} \frac{\partial F_X}{\partial u}$, sec^{-1}

X_α $= \frac{1}{m} \frac{\partial F_X}{\partial \alpha}$, m-rad^{-1}-sec^{-2}

X_{δ_e} $= \frac{1}{m} \frac{\partial F_X}{\partial \delta_e}$, m-rad^{-1}-sec^{-2}

Z_q $= \frac{1}{m} \frac{\partial F_Z}{\partial q}$, m-rad^{-1}-sec^{-2}

Z_u $= \frac{1}{m} \frac{\partial F_Z}{\partial u}$, sec^{-1}

Z_α $= \frac{1}{m} \frac{\partial F_Z}{\partial \alpha}$, m-rad^{-1}-sec^{-2}

$Z_{\dot{\alpha}}$ $= \frac{1}{m} \frac{\partial F_Z}{\partial \dot{\alpha}}$, m-rad^{-1}-sec^{-2}

Z_{δ_e} $= \frac{1}{m} \frac{\partial F_Z}{\partial \delta_e}$, m-rad^{-1}-sec^{-2}

α perturbation angle of attack, rad

α_{tr}	trim angle of attack, rad
Γ	total flight-path angle, $\Gamma_0 + \gamma$, rad
Γ_0	steady-state flight-path angle, rad
γ	perturbation flight-path angle, rad
δ_e	elevator deflection, rad
δ_T	throttle deflection, rad
ζ_D	damping ratio
θ	pitch angle, rad
ξ,η,ζ	moving axis system for airplane
ρ	air density at 0°C and 1 atm, 1.2929 kg-m^{-3}
σ_T	$= \sigma_u + \sigma_w$
σ_u	wind shear parameter for horizontal wind
σ_w	updraft-downdraft parameter
τ_E	engine time constant
ω	circular frequency, rad-sec^{-1}
ω_n	undamped circular frequency, rad-sec^{-1}

Subscripts:

tr	trim
0	steady-state conditions

Dots over a symbol indicate derivatives with respect to time.

An arrow over a symbol indicates a vector. The symbol without the arrow indicates magnitude.

AIRPLANES AND CONDITIONS OF STUDY

Airplanes Represented in Study

Two jet transport airplanes were represented in this study. One was a large four-engine, long-range jet transport referred to as airplane A. The other was a small twin-engine, medium-range jet transport referred to as airplane B. The aerodynamic and physical characteristics of both airplanes are given in table I.

Airplanes A and B were used throughout the study. However, in order to give more data on the shear tolerance factor, two additional four-jet transports were used and are referred to as airplanes C and D. Because of the limited use made of airplanes C and D, their aerodynamic and physical characteristics are not presented.

Wind Conditions

Wind shear is an atmospheric phenomenon and varies with one or more atmospheric parameters. In this paper wind shear was restricted to a variation of the horizontal wind speed with altitude, as shown in figure 1. If an airplane follows the flight path shown in figure 1 and penetrates the wind shear layer, then the airplane will experience a positive wind shear; that is, a head wind changes to a tail wind during transit of the airplane through the wind shear layer. Work reported in references 3 and 4 shows that unstable conditions occur if the wind gradient is large enough to produce a wind shear parameter greater than 1. For the airplanes used in this study a gradient of 0.25 sec^{-1} gives a value of σ_u in the range 1.5 to 2.0. This gradient has been measured in thunderstorms. (See refs. 2 and 7.)

For both stability calculations and time histories, the wind was described by the equation

$$u_w = u_{w,0} + u_w' \ (\Delta z)$$

where $\Delta z = z_{n-1} - z_n$, with z_n being the present value of z and z_{n-1} the previous value of z. In stability calculations $u_{w,0}$ is taken as zero, and in time histories -6.10 m-sec^{-1} head winds are negative. For $u_{w,0} < 0$, $u_w' > 0$, and $z_n < z_{n-1}$, this equation produces winds that provide a positive wind shear situation with respect to the airplane.

In this study wind shear occurred in the altitude interval $106 \geqq H \geqq 56$.

Equations of Motion

The equations of motion developed for stability calculations are derived in appendix A and have $\sigma_u \neq 0$ and $\sigma_w = 0$. (A program for calculating the characteristic equation is given in appendix B.) For the cases studied in this investigation, σ_w is taken as zero and equations (A4) become

$$\begin{bmatrix} \frac{d}{dt} - \frac{g}{2U_0}\sigma_u \sin 2\Gamma_0 - X_u & -X_\alpha & g(\cos\Gamma_0 - \sigma_u \cos 2\Gamma_0) \\ -Z_u - \frac{g}{U_0}(\sigma_u \sin^2\Gamma_0) & -(Z_{\dot{\alpha}} + Z_q)\frac{d}{dt} - Z_\alpha & -(U_0 + Z_q)\frac{d}{dt} + g(\sin\Gamma_0 - \sigma_u \sin 2\Gamma_0) \\ -M_u & \frac{d^2}{dt^2} - (M_{\dot{\alpha}} + M_q)\frac{d}{dt} - M_\alpha & \frac{d^2}{dt^2} - M_q\frac{d}{dt} \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \gamma \end{bmatrix} = \begin{bmatrix} X_{\delta e} \\ Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta_e \tag{1}$$

and are the equations used for the stability calculations. These equations form the basis of program 1 of reference 6. This program and others in reference 6 were used to calculate all stability information.

Time histories of spatial motions of the aircraft were generated using the three-degree-of-freedom nonlinear equations of longitudinal motion (appendix A, eqs. (A13)). The effects of wind shear, updrafts, and downdrafts were incorporated in these equations, as suggested by Etkin (ref. 8).

Gera (ref. 9) has proposed a new set of equations for the analysis of wind shear. These equations are restricted to the condition $\Gamma_0 = 0$. Since the present investigation considered only descending flight, the new equations proposed in reference 9 were not applicable. It should be noted that for $\sigma_w = 0$ and $\Gamma_0 \neq 0$, the equations of this report and those given in reference 9 are the same.

AIRPLANE RESPONSE TO WIND SHEAR

Results presented in reference 4 established the effect of wind shear on the longitudinal stability characteristics of the airplane: Wind shear had little or no effect on the short-period mode, and the long-period mode changed from a lightly damped oscillation to two aperiodic modes, one of which became unstable when $\sigma_u > 1$. A gradient of $u_w' = 0.25\ \text{sec}^{-1}$ was used for the present study. The corresponding values of σ_u were 2.0 for airplane A and 1.7 for airplane B. Table II contains the eigenvalues and stability parameters of airplanes A and B for $u_w' = 0$ (no shear) and $u_w' = 0.25\ \text{sec}^{-1}$. The data presented in table II confirm the results published in reference 4. These results show that for the same gradient, airplane B takes 46 percent more time to double amplitude than does airplane A. Material presented in figures 5 and 6 of reference 4 indicates that the approach speed of an airplane affects its reaction to wind shear. Thus, the longer time required for airplane B to double amplitude is not unexpected, as its still-air approach speed is 10.24 m-sec^{-1} less than that of airplane A.

The wind shear data presented in table II for airplanes A and B are the baseline cases for determining the effects of varying the u derivatives. These cases are labeled "Basic airplane" in tables III and IV.

Effect of Changes in Magnitude of Speed Derivatives on Airplane Response

The speed derivatives $\bar{D}_u$, Z_u, M_u, and T_u have been neglected in the past because these derivatives are small and contribute little to the response of the airplane. Because wind shear can introduce large speed changes, the speed derivatives were varied to determine their effect on airplane response in the presence of wind shear. Four values were used for each derivative: zero, the nominal value, 2 times the nominal value, and 4 times the nominal value. The speed derivatives appear in the first column of equation (1), and because the terms in this column as well as in the first row have little effect on the short-period mode, variations in the speed derivatives should affect only the long-period mode. Calculations showed that there was practically no effect on the short-period mode. Consequently, no short-period mode data are reported.

Increasing $\bar{D}_u$ and T_u above the nominal value caused a slight increase in the time required for airplane A to double amplitude. (See table III.) Setting $\bar{D}_u$ and T_u equal to zero caused only a slight decrease in the time to double amplitude. Variations in M_u and Z_u produced more significant results than did variations in $\bar{D}_u$ and T_u. When the value of M_u was greater than the nominal value, the time to double amplitude increased; and at 4 times the nominal value of M_u, the aperiodic mode turned into an unstable oscillation with a time to double amplitude so large that control of the airplane should not be affected. When Z_u was increased above the nominal value, the aperiodic mode became less stable, an opposite result compared with variations of the other speed derivatives. However, as Z_u became smaller than the nominal value, the airplane became less unstable until at $Z_u = 0$ a stable oscillation with an 80-sec period replaced the aperiodic mode. The process of varying the speed derivatives was repeated for airplane B and the results are presented in table IV. A comparison of the results given in tables III and IV shows that the trends noted for airplane A when the speed derivatives are varied are the same for airplane B, with the exception of the results for M_u. As M_u is increased, the aperiodic mode becomes a long-period stable oscillation; and at 4 times the nominal value of M_u, the oscillation becomes unstable.

The difference in response to the variations of M_u was traced to the signs of coefficients in the characteristic equation. The normalized form of the longitudinal characteristic equation is

$$D^4 + C_3D^3 + C_2D^2 + C_1D + C_0 = 0$$

In the case of airplane A when wind shear is present, both C_0 and C_1 are negative, and increasing M_u causes C_0 to become positive but causes C_1 to become more negative and introduces the unstable oscillation. In the case of airplane B as M_u is increased, C_1 remains positive and a stable oscillation occurs when M_u is twice the nominal value. When this value is doubled, an unstable oscillation with a time to double amplitude of about 42 sec is present. This difference in response to the variations in M_u was traced to the magnitude of $\bar{D}_u$, which is larger for airplane B than for airplane A.

Table V shows the effect on the response of airplane A of increasing M_u when $\bar{D}_u = 0.0844$. In this case the unstable aperiodic response becomes a damped oscillation at 4 times the nominal value of M_u. Further increases in M_u would cause the oscillation to become unstable.

Because M_u and Z_u have so great an influence on the airplane response in wind shear, a further study of these two stability derivatives was made. (See appendix C.) Figures 2 and 3 show the stable and unstable regions of M_u and Z_u as a function of the wind shear parameter σ_u.

The magnitude of the speed stability derivatives, especially M_u and Z_u, has been shown to affect the response of the airplane in wind shear. However, it is not known whether the presence of wind shear influenced the results obtained or not. To obtain information on the influence of wind shear, the speed stability derivatives were varied in the same manner as before but with $\sigma_u = \sigma_w = 0$. The results on the short-period mode were the same as before: there was practically no effect on this mode. The results for the long-period mode of airplane A are presented in table VI. A comparison of tables III and VI clearly indicates that the responses obtained with wind shear present are due to the interaction of the wind shear and the changes in the speed derivatives. Similar results were obtained for airplane B.

Shear Tolerance Factor

The results presented so far show that the magnitudes of M_u and Z_u can have an important effect on the response of the airplane in wind shear. A relationship between these derivatives and M_α and Z_α, called the wind shear tolerance factor S_{TF}, has been developed. (See appendix C.) The shear tolerance factor is dependent only on stability derivatives and is

$$S_{TF} = \frac{M_u Z_\alpha}{M_\alpha Z_u}$$

If $0 < S_{TF} < 1$, the airplane is unstable in wind shear and the degree of instability becomes less as $S_{TF} \rightarrow 1$. If $S_{TF} > 1$, the airplane is stable in wind shear.

