NASA Technical Paper 1857

Solution of Complex Nonlinear Problem by a Generalized Application of the Method of Base and Comparison Solutions With Applications to Aerodynamics Problems

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MAY 1981

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National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1981

SUMMARY

A theory for obtaining approximate solutions to nonlinear problems whose exact solutions require the use of large computational procedures is described. The technique represents in some respects a generalization of the method of base and comparison solutions for flows depending on a parameter. For the generalized problem, the input variable is no longer a parameter but a function that is incremented over its entire domain. After performing calculations for a base configuration and a small number of variations of it, solutions for a large class of configurations can be obtained by forming linear combinations of the solution increments. For a restricted class of problems, approximate solutions can be obtained for general variations of a base configuration by using a function-space derivative estimate obtained from a base solution and a single variation.

INTRODUCTION

Many important aerospace-related problems require the solution of one or more nonlinear differential equations. Although analytic solutions are rare, many such problems have been solved by means of numerical computerized procedures. Often such procedures are long and expensive, when large computing times and storage are required, and they are cumbersome to operate when many input variables and parameters are required.

An alternate to using such programs is to seek approximate solutions by linearizing the differential equations. Such a linearization procedure consists of considering the problem as a perturbation of a "base" problem for which the solution is known. However, even the linear equations often defy solution in analytic form unless the base problem is almost trivially simple, as for example, an undisturbed free-stream flow.

Clearly the usage of large complex computational algorithms could be reduced if there were a way to combine a program for computing a base solution, with a fast, simple procedure for computing the effects of perturbation about that solution. Such a method has been described in reference 1 for flows that depend on some parameter such as Mach number, angle of attack, or thickness ratio, when one of these parameters is perturbed from its base value.

The present analysis represents, in some sense, an extension of the method of reference 1 to problems that involve the perturbation, not of a parameter, but of an entire function. Thus, the method applies to problems for which the input is a function, such as one prescribing a boundary shape; and the output is also a function, such as the pressure distribution over the boundary. The method could also be used to determine variations in the ground noise pattern of an airplane due to variations in its trajectory.

Advantages of the method are that it is applicable to a large class of complex problems, and that it does not require tedious analytic procedures such as solving inner and outer problems, matching, etc. Its primary limitation is that it requires a means of generating base solutions. Furthermore, it is, in general, not directly applicable to problems containing jump discontinuities, although it can be modified to handle such problems by methods similar to those of reference 2.

SYMBOLS

a	constant coefficient
C_p	pressure coefficient
c	local sound speed
F	solution (output) function
f	independent (input) function
G	Gateaux derivative
h	continuous function of small maximum ordinate representing an increment to f
L	denotes a linear operator
M	Mach number
p	parameter
Q	denotes a nonlinear operator
r	nondimensional body radius (in terms of body length)
x,y	nondimensional distance in free-stream direction and perpendicular to free-stream direction, respectively (in terms of body length)
$\tilde{x}$	shifted coordinate
α	angle of attack
Δ	before a variable denotes an increment in the variable
δ	before a function denotes a perturbation in the function
ε	small parameter
ϕ	velocity potential

Subscripts:

i	represents ith quantity in a sequence
o	denotes base value
1,2,3	denotes quantities associated respectively with the first, second, and third variations
x,y	denotes derivatives with respect to x and y

Superscript:

'	Gateaux derivative of an operator

DEVELOPMENT OF THEORY

Background Theory on Nonlinear Problems Depending on a Parameter

Reference 1 treats the problem of reducing computer time by combining calculations which have a full nonlinear numerical solution with calculations obtained by a simple linear interpolation scheme. If the solution depends on a parameter p (e.g., Mach number, angle of attack, thickness ratio), then the effect on the solution F of an incremental change in p is

$$\Delta F = \frac{\partial F}{\partial p} \Delta p \tag{1}$$

provided that F does not change discontinuously with p. The latter situation would occur, for example, in certain transonic flows involving shock waves. The problem that arises in the application of equation (1) is that in the problem treated, it is not possible to obtain an analytic expression for $F(p)$ that can be differentiated to obtain $\frac{\partial F}{\partial p}$. However, if a computer program exists for computing the solution F, then a "base" solution F_o can be calculated for a specified value p_o of the parameter p, and a second solution $F_1 = F_o + \Delta F$ can be calculated for an incremented value $p_1 = p_o + \Delta p$ of the parameter. This second solution is called a calibration solution. The derivative in equation (1), evaluated at p_o can then be approximated by $\frac{\Delta F}{\Delta p}$. Consequently, equation (1) can be used to obtain the variation of this solution from the base solution for any other small variation in p from its base value $\left(\text{if } \Delta_2 p = p_2 - p_o \text{ then } F_2 \cong F_o + \frac{\Delta F}{\Delta p} \Delta_2 p\right)$. Results given in reference 1 indicate that the accuracy of solutions obtained in this manner is often surprisingly good, even when sizable increments in the parameter are taken.

Methods for treating problems with discontinuities have received much attention in the literature. It should be noted that, since the derivative in equation (1) is taken with respect to a parameter and not with respect to the independent distance variable, the presence of the shock itself does not preclude a solution by equation (1). It is the movement of the shock with changes in the parameter that causes the difficulty. Nixon (ref. 2) has treated this problem by using a coordinate-straining method that has been combined with interpolation by equation (1) in reference 1. In the following development, discontinuous flows will not be discussed as such, under the assumption that for such problems the solution obtained can similarly be combined with Nixon's coordinate-straining method (ref. 2).

Theory for Solutions Depending on an Independent Function

By extending the interpolation procedure of reference 1 with a function-space development, one can obtain solutions for a very general class of problems. This class of problems involves perturbations about a base solution which represents a more complex flow than the simple undisturbed free-stream flow.

Consider, for example, the nonlinear potential equation for two-dimensional compressible inviscid flow

$$(c^2 - \phi_x{}^2)\phi_{xy} - 2\phi_x\phi_y\phi_{xy} + (c^2 - \phi_y{}^2)\phi_{yy} = 0 \tag{2}$$

One can assume a solution in the form of a perturbation $\delta\phi$ about a base solution ϕ_o

$$\phi = \phi_o + \delta\phi \tag{3}$$

If equation (3) is substituted into equation (2) and only linear terms in $\delta\phi$ are retained, then the resulting differential equation is a linear equation for $\delta\phi$. However, the coefficients of several terms involve either ϕ_o or its derivatives, and consequently the equation cannot generally be solved analytically. It only becomes tractable in the case in which ϕ_o represents the constant-velocity undisturbed free-stream flow. However, perturbation solutions about this flow pertain only to slender configurations, and such solutions have already been treated extensively.

Consider, however, problems involving nonslender configurations. Solutions for such problems normally are obtained by a numerical procedure. If the solution represents the pressure or velocity distribution on a two-dimensional surface then it can be expressed as a function of the streamwise variable x,

$$F = F(x)$$

The solution can be expressed as an operator Q (in general, nonlinear) operating on the function $f(x)$ that describes the boundary shape

$$F(x) = Q[f(x)]$$

Let $h(x)$ represent a variation of the specific boundary function $f_o(x)$. Then a differential of the operator can be defined with respect to function-space variations of $f(x)$. Formally, if the limit

$$\lim_{\varepsilon \to 0} \frac{Q[f_o + \varepsilon h] - Q[f_o]}{\varepsilon} = L[f_o, h]$$

exists in a neighborhood of f_o, if it is continuous in f at f_o, and if it is continuous in h at $h = 0$, then L is called the Gateaux differential of Q, and it is linear operator operating on the variation h (ref. 3, sec. 3.1).

Although the present theory is oriented toward computer solutions, this concept of the differential of an operator can be illustrated by a simple analytic example. Suppose that $Q(x)$ were expressible in the form

$$Q[f(x)] = f(x) \int_0^x [f(\varepsilon)]^2 \, d\varepsilon$$

Replacing f by $f_o + \varepsilon h$ and taking the limit

$$\lim_{\varepsilon \to 0} \frac{Q[f_o + \varepsilon h] - Q[f_o]}{\varepsilon}$$

yields the result

$$Q = h(x) \int_0^x [f(\varepsilon)]^2 \, d\varepsilon + f(x) \int_0^x 2f(\varepsilon) \, h(\varepsilon) \, d\varepsilon$$

which is linear in h.

When Q is not expressible in analytic form, but only as the output of a computational procedure, then, in a manner somewhat similar to that of reference 1 for incrementing a parameter, this operator can be estimated by obtaining the difference between a calibration solution and a base solution for slightly different boundary functions. Since L is linear, the solutions for other variations proportional to h can be obtained from the formula

$$Q[f_o + ah] = Q[f_o] + L[f_o, ah] = Q[f_o] + aL[f_o, h] \tag{4}$$

This procedure, although somewhat analogous to that of reference 1 for variation of a parameter, differs from it in several notable respects. The first obvious difference is that the change in the solution is accomplished by varying a function rather than incrementing a parameter. Secondly, the differential of Q is calculated directly, without, in general, computing a quantity analogous to the derivative in equation (1). In fact, such a function-space derivative, called the Gateaux derivative $Q'[f] = G[f]$, can be defined by the formula

$$L[f,h] = G[f]h \tag{5}$$

However, when h is given and L is estimated by comparing base and calibration solutions, the calculation of $G[f(x)]$ by dividing by h is generally not applicable. Exceptions to this situation will be discussed in a later section.

