NASA Contractor Report 3431

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NASA

National Aeronautics
and Space Administration

**Scientific and Technical
Information Branch**

1981

ACKNOWLEDGMENTS

ABSTRACT

The purpose of this study is to determine the magnitude of error introduced due to wing vibration when measuring atmospheric turbulence with a wind probe mounted at the wing tip and to determine whether accelerometers mounted on the wing tip are needed to correct for this error. A spectrum analysis approach is used to determine the error. Estimates of the B-57 wing characteristics are used to simulate the airplane wing, and von Karman's cross spectrum function is used to simulate atmospheric turbulence. The major finding of the study is that wing vibration introduces large error in measured spectra of turbulence in the frequency's range close to the natural frequencies of the wing.

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LIST OF SYMBOLS

$a = b/b_R$	Distribution of semi-chord
A_{ij}, B_{ij}	Coefficient for the aerodynamic cross terms
b	Semi-chord (ft)
b_R	Reference semi-chord (ft)
C	Theodorsen's function
D	Characteristic determinate of eigenvalue problem
EI	Beam stiffness (lbf-ft^2)
F	Force on beam (lbf)
h	Response of system to unit impulse
J_0	Bessel function of the first kind order 0
J_1	Bessel function of the first kind order 1
k	Reduced frequency
K	Coefficient function for the lift due to gust
ℓ	Semi-wing span (ft)
L	Turbulence length scale (ft)
m	Mass per unit span along the wing (lbm/ft)
M_i	Generalized mass of mode
p	General time history input to system
s	Spectrum separation distance and non-dimensional time variable
S	Area of wing (ft^2)
t, τ	Time variable (sec)
u	Vertical gust velocity (ft/sec)
V	Mean flight speed (ft/sec)

w	Vertical displacement of wing and general time history of response of system
W,P	Fourier transform of w and p, respectively
y	Spanwise distance (ft)
Y_0	Bessel function of the second kind order 0
Y_1	Bessel function of the second kind order 1
Z	Frequency response function and Fourier transform of $h(t)$
Λ	Width of strips along the wing used in strip theory (ft)
α	Angle of attack (degrees)
η_i	Amplitude of natural mode (ft)
μ_i	Non-dimensional generalized mass
ξ_n	Non-dimensional amplitude of natural mode
ρ	Density of air (lbm/ft^3)
ϕ_i	ith natural bending mode of the wing
ϕ_m	Measured spectrum at the wing tip
ϕ_{wp}	Cross spectrum between w and p
ϕ_w, ϕ_p	Power spectrum of w and p, respectively
ψ_i	Fundamental solution to ordinary differential equation
Ω_i	Reduced natural frequency
ω	Frequency (radians/sec)
ω_i	Natural frequency of wing (radians/sec)

1.0 INTRODUCTION

Progress in applying spectrum analysis in aeronautical engi-
neering is fostering the measuring of the atmospheric turbulence
cross spectra, especially across wing spans. Measuring the atmo-
spheric turbulence spectrum across the wing span requires wind sensors
mounted on the wing tips, where the sensors are exposed to error due
to wing vibrations. Determining this error requires comparison
between the spectrum of the wing tip velocity and spectrum of
atmospheric turbulence. This study demonstrates a procedure to
estimate the wing tip velocity spectrum and the error introduced due
to wing vibration when measuring atmospheric turbulence with a wind
probe mounted at the wing tip of a B-57 type airplane. The wing
characteristics of a B-57 are used for modeling the airplane wing
because NASA is currently planning to use this airplane to acquire
atmospheric gust gradient data across the wing span. The purpose of
this study is to determine whether the error introduced in measuring
atmospheric turbulence is large enough to justify mounting accelero-
meters on the wing tip to correct for the error introduced into the
measured turbulence data due to the wing vibration.

An introduction to spectrum analysis is believed useful in
order to clarify the general solution technique and introduce the
major terms of spectrum analysis. Spectrum analysis is based upon
transferring the system from the time domain to the frequency domain.

The form of the response of a linear system in the time
domain to a single arbitrary input $p(t)$ is given by Duhamel's integral:

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$$w(t) = \int_{-\infty}^{\infty} p(\tau) \, h(t - \tau) d\tau \qquad (1)$$

where $h(t)$ is the response of the system to a unit impulse. If $p(t)$ is a random input, evaluating the integral becomes meaningless because the integral represents only one sample of a random distribution of responses. To overcome this difficulty, Duhamel's integral can be transformed from the time domain to the frequency domain for stationary random inputs. The Fourier transform of Duhamel's integral is:

$$W(\omega) = P(\omega) \, Z(\omega) \qquad (2)$$

where the frequency response function $Z(\omega)$ is the Fourier transform of $h(t)$ and represents the response of the system to a sinusoidal input. In general, the functions in Equation (2) are complex valued functions and contain both magnitude and phase information. In order to study only the magnitude of the response in the frequency domain, the power spectrum of the response is defined by:

$$\phi_w(\omega) = W(\omega) \, \overline{W}(\omega) = P(\omega) \, Z(\omega) \, \overline{P}(\omega) \, \overline{Z}(\omega) = \phi_p(\omega) \, |Z(\omega)|^2 \qquad (3)$$

where ϕ_p is the power spectrum of the input. This equation is the fundamental result of spectral analysis and equates the response spectrum to the product of the input spectrum and the square of the magnitude of the frequency response. References [1] and [2] discuss the derivation of the theory and contain many examples of the application of spectrum analysis.

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The input-output relationship of Equation (3) can be used to outline the general solution technique of spectrum analysis. First, the frequency response function, Z, is determined via the "equation of motion"; second, the input spectrum, ϕ_p, is defined; and, third, the product in the right-hand side of Equation (3) is calculated to obtain the output spectrum.

The complicated system involved with wing vibrations requires simplification in order to become amenable to solution. The major simplification in determining the frequency response function is putting restraints on the aircraft. This study will restrain the aircraft to only vertical motion of the center of gravity and vertical wing bending. The coordinate system used in this study is shown in Figure 1, where the spanwise coordinate, y, is the independent variable and the vertical deflection, w, is the dependent variable. The aerodynamic forces appearing in the equation of motion are those calculated from strip theory and assume the wing is a flat plate having stiffness similar to a B-57 wing. The gust is considered to include only vertical wind variations. The atmospheric spectrum is assumed to be stationary, homogeneous and isotropic; but turbulence varies across the wing span. For calculation purposes, the von Karman cross spectral function as defined by Houbolt and Sen [3] is assumed.

3
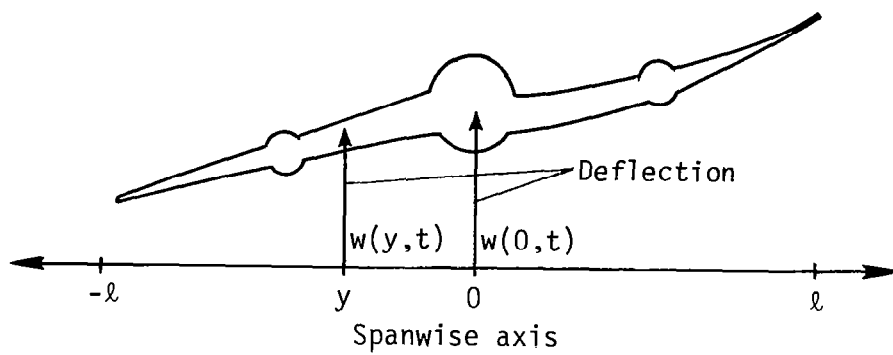
FIGURE 1   COORDINATE SYSTEM

2.0 ANALYSIS

2.1 Spectrum Analysis

The spectrum equation for a single stationary random input, derived in the introduction, is not sufficient for calculating the spectrum of the wing tip velocity for an aircraft with large wing span. Assuming that the aircraft experiences a single input or gust would be tantamount to assuming that the gust field is uniform across the span but random in the flight direction, as depicted by the left-hand sketch in Figure 2. This assumption is unrealistic for an aircraft with large wing span. A better assumption is that the gust is two-dimensional, so that it is also random across the span of the aircraft, as depicted in the right-hand sketch in Figure 2. The assumption of one-dimensional turbulence may lead to an underestimation of the response of the aircraft. Houbolt [4] has shown the the root bending moment spectrum determined by two-dimensional turbulence is significantly higher than the root bending moment spectrum for one-dimensional turbulence. The ratio $2\ell/L$, where 2ℓ is the span of the aircraft and L is the turbulence length scale, determines the validity of the assumption of uniform spanwise turbulence; for $2\ell/L$ approaching unity the assumption of spanwise uniformity becomes invalid. The B-57 has a wing span of $2\ell = 66$ ft, and Houbolt [5] recommends a length scale of atmospheric turbulence of L = 300 ft, in which case the ratio $2\ell/L = 0.22$ is large enough to warrant a two-dimensional spectrum analysis. Near the ground L becomes smaller making the above argument even stronger.
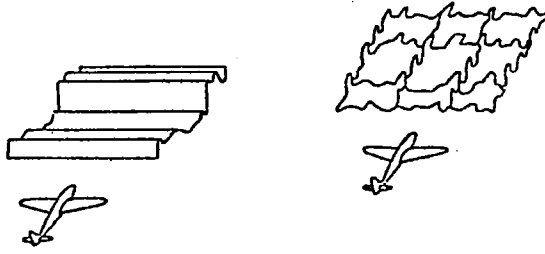
FIGURE 2   ONE-DIMENSIONAL AND TWO-DIMENSIONAL TURBULENCE ENCOUNTER [4]

Assuming a linear system with a continuum of random stationary inputs, the response in the form of Duhamel's integral is:

$$w(t) = \int_{-\ell}^{\ell} \int_{-\infty}^{\infty} p(y,\tau) \ h(y,t - \tau) \ d\tau \ dy \qquad (4)$$

The Fourier transform of the above equation is:

$$W(\omega) = \int_{-\ell}^{\ell} P(y,\omega) \ Z(y,\omega) \ dy \qquad (5)$$

where the frequency response function $Z(y,\omega)$ is a function of both frequency ω and input location y. This study will define $Z(y,\omega)$ as the velocity of the right wing tip due to a sinusoidal gust of frequency ω located at y along the wing and $w(t)$ as the velocity of the right wing tip due to gust excitation along the entire wing.

The output power spectrum as defined by Equation (3) is

$$\phi_w(\omega) = W(\omega) \ \overline{W}(\omega) = \int_{-\ell}^{\ell} \int_{-\ell}^{\ell} \phi_p(y_1,y_2,\omega) \ Z(y_1,\omega) \ \overline{Z}(y_2,\omega) \ dy_1 \ dy_2$$

$$\qquad (6)$$

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The cross spectrum is defined by:

$$\phi_p(y_1, y_2, \omega) = P(y_1, \omega)\ \overline{P(y_2, \omega)} \tag{7}$$

Then it is true for all cross spectra that

$$\phi_p(y_1, y_2, \omega) = P(y_1, \omega)\ \overline{P(y_2, \omega)}$$

$$= \overline{\overline{P(y_1, \omega)}\ P(y_2, \omega)}$$

$$= \overline{\phi}_p(y_2, y_1, \omega) \tag{8}$$

Because the domain of integration is symmetric, an appropriate choice of the limits of integration can be made so that the integrand can be written as:

$$\phi_p(y_1, y_2, \omega)\ Z(y_1, \omega)\ \overline{Z}(y_2, \omega) + \phi_p(y_2,\ y_1, \omega)\ Z(y_2, \omega)\ \overline{Z}(y_1, \omega)$$

$$= \phi_p(y_1, y_2, \omega)\ Z(y_1, \omega)\ \overline{Z}(y_2, \omega) + \overline{\phi}_p(y_1, y_2, \omega)\ \overline{Z(y_2, \omega)\ \overline{Z(y_1, \omega)}}$$

$$= \phi_p(y_1, y_2, \omega)\ Z(y_1, \omega)\ \overline{Z}(y_2, \omega) + \phi_p(y_1, y_2, \omega)\ \overline{Z(y_1, \omega)\ \overline{Z}(y_2, \omega)}$$

$$= 2\mathrm{Re}[\phi_p(y_1, y_2, \omega)\ Z(y_1, \omega)\ \overline{Z}(y_2, \omega)] \tag{9}$$

Therefore the symmetry involved in the domain of the integration and the products of conjugate pairs permits the integral to be rewritten as:

$$\phi_w(\omega) = 2\mathrm{Re}\left[\int_{-\ell}^{\ell} \lim_{y^* \to y_2} \int_{y^*}^{\ell} \phi_p(y_1, y_2, \omega)\ Z(y_1, \omega)\ \overline{Z}(y_2, \omega)\ dy_1 dy_2 \right] \tag{10}$$

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This equation is completely general and requires only that the inputs be stationary. It is reasonable to expect that the spectrum of the input (in particular, turbulent input) is isotropic; in other words, the spectrum is a function of the separation distance [meaning $\phi_p(y_1, y_2) = \phi_p(|y_1 - y_2|)$] only and independent of direction. This assumption can further simplify the integral by making the variable substitutions $s = y_1 - y_2$ and $y = y_2$ and interchanging the order of integration so that:

$$\phi_W(\omega) = \lim_{s^* \to 0} \int_{s^*}^{2\ell} \phi_p(s,\omega) \ 2\text{Re}\left[\int_{-\ell}^{\ell-s} \overline{Z}(y,\omega) \ Z(y+s,\omega) \ dy\right] ds \tag{11}$$

This equation is the final form of the spectrum equation that will be used in this study. Note that $\phi_p(s,\omega)$ is the cross spectrum of the inputs, which within this study is the cross spectrum of atmospheric turbulence. The cross spectrum of turbulence represents the contribution of the frequency ω to the covariance of the vertical velocity at two points along the wing span separated by a distance s.

2.2 Mechanical Analysis

Consider a nonuniform wing (chord, beam stiffness, and mass varying along the span) free to move and bend vertically and subjected to a sinusoidal vertical gust at a spanwise element of width Δ centered at $y = y^*$. The balance of forces then requires that

$$F_S = -F_I - F_M + F_G \ \delta(y,y^*) \tag{12}$$

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where F_S is the force due to beam stiffness, F_I is the inertial force, and F_M and F_G are aerodynamic forces due to translatory wing motion and due to vertical gust, respectively. The function $\delta(y,y*)$ selects the portion of the wing which is subjected to the gust and is zero everywhere except between $y* \pm \Delta/2$, where it has the value of unity. The theory of strength of material states:

$$F_S = \frac{\partial^2}{\partial y^2} \left(EI \frac{\partial^2 w}{\partial y^2} \right) \tag{13}$$

where EI is the beam bending stiffness and w is the vertical deflection of the wing. Newton's law gives:

$$F_I = m\ddot{w} \tag{14}$$

Theodorsen in his famous NACA report [6] showed that the force due to wing motion for a two-dimensional wing is:

$$F_m = \pi \rho b^2 \ddot{w} + 2\pi \rho Vb \, C(k) \, \dot{w} \tag{15}$$

where V is the mean flight speed, ρ is the density of the flight medium, and b is the semi-chord of the wing. The Theodorsen function, $C(k)$, is a function of the reduced frequency ($k = \omega b/V$) of the motion. Both References [2] and [7] develop and apply the Theodorsen function. Defined in terms of the Bessel function, the Theodorsen function is:

$$C = \frac{J_1(J_1 + Y_0) + Y_1(Y_1 - J_0) - i(Y_1 Y_0 + J_1 J_0)}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2} \tag{16}$$

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Note that the argument of the functions is k, the reduced frequency of the motion. The lift due to a gust acting on a two-dimensional wing with semi-chord b as reported in [2] is:

$$F_G = 2\pi\rho V^2 b \left(\frac{u}{V}\right) K(k) \tag{17}$$

where K(k) is the Kussner function. It too is dependent on reduced frequency of the gust. In terms of the Theodorsen function and Bessel functions, the Kussner function is defined:

$$K = C(J_0 - iJ_1) + iJ_1 \tag{18}$$

The forces may be added to yield the partial differential equation:

$$\frac{\partial^2}{\partial y^2} \left(EI \frac{\partial^2 w}{\partial y^2}\right) = -m\ddot{w} - \pi\rho b^2 \ddot{w} - 2\pi\rho Vb \; C(k) \; \dot{w} + 2\pi\rho V^2 b \; K(k) \; \frac{u}{V} \; \delta(y,y*) \tag{19}$$

The boundary conditions are:

$$w''(\ell,t) = w'''(\ell,t) = w''(-\ell,t) = w'''(-\ell,t) = 0 \tag{20}$$

meaning no shear or moment at the wing tips, while the time dependence will be taken to be periodic, as discussed later.

Assume the vertical deflection w(y,t) may be expanded in its natural modes, $\{\phi_i\}_{i=1}^{\infty}$, as:

$$w(y,t) = \sum_{i=1}^{\infty} n_i(t) \; \phi_i(y) \tag{21}$$

Substitution of the solution expanded in its natural modes into the governing Equation (19) yields, after some algebra, an equation for the spatial dependence (the free vibration equation) and another for the time dependence:

$$\frac{\partial^2}{\partial y^2}\left[EI \frac{\partial^2}{\partial y^2} \phi_i\right] = m\omega_i^2\phi_i \qquad \text{for} \qquad -\ell < y < \ell \qquad (22)$$

where ω_i is the natural frequency of the ith mode. The boundary conditions of the differential equations are:

$$\phi_i'''(\ell) = \phi_i'''(-\ell) = \phi_i''(\ell) = \phi_i''(-\ell) = 0 \qquad (23)$$

which corresponds to the shear and bending moment being zero at the ends, as is the case for free ends.

The natural modes are orthogonal [2] because of the choice of boundary conditions so that:

$$\int_{-\ell}^{\ell} (EI\phi_i'')'' \phi_j dy = \omega_i^2 \int_{-\ell}^{\ell} m\phi_i\phi_j dy = \begin{cases} M_i\omega_i^2 & \text{for} \quad i = j \\ \\ 0 & \text{for} \quad i \neq j \end{cases} \qquad (24)$$

Then by using the orthogonality property of the modes, the differential Equation (19) becomes a system of linear differential equations in terms of the natural modes:

$$M_n\omega_n^2 n_n = -M_n\ddot{n}_n - \pi\rho b_R^2 \sum_{j=1}^{N} \overline{A}_{nj}\ddot{n}_j - 2\pi\rho Vb_R \, C(k) \sum_{j=1}^{N} \overline{B}_{nj}\dot{n}_j$$

$$+ 2\pi\rho V^2 b_R \, K(k) \frac{u(y^*)}{V} \phi_i(y^*) \, a(y^*)\Delta \qquad (25)$$

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Where

$$\overline{A}_{nj} = \int_{-\ell}^{\ell} a^2 \phi_n \phi_j \, dy \quad \text{and} \quad \overline{B}_{nj} = \int_{-\ell}^{\ell} a \phi_n \phi_j \, dy$$

are coefficients of the aerodynamic cross terms, the weighting function of the integrals $a(y) = b(y)/b_R$ is the semi-chord distribution. We now introduce the variables:

$$k = \frac{\omega b_R}{V} \quad \text{and} \quad s = \frac{Vt}{b_R} \tag{27}$$

and assume sinusoidal output to sinusoidal input; in other words, we assume the variables are of the form $u = \overline{u} e^{iks}$ and $\eta_n = \overline{\eta}_n e^{iks}$. After substitution of the above variables and division by $\pi \rho V^2 S e^{iks}$, the system of linear differential equations becomes a system of linear algebraic equations:

$$\mu_n \Omega_n^2 \overline{\xi}_n = k^2 \mu_n \overline{\xi}_n + k^2 \sum_{j=1}^{N} A_{nj} \overline{\xi}_j + 2ik \, C(k) \sum_{j=1}^{N} B_{nj} \overline{\xi}_j$$

$$+ \frac{2b_R \, K(k)}{S} \phi_n(y^*) \, a(y^*) \tag{28}$$

where $\Omega_n = \dfrac{\omega_n b_R}{V}$

$$A_{nj} = \frac{b_R \overline{A}_{nj}}{S}$$

$$B_{nj} = \frac{b_R \overline{B}_{nj}}{S}$$

$$\mu_n = \frac{M_n}{\pi \rho b_R S}$$

$$\overline{\xi}_n = \frac{\overline{\eta}_n}{b_R} \frac{V}{\overline{u}}$$

This is the final form of the equations which will be used in the program. Many of the constants of the equations are calculated by integrating products of the natural modes by different weighting functions, and they can be determined as soon as the free vibration problem has been solved for the different modes. The solution $\overline{\xi}_n = \overline{\eta}_n / b_R (V/\overline{u})$ represents the amplitude of the modal response of the deflection of the wing to sinusoidal gust with unit change in angle of attack normalized by the reference semi-chord. The solution, $\xi_n = \overline{\xi}_n e^{iks}$, may be differentiated with respect to time to give the amplitude of the wing tip velocity of the modal response:

$$\frac{d}{dt} \xi_n = ik \frac{\eta_n}{b_R} \frac{V}{\overline{u}} = i\omega \frac{\eta_n}{\overline{u}} \tag{29}$$

For the particular problem of this study, the frequency response function is defined as the velocity of the right wing tip due to a sinusoidal gust located at y* along the wing. The frequency response function is then:

$$Z(y^*,\omega) = i\omega \sum_{j=1}^{N} \frac{\overline{\eta}_j(y^*,\omega)}{\overline{u}} \tag{30}$$

This is the definition used in the spectrum analysis part of the program.