The emphasis has been placed on varying the u derivatives, with the most stress placed on M_u and Z_u. As can be seen from the definition of S_{TF}, both M_α and Z_α can be varied to achieve the same result as obtained from varying M_u and Z_u; however, varying M_α and Z_α causes changes to occur in the short-period mode, which is practically unaffected by changes in Z_u and M_u.

The shear tolerance factor and the time to double amplitude are presented in the following table for four airplanes and a wind gradient of 0.25 sec^{-1}:

Airplane	Shear tolerance factor S_{TF}	Time to double amplitude
A	0.266	5.51
C	.481	6.59
B	.538	8.06
D	.957	28.78

Airplanes C and D were included to extend the calculations with the shear tolerance factor. Both are four-engine commercial jet airliners. Airplane C is medium range; D is long range.

Spatial Motions

Typical controls-fixed spatial motions for α, Γ, h, and V for an encounter with a strong wind shear ($\sigma_u = 2.0$) are presented in figure 4. These motions were generated by integrating the longitudinal equations of motion (A13) on a large digital computer. As can be seen, the principal characteristics of the motion are an increasing angle of attack and a decreasing velocity. Since Γ is related to θ and α through $\Gamma = \theta - \alpha$, the pitch angle θ is also decreasing. These component motions combine to cause a rapid loss of altitude, and for the case shown, the airplane would impact the ground before recovery to an equilibrium flight condition could take place.

Control in Wind Shear

The results presented in reference 4 show that two control systems, attitude and airspeed, are required for adequate control during wind shear encounters. This was confirmed by results presented in reference 9. In a wind shear encounter (fig. 4), the angle of attack α increases as the airspeed V decreases. This type of variation of α and V and an inspection of the eigenvectors suggest that airspeed should be controlled. Airspeed may be controlled by adjusting the engine throttles, drag-only speed brakes, wing flaps, and finally spoilers. The foregoing systems may be used alone or in combinations. Reference 10 discusses and illustrates control systems using throttle position, flaps, and drag-only speed brakes. To evaluate the speed control as a means of smoothing the response of the airplane to wind shear, a control system very similar to the analog throttle control system discussed in reference 4 was used. Figure 5 is a block diagram of the airspeed control system used in this study. The results are presented in figure 6 for several different values of the engine time constants τ_E, a constant that characterizes the time delay between calling for a given thrust level and adhering to the desired level of thrust. Three values of the engine time constant were used: 0.125, which would correspond to thrust modulators (ref. 4); 2.50, which is a good approximation for the engines used on airplanes A and B; and 3.75, a larger engine time constant than would normally be encountered. For the smallest value of the engine time constant, the response is the same as one would expect for an undisturbed airplane. As the engine time constant increases, the response becomes more oscillatory and less acceptable, as shown in figure 6. Although the response deteriorates, it does not become aperiodic up to the maximum value used for the engine time constant.

CONCLUDING REMARKS

An analytical study has been made of the longitudinal response of jet transport airplanes to vertical variation of the horizontal winds. The wind variation occurred between altitudes of 106 and 56 meters and changed a head wind into a tail wind (positive wind shear). The flight condition was landing approach.

When an airplane encounters wind shear, the ensuing response is a result of the interaction between the airplane and its environment. This interaction was investigated by varying the so-called u, or speed, stability derivatives. These derivatives were selected for variation because they affect the long-period mode, the mode that becomes unstable in wind shear, and have little effect on the short-period mode. It was found that increasing the pitching-moment derivative M_u or decreasing the vertical-force derivative Z_u could change an unstable flight situation to a stable one. In order for changes in M_u to be effective in restoring stability, it was found that the coefficient of the first-power term of the characteristic equation must be positive. This coefficient can be maintained greater than zero by increases in the drag derivative $\bar{D}_u$. Alternatively, a decrease in Z_u as M_u increases will achieve the same result.

Another result of the investigation of the effects of varying the u stability derivatives was the development of the shear tolerance factor, which is a function of M_u, Z_u, and Z_α, the derivative of vertical force with respect to angle of attack α. If the shear tolerance factor is less than or equal to 1, the airplane will be unstable in positive wind shear; if the shear tolerance factor is greater than 1, the airplane will be stable in positive wind shear. The wind shear tolerance factor can be used to monitor changes in M_u and Z_u and thus assure the attainment of stable conditions.

Control of airplanes in wind is an important factor in wind shear research. Previous studies have reported that a flight-path control system or a pitch-attitude control system, as well as an airspeed control system was necessary for adequate control in wind shear. Results presented in this paper show that adequate control can be achieved through the use of an airspeed control system, provided the engine time constant is small enough. No other control system is required.

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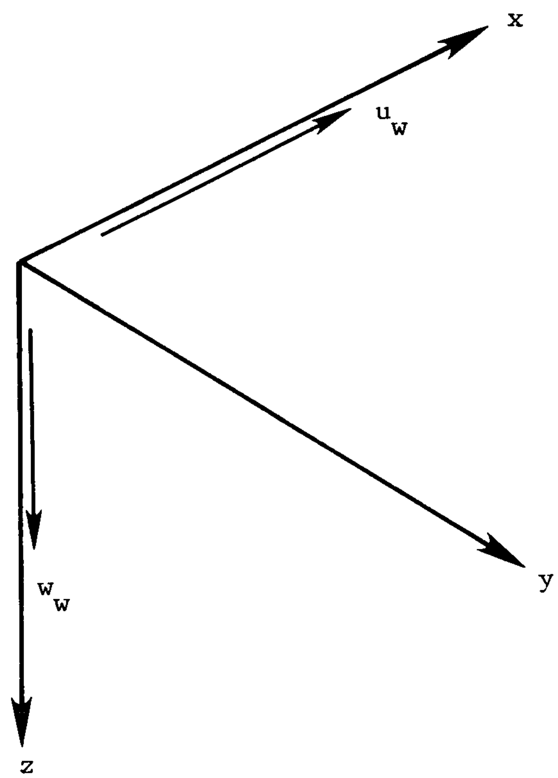
APPENDIX A

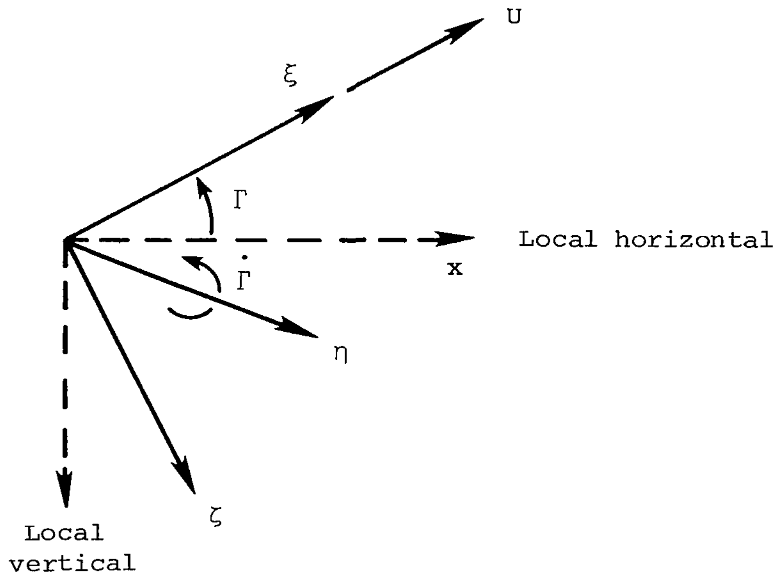
EQUATIONS OF MOTION

The axes systems for the equations derived in this report are shown in sketches (a) and (b).



Sketch (a)

The inertial axes x,y,z are shown in sketch (a); a horizontal wind u_w is flowing in the positive x-direction and a vertical wind w_w is flowing in the positive z-direction. Instead of being referenced to an inertial reference frame, the airplane is referenced to the air-mass axes. The moving axes used for the airplane are the ξ,η,ζ system shown in sketch (b). These are wind as well as stability axes for longitudinal motions, since the ξ-axis is aligned with the single component airplane velocity. As shown in sketch (b), Γ is measured from the local horizontal to U. As was done in references 4 and 9, it is assumed that shear effects in the moment equation are negligible and that $\Gamma = \theta - \alpha$.



Sketch (b)

The inertial acceleration expressed in the moving-axis system ξ, η, ζ is

$$\frac{d}{dt}\left(\vec{V}_A + \vec{V}_W\right) + \vec{\omega}_n \times \left(\vec{V}_A + \vec{V}_W\right) \tag{A1}$$

where $\vec{V}_A = \vec{i}U + \vec{j}(0) + \vec{k}(0)$, $\vec{V}_W = \vec{i}u_W + \vec{j}(0) + \vec{k}w_W$ and $\vec{\omega} = i(0) + j\dot{\Gamma} + k(0)$. The wind velocities u_W and w_W were taken as

$$u_W = u_{W,0} + u_W(z)$$

and

$$w_W = w_{W,0} + w_W(x)$$

There is no loss of generality if $u_{W,0}$ and $w_{W,0}$ are set equal to zero and the wind velocities taken as

$$u_W = u_W(z)$$

and

$$w_W = w_W(x)$$

Thus the wind vector can be written as

$$\vec{V}_W = \vec{i}\,\frac{du_W}{dz}\,z + \vec{k}\,\frac{dw_W}{dx}\,x$$

and the acceleration due to variation in the wind speed is

$$\dot{\vec{V}}_W = u_W'\dot{z} + w_W'\dot{x}$$

where

$$u_W' = \frac{du_W}{dz}$$

and

$$w_W' = \frac{dw_W}{dx}$$

and

$$\dot{z} = U \sin \Gamma + w_W$$

$$\dot{x} = U \cos \Gamma + u_W$$

If the definitions of $\vec{V}_A$, $\vec{\omega}_n$, and $\vec{V}_W$ are substituted into the expression for the acceleration (eq. (A1)), the nonlinear equations can be written as

$$\left.\begin{aligned}
&\dot{U} + u_w' w_w \cos \Gamma - u_w' U \sin \Gamma \cos \Gamma - w_w' U \cos \Gamma \sin \Gamma \\
&\qquad - w_w' u_w' \sin \Gamma + g \sin \Gamma = \frac{F_X}{m} \\
&u_w' w_w \sin \Gamma - u_w' U \sin^2 \Gamma + w_w' U \cos^2 \Gamma + w_w' u_w' \cos \Gamma \\
&\qquad\qquad - \dot{\Gamma} U - g \cos \Gamma = \frac{F_Z}{m} \\
&\ddot{\theta} = \frac{M}{I_Y}
\end{aligned}\right\} \quad \text{(A2)}$$

The following assumptions were used in the linearization of equations (A2):

$$\Gamma = \Gamma_0 + \gamma$$

$$U = U_0 + u$$

$$\gamma = \theta - \alpha$$

Here, θ and α represent either the total angle of attack and pitch angle or perturbations from a trim condition. The latter use occurs only in linear equations such as (A8).

