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SOME ASPECTS OF THE COMPARISON BETWEEN OPTICS AND NONLINEAR ACOUSTICS

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16. Abstract

This paper is devoted to some results concerning nonlinear acoustics deduced from a comparison of nonlinear processes between optics and acoustics. In the first part an aspect of nonlinearity in acoustics connected with the dimensionality of the medium of propagation is emphasized and illustrated by the proof of static instability of an ideal linear solid. In the second part a new phenomenon, which can be called acoustical rectification by analogy with nonlinear optics, is propounded to measure the third order elastic constants and its experimental consequences are predicted in a particular case.

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SOME ASPECTS OF THE COMPARISON BETWEEN OPTICS AND NONLINEAR ACOUSTICS

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1. Introduction /C8-216*

The comparison of nonlinear processes between optics and acoustics permits, on the one hand, broadening the analysis of the properties which determine the essential make-up of nonlinear behavior in these two domains, and on the other hand, exhaustively recording phenomena capable of procuring the measurement of nonlinear characteristics of the propagation medium.

In this paper, confined to a solid medium, there are several consequences, concerning nonlinear acoustics, that can be deduced from such a step:

- The identification of a nonlinear acoustic source connected with the dimensionality of the medium of propagation.

- The proposition of utilizing a new phenomenon (which can be called acoustical rectification, by analogy with nonlinear optics) to measure third order elastic constants.

2. Sources of Nonlinearity

Concerning optics, nonlinearity is totally taken into account by the relation which expresses the electric polarization of a dielectric medium while functioning in the electromagnetic field which produces it. The Maxwell equations are not intrinsically responsible for any nonlinearity and without approximation lead to a linear propagation of electromagnetic fields in a vacuum.

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* Numbers in the margin indicate pagination in the foreign text.

The situation is more complex concerning acoustics where the linear description is still an approximation. Nonlinearity does not result uniquely from the equation of state which permits the expression of anharmonicity of interatomic potentials but also from the constituent relations with acoustics which are intrinsically nonlinear. This last source of nonlinearity is essentially related to the dimensionality of the medium of propagation; this is shown by the introduction of various nonlinear terms:

- Thus, whereas for unidimensional media a satisfactory representation of the state of deformation is realized by the gradient of displacement u_{ij}, for higher media of dimensionality, one must use the Lagrange deformations n_{ij} which are nonlinear expressions of components u_{ij}:

$$n_{ij} = \frac{1}{2} (u_{ij} + u_{ji} + u_{ki} u_{kj}) \qquad (1)$$

($u_{ij} = \frac{\partial u_i}{\partial a_j}$; a_j is a Lagrangian coordinate

- In the same way, various Jacobians introduced to take into consideration the volume mass, and the deformation of the system of coordinates . . . are not simplified in non-unidimensional media. /C8-217

This source of nonlinearity provokes the appearance, in the parameters which permit the description of nonlinear phenomena, of second order elastic constants alongside those of the third order, which represent the anharmonicity of the interatomic potential. Even though the third order constants may often be higher than those of the second order in order of magnitude, the presence of second order constants, and consequently the dimensionality, plays a major role in nonlinear developments; this can be illustrated by studying an ideal linear crystal for which the different nonlinearities would compensate each other (various authors [1,2] have suggested such a definition).

We will show, in effect, that although in a unidimensional case a perfectly linear (or harmonic) model can be conceived, the case is not the same for larger dimensionalities:

The equation of propagation in solids is written:

$$\rho_a \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial a_j}\left[J_{ik} \frac{\partial \phi}{\partial \eta_{jk}}\right] \quad (2)$$

ρ_a and ϕ are respectively the volume mass and deformation energy per unit of volume of nondeformed crystal.

J: Jacobian $J_{ik} = u_{ik} + \delta_{ik}$ (3)

$$\frac{\partial \phi}{\partial \eta_{jk}} = C_{jkmn} \eta_{mn} + \frac{1}{2} C_{jkmnrs} \eta_{mn} \eta_{rs} + \cdots \quad (4)$$

C_{jkmn} and C_{jkmnrs} are, respectively, Brugger's second and third order elastic constants.

If the ideal linear crystal is such that there is no interaction nor distortion of the elastic waves in the course of propagation, the following equation can be written:

$$\frac{\partial}{\partial a_j}\left[J_{ik} \frac{\partial \phi}{\partial \eta_{jk}}\right] = \Gamma_{ijkl} \frac{\partial^2 u_k}{\partial a_j \partial a_l} \quad (5)$$

From this, one deduces:

$$\Gamma_{ijkl} = \frac{1}{2} (C_{ijkl} + C_{ilkj}) \quad (6)$$

and

$$-\frac{1}{2} (C_{ijklmn} + C_{ilkjmn}) = C_{jlmn}\delta_{ik} + \frac{1}{2} (C_{jnkl} + C_{lnkj})\delta_{im} + \frac{1}{2} (C_{ijnl} + C_{ilnj})\delta_{km} \quad (7)$$

By writing out the various symmetries which are verified by third order constants, severe restrictions are obtained concerning those of the second order. Thus, the symmetry by permutation of indices m and n leads to Table 1 of second order elastic constants (one can show that other symmetries do not provide other relations). Such a solid has an elastic hyperisotrope behavior concerning propagation (i.e.: there is a triple degeneration of the modes), and in addition presents a static instability since the module, whether by compressibility or by shear is negative. This result also agrees with that which one obtains on a microscopic level when one imagines a perfectly harmonic crystal [3,4] (i.e.: a crystal for which the development of the interatomic potential does not interfere with the parameters of connection of an order two times higher).

