General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

NASA CR-165347

A SINGLE-EXPRESSION FORMULA FOR INVERTING STRAIN-LIFE AND STRESS-STRAIN RELATIONSHIPS

S.S. Manson
Professor, Mechanical and Aerospace Engineering

and

U. Muralidharan
Research Assistant

N81-23491

(NASA-CR-165347) A SINGLE-EXPRESSION FORMULA FOR INVERTING STRAIN-LIFE AND STRESS-STRAIN RELATIONSHIPS (Case Western Reserve Univ.) 31 p HC A03/MF A01 CSCL 20K

Unclas G3/39 42355

Case Western Reserve University
Cleveland, Ohio 44106

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-165347		

4. Title and Subtitle
A Single-Expression Formula for Inverting Strain-Life and Stress-Strain Relationships

5. Report Date
May 1981

6. Performing Organization Code

7. Author(s)
S. S. Manson and U. Muralidharan

8. Performing Organization Report No.

10. Work Unit No.

9. Performing Organization Name and Address
Case Western Reserve University
10900 Euclid Avenue
Cleveland, OH 44106

11. Contract or Grant No.
NAG3-46

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington DC 20546

13. Type of Report and Period Covered
Interim

14. Sponsoring Agency Code

15. Supplementary Notes
Grant Monitor, Dr. G. R. Halford
Structures & Mechanical Technologies Division
NASA-Lewis Research Center
Cleveland, OH 44135

16. Abstract

Starting with the basic fatigue life formula

$$\frac{\Delta\varepsilon}{\Delta\varepsilon} = \left(\frac{N_f}{N_T}\right)^b + \left(\frac{N_f}{N_T}\right)^c$$

an inversion formula is derived in form

$$\frac{N_f}{N_T} = \left[\left(\frac{\Delta\varepsilon}{\Delta\varepsilon_T}\right)^{\frac{z}{b}} + \left(\frac{\Delta\varepsilon}{\Delta\varepsilon_T}\right)^{\frac{z}{c}}\right]^{\frac{1}{z}}$$

where z is a function of strainrange and the ratio c/b.

The inversion formula is valid over the entire life range of engineering interest for all materials examined. Conformity between the two equations is extremely close, suitable for all engineering problems.

The approach used to invert the life relation is also suitable for the inversion of other formulas involving the sum of two power-law terms.

17. Key Words (Suggested by Author(s))
Fatigue (Metals)
Cumulative Damage
Life Prediction

18. Distribution Statement
Unclassified

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	29	

ABSTRACT

Starting with the basic fatigue life formula

$$\frac{\Delta\varepsilon}{\Delta\varepsilon_T} = \left(\frac{N_f}{N_T}\right)^b + \left(\frac{N_f}{N_T}\right)^c$$

an inversion formula is derived in form

$$\frac{N_f}{N_T} = \left[\left(\frac{\Delta\varepsilon}{\Delta\varepsilon_T}\right)^{\frac{z}{b}} + \left(\frac{\Delta\varepsilon}{\Delta\varepsilon_T}\right)^{\frac{z}{c}}\right]^{\frac{1}{z}}$$

where z is a function of strainrange and the ratio c/b. The inversion formula is valid over the entire life range of engineering interest for all materials examined. Conformity between the two equations is extremely close, suitable for all engineering problems.

The approach used to invert the life relation is also suitable for the inversion of other formulas involving the sum of two power-law terms.

INTRODUCTION

The availability of a formula expressing life directly in terms of strainrange and mean stress is a great convenience in cumulative damage analysis. Data for such an analysis usually appear as a sequence of loading each characterized by a strainrange and mean stress, and the analysis proceeds by establishing the life for each loading condition, and suitably summing cycle ratios based on the number of cycles actually applied at each loading divided by the life that would occur if that loading persisted until failure occurred. Unfortunately, however, the life relationship for each loading condition is formulated by an expression containing two terms with life raised to a fractional exponent. No exact closed-form relation for life can be obtained because the transcendental equation does not lend itself to exact solution.

A number of approximate methods have been obtained in the past for approximate, although accurate, inversion of the life relationship to obtain a direct expression for life N_f. These are discussed in Ref. 1. In one form the equation, defined in Ref. (1) as the Collocation method, is recast to an entirely different form, although numerically quite close to the actual life relation to be inverted. However, the numerical conformity can be achieved over only a few decades of fatigue life; therefore a "floating" relationship is required, changing the range for each application to insure that the life involved falls within it. While the procedure lends itself to easy computer programming, it would obviously be preferable to avoid the need for such a "floating" system.

A second approach, described in Ref. 1 as the spline point method, takes a step toward simplification by providing the inversion relationship in just two analytical expressions, one applicable below the transition strainrange (the strainrange where the elastic and plastic components are equal), the other above the transition strainrange. While quite accurate and simple to program, the inconvenience of having to account for a two-part analytical expression is still present in this method. The authors have therefore continued the search for a single-expression closed-form relationship suitable over the entire life range of interest in common engineering problems.

In this report we draw on the experience gained in Ref. 1 with the development of the two-part inversion expression to establish such a single expression. The report describes the basis of the method, and its applicability to a large number of materials commonly used in engineering design

METHOD

The Life Relation to be Inverted

The form of the basic life relationship was first proposed by Manson (Ref. 2). Later the same basic equation was expressed by Morrow (Ref. 3) with a new notation which is now comonly used

$$\frac{\Delta\varepsilon}{2} = \varepsilon_f' (2N_f)^c + \frac{\sigma_f'}{E} (2N_f)^b \qquad (1)$$

Here $\Delta\varepsilon$ = applied strainrange

N_f = cycles to failure

ε_f' and σ_f' = material constants designated as ductility coefficient and strength coefficient, respectively.

b and c = material constants designated as the elastic and plastic exponents, respectively.

An alternate form of the life relation has been expressed by Manson (Ref. 4)

$$R_\varepsilon = \frac{\Delta\varepsilon}{\Delta\varepsilon_T} = \left(\frac{N_f}{N_T}\right)^b + \left(\frac{N_f}{N_T}\right)^c \qquad (2)$$

where $R_\varepsilon = \frac{\Delta\varepsilon}{\Delta\varepsilon_T}$

$\Delta\varepsilon$ = applied strainrange

N_f = cycles to failure

where N_T and $\Delta\epsilon_T$ are transition life and strainrange given by

$$\Delta\epsilon_T = 2(\epsilon_f')^{b/(b-c)}\left(\frac{\sigma_f'}{E}\right)^{c/(c-b)} \quad (3)$$

$$N_T = \frac{1}{2}\,[E\epsilon_f'/\sigma_f']^{1/(b-c)} \quad (4)$$

Modification of the relation to account for mean stress is discussed in [5]. In Ref. [6] it is shown that, basically, the form of the relation is still given by Eq. (2), except that:

1. N_T is replaced by N_T', where

$$N_T' = \frac{1}{2}\left[(2N_T)^{-b} - \frac{2\sigma_o}{E\Delta\epsilon_T}\right]^{-1/b} \quad (5)$$

where σ_o = mean stress

2. The transition strainrange $\Delta\epsilon_T$ is the same as for completely reversed loading, but can be replaced by $k'\Delta\epsilon_T$ if experimental information is available to indicate that the cyclic stress-strain curve is affected by mean stress. In the absence of such explicit information k_ϵ is taken as unity.

3. The term σ_o in Eq. (5) may be replaced by $k_m\sigma_o$ if experimental evidence exists to provide a more accurate relation between cyclic life and mean stress [7] than that originally proposed by Morrow in [8] that mean stress can be accounted for by replacing σ_f' in Eq. (1) by $\sigma_f' - \sigma_o$.

Thus, whether mean stress is or is not present, the same basic equation of the form of Eq. (2) applies. We shall therefore focus our attention on the inversion of this relation, recognizing that N_T definitely depends on mean stress and $\Delta\epsilon_T$ <u>may</u> be a weak function of mean stress.