The wind shear parameter was also introduced. This parameter is a dimensionless acceleration whose magnitude is proportional to the wind gradient. The parameter $\sigma_u = \frac{U_0 u_w'}{g}$, and $\sigma_w = \frac{U_0 w_w'}{g}$. A value greater than 1 for σ_u or σ_w would represent a severe wind shear, as unstable roots appear in the long-period mode. The linearization process given in reference 8 was followed for the linearization or equations (A2). The resulting linear equations of motion are[1]

[1]The variables x and z that appear in the definition of u_w and w_w transform into ξ and ζ; and since $x_0 = z_{0=0}$, both ξ_0 and ζ_0 are equal to zero.

$$\left.\begin{aligned}
&\left(\frac{d}{dt} - \frac{g\sigma_T}{2U_0}\sin 2\Gamma_0\right)u + g\left(\cos\Gamma_0 - \sigma_T\cos 2\Gamma_0\right)\gamma + \frac{g^2\sigma_u\sigma_w}{U_0{}^2}\xi = \frac{F_X}{m}\\
&\left[-\frac{g}{U_0}\left(\sigma_u\sin^2\Gamma_0 - \sigma_w\cos^2\Gamma_0\right)\right]u + \left[-U_0\frac{d}{dt} + g\left(\sin\Gamma_0 - \sigma_T\sin^2\Gamma_0\right)\right]\gamma\\
&\qquad + \frac{g^2\sigma_u\sigma_w}{U_0{}^2}\xi = \frac{F_Z}{m}\\
&\frac{d^2}{dt^2}(\gamma + \alpha) = \frac{M}{I_Y}
\end{aligned}\right\} \tag{A3}$$

The steady-state parts of equations (A3) are

$$\left.\begin{aligned}
&-g\sigma_T\sin\Gamma_0\cos\Gamma_0 + g\sin\Gamma_0 = \frac{F_{X,0}}{m}\\
&-g\sigma_T\sin^2\Gamma_0 - \sigma_w - g\cos\Gamma_0 = \frac{F_{Z,0}}{m}\\
&0 = \frac{M_0}{I_Y}
\end{aligned}\right\} \tag{A4}$$

These last equations are used to compute the trim values of α and the thrust.

Equations (A3) and (A4) are complete except for the definition of aerodynamic forces and moments. In developing these forces and moments it should be remembered that they are functions of the speed of the airplane with respect to the air mass, so that $U = U_0 + u$. Here U_0 is the still-air approach speed, and u is the small speed deviation from U_0. The lift force will be worked out in detail. Before substitution of $U_0 + u$ for U, the force F_Z has the form

$$\left(C_{L,o} + C_{L_\alpha}\alpha + C_{L_{\dot\alpha}}\dot\alpha + C_{L_q}q + C_{L_{\delta_e}}\delta_e\right)\frac{\rho S U^2}{2}$$

The terms in parentheses represent the lift coefficient of the airplane and, except for $C_{L,o}$ (the lift coefficient at $\alpha = 0$), represent the effect of angular motions on the lift. The effect of speed changes, since it is contained in the dynamic pressure. When $U_0 + u$ is substituted for U and $\alpha_{tr} + \alpha$ is substituted for α, the expression for lift becomes

$$\left[C_{L,o} + C_{L_\alpha}(\alpha_{tr} + \alpha) + C_{L_{\dot{\alpha}}}\dot{\alpha} + C_{L_q}q + C_{L_{\delta_e}}\delta_e\right]\frac{\rho S(U_0^2 + 2U_0u + u^2)}{2}$$

where u^2 is considered a second-order term and, therefore, can be neglected.

The term $\left(C_{L,o} + C_{L_\alpha}\alpha_{tr}\right)\frac{\rho SU_0^2}{2}$ is the lift required to trim the airplane, or $F_{Z,0}$ in equations (A4), and $\left(C_{L_\alpha}\alpha + C_{L_{\dot{\alpha}}}\dot{\alpha} + C_{L_q}q + C_{L_{\delta_e}}\delta_e\right)\frac{\rho SU_0^2}{2}$ is the lift due to angular motions from the trim state. The effect of a change in forward speed is given by $C_L\rho SU_0u$. Thus, the total lift force F_Z for equations (A3) is

$$F_Z = -\left(C_L\rho SU_0u + C_{L_\alpha}\alpha\frac{\rho SU_0^2}{2} + C_{L_{\dot{\alpha}}}\dot{\alpha}\frac{\rho SU_0^2}{2} + C_{L_q}q\frac{\rho SU_0^2}{2} + C_{L_{\delta_e}}\delta_e\frac{\rho SU_0^2}{2}\right) \tag{A5}$$

The longitudinal aerodynamic forces and the pitching moment can be treated in a similar manner. Thus,

$$F_X = -\left(C_{\bar{D}}\rho SU_0 + C_{\bar{D}_\alpha}\alpha\frac{\rho SU_0^2}{2} + C_{\bar{D}_{\delta_e}}\delta_e\frac{\rho SU_0^2}{2}\right) + T_uu \tag{A6}$$

where

$$C_{\bar{D}} = \left(C_{\bar{D},o} + \frac{C_{L,tr}^2}{\pi A} + C_{\bar{D}_\alpha}\alpha + C_{\bar{D}_{\delta_e}}\delta_e\right)$$

and T_u is the change in engine thrust due to a change of forward speed. The aerodynamic pitching moment is given by

$$M = \left(C_m \rho S \bar{c} U_0 u + C_{m_\alpha} \alpha \frac{\rho S \bar{c} U_0{}^2}{2} + C_{m_{\dot{\alpha}}} \dot{\alpha} \frac{\rho S \bar{c} U_0{}^2}{2} + C_{m_q} q \frac{\rho S \bar{c} U_0{}^2}{2}\right) \tag{A7}$$

where

$$C_m = \left[C_{m,o} + \left(C_{m_\alpha}\right)_{tr} \alpha_{tr} + C_{m_\alpha}\alpha + C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_q} q + C_{m_{\delta_e}} \delta_e\right]$$

and $C_{m,o}$ is the pitching-moment coefficient for $\alpha = 0$. Equations (A5), (A6), and (A8) are substituted into equations (A3) to obtain the linearized equations of motion that are used for stability studies. Note that in this substitution $u = \frac{d\xi}{dt}$, $w = \frac{d\zeta}{dt}$, and $\alpha = \frac{1}{U_0}\frac{d\zeta}{dt}$; the equations then take the form

$$\begin{bmatrix} D^2 + a_1 D + a_2 & a_3 D & a_4 \\ a_5 D & a_6 D^2 + a_7 D + a_8 & a_9 D + a_{10} \\ a_{11} D & a_{12} D^3 + a_{13} D^2 + a_{14} D & D^2 + a_{15} D \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \\ \gamma \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \\ b_5 & b_6 \end{bmatrix} \begin{bmatrix} \delta_e & \delta_T \end{bmatrix} \tag{A8}$$

where

$$a_1 = -\left(\frac{g\sigma_T}{2U_0} \sin 2\Gamma_0 - \frac{C_{\bar{D}} \rho S U_0}{m} + \frac{C_{T_u} \rho S U_0}{m}\right) \tag{A9a}$$

$$a_2 = \frac{g^2 \sigma_u \sigma_w}{U_0{}^2} \tag{A9b}$$

$$a_3 = \frac{C_{\bar{D}_\alpha} \rho S U_0{}^2}{2mU_0} \tag{A9c}$$

$$a_4 = g(\cos \Gamma_0 - \sigma_T \cos 2\Gamma_0) \tag{A9d}$$

$$a_5 = -\left[\frac{g}{U_0}\left(\sigma_T \sin^2 \Gamma_0 - \sigma_w\right) - C_{L_u} \frac{\rho S U_0}{m}\right] \tag{A9e}$$

$$a_6 = \left(C_{L_q} + C_{L_{\dot{\alpha}}}\right)\frac{\rho S U_0{}^2}{2mU_0} \tag{A9f}$$

$$a_7 = C_{L_\alpha} \frac{\rho S U_0{}^2}{2mU_0} \tag{A9g}$$

$$a_8 = a_2 \tag{A9h}$$

$$a_9 = -\left(U_0 - C_{L_q} \frac{\rho S U_0{}^2}{2m}\right) \tag{A9i}$$

$$a_{10} = g(\sin \Gamma_0 - \sigma_T \sin 2\Gamma_0) \tag{A9j}$$

$$a_{11} = -\frac{C_{m_u} \rho S \bar{c} U_0}{I_Y} \tag{A9k}$$

$$a_{12} = \frac{1}{U_0} \tag{A9l}$$

$$a_{13} = -\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\frac{\rho S \bar{c} U_0{}^2}{2I_Y U_0} \tag{A9m}$$

$$a_{14} = -C_{m_\alpha} \frac{\rho S \bar{c} U_0{}^2}{2I_Y U_0} \tag{A9n}$$

$$a_{15} = -C_{m_q} \frac{\rho S \bar{c} U_0{}^2}{2I_Y U_0} \tag{A9o}$$

$$b_1 = -C_{\bar{D}_{\delta e}} \frac{\rho S U_0{}^2}{2m} \tag{A9p}$$

$$b_2 = C_{T_{\delta_T}} \frac{\rho S U_0{}^2}{2m} \tag{A9q}$$

$$b_3 = - C_{L_{\delta_e}} \frac{\rho S U_0{}^2}{2m} \tag{A9r}$$

$$b_4 = 0 \tag{A9s}$$

$$b_5 = C_{m_{\delta_e}} \frac{\rho S \bar{c} U_0{}^2}{2 I_Y} \tag{A9t}$$

$$b_6 = C_{m_{\delta_T}} \frac{\rho S \bar{c} U_0{}^2}{2 I_Y} \tag{A9u}$$

If $\sigma_u = \sigma_w = 0$, these equations reduce to the usual form of the linear longitudinal equations of motion.

The characteristic equation of the system described by equations (A8) is obtained by expanding the determinant of the 3 × 3 matrix that occurs on the left side of this equation. The general form of the characteristic equation is

$$C_6 D^6 + C_5 D^5 + C_4 D^4 + C_3 D^3 + C_2 D^2 + C_1 D + C_0 = 0 \tag{A10}$$

The constant term C_0 is equal to 0, so that equation (A10) is a sixth-degree equation with a zero root. Equation (A10) was written

$$C_6 D^5 + C_5 D^4 + C_4 D^3 + C_3 D^2 + C_1 = 0 \tag{A11}$$

for programming purposes. The expressions for the coefficients of the characteristic equation are

$$C_6 = a_6 - a_{12} a_9 \tag{A12a}$$

$$C_5 = a_1 (a_6 - a_{12} a_9) + (a_7 + a_{15} a_6 - a_{13} a_9 - a_{10} a_{12}) \tag{A12b}$$

$$C_4 = a_1(a_7 + a_{15}a_6 - a_{13}a_9 - a_{10}a_{12}) + (a_8 + a_{15}a_7 - a_{14}a_9 - a_{13}a_{10}) + a_2(a_6 - a_{12}a_9) - a_3a_5 + a_4a_5a_{12} \tag{A12c}$$

$$C_3 = a_1(a_8 + a_{15}a_7 - a_{14}a_9 - a_{13}a_{10}) + (a_{15}a_8 - a_{14}a_{10}) + a_2(a_7 + a_{15}a_6 - a_{13}a_9 - a_{10}a_{12}) - a_3(a_{15}a_5 - a_{11}a_9) + a_4(a_{13}a_5 - a_{11}a_6) \tag{A12d}$$

$$C_2 = a_1(a_{15}a_8 - a_{14}a_{10}) + a_2(a_8 + a_{15}a_7 - a_{14}a_9 - a_{13}a_{10}) + a_3a_{11}a_{10} + a_4(a_5a_{14} - a_{11}a_7) \tag{A12e}$$

$$C_1 = a_2(a_{15}a_8 - a_{14}a_{10}) - a_4a_{11}a_8 \tag{A12f}$$

$$C_0 = 0 \tag{A12g}$$

If σ_u and/or $\sigma_w = 0$, the C_1 coefficient becomes 0, and the characteristic equation reduces to a quartic. In this case the C_2 coefficient corresponds to the C_0 coefficient obtained from equations (1).

