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AN ELPLANATION OF THE INSTABILITY OF THE FREE VORTEX CORES OCCURRING OVER DELTA WINGS WITH RAISED EDGES

H. Ludwieg

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AN EXPLANATION OF THE INSTABILITY OF THE FREE VORTEX CORES OCCURRING OVER DELTA WINGS WITH RAISED EDGES[1]

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Summary /242*

By rolling up the surfaces of discontinuity originating from the leading edge of delta wings, free vortex cores are formed above the wing. In case of greater angles of incidence, the flow in these vortex cores shows an instability which abruptly produces strong turbulence. In the present paper an explanation is given of this instability being a "frictionless instability" of the vortex-core flow by increasing helical interference vortices. The occurring vortex-core flows are calculated and investigated for stability by means of a stability criterion concerning flows with helical streamlines given by H. Ludwieg [8,9].

1. Symbols

r, Φ, z	cylinder coordinates
V_r, V_Φ, V_z	corresponding velocity components
p	static pressure
ρ	density
ξ	$= r/z$
ξ_o	ξ value at the vortex core boundary
η	standardized vortex core radius
ηN	vortex core radius with neutral layering of the flow

*Numbers in the margin indicate pagination in the foreign text.

[1]Report of the Aerodynamic Test Institute in Göttingen (AVA). Report presented at the annual conference of the WGL in Freiburg I. Br. in 1961

ζ $\tilde{\zeta}$	vortex core radius or standardized vortex core radius in Section 2
$v_r(\eta), v_z(\eta)$	auxiliary velocities in the transition from Section 1 to Section 2 /243
c_Φ, c_z	velocity gradients dV_Φ/dr or dV_z/dr
$\tilde{c}_\Phi, \tilde{c}_z$	dimensionless velocity gradient
$f(\eta)$	function of shift
C_1, C_2	constants for designating the vortex core profile
C, K	constants for designating the vortex core profile in a standardized form
α	angle of aperture of the tapered vortex core
β_o	angle of inclination of the boundary streamlines of the vortex core flow

Significance of the indices

1,2	quantities in Section 1 or 2
∞	quantities at a large distance from the vortex core axes

2. Introduction

In the case of a raised delta wing with rounded leading edge, it is known that the flow separates near the leading edge at somewhat greater angles of incidence. This also already occurs in pointed delta wings at small angles of incidence. In these cases, surfaces of discontinuity, rolling above the delta wing in the shape of a cone to form a pair of vortices, spread from both separation edges. This phenomenon was first studied by R. Legendre [1] and M. Roy [2]. In Figure 1, a delta wing with the corresponding vortices is illustrated. These vortices exert a considerable influence on the coefficients of air force of the wing, so that a reliable calculation of these coefficients is not possible based on the usual airfoil theory (without taking the vortices into consideration). The shape of the rolled vortex bands, their relative position to the wing and their influence on the velocity field and the forces of the air can be calculated in principle, although extensive simplifications must be introduced for the practical computations. Such calculations were carried out, among others, by K. W. Mangler and J. H. B. Smith [3].

The cores of these free vortices over raised delta wings now demonstrate an unusual phenomenon. They suddenly become instable at a definite point in the flow field without apparent reason, becoming noticeable due to the occurrence of strong, turbulent motions within the vortex core. This phenomenon is so violent that one can almost speak of a "breakdown" of the vortex core. This phenomenon was experimentally studied by H. Werlé [4] and B. J. Elle [5]. H. Werlé employed a water channel for these studies, in which the breakdown can easily be made visible. It is only necessary to introduce a dye on the pressure side of the delta wing very near the point. This then reaches the middle of the vortex core over the separation surface and spreads as color threads along the access of the vortex core. At the point of instability these color threads spread almost abruptly over the entire vortex core. The breakdown of the vortex core over a delta wing subjected to transsonic velocity was made visible by B. J. Elle in a striation photo, and a connection between the position of instability relative to the airfoil and irregularities in the moment course of the wing was demonstrated. The present paper is concerned with the explanation of this instability.

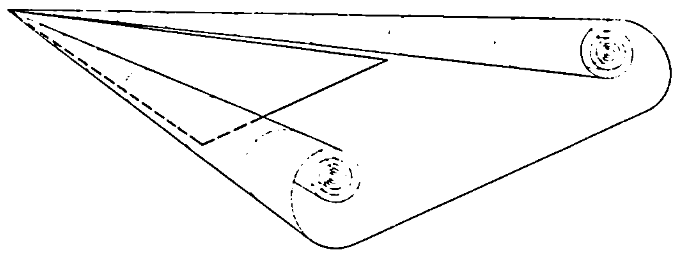

Figure 1: Delta Wing with Rolled Surfaces of Discontinuity

3. Calculation of the Velocity Distribution in the Vortex Core

3.1. Velocity Distribution within the Range of the Tapered Flow

Before we can examine the stability of the vortex core flow, we must first calculate the occurring velocity distributions. For this, we observe a section perpendicular to the direction of flow through a delta wing (Figure 2a) with the typical, rolled vortex bands. This flow can be easily illustrated in such a manner that, as is presented in Figure 2b, a vortex core with homogeneous distribution of rotation is assumed, continuously fed with new rotation from a vortex band stemming the leading edge of the delta wing. When one is only concerned with the flow relationships within the vortex core, the creation of the vortex core can be considered further simplified by assuming that the rotation is supplied to the entire circumference of the vortex core in an even distribution. The flow in the vortex core up to the boundary is then completely rotation-symmetrical.