Choice of the Form of the Inverted Relationship

An intuitive choice of form of the inverted equation is

$$\frac{N_f}{N_T} = R_\epsilon^{\frac{1}{b}} + R_\epsilon^{\frac{1}{c}} \qquad (6)$$

Eq. (6) has the same asymptotes as Eq. (2), so that it is a valid representation of Eq. (2) at extremes of life and strainrange, but unfortunately the two equations do not coincide for intermediate values in the vicinity of the transition point for the parameters associated with most materials. Fig. 1 shows, for example, the results for Ti-6Al-4V. The continuous curve is the proper life relation obtained by adding vertical strain ordinates at any value of $2N_f$, while the dotted curve is determined from Eq. (6), which essentially adds horizontal abscisas at any value of strainrange. Of course the simplified inversion formula is correct at life extremes where the coordinate values on one line are negligible compared to those of the other; in the region of the transition point, where the coordinate values associated with both elastic and plastic lines are numerically significant relative to each other, a considerable difference exists between the results of Eq. (2) and those of Eq. (6).

The next logical step is to introduce the parameter z according to

$$\frac{N_f}{N_T} = \left[R_\epsilon^{z/b} + R_\epsilon^{z/c}\right]^{\frac{1}{z}} \qquad (7)$$

Again it is clear that the asymptotes of Eq. (7) are the same as those of Eq. (2), since they are

$$\frac{N_f}{N_T} = \left[R_\epsilon^{z/b}\right]^{\frac{1}{z}} = R_\epsilon^{1/b} \quad \text{and} \quad \frac{N_f}{N_T} = \left[R_\epsilon^{z/c}\right]^{\frac{1}{z}} = R_\epsilon^{1/c} \qquad (8)$$

However, we now have an adjustable parameter z which can be chosen to produce good fit over a large range about the transition point while retaining the desired asymptotes for all choices of z (except the trivial case of $z = 0$).

Determination of the Parameter z

The first approach was to determine whether z could be taken as a single constant. In this case only a single point on the curve could be force-fitted exactly. Consider, for example, the choice for fit at point (2) in Fig. 2 immediately above the transition point. Here R = 2 (since elastic and plastic strainranges are both equal to $\Delta\epsilon_T$), and $N_f/N_T = 1$. For this point, Eq. (7) becomes

$$1 = [2^{z/b} + 2^{z/c}]^{1/z} \qquad (9)$$

This equation cannot be solved exactly, but a very good approximation can be obtained using the experience gained in Ref. 1. For example, by re-writing the equation in the form

$$1 = [2^{\frac{z}{c} \cdot \frac{c}{b}} + 2^{\frac{z}{c}}] \qquad (9a)$$

it becomes clear that since the only two parameters in this equation are z/c and c/b, that the equation is really a statement z/c = f(c/b). Thus, choosing a series of values of c/b and solving numerically for z/c from Eq. (9a), we can plot the resulting values of z/c vs. the chosen c/b. A linear relation results when the plot is on log-log coordinates, leading to

$$z_2 = -1.117\, c\left(\frac{c}{b}\right)^{-.632} \qquad (10)$$

Another logical point to consider for exact fit is point (1), where $R_\varepsilon = 1$. The value of $\frac{N}{N_T}$ at this point has already been determined in [1] as e^δ

where
$$\delta = \frac{-0.78}{c}\left(\frac{c}{b}\right)^{0.36} \tag{11}$$

Substituting these values into Eq. (7) and solving by the same technique as used for solving Eq. (9) results in

$$z_1 = -0.889c\left(\frac{c}{b}\right)^{-0.36} \tag{12}$$

Finally, a third point (3) in Fig. 2 can be used for fit. For this point N_f is as much lower than N_T as it is higher than N_T at point (1). In other words, N_T is the geometric mean of the life values at (3) and (1). Thus, since $\frac{N_f}{N_T} = e^{-\delta}$ at this point, use of Eq. (2) results in $R_\varepsilon = e^{-b\delta} + e^{-c\delta}$. Again substituting these coordinates into Eq. (7) and solving numerically, results in

$$z_3 = -1.237c\left(\frac{c}{b}\right)^{-.832} \tag{13}$$

Any one of the three values of z from Eqs. (10), (12), or (13) would provide a first-approximation constant value of z that can be used in Eq. (7). Fig. 3 shows the types of fit that can be achieved by using successively points (1), (2), and (3) for the particular alloy Ti-6Al-4V. While all fits are quite good, they are not adequate for applications requiring high accuracy over the entire life range. By taking z as a second-degree polynomial in $\ln R_\varepsilon$, however, rather than a single constant, three adjustable constants become available, permitting fit at all three points (1), (2), and (3), and producing, therefore, an exceptionally good fit over the entire life range. The choice of location of the three points is exceptionally fortuitous, since it permits the final result to be expressed literally in general terms of all the parameters involved. Details of the procedure are outlined in Appendix A, the final result being

$$z = \exp[P(\ln R_\varepsilon)^2 + Q(\ln R_\varepsilon) + S] \quad (14)$$

where

$$P = -.001277(\tfrac{c}{b})^2 + .03893\ (\tfrac{c}{b}) - .0927 \quad (15)$$

$$Q = .004176\ (\tfrac{c}{b})^2 - 0.135\ (\tfrac{c}{b}) + 0.2309 \quad (16)$$

$$S = \ln\left[-.889c\ (\tfrac{c}{b})^{-.36}\right] \quad (17)$$

Final Formulation. The basic procedure is, therefore, as follows:

Starting with the commonly used form of the life relation Eq. (1) calculate the transition point coordinates from Eqs. (3) and (4), casting the basic relation according to Eq. (2). The inverted relation is Eq. (7) where z is given by Eq. (14) with the values of P,Q, and S given in Eqs. (15)-(17). If mean stress is present, use the same procedure, except use Eq. (5) to obtain the transition life. While in most cases the mean stress multiplier k_m, and transition life multiplier k_ε can be taken as unity, actual values of these multipliers can be used if experimentally determined. All the equations necessary for the inversion procedure are summarized in Table II.

DISCUSSION

While the inversion procedure described in Ref. 1 is more compact than that described in this report, its drawback is that it requires two formulas, one valid above the transition strainrange, the other below. The inversion procedure described here is a single-expression relation valid over the entire life and strainrange of the material. Several somewhat lengthy formulas are involved in the procedure, but numerically there is little difficulty involved since they can easily be programmed, even with simple hand-held calculators. The degree of conformity between the basic life relation and the inverted relation is remarkably good, conforming to any reasonable engineering standard likely to be required.

Table I has been prepared to determine the conformity for a large number of materials over the practical life range from 10 to 10^6 cycles to failure. The results are not shown for lives lower than 10 cycles or greater than 10^6 cycles, since agreement becomes even closer as the points involved lie closer to the asymptotic elastic and plastic lines. Data for the table were obtained from Landgraf et al's compilation [9]. For each material the basic fatigue parameters are listed in columns 2 to 6, and the calculated transition point coordinates are shown in columns 7 and 8. The strainrange required to produce lives of 10 to 10^6 cycles, in decade increments, were first calculated from the basic life equations of form as in either Eq. (1) or (2). These strainranges were then used in the inversion formula, Eq. (7) to obtain the lives that the formula would yield, as compared to the "exact" values 10, 10^2, etc. These lives are shown in Columns 9 to 14. An entry of unity means that the inversion formula gives exactly the same results as the basic equation. The degree of discrepancy between the "exact" equation and the inversion formula is indicated by the departure from unity of the number entered at each life level. In most cases the error is only two or three percentage points <u>in life</u>, a degree of accuracy far exceeding the experimental scatter usually associated with the determination of the basic life relation. Only one entry in the entire table involves a difference of more than 10% - material no. 1 at the 10 cycles life level. In most cases the conformity is extremely close.

Figure 4 shows the life relations according to Eqs. (2) and (7) for the four materials for which the discrepancies were the greatest among those listed in Table I. Even for these materials the difference between the two curves can not be discerned on a reasonable graph scale, except for the very lowest lives. Still, the discrepancies are negligible for engineering purposes. It can reasonably be concluded that the inversions formula is sufficiently accurate for all materials for all engineering purposes.