An HP-67 program that calculates the coefficients of the 3 × 3 matrix (eqs. (A8)) and the coefficients of the characteristic equation (eqs. (A10) and (A11)) is given in appendix B.

The equations of motion presented in this appendix model the condition in which an airplane is subjected to a combination of vertical wind shear, updrafts, and downdrafts and contain wind interaction terms $\frac{g^2\sigma_u\sigma_w\xi}{U_0^2}$ and $\frac{g^2\sigma_u\sigma_w\zeta}{U_0^2}$ not present in the previous formulation of this problem. If either σ_u or σ_w is set to zero, these equations reduce to equations given in reference 4 when the same parameter is set equal to zero. If the wind interaction terms are neglected and σ_u and σ_w are not equal to zero, the resulting equations will predict a larger instability (a shorter time to double amplitude) than equations (A8).

Equations (A8) are approximate rather than exact equations. The inexactness occurs because of the assumption that $\Gamma = \theta - \alpha$. The angle Γ, defined in sketch (b), is not the traditional Γ because it is not inertial. The traditional Γ, which will be called Γ_I, can be obtained by applying the correction $\Gamma_I = \Gamma - \varepsilon_e$, where ε_e is given by

$$\varepsilon_e = \tan^{-1}\left(\frac{-u_w \sin \Gamma - w_w \cos \Gamma}{U - u_w \cos \Gamma + w_w \sin \Gamma}\right)$$

Since ε_e can be considered a small angle, it can be set equal to the argument of the inverse tangent. This correction and its first and second derivatives would be applied to equations (A3). However, equations (A3) were used with w_w equal to zero; thus, the correction would be

$$\frac{u_w \sin \Gamma}{U - u_w \cos \Gamma}$$

which approaches zero as Γ approaches zero. The corrections for the first and second derivatives are proportional to $\frac{\Gamma}{U^2}$ and are smaller corrections than ε_e. Because the corrections and Γ are small, no corrections were applied when equations (A8) were used to calculate the stability parameters. If updrafts and downdrafts had been included in the analysis, corrections to Γ would have been necessary. Equations (A8) were not used to compute time histories.

The six-degree-of-freedom equations of motion for principal body axes are

$$m\left(\dot{\vec{V}}_B + \vec{\omega} \times \vec{V}_B\right) = \vec{F}$$

$$\dot{\vec{H}} + \vec{\omega} \times \vec{H} = \vec{M}$$

where $\vec{H} = I_{XX}p + I_{YY}q + I_{ZZ}r$; $\vec{F}$ and $\vec{M}$ are total force and moment vectors; p, q, r are the angular velocities; and I_{ii} are the products of inertia. These equations may be separated into lateral and longitudinal equations of motion; the latter are

$$\left.\begin{aligned} \dot{u} + qw &= -g \sin \theta + \frac{F_X}{m} \\ \dot{w} - qu &= g \cos \theta + \frac{F_Z}{m} \\ \ddot{\theta} &= M/I_Y \end{aligned}\right\} \tag{A13}$$

where u, w, q, and θ are total values, not perturbations and F_X, F_Z, and M are the total aerodynamic forces and moments acting on the airplane. Equations (A13) were used to compute the time histories presented in this report. Wind shear was accounted for by making the wind component of the airspeed vector altitude dependent.

APPENDIX B

PROGRAM FOR CALCULATING THE CHARACTERISTIC EQUATION OF LONGITUDINAL MOTION

This program calculates the coefficients of the 3×3 matrix in equations (A8) using the expressions given in equations (A9). After the calculation of these coefficients has been completed, equations (A12) are used to calculate the coefficients of the characteristic equation (A10) under certain circumstances. The characteristic equation may be fifth order, so that a routine to extract the real root of a quintic is included in the program. Programs 4 and 5 of reference 6 can be used to complete the calculation of the roots of the characteristic equation and to determine the stability parameters.

The two-card program that follows is an improved version of program 1 of reference 6. This program may be checked by using the checks case for program 1 of reference 6.

APPENDIX B

Card 1

Step	Key Entry	Comments	Step	Key Entry	Comments
001	LBLA			RCL5	
	RCL8			x^2	
	STO+7			RCLC	
	STO×9		050	÷	
	RCL0			π	
	x^2			÷	
	CHS			STO+4	
	STO÷(i)			RCL2	
	RCL7			STO+3	
010	RCL6			RCL8	
	SIN			STO+7	
	x^2			RCLE	
	×			STO×4	
	RCL8		060	STO×5	
	−			RCLA	
	STO0			×	
	RCL6			STO×9	
	COS			RCLB	
	STO8			STO×(i)	
020	+			STO×0	
	RCL4			STO×1	
	×			STO×2	
	RCLB			STO×3	
	RCL2		070	RCLA	
	×			STO×6	
	RCL5			STO×7	
	×			STO×8	
	RCL1			P⇌S	
	÷			RCL8	
030	STOE			RCL6	
	RCL5			2	
	×			×	
	2			COS	
	÷		080	RCL7	
	STO×3			STO×8	
	STOB			×	
	÷	C_L		−	
	P⇌S			STOA	
	STO5			RCL6	
040	RCLD			SIN	
	−			STO×8	
	RCL1			RCL8	
	÷	α_{tr}		2	
	RCL6		090	×	
	×			−	
	STO+9			RCL4	

APPENDIX B

Step	Key Entry	Comments	Step	Key Entry	Comments
	STO×(i)			RCL1	
	×			RCL8	
	STOB			RCL3	
	RCL4			×	
	RCL5			+	
	÷			RCL2	
	STO×8			RCL7	
100	STO×0		150	×	
	STO×9			−	
	RCL9			RCLB	
	×			P⇄S	
	STOC			RCL6	
	RCL3			P⇄S	
	RCL8			×	
	+			−	
	RCL0			STOE	
	RCL5			+	
110	P⇄S		160	STO(i)	C5
	CHS			ISZ	
	STO+2			RCLD	
	CHS			RCLC	
	STO÷0			×	
	STO÷1			RCLE	
	STO÷3			RCL4	
	STO÷6			×	
	STO÷7			+	
	1/x			RCL0	
120	P⇄S		170	RCL5	
	STO6			×	
	P⇄S			−	
	R↓			RCLA	
	STO−5			RCL5	
	R↓			P⇄S	
	STO−4			RCL6	
	1			P⇄S	
	0			×	
	STOI			×	
130	RCL3		180	+	
	P⇄S			RCLC	
	RCL6			RCL8	
	P⇄S			RCL1	
	RCL2			×	
	×			+	
	−			RCL6	
	STOD			RCL2	
	P⇄S			×	
	STO5	C6		−	
140	P⇄S		190	RCL7	
	RCL4			RCLB	
	×			×	

APPENDIX B

Step	Key Entry	Comments
	-	
	STOD	
	+	
	STO(i)	C4
	RTN	

APPENDIX B

Storage Map for Card 1

(i) Address	Register	Initial storage	Output storage
0	R_0	k_y	b_5
1	R_1	m	b_4
2	R_2	ρ	
3	R_3	C_{T_u}	
4	R_4	g	
5	R_5	U_0	b_6
6	R_6	Γ_0	
7	R_7	σ_u	
8	R_8	σ_w	
9	R_9	σ_u	
10	S_0	$C_{\bar{D}_\alpha}$	a_3
11	S_1	C_{L_α}	a_7
12	S_2	$C_{L_{\dot{\theta}}}$	a_9
13	S_3	$C_{L_{\dot{\alpha}}}$	a_6
14	S_4	$C_{\bar{D},o}$	a_1
15	S_5	0	a_5
16	S_6	C_{m_α}	a_{14}
17	S_7	$C_{m_{\dot{\alpha}}}$	a_{13}
18	S_8	$C_{m_{\dot{\theta}}}$	a_{15}
19	S_9	$C_{m,o}$	a_{11}
20	R_A	$\bar{c}$	a_4
21	R_B	S	a_{10}
22	R_C	A	a_2, a_8
23	R_D	$C_{L,o}$	
24	R_E	0	
25	I	20	a_{12}

APPENDIX B

Card 2

Step	Key Entry	Comments
001	LBLA	
	ISZ	
	RCLC	
	RCLE	
	×	
	RCLD	
	RCL4	
	×	
	+	
010	RCL8	
	RCLC	
	×	
	RCL6	
	RCLB	
	×	
	−	
	STOE	
	+	
	RCL8	
020	RCL5	
	×	
	RCL9	
	RCL2	
	×	
	−	
	RCL0	
	×	
	−	
	RCL7	
030	RCL5	
	×	
	RCL9	
	RCL3	
	×	
	−	
	RCLA	
	×	
	+	
	STO(i)	C_3
040	ISZ	
	RCLD	
	RCLC	
	×	
	RCLE	
	RCL4	
	×	

Step	Key Entry	Comments
	+	
	RCL0	
	RCL9	
050	RCLB	
	×	
	×	
	+	
	RCL5	
	RCL6	
	×	
	RCL9	
	RCL1	
	×	
060	−	
	RCLA	
	×	
	+	
	STO(i)	C_2
	RCLC	
	RCLE	
	×	
	RCLA	
	RCL9	
070	×	
	RCLC	
	×	
	−	
	P→S	
	STO4	C_1
	RCL5	
	STO÷0	
	STO÷1	
	STO÷2	
080	STO÷3	
	STO÷4	
	RCL4	
	x=0	
	GOTOC	
	GOTOD	
	LBLC	
	4	
	RTN	
	LBLD	
090	5	
	RTN	
	LBLB	

APPENDIX B

Step	Key Entry	Comments	Step	Key Entry	Comments
	RCL0			×	
	STOA			-	
	RCL1			STO6	
	STOB			GOTO0	
	RCL2			LBL1	
	STOC			RCL7	
	RCL3			.	
100	STOD		150	6	
	RCL4			9	
	STOE			3	
	0			CHS	
	STO7			RCL6	
	FIX			÷	
	RCLA			RCL6	
	5			5	
	÷			RTN	
	.			LBLb	
110	0		160	2	
	8			0	
	-			STOI	
	STO5			1	
	STO6			GSBc	
	GSBb			STO0	
	STO8			GSBc	
	.			STO1	
	1			GSBc	
	6			STO2	
120	STO+6		170	GSBc	
	LBL0			STO3	
	1			GSBc	
	STO+7			STO4	
	GSBb			RTN	
	STO9			LBLc	
	RND			RCL6	
	Pause			×	
	x=0			RCL(i)	
	GOTO1			+	
130	RCL6		180	ISZ	
	RCL5			RTN	
	RCL6				
	STO5				
	x↔y				
	-				
	RCL8				
	RCL9				
	STO8				
	x↔y				
140	-				
	÷				
	RCL8				

APPENDIX B

Storage Map for Card 2

(i) Address	Register	Initial storage	Output LBLA[a]	Output LBLB
0	R_0	The initial	C_5	d_3
1	R_1	storage for	C_4	d_2
2	R_2	card 2 LBLA	C_3	d_1
3	R_3	is the	C_2	d_0
4	R_4	output of	C_1	
5	R_5	card 1.	C_6	
6	R_6			
7	R_7			
8	R_8			
9	R_9			
10	S_0		a_3	
11	S_1		a_7	
12	S_2		a_9	
13	S_3		a_6	
14	S_4		a_1	
15	S_5		a_5	
16	S_6		a_{14}	
17	S_7		a_{13}	
18	S_8		a_{15}	
19	S_9		a_{11}	
20	R_A		a_4	
21	R_B		a_{10}	
22	R_C		a_2, a_8	
23	R_D			
24	R_E		a_{12}	
25	I			

[a]At end of label A, the numeral 4 or 5 is displayed in x; 5 indicates a quintic and 4 a quartic. If 5 is displayed, push B; if 4, insert quartic program and proceed.