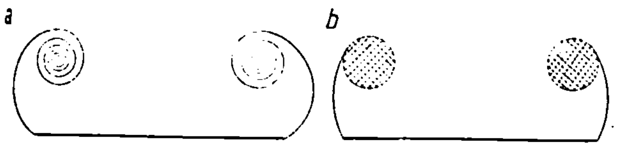

Figure 2: Section through a delta wing with vortex core

a) with rotation into a spiral,

b) with continuously distributed rotation

When we first examined the vortex core only in the area of the front wing section, the entire flow and therefore also the vortex core flow is additionally tapered, especially under the prerequisite of a small angle of the delta wing point, i.e., on straight lines through the wing point the three velocity components and the pressure are constant.

Under these assumptions, the flow field within the vortex core can be easily calculated, when the influence of viscocity is disregarded. We introduce the cylinder coordinates r, Φ ,z and let the z axis coincide with the vortex core axis and the coordinate initial point with the wing point, choosing the sign in such a manner that the z axis points downstream.

When we designate the corresponding velocity components of the vortex core flow with V_r, V_Φ, V_z and the static pressure with p, these quantities then depend only on the parameter $\xi = r/z$ under the given prerequisites, i.e. it applies that

$$V_r = V_r(\xi), \quad V_\varphi = V_\varphi(\xi), \quad V_z = V_z(\xi), \quad p = p(\xi).$$

When this approach is introduced into the Euler equations written in cylinder coordinates, the following differential equations are obtained for the velocity components and the pressure:

$$
(1) \qquad \begin{cases} \xi V_r V_r' - V_\varphi^2 - \xi^2 V_z V_r' = -\dfrac{1}{\varrho}\,\xi p', \\ \xi V_r V_\varphi' + V_r V_\varphi - \xi^2 V_z V_\varphi' = 0, \\ \xi V_r V_z' - \xi^2 V_z V_z' = \dfrac{1}{\varrho}\,\xi^2 p', \\ \xi V_r' + V_r - \xi^2 V_z' = 0. \end{cases}
$$

In the case of a delta wing with an angle of point which is not too great, the angle of aperture of the cone representing the vortex core is small, i.e., in the entire area of the vortex core it applies that $\xi \ll 1$. Under this assumption, then also

$$
V_r \quad V_\varphi, \qquad V' \quad V'_z,
$$

and we can disregard the members $\xi V_r V'_r$ and $\xi^2 V_z V'_r$ in the first of the equations (1). Therefore we obtain from this equation:

$$
(2) \qquad V_\varphi^2 = \frac{1}{\varrho}\,\xi p',
$$

i.e., the radial pressure gradient in the vortex core is essentially determined by centrifugal forces and not by radial acceleration.

When the pressure is now eliminated with equation (2) from the third of the equations (1) and the abbreviation is additionally introduced

$$
V_r - \xi V_z = A
$$

the following differential equations are obtained:

$$
(3) \qquad \begin{cases} \xi V_\varphi' A + V_\varphi A + \xi V_\varphi V_z = 0, \\ V_z' A - V_\varphi^2 = 0, \\ \xi A' + A + 2 \xi V_z = 0. \end{cases}
$$

The equation system (3) can be easily solved by eliminating the individual quantities and integrating the usual differential equations arising. When it is also demanded from the solution that V_r does not become infinite when approaching the axis $\xi = 0$, i.e., that no sources should be situated on the axis, the following velocity field results for the vortex core:

$$(4)\qquad \left\{ \begin{aligned} V_r &= -\frac{1}{2} C_1 \xi, \\ V_\varphi &= C_1 \sqrt{-\ln \xi + \frac{1}{2} + C_2}, \\ V_z &= C_1 (-\ln \xi + C_2). \end{aligned} \right.$$

This solution applies within the vortex core, i.e. for $0 \leqq \xi \leqq \xi_0$, where $\xi_0 = \operatorname{tg}(\alpha/2)$, when the angle of aperture of the tapered vortex core is designated by α. This is in agreement with a velocity distribution for the vortex core gained by M. G. Hall [6] in a similar manner, when the additional condition is introduced there, that no /244 sources or depressions may be present on the axis of the vortex core.[1]

It can be seen from (4) that the velocity component V_r remains small compared to the other components V and V_z for $\xi_0 \ll 1$ in agreement with the assumptions already made.

