NASA Technical Paper 1773

Kinematic Properties of the Helicopter in Coordinated Turns

Robert T. N. Chen and James A. Jeske

APRIL 1981

NASA

NASA Technical Paper 1773

Kinematic Properties of the Helicopter in Coordinated Turns

Robert T. N. Chen and James A. Jeske
Ames Research Center
Moffett Field, California

NASA

D, Y_w, L total aircraft drag, side force, lift along wind axes

g gravitational acceleration

$\dot{h}$ rate of climb

I_{xx} moment of inertia about x-body axis

I_{yy} moment of inertia about y-body axis

I_{zz} moment of inertia about z-body axis

I_{xz}, I_{yz}, I_{xy} product of inertias in body axes

L total aerodynamic and propulsive moment about x-body axis

M total aerodynamic and propulsive moment about y-body axis

m aircraft mass

N total aerodynamic and propulsive moment about z-body axis

n ratio of the total aerodynamic and propulsive force acting on the aircraft to the weight of the aircraft

n_T ratio of the total aerodynamic and propulsive force acting on the aircraft perpendicular to the flightpath to the weight of the aircraft

n_V ratio of the total aerodynamic and propulsive force acting on the aircraft along the flightpath to the weight of the aircraft

n_x, n_y, n_z accelerometer reading along the body axes (also referred to as n-parameters along the body axes)

$n_{x_w}, n_{y_w}, n_{z_w}$ n-parameters along the wind axes

p, q, r angular rates of the aircraft about body axes

R radius of turn

u, v, w airspeed components in the aircraft body axes system

u_I, v_I, w_I components of flight velocity ($\vec{V}_I$) in the body axes system

u_w, v_w, w_w components of wind velocity ($\vec{V}_w$) in the Earth reference axes system

V	airspeed (i.e., speed of flight with respect to air mass)
V_I	speed of flight with respect to earth
V_w	speed of the wind with respect to the Earth
W	weight of the aircraft
X	total aerodynamic and propulsive force acting on the aircraft along x-body axis
x,y,z	body axes system
x_I, y_I, z_I	earth reference axes (regarded as an inertia axes in this report)
x_w, y_w, z_w	wind axes system aligned with D, Y_w, and L
Y	total aerodynamic and propulsive force acting on the aircraft along y-body axis
Z	total aerodynamic and propulsive force acting on the aircraft along z-body axis
α	angle of attack with respect to air mass, $\alpha \triangleq \tan^{-1}\left(\dfrac{w}{u}\right)$
α_I	"angle of attack" with respect to Earth, $\alpha_I \triangleq \tan^{-1}\left(\dfrac{w_I}{u_I}\right)$
β	angle of sideslip with respect to air mass, $\beta \triangleq \sin^{-1}\left(\dfrac{v}{V}\right)$
$\beta*$	alternate definition of angle of sideslip, $\beta* \triangleq \tan^{-1}\left(\dfrac{v}{u}\right)$
β_I	"angle of sideslip" with respect to Earth, $\beta_I \triangleq \sin^{-1}\left(\dfrac{v_I}{V_I}\right)$
Γ	aerodynamic flightpath angle with respect to air mass, $\Gamma = \sin^{-1}\left(\dfrac{\dot{h} - w_w}{V}\right)$
γ	aircraft flightpath angle (with respect to Earth), $\gamma = \sin^{-1}\left(\dfrac{\dot{h}}{V_I}\right)$
$\Delta u, \Delta v, \Delta w$	components of wind velocity ($\vec{V}_w$) in the body axes system
$\delta_e, \delta_c, \delta_a, \delta_p$	longitudinal, collective, lateral, and directional control displacements
Φ, Θ, Ψ	aircraft Euler angles with respect to x_I, y_I, z_I

ϕ_1 tilt angle of the $\vec{n}_T$ from the vertical plane containing the flightpath

$\dot{\psi}$ rate of turn of the aircraft about Earth-referenced vertical axis

KINEMATIC PROPERTIES OF THE HELICOPTER IN COORDINATED TURNS

Robert T. N. Chen and James A. Jeske

Ames Research Center

SUMMARY

This paper describes the results of a study on the kinematic relationship of the variables of helicopter motion in steady, coordinated turns involving inherent sideslip. A set of exact kinematic equations which govern a steady coordinated helical turn about an earth-referenced vertical axis is first developed. A precise and rational definition for the load factor parameter that best characterizes a coordinated turn is proposed. Formulas are then developed which relate the aircraft angular rates and pitch and roll attitudes to the turn parameters, angle of attack, and inherent sideslip. These new closed-form formulas are then used for a detailed evaluation of the effects of sideslip on the kinematic relationship of the helicopter in coordinated turns. Important symmetrical properties that exist in these kinematic relationships are also discussed.

A steep, coordinated helical turn at extreme angles of attack with inherent sideslip is of primary interest in this study. The results show that the bank angle of the aircraft can differ markedly from the tilt angle of the normal load factor and that the normal load factor can also differ substantially from the accelerometer reading along the vertical body axis of the aircraft. Generally, sideslip has a strong influence on the pitch attitude and roll rate of the helicopter. The latter could have a significant impact on handling qualities because of the direct coupling of roll rate to the thrust of the main rotor.

The results of the analysis also indicate that pitch rate is independent of angle of attack in a coordinated turn and that in the absence of sideslip, angular rates about the stability axes are independent of the aerodynamic characteristics of the aircraft.

INTRODUCTION

With the development of armed helicopters for their expanded roles in missions such as ground attack and air to air combat, the question of helicopter maneuverability is receiving increased attention. Better analytical methods are needed to achieve a reliable prediction of the rotor thrust limits and aircraft performance, thereby permitting an accurate simulation of the trajectory and the orientation of the aircraft in maneuvering flight, especially those flights involving extreme conditions. Efforts have been and are still being made (refs. 1-4) to meet this need. Improved flight test techniques are also needed to evaluate and substantiate the actual maneuvering limitations of the helicopter.

Basic to the flight evaluation of helicopter maneuvering capability is steady and coordinated turning flight (refs. 5-6). These maneuvers are often conducted along steep, helical descending paths to achieve a higher load factor (at a given airspeed) by taking advantage of energy maneuverability (ref. 2). Although the establishment of kinematic relationships in this and other maneuvering flight regimes is normally a major objective of the flight investigation for a specific helicopter (e.g., ref. 5), experience has indicated that test data do not correlate well with predictions from simplified theory. An example showing lack of correlation with regard to the kinematic relationship between the bank angle and the normal load factor is depicted in figure 1.

Efforts have been made (refs. 2 and 6) to improve the theory by properly accounting for the inherent sideslip that normally exists in a coordinated turn for single main rotor helicopters. Unfortunately, in the development of the modified theory two important factors have heretofore been ignored:

1. In a steady coordinated turn about an earth-referenced vertical axis, the tilt of the thrust vector (or more accurately the total aerodynamic and propulsive force) of the helicopter from the vertical plane containing the flight velocity can differ appreciably from the aircraft bank angle, especially in a steep and tight turn with substantial inherent sideslip.

2. The difference between the normal load factor and the accelerometer signal along a vertical body axis having its origin at the center of gravity of the aircraft can be significant, especially in a steep and tight coordinated turn with an extreme angle of attack.

The second factor seems to be a result of an imprecise definition of the term "normal load factor." Neglecting the above two factors can therefore introduce substantial errors in the kinematic relationships among the variables of motion in coordinated turns, especially in those turns involving extreme flight conditions. Therefore, there is a need to clearly spell out a set of exact equations governing steady, coordinated turns in order to provide an accurate calculation of trim conditions, such as aircraft pitch and roll attitudes and angular rates.

Accurate knowledge of aircraft attitudes is necessary in planning, conducting, and interpreting the flight experiment. A more important requirement (but less emphasized) is the knowledge of the angular rates of the helicopter in a coordinated turn. The angular rates about the body axes can exert a significant impact on the performance and handling characteristics of the aircraft. The effect of pitch rate on alleviation of stall of the main rotor is well known (ref. 7). As a corollary, the yaw rate can either alleviate or aggravate stall on the tail rotor, depending on the sense of the yaw rate. Roll rate has a direct coupling to the thrust of rotor systems in forward flight because of the asymmetry in dynamic pressure on the advancing and the retreating sides of the rotors. Further, pitch and roll rates significantly influence the control stick trim position in a coordinated turn because of aerodynamic and inertial (gyroscopic) coupling (e.g., refs. 8, 9, and 10).

The objectives of this research are therefore: (1) to develop exact equations governing a general steady, coordinated helical turn, (2) to examine

the various definitions/interpretations of load factor for helicopters in maneuvering flight and establish their relationships to steady, coordinated turns, and (3) to evaluate the effect of inherent sideslip and other key flight parameters on the helicopter attitudes and angular rates in coordinated turns. The development that achieves these objectives is discussed in the following sections, followed by a summary of the results.

KINEMATICS OF THE HELICOPTER IN STEADY, COORDINATED TURNING FLIGHT

In this section a set of kinematic equations governing the helicopter in a steady, coordinated turn about a vertical axis is developed. Throughout the derivation we shall retain the generality of the equations consistent with the flat-earth approximation (ref. 11); no small-angle assumptions will be used.