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3.0 NUMERICAL PROCEDURE

The numerical procedure is basically divided into three sub-
routines. The first subroutine solves the free vibration problem and
determines the natural bending modes and frequencies of the wing. The
second subroutine solves the forced vibration problem and determines
the frequency response function of the wing to a sinusoidal gust at a
point along the span. Finally, the third subroutine performs the arith-
metic in the power spectrum equation and determines the wing tip velocity
spectrum.

3.1 Free Vibration: Structural Character of the Wing

The eigenvalue problem solved by this program is for a wing
vibrating in only bending modes in free space (meaning free ends, i.e.,
moment and shear are zero at the wing tips).

The differential equation is:

$$(EI\phi'')'' = m\omega^2\phi \quad \text{for} \quad -\ell \leq y \leq \ell \tag{31}$$

with boundary conditions:

$$\phi'''(\ell) = \phi''(\ell) = \phi'''(-\ell) = \phi''(-\ell) = 0 \tag{32}$$

where ϕ is the natural mode or eigenfunction and ω is the natural fre-
quency or eigenvalue.

The coefficients of this equation are EI, the bending stiffness
of the wings, and m, the mass per unit length, and both are a function

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of y. Both EI and m are defined symmetrically for most wings, and this implies that the solution of the differential equation should be either symmetric or antisymmetric. The program takes advantage of this so that the range of integration is cut in half (i.e., the problem needs to be solved only for $0 \leq y \leq \ell$). The boundary conditions must then be restated at the origin (or the midspan of the airplane) to be those for a symmetric or antisymmetric function dependent upon whether an even or odd mode is being investigated. The conditions at the origin for a symmetric function are:

$$\phi'(0) = \phi'''(0) = 0 \tag{33}$$

while the conditions for an antisymmetric function are:

$$\phi(0) = \phi''(0) = 0 \tag{34}$$

These will be the boundary conditions, along with those at the wing tip $(y = \ell)$.

If the eigenvalue of the differential equation is specified, the problem is reduced to a linear fourth order, two-point boundary value problem, and a shooting method [10] can be used to determine the numerical solution. The first step of the shooting method is to reduce the single fourth order equation to a system of four first order equations. For Equation (31) the system becomes:

$$\frac{d}{dy}\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{m\omega^2}{EI} & 0 & \frac{-2EI'}{EI} & \frac{-EI''}{EI} \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \tag{35}$$

and boundary conditions for a symmetric mode are:

$$\phi_2(0) = \phi_4(0) = \phi_3(\ell) = \phi_4(\ell) = 0 \tag{36}$$

If all the initial conditions were given, a Runge Kutta scheme could be
used to determine the numerical solution, but because the solution must
match the boundary conditions at $y = \ell$, a shooting method must be used.
The shooting method estimates the complete set of initial conditions
and then uses a Runge Kutta scheme to determine the solution of the
initial value problem. The value of the solution of the initial value
problem at $y = \ell$ is compared with the boundary conditions at $y = \ell$ for the
two-point boundary value problem. The estimate of the complete initial
condition is improved and the procedure is repeated; in this way the
complete set of initial conditions is determined so that the solution
to the initial value problem has the correct value for the boundary
conditions at $y = \ell$. Fortunately, for a linear system the complete set
of initial conditions can be improved to the correct initial conditions
after one trial. This will be shown true later. Because the differen-
tial equation is linear, it can be shown [8] that there exists a vector
base, $\{\overline{\psi}\}_{i=1}^{4}$, of the solution of Equation (35). Then each solution

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$$\overline{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \tag{37}$$

of Equation (35) for the boundary conditions indicated is a linear combination of the base solution:

$$\overline{\phi} = \sum_{i=1}^{4} \overline{\psi}_i C_i \tag{38}$$

By choosing four linearly independent initial conditions, $\{\overline{\psi}\}_{i=1}^{4}$ is guaranteed to be a complete base. A convenient choice of linearly independent initial conditions is:

$$\overline{\psi}_1(0) = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} ; \quad \overline{\psi}_2(0) = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix} ; \quad \overline{\psi}_3(0) = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix} \quad \text{and} \quad \overline{\psi}_4(0) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \tag{39}$$

A base of $\{\overline{\psi}_i\}_{i=1}^{4}$ defined by the above initial conditions and the differential equation is commonly called a fundamental base or the fundamental solutions. The boundary conditions are used to determine the constants $\{C_i\}_{i=1}^{4}$ of Equation (38). In the case of the even mode, the $\{C_i\}_{i=1}^{4}$ must satisfy the system of equations:

$$
\begin{bmatrix}
\psi_{12}(0) & \psi_{22}(0) & \psi_{32}(0) & \psi_{42}(0) \\
\psi_{14}(0) & \psi_{24}(0) & \psi_{34}(0) & \psi_{44}(0) \\
\psi_{13}(\ell) & \psi_{23}(\ell) & \psi_{33}(\ell) & \psi_{43}(0) \\
\psi_{14}(\ell) & \psi_{24}(\ell) & \psi_{34}(\ell) & \psi_{44}(0)
\end{bmatrix}
\begin{bmatrix}
C_1 \\ C_2 \\ C_3 \\ C_4
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
\tag{40}
$$

If the problem was a simple two-point boundary value problem, the boundary conditions would be nonhomogeneous. Then the system of linear Equations (40) would be nonhomogeneous and the $\{C_i\}_{i=1}^{4}$ could be determined after the fundamental solutions $\{\overline{\psi}_i\}_{i=1}^{4}$ were determined from one trial of the shooting method. An eigenvalue problem by definition requires that the boundary conditions be homogeneous, and this leads to a homogeneous system of linear Equations (40) for the $\{C_i\}_{i=1}^{4}$. Linear algebra theory requires that the determinant of the coefficient matrix vanish in order for the homogeneous system to have a nontrivial solution. This determinant:

$$
D(\omega) =
\begin{vmatrix}
\psi_{12}(0) & \psi_{22}(0) & \psi_{32}(0) & \psi_{42}(0) \\
\psi_{14}(0) & \psi_{24}(0) & \psi_{34}(0) & \psi_{44}(0) \\
\psi_{13}(\ell) & \psi_{23}(\ell) & \psi_{33}(\ell) & \psi_{43}(\ell) \\
\psi_{14}(\ell) & \psi_{24}(\ell) & \psi_{34}(\ell) & \psi_{44}(\ell)
\end{vmatrix}
\tag{41}
$$

is commonly called the characteristic determinant, and for $\{C_i\}_{i=1}^{4}$ to be determined it must vanish. Note that the determinant is a function of ω, the unspecified parameter of the differential equation.

Reference [9] shows that the characteristic determinant is an analytic function of ω and the eigenvalues of the differential equation are the zeros of the function $D(\omega)$. When ω is an eigenvalue of the differential equation, the characteristic determinant vanishes and the boundary conditions can be satisfied as a linear combination of the four fundamental solutions. Note that the solution of the system of linear equations for the $\{C_i\}_{i=1}^{4}$ is not unique; therefore an extra condition must be supplied. In this study we demand that the natural modes have the value of unity at the wing tips, which becomes our extra condition imposed on the $\{C_i\}_{i=1}^{4}$. The values of the derivatives of the fundamental solutions appearing as elements of the characteristic determinant are obtained by solving the differential equation by a Runge Kutta method. Note that some of the terms in the determinant are already known from the definition of the fundamental solutions. Substituting in the determinant for these values and simplifying yields:

$$
D(\omega) = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \psi_{13}(\ell) & \psi_{23}(\ell) & \psi_{33}(\ell) & \psi_{43}(\ell) \\ \psi_{14}(\ell) & \psi_{24}(\ell) & \psi_{34}(\ell) & \psi_{44}(\ell) \end{vmatrix} = \begin{vmatrix} \psi_{13}(\ell) & \psi_{43}(\ell) \\ \psi_{13}(\ell) & \psi_{43}(\ell) \end{vmatrix}
$$

(42)

The characteristic determinant is now a function of only the first and fourth fundamental solutions because these are the only two fundamental solutions that satisfy the conditions at the origin for a symmetric function. The program takes advantage of this and determines

only two fundamental solutions. Reference [9] further discusses the theory of linear differential equations.

The general solution procedure can now be outlined. First, the eigenvalue is estimated; then the two fundamental solutions are determined by a Runge Kutta Fehlberg order seven scheme. The value of the characteristic determinant is calculated from the fundamental solutions. A search routine checks if the eigenvalue is bracketed between the current estimate and the previous estimate. In this case, the program is directed to a bisection routine to improve the brackets or continues, taking another step along the frequency line and using this as its next estimate of the eigenvalue. After the eigenvalue has been determined, the natural mode is normalized by a unit displacement at the right wing tip. The new mode is integrated with previous modes determined to calculate the aerodynamic cross terms. The program then steps along the frequency line for its first and second estimates of the next eigenvalue. Another example of this technique for solving eigen-value problems is found in Reference [10].

The calculation was tested against the uniform beam and was found to be very accurate. Three runs were made for different EI distributions, and the program converged very quickly for the lower modes. Figures 3 and 4 show estimates of m and b_R for the B-57 used in all the cases run.

The most difficult parameter to estimate is the beam stiffness, EI. A static analysis was used to estimate the beam stiffness, assuming a loading on the B-57 wing which would be used during extreme operations and a load factor of 10 g. Appendix B shows the details of the analysis;

20
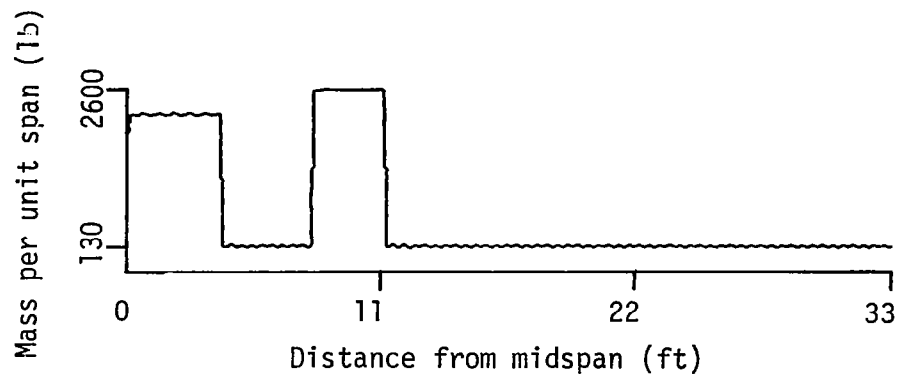
FIGURE 3   DISTRIBUTION OF MASS ACROSS THE SEMISPAN OF THE WING
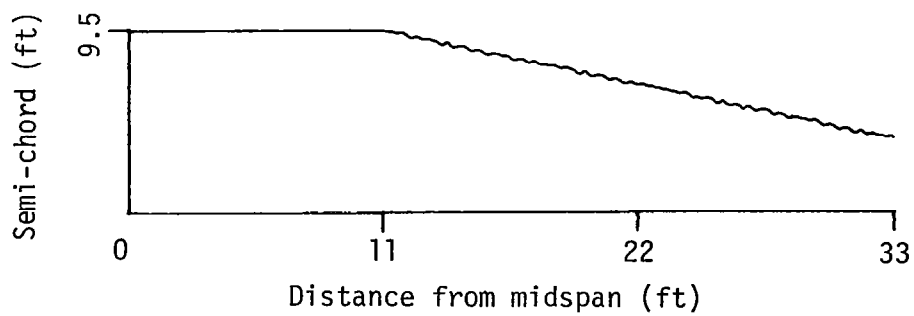
FIGURE 4   DISTRIBUTION OF SEMI-CHORD ACROSS THE SEMISPAN OF THE WING

the results of the analysis show that the beam stiffness is approximately:

$$EI = 9 \times 10^8 \quad \text{for} \quad y \leq |11|$$

$$EI = 9 \times 10^7 \quad \text{for} \quad y \geq |11| \tag{43}$$

This wing is referred to as the "standard wing" throughout the rest of this study. Its natural modes and frequencies are shown in Figure 5. From an in-flight experiment with the B-57, NASA/Langley Research Center determined that the natural frequency of the first bending mode of the B-57 is approximately 7 Hz. In this study, a trial and error method was used with the program to determine the beam stiffness necessary to have a first bending mode at 7 Hz. The beam stiffness was found to be:

$$EI = 3 \times 10^9 \quad \text{for} \quad y \leq |11|$$

$$EI = 3 \times 10^8 \quad \text{for} \quad y \geq |11| \tag{44}$$

The wing with this beam stiffness distribution is referred to as the "stiff wing." For comparison purposes, a third beam stiffness with distribution

$$EI = 9 \times 10^6 \quad \text{for} \quad y \leq |11|$$

$$EI = 9 \times 10^5 \quad \text{for} \quad y \geq |11| \tag{45}$$

was run. Figure 6 shows the natural modes and frequencies of this "flexible wing."

Mode	Natural frequency (Hz)	Reduced natural frequency
1	4.37	0.072
2	12.06	0.199
3	23.35	0.385
4	40.34	0.666
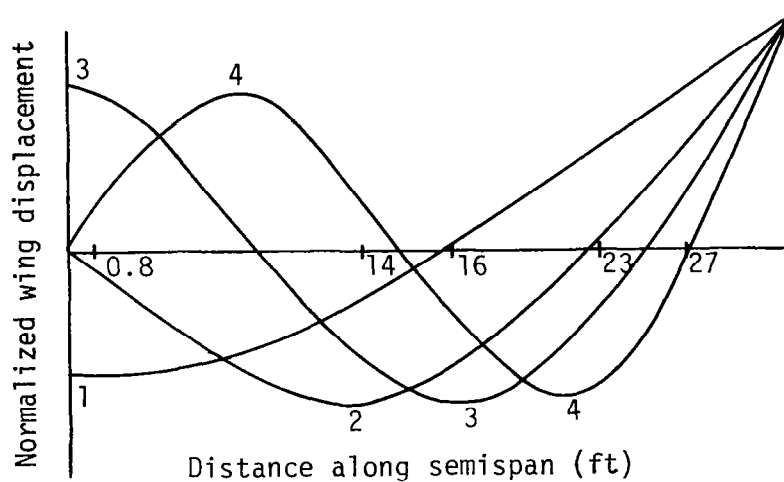
FIGURE 5   NATURAL MODES AND FREQUENCIES FOR THE STIFF WING

Mode	Natural frequency (Hz)	Reduced natural frequency
1	0.44	0.0072
2	1.20	0.0198
3	2.33	0.0385
4	4.03	0.0666
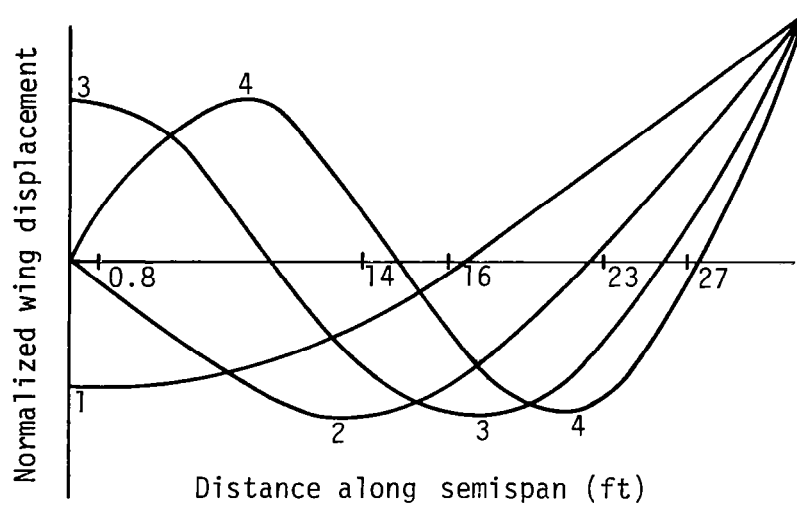
FIGURE 6   NATURAL MODES AND FREQUENCIES FOR THE FLEXIBLE WING

3.2 Forced Vibration: Frequency Response Function

The final form of the equations for the frequency response function of the wing tip deflection due to a sinusoidal gust have been obtained by separating variables, integrating the spanwise influence, and assuming sinusoidal form for the solution. Remaining yet unsolved is a system of linear equations for the amplitudes of the different modal responses. The system of equations is:

$$\left(\Omega_n^2 - k^2\right) \mu_n \xi_n - \sum_{j=1}^{4} \left[k^2 A_{nj} - k\, C(k)\, B_{nj}\right] \xi_j = \frac{2b_R a(y^*)}{S} \Delta\, K(k)\, \phi_n(y^*)$$

$$(46)$$

where $\Omega_i = \dfrac{\omega_i b_R}{V}$; reduced natural frequency of mode

$$\mu_i = \int_{-\ell}^{\ell} m\phi_i \phi_i \, dy / \pi \rho b_R S \quad ; \quad \text{normalized general mass}$$

$$A_{ij} = \frac{b_R}{S} \int_{-\ell}^{\ell} a^2 \phi_i \phi_j \, dy \quad \text{and} \quad B_{ij} = \frac{b_R}{S} \int_{-\ell}^{\ell} a \phi_i \phi_j \, dy \quad ; \quad \text{normalized}$$

aerodynamic cross terms

The unknowns, $\overline{\xi}_i = \overline{n}_i / b_R (V/\overline{u})$, are normalized amplitudes of the modal wing tip deflection in terms of the reference semi-chord, b_R, and the magnitude of the disturbing force, $\alpha = \overline{u}/V$. The remaining terms are the reduced forcing frequency, Ω_j, and the gust location, y^*, which for this study are spaced evenly at 38 locations along the span. The above system of equations is solved by Gaussian elimination for each forcing frequency and gust location. Fortunately, the coefficient matrix does

not change for each gust location, and the coefficient matrix is reduced only once in subroutine GAUSS for each forcing frequency. The nonhomogeneous part is reduced for each gust location and the amplitudes are determined by back substitution in subroutine BACKS.

Careful examination of the cross terms, A_{ij} and B_{ij}, shows that they vanish when the integration is of the product of an even mode and an odd mode. This means that the linear system of equations becomes uncoupled between even and odd modes, and the response of the system to a sinusoidal input as separated into response of even modes and response of odd modes can then be examined. Obviously, the response of the even modes to a gust located at y is the same as the response to a gust located at -y. The response of the odd modes to a gust is antisymmetric. In other words, the response of the odd modes to a gust at y is the negative of the response at -y. The system of linear equations is solved for gust locations on only half of the wing. Shown in Figures 7-11 are the plots of the frequency response function versus the reduced frequency of the gust for several gust locations along the nonuniform wings.

