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INTRODUCTION

Quite frequently when analyzing time series, gaps are encountered in the data. For example, time records of three-day averages of the brightness temperatures of the earth's surface made by a microwave radiometer onboard an earth-monitoring satellite contain a number of missing data arrays (Wilheit 1972; Zwally and Gloersen 1977; Rayner and Howarth 1979). In the analysis of tides, observations occur randomly in time (Zetler 1965). In astronomy and oceanography, factors such as weather and the availability of measuring instruments dictate when observations can be made. Even with portions of a data record missing, there may be a random element in the spacing of the points. Unfortunately, standard formulas for calculating power spectrums (in order to detect periodicities in the data) are based upon the measurements or observations being equally spaced.

The problem of estimating the power spectrum for unequally spaced data has been studied by various authors. A number of articles and papers dealing with the subject have appeared in the literature within the last ten years.

Meisel (1978, 1979) discusses the problem of unequal spacing and describes procedures which interpolate in the time domain. He also describes a general procedure for processing segmented data sets and for obtaining the Fourier transform of a time series with arbitrary spacing. His procedure is most efficient for data where large gaps occur between the data sets and the data sets are made up of equal or nearly equal intervals (within 20%) of time. An excellent set of references on the subject of unequally spaced data can be found in both of his articles.

Deeming (1975) studied the Discrete Fourier Transform of a function defined for arbitrary data spacing and also discussed properties of spectral windows for various kinds of data spacing.

Vanicek (1969, 1971) combined least squares fitting procedures and Fourier analysis in order to remove undesired influences on the spectrum, such as biases, linear trends, etc. His method can also be used to uncover periodicities from unequally spaced time series. Other references which discuss the least squares aspect of the problem are Barning (1963), Taylor and Hamilton (1972), Lomb (1976), Ferraz-Mello (1977), and Wells and Vanicek (1978).

Ferraz-Mello (1980) defines a "Date Compensated Discrete Fourier Transform" for estimating the power spectrum. His estimate is obtained by orthonormalizing the three basis functions – cos ωt, sin ωt, and 1, using a Gram-Schmidt procedure. Since the set of basis functions is complete, harmonic filtering of the time series is made possible.

The present analysis treats the problem of missing data from the autocorrelation function (or lagged products) point of view. It is assumed that there are gaps in the data. However, data points which are separated by gaps are assumed to be nearly equally spaced (to within approximately 20% of an equal sampling interval).

Two algorithms are developed for calculating the power spectrum from the autocorrelation function when there are missing data points. Both procedures use an average sampling interval in order to compute the lagged products where the sampling interval has been calculated over sets of data points separated by gaps. The intervals between all adjacent data points are then calculated to the nearest multiple of this average spacing. Two counters are set up. One counter sums the number of lags of a given size, the other the corresponding cross product terms.

One procedure, the Correlation Function Power Spectrum, takes the Fourier transform of the lagged products function directly to obtain the spectrum, and is equivalent to the Fourier Transform Power Spectrum method when there are only missing data points (no random or arbitrary spacing of points). The other procedure, the Modified Blackman-Tukey Power Spectrum, takes the Fourier transform of the *mean* lagged products (obtained from the ratio of the two counters mentioned above) to calculate the power spectrum.

Both algorithms are compared with the Fourier Transform Power Spectrum method (Deeming 1975) and two least squares procedures (one by Vanicek 1971 and the other by Ferraz-Mello 1980).

Examples are given showing recovery of frequency components by all the techniques from simulated periodic data where portions of data are missing and where random noise has been added to both the time points and to the values of the time series. In addition, the methods are compared using real data. All procedures performed equally well in detecting periodicities in the data.

<u>Power Spectrum From the Autocorrelation Functions</u>

Let the time series be given by the pairs of points (t_i, x_i) for $i = 1,2,\ldots N$.