In steady, turning flight (i.e., $\dot{u}_I = \dot{v}_I = \dot{w}_I = 0$; $\dot{p} = \dot{q} = \dot{r} = 0$) the Euler equations (ref. 11) reduce to

$$\left.\begin{aligned}
X &= mg \sin\theta + m(qw_I - rv_I) \\
Y &= -mg \cos\theta \sin\Phi + m(ru_I - pw_I) \\
Z &= -mg \cos\theta \cos\Phi + m(pv_I - qu_I)
\end{aligned}\right\} \tag{1a}$$

$$\left.\begin{aligned}
L &= -I_{yz}(q^2 - r^2) - I_{xz}pq + I_{xy}rp - (I_y - I_z)qr \\
M &= -I_{xz}(r^2 - p^2) - I_{xy}qr + I_{yz}pq - (I_z - I_x)rp \\
N &= -I_{xy}(p^2 - q^2) - I_{yz}rp + I_{xz}qr - (I_x - I_y)pq
\end{aligned}\right\} \tag{1b}$$

When the aircraft is making a steady turn about an earth-referenced vertical axis with a steady turning rate of $\dot{\Psi}$, the three components of angular velocity about the body axes of the aircraft become

$$\left.\begin{aligned}
p &= -\dot{\Psi} \sin\theta \\
q &= \dot{\Psi} \cos\theta \sin\Phi \\
r &= \dot{\Psi} \cos\theta \cos\Phi
\end{aligned}\right\} \tag{2}$$

Now replacing the three force components X, Y, and Z along the body axes in equation (1a) by the accelerometer readings at the center of gravity of the aircraft, i.e., $n_X = X/mg$, $n_y = Y/mg$, and $n_z = Z/mg$, and expressing the three components of the flight velocity V_I by

$$\left.\begin{aligned}
u_I &= V_I \cos\alpha_I \cos\beta_I \\
v_I &= V_I \sin\beta_I \\
w_I &= V_I \sin\alpha_I \cos\beta_I
\end{aligned}\right\} \tag{3}$$

3

Equation (1a), by virtue of (2) and the substitution above, becomes

$$n_x = \sin \Theta + \tan \phi_1 (\sin \alpha_I \cos \beta_I \cos \Theta \sin \Phi - \sin \beta_I \cos \Theta \cos \Phi)$$

$$n_y = -\cos \Theta \sin \Phi + \tan \phi_1 \cos \beta_I (\cos \alpha_I \cos \Theta \cos \Phi + \sin \alpha_I \sin \Theta) \qquad (4)$$

$$n_z = -\cos \Theta \cos \Phi - \tan \phi_1 (\sin \beta_I \sin \Theta + \cos \alpha_I \cos \beta_I \cos \Theta \sin \Phi)$$

where

$$\tan \phi_1 = \frac{\dot{\psi} V_I}{g} \qquad (5)$$

For a coordinated turn, $n_y = 0$, and the second equation in equations (4) yields the following constraint equation

$$\sin \Phi = \tan \phi_1 (\cos \alpha_I \cos \Phi + \sin \alpha_I \tan \Theta) \cos \beta_I \qquad (6)$$

Further, the aircraft pitch and roll attitudes Θ, Φ, and the angle of attack and sideslip are related to a given flightpath angle γ by the kinematic relationship:

$$\cos \alpha_I \cos \beta_I \sin \Theta - (\sin \beta_I \sin \Phi + \sin \alpha_I \cos \beta_I \cos \Phi) \cos \Theta = \sin \gamma$$

$$(7)$$

Note that in steady flight, the force components X, Y, Z (or n_x, n_y, n_z) and the three moment components, L, M, and N are functions of the aerodynamic flight parameters, V, α, β; p, q, r, and control positions (e.g., δ_e, δ_c, δ_a, δ_p). Symbolically,[1]

$$n_x = f(V, \alpha, \beta; p, q, r; \delta_e, \delta_c, \delta_a, \delta_p)$$

with similar functional relationships for n_y, n_z, L, M, and N. It should be emphasized here that V, α, and β are related to air mass; V_I, α_I, β_I and γ are related to earth. Figure 2 shows the relationship of the two sets of parameters for the case when $\beta = \beta_I = 0$. Let $(u_w, v_w, w_w)^T$ be the steady wind velocity V_w (relative to earth) as expressed in the earth referenced axes x_I, y_I, z_I. Then the three components of the flight velocity (relative to earth) as expressed in the body axes are given by[2]

_____

[1]For the steady flight considered here, the dependency on $\dot{V}$, $\dot{\alpha}$, $\dot{\beta}$ has been dropped.

[2]The definition for the angle of attack and angle of sideslip is a standard one (ref. 11). An alternate definition for the angle of sideslip, which is sometimes used in the helicopter technical community, is

$$\beta^* = \tan^{-1}\left(\frac{v}{u}\right)$$

We recommend that an asterisk superscript be assigned for this particular definition to avoid unnecessary confusion with β.

$$u_I = V \cos \alpha \cos \beta + \Delta u$$

$$v_I = V \sin \beta + \Delta v$$

$$w_I = V \sin \alpha \cos \beta + \Delta w$$

where

$$
\begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix} =
\begin{bmatrix}
\cos \Theta \cos \Psi & \cos \Theta \sin \Psi & -\sin \Theta \\
\begin{array}{c} \sin \Theta \sin \Phi \cos \Psi \\ - \cos \Phi \sin \Psi \end{array} & \begin{array}{c} \sin \Theta \sin \Phi \sin \Psi \\ + \cos \Phi \cos \Psi \end{array} & \sin \Phi \cos \Theta \\
\begin{array}{c} \sin \Theta \cos \Phi \cos \Psi \\ + \sin \Phi \sin \Psi \end{array} & \begin{array}{c} \sin \Theta \cos \Phi \sin \Psi \\ - \sin \Phi \cos \Psi \end{array} & \cos \Phi \cos \Theta
\end{bmatrix}
\begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix}
$$

and

$$V_I = (u_I^2 + v_I^2 + w_I^2)^{1/2}$$

$$\alpha_I = \tan^{-1} \frac{w_I}{u_I}$$

$$\beta_I = \sin^{-1} \frac{v_I}{V_I}$$

Clearly, in the calm air situation, $V_I = V$, $\alpha_I = \alpha$, and $\beta_I = \beta$. Thus for a given set of $\dot{\Psi}$, V_I, and γ, the eleven algebraic equations (1b), (2), (4), (6), and (7) completely determine the eleven unknown trim values in a steady coordinated turn, i.e., Euler attitudes Θ, Φ; angular rates p, q, r; the angle of attack α, angle of sideslip β, and the four control variables of the aircraft. For convenience, this set of equations is summarized in table 1.

It should be noted that equations (6) and (7) are the two important kinematic equations that relate the body axis Euler angles Θ and Φ to the flight parameters γ, α_I, β_I, and ϕ_1. As discussed earlier in the introduction, a single main rotor helicopter normally exhibits some sideslip in a coordinated turn, the amount of which depends on the aircraft configuration as well as the side-force characteristics.

The turn parameter ϕ_1 that relates the turn rate and the speed of flight as given by equation (5) is kinematically related to the load factor which is discussed next.

5

LOAD FACTOR AND TURNING PERFORMANCE

The lack of a precise and rational definition of the term "load factor" and the fact that the roll attitude and the thrust vector tilt are generally different may well be the two major sources that contribute to the discrepancies between theory and flight test results relating aircraft bank angle to load factor. In this section we shall first describe analytically the various uses of the term "load factor" for a helicopter in maneuvering flight. The differences between these definitions are then examined in both steady straight flight and coordinated turns. Finally, a definition of load factor with its rationale is then proposed for universal use for the helicopter in maneuvering flight.

A review of the helicopter literature related to maneuvering flight reveals that there are at least three definitions (or interpretations) for the term "load factor." They are: (1) the normal acceleration in units of g, n_T (refs. 2, 5, 6, and 12), (2) the accelerometer reading at the c.g. of the aircraft along the vertical body axis, $-n_z$ (refs. 2, 5, and 6), and (3) the thrust to weight ratio $n' = T/W$ (refs. 4, 10, 12).

The lack of a unified definition or interpretation of load factor for a pure helicopter in maneuvering flight may stem from the fact that the thrust of the main rotor is used for lifting the aircraft as well as for providing the propulsive force. In the case of a conventional fixed-wing aircraft (CTOL), the propulsive force is not obtained from the main lifting device, but rather from a jet or an air screw. The definition of load factor for a CTOL aircraft has been uniquely based on the lift of the aircraft.

For the sake of comparison, we will add to the above three definitions, a fourth one derived from the fixed-wing practice, i.e., (4) the lift to weight ratio of the aircraft, $n_{z_W} = L/W$ (refs. 11, 12).

Note that the thrust to weight ratio, n', generally differs from the ratio of the total aerodynamic and propulsive force to weight of the aircraft, n, which is the vector sum of the signals of the three orthogonal accelerometers at the center of gravity of the aircraft and they are normally available in the standard test instrumentation complement. For preciseness, n instead of n' will be used in the subsequent discussions. For a pure helicopter (i.e., conventional rather than compound helicopter) the two quantities, namely, n and n', are approximately equal.

The four definitions of load factor in a steady coordinated turn are interrelated by

$$n_T = \frac{\cos \gamma}{\cos \phi_1} = \left[1 + \left(\frac{\dot{\psi} V_I}{g}\right)^2\right]^{1/2} \cos \gamma \qquad (8a)$$

$$= (n_{z_W}^2 + \tan^2 \beta_I \sin^2 \gamma)^{1/2} \qquad \text{(in calm air)} \qquad (8b)$$

$$-n_z = \cos \Theta \cos \Phi \pm \frac{(n_T^2 - \cos^2 \gamma)^{1/2}}{\cos \gamma} (\sin \beta_I \sin \Theta + \cos \alpha_I \cos \beta_I \cos \Theta \sin \Phi)$$

$$(9)$$

To obtain (8b), one first notes that $n_{x_W} = -\sin \gamma$ and that $n_T^2 = n_{y_W}^2 + n_{z_W}^2$. It can be shown then that $n_{y_W} = -\tan \beta_I \sin \gamma$. In equation (9) it should be noted that $+$ is for right turns and $-$ is for left turns. Further, Θ and Φ satisfy (6) and (7). Also,

$$n = (n_T^2 + \sin^2 \gamma)^{1/2} \tag{10a}$$

$$= (n_x^2 + n_z^2)^{1/2} \tag{10b}$$

and

$$n_{z_W} = n_x \sin \alpha - n_z \cos \alpha \tag{11a}$$

$$= \frac{\tan \alpha_I \sin \gamma}{\cos \beta_I} - \frac{n_z}{\cos \alpha_I} \quad \text{(in calm air)} \tag{11b}$$

From the above relationships, we observe that n is always greater than or equal to either n_T or $-n_z$. In calm air, $n_T \geq n_{z_W}$ always, and $n_{z_W} \geq -n_z$ whenever α_I and γ have the same sign. In a steady coordinated level turn, in particular, it is seen that n is always equal to n_T, and that in calm air,

$$n_T = n_{z_W} = \frac{-n_z}{\cos \alpha_I}$$

indicating that the first, third, and fourth definitions coincide.

The effects of α and β on the four definitions of load factor in steady coordinated 2g (n_T) right turns in calm air are illustrated in figure 3. For level turns note that $n = n_{z_W} = n_T = 2$ throughout and that $-n_z$ is independent of β as confirmed by the above equation. For climbing and descending turns, n is independent of α and β (by the same token n_T is also); n_{z_W} is independent of α, and is symmetrical with respect to γ and β as confirmed by equation (8b); $-n_z$ is symmetrical with respect to β and is also symmetrical with respect to the pair (α,γ) as evident from equations (8b) and (11b). These effects are independent of the direction of the turn.

Figures 4 and 5 show, respectively, the relationships between $-n_z$ and n_T, and n_{z_W} and n_T for a range of n_T from 1 to 3g. The aircraft is flown in calm air and the coordinated turns are right descending with $\gamma = -20°$. Note

that $-n_z$ is strongly influenced by α, but n_{z_W} is independent of α. Both $-n_z$ and n_{z_W} are only weakly affected by the inherent sideslip.