If it were possible to compute $G[f]$, then the effect of making an arbitrary variation in f could be used to obtain the effect of making any other type of variation. In the usual case, however, only the solution corresponding to boundary variations proportional to $h(x)$ can be obtained from a single calibration solution. If a different type of variation is considered, then a second calibration calculation is required. Thus, for the two types of variations, equation (4) gives

$$Q[f_o + a_1 h_1] \cong Q[f_o] + a_1 L[f_o, h_1]$$

$$Q[f_o + a_2 h_2] \cong Q[f_o] + a_2 L[f_o, h_2]$$

Furthermore, since L is a linear operator on h,

$$Q[f_o + a_1 h_1 + a_2 h_2] = Q[f_o] + a_1 L[f_o, h_1] + a_2 L[f_o, h_2] \tag{6}$$

Thus, if two calibration calculations have been performed, a large variety of shapes determined by the various linear combinations of h_1 and h_2 can be analyzed. Clearly this procedure can be extended to any number n of linearly independent variations h_i $(i = 1, n)$ provided that the total variation

$$\sum_{i=1}^{n} a_i h_i$$ remains small. Thus, even in the absence of analytic expressions a partial synthesis of the operator L can be generated.

Application of Theory

Airfoil pressure distributions.- To illustrate the application of the method, two types of problems have been selected. The first problem involves the calculation, by the method of reference 4 of the upper surface pressure distribution on an airfoil at $M = 0.56$ and $\alpha = 2.0^\circ$. The base design, with its pressure distribution, is shown in figure 1(a). The airfoil is 17.5 percent thick with its maximum upper surface ordinate at the 30-percent-chord station. A variation $f_o + h_1$ of the upper surface shape is shown in figure 1(b), together with its pressure distribution. This variation is 17.3 percent thick with its maximum upper surface ordinate at the 40-percent station. A second variation $f_o + h_2$ is shown in figure 1(c). It is 19 percent thick with its maximum upper surface ordinate at the 30-percent station.

The first variation was obtained by shifting the abscissa,

$$\tilde{x} = x - 0.476x(1 - x)$$

and defining the new surface by

$$f_o + h_1 = f_o(\tilde{x})$$

Thus,

$$h_1 = f_o(\tilde{x}) - f_o(x)$$

The other variation was defined by

$$h_2 = 0.04x(1 - x)$$

Many possible airfoil shapes can be obtained by changing the ordinate of the f_o design in increments proportional to h_1, to h_2, or to some linear combination of the two. An example is shown in figure 2. It is represented by the combination $f_o + 0.6h_1 - 0.4h_2$, which is 16.7 percent thick with its maximum upper surface ordinate at the 37-percent station.

Figure 2 also shows a comparison of the pressure distribution computed by the full nonlinear algorithm with that obtained by the present method. It is seen that, for this example, the accuracy of the approximation is very good. The actual pressure calculations for this example are given in table I.

For many problems, the output function is not uniformly sensitive to small variations in the input function. For example, airfoil pressures tend to be sensitive to variations in the geometry very near the nose. If, for a particular case, this sensitivity causes a problem, then smaller variations in the geometry shape must be taken.

Supersonic forebody pressure distributions.- The second type of problem treated is that of computing the pressure distribution on pointed axisymmetric forebodies at supersonic speeds. Figure 3(a) shows the base configuration $f_o(x)$ with its pressure distribution, calculated by the method of reference 5, for a free-stream Mach number of 3.0. These examples are relatively straightforward, and not representative of the large complex computer calculations for which the method is intended. They are, however, adequate for purposes of illustration; and they possess the further advantage that they do not involve specialized knowledge or notation, as would be required, for example, in a boundary-layer calculation.

Figures 3(b) and 3(c) give the body shapes and pressure distributions, respectively, for two variations: $f_o + h_1$ ($h_1 = 0.04x(1 - x)$), and $f_o + h_2$ ($h_2 = -0.02 \sin \pi x$). Figure 4(a) gives results for the linear combination $f_o + 0.3h_1 + 0.7h_2$, both by direct calculations and by linearly combining the pressure distributions of figures 3(a), 3(b), and 3(c). Figure 4(b) gives a similar comparison for the combination $f_o + 0.3h_1 - 0.7h_2$. In both cases, the approximate calculations are nearly indistinguishable from the full nonlinear results.

The magnitude of the variation that can be taken with the calculation remaining in the linear range depends on the type of problem considered. If small increments in the input function $f(x)$ cause large changes in the output function $F(x)$, then the variations taken must be very small. If small increments in $f(x)$ cause discontinuous jumps in $F(x)$, then the linear theory is not applicable.

An example of a sizable variation h_3 ($h_3 = -0.02 \sin 2\pi x$) of the base configuration $f_o(x)$ (fig. 3(a)) is shown in figure 5. The resulting pressure distribution differs radically from that of the base shape. In this case, the variation cannot be considered to be an incremental change in the base design. Nevertheless, when it is treated as such, as in the combination $f_o + 0.3h_1 + 0.7h_3$ shown in figure 6, the results show remarkably good agreement with the exact calculation.

Implementation of the Method

Application of the theory described in the previous section requires some simple software in addition to the basic computer algorithm that computes the result $F(x) = Q[f(x)]$ from the input distribution $f(x)$. For each case, the input x and f arrays as well as the output x and F arrays must be stored with distinguishing case labels. Then a short auxiliary program can perform the following functions:

1. Read the x, f, and F arrays for each case.

2. Establish a common array of the independent variable x for all cases, by interpolation, if necessary.

3. Distinguish one configuration as the base $f_o(x)$.

4. Calculate the incremental functions $L[f_o,h_i]$ by obtaining the differences $Q[f_o + h_i] - Q[f_o] = L[f_o,h_i]$

5. Compute and output results for any linear combination

$$Q[f_o + \sum a_i h_i] = Q[f_o] + \sum a_i L[f_o,h_i]$$

Considerations on Approximating Gateaux Derivative

It was pointed out previously that the function-space differential is normally computed directly, whereas, for an ordinary function of real variables, the differential is defined in terms of the derivatives (eq. (1)). The question arises as to whether a function-space analogue of the derivative exists. As mentioned earlier, under certain conditions for the existence of the linear operator $L[f_o,h]$ (ref. 3, sec. 3.4), the Gateaux derivative $G = Q'$ of the nonlinear operator Q can be defined by

$$L[f_o,h] = G[f_o]h$$

If $L[f_o,h]$ were known in the form of an analytic expression containing a factor of h, then an expression for G could be obtained by a simple division. The linear differential L could then be computed for any variation $f_o + h$ by this equation. Thus, the linear approximations to the solutions for all configurations near f_o could be obtained with only two calculations.

For the type of problem under consideration, in which the operators are not expressible in analytic form, such a procedure is generally not applicable. However, in the next section an example will be provided for which the derivative formulation appears to be useful.

Example of Approximating the Derivative

The advantages of the derivative formulation of the problem (eq. (5)) provide motivation for exploring the possibility of obtaining an estimate for the derivative when the solution for F is provided in the form of a numerical computer procedure. When a problem can be formulated in several ways, one of these possibilities may lead more naturally to a relation of the form of equation (5) from which the function-space derivative can be approximated. Of course, in such a procedure division by h at its zeros must be avoided, with values of the derivative at these points obtained by interpolation.

Consider, for example, the above illustrative problem of computing the pressure distribution on a supersonic body of revolution. For this problem, the input function $f(x)$ can be chosen to be the slope distribution of the body meridian line. Since it is well known that small local changes in flow angle are approximately proportional to the corresponding changes in pressure at supersonic speeds, one might expect that the quantities $h(x)$ and $L[f_o,h]$ representing the variation in body slope and the corresponding variation in pressure, respectively, would vary in a similar manner. Thus, a valid approximation to the derivative according to equation (5) appears possible.

In order to illustrate this procedure, a calculation was performed using the same base body shape as in the preceding example. The variation $-0.01 \sin 2\pi x$ was arbitrarily chosen. The slope variation is $h_1 = -0.02 \cos 2\pi x$. The result $L[f_o,h_1]$ was divided by h_1 to provide an estimate of the derivative $G[f_o]$ in accordance with equation (5). A second variation $0.04x(1 - x)$ which corresponds to a slope variation of $h_2 = 0.04(1 - 2x)$ was then chosen, and the pressure increment was estimated by multiplying the function G by $h_2(x)$. Finally, for comparison, the pressure increment was determined exactly by calculating directly the pressure for the slope $f_o + h_2$.

The results are compared in figure 7(a) for the pressure increments, and the total pressures are compared in figure 7(b). It is seen that the approximate method provides a good approximation to the exact theory for this particular problem. Such calculations with the function-space derivative should be useful when a rapid rough approximation is acceptable and in certain design problems.

Limitations of the Theory

It should be clear that any theory involving differentials or small variations are limited to problems for which the output quantities are continuous functions of the input quantities that are to be incremented. If the output is relatively sensitive to variation in the input, then relatively small increments must be taken. However, if the solution for the incremented input differs qualitatively from the base solution, then the method generally fails. Some examples for which the latter problem occurs are the sudden appearance of a shock, the appearance of leading-edge separation, and the movement of a shock wave or a separation point. In the case of shock movement, the linear theory can be applied in combination with the coordinate straining procedure of Nixon, which has already been noted. This procedure involves applying a shift operator to the incremented input function in order to "line up" the shocks for the base and incremented solutions. The coordinates are then shifted back a proportional distance to obtain the interpolated solutions.