The monotonic and cyclic stress-strain curves represent other examples of relations that can be inverted by similar procedures. Appendix B outlines the application to the cyclic stress-strain curve. Since stress is expressed directly in terms of strain, closed form analytical expressions can easily be obtained for stress in terms of strain for double amplitude stress strain curves, for both increasing and decreasing directions. With the proper rule for incorporating memory, it is possible to find the stress response of a material for a given complex strain history.

The basic inversion approach has utility for other applications, as well, whenever one variable is expressed as the sum of two negative power-law expressions of a second variable. The Smith-Watson-Topper [10] relationship for treating mean stress effects also involves such a formulation, and is therefore amenable to treatment by the method of this report. Appendix C presents the basic outline of the approach. Other potential applications are creep strain analysis, wherein a creep or creep-rupture curve is sometimes represented by the sum of two power-law relations. Numerous other applications can be envisioned.

CONCLUDING REMARKS

The method presented in this report for inverting the life relationship provides extremely accurate results for all materials examined over the entire range of fatigue lives of interest in common engineering applications. The formulas involved can easily be programmed on a computer or hand-held calculator. No trial-and-error calculations are involved. All the relations involved are summarized for convenience in Table II. The basic inversion method may also find utility in other applications such as inverting the stress-strain curve, analytical treatment of alternative methods of calculating mean-stress effects on fatigue life -- such as the Smith-Watson-Topper parameter, and inverting creep and creep-rupture relations involving two power-laws. The approach is even suggestive of how to treat relationships involving three or more power-law terms, although details would require further study.

TABLE I

APPLICATION OF INVERSION FORMULA FOR FIFTY CHARACTERIZED MATERIALS OF ENGINEERING INTEREST

1	2	3	4	5	6	7	8	INVERTED LIFE AT 9	10	11	12	13	14
MATERIAL	σ_f' ksi	ϵ_f'	b	C	E ksi	$\Delta\epsilon_T$	N_T Cycles	X10	$X10^2$	$X10^3$	$X10^4$	$X10^5$	$X10^6$
#1 SAE1005-1009 HRLC	78	.11	-.073	-.41	29000	$2.41X10^{-3}$	30300	.89	.92	.99	1.00	.97	.95
#2 GA:NEX (HRLC)	117	.86	-.071	-.65	29200	$4.15X10^{-3}$	5322	.96	.92	1.07	.97	.98	1.05
#3 MAN-TEN-150BHN	141	.85	-.11	-.59	30000	$2.86X10^{-3}$	25219	.96	.92	.97	1.00	.97	.98
#4 SAE1045-225 BHN	178	1.0	-.095	-.66	29000	$5.21X10^{-3}$	4111	.95	.94	1.026	.97	.98	1.00
#5 SAE1045-390 BHN	230	.45	-.074	-.68	30000	$9.33X10^{-3}$	414.	.93	1.07	.96	1.00	1.05	1.02
#6 SAE1045-410 BHN	270	.60	-.073	-.70	2900.	.01146	384	.93	1.07	.95	1.00	1.06	1.02
#7 SAE1045-450 BHN	260	.35	-.07	-.69	30000	.01142	195	.97	1.05	.94	1.04	1.06	1.01
#8 SAE1045-595 BHN	395	.07	-.081	-.60	30000	.0203	12.5	1.01	.95	1.00	1.00	.99	.99
#9 10862-430 BHN	258	.32	-.067	-.56	28000	.0114	667	.93	1.06	.99	.95	1.02	1.03
#10 AISI4130-365 BHN	246	.89	-.081	-.69	29000	$9.14X10^{-3}$	1041	.93	1.01	1.01	.96	1.03	1.02
#11 SAE4142-475 BHN	315	.09	-.081	-.61	30000	.01511	29	1.04	.96	.98	1.01	1.00	.99
#12 SAE4142-560 BHN	385	.07	-.089	-.76	30000	.0205	6.27	.98	.99	1.03	1.00	.99	1.00
#13 Ti-6Al-4V	552.4	1.053	-.1052	-.6903	17000	.0348	191	.96	1.01	.96	.99	.99	.98
#14 SAE4142-400 BHN	275	.5	-.09	-.75	29000	.011	203.3	.95	1.04	.96	1.02	1.01	.99

TABLE I - continued

1 MATERIAL	2 σ_f' ksi	3 ϵ_f'	4 b	5 C	6 E ksi	7 $\Delta\epsilon_T$	8 N_T Cycles	INVERTED LIFE AT 9 X10	10 $X10^2$	11 $X10^3$	12 $X10^4$	13 $X10^5$	14 $X10^6$
#15 SAE4142-450 BHN	290	.40	-.08	-.73	30000	.0122	154	.98	1.04	.96	1.04	1.03	1.00
#16 SAE4142-475 BHN	300	.20	-.082	-.77	29000	.0145	37	1.06	.96	1.02	1.04	1.00	1.00
#17 AISI4340-243 BHN	174	.45	-.095	-.54	28000	$4.98X10^{-3}$	7557	.92	.94	1.00	.99	.96	.98
#18 AISI4340-409 BHN	290	.48	-.091	-.60	29000	.01	1005	.92	1.00	1.00	.96	.99	.99
#19 SAE5160-430 BHN	280	.40	-.071	-.57	28000	.0118	812	.92	1.05	1.00	.95	1.01	1.02
#20 SAE9262-260 BHN	151	.155	-.071	-.47	30000	$5.47X10^{-3}$	2688	.9	1.00	1.02	.96	.96	.99
#21 SAE9262-410 BHN	269	.38	-.057	-.65	29000	.013	262	.96	1.00	.92	1.04	1.12	1.06
#22 AISI304-160 BHN	350	1.02	-.15	-.77	27000	$9.02X10^{-3}$	571.3	.94	.99	.99	.98	.98	.98
#23 AISI310-145 BHN	240	.60	-.15	-.57	28000	$3.76X10^{-3}$	12361	.95	.97	.99	1.00	.99	.99
#24 AM35 Annealed	406	.33	-.14	-.84	28000	.016	43.4	1.00	.98	.99	.98	.99	1.00
#25 18% Ni-Maraging 460 BHN	310	.80	-.071	-.79	27000	.015	183	.95	1.05	.94	1.00	1.06	1.011
#26 18% Ni-Maraging 480 BHN	325	.60	-.07	-.75	26000	.017	148	.98	1.04	.95	1.00	1.06	1.01
#30 2014-Al-T6	123	.42	-.106	-.65	10000	.0124	329	.95	1.01	.97	.98	.99	.98
#31 2014-Al-T4	147	.21	-.11	-.52	10200	.01405	344	.97	1.00	.98	.98	.97	.96

TABLE I - continued

1 MATERIAL	2 σ_f' ksi	3 ϵ_f'	4 b	5 C	6 E ksi	7 $\Delta\epsilon_T$	8 N_T Cycles	INVERTED LIFE AT 9 X10	10 $X10^2$	11 $X10^3$	12 $X10^4$	13 $X10^5$	14 $X10^6$
#32 5456 Aluminum	105	.46	-.11	-.67	10000	9.99×10^{-3}	427	.94	1.01	.98	.98	.99	.98
#33 SAE1015-80 BHN	120	.95	-.11	-.64	30000	2.57×10^{-3}	15183	.97	.93	.98	1.00	.97	.98
#34 SAE950X-150 BHN	91	.35	-.075	-.54	30000	2.82×10^{-3}	13604	.94	.91	1.02	1.01	.95	.99
#35 VAN80-225 BHN	153	.21	-.08	-.53	28200	5.67×10^{-3}	1688	.91	1.00	1.01	.96	.98	.99
#36 RQC100-298 BHN	147	.60	-.076	-.67	29400	5.42×10^{-3}	1582	.93	.99	1.04	.95	1.03	1.03
#37 SAE1045-500 BHN	330	.25	-.08	-.68	30000	.0145	91	1.02	1.00	.96	1.03	1.01	.995
#38 AISI4130-258 BHN	185	.92	-.083	-.63	32000	5.36×10^{-3}	5298	.96	.93	1.04	.98	.97	1.01
#39 SAE4142-380 BHN	265	.45	-.08	-.75	30000	.01105	177	.96	1.05	.95	1.04	1.03	1.00
#40 SAE4142-450 BHN	305	.60	-.09	-.76	29000	.0122	209	.95	1.04	.96	1.02	1.01	.994
#41 SAE4340-350 BHN	240	.73	-.076	-.62	28000	9.21×10^{-3}	1767	.92	.99	1.03	.95	1.00	1.02
#42 AISI52100-518 BHN	375	.18	-.09	-.56	30000	.015	146	.99	1.00	.96	.98	.99	.98
#43 SAE9262-280 BHN	177	.41	-.073	-.60	28000	7.09×10^{-3}	1372	.91	1.02	1.02	.95	1.01	1.03
#44 H-11 660 BHN	460	.08	-.077	-.74	30000	.0253	6	.98	1.00	1.05	1.02	1.00	1.00
#45 AISI304-327 BHN	330	.89	-.12	-.69	25000	.0109	808	.93	.99	.99	.97	.98	.98