When the radius of the vortex core is now standardized to 1, i.e. when

$$\xi/\xi_0 = \eta,$$

the result for the main components V_Φ and V_z, when $C_1 = C$ and $C_2 - \ln \xi_0 = K$:

[1]In a more recent paper by M. G. Hall [13], which first became known to the author after completion of this work, this condition is taken into consideration. The formulas given there for the velocity field of the vortex core are in complete agreement with the exception of the designations with the equations [4].

$$
(5) \quad \left\{ \begin{array}{l} V_\varphi = C \sqrt{-\ln \eta + \frac{1}{2} + K}, \\ V_z = C(-\ln \eta + K) \end{array} \right.
$$

with $0 \leqq \eta \leqq 1$. In this representation, C indicates the size of the velocities and K the shape of the velocity profiles.

It can easily be seen from (5), that the various velocity profiles, characterized by the parameter K, can be interpreted as various sections of a unit distribution, standardized to the radius 1.

The angle of inclination of the boundary streamline of the vortex core β_o also depends only on K through the relationship

$$
\beta_0 = \operatorname{arc\,tg} \frac{K}{\sqrt{\frac{1}{2} + K}}
$$

and can therefore be employed as profile parameter.

Infinitely high velocities and also velocity gradients result on the axis of the vortex core according to (5). Here, disregarding the viscosity is certainly no longer permissible; however, it can be assumed that this influence only extends to a narrow area near the vortex core axis with sufficiently high Reynolds numbers. This influence of friction on the velocity field of the vortex core flow was more closely studied by M. G. Hall [7, 13] and compared with experiments. In this study it was shown that this influence is actually only substantial near the vortex core axis.

Moreover, the internal friction becomes apparent when boundary layer material flows over the separation surface in the vortex core, so that the assumption made on constant total pressure in the vortex core no longer completely applies; however, as measurements made by J. K. Harvey show, reproduced in the work of M. G. Hall [6], this affect is only slight, when the areas of the vortex core close to the axis are neglected.

Near the outer boundary, the vortices occurring in the case of delta wings no longer demonstrate the rotation-symmetrical behavior of our substitute vortex, since the assumption of homogeneous and rotation-symmetrical rotation distribution no longer applies.

Excluding these effects, however, it can be expected that the equations (5) supply the velocity profiles occurring in the actual vortex cores.

3.2. Distribution of Velocity in the Area of the Non-Tapered Flow

When the vortex core flow in the area of the trailing edge of the delta wing is examined, it can no longer be tapered here, since the entire flow field around the delta wing is not tapered in this area because the wing stops. In this case there are essentially two influences determining the non-tapered further development of the vortex core flow. On the one hand, the previously even feeding of the vortex core with new rotation in this area becomes less due to the rolling of the vortex band and then it stops completely. On the other hand, the vortex, embedded in the underpressure field on the suction side of the wing in the tapered area of the flow, is released from this underpressure field in the area of the trailing wing edge. For this reason, the pressure on the surface of the vortex core is increased, also influencing the flow within the vortex core.

The first effect does now alter the tapered character of the flow field in the vortex core, but without substantially altering the velocity distribution in the main components V_Φ and V_z. This can be easily understood when the situation is pictured that feeding new rotation to the surface suddenly stops at a defined point in the case of a vortex core developing in a tapered shape. The vortex will then simply continue and a purely cylindrical vortex with the V_Φ and V_z distribution present upon cessation of the rotation supply. In the case of a slow drop in rotation supply, the vortex core profiles will be somewhat altered in the outer range, but this effect can only become noticeable relatively far downstream, since the vortex band leading the wing near the trailing wing edge must first reach the vortex core.

The conditions are somewhat different in the second effect. In this case, a substantial alteration of the previously present V_Φ and V_z velocity profiles occurs with an immediate effect in the area of pressure increase. For approximate calculation of this alteration a purely cylindrical vortex core is examined with a V_Φ, V_z distribution according to (5), embedded in an also cylindrical rotation-free external flow, connected to the vortex core surface with continuous velocity components (Figure 3). It can be considered that the pressure increase in the external flow is produced by a sudden expansion of an external rotation-symmetrical boundary surface, as can be seen in Figure 3. Through the increase in the pressure on the edge of the vortex core, the vortex core diameter will be increased, and the vortex core velocity profiles will be altered in a similar manner to that in a delta wing in the area of the pressure increase.