Other Types of Problems Amenable to Treatment by Local Superposition

Although the preceding discussion is restricted to problems for which a change in the output function is effected by varying an input function, a similar treatment is applicable to other kinds of problems. In reference 1, a change in the output function resulted from incrementing an input parameter, such as Mach number, angle of attack, or thickness ratio. Using the present method, it would be possible to compute the effect of incrementing each of these parameters separately, and then approximate the effects of incrementing various combinations of them simultaneously by linearly superimposing the separate increments.

Another type of problem involves the increment in a single output quantity as a result of incrementing an input parameter. Examples of such problems include computing the lift of an airfoil or the wave drag of a supersonic configuration as a function of Mach number, angle of attack, or thickness ratio. Since, in this case, the output quantity is a simple function of several variables, the partial derivatives with respect to each independent variable can be approximated, and the total differential due to varying combinations of them can be computed by the linear nature of the derivatives.

Suppose, on the other hand, that the increments in an output quantity such as lift or drag were due, not to incrementing a parameter, but to varying the configuration shape. The output quantity is then denoted a functional (ref. 3, p. 9), depending on the input function that describes the boundary shape. By computing the increments due to variation in the boundary shape, one can obtain the effects of linearly combining the variations. An interesting problem of this type is that of studying the changes in the effective perceived noise level at a ground point due to variations in the flight pattern or engine operation of a jet airplane.

Finally, it should be pointed out that the inputs to this type of procedure are by no means limited to those obtainable from an analytic formula, or a computational procedure, but could just as well represent experimental data.

CONCLUDING REMARKS

A theory has been presented for reducing the usage of large computational algorithms. Such algorithms are treated in this theory as nonlinear operators operating on an input function. The method requires a procedure for obtaining computer solutions for a base problem, and for a limited number of variations of this problem. The solutions of other problems that differ slightly from the base problem are then found by forming linear combinations of the increments obtained for the computed variations. The examples given involved nonlinear analysis of subsonic airfoils and of supersonic forebody shapes.

For a restricted class of problems, a function-space derivative can be approximated. For such cases, only one variation from the base problem is required in order to approximate solutions for general variations. The example that was described treated the solution for the pressure distribution on an axisymmetric supersonic forebody as an operator on the slope distribution of the body meridian.

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March 31, 1981

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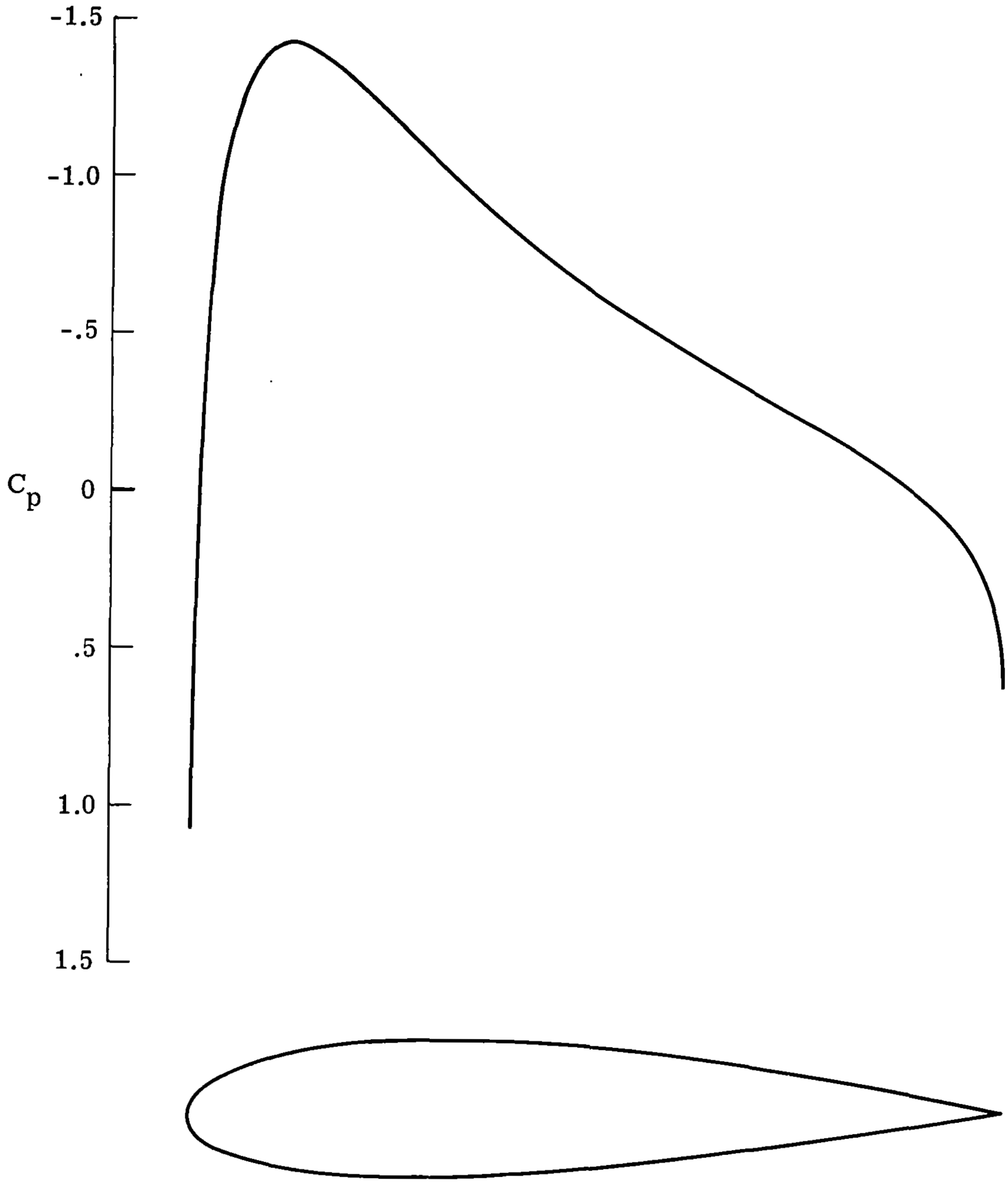
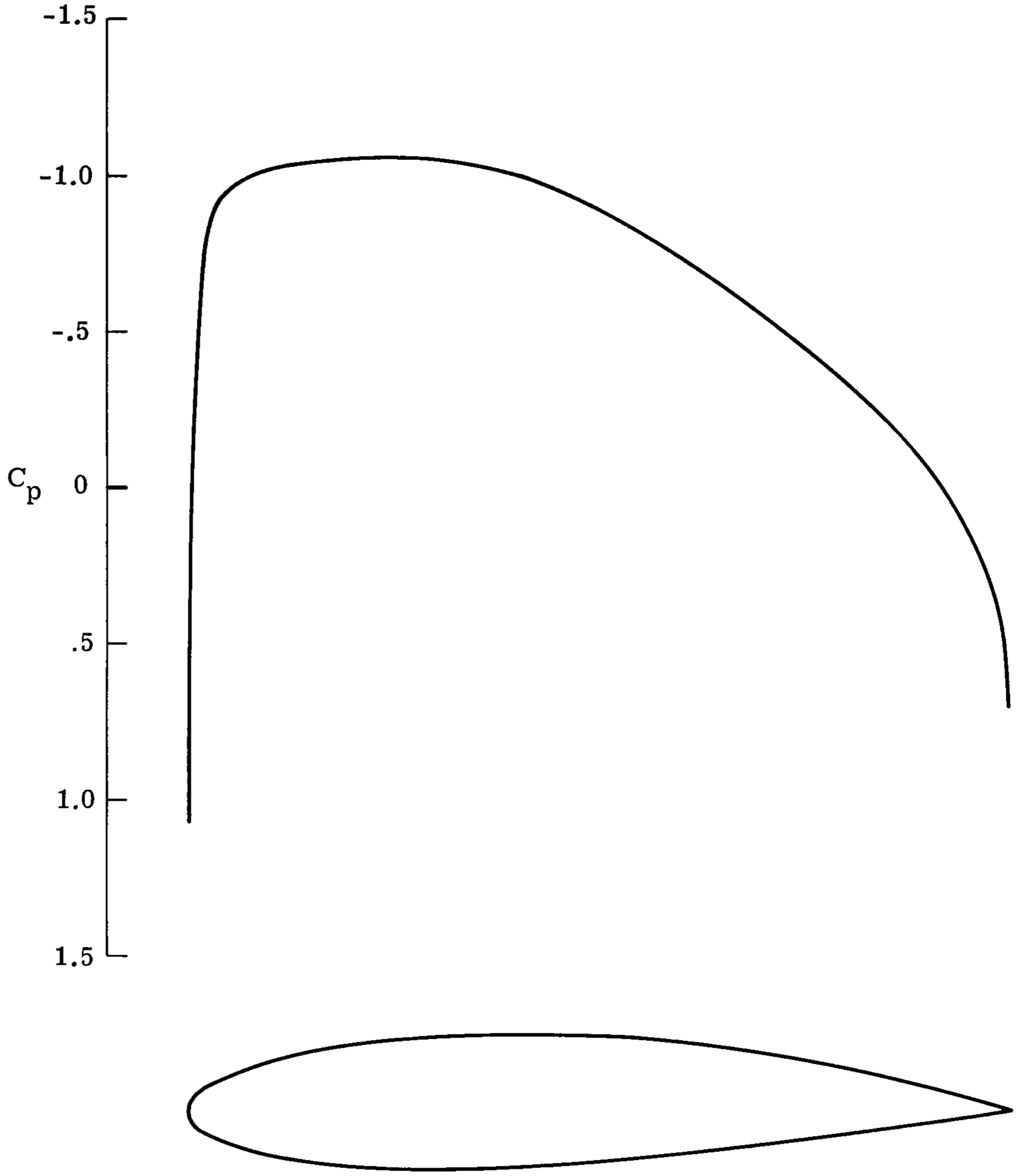
TABLE I.- PRESSURE CALCULATIONS FOR AIRFOIL DEFINED BY $F_o + 0.6h_1 - 0.4h_2$