The discrete Fourier transform $X_N(\omega)$ of x_i is defined by (Deeming 1975 and Appendix B)

$$X_N(\omega) = \sum_{i=1}^{N} x_i e^{-j\omega t_i}$$

$$= (\phi_1^T - j\phi_2^T)x \qquad (1)$$

where

$$\phi = (\phi_1, \phi_2) = \begin{pmatrix} \cos \omega t_1 & \sin \omega t_1 \\ \cos \omega t_2 & \sin \omega t_2 \\ \cdots & \\ \cdots & \\ \cos \omega t_N & \sin \omega t_N \end{pmatrix}$$

and

$$x^T = (x_1, x_2, \ldots, x_N)$$

The normalized Fourier Transform Power Spectrum $S_F(\omega)$ is then (Appendix C)

$$S_F(\omega) = \frac{2x^T\phi\phi^T x}{Nx^T x} \qquad (2)$$

and the Least Squares Power Spectrum is (Vanicek 1971 and Appendix C)

$$S_L(\omega) = \frac{x^T \phi(\phi^T \phi)^{-1} \phi^T x}{x^T x} \tag{3}$$

In the case of equal spacing the expressions in (2) and (3) are equivalent since

$$(\phi^T \phi) = (N/2)\, I \qquad \text{with I the } 2 \times 2 \text{ identity matrix} \tag{4}$$

The real symmetric matrix $\phi\phi^T$ can be written as

$$\phi\phi^T = I + C + C^T \tag{5}$$

where I is the (N X N) identity matrix and

$$C = \begin{pmatrix} 0 & \cos(\omega\Delta t_{12}) & \cos(\omega\Delta t_{13}) \ \ldots & \cos(\omega\Delta t_{1N}) \\ 0 & 0 & \cos(\omega\Delta t_{23}) & \cos(\omega\Delta t_{2N}) \\ \ldots & \ldots & \ldots & \ldots \\ & & & \cos\omega(\Delta t_{N-1,N}) \\ & \text{Zeros} & & \\ & & & 0 \end{pmatrix}$$

with $\Delta t_{ij} = t_j - t_i$

Since

$$x^T(C + C^T)x = 2x^T Cx \tag{6}$$

the spectrum in (2) can also be written as

$$S_F(\omega) = \frac{2}{Nx^T x}(x^T x + 2x^T Cx) \tag{7}$$

or as

$$S_F(\omega) = \frac{2}{Nx^T x}\left(2 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \cos\omega\Delta t_{ij} - x^T x\right) \tag{8}$$

Now let

$$\begin{aligned} y_k &= x_i x_j & i &= 1,2,\ldots,N \\ \Delta t_k &= \Delta t_{ij} & j &= 1,2,\ldots,N \\ & & k &= 1,2,\ldots N(N+1)/2 \end{aligned} \tag{9}$$

and define y_{-k} and Δt_{-k} by

$$
\begin{aligned}
y_{-k} &= x_j x_i = y_k \\
\Delta t_{-k} &= \Delta t_{ji} \qquad (10) \\
\cos(\omega \Delta t_k) &= \cos(\omega \Delta t_{-k})
\end{aligned}
$$

Then the expression in (8) can be written as the discrete Fourier transform of the set of cross product terms y_k

$$
\begin{aligned}
S_F(\omega) &= \frac{2}{Nx^Tx} \sum_{k=-K}^{K} y_k e^{-j\omega\Delta t_k} \\
&= \frac{2}{Nx^Tx} \sum_{k=-K}^{K} y_k \cos(\omega \Delta t_k) \qquad (11)
\end{aligned}
$$

where $K = N(N+1)/2$.

Let Δt be an average spacing calculated over data points which are separated by gaps and let $\Delta\tau_k$ be an approximation to Δt_k obtained by rounding Δt_k to the nearest multiple of Δt.

$$
\Delta\tau_k = \left[\frac{\Delta t_k}{\Delta t} + 0.5 \right] \Delta t \qquad (12)
$$

and $[x]$ is the largest interger contained in x.