Note: In addition to the stored output, the stack contains the following information:

Stack register	
T	number of iterations to obtain root
z	time to halve or double amplitude
y	root of quintic equation
x	5 indicates that a real root of a fifth-order equation has been found

EXPRESSIONS FOR $C_0 = 0$

The nonnormalized characteristic equation of longitudinal motion is

$$C_4D^4 + C_3D^3 + C_2D^2 + C_1D + C_0 = 0 \tag{C1}$$

When wind shear is introduced in the airplane stability and control problem, the coefficients affected most are C_1 and C_0. The coefficient C_0 is the dominant one. For negative wind shear, the airplane remains stable; however, for positive wind shear, the airplane becomes unstable when the wind shear parameter σ_u exceeds 1. (See ref. 4.) The expression for C_0 (when $\sigma_w = 0$) is

$$a_{10}(a_3a_{11} - a_1a_{14}) + a_4(a_5a_{14} - a_7a_{11}) = C_0 \tag{C2}$$

If C_0 is set equal to zero, equation (C2) may be solved for Z_u and M_u. The expression for Z_u that satisfies $C_0 = 0$ is

$$Z_u = \frac{a_4a_7a_{11} - a_{10}(a_3a_{11} - a_1a_{14})}{a_4a_{11}} + \frac{g\sigma_u}{U_0}\sin^2 \Gamma_0 \tag{C3}$$

and a similar expression can be obtained for M_u. Equation (C3) was used to make figure 2.

If equation (C2) is solved for the ratio $\frac{a_{11}}{a_{14}} = \frac{C_{m_\alpha}}{C_{m_{\dot\alpha}}}$, the following expression results:

$$\frac{a_{11}}{a_{14}} = \frac{a_1 - \frac{a_4}{a_{10}}a_5}{a_3 - \frac{a_4}{a_{10}}a_7} \tag{C4}$$

Equation (C4) may be written in the form

$$\frac{a_{11}}{a_{14}}\left(\frac{a_3 - \dfrac{a_4}{a_{10}} a_7}{a_1 - \dfrac{a_4}{a_{10}} a_5}\right) = 1 \tag{C5}$$

If the equality of equation (C5) is fulfilled, $C_0 = 0$. If the left-hand side of equation (C5) is less than zero, unstable conditions exist; if greater than zero, the airplane is stable. The matrix elements a_1 and a_3 may be a negligible contribution to the term in parentheses of equation (C5). If a_1 and a_3 are dropped, the ratio $\dfrac{a_4}{a_{10}}$ in the parenthetical term cancels, and equation (C5) may be written as

$$\frac{a_{11}}{a_{14}} \frac{a_7}{a_5} \approx 1 \tag{C6}$$

If a_1 and a_3 are retained, the value in parentheses for airplane A is 0.00466; if dropped, the value is 0.00463. This small difference justifies dropping a_1 and a_3. Except for a_5, all the a-terms in equation (C6) are functions solely of the stability derivatives. Wind shear is thus introduced through the inclusion of the term $-\dfrac{g}{U_0} \sigma_T \sin^2 \Gamma_0 - \sigma_w$ in Z_u. For reasonable values of Γ_0 (that is, $-0.17453 \leqq \Gamma_0 \leqq 0.17453$) and $\sigma_w = 0$, the expression $-\dfrac{g}{U_0}\left(\sigma_T \sin^2 \Gamma_0 - \sigma_w\right)$ will make a negligible contribution to a_5 because of $\sin^2 \Gamma_0$ in the expression. For airplane A with $\sigma_w = 0$, $\Gamma_0 = -0.05236$ rad, and $\sigma_u = 2.0$, this term has a value of -0.0007; and since Z_u is 0.2553665, the wind shear term may be safely dropped when $\sigma_w = 0$. If σ_w is nonzero, the wind shear term for σ_w is $\sigma_w g/U_0$ and, unless σ_w is very small, may not be neglectable with respect to Z_u. The term $\dfrac{a_{11}}{a_{14}} \dfrac{a_7}{a_5}$ is called the wind shear tolerance factor S_{TF} and may be written as

$$\left.\begin{aligned} &\frac{M_u Z_\alpha}{M_\alpha a_5} \approx 1 \\ \text{or} \\ &\frac{M_u Z_\alpha}{M_\alpha Z_u} \approx 1 \qquad (\sigma_w = 0) \end{aligned}\right\} \tag{C7}$$

The interpretation of the wind shear tolerance factor is discussed in the text.

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TABLE I.- PHYSICAL AND AERODYNAMIC CHARACTERISTICS

FOR AIRPLANES USED IN THIS STUDY

[A constant air density of 1.2929 $kg\text{-}m^{-3}$ was used for all calculations]

Characteristic	Airplane A	Airplane B
k_y, m	10.463784	5.38
$\bar{c}$, m	7.0104	3.41
S, m^2	267.1	91.04
A	7.03	8.83
m, kg	90909.1	40916.87
g, $m\text{-}sec^2$	9.80665	9.80665
σ_u	0 to 2.0	0 to 1.7049
σ_w	0	0
U_0, $m\text{-}sec^{-1}$	77.12	66.88
Γ, rad	-0.05236	-0.05236
$C_{\bar{D}\alpha}$, rad^{-1}	0.529	0.9168
$C_{L\alpha}$, rad^{-1}	4.87	6.59
$C_{L\dot{\theta}}$	0.283	0.2208
$C_{L\dot{\alpha}}$, $rad^{-1}\text{-}sec$ . . .	0.0889	0.20893
$C_{\bar{D},o}$	0.038	0.13778
$C_{m\dot{\alpha}}$, $rad^{-1}\text{-}sec$ . . .	-0.241	-0.0656
$C_{m\dot{\theta}}$	-0.707	-0.6884
$C_{m\alpha}$, rad^{-1}	-1.115	-1.369
$C_{m,o}$	-0.015	-0.14
C_{T_u}, $m^{-1}\text{-}sec$	-0.00025	-0.0055409
$C_{L\delta_e}$, rad^{-1}	0.23302	0.46413
$C_{\bar{D}\delta_e}$, rad^{-1}	0.01387	-----------
$C_{m\delta_e}$	-1.017	-1.656

TABLE II.- RESPONSE OF AIRPLANES A AND B TO WIND SHEAR

[Γ_0 = 0.0524 rad]

Mode	Root	$t_{1/2}$, sec	t_D, sec	P, sec	ω_n, rad-sec^{-1}	ζ_D	Remarks
			Airplane A, U_0 = 77.12 m-sec^{-1}				
Short-period	-0.6990 ± 0.8061i	0.991	----	7.79	1.07	0.66	No wind shear
Long-period	-0.00648 ± 0.1291i	106.97	----	48.66	.129	.050	No wind shear
Short-period	-0.6933 ± 0.7939i	.995	----	7.92	1.054	.66	u_W' = 0.25 sec^{-1}
Long-period	-0.1373, 0.1258	-------	5.51	-----	-----	-----	u_W' = 0.25 sec^{-1}
			Airplane B, U_0 = 66.88 m-sec^{-1}				
Short-period	-0.6082 ± 0.9882i	1.14	----	6.36	1.16	0.52	No wind shear
Long-period	-0.0298 ± 0.1202i	23.33	----	52.36	.12	.24	No wind shear
Short-period	-0.6074 ± 0.9721i	1.14	----	6.46	1.15	.53	u_W' = 0.25 sec^{-1}
Long-period	-0.1352, 0.0863	------	8.06	-----	----	----	u_W' = 0.25 sec^{-1}

TABLE III.- EFFECT OF VARYING THE u-STABILITY DERIVATIVES ON THE

LONG-PERIOD MODE OF AIRPLANE A

[U_0 = 77.12 m-sec^{-1}; u_w' = 0.25 sec^{-1}; Γ_0 = -0.0524]

Parameter varied	Root	$t_{1/2}$, sec	t_D, sec	P, sec	ω_n, rad-sec^{-1}	ζ_D	Remarks
Effect of varying the magnitude of $\bar{D}_u$							
$\bar{D}_u$ = 0	-0.1253, 0.1354	------	5.12	------	------	------	
$\bar{D}_u$ = 0.0212	-0.1373, 0.1258	------	5.51	------	------	------	Basic airplane
$\bar{D}_u$ = 0.0424	-0.1502, 0.1171	------	5.92	------	------	------	
$\bar{D}_u$ = 0.0848	-0.1784, 0.1021	------	6.79	------	------	------	
Effect of varying the magnitude of T_u							
T_u = 0	-0.1356, 0.1271	------	5.45	------	------	------	
T_u = -0.00281	-0.1373, 0.1258	------	5.51	------	------	------	Basic airplane
T_u = -0.00562	-0.1389, 0.1246	------	5.56	------	------	------	
T_u = -0.01124	-0.1423, 0.1223	------	5.67	------	------	------	
Effect of varying the magnitude of M_u							
M_u = 0	-0.1666, 0.1425	------	4.86	------	------	------	
M_u = 0.000997	-0.1373, 0.1258	------	5.51	------	------	------	Basic airplane
M_u = 0.00199	-0.1040, 0.1050	------	6.60	------	------	------	
M_u = 0.00399	0.0125 ± 0.0356i	------	55.35	176.67	0.0377	-0.31	
Effect of varying the magnitude of Z_u							
Z_u = 0	-0.00639 ± 0.0782i	108.42	-----	80.34	0.0785	0.0815	
Z_u = 0.255	-0.1373, 0.1258	------	5.51	------	------	------	Basic airplane
Z_u = 0.502	-0.2103, 0.1970	------	3.52	------	------	------	
Z_u = 1.021	-0.3186, 0.2959	------	2.34	------	------	------	

TABLE IV.- EFFECT OF VARYING THE u-STABILITY DERIVATIVES ON

LONG-PERIOD MODE OF AIRPLANE B

[U_0 = 66.88 m-sec^{-1}; u_w' = 0.25 sec^{-1}; Γ_0 = -0.0524 rad]

Parameter varied	Root	$t_{1/2}$, sec	t_D, sec	P, sec	ω_n, rad-sec^{-1}	ζ_D	Remarks
Effect of varying the magnitude of $\bar{D}_u$							
$\bar{D}_u$ = 0	-0.1090, 0.1023	-----	6.79	------	------	------	
$\bar{D}_u$ = 0.0427	-0.1352, 0.0863	-----	8.06	------	------	------	Basic airplane
$\bar{D}_u$ = 0.0955	-0.1662, 0.0740	-----	9.36	------	------	------	
$\bar{D}_u$ = 0.1709	-0.2365, 0.0573	-----	12.16	------	------	------	
Effect of varying the magnitude of T_u							
T_u = 0	-0.1122, 0.0993	-----	7.00	------	------	------	
T_u = -0.0356	-0.1352, 0.0863	-----	8.06	------	------	------	Basic airplane
T_u = -0.0713	-0.1608, 0.0758	-----	9.12	------	------	------	
T_u = -0.1426	-0.2182, 0.0607	-----	11.36	------	------	------	
Effect of varying the magnitude of M_u							
M_u = 0	-0.2012, 0.1236	-----	5.59	------	------	------	
M_u = 0.0039	-0.1352, 0.0863	-----	8.06	------	------	------	Basic airplane
M_u = 0.0078	-0.0104 ± 0.0321i	66.63	-----	195.74	0.034	0.308	
M_u = 0.1425	0.0166 ± 0.1590i	-----	41.75	39.52	0.160	-.1038	
Effect of varying the magnitude of Z_u							
Z_u = 0	-0.0247 ± 0.1034i	27.72	-----	254.38	0.106	0.232	
Z_u = 0.2942	-0.1352, 0.0863	-----	8.06	------	------	------	Basic airplane
Z_u = 0.5884	-0.2155, 0.1660	-----	4.17	------	------	------	
Z_u = 1.1769	-0.3245, 0.2704	-----	2.57	------	------	------	