TABLE I - continued

1 MATERIAL	2 σ_f' ksi	3 ϵ_f'	4 b	5 C	6 E ksi	7 $\Delta\epsilon_T$	8 N_T Cycles	INVERTED LIFE AT 9 X10	10 $X10^2$	11 $X10^3$	12 $X10^4$	13 $X10^5$	14 $X10^6$
#46 AM350-496 BHN	390	.098	-.102	-.42	26000	.0164	183	.99	.99	.98	.98	.98	.96
#47 18% Nickel Marraging 450 BHN	240	.3	-.065	-.62	27000	.0118	284	.97	1.07	.94	1.00	1.07	1.03
#48 2024-T351 Aluminum	160	.22	-.124	-.59	10600	.0148	157	.98	.99	.98	.98	.97	.97
#49 7075-T6 Aluminum	191	.19	-.126	-.52	10300	.0176	184	.99	1.00	.99	.98	.97	.96
#50 SAE1005-HRLC	93	.1	-.109	-.39	29000	$1.69X10^{-3}$	103608	.95	.97	.99	1.00	1.00	1.00

TABLE II

LIST OF FORMULAS USED IN INVERSION METHOD

For completely reversed cycling:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

Transition strain-range $\Delta\varepsilon_T = 2(\varepsilon_f')^{\left(\frac{b}{b-c}\right)} \cdot \left(\frac{\sigma_f'}{E}\right)^{c/(c-b)}$

Transition life $N_T = \frac{1}{2}\left(\frac{E\ \varepsilon_f'}{\sigma_f'}\right)^{\frac{1}{(b-c)}}$

If the mean stress of σ_0 is present, transition life

$$N_T' = \frac{1}{2}\left[(2N_T)^{-b} - \frac{2\sigma_0}{E\Delta\varepsilon_T}\right]^{-1/b}$$

$$R_\varepsilon = \frac{\Delta\varepsilon}{\Delta\varepsilon_T} = \left(\frac{N}{N_T}\right)^b + \left(\frac{N}{N_T}\right)^c$$

Inverted relation:

$$N_f = N_T \cdot \left[R_\varepsilon^{\frac{z}{b}} + R_\varepsilon^{\frac{z}{c}}\right]^{\frac{1}{z}}$$

where

$$z = \exp[P\ \ln^2 R_\varepsilon + Q\ \ln R_\varepsilon + S]$$

$$P = -.001277\left(\frac{c}{b}\right)^2 + .03693\left(\frac{c}{b}\right) - .0927$$

$$Q = .004176\left(\frac{c}{b}\right)^2 - .135\left(\frac{c}{b}\right) + .2309$$

$$S = \ln\left[-.889c\left(\frac{c}{b}\right)^{-.36}\right]$$

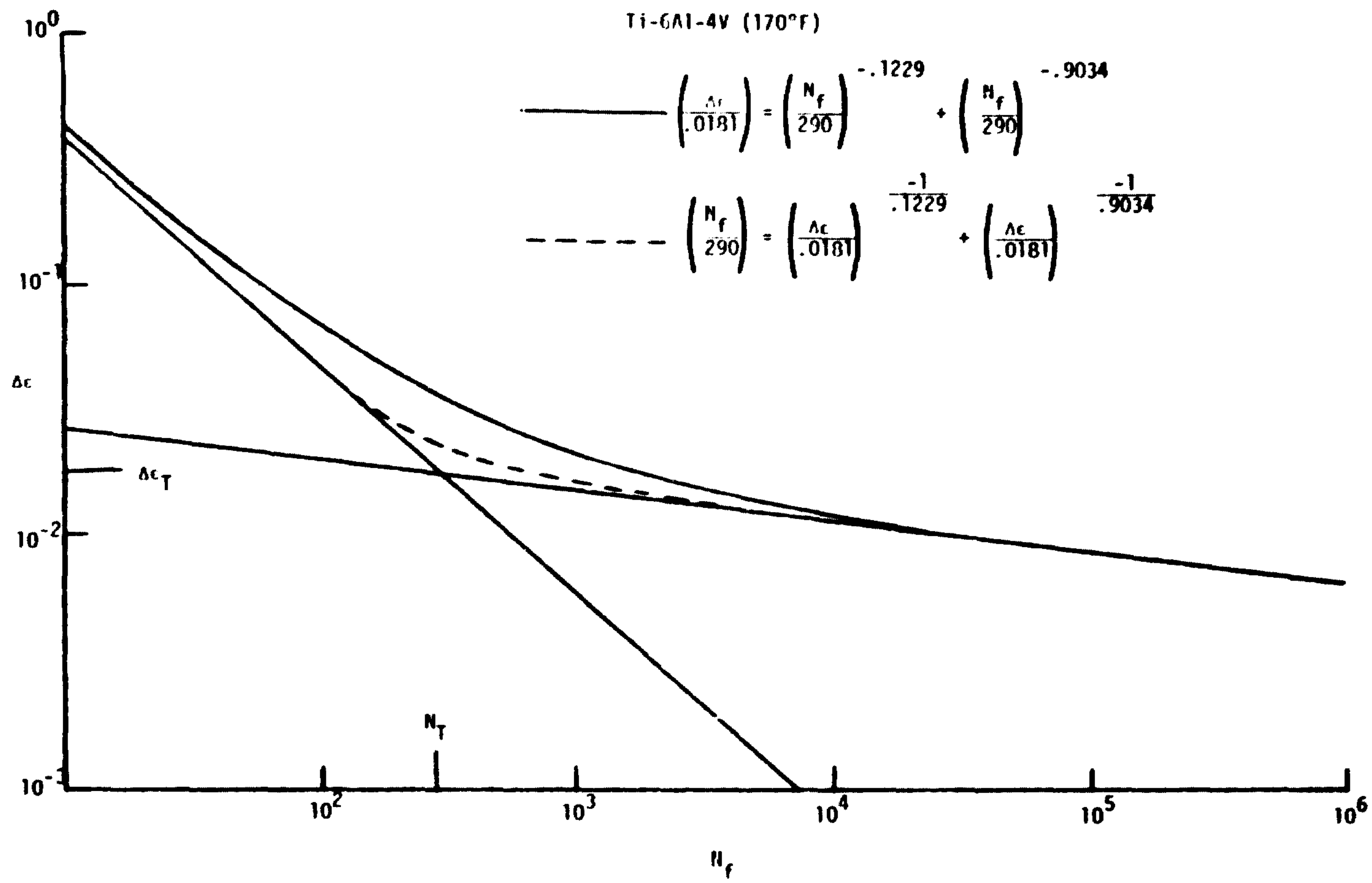


Fig. 1. Actual and Inverted Strain-life curves for Ti-6Al-4V by equations shown above.