Of fundamental importance is the fact that n and n_T depend upon flight-path and therefore are independent of α and β. On the other hand, both $-n_z$ and n_{z_W} are functions of the inherent sideslip and n_z is also dependent upon angle of attack in a given coordinated turn. They are therefore dependent on the specific aerodynamic characteristics of the helicopter. It is only logical, therefore, to adopt n as the definition of load factor for the helicopter in maneuvering flight. As a corollary the term normal load factor should be given to n_T, not $-n_z$. The value of n_T can be calculated from n using equation (10a) or from $\dot{\Psi}$ using (8a).

To further strengthen the above statement, we shall now consider a special case of coordinated turn, namely, a coordinated turn where the rate of turn $\dot{\Psi}$ approaches zero. This is steady "coordinated" straight flight, which can readily be shown to be identical to steady straight wing-level flight.

Coordinated Steady Straight Flight

With $\dot{\Psi} = 0$ equations (8a), (9), (10a), and (11a) reduce to

$$\left.\begin{aligned} n_T &= \cos \gamma \\ -n_z &= \cos \Theta \\ n &= 1 \\ n_{z_W} &= \cos(\Theta - \alpha) \end{aligned}\right\} \tag{12}$$

The aircraft pitch attitude Θ is strongly influenced by the inherent sideslip in this flight condition. In fact,

$$\sin(\Theta - \alpha_I) = \frac{\sin \gamma}{\cos \beta_I} \tag{13}$$

Thus, while n_T and n are independent of the aerodynamic characteristics of the helicopter, $-n_z$ and n_{z_W} depend on the aerodynamic characteristics as well as wind conditions. In the calm air situation, $\alpha = \alpha_I$, and n_{z_W} becomes

$$n_{z_W} = \left(1 - \frac{\sin^2 \gamma}{\cos^2 \beta_I}\right)^{1/2} \tag{14}$$

When sideslip is absent in coordinated straight flight (generally in the case of fixed-wing aircraft) equation (14) yields $n_{z_W} = \cos \gamma$, which is seen to be identical to n_T.

The amount of inherent sideslip depends on the lateral-directional aero-
dynamic characteristics of the helicopter. Figure 6 shows an example of the
flight test results of three helicopters (from ref. 13) in straight wing-level
flight. As indicated in this figure, substantial inherent sideslip can be
present in low speeds.

Now we return to equation (12). If we use n as the definition of load
factor as proposed earlier in the report, then n = 1 in a steady straight
flight (level or not, and, in fact, coordinated or not). Since steady straight
flight is unaccelerated flight, the choice of n as load factor is indeed
appropriate.

Having determined a rational definition of load factor we now proceed to
examine the rate and radius of turn in a steady helical turn.

Rate and Radius of Turns

In a steady turn at a given speed, an increase in load factor increases
turn rate and reduces turn radius. At a given load factor, an increase in
speed decreases turn rate and increases turn radius. Furthermore, for a given
load factor, the helicopter turns at a faster turn rate and on a smaller
radius in a helical turn than in a level turn.

In a steady helical turn, the turn rate can be derived from equations (8a)
and (10a)

$$\dot{\psi} = \pm \frac{g}{V_I \cos \gamma} (n_T^2 - \cos^2 \gamma)^{1/2} \tag{15a}$$

$$= \pm \frac{g}{V_I \cos \gamma} (n^2 - 1)^{1/2} \tag{15b}$$

where the positive sign is for a right turn and the negative sign is for a
left turn. It follows that the turn radius is

$$R = \frac{V_I^2 \cos^2 \gamma}{g(n_T^2 - \cos^2 \gamma)^{1/2}} \tag{16a}$$

$$= \frac{V_I^2 \cos^2 \gamma}{g(n^2 - 1)^{1/2}} \tag{16b}$$

Figure 7 shows the radius and the rate of turns for several values of the
flightpath angle over a range of flight speeds. The time to complete a 180°
turn for a range of V_I and γ is shown in figure 8.

HELICOPTER ANGULAR RATES AND ATTITUDES IN STEADY COORDINATED TURNS

The aircraft angular velocities about the body axes have significant influence on the thrust capability and stall characteristics of both the main rotor and the tail rotor. Therefore, it is important to examine the effects of the flight parameters such as γ, α, β, and V, as well as load factor on angular velocities of the helicopter in a coordinated turn.

The helicopter pitching velocity, for example, has a well-known effect on the thrust capability of the rotor. A positive-pitching velocity, such as that which exists during a coordinated turn, has been shown to provide an increased thrust or g-capability due to loading of the advancing blade and unloading of the retreating blade, thereby providing a stall-alleviating effect for a lifting rotor. The principal mechanism causing this effect is due to a gyroscopic moment acting on the rotor system (ref. 7). Conversely, a negative pitching velocity will aggravate stall of the rotor system. As a corollary, yaw rate has a significant effect on the stall characteristics of the tail rotor. In fact, it is an important factor to be considered in the design of the tail rotor system (ref. 14). The helicopter roll rate couples directly to the thrust of the main rotor system (ref. 8). The effect is due primarily to the change in the rotor inflow distribution. As such, it is primarily an aerodynamic rather than an inertia effects, as is the case for the pitching velocity discussed previously.

Helicopter Angular Rates in Steady Coordinated Turns

The aircraft angular velocities in the body-axes system in a coordinated helical turn can be developed using equations (2), (6), and (7). From equations (2), it is clear that p, q, and r are functions of aircraft turn rate and the body-axes Euler angles Θ and Φ. Since in a coordinated turn, Θ and Φ are functions of γ, α_I, and β_I as indicated in equations (6) and (7), and the rate of turn is a function of n_T, V_I, and γ, it would seem then that p, q, r would also be functions of n_T, γ, V_I, α_I, and β_I. However, it will be shown in the following that although such is the case for p and r, the pitch rate q is independent of the angle of attack in a coordinated turn.

Equations (6) and (7), respectively, become

$$q = \tan \phi_1 \cos \beta_I (r \cos \alpha_I - p \sin \alpha_I) \tag{16}$$

$$q \tan \beta_I = - \frac{\dot{\psi} \sin \gamma}{\cos \beta_I} - (r \sin \alpha_I + p \cos \alpha_I) \tag{17}$$

by virtue of equations (2).

Denoting the two terms in the parentheses in the above euqations by r' and p' respectively, namely, r' = r cos α_I - p sin α_I, and p' = r sin α_I + p cos α_I, equations (16) and (17) become

10

$$r' = \frac{q}{\tan \phi_1 \cos \beta_I} \tag{18}$$

$$p' = -\frac{\dot{\Psi} \sin \gamma}{\cos \beta_I} - q \tan \beta_I \tag{19}$$

where p' and r' are related to p and r by a rotation in α_I (i.e., loosely, p' and r' are the roll and yaw rates about the stability axes),

$$\begin{pmatrix} p \\ r \end{pmatrix} = \begin{pmatrix} \cos \alpha_I & -\sin \alpha_I \\ \sin \alpha_I & \cos \alpha_I \end{pmatrix} \begin{pmatrix} p' \\ r' \end{pmatrix} \tag{20}$$

Note from equation (20) that $p'^2 + r'^2 = p^2 + r^2$. Therefore, from (2), $p'^2 + r'^2 = \dot{\Psi}^2 - q^2$. When this identity is utilized, equations (18) and (19) will yield the following quadratic equation for the pitch rate:

$$(\cosec^2 \phi_1)q^2 + (2\dot{\Psi} \sin \gamma \sin \beta_I)q + \dot{\Psi}^2(\sin^2 \gamma - \cos^2 \beta_I) = 0 \tag{21}$$

The solution for q is therefore given by

$$q = \dot{\Psi} \sin^2 \phi_1 \left[-\sin \gamma \sin \beta_I \pm \left(\sin^2 \gamma \sin^2 \beta_I - \frac{\sin^2 \gamma - \cos^2 \beta_I}{\sin^2 \phi_1} \right)^{1/2} \right] \tag{22}$$

Since q is always positive in a steady turn, regardless of the turn direction, there is no ambiguity in selecting the proper sign in equation (22). From equation (22), it is clear that in a coordinated turn, pitch rate is independent of angle of attack. However, roll and yaw rates are affected by α_I, which is evident from equations (18), (19), and (20).

In equation (22), the rate of turn is related to V_I, γ, and n_T by equation (15). The two terms $\tan \phi_1$ and $\sin \phi_1$ that are required in equations (18) and (22) can be expressed in terms of the load factor, n, and the flightpath angle, γ, as follows:

$$\tan \phi_1 = \pm \frac{1}{\cos \gamma} (n_T^2 - \cos^2 \gamma)^{1/2} \tag{23a}$$

$$= \pm \frac{1}{\cos \gamma} (n^2 - 1)^{1/2} \tag{23b}$$

and

$$\sin \phi_1 = \pm \frac{1}{n_T} (n_T^2 - \cos^2 \gamma)^{1/2} \tag{24a}$$

$$= \pm \left(\frac{n^2 - 1}{n^2 - \sin^2 \gamma} \right)^{1/2} \tag{24b}$$

11

In equations (23a) through (24b), the positive sign is for a right turn and the negative sign is for a left turn.

Figures 9 and 10 show angular velocities and pitch and roll attitudes in steady, coordinated right turns[3] at $n_T = 2g$ and flight speed of 60 knots with various combinations of α, β, and γ. Note that the aircraft pitch rate is not dependent on α, but the roll and yaw rates are. Note also that roll rate changes sign from positive to negative when the aircraft pitch attitude changes from positive to negative. The effect of the inherent sideslip on the aircraft roll rate is much more pronounced than are either the pitch or the yaw rates, over the range of γ and α considered.

Some symmetrical properties are noteworthy: a simultaneous change in the sign of γ, α, and β results in only a sign change for the roll rate; the signs and the magnitudes of the pitch and yaw rates remain unchanged. When the direction of the turn changes, say from right to left, the magnitudes of the angular rates are symmetrical with respect to the sideslip and the signs of the roll and the yaw rates are changed. Thus, all left turn parameters may be derived from the right turn computations by making the proper sign changes. Symbollically:

$$
\begin{array}{ll}
\text{Right turn} & \text{Left turn} \\
(@\ n_T,\ V_I) & (@\ n_T,\ V_I)
\end{array}
$$

$$
\left.\begin{array}{c}\gamma \\ \alpha \\ \beta\end{array}\right\} \rightarrow \begin{array}{c}p \\ q \\ r\end{array} \qquad \left.\begin{array}{c}\gamma \\ \alpha \\ -\beta\end{array}\right\} \rightarrow \begin{array}{c}-p \\ q \\ -r\end{array}
$$

$$
\left.\begin{array}{c}-\gamma \\ -\alpha \\ -\beta\end{array}\right\} \rightarrow \begin{array}{c}-p \\ q \\ r\end{array} \qquad \left.\begin{array}{c}-\gamma \\ -\alpha \\ \beta\end{array}\right\} \rightarrow \begin{array}{c}p \\ q \\ -r\end{array}
$$

It is of additional interest to examine the behavior of pitch rate for two specific flight conditions, namely (a) coordinated level turns and (b) coordinated turns that have no inherent sideslip (as normally the case for fixed-wing aircraft), and further to observe the influence on roll and yaw rates that follow based on equations (18), (19), and (20).