3.3 Spectrum Equation: Wing Tip Velocity Power Spectrum

The final form of the output power spectrum due to a continuum of stationary, homogeneous, isotropic input is obtained from Equation (11), which can be integrated numerically using the trapezoidal rule to yield:

$$\phi_w(s,w) = \phi_p(0,\omega) \sum_{j=1}^{N} Z_i Z_i + \sum_{j=1}^{N-1} \phi_p(j\Delta,\omega) \; 2\mathrm{Re}\left(\sum_{i=1}^{N-j} Z_i \overline{Z}_j \right) \qquad (47)$$

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Tip velocity of wing normalized by gust velocity

0.382

0.0

Flexible wing

Standard wing

0.001 0.01 0.1 1.0 10.0

Reduced frequency $\dfrac{\omega b_R}{V}$

Gust location at y (0.86 ft)
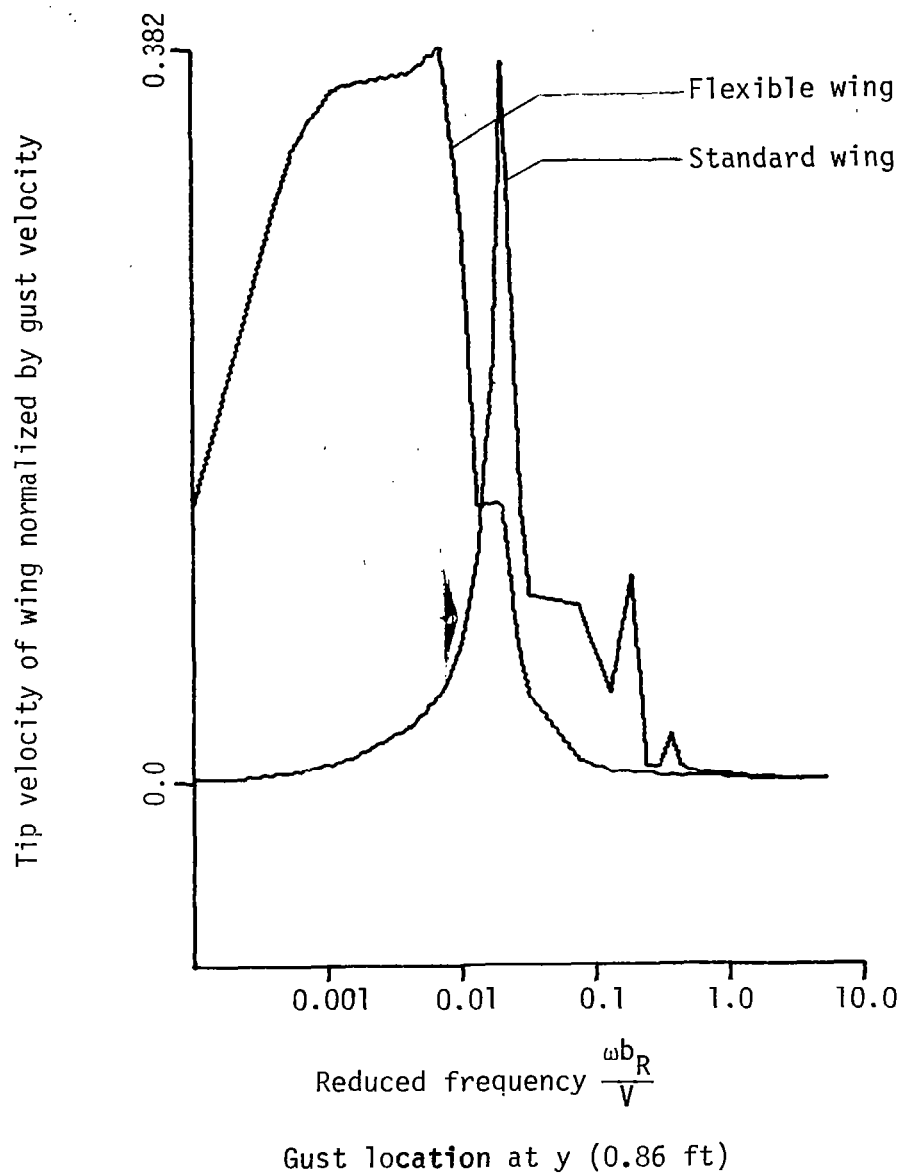
FIGURE 7   FREQUENCY RESPONSE FUNCTIONS DUE TO GUST EXCITATION AT 0.86 FEET FROM MIDSPAN
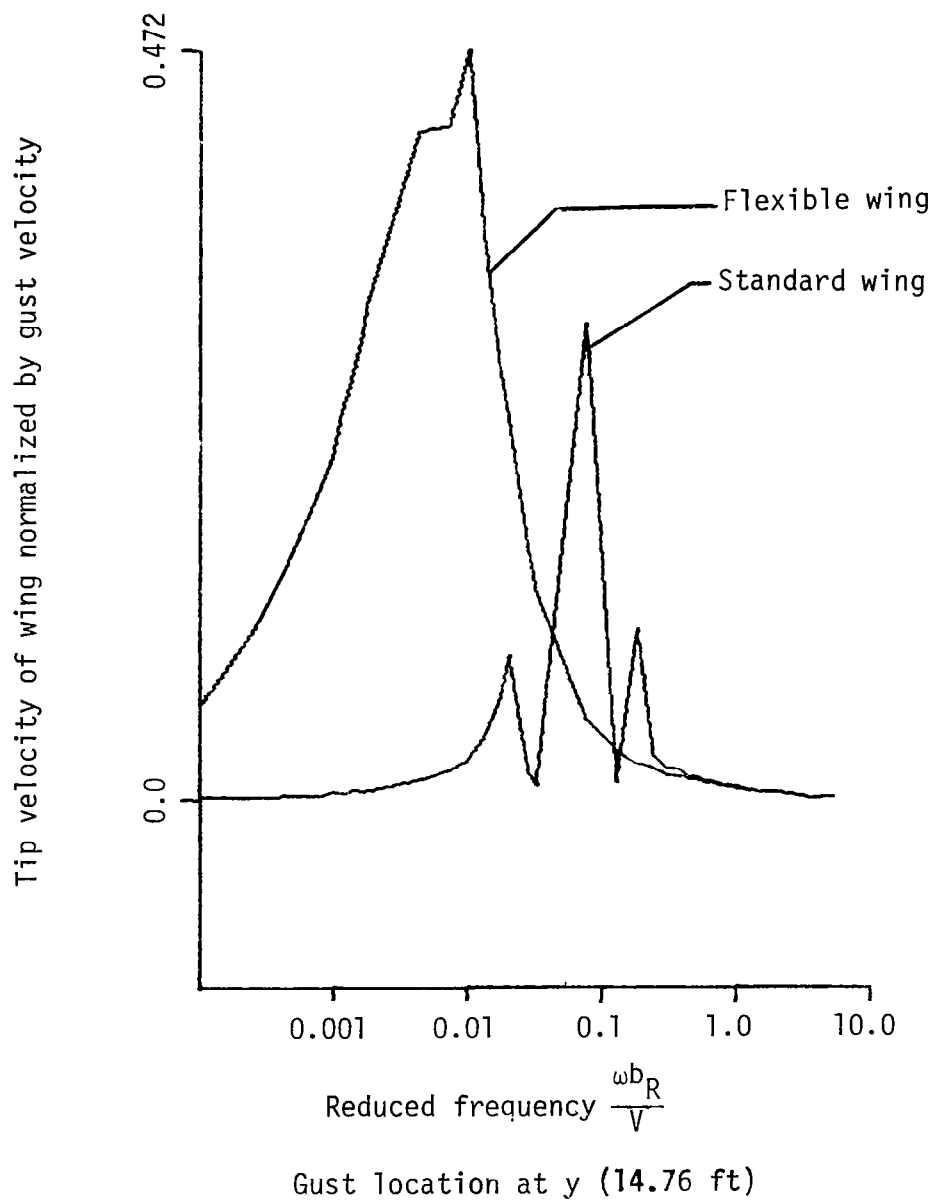
FIGURE 8 FREQUENCY RESPONSE FUNCTIONS DUE TO GUST EXCITATION AT
14.76 FEET FROM MIDSPAN
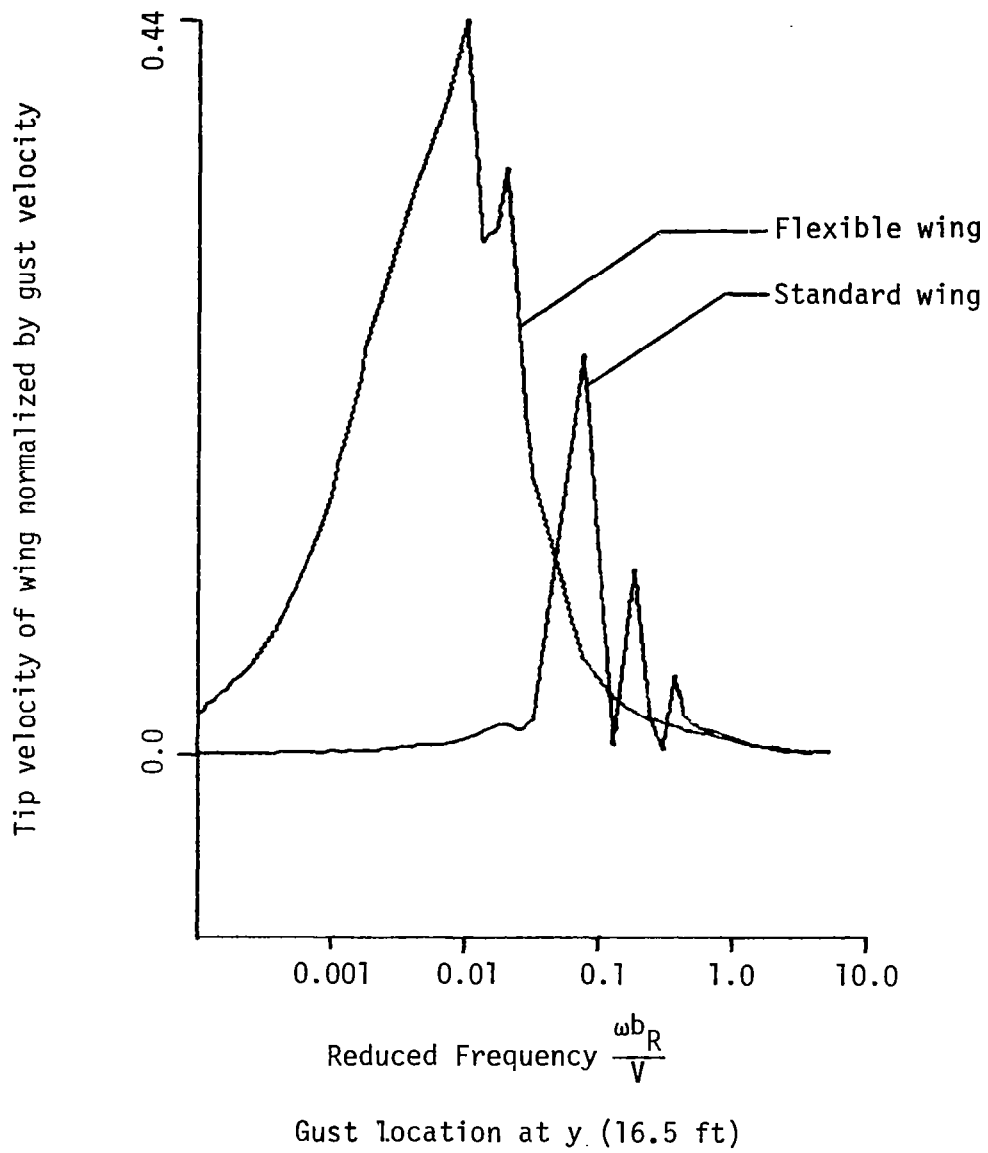
FIGURE 9   FREQUENCY RESPONSE FUNCTIONS DUE TO GUST EXCITATION AT
16.5 FEET FROM MIDSPAN
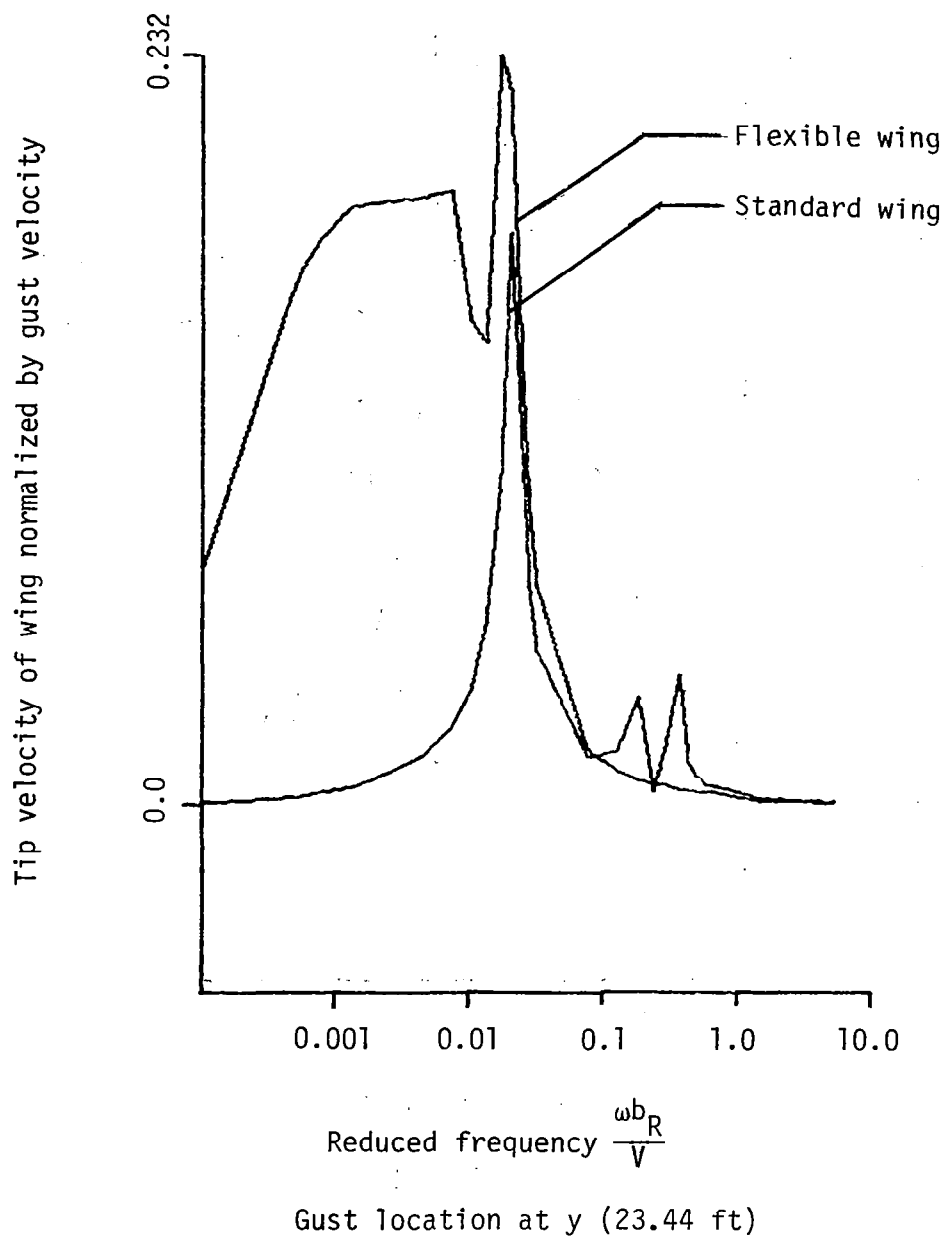
FIGURE 10 FREQUENCY RESPONSE FUNCTIONS DUE TO GUST EXCITATION AT
23.44 FEET FROM MIDSPAN

30
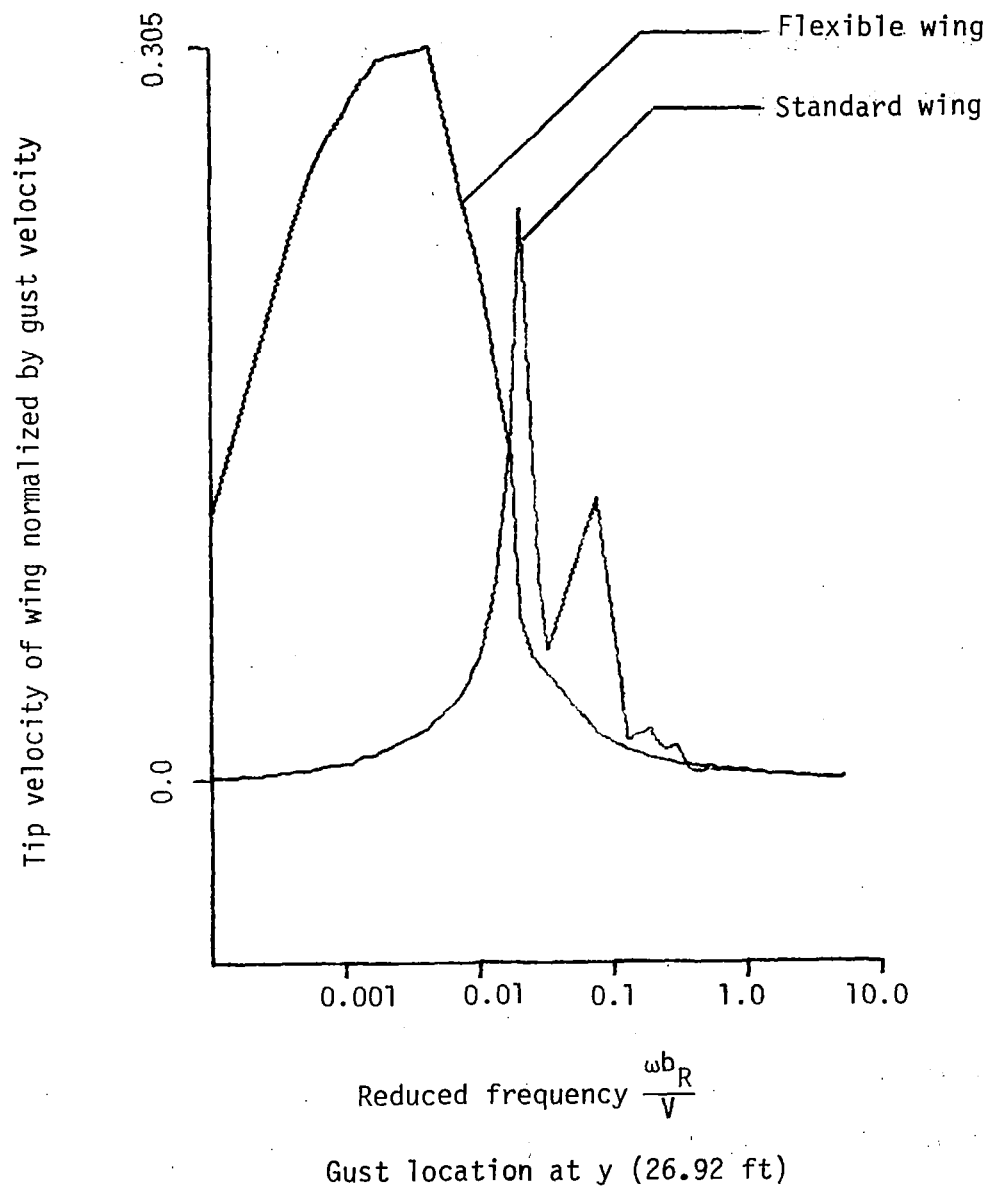
FIGURE 11 FREQUENCY RESPONSE FUNCTIONS DUE TO GUST EXCITATION AT
26.92 FEET FROM MIDSPAN

31

where N is the number of gust stations and Δ is the gust station width. The spectrum program determines the wing tip velocity spectrum, therefore, Z_i must represent the velocity at the right wing tip due to a sinusoidal gust at station i. The frequency response function is defined in the program as:

$$Z = \frac{i\omega}{\overline{u}} \sum_{j=1}^{NM} \overline{n}_j \qquad (48)$$

where NM is the number of elastic modes considered. The summation includes the response of only the elastic modes; the response of the rigid body mode is not included in the summation because the navigation system, located at the airplane's center of gravity, should be able to subtract the motion of the center of gravity from the turbulence data taken at the wing tip. There is a difference between the motion of the center of gravity and the motion of the rigid body mode; the center of gravity motion includes the motion of the even modes at the center of gravity. An attempt should be made to filter out the elastic mode motion from the center of gravity motion before correcting the turbulence data because the elastic mode motion at the center of gravity can be 180 degrees out of phase with the elastic mode motion at the wing tip and can therefore introduce a larger error in the turbulence data. This study will assume that the motion of the center of gravity measured by the aircraft navigation system has been filtered so that it contains only the rigid body motion of the airplane before correcting the turbulence data. In this study wing vibration is defined as the motion of only the elastic modes of the wing.