Station, x	$C_{p,o}$	$C_{p,1}$	$C_{p,2}$	$\Delta_1 C_p$	$\Delta_2 C_p$	$C_p + 0.6\ \Delta_1 C_p - 0.4\ \Delta_2 C_p$	Exact C_p
0.05	-1.130	-0.955	-1.140	0.175	-0.010	-1.021	-1.014
.10	-1.400	-1.015	-1.566	.385	-.166	-1.103	-1.090
.14	-1.406	-1.033	-1.674	.373	-.268	-1.075	-1.088
.20	-1.285	-1.051	-1.565	.234	-.280	-1.033	-1.055
.25	-1.164	-1.0575	-1.410	.1065	-.246	-1.002	-1.016
.30	-1.038	-1.051	-1.250	-.013	-.212	-.961	-.968
.35	-.916	-1.031	-1.105	-.115	-.189	-.909	-.913
.40	-.795	-.997	-.970	-.202	-.175	-.841	-.847
.50	-.607	-.880	-.730	-.273	-.123	-.722	-.717
.60	-.440	-.735	-.530	-.295	-.090	-.581	-.570
.69	-.301	-.570	-.365	-.269	-.064	-.437	-.430
.80	-.135	-.339	-.167	-.204	-.032	-.245	-.240
.90	.038	-.069	.043	-.107	.005	-.024	-.020



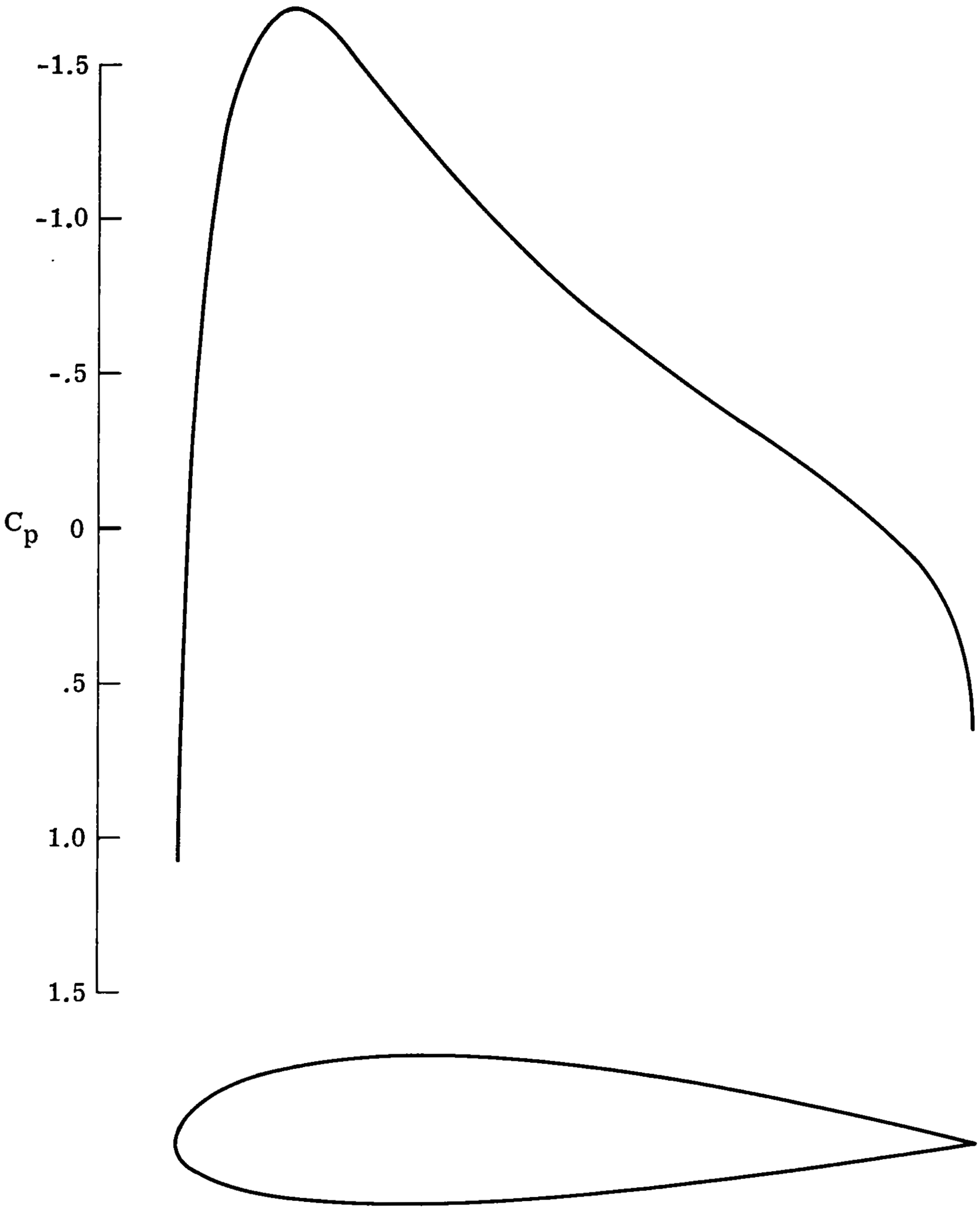
(a) Base configuration.

Figure 1.- Original (base) airfoil configuration and two upper surface variations with corresponding pressure distributions. $M = 0.56$; $\alpha = 2.0^{\circ}$.



(b) First variation of airfoil configuration.

Figure 1.- Continued.



(c) Second variation of airfoil configuration.

Figure 1.- Concluded.