<u>Coordinated level turn</u>- With $\gamma = 0$, equation (22) becomes

$$q = \dot{\psi} \sin \phi_1 \cos \beta_I \tag{25}$$

The corresponding formulas for p and r are given in the appendix. Using equations (15) and (24a) and noting that for $\gamma = 0$ and $n_T = n$, equation (25) becomes

$$q = \frac{g}{nV_I} (n^2 - 1)\cos \beta_I \tag{26}$$

---

[3]Unless noted otherwise, all the turns are flown in calm air.

Thus, for a coordinated, level turn at a given g-level and speed V_I, the presence of sideslip decreases the aircraft pitch rate, thereby reducing the stall alleviation effect discussed earlier.

More symmetrical properties relevant to the coordinated level turn may be of interest to note. It is evident from equations (25) and (26) that q is independent of α_I and is an even function of β_I, i.e., symmetrical with respect to β_I; that r is symmetrical with respect to the pair (α_I, β_I); and that the roll rate is skew-symmetrical with respect to the pair (α_I, β_I) (i.e., p is an odd function). The last two properties can be seen from equations (18), (19), and (20), and all of the above symmetrical properties are shown in figure 9.

Figure 11 shows the effect of the inherent sideslip on the aircraft angular rates in coordinated, level right turns over a range of normal load factor from 1 g to 3 g. Again, the flight speed is 60 knots, and several values of angle of attack are shown.

<u>Coordinated turn without sideslip</u>- If sideslip is not present in a coordinated turn, then the pitch rate becomes

$$q = \dot{\Psi} \sin \phi_1 \cos \gamma \qquad (27)$$

Formulas for p and r are given in the appendix. Expressed in terms of the load factor, the above equation can also be written as:

$$q = \frac{g(n^2 - 1)}{V_I (n^2 - \sin^2 \gamma)^{1/2}} \qquad (28)$$

It is readily seen from equations (26) and (28) that for the same load factor and speed of flight, a coordinated helical turn (either climbing or descending) increases the aircraft pitch rate over its counterpart in a coordinated level turn. Thus by taking advantage of the rate of change of potential energy of the aircraft, a helicopter at a given airspeed can generate more "g" in a helical turn than in a level turn before encountering stall.

It is of fundamental importance that the pitch rate q is independent of the aerodynamic characteristics of the helicopter. It depends only on the turning parameters, i.e., V_I, n_T, γ, and the direction of the turn.[4] By the same token, the roll and yaw rates about the stability axes, i.e., p' and r', are also independent of the aerodynamic characteristics of the aircraft. In fact, for a coordinated turn, all aircraft have identical angular rates about their stability axes, which are characterized by the same set of four turn parameters if no sideslip is present.

This important kinematic property has far reaching ramifications. Since fixed-wing aircraft normally exhibit no inherent sideslip in a coordinated

---

[4]The turn of interest here is characterized by a set of four turn parameters or their equivalence such as n_T, γ, $\dot{\Psi}$, counting the magnitude and the direction of $\dot{\Psi}$ as two parameters.

turn, the angular rates about the stability axes will be identical for all these aircraft, i.e.,

$$p' = -\dot{\Psi} \sin \gamma$$

$$q' = \dot{\Psi} \sin \phi_1 \cos \gamma \left.\begin{array}{c}\\\\\\\end{array}\right\} \qquad (29)$$

$$r' = \dot{\Psi} \cos \phi_1 \cos \gamma$$

Note that (29) can also be derived directly by transforming $\dot{\Psi}$ through γ and ϕ_1. It follows that all the kinematic terms in the Euler equations (1a) and (1b) will also be identical if these aircraft have the same weight and inertia characteristics referenced to their stability axes. It is only logical, then, to use the stability axes to isolate the influence of inertia effect and to study the influence of aerodynamic characteristics on dynamic behavior during steep, turning flight.

Helicopter Pitch and Roll Attitudes in Coordinated Turns

With the angular rates calculated as shown in the preceding subsection, the aircraft pitch and roll attitudes can be obtained as follows:

$$\Theta = \sin^{-1}\left(\frac{-p}{\dot{\Psi}}\right) \qquad (30a)$$

$$\Phi = \tan^{-1}\left(\frac{q}{r}\right) \qquad (30b)$$

If we substitute the roll and yaw rates into the above equations, the results become

$$\sin \Theta = \frac{\sin \gamma \cos \alpha_I}{\cos \beta_I} + \frac{q}{\dot{\Psi}} \left(\cos \alpha_I \tan \beta_I + \frac{\sin \alpha_I}{\tan \phi_1 \cos \beta_I}\right) \qquad (31a)$$

$$\tan \Phi = \frac{\tan \phi_1 \cos \beta_I}{\cos \alpha_I - \tan \phi_1 \sin \alpha_I \sin \beta_I - (\dot{\Psi}/q)\tan \phi_1 \sin \gamma \sin \alpha_I} \qquad (31b)$$

where q is given by equation (22). For convenience, these equations along with the formulas for the angular rates are summarized in table 2.

Unlike the angular rates, which are functions of α_I, β_I, and the complete set of four turn parameters, the pitch and roll attitudes depend on α_I, β_I, and only the three turn parameters, namely γ, n_T (or ϕ_1), and the direction of the turn. The pitch and roll attitudes in a coordinated turn are independent of the flight speed or the rate of turn. As in the case of the angular rates, there are also interesting symmetrical properties for the pitch and roll attitudes with respect to the direction of the turn, γ, α_I, and β_I as summarized in table 3. Also, it is important to note that $\Phi \neq \phi_1$, unless $\alpha_I = \beta_I = 0$.

14

Figure 10 shows the influence of sideslip on the aircraft attitudes in steep helical turns at $n_T = 2g$. Figure 12 shows its effect over a range of the normal load factor from 1g to 3g. In addition one can see from these illustrations that ϕ_1 and Φ can differ markedly in a coordinated turn at a higher value of n_T, especially at extreme values of γ, α_I, and β_I; and that sideslip has a strong influence on the pitch attitude of the aircraft. The symmetrical properties from these figures indicate that if the pitch and roll attitudes Θ and Φ have been calculated for a right turn at n_T, γ, α_I, and β_I, then their values for a left turn at n_T, γ, α_I, and $-\beta_I$ will be Θ and $-\Phi$. This and other symmetrical properties are shown in table 3.

For a coordinated level turn, equations (30a) and (30b) may be reduced to

$$\sin \Theta = \sin \phi_1 \cos \alpha_I \sin \beta_I + \cos \phi_1 \sin \alpha_I \tag{32a}$$

$$\tan \Phi = \frac{\tan \phi_1 \cos \beta_I}{\cos \alpha_I - \tan \phi_1 \sin \alpha_I \sin \beta_I} \tag{32b}$$

by virtue of equation (25). Figures 10 and 12 show the influence of sideslip on the aircraft attitudes in coordinated level right turns at $n_T = 2g$ and over a range of the normal load factor from 1g to 3g, respectively. Note that the effect of sideslip in a level turn is somewhat weaker as compared to the steeper turn case.

Additional symmetrical properties for a coordinated level turn are (1) the roll attitude is symmetric with respect to the pair (α_I, β_I) and (2) the pitch rate is skew-symmetric with respect to this pair. This can be seen from equations (32a) and (32b).

Further Discussions of the Results

The new formulas developed in the two preceding subsections, which directly connect the aircraft angular rates and pitch and roll attitudes to the turn parameters, angle of attack, and sideslip can be used to drastically simplify the trim computation for the helicopter in a steady coordinated helical turn. This simplification is due to equations (20), (22), (31a), and (31b) essentially decoupling the 11 governing equations shown in table 1.

For a steady coordinated helical turn,[5] the 11 governing equations uniquely determine the trim values of the following 11 flight parameters:

Angle of attack and sideslip α, β (or α_I, β_I)
Aircraft angular rates p, q, r
Aircraft pitch and roll attitudes Θ, Φ
Control positions, e.g., δ_e, δ_c, δ_a, δ_p

---

[5]Recall that the turn in question is characterized by a set of four turn parameters as discussed earlier.

It would be necessary, without those five new formulas for aircraft angular rates and pitch and role attitudes, to invert an associated 11×11 Jacobian matrix in each iterative cycle in the numerical solution of the eleven (non-linear algebraic) governing equations. With the aircraft angular rates and Θ and Φ expressed explicitly in terms of α_I and β_I (see equations (31a) and (31b), the associated Jacobian matrix can be compressed into a simpler 6×6 as normally is the case for a steady straight-flight condition.

Let $\underline{c}$, $\underline{f}$, and $\underline{g}$ be denoted by

$$\left. \begin{aligned} \underline{c} &= (\alpha_I, \ \beta_I; \ \delta_e, \ \delta_c, \ \delta_a, \ \delta_p)^T \\[6pt] \underline{f} &= (f_1, \ f_2, \ f_3, \ f_4, \ f_5, \ f_6)^T \\[6pt] \underline{g} &= (p, \ q, \ r; \ \Theta, \ \Phi)^T \end{aligned} \right\} \tag{33}$$

where $f_1, f_2, \ldots, f_6$ are the first six steady state Euler equations in table 1. Note that $\underline{g} = \underline{g}(\underline{c})$. Then $\underline{f}$ takes the following form

$$\underline{f} = F[\underline{c}, \ \underline{g}(\underline{c})] = 0 \tag{34}$$

The associated six by six Jacobian matrix $\partial \underline{f}/\partial \underline{c}$ becomes

$$\frac{\partial \underline{f}}{\partial \underline{c}} = \frac{\partial F}{\partial \underline{c}} + \frac{\partial F}{\partial \underline{g}} \frac{\partial \underline{g}}{\partial \underline{c}} \tag{35}$$

The Jacobian matrix $\partial \underline{f}/\partial \underline{c}$ is a necessary ingredient in the iterative methods for numerical solution of the trim equations. If, for example, a Newton-type procedure is used, then an algorithm for determining the trim values for the vector $\underline{c}$ beginning with an initial guess $\underline{c}_0$ is of the following sort

$$\underline{c}_{i+1} = \underline{c}_i + \delta \underline{c}_i$$

$$\delta \underline{c}_i = -\lambda_i \left(\frac{\partial \underline{f}}{\partial \underline{c}_i} \right)^{-1} \underline{f}(\underline{c}_i)$$

where $\lambda_i \leq 1$ is a damping parameter for a stable iteration. With the trim value for $\underline{c}$ determined, say $\underline{c}_t$, the desired trim values for the angular rates and Θ and Φ follow immediately, which are

$$\underline{g}_t = \underline{g}_t(\underline{c}_t)$$

In the absence of an inherent sideslip in a steady coordinated turn, as is normally the case for fixed-wing aircraft, the trim computation can be made even simpler. As discussed earlier, the pitch rate, under this condition, is now only a function of the turn parameters; p, r, Θ, and Φ are functions of turn parameters and α_I only. These fortuitous properties should be beneficial in applications for fixed-wing aircraft.