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Note that the frequency response function is normalized by the
gust magnitude, $\bar{u}$, and therefore represents the velocity at the wing
tip due to a unit sinusoidal gust. The input spectrum is taken as
von Karman's cross spectrum function of atmospheric turbulence [3] and
is normalized in the program by the root mean square of the turbulence,
σ_u^2. Therefore, the wing tip velocity power spectrum is normalized by
the root mean square of the turbulence. Reference [2] gives further
examples of spectrum analysis problem solving.

4.0 RESULTS AND CONCLUSIONS

A study of the frequency response function will aid in under-
standing the power spectrum. An examination of the frequency response
in the high frequency (reduced frequencies above one) domain displayed
in Figures 7-11 shows that regardless of gust location, the wing tip
velocity vanishes rapidly with increasing gust frequency. Physically,
the wing is expected to respond less to higher frequencies because the
higher modes have low response amplitude. Another gust location
independent trend in the frequency response function is the extreme
maximum at low frequency, also displayed in Figures 7-11. This is
physically explainable because the gust frequency becomes close to the
natural frequency of the wing.

The more interesting trends of the frequency response functions
of the wing are the gust location dependent trends. Figures 7-11 show
the frequency resonse functions for both the standard and the flexible
wing excited at five different locations along the wing. The locations
of the gust are marked along the abscissa in Figures 5 and 6. Figure 7
shows the response of the wing due to a gust near the midspan.
Because the odd modes have their nodal point at the midspan, they
should not respond as much as the even modes. The response is particu-
larly clear in the curve for the standard wing where only the first and
third elastic modes have large peaks. The response of the flexible wing
is not quite as clear as the response of the standard wing because its
natural frequencies are so close together the curve tends to be washed

out. Figure 8 shows the response of the wing due to gust excitation at 14.76 feet from midspan. This gust location is at the maximum of second elastic mode deflection and close to the nodal points of the first and third elastic modes. The response curve should be like that shown for the standard wing. The frequency response curve for the flexible wing shows a much more pointed maximum than the response curve for the flexible wing shown in Figure 7 because in Figure 8 only the second elastic mode is participating while in Figure 7 the maximum consists of both the first and third elastic mode responses. The first elastic mode has its nodal point approximately 16.5 feet from the mid span. Figure 9 shows the frequency response function due to gust excitation at this point. The response of the flexible wing shows a pointed maximum characteristic of low first elastic mode participation, while the third mode shows a great deal of participation since the excitation is at its maximum deflection for the mode. As expected, the standard wing shows a very small peak for its first mode response. Figure 10 shows the response curve due to gust excitation near the nodal point for the second elastic mode. The response of the flexible wing shows a local minimum for the second elastic mode response, while the first and third elastic modes show peaks for their response. The standard wing shows no response around the second elastic mode. Figure 11 shows the response curve for gust excitation at 26.92 feet from mid span, which is near the nodal point for the third and fourth elastic modes. The curve shows minimum participation of the third and fourth modes for both standard and flexible wings.

The wind velocity measured at the wing tip is the sum of the turbulence, $U_p(t)$, at the wing tip and the velocity of the wing tip, $U_w(t)$, due to wing vibration. The measured wind velocity is then:

$$U_m(t) = U_p(t) + U_w(t) \qquad (49)$$

The correlation function of the measured wind is:

$$R_m(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} U_m(t) \, V_m(t + \tau) \, d\tau$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left[U_w(t) \, U_w(t + \tau) + U_p(t) \, U_p(t + \tau) \right.$$

$$\left. + U_w(t) \, U_p(t + \tau) + U_p(t) \, U_w(t + \tau) \right] d\tau$$

$$= R_w(t) + R_p(t) + R_{wp}(t) + R_{pw}(t) \qquad (50)$$

The Fourier transform of the correlation equation gives the spectrum equation

$$\phi_m = \phi_w + \phi_p + \phi_{wp} + \phi_{pw} \qquad (51)$$

Since $\phi_{wp} = \overline{\phi}_{pw}$, the equation can be simplified:

$$\phi_m = \phi_w + \phi_p + 2\mathrm{Re}\phi_{wp} \qquad (52)$$

where ϕ_p is the power spectrum of turbulence at the wing tip and ϕ_w is the power spectrum of the wing tip velocity. Figures 12-15 show

36
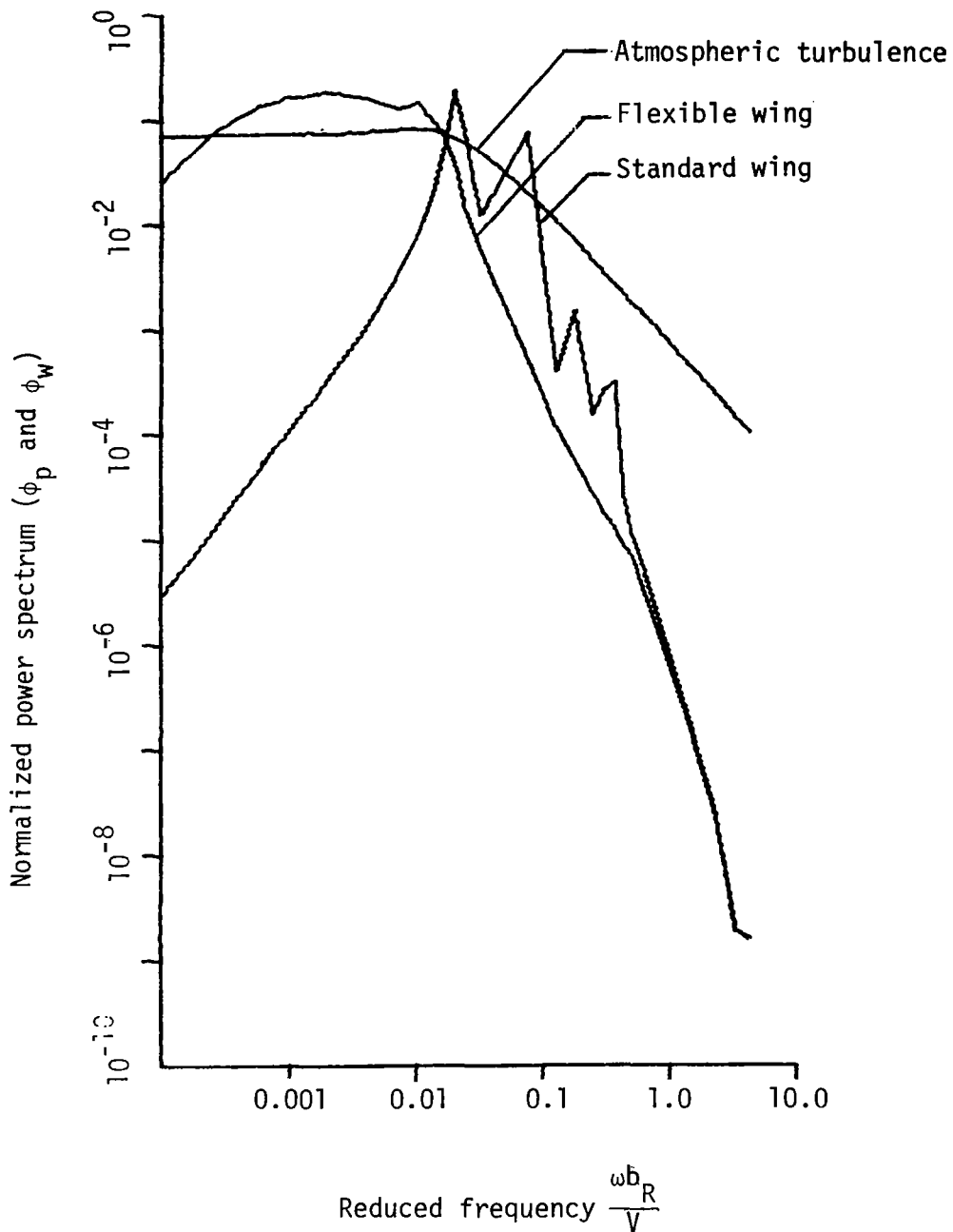
FIGURE 12 SPECTRUM OF ATMOSPHERIC TURBULENCE FOR LENGTH SCALE
132 FEET AND SPECTRUM OF WING TIP VELOCITY FOR FLEXIBLE
AND STANDARD WINGS
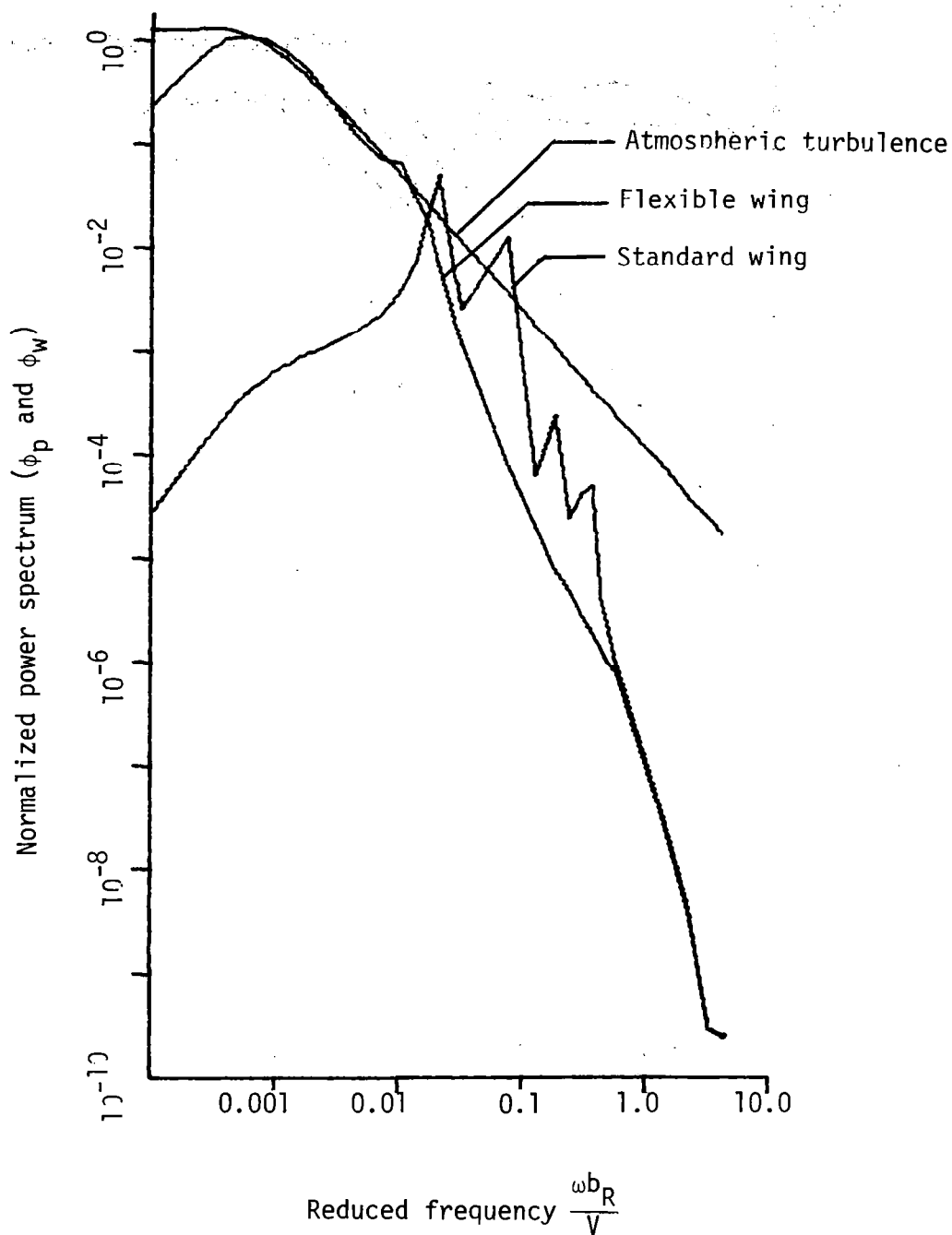
FIGURE 13   SPECTRUM OF ATMOSPHERIC TURBULENCE FOR LENGTH SCALE
2112 FEET AND SPECTRUM OF WING TIP VELOCITY FOR FLEXIBLE
AND STANDARD WINGS

38
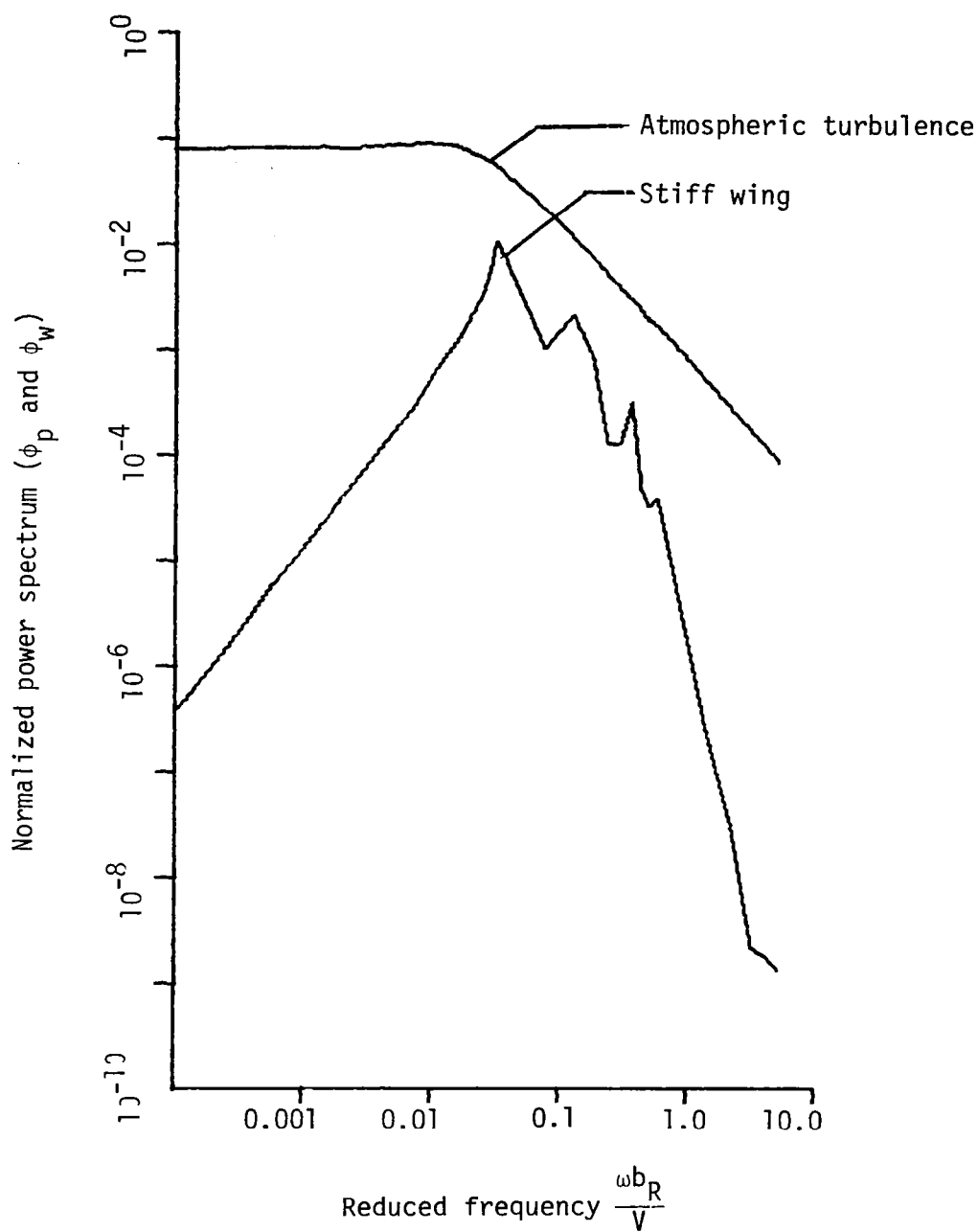
FIGURE 14 SPECTRUM OF ATMOSPHERIC TURBULENCE FOR LENGTH SCALE 132 FEET AND SPECTRUM OF WING TIP VELOCITY FOR STIFF WING

39

Reduced frequency $\frac{\omega b_R}{V}$
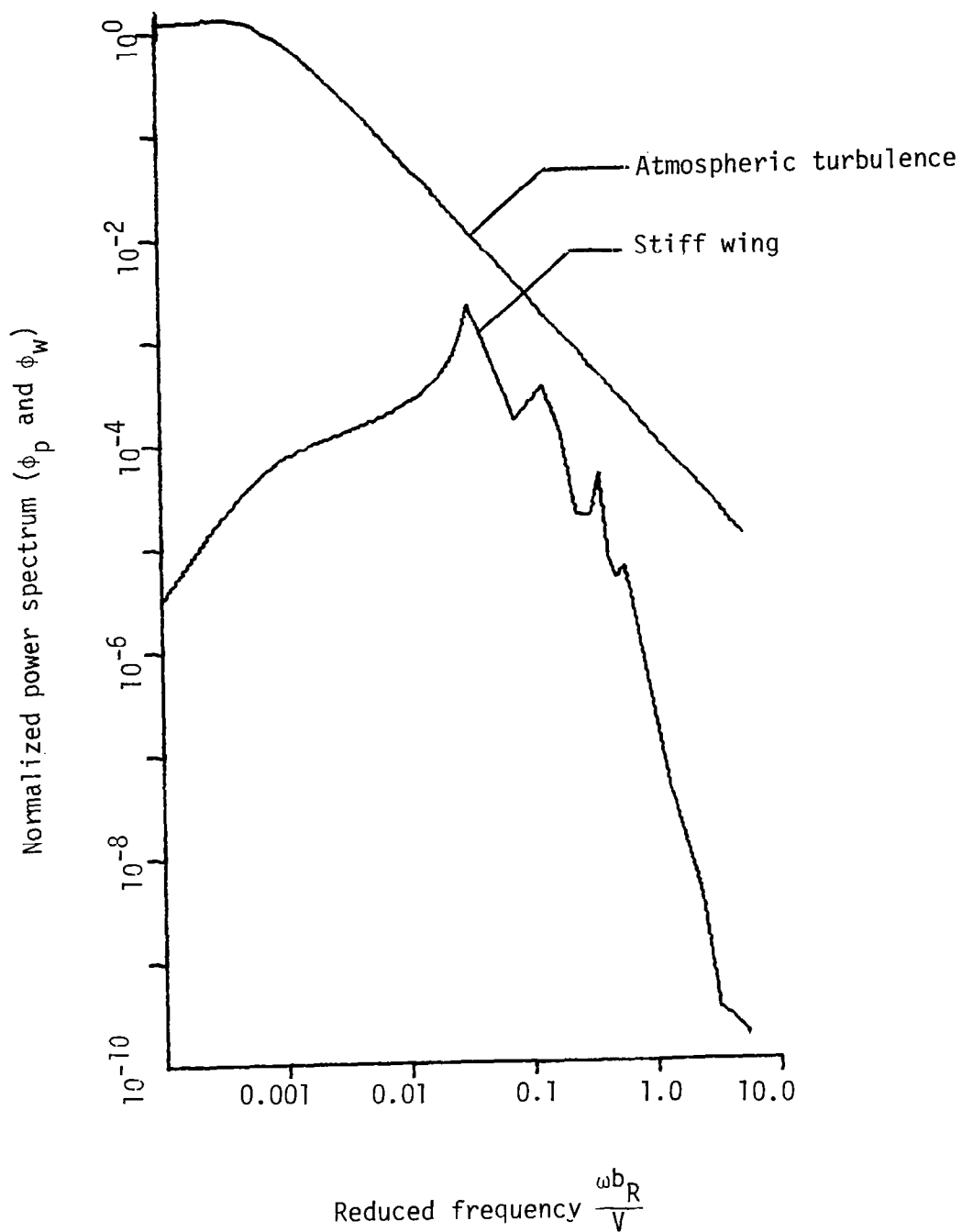
FIGURE 15   SPECTRUM OF ATMOSPHERIC TURBULENCE FOR LENGTH SCALE
2112 FEET AND SPECTRUM OF WING TIP VELOCITY FOR STIFF
WING

the wing tip velocity power spectrum and, for comparison, the atmospheric turbulence power spectrum. The cross spectrum ϕ_{wp} contains phase information between the wing tip velocity and the atmospheric turbulence velocity. The equation for ϕ_{wp} used in the program is:

$$\phi_{wp} = \int_0^{2\ell} \phi_p(s) \, Z(\ell - s) \, ds \qquad (53)$$

The relative error introduced due to wing tip velocity is:

$$E = \frac{|\phi_m - \phi_p|}{\phi_p} = \frac{|\phi_w + 2\mathrm{Re}\phi_{wp}|}{\phi_p} \qquad (54)$$