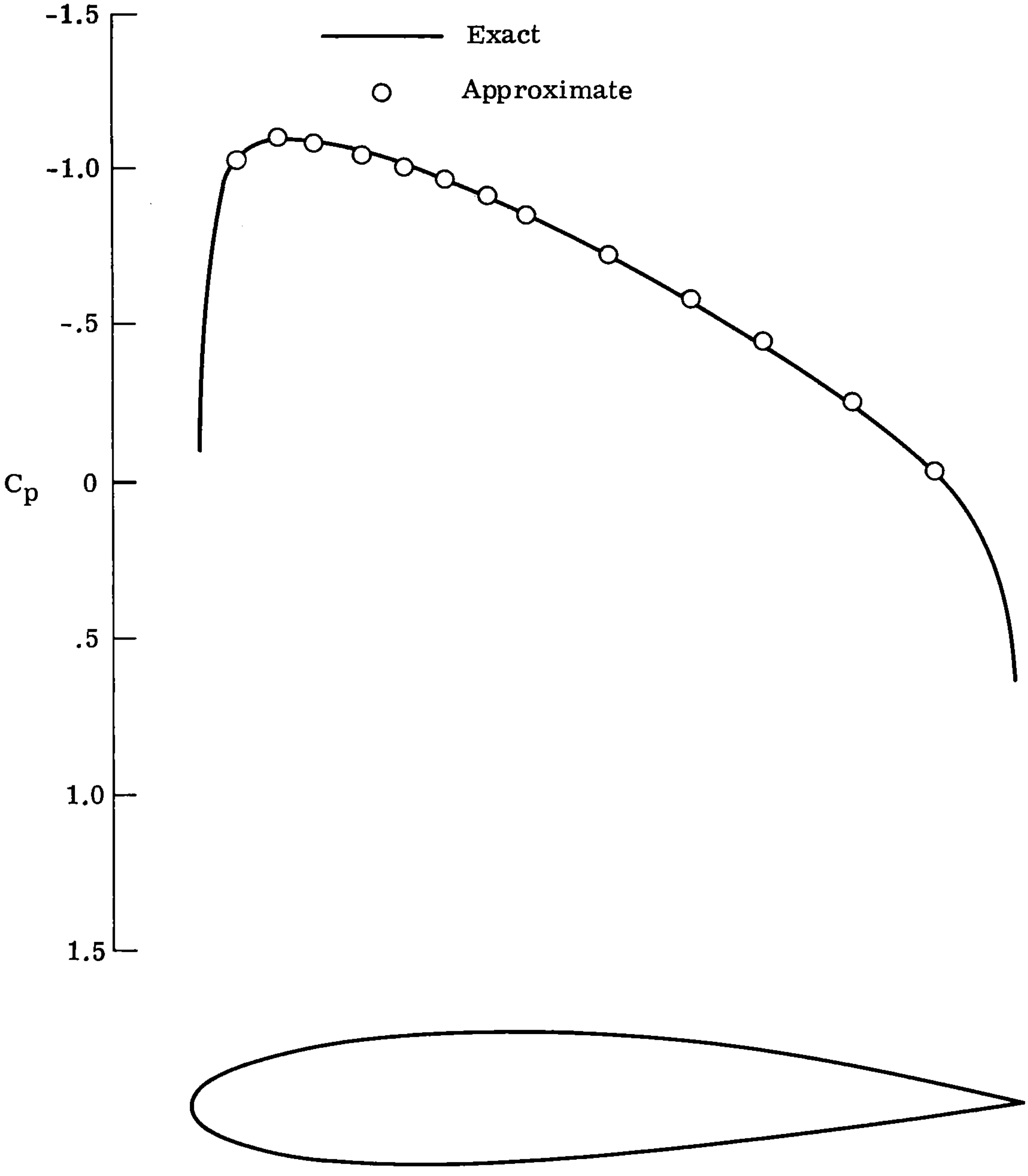
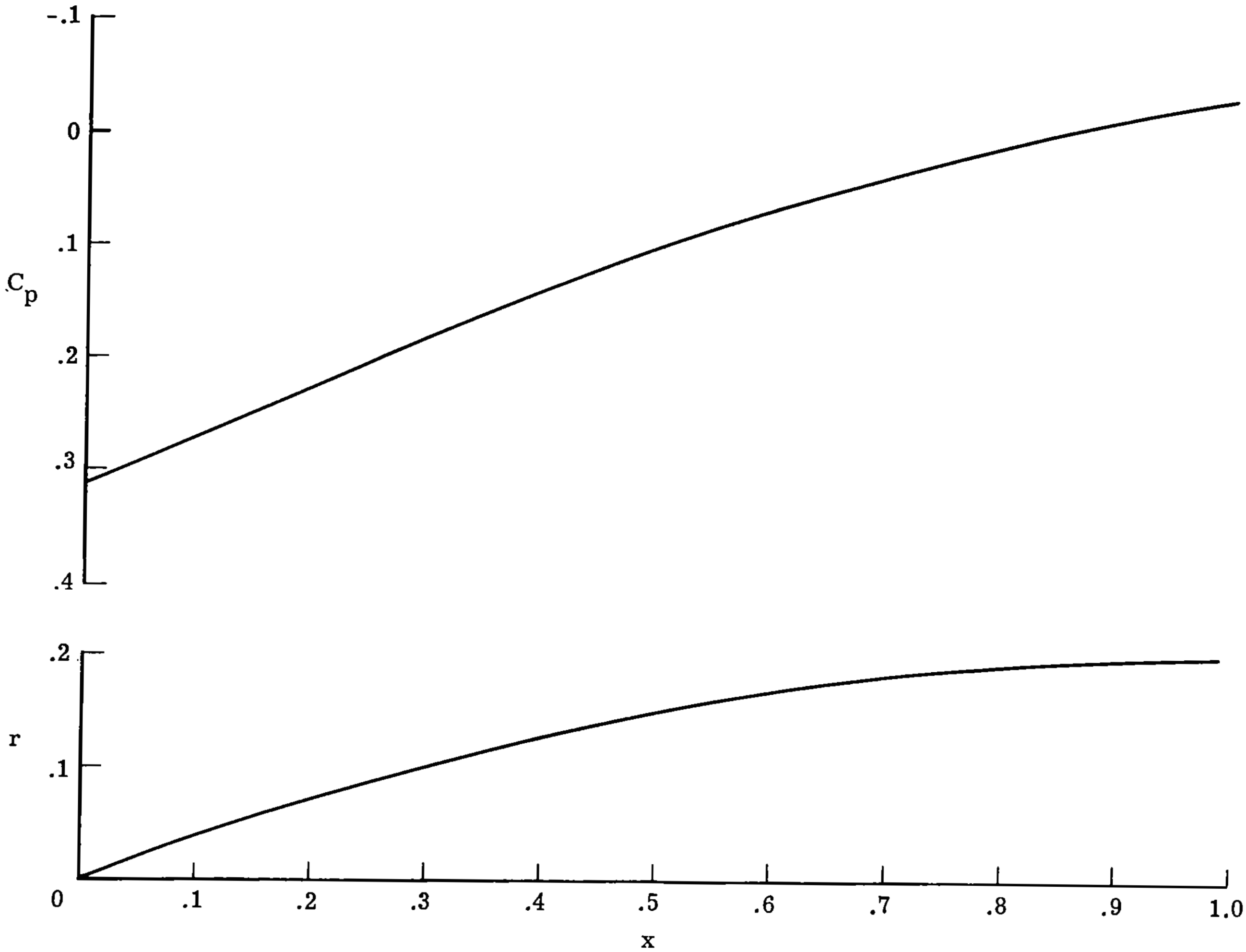
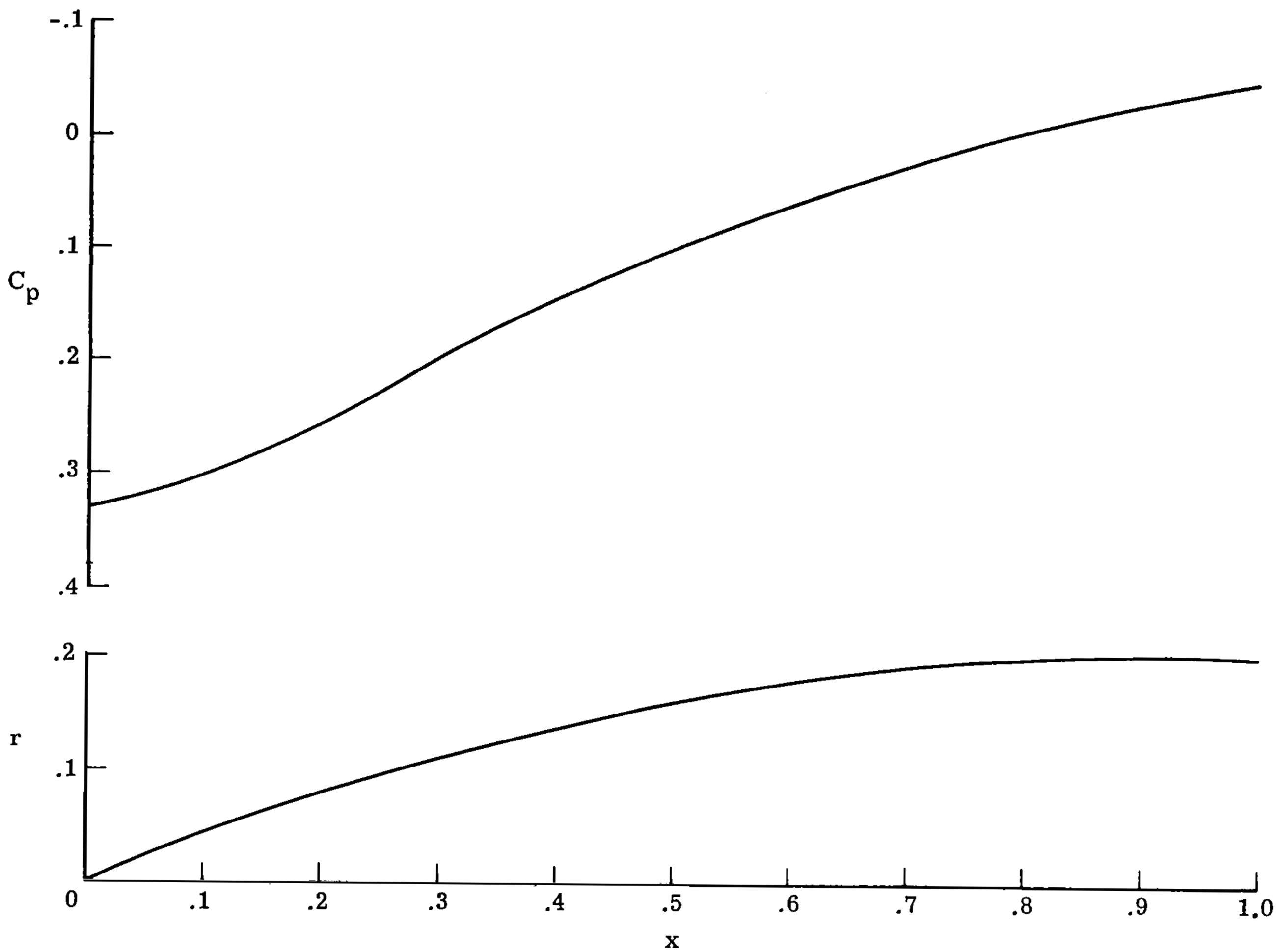


Figure 2.- Comparison of exact and approximate airfoils for composite upper surface $f_o + 0.6h_1 - 0.4h_2$.