16

In the preceding paragraphs, we have dwelled on the calm air situation. In the presence of a steady wind, the flight parameters at trim $\underline{c}_t$ and $\underline{g}_t$ will no longer be constant. In fact they will depend on the heading of the aircraft except when the wind is purely vertical in the earth-referenced axes system. As such they will become periodic time-varying functions.

In the presence of a horizontal wind, for example, the body axes components of the wind, Δu, Δv, Δw will be functions of the aircraft heading, which is constantly changing with time ($\dot{\Psi}$ = constant). For given wind conditions (direction and magnitudes), the trim computation procedure described above applied at a specific heading of the aircraft, say $\Psi = \Psi_1$, will result in a quasi-steady trim value $(\underline{c}_t, \underline{g}_t)_{\Psi=\Psi_1}$. When the aircraft heading changes from Ψ_1 to Ψ_2 the corresponding quasi-steady trim values $(\underline{c}_t, \underline{g}_t)_{\Psi=\Psi_2}$ will differ from those for the heading at Ψ_1. Thus, a steady flight with constant flight parameters will no longer exist in a coordinated turn in the presence of a steady horizontal wind.

CONCLUDING REMARKS

A set of eleven exact kinematic equations which govern a steady, coordinated, helical turn involving inherent sideslip has been developed in this analytical study. A variety of definitions and interpretations of load factor for the helicopter in maneuvering flight has been examined and interrelationships established for steady coordinated turns. It is concluded that the most logical definition of load factor to be used for the helicopter is the ratio of the total aerodynamic and propulsive force to the weight of the aircraft. This ratio is also the vector sum of the signals of the three orthogonal accelerometers at the c.g. of the aircraft. In a steady coordinated turn, load factor is independent of angle of attack and sideslip; it depends only on the turn parameters. Likewise the load factor normal to the flightpath exhibits properties similar to those of the total load factor. Furthermore, normal load factor, instead of the accelerometer signal along the vertical body axis is more appropriate to associate with turn performance in the presence of sideslip.

New formulas that explicitly relate aircraft angular rates and pitch and roll attitudes to the turn parameters, angle of attack, and sideslip have been obtained. These formulas decouple the eleven governing equations, thus drastically simplifying the computation of the kinematics for the helicopter in steady coordinated turns. Incorporation of these equations into the standard helicopter simulation computer code may improve the accuracy of the trim computation for coordinated turns. A detailed evaluation of the effects of sideslip on the kinematic relationships in calm air indicates the following:

1. In a steep, helical, coordinated turn at high normal load factor and large angles of attack and sideslip, the bank angle of the aircraft can differ markedly from the tilt of the normal load factor. Likewise, the normal load factor can differ substantially from the accelerometer signal along the vertical body axis with the origin at the center of gravity of the aircraft.

17

2. Sideslip has a strong influence on the roll rate and the pitch attitude of the helicopter. It therefore exerts influence on the performance as well as the handling qualities of the helicopter.

3. The pitch rate of the helicopter is dependent on the turn parameters and sideslip; it is independent of angle of attack. The presence of sideslip reduces the pitch rate, thereby reducing the stall alleviation effect on the main rotor system. Also, for the same load factor and speed of flight, an increase in flightpath angle results in an increased pitch rate and augments the stall alleviation.

4. Important symmetrical properties exist for the angular rates and the pitch and roll attitudes with respect to the direction of turn, γ, α_I, and β_I. These properties further simplify analysis and computation. They are summarized in table 3.

In the presence of a steady, horizontal wind there no longer exists a coordinated turn with steady (constant) flight parameters. All of the pertinent variables of the motion, i.e., Θ, Φ; p, q, r; α, β (or α_I and β_I) and the control variables δ_e, δ_c, δ_a, δ_p (e.g.) will change with the heading of the aircraft for a specific turn, characterized by the four turn parameters described earlier in the report.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, January 1981

18

APPENDIX

AIRCRAFT ANGULAR RATES IN COORDINATED LEVEL TURNS AND COORDINATED TURNS WITHOUT SIDESLIP

In this appendix formulas for aircraft angular rates for the two special flight conditions that were examined at some length in the text, namely, (a) coordinated level turns and (b) coordinated turns that have no sideslip, will be derived. It will also be shown here that, under these special conditions, the formulas can be obtained in a more direct manner.

Coordinated Level Turns

The angular rates can readily be obtained by substituting equation (25) into equations (18), (19), and (20), as follows:

$$\left.\begin{array}{l} p = -\dot{\Psi}(\sin \phi_1 \cos \alpha_I \sin \beta_I + \cos \phi_1 \sin \alpha_I) \\[2mm] q = \dot{\Psi} \sin \phi_1 \cos \beta_I \\[2mm] r = -\dot{\Psi}(\sin \phi_1 \sin \alpha_I \sin \beta_I - \cos \phi_1 \cos \alpha_I) \end{array}\right\} \tag{A1}$$

Equation (A1) can also be obtained in a different, but more direct way. From equation (8b) we observe that when $\gamma = 0$ or $\beta_I = 0$, or $\gamma = \beta_I = 0$, then $n_{z_W} = n_T$, which indicates that $n_{y_W} = 0$ and that therefore $\vec{n}_T$ and $-\vec{n}_{z_W}$ coincide. Thus, it follows that the directions of $\vec{V}_I$, $\vec{n}_T$, and $\vec{y}_W$ (which is perpendicular to both $\vec{V}_I$ and $\vec{n}_T$) form the stability axes system ($-x_W$, y_W, $-z_W$). This property for these special cases can now be used to obtain (A1) directly from (29) through rotations in β_I and α_I. For $\gamma = 0$, we have

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{bmatrix} \cos \alpha_I \cos \beta_I & -\cos \alpha_I \sin \beta_I & -\sin \alpha_I \\ \sin \beta_I & \cos \beta_I & 0 \\ \sin \alpha_I \cos \beta_I & -\sin \alpha_I \sin \beta_I & \cos \alpha_I \end{bmatrix} \begin{pmatrix} 0 \\ \dot{\Psi} \sin \phi_1 \\ \dot{\Psi} \cos \phi_1 \end{pmatrix} \tag{A2}$$

which is indeed identical to (A1).

Coordinated Turns Without Sideslip

Under this condition, pitch rate is given by equation (27). By virtue of equations (18), (19), and (20), the angular rates can be shown to be

$$p = -\dot{\Psi}(\cos \alpha_I \sin \gamma + \cos \phi_1 \sin \alpha_I \cos \gamma)$$

$$q = \dot{\Psi} \sin \phi_1 \cos \gamma$$

$$r = -\dot{\Psi}(\sin \alpha_I \sin \gamma - \cos \phi_1 \cos \alpha_I \cos \gamma)$$

(A3)

As in the case of coordinated, level turns, equation (A3) can also be obtained directly from (29) as follows:

$$
\begin{pmatrix} p \\ q \\ r \end{pmatrix} =
\begin{bmatrix} \cos \alpha_I & 0 & -\sin \alpha_I \\ 0 & 1 & 0 \\ \sin \alpha_I & 0 & \cos \alpha_I \end{bmatrix}
\begin{pmatrix} -\dot{\Psi} \sin \gamma \\ \dot{\Psi} \sin \phi_1 \cos \gamma \\ \dot{\Psi} \cos \phi_1 \cos \gamma \end{pmatrix}
$$

(A4)

It should be emphasized that except for these two special cases the angular rates in the body axes system cannot be obtained directly from (29).

REFERENCES

1. Livingston, C. L.: Maneuverability Prediction. Paper presented to the Southwest Region of the American Helicopter Society, Dec. 16, 1966, American Helicopter Society J., vol. 14, no. 1, Jan. 1969, pp. 49-66.

2. Wells, C. D.; and Wood, T. L.: Maneuverability — Theory and Application. American Helicopter Society J., vol. 18, no. 1, Jan. 1973, pp. 10-22.

3. Livingston, C. L.; and Murphy, M. R.: Flying Qualities Considerations in the Design and Development of the Hueycobra. American Helicopter Society J., vol. 14, no. 1, Jan. 1969, pp. 49-66.

4. Keys, C. N.: Rotary-Wing Aerodynamics Vol. II — Performance Prediction of Helicopters. NASA CR-3083, Jan. 1979.

5. Lewis, R. B., II: Hueycobra Maneuvering Investigations. In: 26th American Helicopter Society, Annual National Forum, Washington, D.C., June 16-18, 1970.

6. Lewis, R. B., II; Bailes, Edward E.; McClellan, Randy D.; Claxton, John D.; and Buss, Marvin W.: Engineering Flight Test AH-1G Helicopter (Hueycobra) Maneuvering Limitations Final Report. U.S. Army Aviation Systems Test Activity, Edwards Air Force Base, Edwards, Calif., March 1971. (USAASTA-69-11)

7. Brown, E. L.; and Schmidt, P. S.: The Effect of Helicopter Pitching Velocity on Rotor Lift Capability. American Helicopter Society J., vol. 8, no. 4, Oct. 1963, p. 1-6.

8. Chen, R. T. N.: A Simplified Rotor System Mathematical Model for Piloted Flight Dynamics Simulation. NASA TM-78575, 1979.