In the transition from the original pressure (Section 1) to the increased pressure (Section 2), the following conditions apply in the case of friction-free flow:

1. On a rotation-symmetrical flow surface, the V_Φ component is altered according to the theorem of spin, i.e., V_Φ is inversely proportional to the individual radius. /245
2. For a rotation-symmetrical flow tube with ring shaped cross-section, the continuity equation applies (i.e. V_z is altered inversely proportional to the flow tube cross-section.
3. Exactly as in cross-section 1, radial balance must again be established in cross-section 2, i.e., the forces, determined by the radial pressure gradient, must also be in balance with the centrifugal forces in this case.

4. Exactly as in cross-section 1, the equation of Bernoulli must again apply in cross-section 2 with identical total pressure for all stream lines.

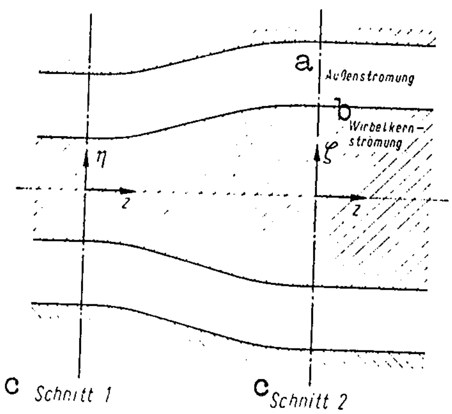

Figure 3: Diagram of the vortex core flow with a pressure increase in the external flow

Key:
a. external flow
b. vortex core flow
c. section

The pressure can be eliminated from the third and fourth conditions, so that they supply a condition together for the components of velocity.

For the quantitative formulation of the above-mentioned conditions the radius in Section 1 is designated by η and in Section 2 by ζ. and a function of shift $f(\eta)$ is introduced, indicating by how much the radius η of a rotation-symmetrical flow surface in Section 1 is altered in the transition to Section 2. The diameter of the vortex core in Section 1 is chosen equal to number 1, so that f(1) is the enlargement of the vortex core radius in the transition to Section 2. The velocity components in the Sections 1 and 2 are designated by

$$V_{\varphi 1}(\eta),\quad V_{z1}(\eta) \quad \text{or} \quad V_{\varphi 2}(\zeta),\quad V_{z2}(\zeta).$$

The differences $V_{\Phi 2} - V_{\Phi 1}$ and $V_{z2} - V_{z1}$ for points, situated on the same flow surface (not on the same radius), are designated by $v_{\Phi}(\eta)$ and $v_z(\eta)$. It therefore applies that:

$$\left.\begin{aligned} V_{\varphi 2}(\zeta) &= V_{\varphi 1}(\eta) + v_{\varphi}(\eta), \\ V_{z2}(\zeta) &= V_{z1}(\eta) + v_z(\eta) \end{aligned}\right| \text{for } \zeta = \eta + f.$$

Furthermore, it is assumed that the pressure alteration in the external flow is small. Then $f(\eta)$ remains small compared to 1, furthermore v_Φ and v_z are small compared to $V_{\Phi 1}$, V_{z1}, $V_{\Phi 2}$ and V_{z2}, so that the resulting equations may be treated in a linear manner. Therefore, it then follows from the first condition:

$$V_{\varphi 1}(\eta)\,\eta = V_{\varphi 2}(\eta + f)\,(\eta + f)$$

or after conducting the linearization

$$(6) \qquad V_{\varphi 1} f + \eta\, v_\varphi = 0\,.$$

It follows from the second condition:

$$\eta\, \Delta\eta\, V_{z1}(\eta) = (\eta + f)\,(\Delta\eta + \Delta f)\, V_{z2}(\eta + f)$$

or after conducting the linearization

$$(7) \qquad V_{z1} f + \eta\, V_{z1} f' + \eta\, v_z = 0\,.$$

It follows from the third condition

$$\varrho\, \frac{V_{\varphi 1}^2(\eta)}{\eta} = \frac{dp_1}{d\eta}$$

or

$$\varrho\, \frac{V_{\varphi 2}^2(\zeta)}{\zeta} = \frac{dp_2}{d\zeta}$$

and from the fourth condition:

$$p_1(\eta) + \frac{\varrho}{2}\,[V_{\varphi 1}^2(\eta) + V_{z1}^2(\eta)] = \text{const}\,,$$

$$p_2(\zeta) + \frac{\varrho}{2}\,[V_{\varphi 2}^2(\zeta) + V_{z2}^2(\zeta)] = \text{const}\,,$$

where the constants are identical in both equations. Through the elimination of p_1 or p_2, it follows from the last four equations:

$$(8\,\mathrm{a}) \qquad \frac{V_{\varphi 1}^2(\eta)}{\eta} + V_{\varphi 1}(\eta)\, V'_{\varphi 1}(\eta) + V_{z1}(\eta)\, V'_{z1}(\eta) = 0\,,$$

$$(8\,\mathrm{b}) \qquad \frac{V_{\varphi 2}^2(\zeta)}{\zeta} + V_{\varphi 2}(\zeta)\, V'_{\varphi 2}(\zeta) + V_{z2}(\zeta)\, V'_{z2}(\zeta) = 0\,.$$