Then (11) can be approximated by the Correlation Function Power Spectrum $S_C(\omega)$ defined as

$$
S_C(\omega) = \frac{2}{Nx^Tx} \left(2 \sum_{k=0}^{M} c_k \cos(k\omega\Delta t) - x^Tx \right) \qquad (13)
$$

where the lagged product terms c_k are given by

$$
c_k = \sum_{i=0}^{K} y_i \delta(\Delta\tau_i) \qquad (14)
$$

and

$$
\delta(\Delta\tau_i) = \begin{cases} 1 \text{ for } \Delta\tau_i = k\Delta t \\ 0 \text{ for } \Delta\tau_i \neq k\Delta t \end{cases} \qquad (15)
$$

and M, the maximum lag in the data is

$$M = \left[\frac{\Delta t_{1,N}}{\Delta t} + 0.5\right] \tag{16}$$

If there are only gaps in the data and the points are separated by multiples of the average spacing, Δt, then (13) will be the same as the Fourier Transform Power Spectrum in (2). Furthermore, if the maximum lag M is replaced by m where m is 70% to 80% of M, the resulting spectrum will be about the same as (13).

It should be noted that for equally spaced data, using the Blackman-Tukey procedure (Blackman 1958) mean lagged products R_r are first calculated

$$R_r = \frac{1}{N-r} \sum_{i=1}^{N-r} x_i x_{i+r} \qquad (i=0,1,\ldots,m) \tag{17}$$

Then the spectrum is given by

$$v_k = 2\Delta t \left(R_o + 2 \sum_{r=1}^{m-1} R_r \cos(\frac{\pi kr}{m}) + R_m \cos(\pi k)\right) \tag{18}$$

We can modify this standard procedure of Blackman and Tukey by defining two counters n_k and s_k

$$n_k = \sum_{i=0}^{K} \delta(\Delta\tau_i)$$

$$s_k = \sum_{i=0}^{K} y_i \delta(\Delta\tau_i) \tag{19}$$

where $\delta(\Delta\tau_i)$ has been previously defined in (15) above and y_i in (9)

Then, mean lagged products c'_k will be given by

$$c'_k = \frac{s_k}{n_k} \tag{20}$$

and we will have a modified Blackman-Tukey spectrum $S_{MBT}(\omega)$

$$S_{MBT}(\omega) = \frac{2}{x^T x}\left(2 \sum_{k=0}^{m}{}' \; c_k' \cos(k\omega\Delta t) - \frac{1}{N} x^T x \right) \tag{21}$$

Comparing (17) and (21) we see that the counters n_k and s_k ensure that when the time differences across all time points are calculated (multiples of the average time difference or spacing), the cross products are summed with their proper lag number k. No interpolation is done and only known data points are used. Also, no zeros are substituted for missing data points. Thus, for a lag of k, instead of N-k points in the sample there will be something less. If not too many points are missing, the c_k' terms will not be greatly affected. For a value of m equal to about 60% of the maximum lag M, the results obtained by using (21) will be about the same as (13). It should be noted that in computing $S_{MBT}(\omega)$ some negative amplitudes will occur which is due to the fact that we are dividing by a number less than or equal to N-k (instead of N) and we are not using all the lagged product terms.

Frequency Aliasing

If a period function, $\cos(2\pi ft)$, is sampled at equally spaced intervals of time, Δt, the following values of the function are obtained

$$x_i = \cos(2\pi fi\Delta t) \tag{22}$$

$$(i = \ldots -2, -1, 0, 1, 2, \ldots)$$

A sampling interval of Δt seconds gives rise to a maximum resolvable (cutoff) frequency of $f_c = (1/2\Delta t)$ Hz. The function x_i can be written as

$$x_i = \cos\left\{2\pi[(2kf_c - f)i\Delta t]\right\} \tag{23}$$

$$(k = 1, 2, \ldots)$$

$$(i = \ldots -1, 0, 1 \ldots)$$

From (23) it can be seen that for $|2kf_c - f| \leq f_c$, the frequencies $|2kf_c - f|$ and f are indistinguishable and are called aliases of each other.

EXAMPLES

In order to demonstrate the algorithms developed in (13) and (21), data points x_i were generated using the following function

$$x_i = \sin(2\pi f_1 t_i) + \cos(2\pi f_2 t_i) + \sin(2\pi f_3 t_i) + \cos(2\pi f_4 t_i) + \sin(2\pi f_5 t_i) \tag{22}$$

where $f_1 = 0.37$ Hz, $f_2 = 0.14$ Hz, $f_3 = 0.42$ Hz, $f_4 = 0.65$ Hz, and $f_5 = 1.09$ Hz.