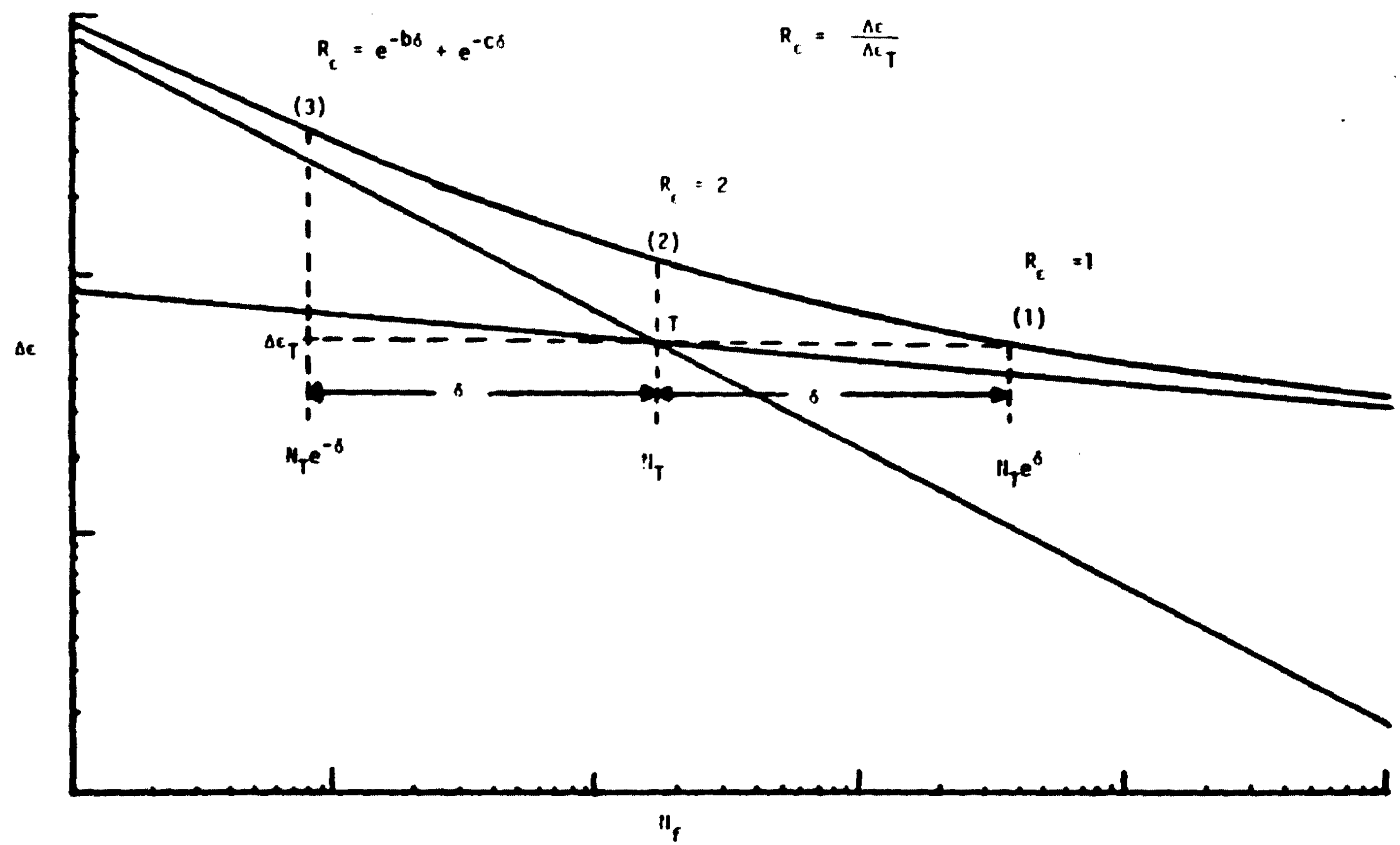


Fig. 2. Selection of points on life curve which can be forced fit to determine z as a single constant.

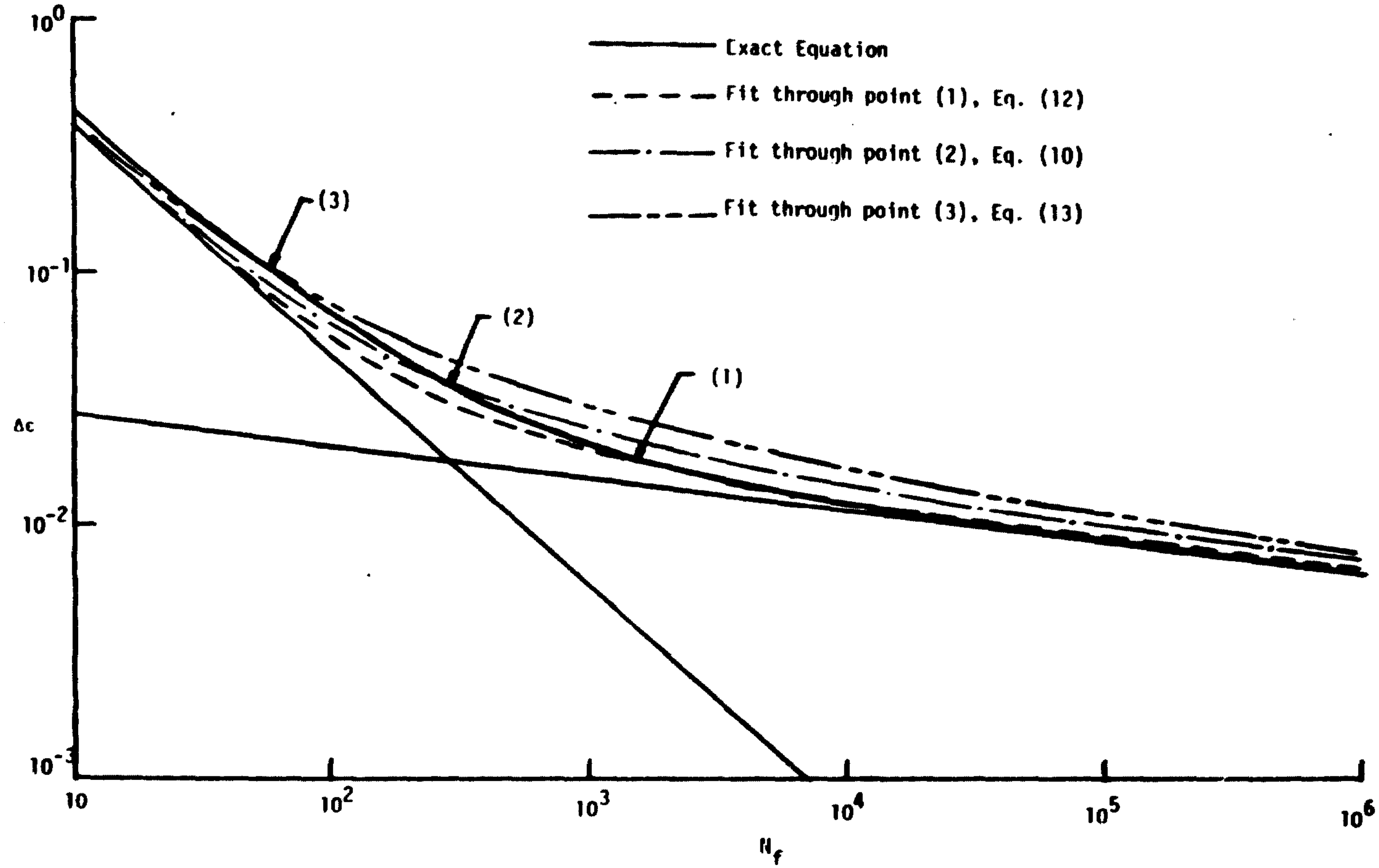
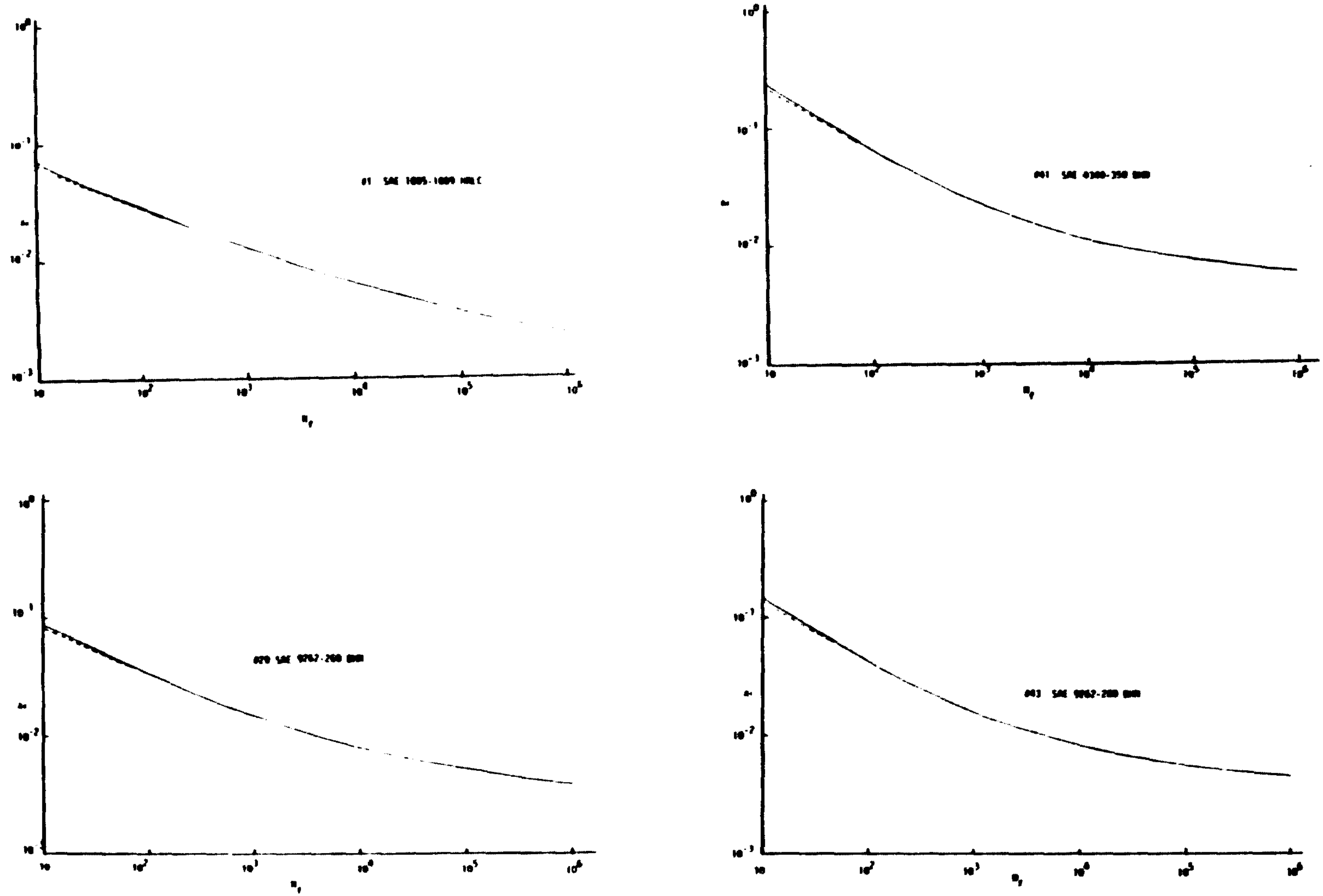