3. Acoustical Rectification

Even though the analysis of the causes of nonlinearity may produce a greater complexity in acoustics than in optics, macroscopic manifestations should be analogous to them. Effects of the second order can be summed up for the most part as an interaction of two fields of radiation of frequencies ω_1 and ω_2, which creates a new field of frequency ω_3. A first rule of selection dictates:

$$\omega_3 = \omega_1 + \omega_2 \tag{8}$$

It suffices to take several values for ω_1 and ω_2 to obtain second order nonlinear phenomena (Table 2). Thus, for frequencies ω_1 and ω_2 equal, respectively, to ω and to $-\omega$, one obtains the phenomenon called rectification. /C8-218

Rectification is expressed in optics as the appearance of a continuous polarization while passing an intense laser beam through several crystals; this effect is sometimes called a dc effect.

In acoustics the tensor σ of Cauchy stress (or stress deformed surfaces) is written:

$$\sigma = \frac{1}{\det J} \; J \frac{\partial \phi}{\partial \eta} t_J \qquad (9)$$

One can deduce from (1,4,9) a nonlinear relation between σ_{ij} and u_{ij}:

$$\sigma_{ij} = C_{ijkl} u_{kl} + \frac{1}{2} A_{ijklmn} u_{kl} u_{mn} \qquad (10)$$

Table 1

SECOND ORDER ELASTIC CONSTANTS
OF AN "IDEAL LINEAR CRYSTAL"

$$\begin{bmatrix} c_{11} & -c_{11} & -c_{11} & 0 & 0 & 0 \\ -c_{11} & c_{11} & -c_{11} & 0 & 0 & 0 \\ -c_{11} & -c_{11} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{11} \end{bmatrix}$$

Table 2

SOME NONLINEAR EFFECTS
OF THE SECOND ORDER

ω_1	ω_2	ω_3	
ω	ω	2ω	Generation of second harmonic
ω	0	ω	Influence of a static field on the propagation of a radiation field with frequency ω
ω	$-\omega$	0	Rectification

A_{ijklmn} is a linear combination of second and third order elastic constants.

If a deformation field of fundamental frequency ω is propagated, we may write ($\varepsilon \ll 1$):

$$u_{kl} = \varepsilon\, \xi_{kl}\,(\omega)e^{i\omega t} + \xi_{kl}(-\omega)\, e^{-i\omega t} + \varepsilon^2\, \xi_{kl}\,(0) + \varepsilon^2 \xi_{kl}(2\omega)\, e^{2i\omega t} + \varepsilon^2 \xi_{kl}(-2\omega)\, e^{-2i\omega t} + 0\,(\varepsilon^2) \tag{11}$$

thus, $\sigma(o)$ being the static component of the spectrum of stresses:

$$\sigma_{ij}(o) = \varepsilon^2 \left[C_{ijkl}\xi_{kl}(o) + A_{ijklmn}\xi_{kl}\,(\omega)\xi_{mn}\,(-\omega) \right] + 0(\varepsilon^2) \tag{12}$$

We call acoustical rectification the effect which associates static stress $\left[\varepsilon^2\, A_{ijklmn}\, \xi_{kl}\,(\omega)\, \xi_{mn}\,(-\omega)\right]$ with a deformation $\varepsilon\xi(\omega)$.

Remarks

1. This effect is very close to the phenomenon of pressure of acoustical radiation originally foreseen by Rayleigh [5]; its interpretation was somewhat erroneous. It was not until around 1920 that Brillouin [6] clarified this phenomenon, in a remarkable way, by the following two aspects:

- the pressure of radiation has a tensorial nature.

- there are two distinct contributions to this effect:

· on the one hand, distortion of the wave during propagation,

· on the other hand, the flux of the quantity of movement across the immobile surface upon which the observation is made.

In addition, Brillouin is the only person, to our knowledge, to have considered the case of solids.

2. The term acoustic rectification has already been used by Mathur and Sagoo [7] to describe a phenomenon which is very different from the one which we present, and is more connected to that of autofocalization: they have foreseen and observed that an ultrasonic beam whose profile is not initially uniform can, under certain conditions, make itself uniform in the course of propagation. The term rectification used in this case thus has a very different meaning from the same term used in relation to optics or electronics. /C8-219

Experimental consequences of acoustical rectification essentially depend on the conditions of the ends of the solid. We will now consider that static stresses are zero on the surfaces of a solid (or at least remain unchanged by the application of an ultrasonic field). Consequently, the relation (12) provides for the appearance of a static deformation of the solid when an ultrasonic wave is passed through it:

$$\varepsilon^2 \xi_{ij}(o) = -\varepsilon^2 K_{ijkl} A_{klmnrs} \xi_{mn} \xi(\omega)_{\xi rs} (-\omega) + 0 (\varepsilon^2) \qquad (13)$$

K_{ijkl} represents the tensor of elastic compliances.