9. Chen, R. T. N.: Effect of Primary Rotor Parameters on Flapping Dynamics. NASA TP-1431, 1980.

10. Saunders, George, H.: Dynamics of Helicopter Flight. John Wiley & Sons, Inc., New York, 1975.

11. Etkin, Bernard: Dynamics of Atmospheric Flight. John Wiley & Sons, Inc., New York, 1972.

12. Seckel, Edward: Stability and Control of Airplanes and Helicopters. Academic Press, New York, 1964.

13. Winn, A. L.; and Kishi, J. S.: Study: Pilot Perceived Jettison Envelope. Final Report. U.S. Army Aviation Systems Test Activity, Edwards Air Force Base, Calif., June 1973. (AD-768171)

14. Lynn, R. R.; Robinson, F. D.; Batra, N. N.; and Duhon, J. M.: Tail Rotor Design. Part I: Aerodynamics. American Helicopter Society J., vol. 15, no. 4, Oct. 1970, pp. 2-15.

TABLE 1.- EQUATIONS GOVERNING A COORDINATED HELICAL TURN

Steady-state Euler equations:

$$n_x - \sin\theta - \tan\phi_1(\sin\alpha_I \cos\beta_I \cos\theta \sin\phi - \sin\beta_I \cos\theta \cos\phi) = 0$$

$$n_y = 0 \tag{1a}$$

$$n_z + \cos\theta \cos\phi + \tan\phi_1(\sin\beta_I \sin\theta + \cos\alpha_I \cos\beta_I \cos\theta \sin\phi) = 0$$

$$L + I_{yq}(q^2 - r^2) + I_{xz}pq - I_{xy}rp + (I_y - I_z)qr - 0$$

$$M + I_{xz}(r^2 - p^2) + I_{xy}qr - I_{yz}pq + (I_z - I_x)rp = 0 \tag{1b}$$

$$N + I_{yx}(p^2 - q^2) + I_{yz}rp - I_{xz}qr + (I_x - I_y)pq = 0$$

Kinematic relationship:

$$\sin\phi = \tan\phi_1(\cos\alpha_I \cos\phi + \sin\alpha_I \tan\theta)\cos\beta_I \tag{6}$$

$$\sin\gamma = \cos\alpha_I \cos\beta_I \sin\theta - (\sin\beta_I \sin\phi + \sin\alpha_I \cos\beta_I \cos\phi)\cos\theta \tag{7}$$

$$p = -\dot\psi \sin\theta$$

$$q = \dot\psi \cos\theta \sin\phi \tag{2}$$

$$r = \dot\psi \cos\theta \cos\phi$$

where

$$\tan\phi_1 = \frac{\dot\psi V_I}{g} = \pm \frac{1}{\cos g}(n^2 - 1)^{1/2}, \quad + \text{ right turn; } - \text{ left turn.}$$

TABLE 2.- ALGORITHMS FOR A/C ANGULAR RATES AND EULER ANGLES IN A
COORDINATED HELICAL TURN

Given:	n_T (or $\dot{\psi}$), γ, V_I, α_I, β_I

Then:

$$\dot{\psi} = \pm \frac{g}{V_I \cos \gamma} (n_T^2 - \cos^2 \gamma)^{1/2}$$

$$\tan \phi_1 = \pm \frac{1}{\cos \gamma} (n_T^2 - \cos^2 \gamma)^{1/2} = \frac{\dot{\psi} V_I}{g}$$

$$\phi_1 = \tan^{-1}\left[\pm \frac{1}{\cos \gamma} (n_T^2 - \cos^2 \gamma)^{1/2}\right] = \tan^{-1}\left(\frac{\dot{\psi} V_I}{g}\right)$$

$\left.\begin{array}{c}\\ \\ \\ \\ \\ \end{array}\right\}$ + right turn − left turn

$$n_T = \frac{\cos \gamma}{\cos \phi_1}$$

$$q = \dot{\psi} \sin^2 \phi_1\left[-\sin \gamma \sin \beta_I \pm \left(\sin^2 \gamma \sin^2 \beta_I - \frac{\sin^2 \gamma - \cos^2 \beta_T}{\sin^2 \phi_1}\right)^{1/2}\right],$$
$$q \geq 0$$

$$r' = \frac{q}{\tan \phi_1 \cos \beta_I} ; \quad p' = -\frac{\dot{\psi} \sin \gamma}{\cos \beta_I} - q \tan \beta_I$$

$$p = p' \cos \alpha_I - r' \sin \alpha_I$$

$$r = p' \sin \alpha_I + r' \cos \alpha_I$$

$$\theta = \sin^{-1}\left(\frac{p}{\dot{\psi}}\right)$$

$$\phi = \tan^{-1}\left(\frac{q}{r}\right)$$

TABLE 3.- SUMMARY OF SYMMETRICAL PROPERTIES
OF AIRCRAFT ANGULAR RATES, ATTITUDES AND
n-PARAMETERS (AT n_T, V_I)

Right turn	Left turn
$\left.\begin{array}{l}\gamma\\[4pt]\alpha_I\\[4pt]\beta_I\end{array}\right\} \to \phi,\theta;\ p,q,r \quad n_z,n_{z_w},n$	$\left.\begin{array}{l}\gamma\\[4pt]\alpha_I\\[4pt]-\beta_I\end{array}\right\} \to -\phi,\theta;\ -p,q,-r \quad n_z,n_{z_w},n$
$\left.\begin{array}{l}-\gamma\\[4pt]-\alpha_I\\[4pt]-\beta_I\end{array}\right\} \to \phi,-\theta;\ -p,q,r \quad n_z,n_{z_w},n$	$\left.\begin{array}{l}-\gamma\\[4pt]-\alpha\\[4pt]\beta_I\end{array}\right\} \to -\phi,-\theta;\ p,q,-r \quad n_z,n_{z_w},n$

25

Figure 1.- Bank angle normal load factor relationships. (From ref. 5)
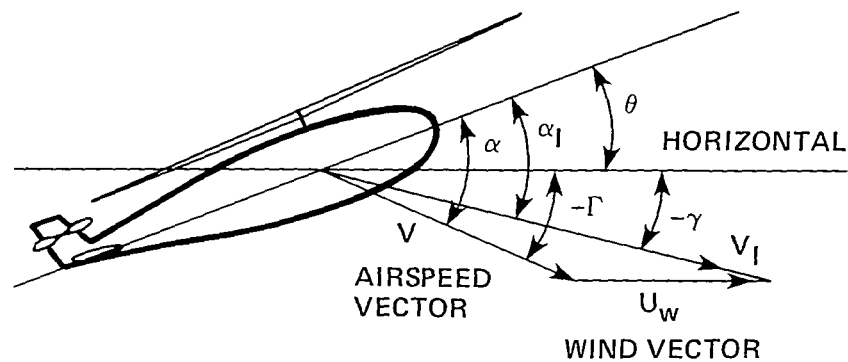
Figure 2.- Air-mass referenced and earth-referenced flight parameters
$(\beta = \beta_I = 0)$.
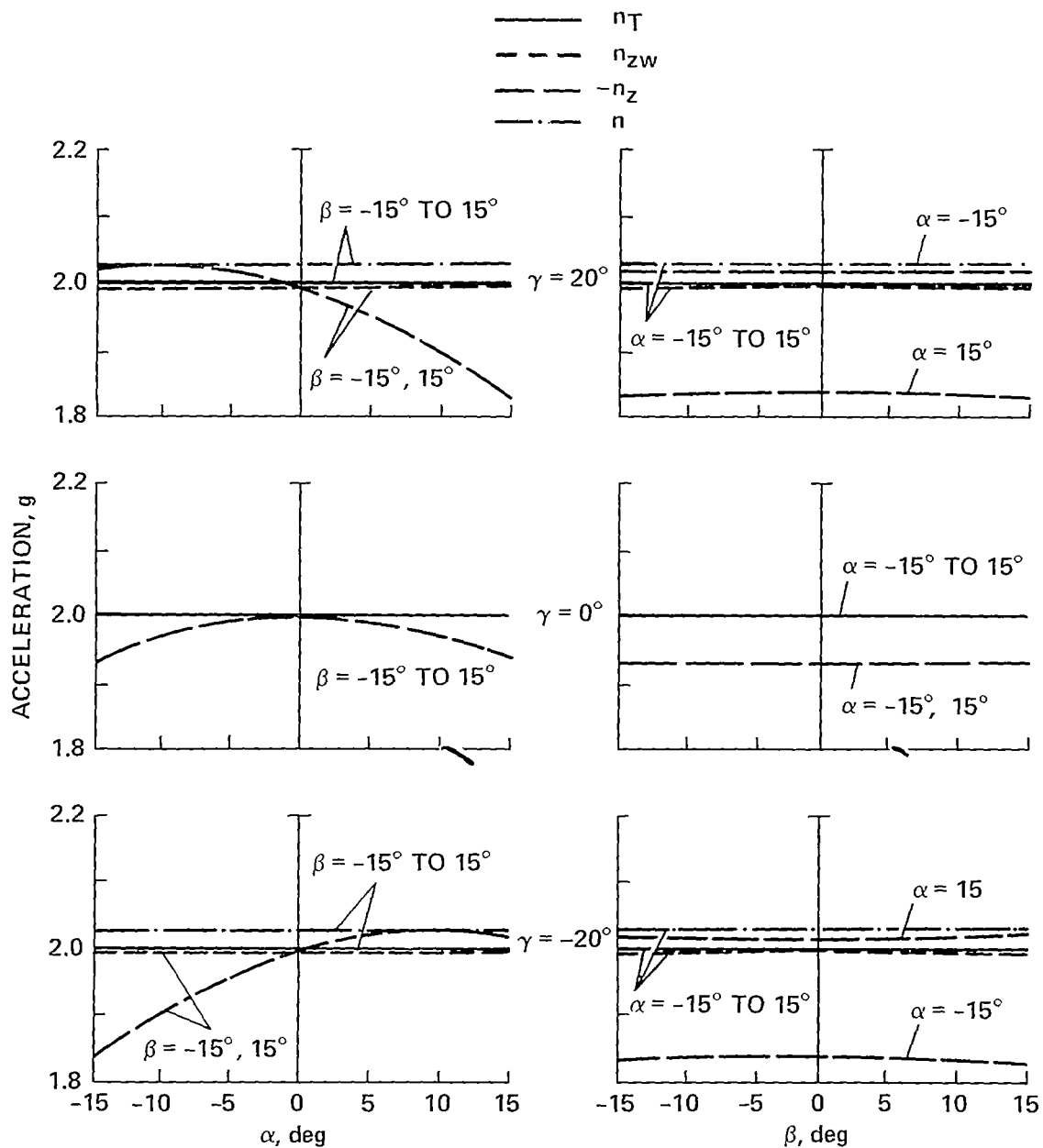
Figure 3.- Comparison of the values of the four load factors in steady coordinated right turns. $V = 60$ knots; $n_T = 2$ g.
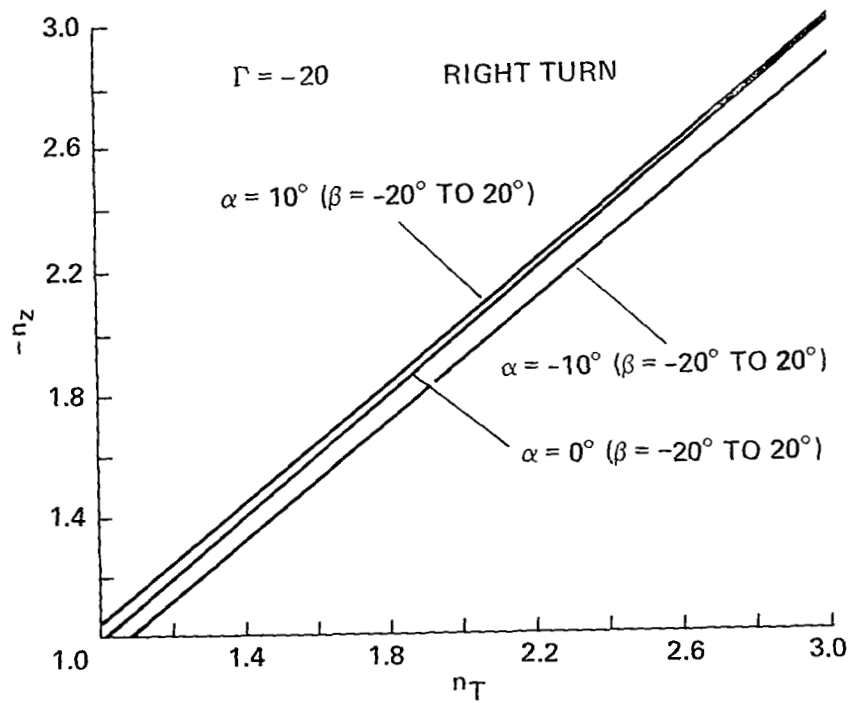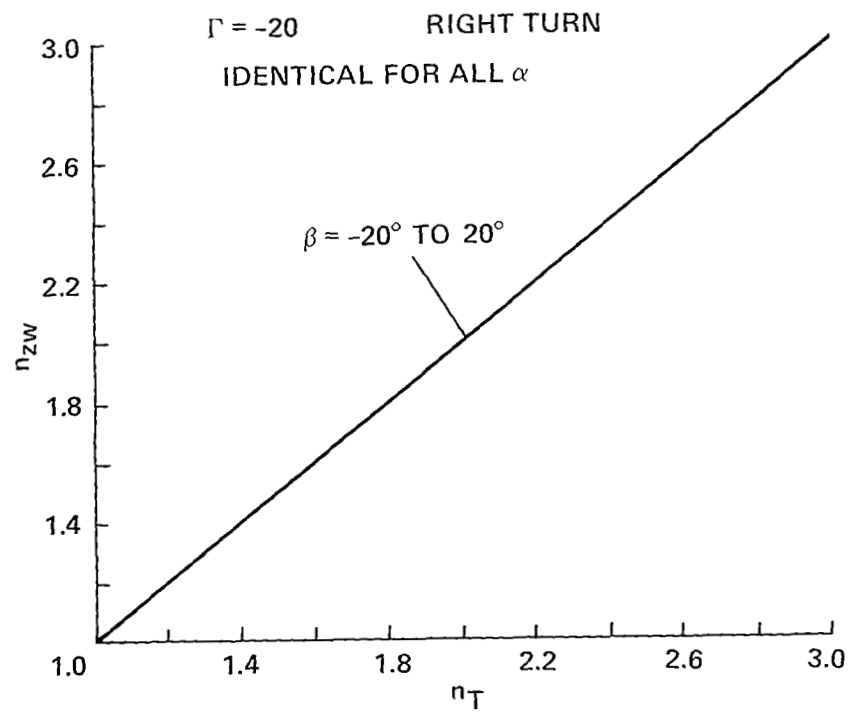
Figure 4.- Relationship between $-n_z$ and n_T.
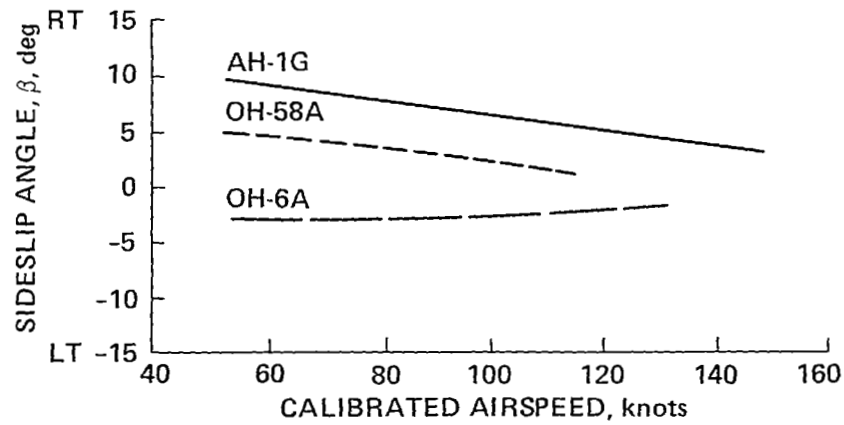