By means of linearization and consideration of

$$
\begin{aligned}
V'_{\varphi 2}(\zeta) &= V'_{\varphi 1}(\eta) + v'_{\varphi}(\eta) - V'_{\varphi 1}(\eta)\, f'(\eta)\,, \\
V'_{z 2}(\zeta) &= V'_{z 1}(\eta) + v'_{z}(\eta) - V'_{z 1}(\eta)\, f'(\eta)
\end{aligned}
$$

it results that

$$
(9) \qquad \left|\; \begin{aligned} & -\frac{V_{\varphi 1}^{2}}{\eta^{2}} f + \frac{V_{\varphi 1}^{2}}{\eta} f' + \left(V'_{\varphi 1} + \frac{2}{\eta} V_{\varphi 1}\right) v_{\varphi} + \\ & \qquad + V_{\varphi 1} v_{\varphi}{}' + V_{z 1} v_{z}{}' + V'_{z 1} v_{z} = 0\,. \end{aligned} \right.
$$

The equations (6), (7) and (9) form three linear differential equations for v_Φ, v_z and f. Through the elimination of v_Φ and v_z, the differential equation is obtained from these, taking into consideration (8a) for the function of shift $f(\eta)$

$$
(10) \qquad f'' + \frac{V_{z 1} + 2\,\eta\, V'_{z 1}}{\eta\, V_{z 1}} f' - \frac{1}{\eta^{2}} f = 0\,.
$$

When the value from (5) is introduced here for v_{z1}, (10) can be solved, and the result with the boundary condition $f(0) = 0$ for $f(\eta)$, when the constant K is designated by K_1 for the profile in Section 1,

$$
(11) \qquad f(\eta) = \frac{K_1 f(1)\, \eta}{-\ln \eta + K_1}\,.
$$

In this case, f(1) can be freely selected as an integration constant. Because of the linearization employed, it must be small compared to 1.

v_Φ and v_z and therefore also $V_{\Phi 2}$ and V_{z2} can be calculated from (6), (7) and (11). The result is:

$$
(12)\quad \begin{cases} V_{\eta 2}(\zeta) = \\ \quad = C\sqrt{-\ln \eta + \dfrac{1}{2} + K_1\left(1 - \dfrac{K_1 f(1)}{-\ln \eta + K_1}\right)}, \\ V_{z2}(\zeta) = C(-\ln \eta + K_1) \times \\ \quad \times \left(1 - \dfrac{2K_1 f(1)}{-\ln \eta + K_1} - \dfrac{K_1 f(1)}{(-\ln \eta + K_1)^2}\right) \end{cases}
$$

with /246

$$
\zeta = \eta\left(1 + \frac{K_1 f(1)}{-\ln \eta + K_1}\right).
$$

When η is now introduced in the place of ζ on the right-hand side, the result is, when only members of the zero and first order are taken into consideration again:

$$
(13)\quad \begin{cases} V_{\eta 2}(\zeta) = C\sqrt{-\ln \zeta + \dfrac{1}{2} + K_1[1 - 2f(1)]}, \\ V_{z2}(\zeta) = C[-\ln \zeta + K_1[1 - 2f(1)]] \end{cases}
$$

with

$$
0 \leq \zeta \leq 1 + f(1).
$$

When the profile is again standarized to the radius 1, i.e. when

$$
\tilde{\zeta} = \frac{\zeta}{1 + f(1)},
$$

it applies in a linearized approximation

$$
(14)\quad \begin{cases} V_{\eta 2}(\tilde{\zeta}) = C\sqrt{-\ln \tilde{\zeta} + \dfrac{1}{2} + K_2}, \\ V_{z2}(\tilde{\zeta}) = C(-\ln \tilde{\zeta} + K_2) \end{cases}
$$

with

$$
0 \leq \tilde{\zeta} \leq 1,
$$

where

$$(15) \qquad K_2 = K_1 [1 - 2f(1)] - f(1)$$

is set as an abbreviation. From these equations, it can be seen that the velocity profiles of the resulting vortex core flow again belong to the same class given by (5), when a cylindrical vortex core flow with velocity profiles according to the equation (5) is taken as a basis and this vortex core flow is permitted to proceed through a pressure increase. Only the constant K characterizing the standardized vortex core profile is altered according to equation (15).

The present derivations are carried out under the assumption that the pressure increase weak and therefore $f(1) \ll 1$ remains. With greater pressure increases, the same calculation can be carried out in individual steps, for which $f(1) \ll 1$ applies each time. From this it immediately follows that the occurring velocity profiles belong to the profile class given by (5) even in this case. For the K_2 values occurring in this case the relationship is obtained by integration, when (15) is interpreted as a differential equation for the K alteration:

$$(16) \qquad K_2 = \frac{(K_1 + 1/2)}{[1 + f(1)]^2} - \frac{1}{2}.$$

With this, the vortex core profile alterations are also determined for finite increases in pressure.