TABLE V.- LONGITUDINAL RESPONSE, AIRPLANE A

[$\bar{D}_u = 0.0844$]

Parameter varied	Root	$t_{1/2}$, sec	t_D, sec	P, sec	ω_n, rad-sec^{-1}	ζ_D	Remarks
Effect of varying the magnitude of M_u							
$M_u = 0$	-0.2083, 0.1787	-----	5.84	------	------	-----	
$M_u = 0.000997$	-0.1784, 0.1021	-----	6.79	------	------	-----	
$M_u = 0.00199$	-0.1949, 0.0817	-----	8.59	------	------	-----	
$M_u = 0.00399$	-0.0189 ± 0.0134i	36.59	----	467.17	0.0232	0.815	

TABLE VI.- EFFECT OF VARYING THE u-STABILITY DERIVATIVES ON

LONG-PERIOD MODE OF AIRPLANE A

[U_0 = 77.12 m-sec^{-1}; u_w' = 0.0 sec^{-1}; Γ_0 = -0.0524 rad]

Parameter varied	Root	$t_{1/2}$, sec	t_D, sec	P, sec	ω_n, rad-sec^{-1}	ζ_D	Remarks
Effect of varying the magnitude of $\bar{D}_u$							
$\bar{D}_u$ = 0	-0.0039 ± 0.1288i	------	-177.17	48.76	0.1289	-0.0303	
$\bar{D}_u$ = 0.0211	-0.0065 ± 0.1291i	106.97	-------	48.66	.1293	.050	Basic airplane
$\bar{D}_u$ = 0.0422	-0.0169 ± 0.1296i	41.09	-------	48.88	.1296	.1301	
$\bar{D}_u$ = 0.0844	-0.0376 ± 0.1248i	18.41	-------	50.33	.1304	.289	
Effect of varying the magnitude of T_u							
T_u = 0	-0.00510 ± 0.1291i	135.96	-------	48.66	0.1292	0.039	
T_u = -0.00281	-0.00648 ± 0.1291i	106.97	-------	48.66	.1293	.050	Basic airplane
T_u = -0.00562	-0.00786 ± 0.1291i	88.17	-------	48.67	.1293	.061	
T_u = -0.01124	-0.0106 ± 0.1290i	65.24	-------	48.71	.1294	.082	
Effect of varying the magnitude of M_u							
M_u = 0	-0.00543 ± 0.1507i	127.57	-------	41.71	0.1508	0.036	
M_u = 0.000977	-0.00648 ± 0.1291i	106.97	-------	48.66	.1293	.050	Basic airplane
M_u = 0.00195	-0.00749 ± 0.1033i	92.55	-------	60.81	.1036	.072	
M_u = 0.00391	-0.0443, 0.0255	------	27.18	-----	------	-----	
Effect of varying the magnitude of Z_u							
Z_u = 0	-0.0938, -0.0631	------	10.98	-----	------	------	
Z_u = 0.2540	-0.00648 ± 0.1291i	106.97	-------	48.66	.1293	0.050	Basic airplane
Z_u = 0.5079	0.00211 ± 0.1969i	------	328.68	31.91	.1969	-.011	
Z_u = 1.016	0.0185 ± 0.2893i	------	37.45	22.10	.2850	-.064	

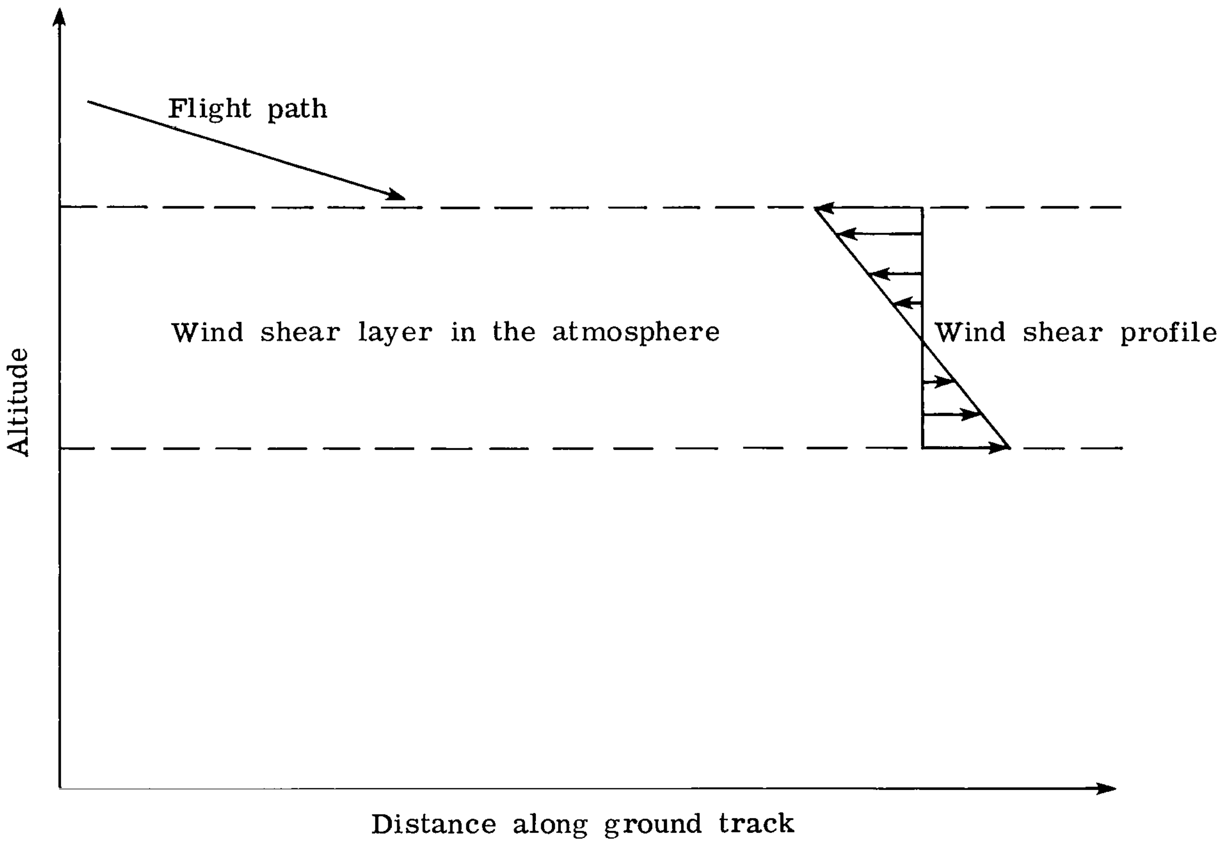
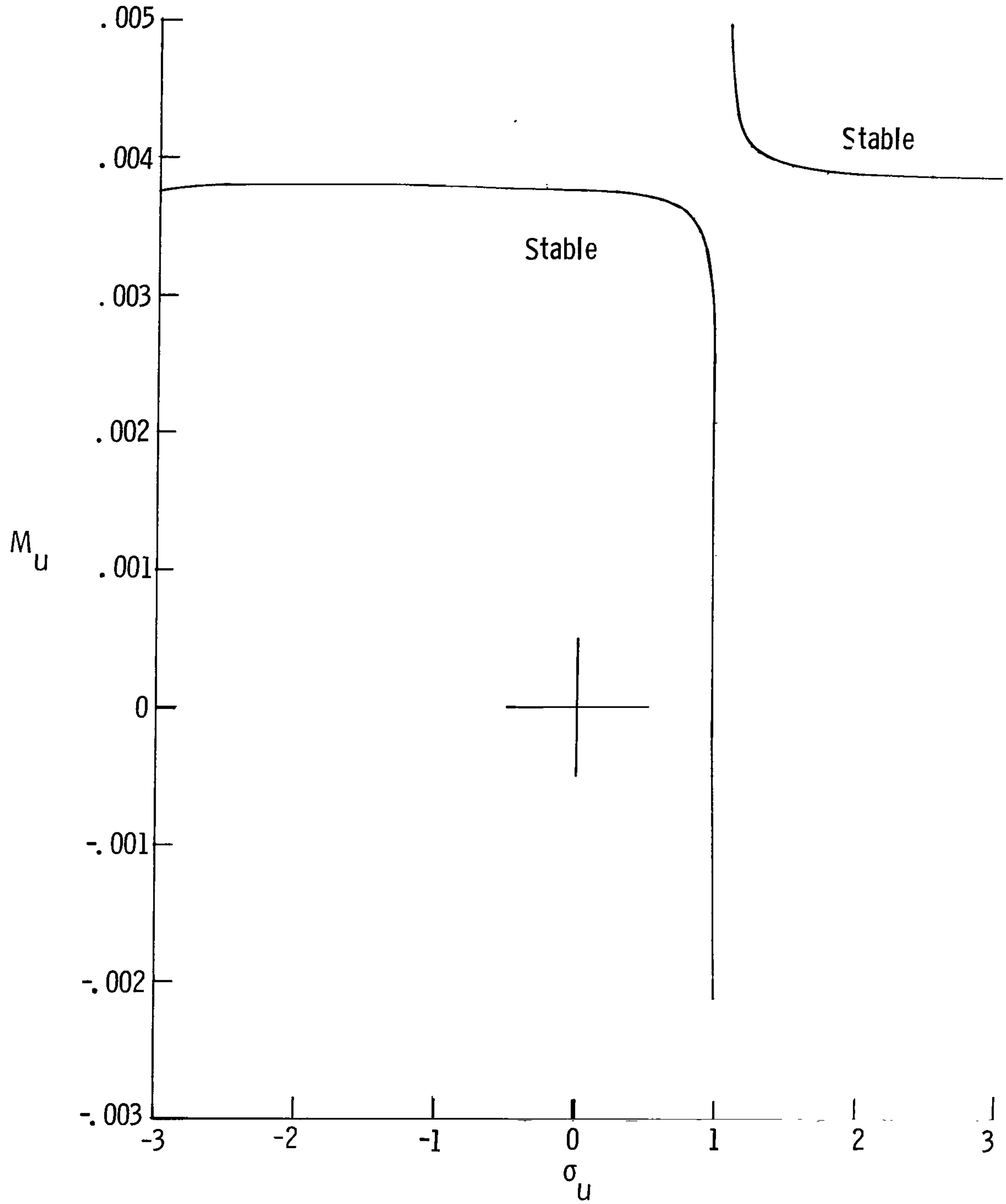
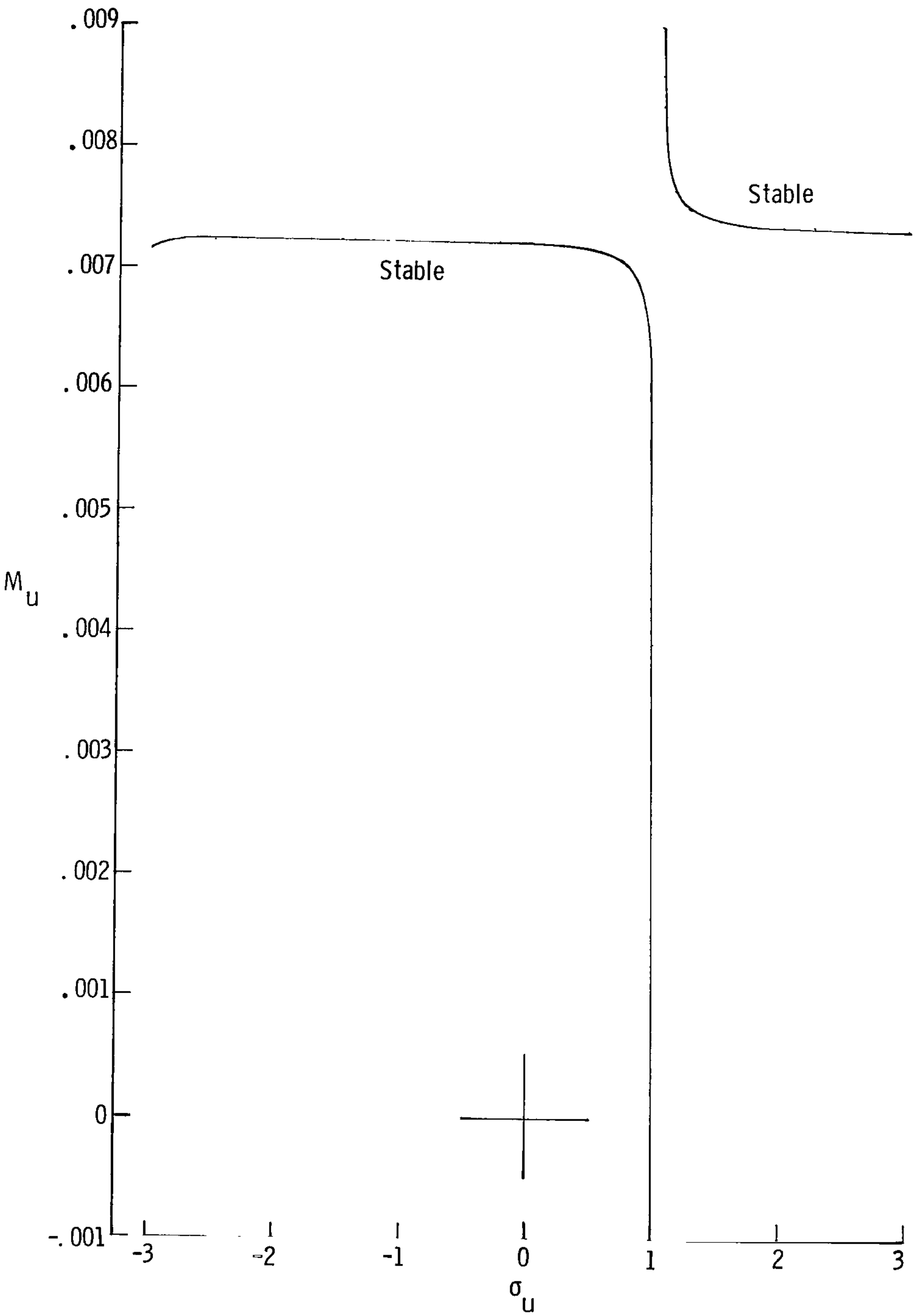