Figures 16-21 show this relative error as a function of frequency for the three different wings and different turbulence length scales. The flexible wing shows a large relative error throughout all the lower frequencies. An airplane having these wing characteristics is not recommended for measuring atmospheric cross spectra across its wing span. The standard wing shows a large relative error close to its first natural bending frequency. The first natural frequency can be close to the lower frequencies of atmospheric turbulence (approximately 0.04 Hz) if the turbulence length scale is small enough. For large turbulence length scale, the maximum error introduced due to wing vibration of the standard wing is reduced but still amounts to some 50 percent more over the turbulence spectra scale as $\omega L/V$ and; hence, for higher length scales the frequency of the wing vibration is out of the frequency range which contains significant turbulence energy. If the wing has characteristics

41
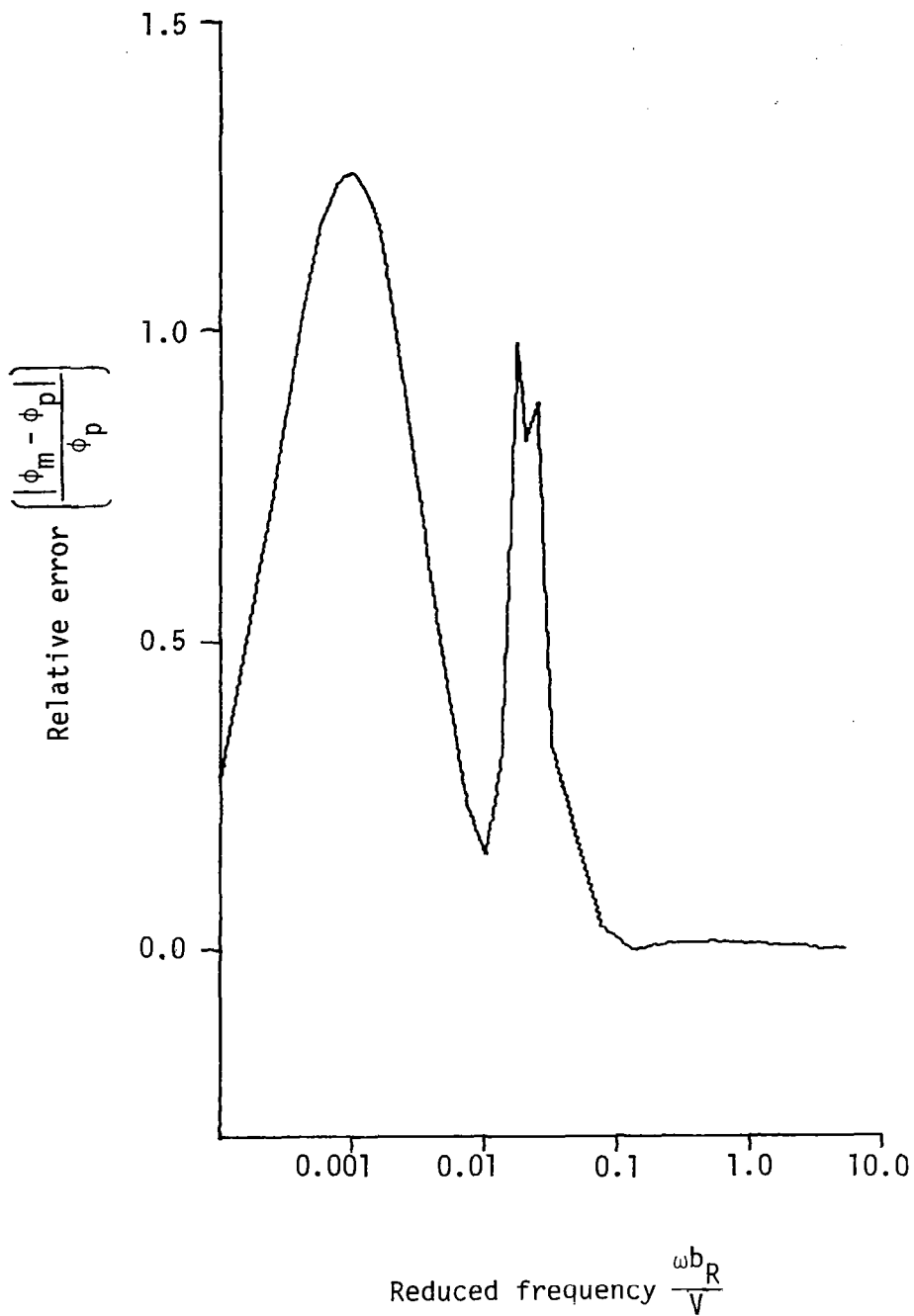
FIGURE 16   RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
132 FEET FOR FLEXIBLE WING
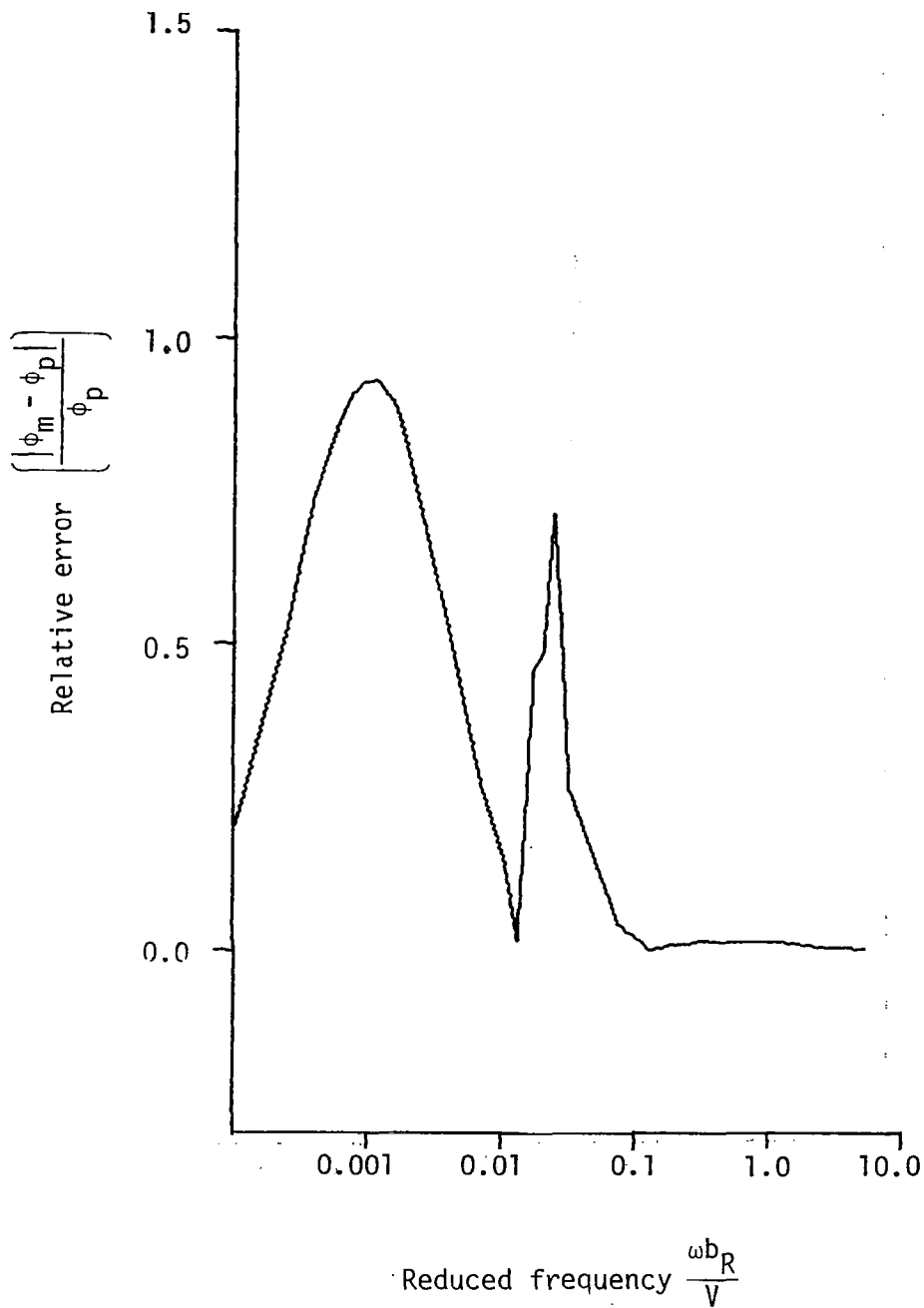
FIGURE 17 RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
2112 FEET FOR FLEXIBLE WING
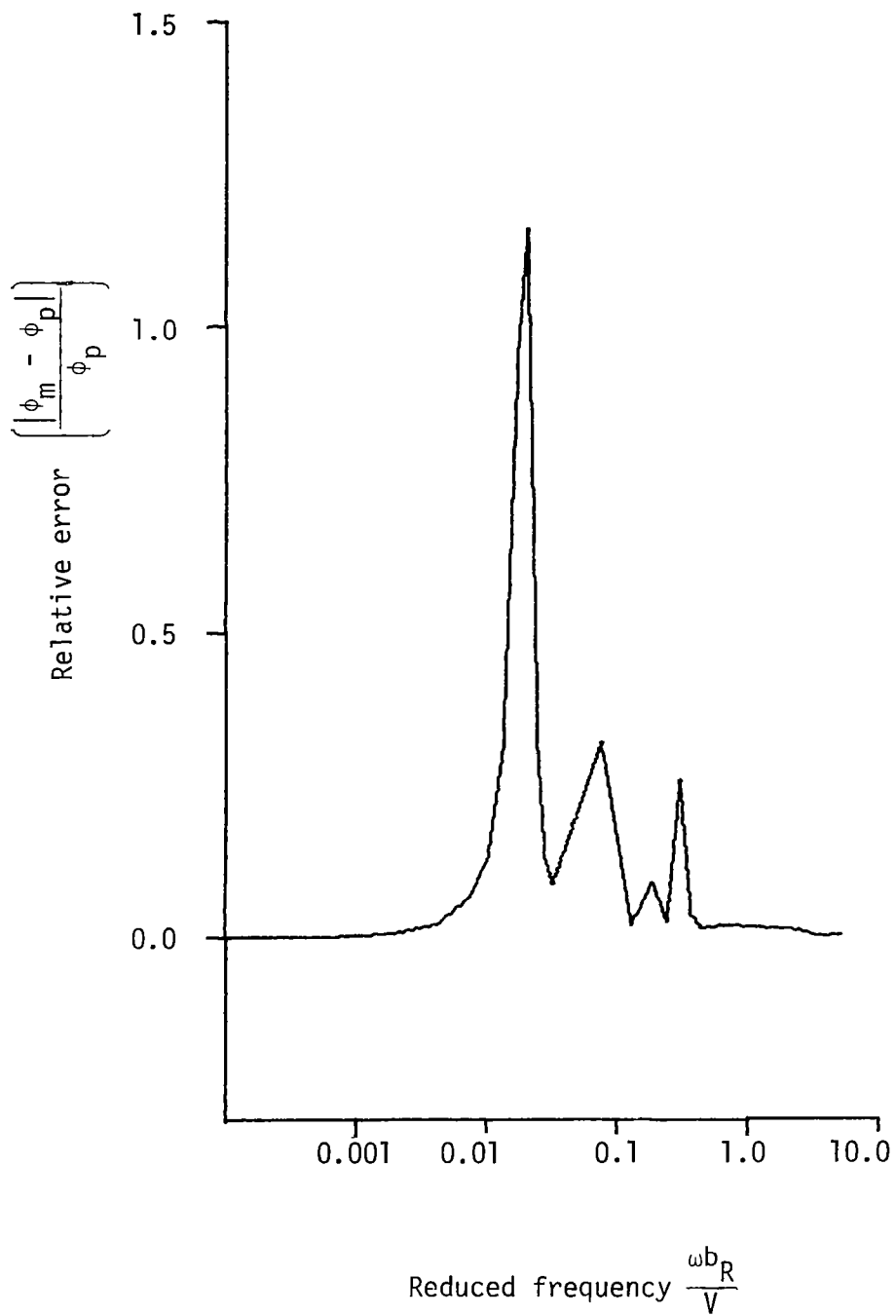
FIGURE 18   RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
132 FEET FOR STANDARD WING

44
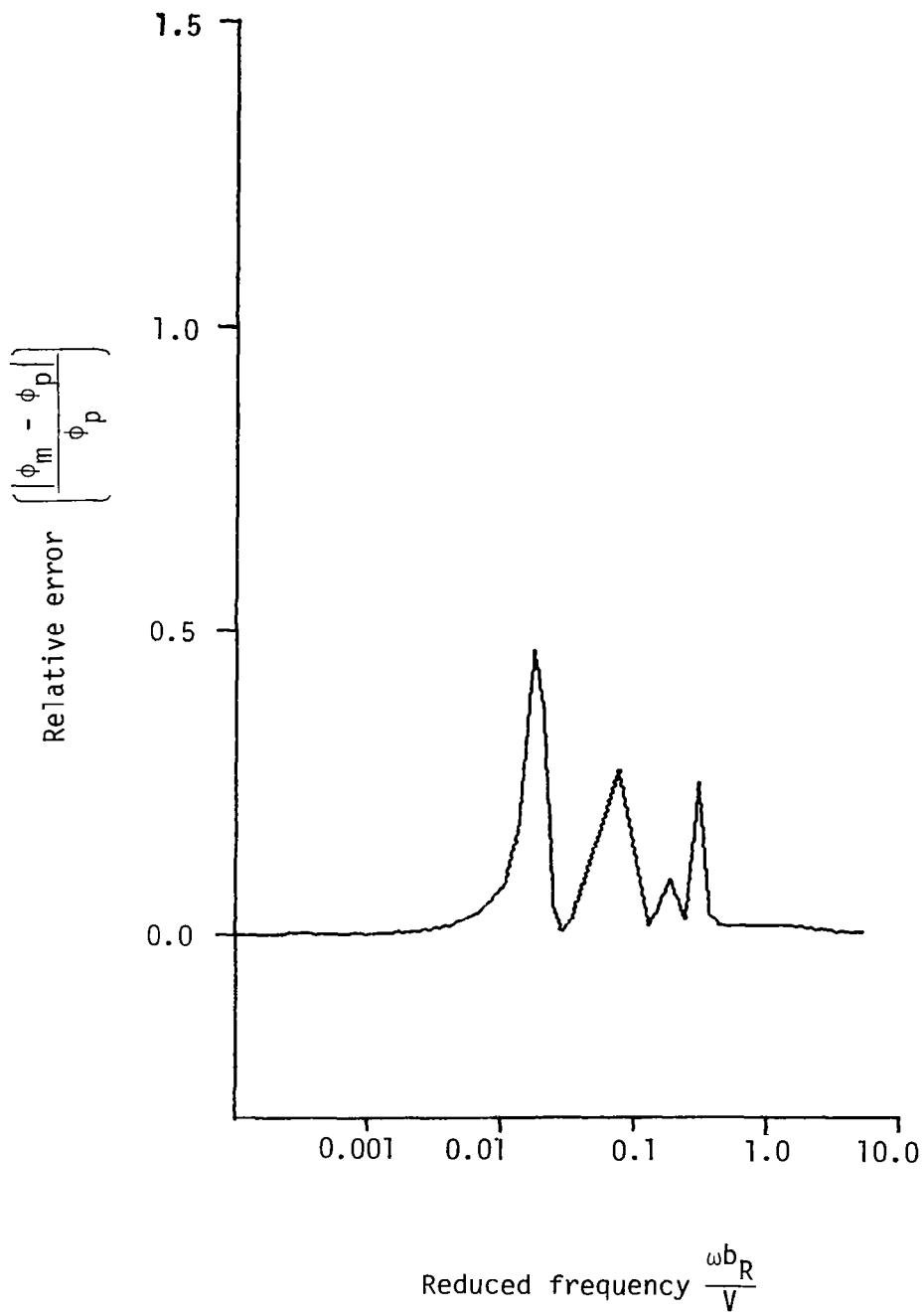
FIGURE 19 RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
2112 FEET FOR STANDARD WING
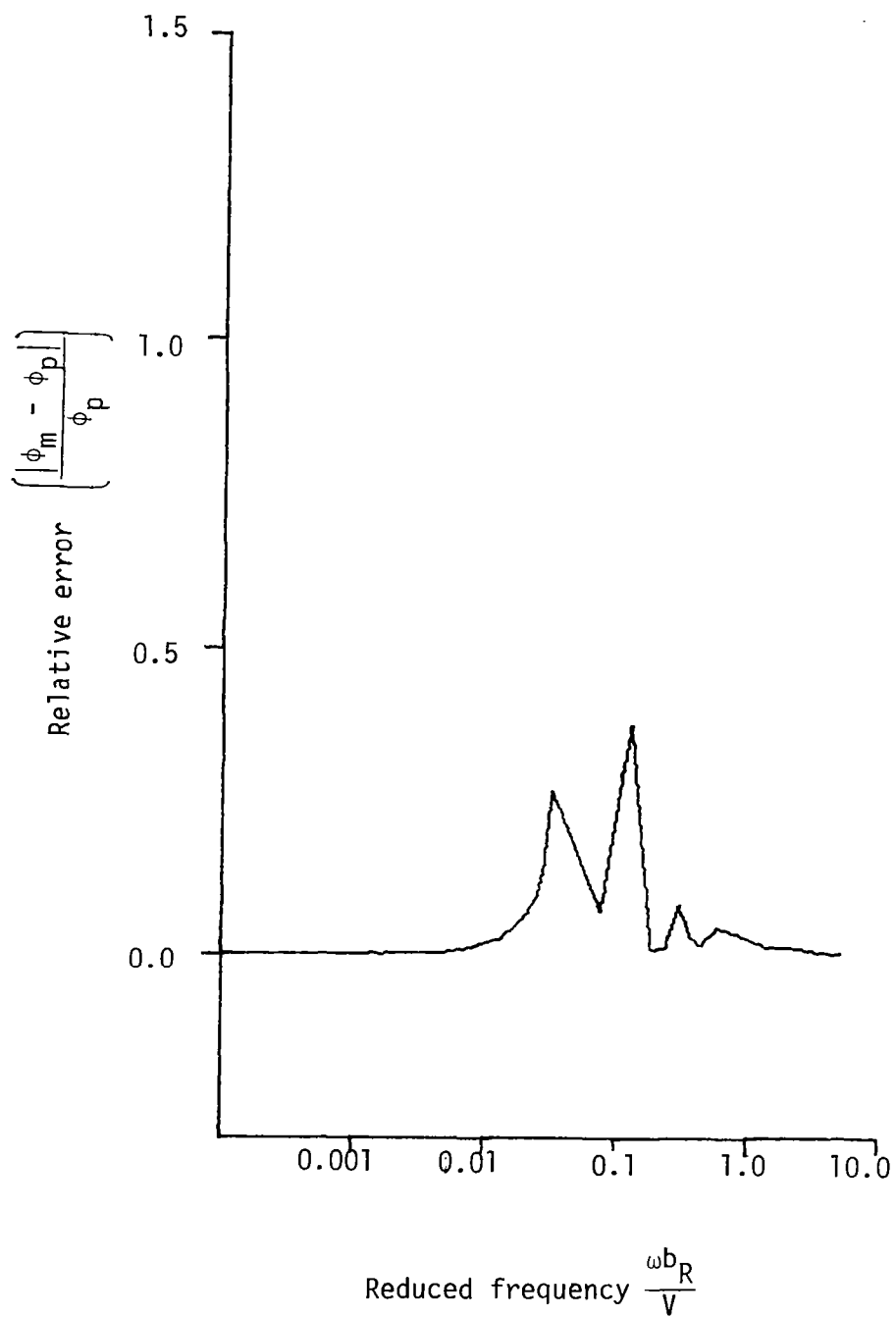
FIGURE 20   RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
132 FEET FOR STIFF WING
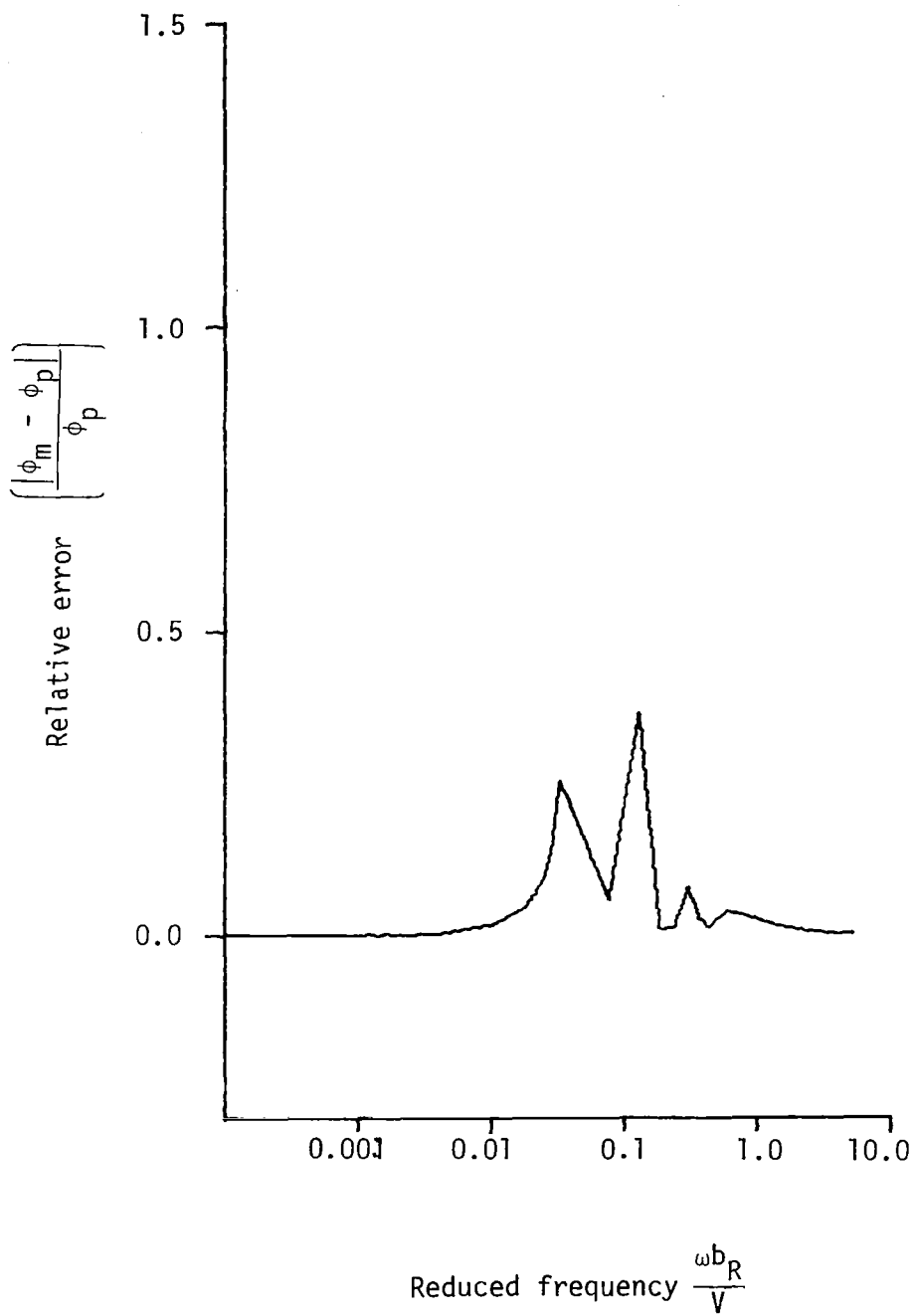
FIGURE 21   RELATIVE ERROR IN MEASURING TURBULENCE WITH LENGTH SCALE
2112 FEET FOR STIFF WING