If it is possible to measure the deformation of the solid which results from rectification, one can thus determine several combinations of third order elastic constants [4].

Example

Cubic crystals of the highest symmetry possess six third order elastic constants. If one considers the propagation of a longitudinal wave A cos $(t - \frac{a}{v})$ following an axis of order 4, the relation (13) becomes:

$$\varepsilon^2\xi_{11}(0) = -\frac{A^2\omega^2}{4V^2}$$

$$\frac{3C_{11}(C_{11}+C_{12}) + 2C_{12}^2 + (C_{11}+C_{12})C_{111} - 2C_{12}C_{112}}{(C_{11}-C_{12})(C_{11}+2C_{12})} \tag{14}$$

$$\varepsilon^2\xi_{22}(0) = \varepsilon^2\xi_{33}(0)$$

$$= -\frac{A^2\omega^2}{4V^2}\,\frac{-4C_{11}C_{12} + (C_{11}C_{112} - C_{12}C_{111})}{(C_{11}-C_{12})(C_{11}+2C_{12})} \tag{15}$$

$$\xi_{12}(0)+\xi_{21}(0) = \xi_{23}(0)+\xi_{32}(0) = \xi_{13}(0)+\xi_{31}(0) = 0 \tag{16}$$

Detection of static deformations can be facilitated by using an ultrasonic wave modulated in impulse $A\left(t - \frac{a}{V}\right)\cos\left(t - \frac{a}{V}\right)$. Thus:

$$\varepsilon^2\xi_{11}(0) = \lambda\left[\omega^2 A^2\left(t - \frac{a}{V}\right) + A'^2\left(t - \frac{a}{V}\right)\right] \tag{17}$$

In general:

$$A'^2 \ll \omega^2 A^2 \tag{18}$$

(this corresponds to a parametric approximation).

Longitudinal acoustic rectification will correspond to the propagation of signal $l(t)\lambda\omega^2A^2\left(t - \frac{a}{V}\right)$ where $l(t)$ is the length of the impulse in the solid (Fig. 1). One can thus foresee that the order of magnitude of the displacement due to rectification will be comparable to that of the second harmonic generated on a distance equal to that of the impulse. Methods permitting the detection of harmonics can thus be retained *a priori* to emphasize rectification. Thus capacitive detectors such as those developed by Gauster and

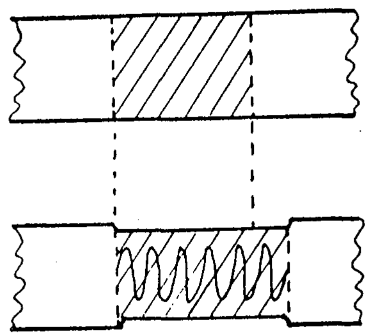

Fig. 1 Acoustical rectification due to the passing of an ultrasonic impulse through a solid.

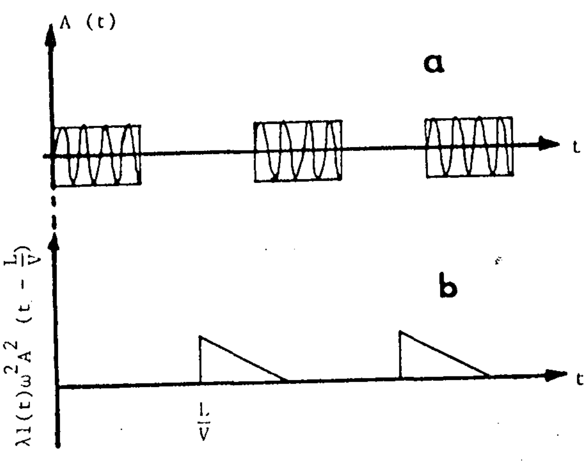

Fig. 2 (a) Ultrasonic signal at beginning of propagation; (b) rectification signal detected at end of the solid.

Breazeale [8] seem particularly well-adapted: in effect, they permit measuring amplitude of ultrasonic displacement at one end of the solid. An analysis as to the frequency of recurrence of the impulse must then pick up a signal, corresponding to the integration of the square of the envelope (Fig. 2), from which a linear combination of the two constants C_{111} and C_{112} can be deduced.

Contingent upon whether one can experimentally reproduce conditions which are sufficiently close to previously explicit limits, acoustical rectification can become a new approach to measuring third order elastic constants. /C8-220

4. Conclusion

The study of nonlinear optical acoustics has permitted us, on the one hand, to show an essential aspect of acoustical nonlinearity related to the dimensionality of the propagation medium (we have illustrated this fact by demonstrating the static instability of an ideal linear crystal), and on the other hand, to outline a new method of measuring third order elastic constants by using a phenomenon which we propose to call acoustical rectification.

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