It will be subsequently demonstrated how the alteration of the K value is related to the amount of increased pressure outside of the vortex. For this purpose, we consider Figure 3 and designate the pressure with $p_{1\infty}$, in Section 1 far in front of the vortex core axis, where the potential vortex surrounding the vortex core has already disappeared. The corresponding z component of the velocity is designated by $V_{z1\infty}$ and the corresponding quantities in Section 2 by $p_{2\infty}$ and $V_{z2\infty}$. Then we introduce a pressure number c_p in the following manner:

$$c_p = \frac{p_{1\infty} - p_{2\infty}}{(\varrho/2)\, V_{z2\infty}^2} . \tag{17}$$

From this, it follows with the aid of the Bernoulli equation:

$$c_p = 1 - \frac{V_{z1\infty}^2}{V_{z2\infty}^2} .$$

Since the quantities V_{z1} and V_{z2} do not change within the potential vortex surrounding the vortex core and also do not jump at the edge of the vortex core, $V_{z1\infty}$ and $V_{z2\infty}$ can be replaced by the corresponding V_z components at the edge of the vortex core. Taking (5) into consideration, the equation is then obtained:

$$c_p = 1 - \frac{K_1^2}{K_2^2} . \tag{18}$$

This equation determines the value K_2 with a predetermined K_1 and c_p, and the corresponding value f(1) can also be determined with the aid of (16).

The pressure number c_p characterizing the increase in pressure has been defined in such a manner that the increase in pressure outside of the vortex, occurring from the tapered portion of the flow field up to a point far behind the delta wing, is designated by c_p, identical to the usual pressure number c_p for the underpressure, in which the vortex is embedded in the tapered flow area.

4. Stability of the Calculated Vortex Core Flows

It was seen in Section 3 that in the case of free vortices over separated delta wings for the circumference and longitudinal components of the vortex core flow, both in the tapered and in the non-tapered area, velocity profiles occur, represented approximately by the equations (5). These velocity profiles are changed only slowly in the vortex longitudinal direction, and the radial component of the velocity therefore remains small, so that stability observations can be conducted with cylinder-symmetrical vortex core flows.

For those cylindrical flows with helical streamlines, a stability criterion was proposed by H. Ludwieg [8], indicating under which conditions such a spatial flow - in the sense of the Rayleigh stability criterion for smooth flows [9] - is stable or instable. This criterion states, that for

$$(19a) \qquad (1-\tilde{c}_\varphi)(1-\tilde{c}_\varphi^2)-\left(\frac{5}{3}-\tilde{c}_\varphi\right)\tilde{c}_z^2 > 0$$

the flow is layered in a stable manner and for

$$(19b) \qquad (1-\tilde{c}_\varphi)(1-\tilde{c}_\varphi^2)-\left(\frac{5}{3}-\tilde{c}_\varphi\right)\tilde{c}_z^2 < 0$$

the flow is layered in an instable manner. In this case $\tilde{c}_\Phi$ and $\tilde{c}_z$ signify the radial velocity components dV_Φ/dr and dV_z/dr, which became dimensionless with the circumference component V_Φ and the radius r, i.e.

$$\tilde{c}_\varphi = \frac{dV_\varphi}{dr}\frac{r}{V_\varphi}, \qquad \tilde{c}_z = \frac{dV_z}{dr}\frac{r}{V_\varphi}.$$

When a $\tilde{c}_\Phi$ or $\tilde{c}_z$ plane is employed for the description of the various flows, the stability and instability areas shown in Figure 4 are obtained.

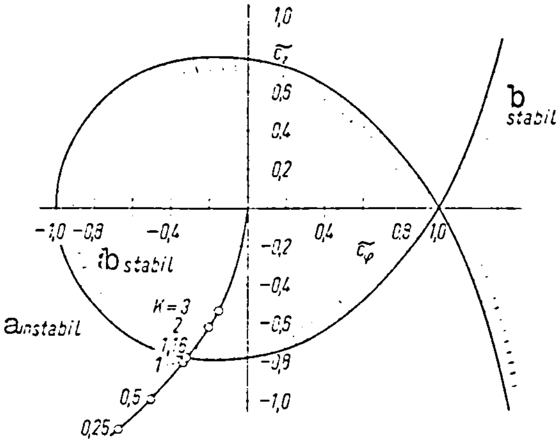

Figure 4: Stable and instable areas in the $\tilde{c}_\Phi$, $\tilde{c}_z$ plane and $\tilde{c}_\Phi$, $\tilde{c}_z$ values for vortex core velocity profiles according to the equations (5) as a function of profile parameter K.