of the standard wing, it is recommended that accelerometers be mounted on the wing tip to obtain accurate measurements of atmospheric turbulence. The stiff wing shows a smaller relative error than the standard wing and the range of larger error is slightly shifted to the high frequencies. The relative error still reaches 50 percent and for small turbulence length scale can get close to the lower frequencies of atmospheric turbulence. If accurate measurements of atmospheric turbulence frequencies on the order of 0.22 Hz are desired when using an airplane with the stiff wing characteristics, it is recommended that accelerometers be mounted on the wing tip. This study concludes that to measure atmospheric cross spectra across an airplane wing span, a stiff wing is required and that to measure accurately the whole range of the atmospheric turbulence spectrum, accelerometers mounted on the wing tips are required.

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APPENDIX A

USER'S GUIDE

This appendix contains details of the computer code used for this study. The four flow charts illustrate the complete computer code, the free vibration subroutine, the forced vibration subroutine, and the spectrum analysis subroutine. Following the charts is an explanation of how to modify the computer code for different aircraft. Finally, the complete computer code listing is provided.

A.1 FLOW CHART OF THE COMPLETE PROGRAM

NC-Number of Mode To Start On
N ~ Total Number of Mode To Determine
NN ~ Number Modes To Determine Mode On
W ~ First Guess of Frequency

Free Vibration Program

W_i ~ Frequency

M_i ~ Generalized Mass

ϕ_i ~ Mode

A_{ij} & B_{ij} ~ Aerodynamic Cross Product

Forced Program

k ~ Driving Frequency
$Z_i(y)$ ~ Response Of i Mode To Gust

Input/Output

Program

Spectrum Program

k ~ Frequency
ϕ_W ~ Power Spectrum Of Wing Tip Velocity

A.2 FLOW CHART FOR THE FREE VIBRATION PROGRAM

MAIN

NC ~ Number of The First Mode To Determine

N ~ Number of Modes To Determine

NN ~ Number of Modes To Determine The Mode On

W ~ Guess Of Frequency

H ~ Step Size

NN

NC,N,NN,W,H

SUBROUTINE STRK

1. Determines Coef. For Runge-Kutta
2. Fixes Nodes and Wing Plan X, RA

A,B,C,CH

SUBROUTINE SZERO

Steps Along Frequency Line Until Brackets Root
F ~ Value of Characteristic Determinant

NN, YY

SUBROUTINE RK7

Integrates Differential Equation
EL & EU ~ Error Bands
Y ~ Solution
X ~ Node Location

W_1 W_2 W_2

SUBROUTINE BISEC and SUBROUTINE SECANT

SUBROUTINE DEMODE
Determines Mode

Y   Y

W   F

X,Y   F

W   F

YY

SUBROUTINE FUN

1. Fixes Initial Condition Victor
2. Determines Value of Characteristic Determinant

FUNCTION SIMPSON
Integrates

SUBROUTINE COF
Sets Up Integration

SUBROUTINE FUNEV
Make Derivative
Evaluations

X   RM, SEI

FUNCTION SET and Function RM

PRINT OUT

W ~ Natural Frequency

GM ~ Generalized Mass

RP ~ Modes

A,B ~ Aerodynamic Cross Terms

○ Input/Output

☐ Subroutine

↑ Variables Passed
YY Between Subroutines

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A.3 FLOW CHART OF THE FORCED VIBRATION PROGRAM

MAIN

NN ~ Number of Modes

N2 ~ Number of Gust Station

NN, N2

INPUT

From
Free Vibra.

OUTPUT

RK ~ Frequency
Y ~ Amplitude

SUBROUTINE DO

1. Read Inputs
   A, B, W, GM, RP

2. Performs Operations
   That Are Frequency
   Independent: GAMA OMEG

3. Fixes Driving Freq: RK

C

C,D

**SUBROUTINE
GAUSS**
Does Gauss
Elimination
On Coef. Mat.

Y

OMEGA, GAMA
A,B
RK

C,D

**SUBROUTINE
BACKS**
Reduces Non-hom
Victor and Back
Substitute

SUBROUTINE COF

1. Sets Up Coef. Matrix: C
2. Sets Up Gust Force: D

RK

CC, RKK

Fun CC & RKK

○ Input/Output

▭ Subroutine

↑ Variables Passed Between Subroutines
RK

A.4 FLOW CHART OF THE SPECTRUM PROGRAM


```
              INPUT        RK │ MAIN                                    │ RK │ OUTPUT
                              │ TL ~ Turbulence Length Scale            │    │ RK ~ Driving
                              │  N ~ Number of Driving Frequency        │    │    Frequency
                              │ N2 ~ Number of Gust Location            │ RR │ RR ~ Response
                              │ RK ~ Driving Frequency                  │    │ To Turbulence
                              │ RR ~ Total Response                     │
                              │   1.  Reads Driving Frequency           │
                              │   2.  Outputs RK & RR                   │
```

RK   RR

```
  SUBROUTINE          SUBROUTINE SPEC
  Coeff. 1&2          Determines Spectrum
  Setup Coef.         of Wing Tip Velocity
  For Bessel fctn
  approximation
```

RK, TL   RR

```
          FUNCTION TSPEC
          Determines Atmospheric
          Cross Spectrum Between
          Gust Location
```

RK   BSL2
                      BSL1

```
          FUN BSL2 and BSL1
  Evaluates Modified Bessel Function of Second Kind
          5/6 and LS/6 Order
```

```
          FUNCTION POLY
  Does Polynomial Evaluation
```

○ Input/Output

▢ Subroutine

↑RK   Variables Passed Between Subroutines

A.5 PROGRAM MODIFICATION

The computer code is actually three programs: the free vibra-
tion program, run on the IBM 360 at The University of Tennessee,
Knoxville; and the forced vibration and spectrum analysis programs,
both run on the PDP 11 at The University of Tennessee Space Institute.
The input-output data format is compatible between programs, and all
three programs may be changed and extended. Listed below are changes
that must be made to each program for different wings.

The free vibration program was written for application to any
wing. To adapt this program to a different wing, the function EI,
specifying bending stiffness along the span, must be changed. Also, the
functions EIP and EIPP, which are the first and second derivatives of
EI, might need to be changed if EI is more complicated than a step func-
tion. The array RA, which defines the wing's semi-chord along the span,
and the array X, which defines the mode position, must be changed.
They are defined in subroutine STRK. Finally, the function RM, the mass
per unit length, must be corrected.

The forced vibration program is extremely simple to modify for
other aircraft. The array X, specifying the gust locations, must be
changed in MAIN. Also, in MAIN the constants BR, reference semi-chord;
S, surface area of wing; and U, mean flight speed; must be changed to
fit the airplane. Finally, the function RA, wing semi-chord distribution,
must be altered.

There are only two constants that must be changed to modify the
spectrum analysis program for other airplanes. These are the reference

semi-chord, BR, and the mean flight speed, U. This program is most likely to require change due to a new distribution of the atmospheric turbulence cross spectrum, which will require that function TSPEC be rewritten. The program can be modified to give the power spectrum of the root bending or wing deflection at any point of the span by redefining the response function, the array Z.

```
      COMMON/EDAT/EEE
      EEE=3000000000.0
      CALL D1
      CALL D2
      CALL D3
      STOP
      END



      SUBROUTINE D1
C*** THIS PROGRAM IS USED FOR DETERMING WING'S NATURAL
C     FREQUENCY AND MODE.
C
C     IT USES A BISECTION LIKE METHOD TO IMPROVE THE GUESS
C     ON THE FREQUENCY AND MAKES THE CHARACTERISTIC DETERMINANT
C     VANISH.
C
C     RUNGE-KUTTA FEHLBERG 7&8 IS USED TO DETERMINE SOLUTION
C     TO DIFFERENTIAL EON IT HAS VAIRABLE SIZING BETWEEN
C     FIXED NODES.
C
C     SIMPSONS METHOD IS USED TO INTEGRATE FOR THE AREODYNAMIC
C     CROSS TERMS AND GENERALIZED MASS.
C
C     THE PROGRAM MAYBE ADOPTED BY CHANGING FUN...SEI
C     (THE FUNCTION DESCRIBING THE DISTRIBUTION OF THE PRODUCT
C     YOUNG'S ELASTICITY AND SECTIONAL MASS MUENT, IF SEI IS TO
C     HAVE DERIVATIVES UP TO THE SECOND ORDER FUN...FUNEV NEEDS TO BE
C     CHANGED TO INCLUDE THEM IN THE DIFFERENTIAL EQN.  THE WING
C     PLAN ARRAY, RA, WILL NEED TO BE CHANGED.
C***NC=THE NUMBER OF MODE TO START CALCULATING
C***W=THE FRIST GUESS OF THE NATURAL FREQUENCY OF THE NC MODE
C***H=STEP SIZE FOR SEARCH ROUTINE
C***N=TOTAL NUMBER OF MODES TO CALCULATE
C***NN=NUMBER OF NODES TO CALCULATE MODE ON
      REAL*8  W,H
      NC=2
      H=2.0D0
      W=4.0D0
      N=4
      NN=151
      CALL STRK(NN)
      CALL SZERO(W,H,N,NC,NN)
      RETURN
      END
```

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```
      SUBROUTINE SZERO(W,H,N,NC,NN)
C     THIS SUBROUTINE STEPS ALONG THE FREQUENCY DOMAIN UNTILL
C     IT BRACKETS THE NATURAL FREQUENCY AND THE CALLS A BISECTION
C     ROUTINE AND A SECANT ROUTINE TO IMPROVE THE WIDTH OF THE BRACKET
C     THE SUBROUTINE FINALLY CALLS SUB...DMODE TO DETERMINE THE
C     MODE, AND THEN CALLS COEF TO SET UP THE INTERGRATIONS OF MODES
C     AND THEIR PRODUCTS.
C**** EL & EU=ERROR BOUNDS FOR RUNGE KUTTA
C**** W1 & W2=BRACKETS FOR FREQUENCY AND STEPS FOR THE SEARCH ROUTINE
C**** F1 & F2=VALUES OF THE CHARACTERISTIC DETERMINTE FOR W1 & W2
C**** N=NUMBER OF EIGENVALUES TO BE SEARCHED
C**** W=GUESS FOR EIGEN VALUE
C**** H=STEP SIZE FOR SEARCH
C**** NC=NUMBER OF THE FRIST MODE TO CALCULATE
C**** NN=NUMBER OF NODES TO CALCULATE MODES ON
C**** I=NUMBER OF MODES CALCULATED DURING PROCESS
      REAL*8 W,H,W1,W2,F1,F2,EL,EU,Y,X
      COMMON/FACT/Y(8,151),X(151)
      DIMENSION YY(151)
      I=0
      W1=W
      EL=.0001D0
      EU=.001D0
  103 CALL FUN(W1,F1,EL,EU,NC,NN)
  101 W2=W1+H
      CALL FUN(W2,F2,EL,EU,NC,NN)
      IF(F1*F2.LT.0) GO TO 102
      W1=W2
      F1=F2
      GO TO 101
  102 CONTINUE
      CALL BISEC(W1,F1,W2,F2,NC,NN)
      CALL SECANT(W1,F1,W2,F2,NC,NN)
      CALL DMODE(YY,NN)
      WRITE(6,200) W2
  200 FORMAT(2X,F20.10)
      WRITE(6,200)(YY(J),J=1,NN)
      I=I+1
      WS=W2
      CALL COF(NC,YY,WS,NN,I)
      NC=NC+1
      IF(I.GE.N) GO TO 104
      W1=W2+H
      GO TO 103
  104 RETURN
      END



      SUBROUTINE FUN(W,F,EL,EU,NC,NN)
C     THIS SUBROUTINE FIXES THE INITIAL VALUE FOR THE SOLUTION
C     AND THEN CALLS SUB...RK7(RUNGE KUTTA ROUTINE) TO DETERMINE SOLUTION
C     AFTER WHICH THIS SUB CALCULATES THE VALUE OF THE CHARACTERISTIC
C     DETERMINANT.
C*** EL & EU =ERROR BOUNDS FOR RUNGE KUTTA
C*** NC=WHICH MODE WORKING ON DETERMING
C*** Y=FUNDAMENTAL SOLUTIONS TO DIFFERENTIAL EQN OUTPUT FROM RK7
C*** F=VALUE OF CHARACTERISTIC DETERMINTE
      REAL*8 RK,W,F,EL,EU,Y,X
      COMMON/FACT/Y(8,151),X(151)
      COMMON/EGNVL/RK
      RK=W
```

```
C INITIAL CONDITION TEST FOR EVEN OR ODD MODE
      DO 100 I=1,8
 100  Y(I,1)=0.DO
      IF(NC.EQ.2.OR.NC.EQ.4) GO TO 101
C INITIAL CONDITION FOR ODD MODE
      Y(2,1)=1.DO
      Y(8,1)=1.DO
      GO TO 500
C INITIAL CONDITION FOR EVEN MODE
 101  Y(1,1)=1.DO
      Y(7,1)=1.DO
 500  CALL RK7(8,NN,EL,EU)
      F=Y(3,NN)*Y(8,NN)-Y(7,NN)*Y(4,NN)
      RETURN
      END


      SUBROUTINE BISEC(X1,F1,X2,F2,NC,NN)
      REAL*8 X1,X2,F1,F2,FM,XM,EL,EU
      CCB=.01
      EU=.001DO
      EL=.0001DO
 102  XM=(X1+X2)/2.DO
      CALL FUN(XM,FM,EL,EU,NC,NN)
      IF(F1*FM.LE.0.DO) GO TO 100
      X1=XM
      F1=FM
      GO TO 101
 100  X2=XM
      F2=FM
 101  RE=DABS(X1-X2)/X1
      IF(RE.GT.CCB) GO TO 102
      RETURN
      END