Figure 5.- Relationship between n_{zw} and n_T.

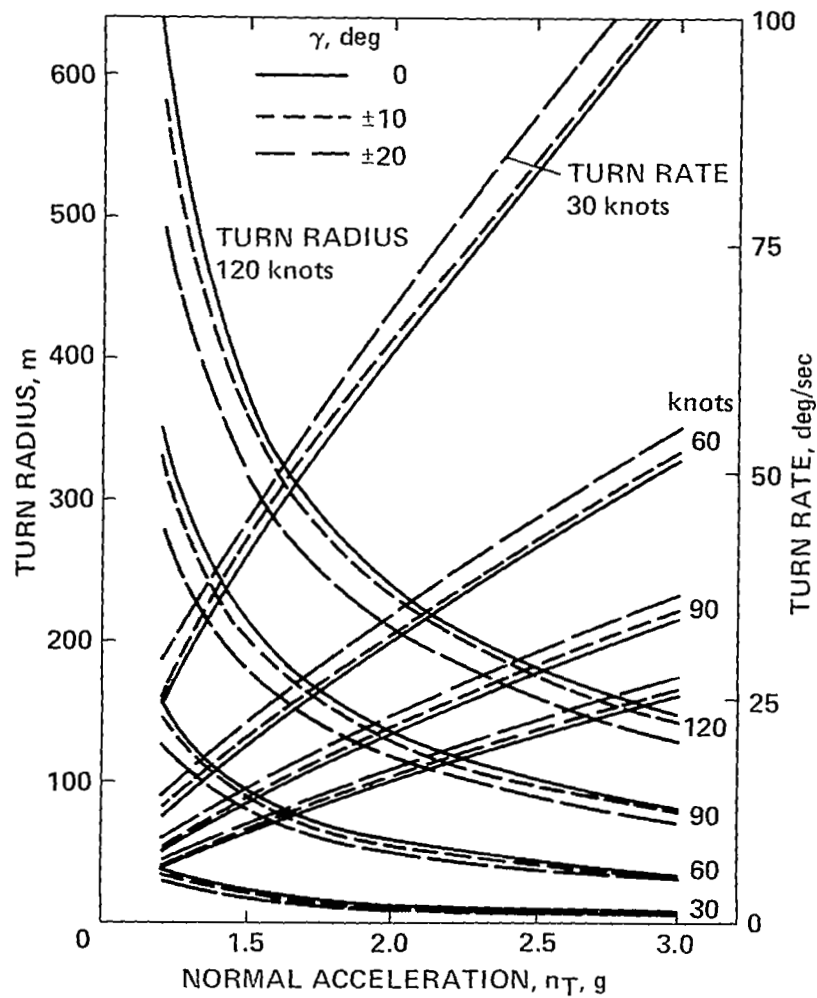Figure 6.- Inherent sideslip in steady wing-level straight flight.
(From ref. 13)