First 101 points were calculated at equal intervals of 1 second with $t_i = i - 1$ ($i=1,2,\ldots 101$). With this choice of sampling interval aliasing can be expected. This was done in order to assess the effects of the sample interval upon recovery of the frequency components.

The power spectrum was then calculated by the Correlation Function method in (13) with M (the maximum lag in the data) replaced by m = 0.6M, the Modified Blackman-Tukey procedure in (21) with M replaced by m = 0.5M, the Fourier Transfer method (Deeming 1975) in (2), and the Least Squares procedure (Vanicek 1971) in (3). The results can be seen in Figures 1 through 4. Note that the frequencies 0.09 Hz and 0.65 Hz which show up in the spectrums are aliases of 1.09 Hz and 0.65 Hz. These particular frequencies, since they are greater than the Nyquist or cutoff frequency ($f_c = 0.5$ Hz), beat with twice the cutoff frequency. Thus, $(f_5 - 2f_c)$ and $(2f_c - f_4)$ show up in the spectrum.

Next, approximately 30% of the data points were deleted. The spectrum for each method was calculated and can be seen in Figures 5 through 8. Note that the frequency components have been recovered (except for aliasing).

Figures 9, 10, 11, and 12 demonstrate recovery of frequency components when random noise from a Gaussian distribution with zero mean and standard deviation of 0.10 has been added to the time points t_i. The function in (22) was calculated for these pseudo-random time points and the spectrum obtained using all four methods. It should be noted that with a standard deviation of 0.1 (and a sampling interval of 1 second), approximately 95% of the noise will be less than or equal to 0.2 or two sigmas. This is in agreement with Meizel (1978) who concluded that as long as data spacing was within 20% of equal intervals ordinary sinc [(sin x)/x] interpolation is satisfactory.

Finally, Figures 13, 14, 15, and 16 show recovery when there are both missing data (the same points being deleted as in Figures 5 through 8) and random noise (the same noise as in Figures 9 through 12).

In Appendix D there are additional examples using the same time series but with a sampling interval of 0.72 seconds, noise having a standard deviation of 0.2 seconds on the time points and amplitude noise with a standard deviation of 20% of the value of the function. Also the method of Ferraz-Mello (1980) is compared with the above methods. All of the procedures produced comparable results and the frequencies in the data were recovered equally well.

As a further example to demonstrate both the Correlation Function Power Spectrum method in (13) and the Modified Blackman-Tukey spectrum in (21) and compare them with other spectrum calculations, we have used real data.

A covariance matrix was built up from 170 three-day average brightness temperature maps (Wilheit 1972; Zwally and Gloersen 1977; Rayner and Howarth 1979) of the earth's surface where the temperatures were obtained from an Electrically Scanning Microwave Radiometer (ESMR) onboard an orbiting NIMBUS 5 satellite. The raw data was averaged in both time and space. The time period extended from September 1973 through May 1975 and the spatial data consisted of an area in the South Polar region approximately 3780 kilometers by 4380 kilometers. Land masses were masked out. Thus the observations are essentially the brightness temperatures of the water and sea ice. Out of a total of 201 possible three-day average maps covering the time period, there were 31 missing data arrays. An eigenvalue/eigenvector analysis of the covariance matrix was made and the principal components calculated. The first ten principal components accounted for more than 91% of the total variance in the data. These principal components (varying in time across the 170 time points) were then normalized. The second principal component time series is plotted in Figure 17. Figures 18 through 25 are normalized power spectrums which have been calculated by various methods and which show the same periodicities in the data. The first peak at a frequency of about 0.0028 cycles/day corresponds with a period of about 360 days. The second peak of about 0.006 cycles/day corresponds with a period of about 167 days while the third peak at 0.0083 cycles/day corresponds with a period of about 120 days. These particular periods can be related to the time variation of the spatial mean latitude of the outer ice boundary (360 days), the warm season (167 days) and the spring summer removal of ice (120 days) reported by Rayner and Howarth (1979). Figure 18 is the power spectrum computed by least squares using (3). Figure 19 is the power spectrum computed by the Fourier Transform method using (2). Figure 20 is the spectrum computed by the method of Ferraz-Mello (1980). Figure 21 has been calculated using (13) and is exactly the same as the Fourier Transform spectrum as there are only gaps in the data. Figure 22 has been calculated using instead of M, a value of m equal to 80% of M and using (13). Figures 23 and 24 are for values of m equal to 70% and 60% of M respectively. Finally, Figure 25 has been calculated by the Modified Blackman-Tukey Power Spectrum algorithm in (21) with the lag number m equal to 60% of M.