Fig. 3. Exact and inverted life relations which are forced fit at locations (1), (2) or (3) for Ti-6Al-4V (170°F).



—— Eq. (2) ------ Eq. (7)

Fig. 4. Comparison of actual and inverted life relations for above four materials which show maximum discrepancy as shown in Table I.

REFERENCES

1. S.S. Manson, "Inversion of the Strain-Life and Strain-Stress Relationships For Use in Metal Fatigue Analysis," Journal of Fatigue of Engineering Materials and Structures, Vol. 1 (1979) pp. 37-57.

2. S.S. Manson, "Thermal Stress in Design, Part 19, Cyclic Life of Ductile Materials," Machine Design (July 7, 1960) pp. 139-144.

3. J. Morrow, "Cyclic Plastic Strain Energy and Fatigue of Metals," ASTM STP 378 (1965) pp. 45-87.

4. S.S. Manson, "Predictive Analysis of Metal Fatigue in the High Scycle Life Range," Methods of Predicting Material Life in Fatigue, ASME (1979) pp. 145-183.

5. S.S. Manson and G.R. Halford, "Practical Implementation of the Double-Linear Damage Rule for Cumulative Fatigue Damage Analysis," International Journal of Fracture, Vol. 17, No. 2 (April 1981) (See also NASA TM-81517, 1980).

6. S.S. Manson and G.R. Halford, "Recent Developments in Practical Implementation of the Double Linear Damage Rule," International Journal of Fracture, Vol. 17, No. 3 (1981) (in press).

7. J. Walcher, D. Gray, and S.S. Manson, "Aspects of Cumulative Fatigue Damage Analysis of Cold End Rotating Structures," AIAA/SAE/ASME 15th Joint Propulsion Conference, June 18-20, Las Vegas, Nevada, AIAA Paper 79-1190.

8. JoDean Morrow, "Fatigue Properties in Metals," Section 3.2, Fatigue Design Handbook, Advances in Engineering, Volume 4, J.A. Grahm, ed., Society of Automotive Engineers, Incorporated, Warrendale (1968) pp. 21-29.

9. R.W. Landgraf, M.R. Mitchell and N.R. LaPointe, "Monotonic and Cyclic Properties of Engineering Materials," Ford Motor Company (1972).

10. K.N. Smith, P. Watson, and T.H. Topper, "A Stress-Strain Function for the Fatigue of Metals," Journal of Materials, ASTM, Vol. 5, No. 4 (December 1970) pp. 767-778.

Appendix A

DETERMINATION OF THE Z-PARAMETER

To fit z through three known points represented by Eqs. (10), (12), and (13), we start by noting that the values of z/c for each of these equations is a function of only $(\frac{c}{b})$. Furthermore, the values of R_ε at points (1) and (2) are constants, while at point (3) it also depends on only c/b (since $\delta = \frac{-0.78}{c}(\frac{c}{b})^{0.36}$ from Ref. [1], $e^{-b\delta} = \exp\left[0.78\ (\frac{c}{b})^{-0.64}\right]$ and $e^{-c\delta} = \exp\left[0.78\ (\frac{c}{b})^{0.36}\right]$, thus both terms are functions of only c/b). As a result, if we formulate the relation

$$\ln\frac{z}{c} = P\ (\ln R_\varepsilon)^2 + Q(\ln R_\varepsilon) + S' \qquad \text{(A-1)}$$

it is clear that satisfying the three equations at point (1), (2) and (3) of Fig. 2 will produce relations for P, Q and S' that will be functions of only c/b. However, since these functions are rather complicated because of the exponentials involved, we proceeded in more simplified fashion. Choosing specific values of c, c/b for actual materials, we applied the coordinates expressed in Eqs. (10), (12) and (13), to substitute into Eq. (A-1) and solved for P, Q, and S'. The value of S' could easily be determined in closed form, since at $R_\varepsilon = 1$, $\ln R_\varepsilon = 0$, so S' depends only on c/b, resulting in

$$S' = \ln\left[-\ .889c(\frac{c}{b})^{-0.36}\right] \qquad \text{(A-2)}$$

However, for determination of P and Q the graphs shown in Fig. A-1 were used. Calculations for P and Q are shown for various materials, and are plotted against (c/b). As expected, a single curve results. Passing a second degree polynomial through each of these two curves, using least-squares analysis resulted in the relations shown in Eqs. (15) and (16). By combining the $\ln\frac{1}{c}$ term of Eq. (A-1) with the S' term of Eq. (A-2), the resulting value of S shown in Eq. (17) was obtained, leading to the equation for z in Eq. (14) as a direct consequence of Eq. (A-1).

Although the above method was adopted for final use, other approaches were investigated during the study. Among them was one in which expressions for z similar to Eqs. (10), (12) and (13) were written for a number of additional points along the curve. Least squares analysis was then applied to obtain a fit of an equation similar to Eq. (14) through the redundancy of points. The resulting equations were less accurate than the method finally adopted. Other approaches were more complicated but not more accurate. The method adopted provides a high degree of accuracy with a relative simplicity of underlying basis and ease of final application, as discussed in the report.