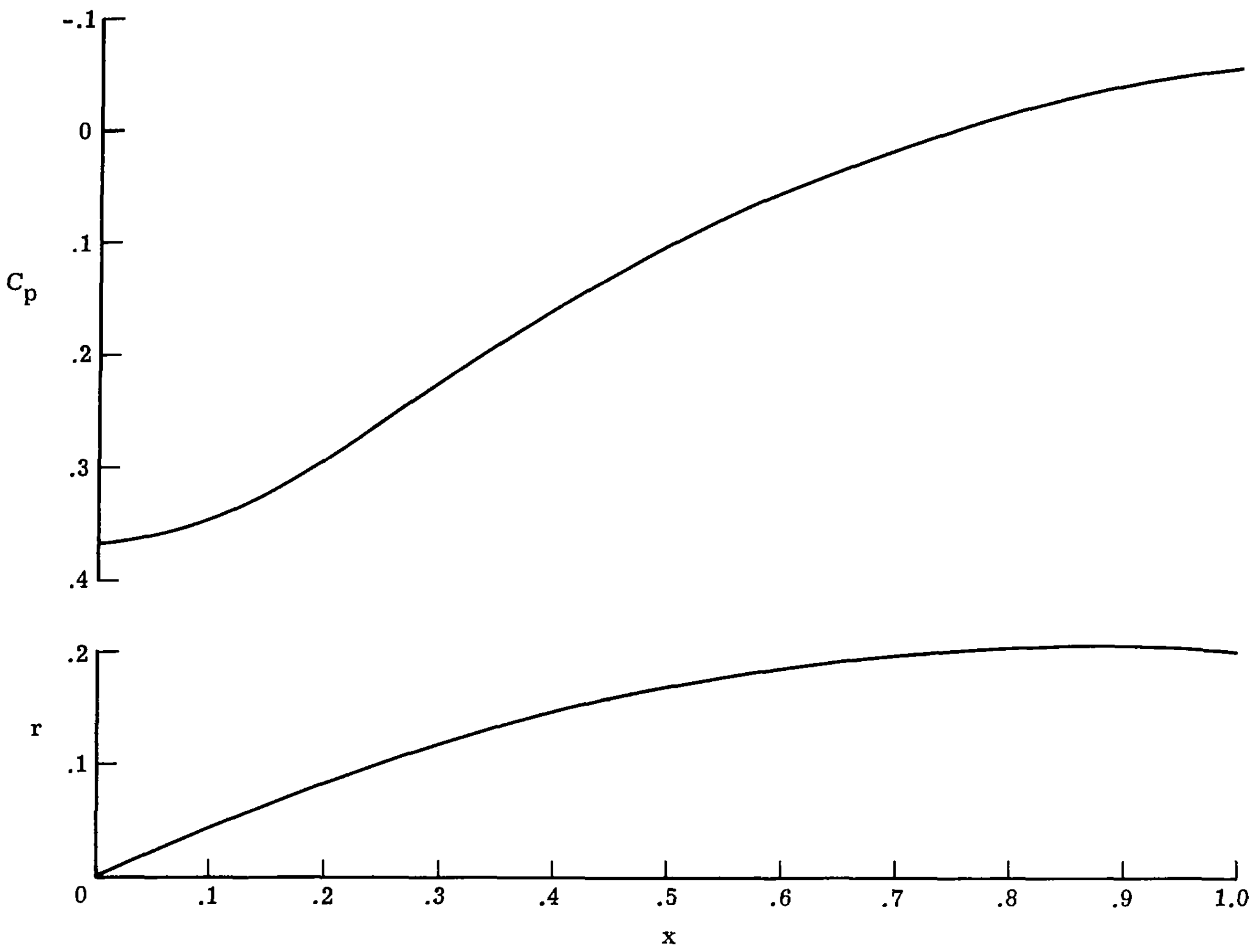
(a) Base configuration.

Figure 3.- Supersonic forebody (base) configuration and two variations with corresponding pressure distributions. $M = 3.0$; $\alpha = 0°$.



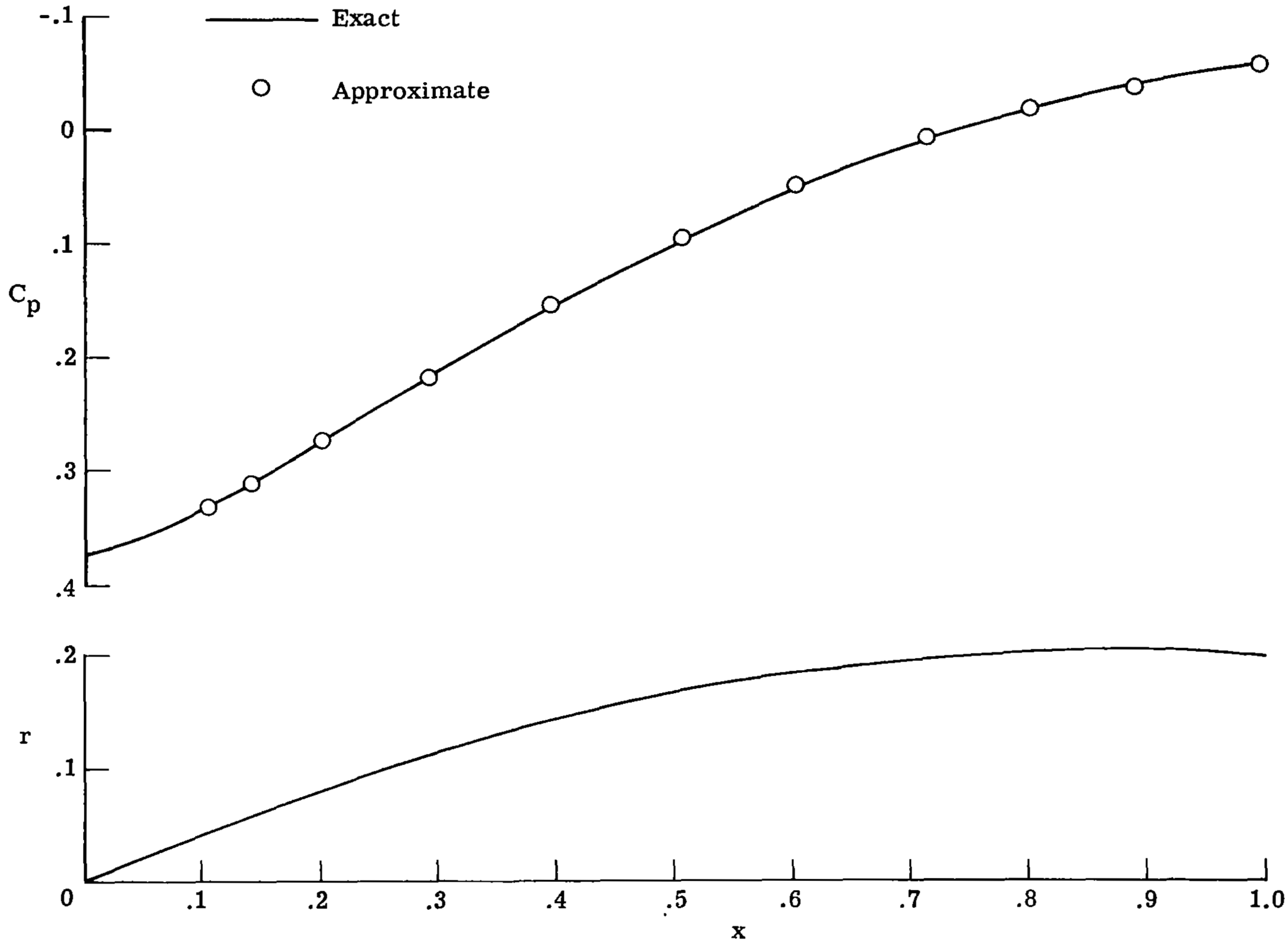
(b) First variation $h_1(x)$.

Figure 3.- Continued.



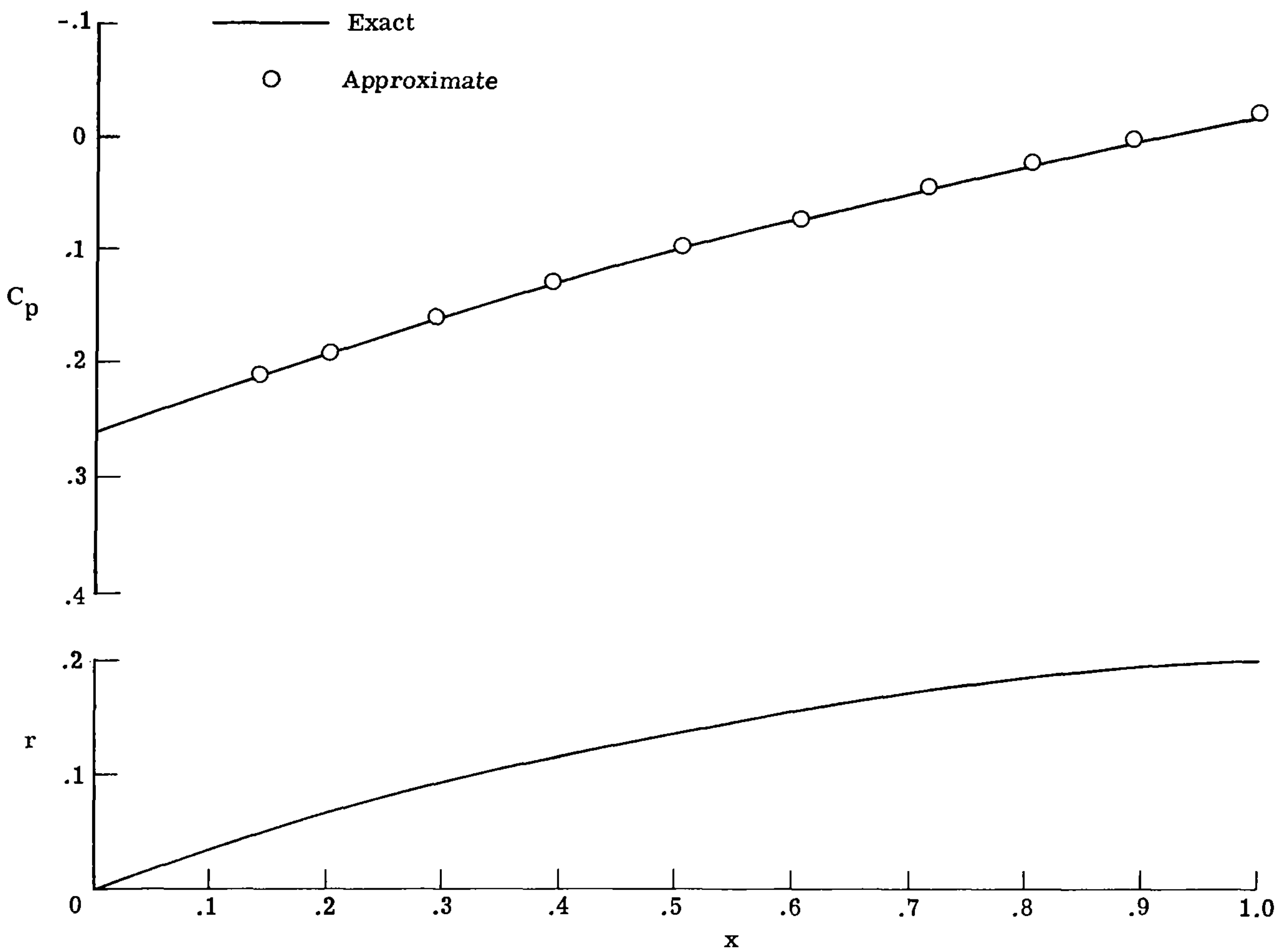
(c) Second variation.