Key:
a. instable
b. stable

When the $\tilde{c}_\Phi$ and $\tilde{c}_z$ values are now calculated for the vortex core flows in accordance with (5), the result is /247

$$
(20) \quad \left|\begin{aligned} \tilde{c}_\varphi &= -\frac{1}{2\left(-\ln\eta + \frac{1}{2} + K\right)}, \\ \tilde{c}_z &= -\frac{1}{\sqrt{-\ln\eta + \frac{1}{2} + K}}. \end{aligned}\right.
$$

It can be recognized from these equations that the $\tilde{c}_\Phi$ and $\tilde{c}_z$ values of the vortex core flow according to (5) depend only on the profile parameter K in addition to the radius η. When the $\tilde{c}_\Phi$ and $\tilde{c}_z$ values, coordinated to the various η values of the vortex core, are plotted in Figure 4, parabolic curves are obtained, which are situated for all K values on a unified parabola, which is plotted in Figure 4. In this case the terminal points of the individual parabolic curves, coordinated to the vortex core axis ($\eta = 0$) are all situated in the coordinate initial point, while the points, coordinated to the vortex core edge ($\eta = 1$), are situated at various points of the parabola. In Figure 4, the terminal points corresponding to the vortex edge are plotted for five K values. It can be seen from this that in a vortex core with a K value greater than 1.16, the flow is layered in a stable manner in the entire range from $\eta = 0$ to $\eta = 1$, while in the case of a vortex core with a K value smaller than 1.16, the external portion of the flow is layered in an instable manner. In the latter case, the transition from the stably layered to the instably layered flow occurs at a radius ηN, given on the basis of the equations (19) and (20) by

At this critical radius ηN, at which the flow is layered just neutrally, the helical streamlines of the vortex core flow have an angle of inclination of 42.0° according to (5).

The result of the stability study can therefore also be summarized in the following manner: In the case of vortex core flows according to equations (5), the flow in areas with a streamline angle of inclination greater than 42.0° is stably layered and in areas with an angle of inclination smaller than 42.0° it is instably layered.

We have therefore obtained the important result that the spatial vortex core flow according to (5) becomes instable in the outer area for small K values, although it has the same total pressure on all streamlines, calculated as a frictionless flow. This is of interest, because there is no analogy for this in the case of smooth flows. At constant total pressure, namely, these are always potential flows, always layered in a neutral manner in the sense of the Rayleigh stability criterion.

5. Explanation of the Instability and Comparison with the Experimental Observations

According to the preparatory studies of the previous sections, we can now construct the following simple, somewhat schematic picture about the "breakdown" of the cores of the free vortices over delta wings. In the case of a wing subjected to a flow with such an angle of incidence, at which the phenomenon of breakdown occurs, a vortex core flow is formed due to rolling of the separation surfaces, which has the velocity profiles of the type of the equations (5). The profile parameter K of these profiles is first situated in this case in the tapered range above the critical value of K = 1.16, so that the flow is stable. Behind the tapered area, an increase in pressure occurs in the external flow, in which the vortex is embedded, shaping the vortex core profile in such a manner that the K value becomes smaller. Upon exceeding the critical K value, the vortex core flow becomes instable, beginning from the external edge. According to the amount of pressure increase, a more or less large ring area of the core flow becomes instable in the sense that in the case of frictionless flow, helical disturbance vortices occur with exponentially increasing intensity. The instability is therefore of the same type as the known Rayleigh instability for smooth, rotating flows, only with the difference that helical vortices now replace the ring-shaped vortices. The internal friction shifts the stability boundaries in the same manner, as was shown by G. I. Taylor [11] in the Rayleigh case. With the large Reynold numbers, as normally occur in the case of delta wing flows, however, this friction effect is negligible. After exceeding the stability limit, the flow is therefore almost simultaneously instable compared to a large number of disturbance vortices of various dimensions. Disturbances of different wave lengths, randomly present in the flow, will rapidly grow and cause strong turbulence in the already instable area without the formation of marked helical vortices. This turbulence then spreads over the edges of the already instable area and includes especially the still stable inner portion of the vortex core.

The increase in pressure causing the destabilization of the velocity profiles intensifies automatically due to the fact that the occurring turbulence shapes the vortex core profiles in the sense that the drop in pressure caused by the centrifugal forces is reduced from the vortex core edge to the vortex axis, supplying an additional pressure increase for the internal, still stable portions, of the vortex core. A further additional destabilizing effect is caused by the fact that the actual vortex core expands more greatly than our substitute vortex with constant pressure due to the reduced total pressure near the vortex core /248
axis. These two effects contribute to the fact that the transition to a completely turbulent flow occurs abruptly.