Figure 1.- Wind shear.



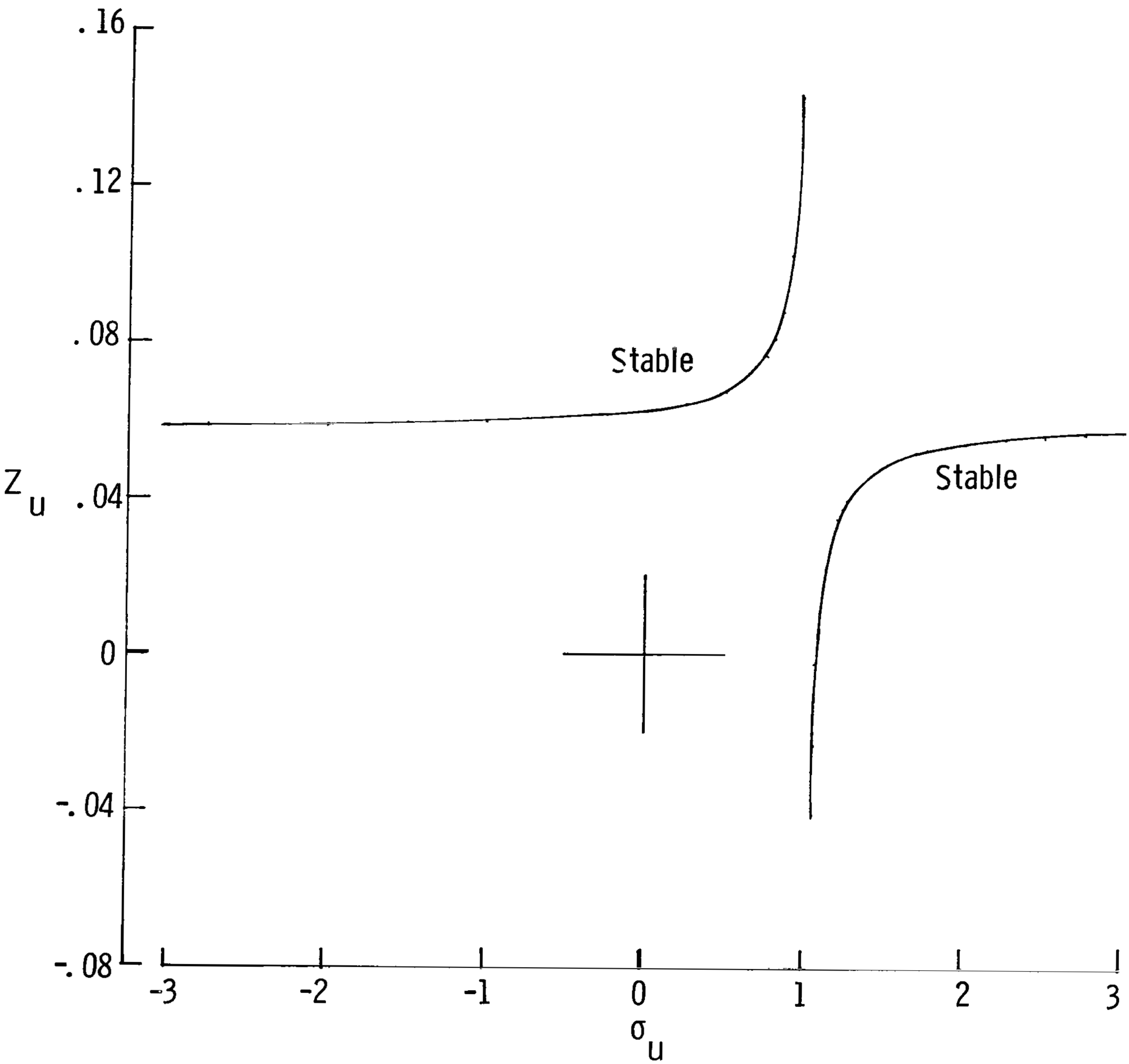
(a) Airplane A.

Figure 2.- Curves for $C_0 = 0$ for airplanes A and B as a function of σ_u and M_u.



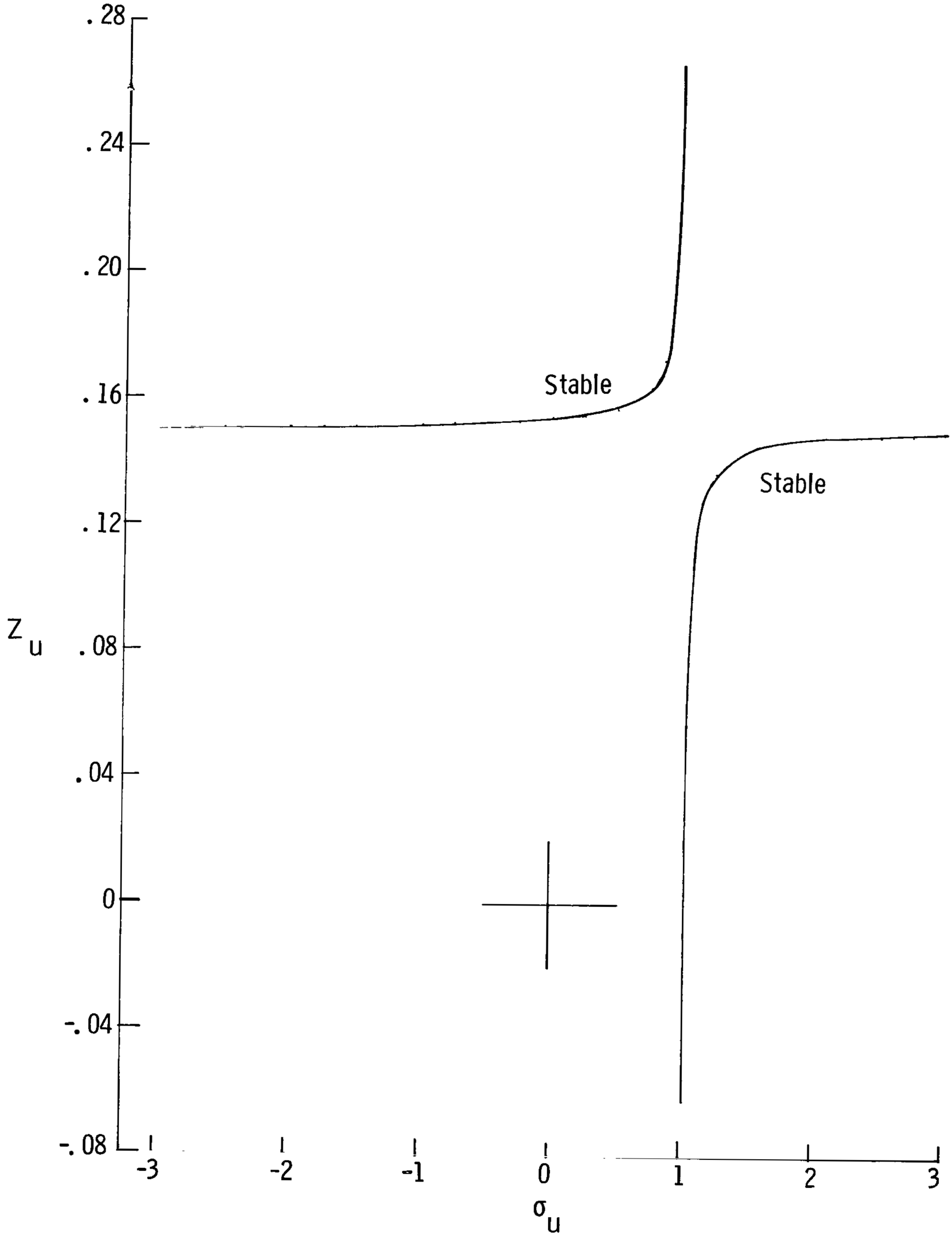
(b) Airplane B.

Figure 2.- Concluded.