      SUBROUTINE FUNEV(K,X,Y,F)
C     THIS SUBROUTINE SUPPORTS THE RUNGE KUTTA AND DESCRIBES THE
C     DIFFERENTIAL EQN
C
C     THIS SUB WILL NEED CHANGING IF SEI IS TO HAVE DERIVATIVES
C     OR IF THE TORSIONAL MODES ARE BEING DETERMINED
C**** K=ORDER OF THE TAYLOR SERIS TERMS
C**** F=DERIVATIVES VALUES
C**** Y=FUNDAMENTAL SOLUTION
C**** W=FREQUENCY
      REAL*8 F,Y,DRM,W,SEI,X
      DIMENSION F(8,13),Y(8)
      COMMON/EGNVL/W
      F(1,K)=Y(2)
      F(2,K)=Y(3)
      F(3,K)=Y(4)
      F(4,K)=DRM(X)*W**2*Y(1)/SEI(X)
      F(5,K)=Y(6)
      F(6,K)=Y(7)
      F(7,K)=Y(8)
      F(8,K)=DRM(X)*W**2*Y(5)/SEI(X)
      RETURN
      END
```

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```
      FUNCTION SEI(X)
C*** X=DISTANCE FROM SEMI SPAN
      IMPLICIT REAL*8 (A-H,O-Z)
      SEI=3000000000.D0
      IF(X.GE.11.D0) SEI=300000000.D0
      RETURN
      END
      SUBROUTINE SECANT(X1,F1,X2,F2,NC,NN)
      REAL*8 X1,F1,X2,F2,XM1,FP,DX,XM,FM,CCS,EU,EL
      EL=.000000001D0
      EU=.00000001D0
      CCS=.0001D0
      XM1=X1
      CALL FUN(X1,F1,EL,EU,NC,NN)
      CALL FUN(X2,F2,EL,EU,NC,NN)
  103 FP=(F2-F1)/(X2-X1)
      DX=-F1/FP
      XM=X1+DX
      CALL FUN(XM,FM,EL,EU,NC,NN)
      WRITE(6,200)X1,F1,X2,F2
  200 FORMAT(2X,2(D21.14,2X,D21.14,2X))
C     IF(DABS(XM-XM1)/XM).LT.CCS) GO TO 101
      IF(DABS(FM).LT.CCS) GO TO 101
      IF(FM*F1.LE.0.D0) GO TO 500
      X1=XM
      F1=FM
      XM1=XM
      GO TO 103
  500 X2=XM
      F2=FM
      XM1=XM
      GO TO 103
  101 X2=XM
      F2=FM
      WRITE(6,201) X2,F2
  201 FORMAT(2X,D21.14,2X,D21.14)
      RETURN
      END
      SUBROUTINE DMODE(YY,NN)
      REAL*8 Y,X,C1,C2,D
      COMMON/FACT/Y(8,151),X(151)
      DIMENSION YY(151)
      D=Y(1,NN)*Y(7,NN)-Y(5,NN)*Y(3,NN)
      C1=Y(7,NN)/D
      C2=-Y(3,NN)/D
      DO 100 I=1,NN
  100 YY(I)=C1*Y(1,I)+C2*Y(5,I)
      RETURN
      END
      FUNCTION DRM(X)
      IMPLICIT REAL*8 (A-H,O-Z)
      DRM=130.D0
      IF(X.LE.4.D0) DRM=2205.D0
      IF(X.LE.11.D0.AND.X.GE.8.D0) DRM=2600.D0
      RETURN
      END
      FUNCTION RM(X)
      RM=130.
      IF(X.LE.4.) RM=2205.
      IF(X.LE.11..AND.X.GE.8.) RM=2600.
      RETURN
      END
```

```
       FUNCTION SIMP(Y,H,N)
       DIMENSION Y(N)
       T1=0.
       J1=N-2
       T2=0.
       DO 100 I=3,J1,2
 100   T1=T1+Y(I)
       DO 200 I=2,N,2
 200   T2=Y(I)+T2
       SIMP=H*(Y(1)+Y(N)+2.*T1+4.*T2)/6.
       RETURN
       END
       SUBROUTINE COF(N,YY,W2,NN,I1)
C         THIS SUBROUTINE SETS UP THE FUNCTIONS FOR INTEGRATION
C***    YY=NEW MODE THEN LATER USED AS SCRATCHED ARRAY
C***    RP=ARRAY OF MODES
C***    W=FREQUENCY
C***    GM=GENERALIZED MASS
C***    A & B=AREODYNAMMIC CROSS TERMS
C***    RA=WING PLAN
C***    N=NUMBER OF MODE BEING WROKED ON
C***    NN=NUMBER OF NODES
C***    I1=NUMBER OF TIMES COF CALLED
       COMMON/DATARA/RA(151)
       COMMON/TRAN1/W(5),GM(5),RP(5,151),A(5.5),B(5,5)
       DIMENSION YY(151),XX(151),RR(151)
C*** READ DATA
       H=33./(NN-1)
       IF(I1.GT.1) GO TO 500
       DO 601 I=1,NN
 601   XX(I)=(I-1)*H
       NM=N-1
       W(1)=0.
       GM(1)=40000.0
       DO 600 I=1,NN
 600   RP(1,I)=1.
       A(1,1)=45.038925
       B(1,1)=52.939763
C*** CALCULATE NEW DATA
 500   CONTINUE
       DO 101 I=1,NN
 101   RP(N,I)=YY(I)
       DO 102 I=1,NN
       A1=RM(XX(1))
       B1=RP(N,1)*RP(N,I)
       YY(1)=A1*B1
 102   CONTINUE
       GM(N)=SIMP(YY,H,NN)*2.
       W(N)=W2
       DO 104 J=1,N
       NR2=N/2.
       NR1=J/2.
       N2=N-2*NR2
       N1=J-2*NR1
       IF(N2.EQ.0.AND.N1.EQ.0) GO TO 400
       IF(J.EQ.1) GO TO 400
       IF(N2.EQ.1.AND.N1.EQ.1) GO TO 400
       A(N,J)=0.0
       B(N,J)=0.0
       GO TO 105
 400   CONTINUE
```

```
          DO 103 I=1,NN
          YY(I)=RA(I)*RP(J,I)*RP(N,I)
  103     RR(I)=RA(I)*YY(I)
          A(N,J)=SIMP(RR,H,NN)
          B(N,J)=SIMP(YY,H,NN)
  105     CONTINUE
  104     CONTINUE
          RETURN
          END
          SUBROUTINE STRK(NN)
C         THIS SUBROUTINE INITIALIZES THE COEFICIENTS FOR RK7
C         FIXES THE NODES AND WING PLAN
C****     NN=NUMBER OF NODES
C****     A,B,C = COEFFICINTS FOR RK7
C****     X=NODE VALUES
C****     RA=WING PLAN ARRAY
          REAL*8 A,B,C,CH,X,Y,DEL
          COMMON/DATARA/RA(151)
          COMMON/RKC/A(13),B(13,12),C(13),CH(13)
          COMMON/FACT/Y(8,151),X(151)
          DEL=33.D0/(NN-1)
          DEL2=33./(NN-1)
          ND=(NN-1)/3.
          RM1=(NN-1)/3.
          DO 100 I=1,NN
          X(I)=(I-1)*DEL
          RA(I)=1.0
          IF(I.GT.ND)RA(I)=1.-.027*(I-RM1)*DEL2
  100     CONTINUE
          A(1)=0.0D0
          A(2)=2.D0/27.D0
          A(3)=1.D0/9.D0
          A(4)=1.D0/6.D0
          A(5)=5.D0/12.D0
          A(6)=1.D0/2.D0
          A(7)=5.D0/6.D0
          A(8)=1.D0/6.D0
          A(9)=2.D0/3.D0
          A(10)=1.D0/3.D0
          A(11)=1.D0
          A(12)=0.D0
          A(13)=1.D0
          B(1,1)=0.0D0
          B(2,1)=2.D0/27.D0
          B(3,1)=1.D0/36.D0
          B(3,2)=1.D0/12.D0
          B(4,1)=1.D0/24.D0
          B(4,2)=0.D0
          B(4,3)=1.D0/8.D0
          B(5,1)=5.D0/12.D0
          B(5,2)=0.0D0
          B(5,3)=-25.D0/16.D0
          B(5,4)=25.D0/16.D0
          B(6,1)=1.D0/20.D0
          B(6,2)=0.D0
          B(6,3)=0.D0
          B(6,4)=1.D0/4.D0
          B(6,5)=1.D0/5.D0
          B(7,1)=-25.D0/108.D0
          B(7,2)=0.D0
          B(7,3)=0.0D0
          B(7,4)=125.D0/108.D0
          B(7,5)=-65.D0/27.D0
```

```
B(7,6)=125.D0/54.D0
B(8,1)=31.D0/300.D0
B(8,2)=0.0D0
B(8,3)=0.D0
B(8,4)=0.D0
B(8,5)=61.D0/225.D0
B(8,6)=-2.D0/9.D0
B(8,7)=13.D0/900.D0
B(9,1)=2.D0
B(9,2)=0.D0
B(9,3)=0.D0
B(9,4)=-53.D0/6.D0
B(9,5)=704.D0/45.D0
B(9,6)=-107.D0/9.D0
B(9,7)=67.D0/90.D0
B(9,8)=3.D0
B(10,1)=-91.D0/108.D0
B(10,2)=0.0D0
B(10,3)=0.D0
B(10,4)=23.D0/108.D0
B(10,5)=-976.D0/135.D0
B(10,6)=311.D0/54.D0
B(10,7)=-19.D0/60.D0
B(10,8)=17.D0/6.D0
B(10,9)=-1.D0/12.D0
B(11,1)=2383.D0/4100.D0
B(11,2)=0.D0
B(11,3)=0.D0
B(11,4)=-341.D0/164.D0
B(11,5)=4496.D0/1025.D0
B(11,6)=-301.D0/82.D0
B(11,7)=2133.D0/4100.D0
B(11,8)=45.D0/82.D0
B(11,9)=45.D0/164.D0
B(11,10)=18.D0/41.D0
B(12,1)=3.D0/205.D0
B(12,2)=0.D0
B(12,3)=0.D0
B(12,4)=0.D0
B(12,5)=0.D0
B(12,6)=-6.D0/41.D0
B(12,7)=-3.D0/205.D0
B(12,8)=-3.D0/41.D0
B(12,9)=3.D0/41.D0
B(12,10)=6.D0/41.D0
B(12,11)=0.0D0
B(13,1)=-1777.D0/4100.D0
B(13,2)=0.D0
B(13,3)=0.D0
B(13,4)=-341.D0/164.D0
B(13,5)=4496.D0/1025.D0
B(13,6)=-289.D0/82.D0
B(13,7)=2193.D0/4100.D0
B(13,8)=51.D0/82.D0
B(13,9)=33.D0/164.D0
B(13,10)=12.D0/41.D0
B(13,11)=0.D0
B(13,12)=1.D0
C(1)=41.D0/840.D0
C(2)=0.D0
C(3)=0.D0
C(4)=0.D0
C(5)=0.D0
```

```
          C(6)=34.D0/105.D0
          C(7)=9.D0/35.D0
          C(8)=9.D0/35.D0
          C(9)=9.D0/280.D0
          C(10)=9.D0/280.D0
          C(11)=41.D0/840.D0
          C(12)=0.D0
          C(13)=0.D0
          CH(1)=0.D0
          CH(2)=0.D0
          CH(3)=0.D0
          CH(4)=0.D0
          CH(5)=0.D0
          CH(6)=34.D0/105.D0
          CH(7)=9.D0/35.D0
          CH(8)=9.D0/35.D0
          CH(9)=9.D0/280.D0
          CH(10)=9.D0/280.D0
          CH(11)=0.D0
          CH(12)=41.D0/840.D0
          CH(13)=41.D0/840.D0
          RETURN
          END
          SUBROUTINE RK7(NS,NN,EL,EU)
C*** RUNGE KUTTA FEHLBERG SEVENTH ORDER
C*** EL=ERROR LOWER BOUND
C*** EU=ERROR UPPER BOUND
C*** NS=NUMBER OF SYSTEM OF EQN
C*** Y=SOLUTION
C*** NN=NUMBER OF PTS TO DETERMINE THE SOLUTION
C*** RL=LENGTH OF INTERVAL
          DIMENSION Y0(8),F(8,13),YY(8)
          DIMENSION DY4(8),DY5(8),Y1(8)
          COMMON/RKC/A(13),B(13,12),C(13),CH(13)
          COMMON/FACT/Y(8,151),X(151)
          REAL*8 DY4,DY5,XX,YY,TH,A,B,C,F,DD1,DD2,H,Y1,Y0,CH,EL,EU,X,Y
          DO 101 I=1,NS
  101     Y0(1)=Y(I,1)
          NT=NN-1
          L=1
C*** MAIN DO LOOP INCREAMENT TO EACH NODE
          DO 100 I1=1,NT
          NC=0
          H=X(I1+1)-X(I1)
          GO TO 203
  207     L=L-1
          GO TO 203
  206     L=L+1
          NC=1
  203     DO 201 I=1,NS
  201     Y1(I)=Y0(I)
          TH=H/L
C*** DO LOOP FOR STEPS BETWEEN NODES
          DO 200 I2=1,L
C*** DETERMINE THE NEEDED FUNCTION EVALURATION
          DO 300 K=1,13
          KM=K-1
          DO 301 J=1,NS
          XX=X(I1)+TH*(I2-1)+A(K)*TH
          YY(J)=Y1(J)
          IF(KM.EQ.0) GO TO 303
          DO 302 I3=1,KM
  302     YY(J)=TH*B(K,I3)*F(J,I3)+YY(J)
```

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```
303   CONTINUE
301   CONTINUE
300   CALL FUNEV(K,XX,YY,F)
C*** DETERMINE SOLUTION VALUE FOR END OF STEP
      DO 500 I=1,NS
      DY4(I)=0.0
500   DY5(I)=0.0
      DO 401 I=1,NS
      DO 402 K=1,13
      DY4(I)=TH*C(K)*F(I,K)+DY4(I)
402   DY5(I)=TH*CH(K)*F(I,K)+DY5(I)
401   CONTINUE
C*** ERROR AND STEP SIZE CONTROL
      DD1=DABS((DY4(1)-DY5(1))/(DY4(1)+Y1(1)))
      DD2=DABS((DY4(5)-DY5(5))/(DY4(5)+Y1(5)))
      IF(DD1.LT.EU.AND.DD2.LT.EU) GO TO 202
      GO TO 206
202   IF(DD1.GT.EL.AND.DD2.GT.EL) GO TO 204
      IF(L.EQ.1) GO TO 204
      IF(NC.EQ.1) GO TO 204
      GO TO 207
204   CONTINUE
      DO 205 I=1,NS
205   Y1(I)=Y1(I)+DY4(I)
200   CONTINUE
      DO 102 I=1,NS
      YO(I)=Y1(I)
102   Y(1,I1+1)=YO(I)
100   CONTINUE
      RETURN
      END
      SUBROUTINE D2
C     THIS PROGRAM DETERMINES THE AMPLITUDES OF THE DIFFERENT
C     MODES TO A SINUSODAL GUST AT THE DIFFERENT STATIONS ALONG
C     THE WING. SUB...DO TAKES CARE OF INPUT AND OUTPUT PLUS
C     SETS UP THE COEFFICIENTS THAT ARE DRIVING FREQUENCY INDEPENDENT
C     SUBROUTINE COEF SET UP THE COEFFICIENT MATRIX FOR EACH DRIVING
C     FREQUENCY. WHILE SUB...GAUSS DOES HALF OF THE REDUCTION AND
C     SUB...BACKS FINISHES THE REDUCTION AND DOES BACK SUBSITUTION
C     FOR THE DIFFERENT NON-HOMOGENOUS VICTORS CORRESPONDING TO
C     DIFFERENT GUST LOCATIONS.
C*** NN=NUMBER OF NODES THE MODES ARE DETERMINE ON
C*** N2=NUMBER OF GUST LOCATION
      NN=151
      N2=20
      CALL DO(NN,N2)
      RETURN
      END



      SUBROUTINE DO(NN,N2)
C     THIS SUBROUTINE TAKES CARE OF INPUT AND OUTPUT
C     AND PERFORMS OPERATIONS THAT ARE DRIVING FREQUENCY
C     INDEPENDENT, MEANING GAMA AND OMEG.
C*** Y=SOLUTION, AMPLITUDES OF MODES
C*** RP=MODE ARRAY
C*** X=GUST LOCATION
C*** A,B=AERODYNAMIC CROSS PRODUCTS
C*** W=NATURAL FREQUENCIES OF MODES
C*** GM=GENERALIZED MASS OF MODES
C*** OMEG=REDUCED NATURAL FREQUENCY
C*** GAMA=NON DIMENSIONAL GM
      COMMON/SOL/Y(20,5)
      COMMON/TRAN1/W(5),GM(5),RP(5,151),A(5,5),B(5,5)
```

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```
            COMMON/TRAN2/RRK(37)
            COMMON/TRAN3/SY(37,19,5)
            COMMON/EDAT/EEE
            COMMON/FAC/PI,BR,S,RO,U
            DIMENSION X(20)
            COMPLEX Y,SY
            COMPLEX CMPLX
            COMMON/DAT/GAMA(5),OMEG(5)
            N=37
C50         FORMAT(2X,E14.7)
C           WRITE(7,50) EEE
            PI=3.14159
            BR=19.0/2.0
            S=960.0
            RO=.0765
            U=575.0
C***  PERFORM ARITHMATIC
            DO 601 I=1,5
            GAMA(I)=GM(I)/(PI*RO*S*BR)
            OMEG(I)=W(I)*BR/U
            JN=I
            DO 602 J=1,JN
            A(I,J)=A(I,J)*BR/S
            B(I,J)=B(I,J)*BR/S
            A(J,I)=A(I,J)
            B(J,I)=B(I,J)
  602       CONTINUE
  601       CONTINUE
            DO 500 I=1,37
            IF(I.LE.10) RK=I/1000.
            IF(I.LE.19.AND.I.GT.10) RK=(I-9)/100.
            IF(I.LE.28.AND.I.GT.19) RK=(I-18)/10.
            IF(I.GT.28) RK=(I-27)
            DEL=33./(N2-1)
            N2M=N2-1
            DO 100 I1=1,N2M
  100       X(I1)=DEL*(I1-.5)
            CALL COEF(RK,X,DEL,N2M,NN)
            CALL GAUSS(5,N2M)
            CALL BACKS(N2M,5)
            RRK(I)=RK
            DO 1 J=1,N2M
            DO 2 J2=1,5
            SY(I,J,J2)=Y(J,J2)
  2         CONTINUE
  1         CONTINUE
C           WRITE(7,101) RK
C101        FORMAT(2X,E14.7)
C           DO 102 J=1,N2M
C           WRITE(7,103)(Y(J,I1),I1=1,5)
C103        FORMAT(1X,3(2E13.6))
C102        CONTINUE
  500       CONTINUE
            RETURN
            SUBROUTINE COEF(RK,X,DEL,N2M,NN)
C    THIS SUBROUTINE SETS UP COEFFICIENT MATRIX AND THE DIFFERENT
C    NON HOMOGENOUS VICTORS.
C** C=COEFFICIENT MATRIX
C** D=ARRAY OF NON HOMOGENOUS VICTOR
C*** C=COEFICIENT ARRAY
C*** RK=REDUCED FREQUENCY
C*** X=LOCATION OF GUST
            COMMON/LS/C(5,5),D(20,5)
```

```fortran
      COMMON/DAT/GAMA(5),OMEG(5)
      COMMON/TRAN1/W(5),GM(5),RP(5,151),A(5,5),B(5,5)
      COMMON/FAC/PI,BR,S,RO,U
      DIMENSION X(20),S1(5,5)
      COMPLEX C,CI,D,CC,RKK
      COMPLEX CMPLX
C*** READ DATA
      CI=CMPLX(0.,1.)
      DO 103 I=1,5
      DO 104 J=1,5
      C(I,J)=-RK**2*A(I,J)+2*CI*RK*CC(RK)*B(I,J)
      IF(I.NE.J) GO TO 901
      C(I,J)=C(I,J)+GAMA(I)*(OMEG(I)**2-RK**2)
  901 S1(I,J)=CABS(C(I,J))
  104 CONTINUE
  103 CONTINUE
C     DO 902 I=1,5
      DO 106 J=1,N2M
      DO 105 I=1,5
      D(J,I)=2*BR/S*RA(X(J))*RPH(I,X(J),NN)*RKK(RK)*DEL
  105 CONTINUE
  106 CONTINUE
      RETURN
      END
      FUNCTION RPH(I,Y,N)
      COMMON/TRAN1/W(5),GM(5),RP(5,151),A(5,5),B(5,5)
      DEL=33./(N-1)
      NN=ABS(Y)/DEL+1
      RS=ABS(Y)/DEL+1.-NN
      RPH=RP(I,NN)+RS*(RP(I,NN+1)-RP(I,NN))/DEL
      IF(Y.LT.0.0) GO TO 500
      GO TO 600
  500 CONTINUE
      IF(I.EQ.3.OR.I.EQ.5) RPH=-RPH
  600 RETURN
      END
      FUNCTION RA(Y)
      RA=1-.027*(ABS(Y)-11.)
      IF(ABS(Y).LE.11) RA=1.
      RETURN
      END
      FUNCTION RY1(X)
      Z=(X/3.)**2
      RY1=((((((.0027873*Z-.0400976)*Z+.3123951)*Z-1.3164827)*Z
     ++2.1682709)*Z+.2212091)*Z-.6366198+.6366198*X*ALOG(X/2.)*RJ1(X)
     +)/X
      RETURN
      END
      COMPLEX FUNCTION CC(RK)
C     THIS FUNCTION CALCULATES THE THEODORSEN FUNCTIONS
      COMPLEX CI
      COMPLEX CMPLX
      CI=CMPLX(0.,1.)
      PJ1=RJ1(RK)
      PJ0=RJ0(RK)
      PY1=RY1(RK)
      PY0=RY0(RK)
      F=PJ1*(PJ1+PY0)+PY1*(PY1-PY0)
      G=PY1*PY0+PJ1*PJ0
      H=(PJ1+PY0)**2+(PY1-PJ0)**2
      CC=(F+CI*G)/H
      RETURN
      END
```

```
      COMPLEX FUNCTION RKK(RK)
C     FUNCTION DETERMINES THE GUST FORCE FUNCTION
      COMPLEX CI,CC
      COMPLEX CMPLX
      CI=CMPLX(0.,1.)
      PJ1=RJ1(RK)
      RKK=CC(RK)*(RJ0(RK)-CI*PJ1)+CI*PJ1
      RETURN
      END
      FUNCTION RJ1(X)
      Z=(X/3.)**2
      RJ1=((((((.00001109*Z-.00031761)*Z+.00443319)*Z-.03954289)*Z
     ++.21093573)*Z-.56249985)*Z+.5)*X
      RETURN
      END
      FUNCTION RJ0(X)
      Z=(X/3.)**2
      RJ0=((((((.00021*Z-.0039444)*Z+.0444479)*Z-.3163866)*Z+1.2656208)*
     +Z-2.2499997)*Z+1.0
      RETURN
      END
      FUNCTION RY0(X)
      Z=(X/3.)**2
      RY0=((((((-.00024846*Z+.00427916)*Z-.04261214)*Z+.25300117)*Z
     +-.74350384)*Z+.6055936)*Z+.36746691+.6366198*RJ0(X)*ALOG(X/2.)
      RETURN
      END
      SUBROUTINE GAUSS(N,N2)
C     THIS SUBROUTINE DOES GAUSSIAN ELIMINATION FOR ONLY THE COEFFICIENT
C     MATRIX.    SCALED PARTIAL PIVOTING IS USED.
      COMMON/LS/C(5,5),D(20,5)
      COMMON/PIVOT/IPEN(5)
      DIMENSION S(5)
      COMPLEX C,D
C*** K=PIVOT INDEX
C*** C=COEFFICIENT ARRAY
C*** D=INHOMOGENOUS VICTOR
C*** N=NUMBER OF EQN
      DO 103 I=1,N
      IPEN(I)=I
      S(I)=0.
      DO 104 J=1,N
  104 IF(CABS(C(I,J)).GT.S(I)) S(I)=CABS(C(I,J))
  103 CONTINUE
      NM=N-1
      DO 100 KK=1,NM
      IS=KK+1
      IP=IPEN(KK)
      J=KK
      CM=CABS(C(IP,KK))/S(IP)
      DO 105 I=IS,N
      IP=IPEN(I)
      T=CABS(C(IP,KK))/S(IP)
      IF(T.LE.CM) GO TO 105
      CM=T
      J=I
  105 CONTINUE
      IPK=IPEN(J)
      IPEN(J)=IPEN(KK)
      IPEN(KK)=IPK
      DO 101 II=IS,N
       I=IPEN(II)
       K=IPEN(KK)
      C(I,KK)=C(I,KK)/C(K,KK)
```

```
          DO 102 J=1S,N
  102   C(1,J)=C(I,J)-C(I,KK)*C(K,J)
  101   CONTINUE
  100   CONTINUE
        RETURN
        END
        SUBROUTINE BACKS(N1,N2)
C       DOES REDUCTION ON THE NON HOMOGENOUS VICTOR AND THEN DOES
C       BACK SUBSITUTION.
C***   N1=NUMBER OF NON HOMOGENOUS VICTOR
C***   N2=DIMENSION OF NON HOMOGENOUS VICTOR
        COMMON/PIVOT/IPEN(5)
        COMMON/LS/C(5,5),D(20,5)
        COMMON/SOL/Y(20,5)
        COMPLEX C,D,Y
C***   K1=SOLUTION INDEX
C***   REDUCTION ON NON HOMOGENOUS VICTOR
        DO 100 K1=1,N1
        IP=IPEN(1)
        Y(K1,1)=D(K1,IP)
        DO 101 KK=2,N2
        K=IPEN(KK)
        T=0.0
        JN=KK-1
        DO 102 J=1,JN
  102   T=C(K,J)*Y(K1,J)+T
  101   Y(K1,KK)=D(K1,K)-T
        Y(K1,N2)=Y(K1,N2)/C(K,N2)
C***   BACK SUBSITUTION
        JJ=N2
        DO 103 K=2,N2
        JS=JJ
        JJ=JJ-1
        KK=IPEN(JJ)
        T=0.0
        DO 104 J=JS,N2
  104   T=C(KK,J)*Y(K1,J)+T
  103   Y(K1,JJ)=(Y(K1,JJ)-T)/C(KK,JJ)
  100   CONTINUE
        RETURN
        END
        SUBROUTINE D3
C       THIS PROGRAM PERFORMES THE ARTHIMETIC TO DETERMINE WING TIP
C       VELOCITY POWER SPECTRUM.   SUB...SPEC PERFORMS THE CALCULATION
C       AND FUN...TSPEC EVALURATES ATMOSPHERIC TURBULENCE SPECTRUM.
C***   N2=NUMBER OF GUST STATIONS
C***   N=NUMBER FOR DRIVING FREQUENCIES
C***   TL=TURBULENCE LENGTH SCALE
        COMMON/TRAN2/RRK(37)
        COMMON/TRAN3/SY(37,19,5)
        COMMON/EDAT/EEE
        COMPLEX SY
        CALL COEF1
        CALL COEF2
        BR=19./2.
        DO 101 J1=1,5
        TL=66.*(2**J1)
        N=37
        N22=20
        N22=N22-1
```