The assumption that the breakdown of the vortex cores is an instability of the vortex core flow was already expressed by B. J. Elle [5] and J. P. Jones [12]. The latter author also made a disturbance wave approach and derived the corresponding disturbance differential equations. In this case, however, only ring-shaped disturbance waves were assumed, which probably would not supply the stability limits of such spatial vortex core flows, if the differential equations were solved.

An examination will subsequently be undertaken on how the proposed theory for the breakdown of the vortex cores agrees with the still somewhat thin experimental findings.

According to our theory, the instability begins, when the critical K value of 1.16 is reached, coordinated to a streamline angle of inclination of 42.0° at the vortex core edge. This statement is so difficult to check precisely, because the vortex core edge is actually only exactly defined in the case of our idealized substitute vortices. Rough estimations on the basis of so-called coating pictures, which make the inclination of the streamlines visible on the wing surface, demonstrate that the breakdown occurs at angles of inclination which are approximately in the range of the critical angle.

Since the conversion to instable vortex core profile takes place according to our theory by means of an increase in pressure in the external flow, this breakdown would also have to occur in areas with a pressure increase. This is apparently the case for the observed delta wings, since a pressure increase is present in the entire rear area of the wing - deviating from the so-called slender-body theory - because of fulfilling the Kutta-Joukowski condition for flow off. Moreover, the dependency of instability on a pressure increase is confirmed by the fact that obstacles inducing a pressure increase in the area of the vortex core shift the instability point towards the front, as was shown in the studies made by H. Werlé [4]. The question on whether the pressure increases occurring in the case of delta wings are sufficient to cause instability in essential portions of the vortex core flow in the manner described in Section 3.2 can also be answered completely positively, as the following example demonstrates. When only a pressure number of $c_p = -1$ is assumed for the underpressure, in which the vortex is embedded in the tapered area of the delta wing, a very conservative estimate when considering the large angle of incidence at which the breakdown occurs, just those stable vortex core profiles with K = 1.16 would convert into those with K = 0.58 according to equation (18). In this case, approx. 50% of the vortex core cross-sectional area would then be instable according to equation (21). As already mentioned, the destabilization effect is actually still greater because of the reduction in total pressure due to friction in the interior of the vortex core and because of the already mentioned automatic intensification of the pressure increase.

The substantial lack of a relationship between instability and Reynolds number, observed in the experimental studies, is also in agreement with our theories, which explain the phenomenon on the basis of a so-called frictionless instability. The shift in the instability point to the rear observed by H. Werlé [4], occurring at very small Reynolds numbers, can also be easily explained. For these small Reynolds numbers, a similar, stabilizing effect of the internal friction must be made noticeable, as was calculated for smooth flows by G. I. Taylor [11]. An estimation on the basis of the Taylor calculations demonstrates that this effect can be approximately expected in the case of the Reynolds numbers, for which it was also observed.

The fact concluded from our theory, that the vortex core flow becomes increasingly instable beginning from the outside to the inside, is also supported by observations. It can be seen, namely, from the experiments of H. Werlé [4], that a dye spreads very abruptly over the entire vortex core area when the flow becomes instable, if the fluid particles near the vortex core axis are marked by a dye material. This is only understandable, when all other particles of the vortex core are already intensely turbulent when the flow becomes turbulent near the vortex core axis.

The effect of the breakdown in the vortex on the coefficient of momentum of a delta wing observed by B. J. Elle [5] can easily be explained by a displacement effect, occurring in the conversion of the vortex core profiles upon breaking down. The intensification of the effect with increasing mach number is also understandable due to the greater, lateral spreading of disturbances in the transsonic range.

It can therefore be stated in summary that our theory on the process of instability of the vortex core flow is in good agreement with the observed facts.

6. Summary

Free vortex cores are formed in the flow above the wing due to rolling of the surfaces of discontinuity extending from the leading edge of delta wings. The velocity field of this vortex core flow is first calculated for the tapered portion of the delta wing flow under several idealized assumptions. A family of vortex core flows with a single parameter is obtained in this case, which agree with a solution given by M. G. Hall [6], when an additional boundary condition is taken into consideration there. Subsequently, the influence of an increase in pressure in the flow outside of the vortex core, as occurs in the area of the trailing edge of delta wings, on the vortex core flow is studied. It is demonstrated in this study that the vortex core flows created by the increase in pressure belong to the identical single-parameter family. The parameter describing the flow, however, is altered considerably by the increase in pressure.

The vortex core flows obtained are examined for stability with the aid of stability criterion for helical flows given by H. Ludwieg [8, 9]. It is demonstrated in this case that the vortex core flow becomes instable in the outermost range of radii, when the parameter describing the flows fall below a critical value. The instability occurring is a "frictionless instability" of the type of the known Rayleigh instability for plane vortex flows, only with the difference that helical dis- /249
turbance vortices now replace the ring-shaped vortices. The variously observed breakdown of the vortex cores is explained by this instability. The experimental findings agree well with this theory.

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