Figure 7.- Turn radius and turn rate in steady helical turns.
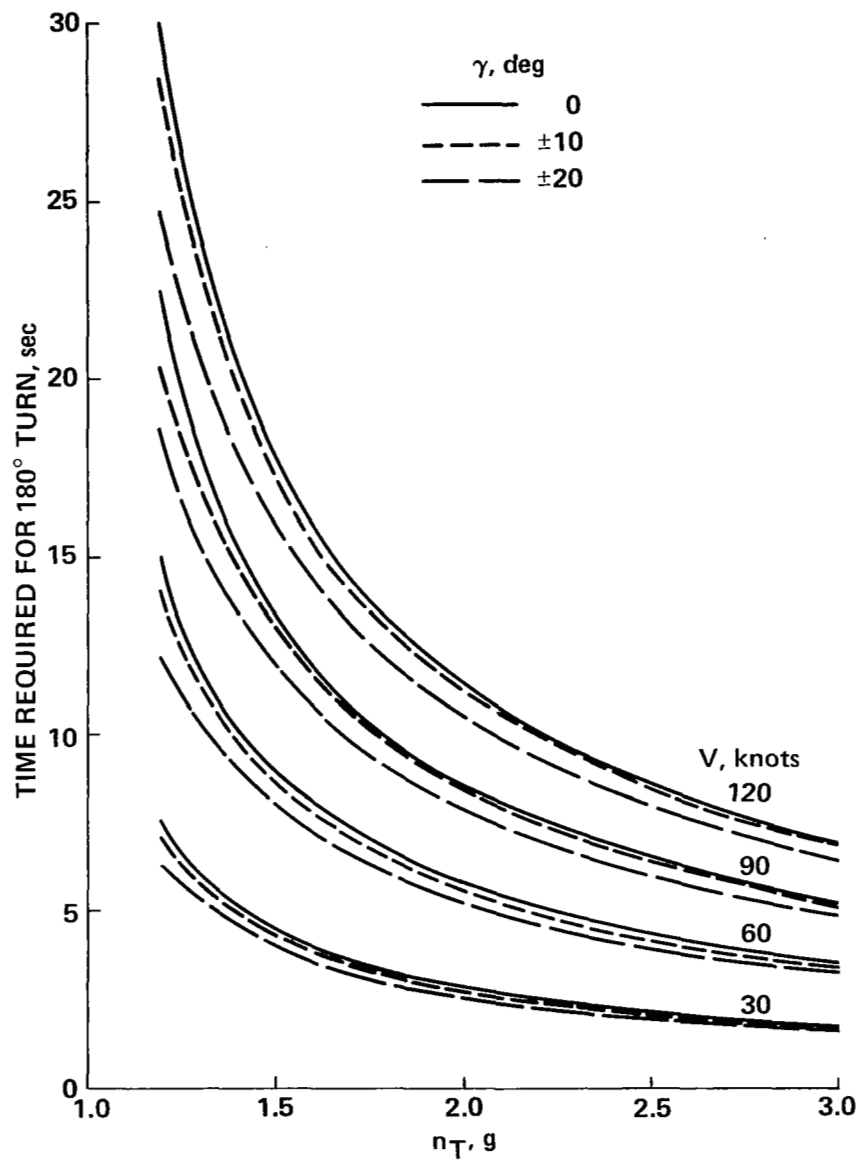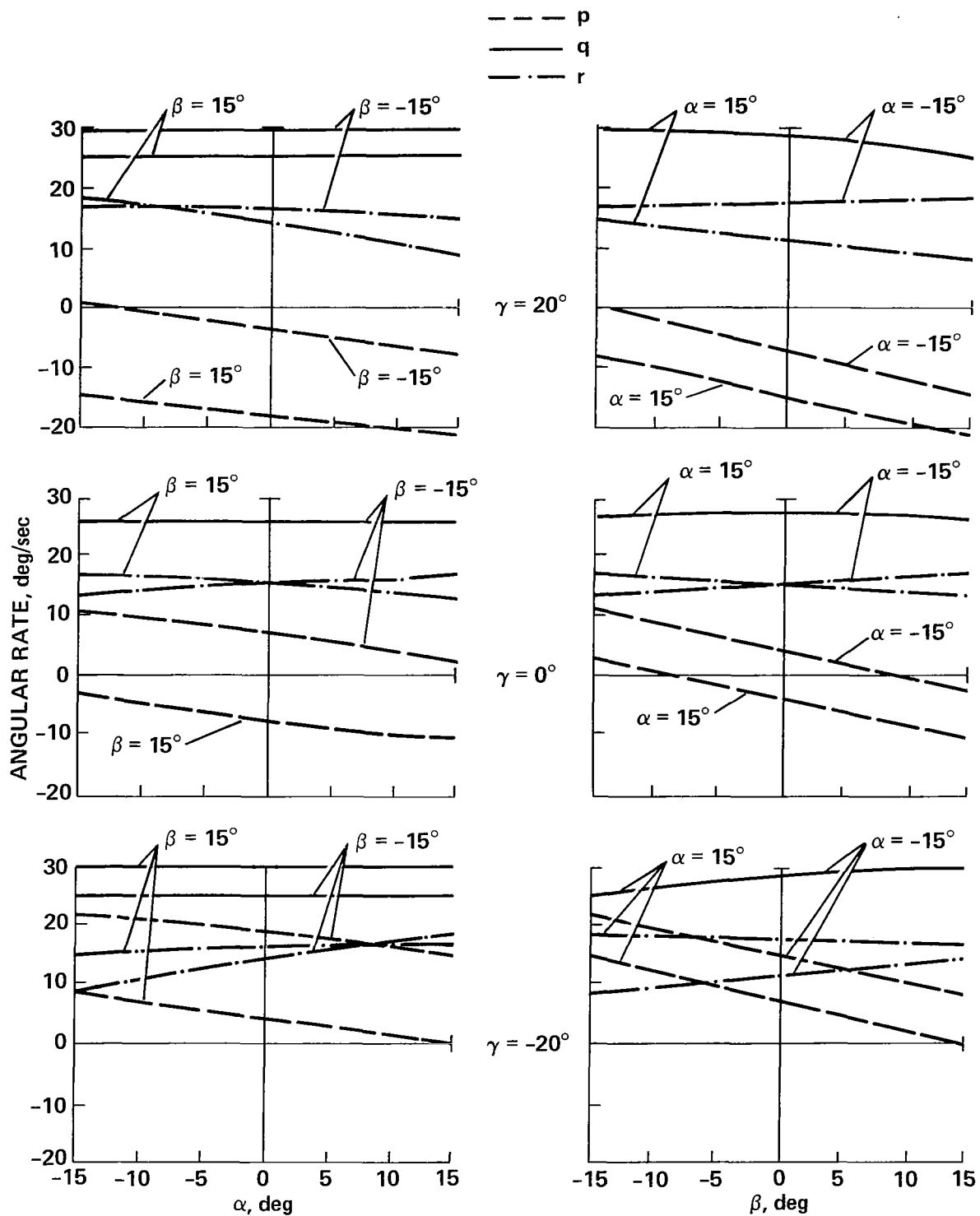
Figure 8.- Time required to complete 180° turn.

Figure 9.- Effect of α and β on angular rates in steady coordinated right turns. $V = 60$ knots; $n_T = 2$ g.
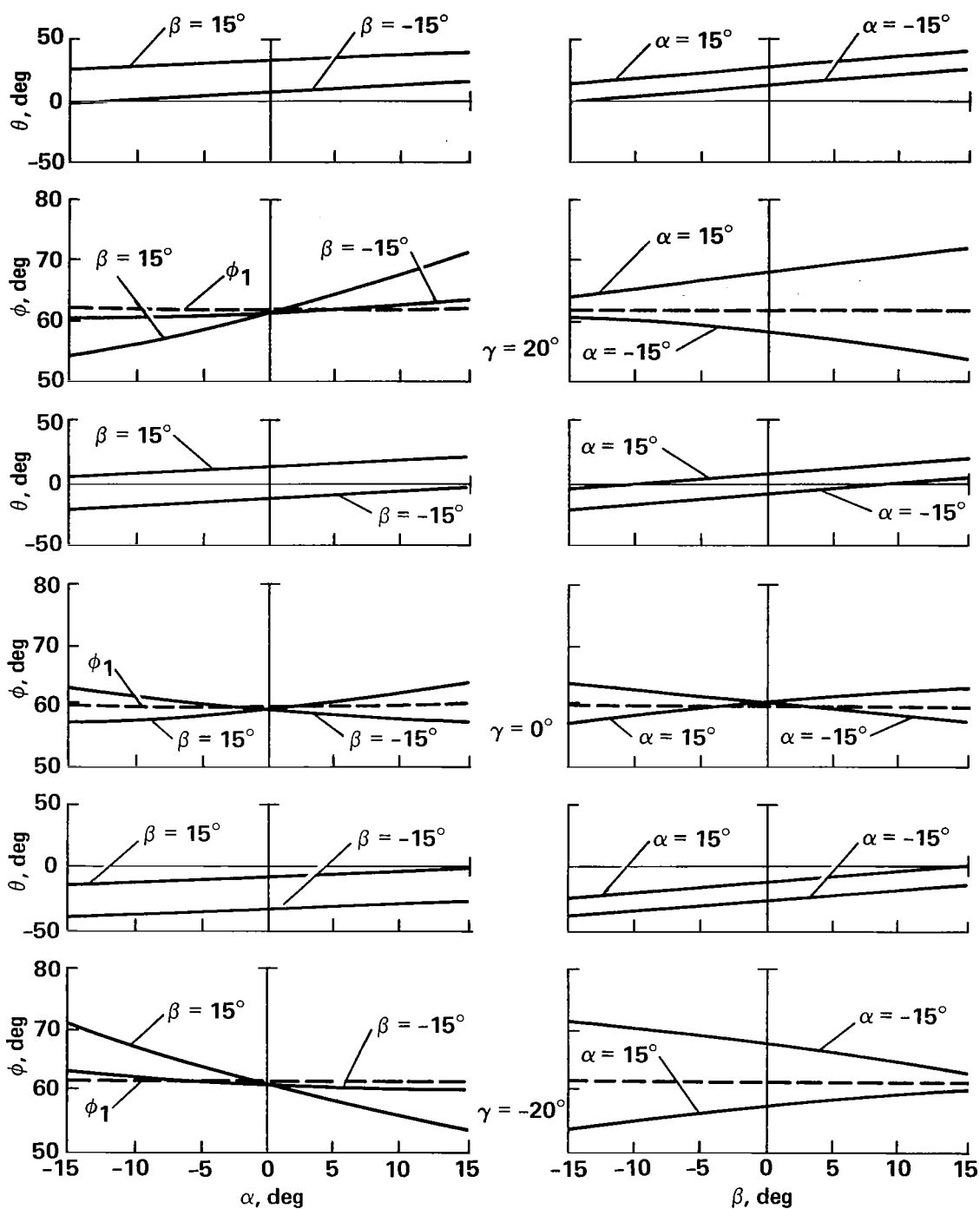
Figure 10.- Effect of α and β on the aircraft attitude in steady coordinated right turns; n_T = 2 g.

1 Report No. NASA TP-1773	2. Government Accession No	3. Recipient's Catalog No.
4 Title and Subtitle KINEMATIC PROPERTIES OF THE HELICOPTER IN COORDINATED TURNS		5. Report Date April 1981
		6. Performing Organization Code
7. Author(s) Robert T. N. Chen and James A. Jeske		8. Performing Organization Report No. A-8399
		10 Work Unit No 505-42-21
9. Performing Organization Name and Address NASA-Ames Research Center Moffett Field, CA 94035		11 Contract or Grant No
		13. Type of Report and Period Covered Technical Paper
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14 Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This paper describes the results of a study on the kinematic relationship of the variables of helicopter motion in steady, coordinated turns involving inherent sideslip. A set of exact kinematic equations which govern a steady coordinated helical turn about an earth-referenced vertical axis is first developed. A precise and rational definition for the load factor parameter that best characterizes a coordinated turn is proposed. Formulas are then developed which relate the aircraft angular rates and pitch and roll attitudes to the turn parameters, angle of attack, and inherent sideslip. These new closed-form formulas are then used for a detailed evaluation of the effects of sideslip on the kinematic relationship of the helicopter in coordinated turns. Important symmetrical properties that exist in these kinematic relationships are also discussed.

A steep, coordinated helical turn at extreme angles of attack with inherent sideslip is of primary interest in this study. The results show that the bank angle of the aircraft can differ markedly from the tilt angle of the normal load factor and that the normal load factor can also differ substantially from the accelerometer reading along the vertical body axis of the aircraft. Generally, sideslip has a strong influence on the pitch attitude and roll rate of the helicopter. The latter could have a significant impact on handling qualities because of the direct coupling of roll rate to the thrust of the main rotor.

The results of the analysis also indicate that pitch rate is independent of angle of attack in a coordinated turn and that in the absence of sideslip, angular rates about the stability axes are independent of the aerodynamic characteristics of the aircraft.

17. Key Words (Suggested by Author(s)) Helicopter kinematics Coordinated helical turns Load factor Sideslip Angular rates and attitudes	18. Distribution Statement Unclassified - Unlimited Subject Category 05

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 41	22. Price* A03