Analysis of Figures 18 through 25 show that all of the methods perform about the same. They differ only in the amount of calculation necessary to obtain the spectrum. Using the Correlation Method in (13) where the lagged products are summed to about 70% of the maximum lag M and the Modified Blackman-Tukey procedure in (21) with an m equal to about 60% of the maximum lag M involves less computation and computer time than the other procedures.

SUMMARY

Two algorithms have been developed for calculating power spectrums from the autocorrelation function for time series with missing data points. Both methods use an average sampling interval to compute the lagged products. One procedure, the Correlation Function Power Spectrum, calculates the Fourier transform of the lagged products directly to obtain the spectrum. This method is equivalent to the Discrete Fourier Transform spectrum (Deeming 1975) when there are only missing data points (that is, the time series is equally spaced except for gaps) and all lagged products are included in the computation. The other procedure, the Modified Blackman-Tukey Spectrum, calculates the Fourier transform of the *mean* lagged products in order to obtain the spectrum. Satisfactory results can be obtained by summing lagged product terms up to approximately 50% to 80% of the maximum lag in the data.

The algorithms have been compared with the Discrete Fourier Transform method (Deemin 1975) and two least squares procedures (Vanicek 1971 and Ferraz-Mello 1980) using both simulated and real data. In the case of the simulated data, there are portions of the time series missing and random noise with a Gaussian distribution has been added to both the time points and to values of the function. All of the methods performed satisfactorily and frequency components in the data were recovered equally well.

Recovery of frequency components in the presence of noise having a standard deviation of 10% of the sampling interval (Gaussian noise with 95% less than 20% or 2 standard deviations) supports Meisel (1978) that as along as the data spacing is within 20% of equal intervals, ordinary sinc [(sinx)/x] interpolation is satisfactory.