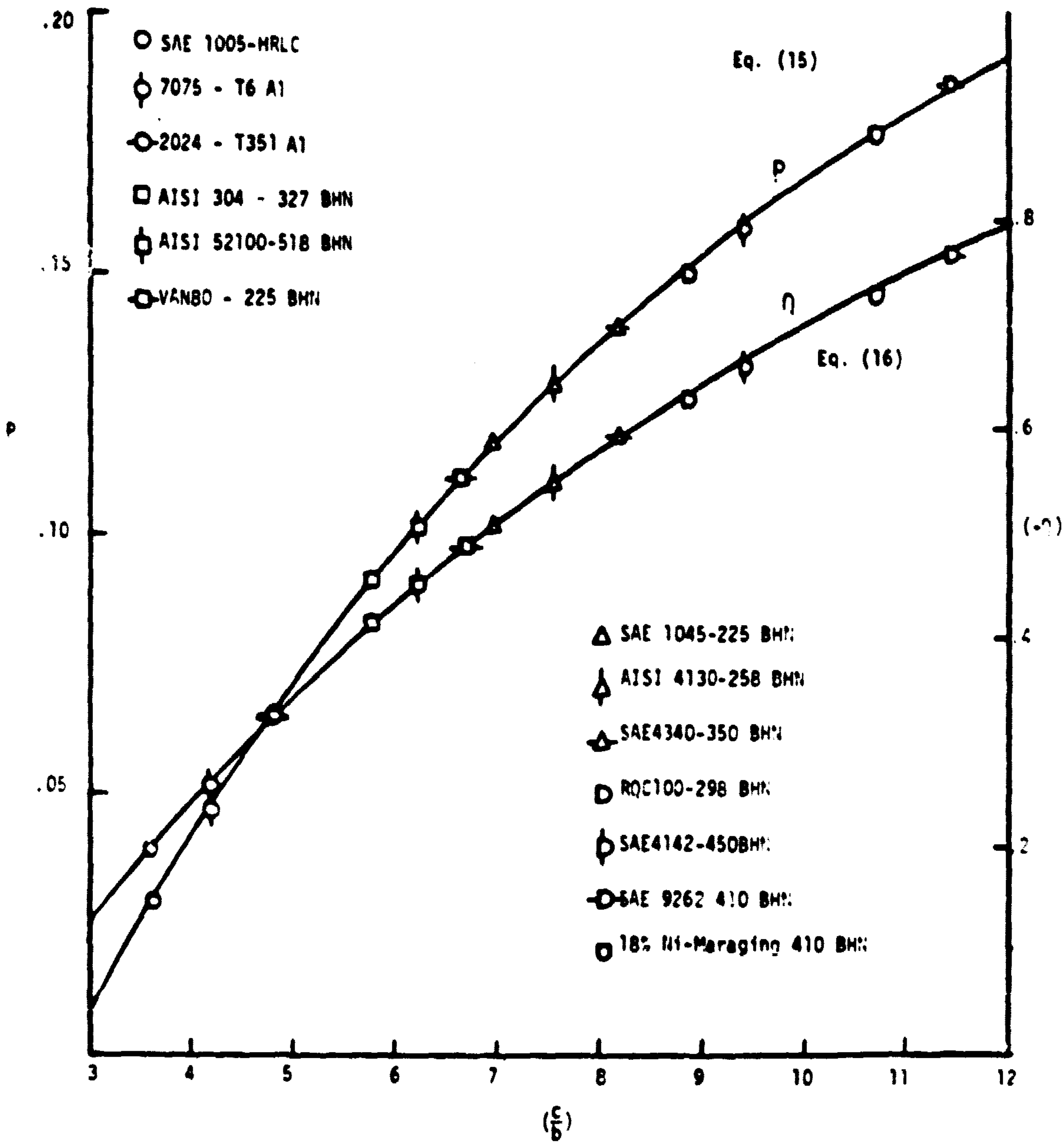


Fig. A-1. Determination of Constants P and Q in Eq. (A-1) for various materials shown.

APPENDIX B

INVERSION OF THE CYCLIC STRESS-STRAIN CURVE

The cyclic stress-strain curve can be expressed by the following:

$$\Delta\sigma = \Delta\varepsilon_e E \tag{B-1}$$

$$\Delta\varepsilon_e = \left(\frac{N_f}{N_T}\right)^b \cdot \Delta\varepsilon_T \tag{B-2}$$

$$\Delta\sigma = E\Delta\varepsilon_T \left(\frac{N_f}{N_T}\right)^b \tag{B-3}$$

$(\frac{N_f}{N_T})$ can be expressed in terms of R_ε by Eq. (7). Here Eq. (B-3) becomes,

$$\Delta\sigma = E\Delta\varepsilon_T \left[R_\varepsilon^{\frac{z}{b}} + R_\varepsilon^{\frac{z}{c}}\right]^{\frac{b}{z}} \tag{B-4}$$

where z is computed by Eqs. (14) to (17).

Fig. (B-1) shows the basic cyclic stress-strain curve for Ti-6Al-4V, and the inverted relation by Eq. (B-4), which are essentially identical. Hence stress is known directly in terms of applied strain-range.

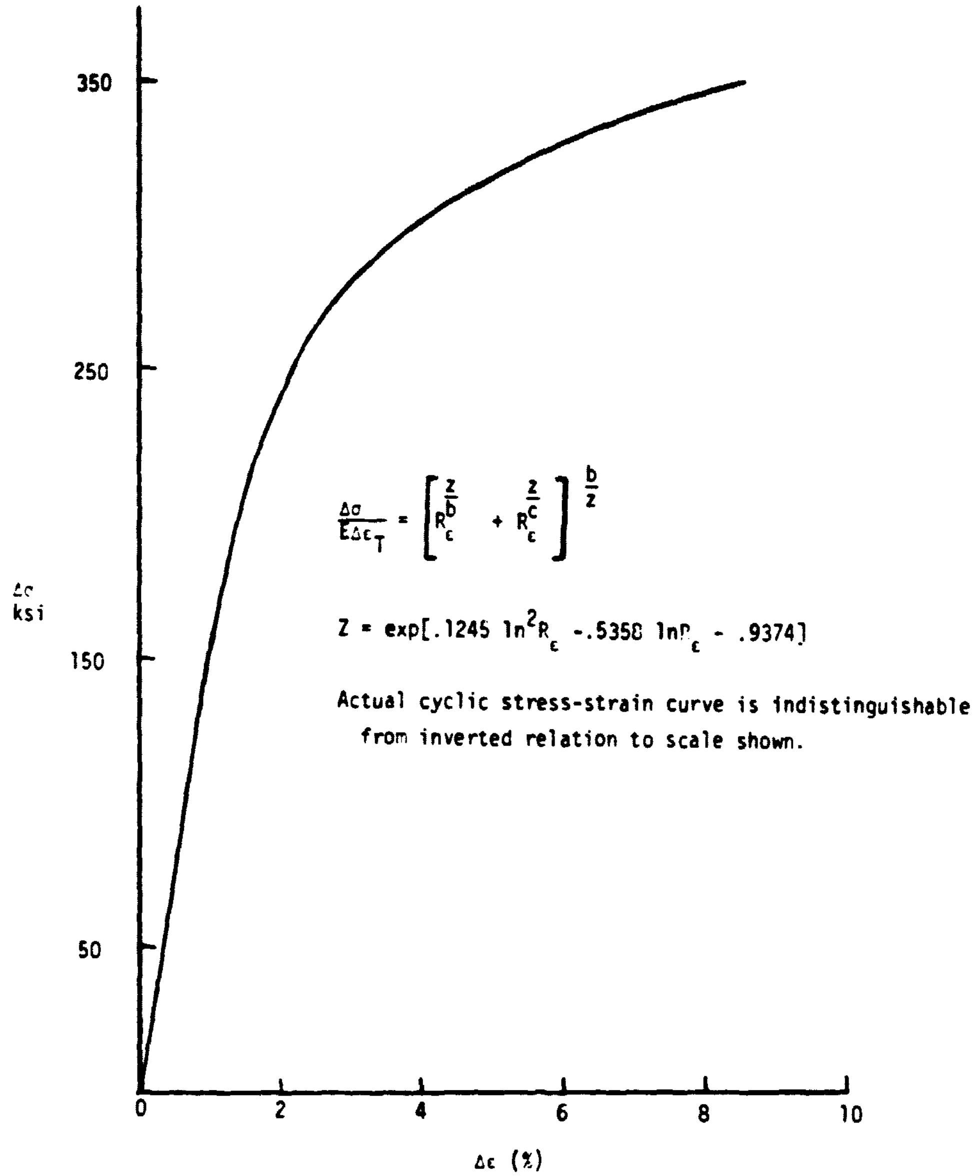


Fig. B-1. Comparison of basic cyclic stress-strain curve and inverted relation for Ti-6Al-4V (170°F).