Figure 3.- Concluded.



(a) First composite body $f_o + 0.3h_1 + 0.7h_2$.

Figure 4.- Comparison of exact and approximate theories for two composite configurations synthesized from the base configuration f_o and the two variations h_1 and h_2. $M = 3.0$; $\alpha = 0^o$.



(b) Second composite body $f_o + 0.3h_1 - 0.7h_3$.

Figure 4.- Concluded.

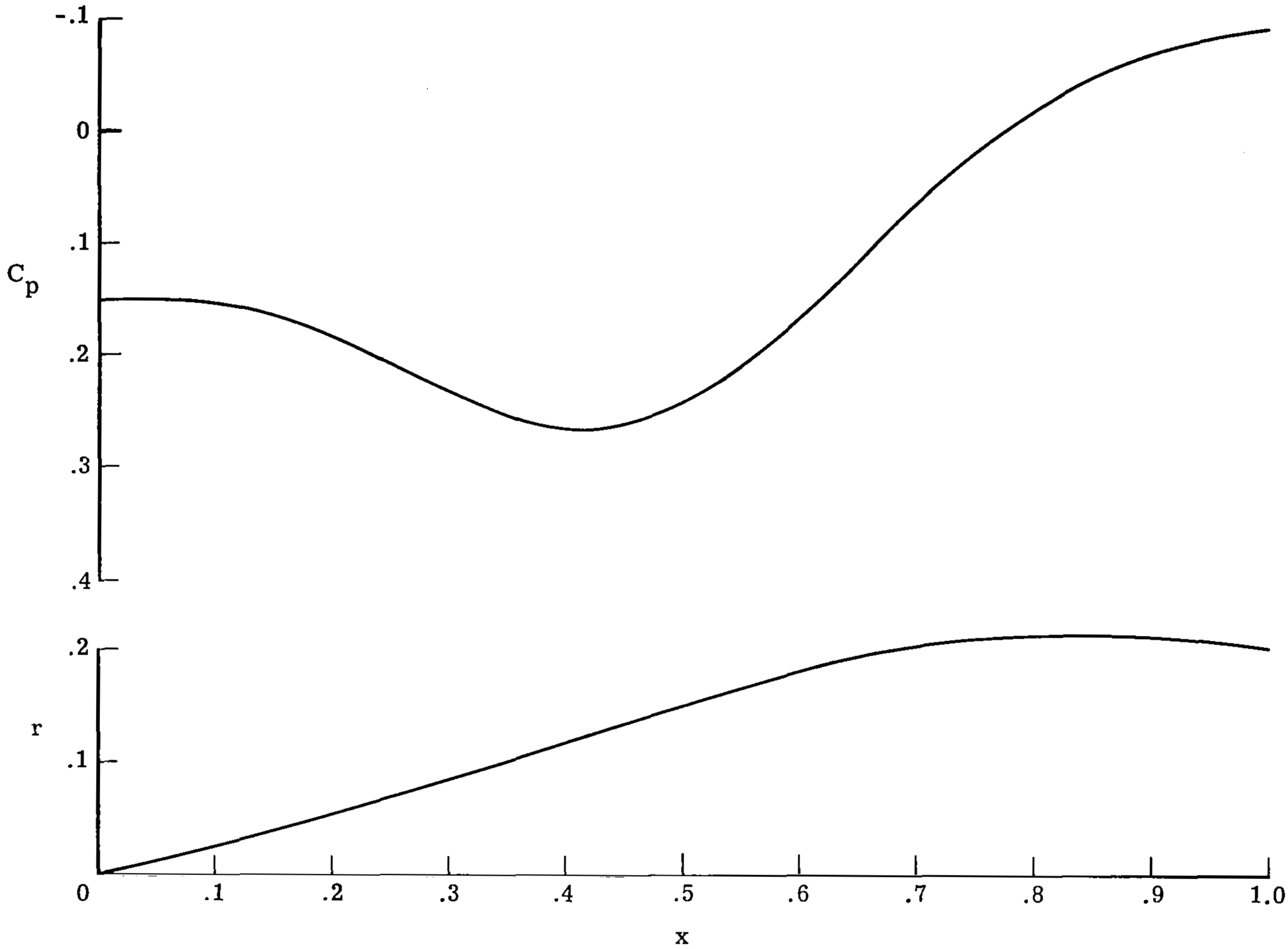


Figure 5.- Example of a variation h_3 representing a relatively large deviation from the base configuration f_o, with corresponding pressure distribution. $M = 3.0$; $\alpha = 0^o$.