Acknowledgement

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CORRELATION FUNCTION SPECTRUM 50% LAGS, NO NOISE, NO HOLES

PLOT OF X2*X1 SYMBOL USED IS 0

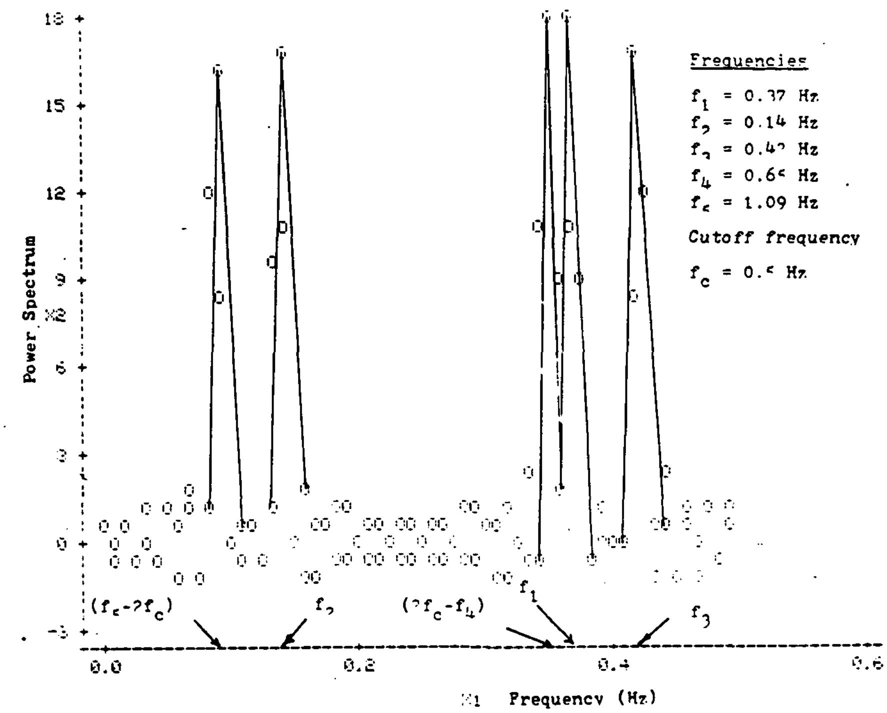

Figure 1

NOTE: 6 OBS HIDDEN

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MODIFIED B-T SPECTRUM 50% LAGS, NO NOISE, NO HOLES