APPENDIX C

INVERSION OF SMITH-WATSON-TOPPER RELATIONSHIP

Life associated with a given hysteresis loop with mean stress can be computed by the knowledge of maximum stress and strain range by the SWT procedure [7]. According to this method a universalized curve results when we plot $(\frac{E\Delta\varepsilon}{2} \cdot \sigma_{max})$ vs. N_f for all combinations of $\Delta\varepsilon$ and mean stress. Thus if we start with case for completely reversed loading,

$$\sigma_{max} = \frac{\sigma_f'}{E} (2N_f)^b$$

Using Eq. (1), we get

$$(\frac{\Delta\varepsilon}{2} \cdot E\ \sigma_{max}) = \sigma_f'^{\,2}(2N_f)^{2b} + \varepsilon_f'\sigma_f'E(2N_f)^{c+b} \qquad \text{(C-1)}$$

Expressing Eq. (C-1) in the form of Eq. (2),

$$\frac{\left(\frac{\Delta\varepsilon}{2}\ E\ \sigma_{max}\right)}{\left(\frac{\Delta\varepsilon}{2}\ E\ \sigma_{max}\right)_T} = \left(\frac{N_f}{N_T}\right)^{2b} + \left(\frac{N_f}{N_T}\right)^{(c+b)} \qquad \text{(C-2)}$$

where N_T is given by Eq. (4)

$$\text{and} \left(\frac{\Delta\varepsilon\ E\ \sigma_{max}}{2}\right)_T = \sigma_f'^{\,2}\left[\frac{E\varepsilon_f'}{\sigma_f'}\right]^{\frac{2b}{(b-c)}} \qquad \text{(C-3)}$$

Eq. (C-2) can be easily inverted by using the procedure described for inverting strain-life relationship

$$\text{Let} \quad R = \frac{\left(\frac{\Delta\varepsilon\ E\ \sigma_{max}}{2}\right)}{\left(\frac{\Delta\varepsilon\ E\ \sigma_{max}}{2}\right)_T}$$

Then

$$\frac{N_f}{N_T} = \left[R^{\frac{z}{2b}} + R^{\frac{z}{c+b}}\right]^{\frac{1}{z}} \qquad (C-4)$$

$$z = \exp[P \ln^2 R + Q \ln R + S]$$

$$P = -.001277[\frac{c+b}{2b}]^2 + .03893[\frac{c+b}{2b}] - .0927$$

$$Q = .004176 [\frac{c+b}{2b}]^2 - .135[\frac{c+b}{2b}] + .2309$$

$$S = \ln\left[-.889(c+b)(\frac{c+b}{2b})^{-.36}\right]$$

Actual $(\frac{\Delta\varepsilon}{2} E \sigma_{max})$ vs. N_f curve and inverted curve for Ti-6Al-4V as shown in Fig. C-1 are essentially indistinguishable. For a given material $\left(\frac{\Delta\varepsilon E \sigma_{max}}{2}\right)_T$; N_T, P,Q,S can be easily calculated. Then for any given hysteresis loop, if σ_{max} and $\frac{\Delta\varepsilon}{2}$ are known, N_f can be determined by Eq. (C-4).

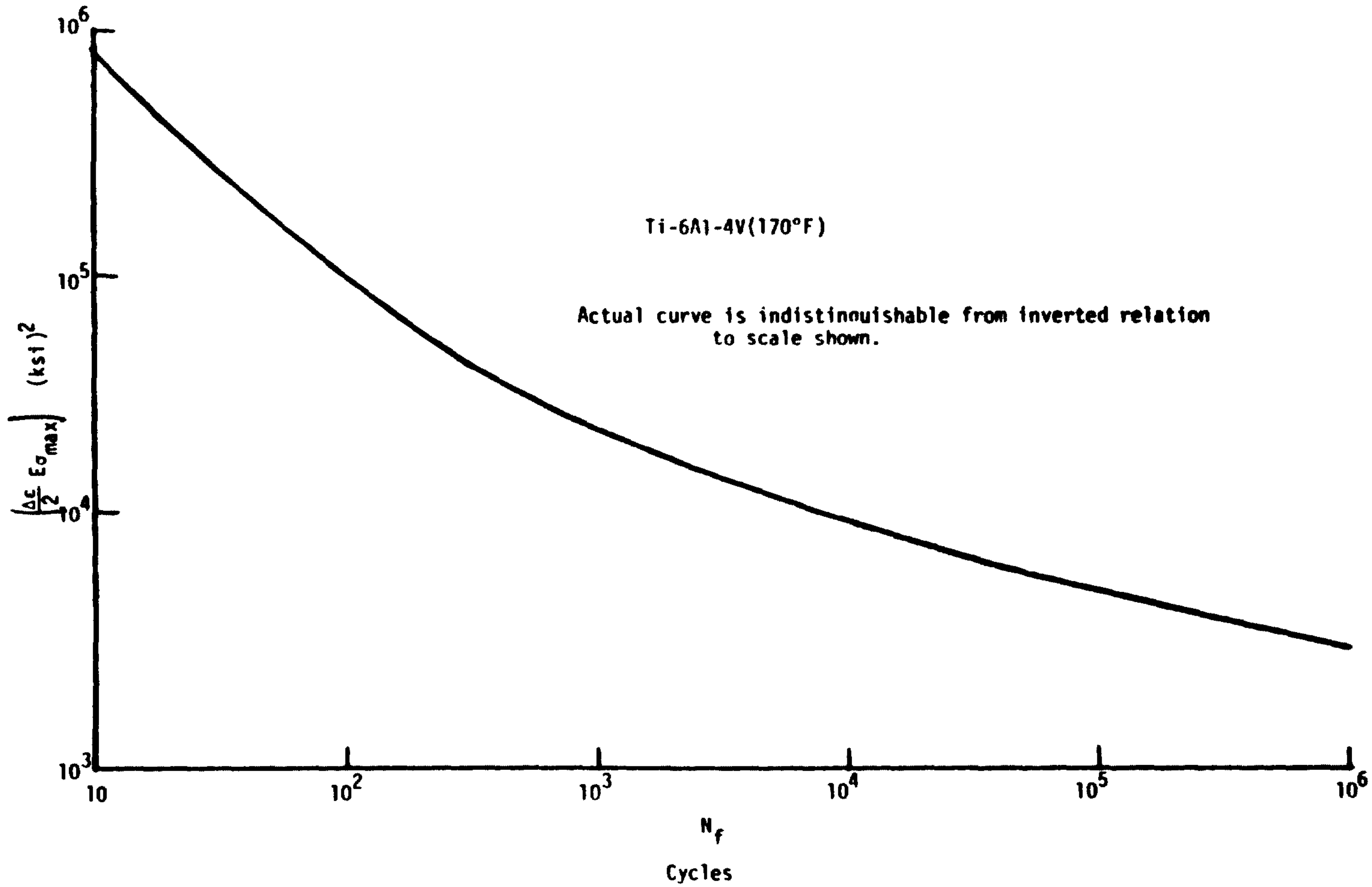


Fig. C-1. Comparison of actual and inverted curve used in Smith-Watson-Topper method for estimating life under mean stress and strainrange.