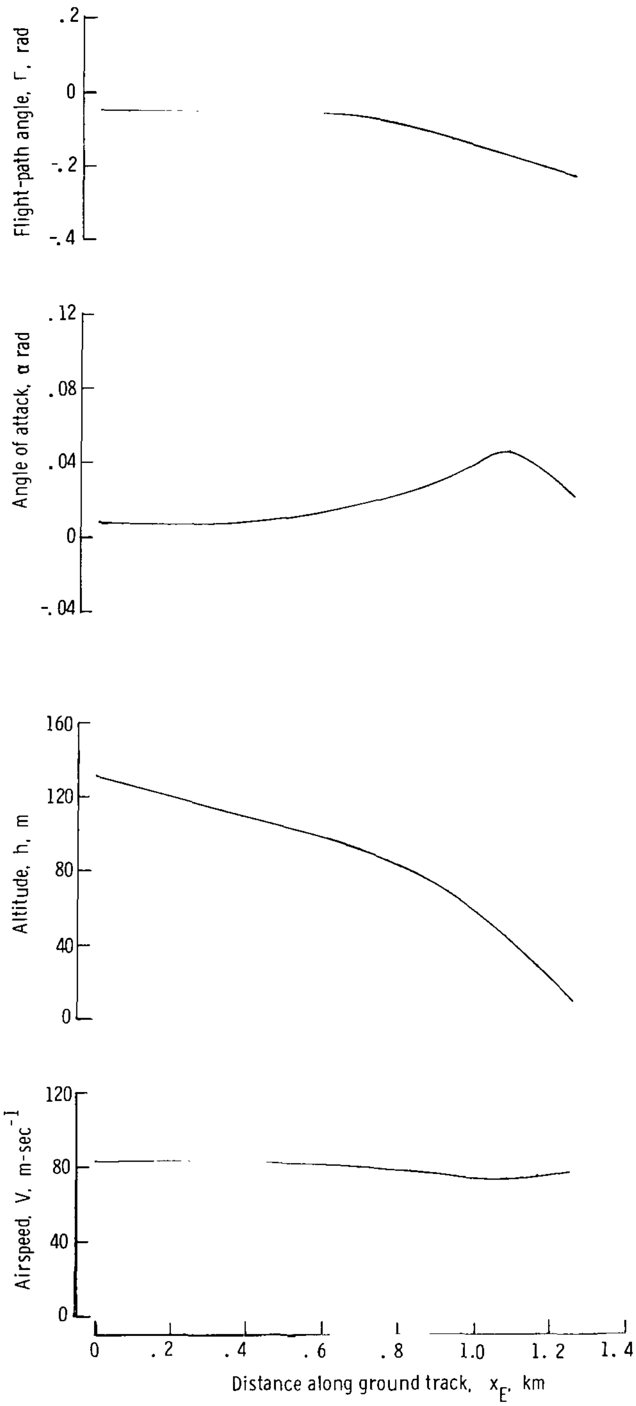
(a) Airplane A.

Figure 3.- Curves for $C_0 = 0$ for airplanes A and B as a function of σ_u and Z_u.



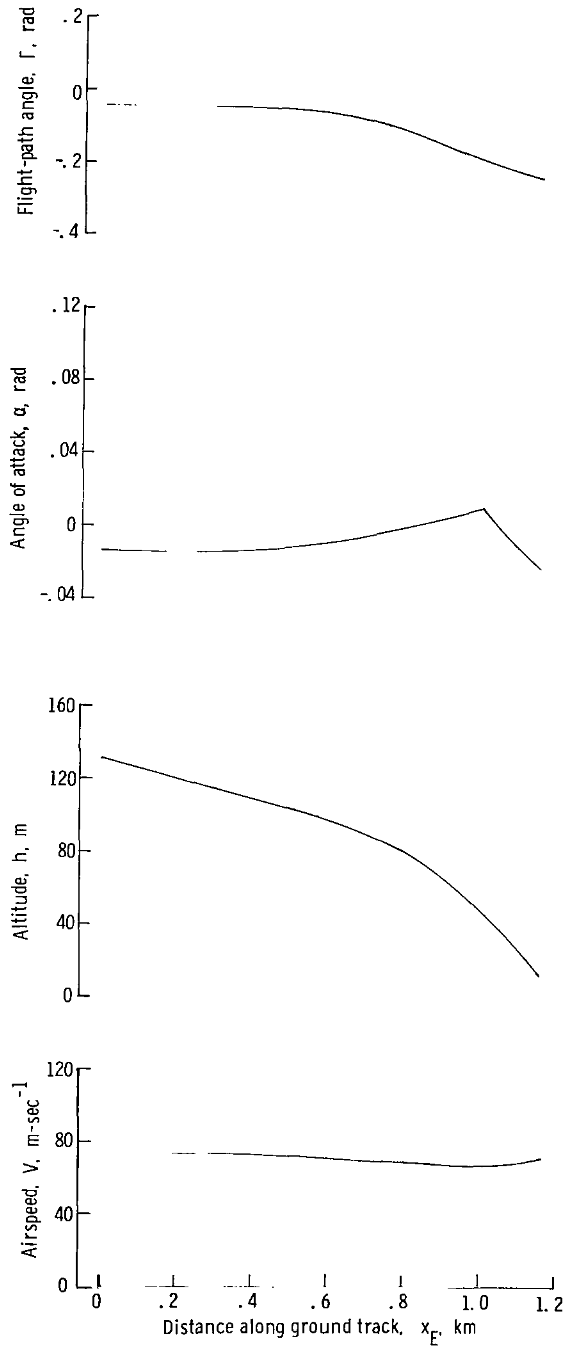
(b) Airplane B.

Figure 3.- Concluded.



(a) Airplane A.

Figure 4.- Response of airplane to wind shear with winds of -6.10 m-sec^{-1}. Gradient $u_W' = 0.25$ sec^{-1}.



(b) Airplane B.

Figure 4.- Concluded.

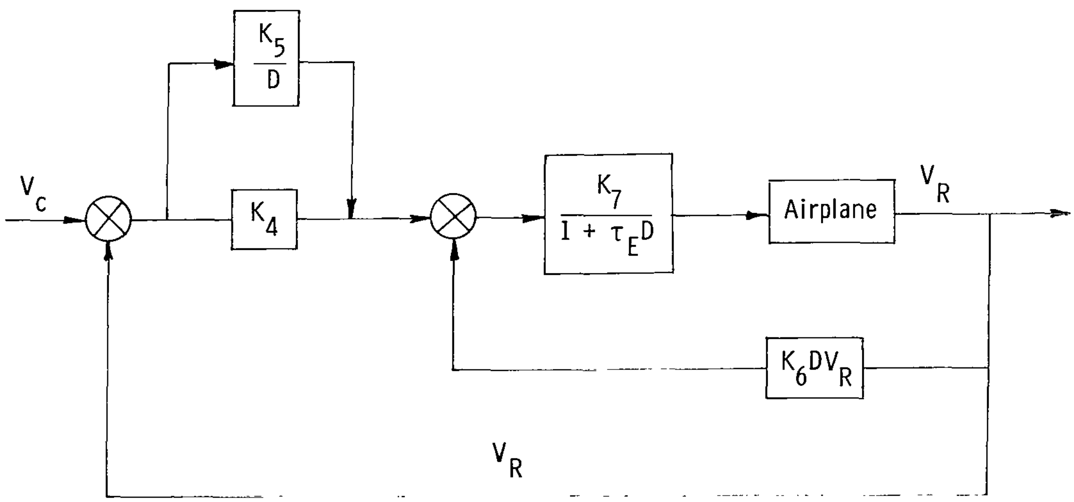


Figure 5.- Block diagram of airspeed control system.

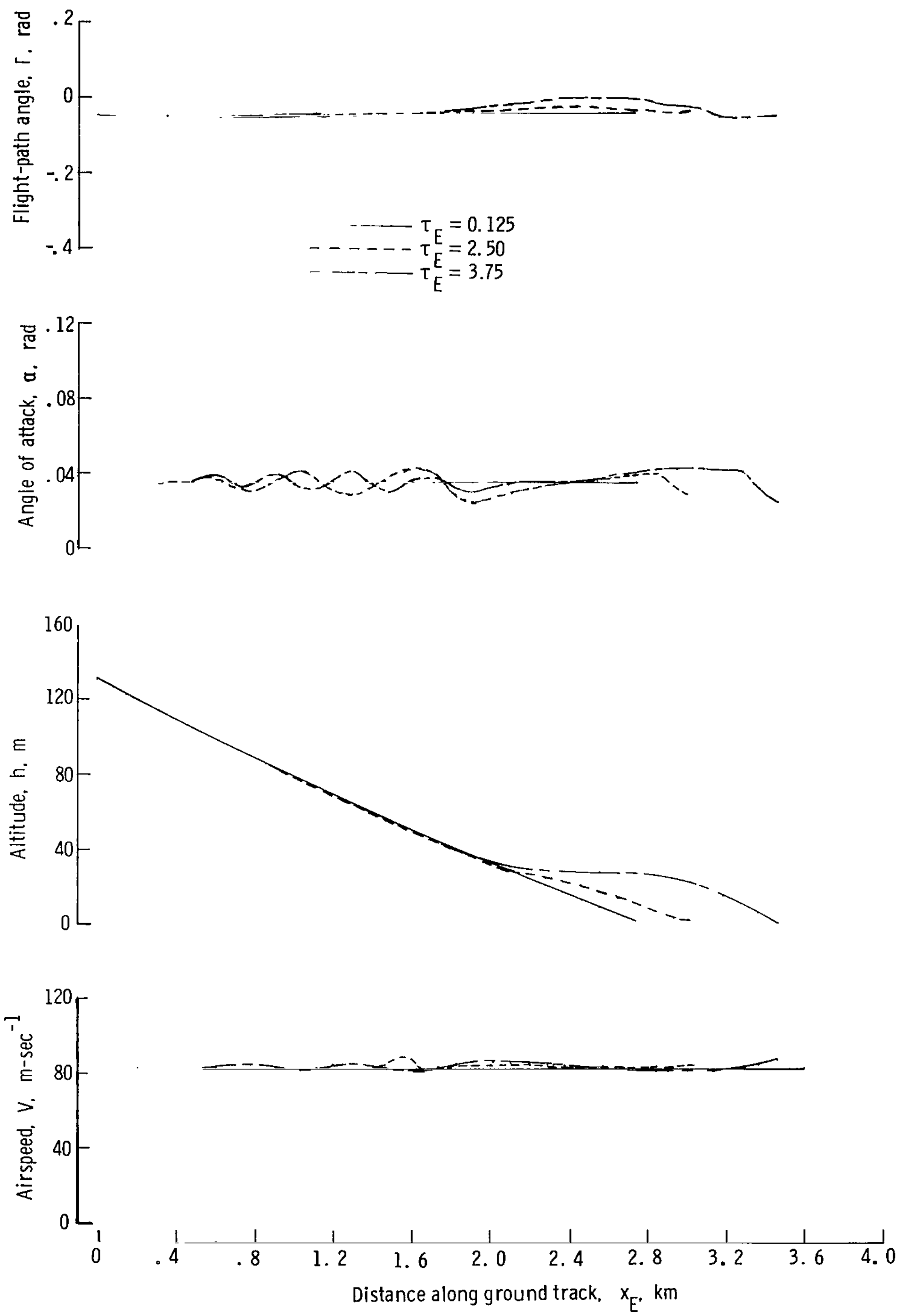


Figure 6.- Airplane responses when equipped with speed control system. Engine time constant varied; airplane A.

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16. Abstract

Wind shear, the vertical variation of the horizontal wind, has been a causative factor in several airplane accidents and may have been a contributing factor in others. The study reported herein concentrates on the longitudinal mode of the airplane with specific reference to the role of the speed (u) stability derivatives in the interaction of the airplane and its environment. The relative importance of the u-stability derivatives was determined. As a result of this study, the wind shear tolerance factor was found. This factor can be used to determine, in a qualitative manner, the stability (tolerance) of an airplane to wind shear. A further study of the control problem showed that the criteria for good control could be reduced from two to one automatic control systems. Only a speed control system is necessary for good control in wind shear.

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