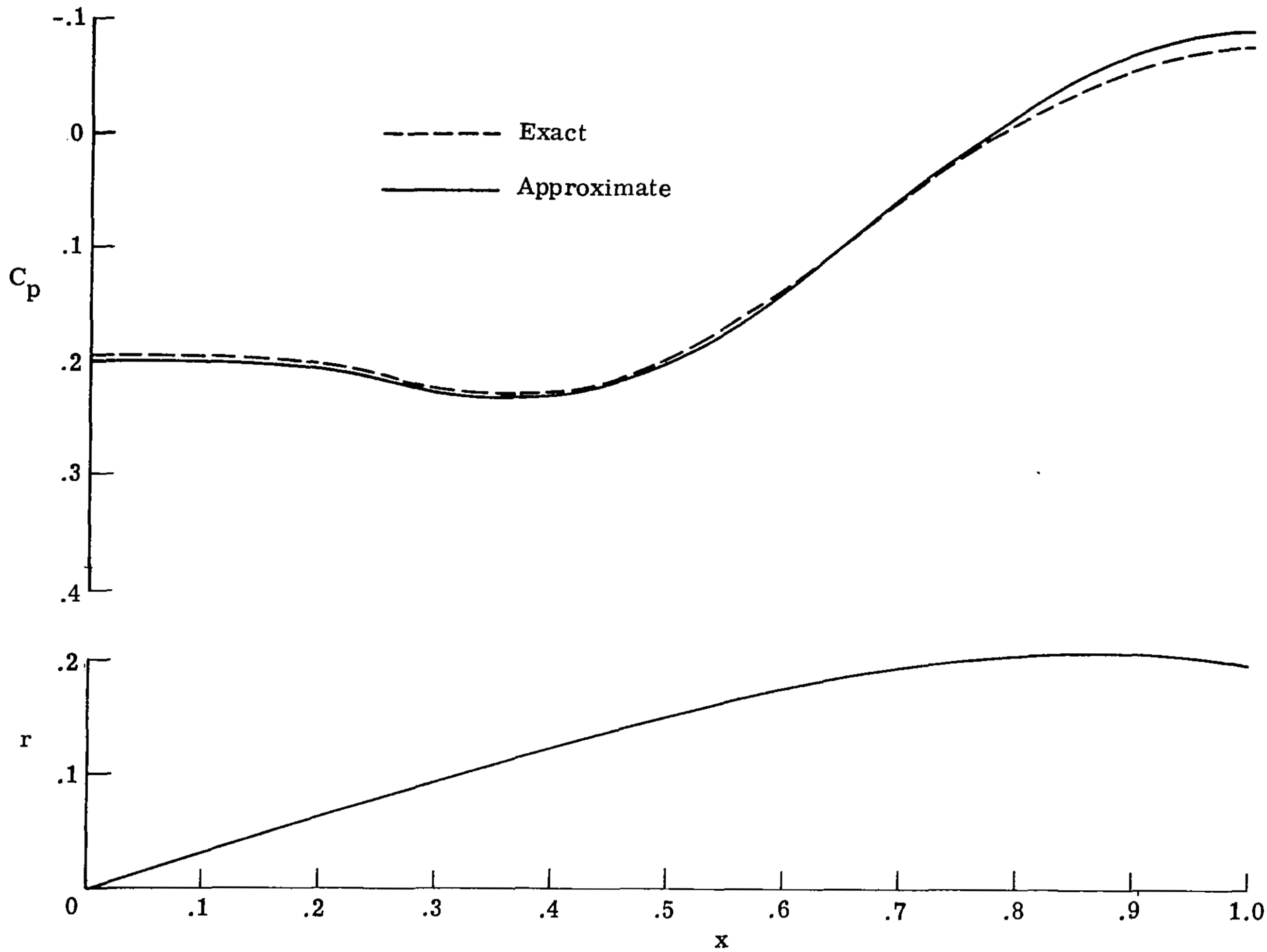
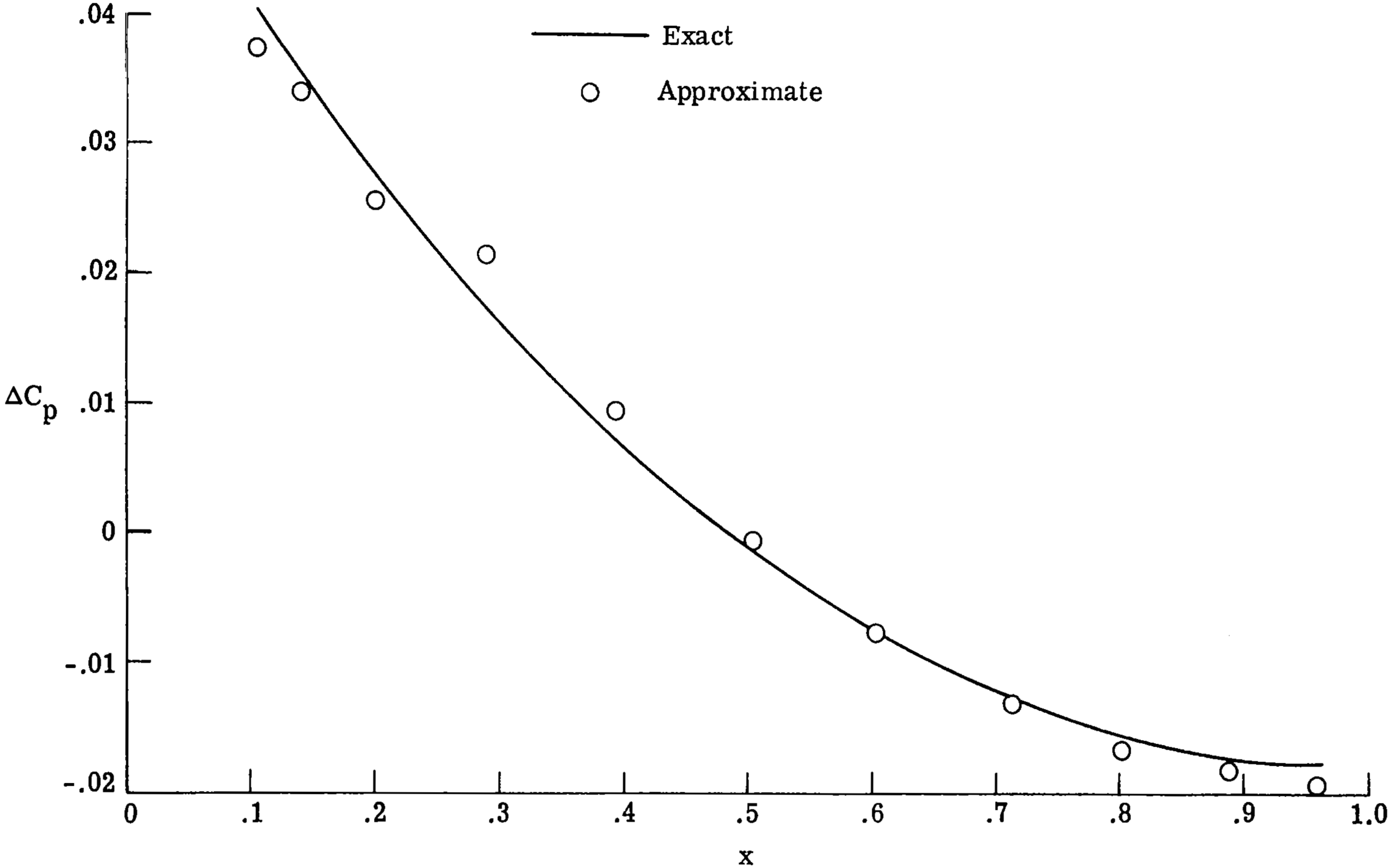
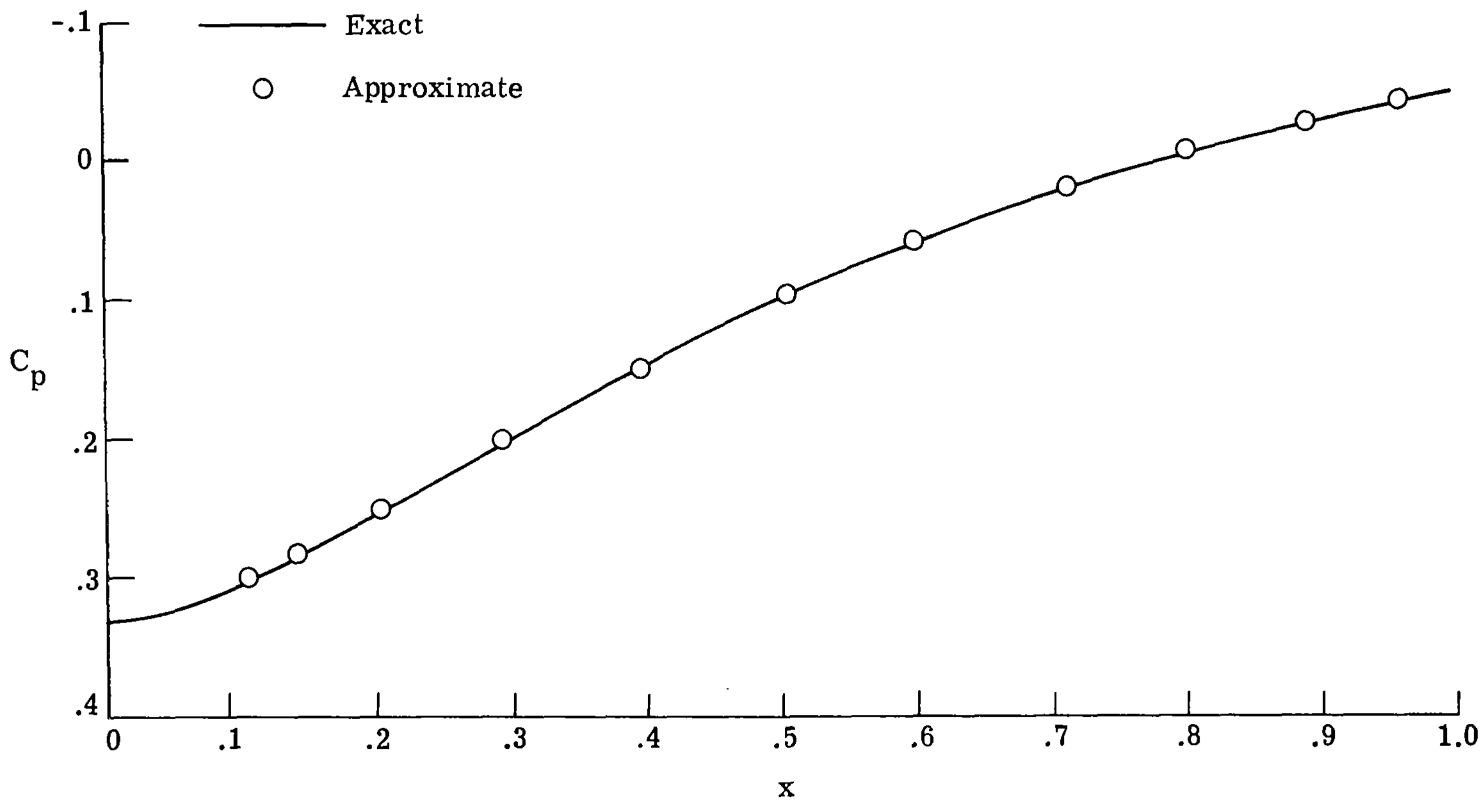


Figure 6.- Comparison of exact and approximate theories for the composite configuration $f_o + 0.3h_1 + 0.7h_3$. $M = 3.0$; $\alpha = 0^o$.



(a) Pressure increment.

Figure 7.- Comparison of exact and approximate calculations utilizing function-space derivation approximation for the h_1 variation. $M = 3.0$; $\alpha = 0^{\circ}$.



(b) Pressure coefficient.

Figure 7.- Concluded.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TP-1857		

4. Title and Subtitle
SOLUTION OF COMPLEX NONLINEAR PROBLEMS BY A GENERALIZED APPLICATION OF THE METHOD OF BASE AND COMPARISON SOLUTIONS WITH APPLICATIONS TO AERODYNAMICS PROBLEMS

5. Report Date
May 1981

6. Performing Organization Code
505-43-23-02

7. Author(s)
Raymond L. Barger

8. Performing Organization Report No.
L-14197

9. Performing Organization Name and Address
NASA Langley Research Center
Hampton, VA 23665

10. Work Unit No.

11. Contract or Grant No.

13. Type of Report and Period Covered
Technical Paper

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, DC 20546

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A theory for obtaining approximate solutions to nonlinear problems whose exact solutions require the use of large computational procedures is described. The technique represents in some respects a generalization of the method of base and comparison solutions for flows depending on a parameter. For the generalized problem, the input variable is no longer a parameter but a function that is incremented over its entire domain. After performing calculations for a base configuration and a small number of variations of it, solutions for a large class of configurations can be obtained by forming linear combinations of the solution increments. For a restricted class of problems, approximate solutions can be obtained for general variations of a base configuration by using a function-space derivative estimate obtained from a base solution and a single variation.

17. Key Words (Suggested by Author(s))
Computer solutions
Aerodynamics problems
Nonlinear problems

18. Distribution Statement
Unclassified - Unlimited

Subject Category 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	26	A03

National Aeronautics and
Space Administration

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