PLOT OF X2*X1 SYMBOL USED IS 0

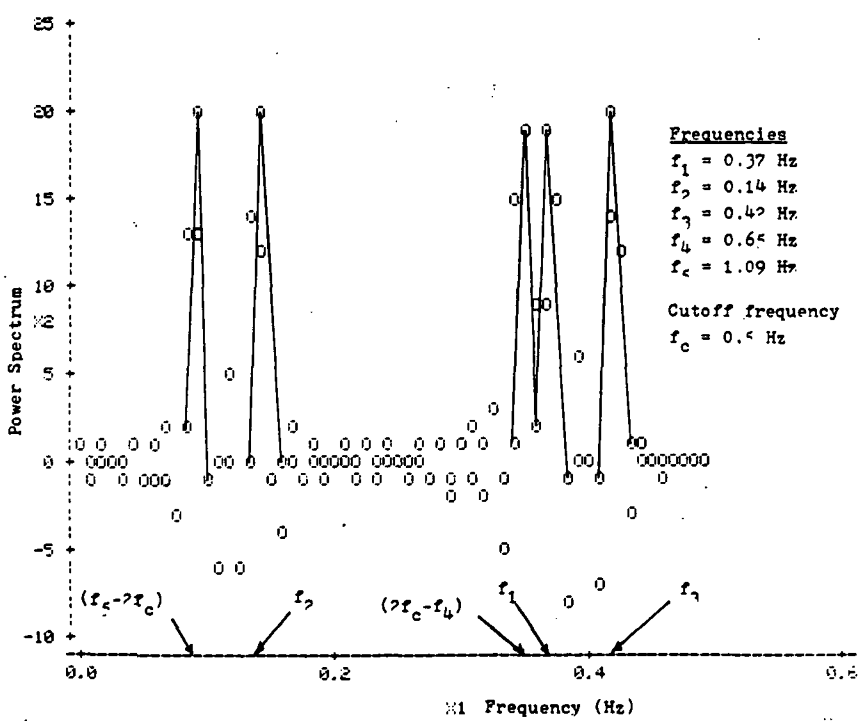

Figure 2

NOTE: 5 OBS HIDDEN

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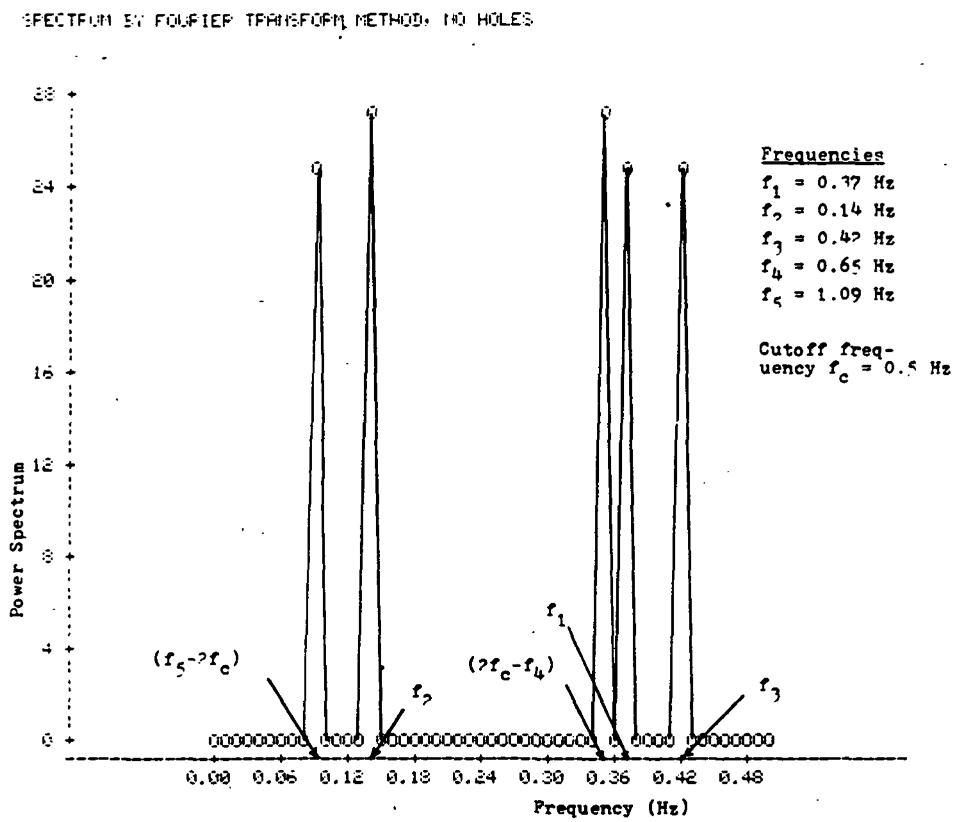

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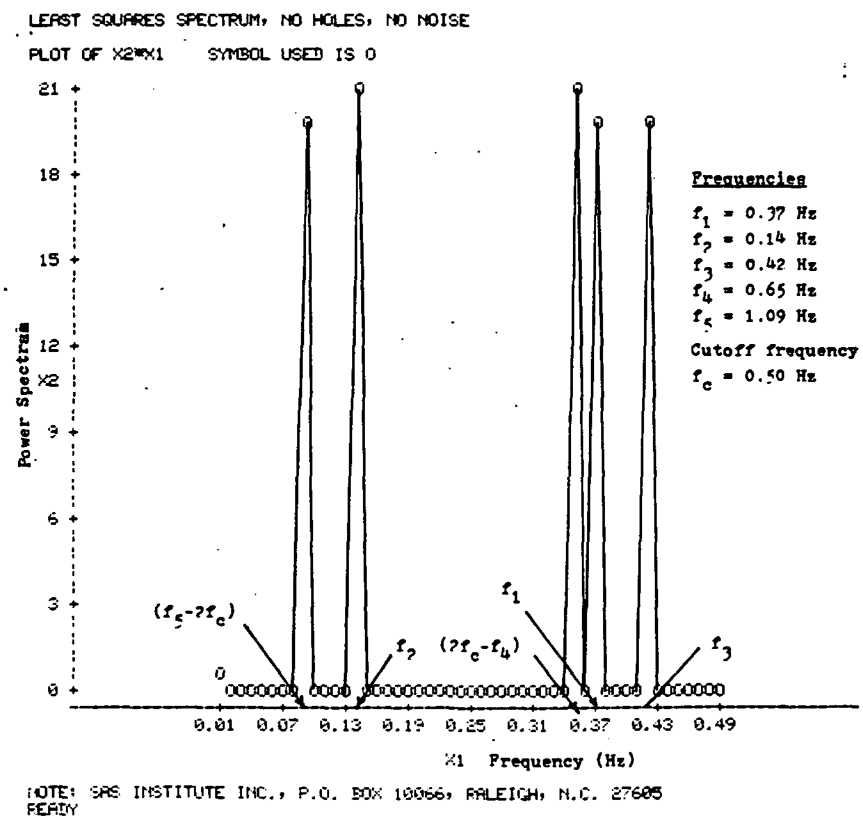