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```
      N2=N22*2
      WRITE(7,202)EEE,TL,N2
  202 FORMAT(2X,'EI=',F10.1,'TL=',F10.2,'N2=',I3)
      DU 100 I=1,N'
      RK=RKK(I)
      RNU=RK*TL/BR
      SS=0.0
      U=575.
      TS=TSPEC(SS,RNU,U,TL)
      CALL SPEC(RK,RR,TL,N2,I)
      WRITE(7,201) RK,RR,TS
  201 FORMAT(2X,E14.7,2X,E14.7,2X,E14.7)
  100 CONTINUE
  102 CONTINUE
  101 CONTINUE
      RETURN
      END
      SUBROUTINE SPEC(RK,RR,TL,N2,IC)
C     THIS SUBROUTINE DETERMINES THE SPECTRUM OF THE WING
C     TIP VELOCITY.
C*** RR=TOTAL AIRPLANE RESPONSE
C*** RK=REDUCE FREQUENCY WB/U
C*** Z=RESPONSE TO GUST AT ONE STATION
C*** N2=NUMBER OF GUST STATIONS
      COMMON/TRAN2/RRK(37)
      COMMON/TRAN3/SY(37,19,5)
      DIMENSION Y(20,5),Z(40)
      COMPLEX Y,CI,Z,T,T2,SY
C*** READ DATA
      CI=CMPLX(0.,1.)
      BR=19./2.
      N22=N2/2
      DEL=66./N2
      U=575.
      DO 100 J=1,N22
      DO 2 J2=1,5
      Y(J,J2)=SY(IC,J,J2)
    2 CONTINUE
  100 CONTINUE
C*** DETERMINE PLANES RESPONSE
      DO 101 J=1,N22
      T=0.0
      T2=0.0
      DO 102 I=2,5
      T2=Y(J,I)+T2
      IF(I.EQ.3.OR.I.EQ.5) T2=T2-2.*Y(J,I)
  102 T=Y(J,I)+T
      Z(J)=T2*RK*CI
  101 Z(J+N22)=CI*RK*T
C*** DETERMINE PLNES TOTAL RESONSE
      TT=0.0
      DO 300 I=1,N2
      IS=I-1
      T=0.0
      JN=N2-IS
      DO 301 J=1,JN
  301 T=2*REAL(Z(J)*CONJG(Z(J+IS)))+T
      SS=IS*DEL/TL
      RNU=RK*TL/BR
      IF(IS.EQ.0)T=T/2.
  300 TT=TSPEC(SS,RNU,U,TL)*T+TT
      RR=TT
      RETURN
      END
```

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```
      FUNCTION TSPEC(SS,RNU,U,TL)
C     THIS FUNCTION DETERMINES TURBULENCE CROSS AND POWER
C     SPECTRUM FROM THE VON KARMAN SPECTRUM FUNCTION.
C***  SS=SEPERATION DIVIDED BY TL(TURBLONCE LENGTH SCALE)
C***  RNU= W*TL/U THE REDUCED FROQUENCY OF TURBULENCE
C***  U=FLIGHT SPEED OR MEAN WIND SPEED
C***  TL=TURBULENCE LENGTH SCALE
      IF(SS.EQ.0.0) GO TO 500
C     CROSS SPECTRUM
      Z=SS*SQRT(1.+(1.339*RNU)**2)/1.339
      TSPEC=TL*.10853/U*(4.78112*SS**(5./3.)/Z**(5./6.)*BSL1(Z)
     +-SS**(11./3.)/Z**(11./6.)*BSL2(Z))
      RETURN
 500  CONTINUE
C     POWER SPECTRUM
      Z=(1.339*RNU)**2
      TSPEC=TL*(1+(8./3.)*Z)/(1+Z)**(11./6.)/3.14159/U
      RETURN
      END
      SUBROUTINE COEF1
C     THIS SUBROUTINE SETS UP THE COEFFIENETS FOR THE POLYNOMIAL
C     APPROXIMATION FOR THE MODIFRIED BESSEL FUNCTION OF THE
C     SECOND KIND 5/6 ORDER.
      COMMON/K13/A(10),B(10),A2(10)
      F=5./6.
      A(1)=1.0/.9405612296
      DO 100 I=1,9
 100  A(I+1)=A(I)/I/(F+1)
      F=1.0-F
      B(1)=1.0/5.56756615
      DO 101 I=1,9
 101  B(I+1)=B(I)/I/(F+I-1.0)
      S=4.*(5./6.)**2
      A2(1)=1.
      DO 200 I=1,9
 200  A2(I+1)=A2(I)*(S-(2*I-1)**2)/8./I
      RETURN
      END
      FUNCTION BSL1(Z)
C     THIS FUNCTION EVALURATES THE MODIFRIED BESSEL FUNCTION
C     OF THE SECOND KIND 5/6 ORDER
      COMMON/K13/A(10),B(10),A2(10)
      IF(Z.LE.2) GO TO 100
      Y=1./Z
      BSL1=SQRT(1.5707*Y)*EXP(-Z)*POLY(A2,10,Y)
      RETURN
 100  Y=(Z/2.0)**2.0
      RIP=(Z/2.0)**(5./6.)*POLY(A,10,Y)
      RIN=POLY(B,10,Y)/((Z/2.0)**(5./6.))
      BSL1=(3.141/2/SIN(5.0*3.141/6.0))*(RIN-RIP)
      RETURN
      END
      FUNCTION POLY(A,N,Z)
C     THIS FUNCTION DOES THE POLYNOMIAL EVALURATIONS
      DIMENSION A(N)
      T=A(N)*Z
      NN=N-2
      DO 100 I=1,NN
 100  T=(T+A(N-I))*Z
      POLY=T+A(1)
      RETURN
      END
```
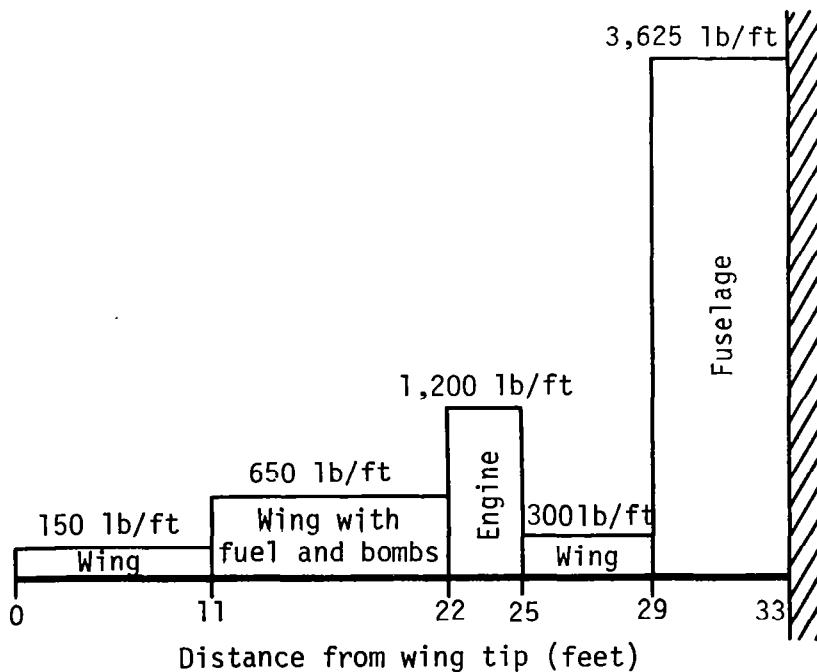
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```
      FUNCTION BSL2(Z)
C    THIS FUNCTION EVALURATES THE MODIFRIED BESSEL FUNCTION OF THE
C    SECOND KIND 11/6 ORDER.
      COMMON/K23/E(10),G(10),E2(10)
      IF(Z.LE.2) GO TO 100
      Y=1./Z
      BSL2=SQRT(1.5707*Y)*EXP(-Z)*POLY(E2,10,Y)
      RETURN
  100 Y=(Z/2.0)**2.0
      RIP=(Z/2.0)**(11./6.)*POLY(E,10,Y)
      RIN=POLY(G,10,Y)/((Z/2.0)**(11./6.))
      BSL2=(3.141/2/SIN(11.0*3.141/6.0))*(RIN-RIP)
      RETURN
      END
      SUBROUTINE COEF2
C    THIS SUBROUTINE SETS UP THE COEFFICIENTS FOR THE POLYNOMIAL
C    APPROXIMATIONS OF THE MODIFRIED BESSEL FUNCTION OF THE
C    SECOND KIND 11/6 ORDER.
      COMMON/K23/E(10),G(10),E2(10)
      F=11./6.
C***** ONE OVER THE GAMMA VALUE OF 1+ORDER   *******
      E(1)=1.0/1.724362254
      DO 100 I=1,9
  100 E(I+1)=E(I)/I/(F+I)
C***** ONE MINUS THE ORDER OF THE MODFRIED BESSEL **
      F=1.0-F
      G(1)=1.0/(-6.68107938)
      DO 101 I=1,9
  101 G(I+1)=G(I)/I/(F+I-1.0)
      S=4*(11./6.)**2
      E2(1)=1.
      DO 200 I=1,9
  200 E2(I+1)=E2(I)*(S-(2*I-1)**2)/8./I
      RETURN
      END
```

APPENDIX B

ESTIMATION OF WING STIFFNESS

A major parameter of this study is the beam stiffness. Because structural information about the B-57 could not be located, this parameter had to be estimated. A static analysis is used in this appendix to estimate the beam stiffness. We assume that the deflection at the wing tip is one foot for a 10 g loading of the aircraft in wartime operation (meaning fuel tanks completely full and aircraft loaded with bombs both in the fuselage and on the wings). The wing is modeled as a cantilever beam. The loading on the wing, including structural weight, fuel and bombs, is depicted:

3,625 lb/ft

Fuselage

1,200 lb/ft

Engine

650 lb/ft

Wing with fuel and bombs

150 lb/ft

Wing

300 lb/ft

Wing

| 0 | 11 | 22 | 25 | 29 | 33 |

Distance from wing tip (feet)

LOADING OF B-57 WING

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The values for the loading diagram were determined from the Air Force Basic Flight Manual for the B-57. The loading can be written in functional form using singularity functions:

$$\ell(y) = -150\langle y\rangle^0 - 500\langle y - 11\rangle^0 - 550\langle y - 22\rangle^0 + 900\langle y - 25\rangle^0$$

$$- 3325\langle y - 29\rangle^0 + Q\langle y\rangle^{-1}$$

where Q represents a hypothetical force at the wing tip and is used for Castigliano's Theorem to calculate the deflection at the wing tip. The loading equation is integrated twice to give the moment equation:

$$2M(y) = -150\langle y\rangle^2 - 500\langle y - 11\rangle^2 - 550\langle y - 22\rangle^2 + 900\langle y - 25\rangle^2$$

$$- 3325\langle y - 29\rangle^2 + Q/2\langle y\rangle^0$$

The form of the beam stiffness must also be specified:

Distance from wing tip (feet)
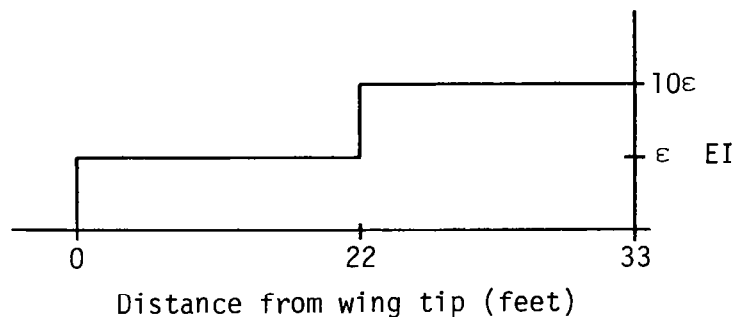
FORM OF EI FUNCTION

ε is the characteristic beam stiffness and must be determined from this analysis. The internal strain energy may be written:

$$\frac{U}{n} = \int_0^{33} \frac{M^2}{2EI}\, dy = \int_0^{22} \frac{M^2}{2\varepsilon}\, dy + \int_{22}^{33} \frac{M^2}{20\varepsilon}\, dy$$

where n represents the load factor, assumed to be 10 g. According to Castigliano's Theorem, the deflection is:

$$\frac{\Delta}{h} = \frac{\partial U}{\partial Q} = \int_0^{33} \frac{M \frac{\partial M}{\partial Q}}{EI}\, dy = \frac{1}{\varepsilon}\int_0^{22} M\,\frac{\partial M}{\partial Q}\, dy + \frac{1}{10\varepsilon}\int_{22}^{33} M\,\frac{\partial M}{\partial Q}\, dy$$

Substituting the equation for the moment and the derivative of the moment with respect to the hypothetical force, the equation becomes:

$$\frac{\Delta\varepsilon}{h} = \int_0^{22} y\left(\frac{-150}{2} <y>^2 - \frac{500}{2} <y - 11>^2\right) dx$$

$$+ \frac{1}{10}\int_{22}^{33} y\left(\frac{-150}{2} <y>^2 - \frac{500}{2} <y - 11>^2 - \frac{550}{2} <y - 22>^2\right.$$

$$\left. + \frac{900}{2} <y - 25>^2 - \frac{3325}{2} <y - 29>^2\right) dy$$

Performing the integration, the equation is approximately:

$$\frac{\Delta\varepsilon}{h} \approx 9 \times 10^6$$

Assuming a one foot deflection at the wing tip due to a 10 g loading

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we get $\varepsilon \approx 9 \times 10^7$. Therefore the beam stiffness distribution due to design characteristics is estimated as:

$$EI = 9 \times 10^8 \quad \text{for} \quad y \leq |11|$$

$$EI = 9 \times 10^7 \quad \text{for} \quad y \geq |11|$$

1. REPORT NO. NASA CR-3431	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.	
4. TITLE AND SUBTITLE Airplane Wing Vibrations Due to Atmospheric Turbulence		5. REPORT DATE June 1981	
		6. PERFORMING ORGANIZATION CODE	
7. AUTHOR(S) Robert L. Pastel, John E. Caruthers, and Walter Frost		8. PERFORMING ORGANIZATION REPORT #	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Atmospheric Science Division The University of Tennessee Space Institute Tullahoma, Tennessee		10. WORK UNIT NO. M-347	
		11. CONTRACT OR GRANT NO. NAS8-32692	
		13. TYPE OF REPORT & PERIOD COVERED	
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D.C. 20546		Contractor (Interim Report)	
		14. SPONSORING AGENCY CODE 989 13 22	

15. SUPPLEMENTARY NOTES

Marshall Technical Monitor: Dennis W. Camp

16. ABSTRACT

The purpose of this study is to determine the magnitude of error introduced due to wing vibration when measuring atmospheric turbulence with a wind probe mounted at the wing tip and to determine whether accelerometers mounted on the wing tip are needed to correct for this error. A spectrum analysis approach is used to determine the error. Estimates of the B-57 wing characteristics are used to simulate the airplane wing, and von Karman's cross spectrum function is used to simulate atmospheric turbulence. The major finding of the study is that wing vibration introduces large error in measured spectra of turbulence in the frequency's range close to the natural frequencies of the wing.

17. KEY WORDS Wind gust Wind turbulence Gust gradient Wind spectra Aircraft design Aviation meteorology	18. DISTRIBUTION STATEMENT Unclassified - Unlimited Subject Category 47
19. SECURITY CLASSIF. (of this report) Unclassified	20. SECURITY CLASSIF. (of this page) Unclassified	21. NO. OF PAGES 84	22. PRICE A05