Figure 4

CORRELATION POWER SPECTRUM 70% LAGS, HOLES, NO NOISE

PLOT OF X2*X1   SYMBOL USED IS 0

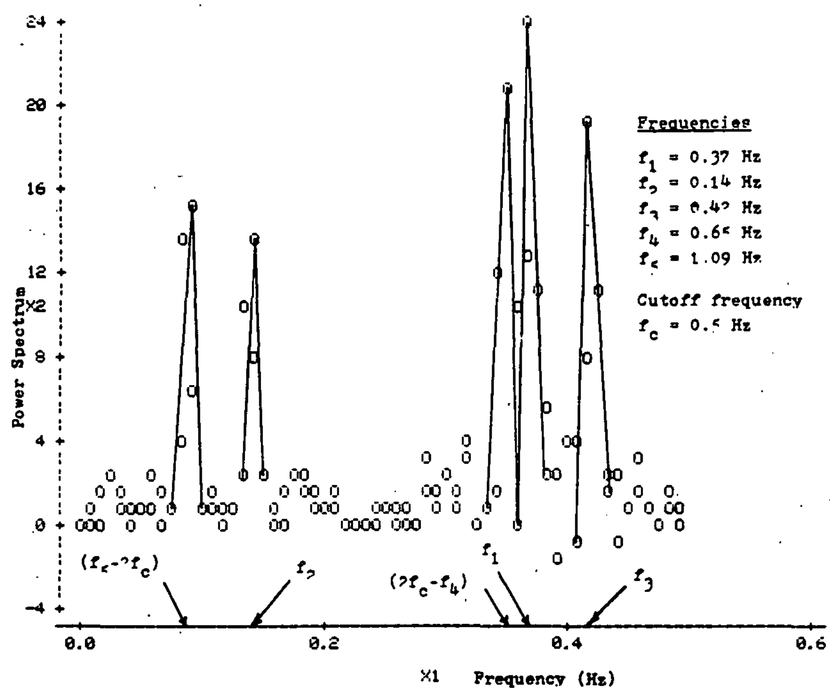

Figure 5

MODIFIED B-T SPECTRUM 60% LAGS, HOLES, NO NOISE

PLOT OF X2*X1   SYMBOL USED IS 0

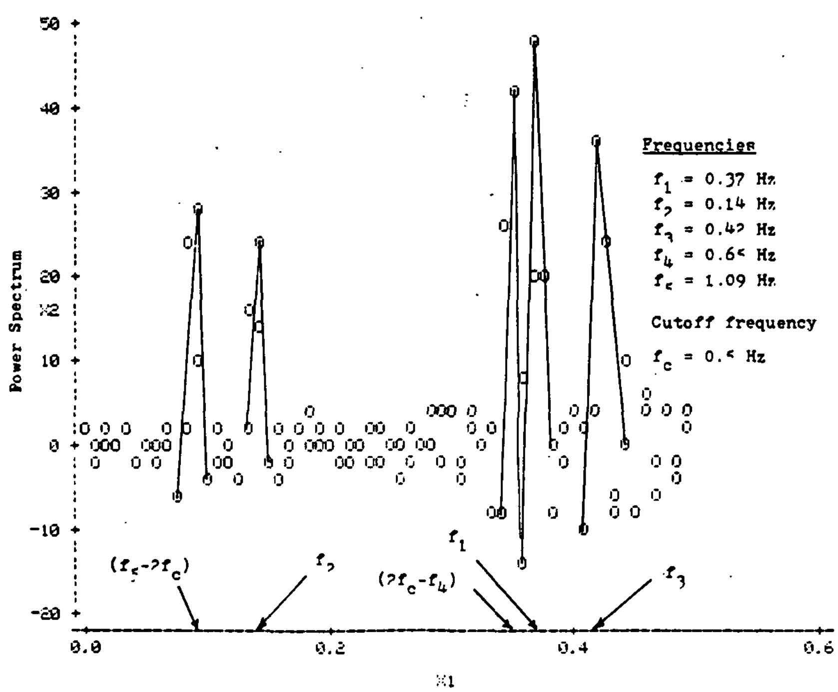

Figure 6

NOTE: 2 OBS HIDDEN

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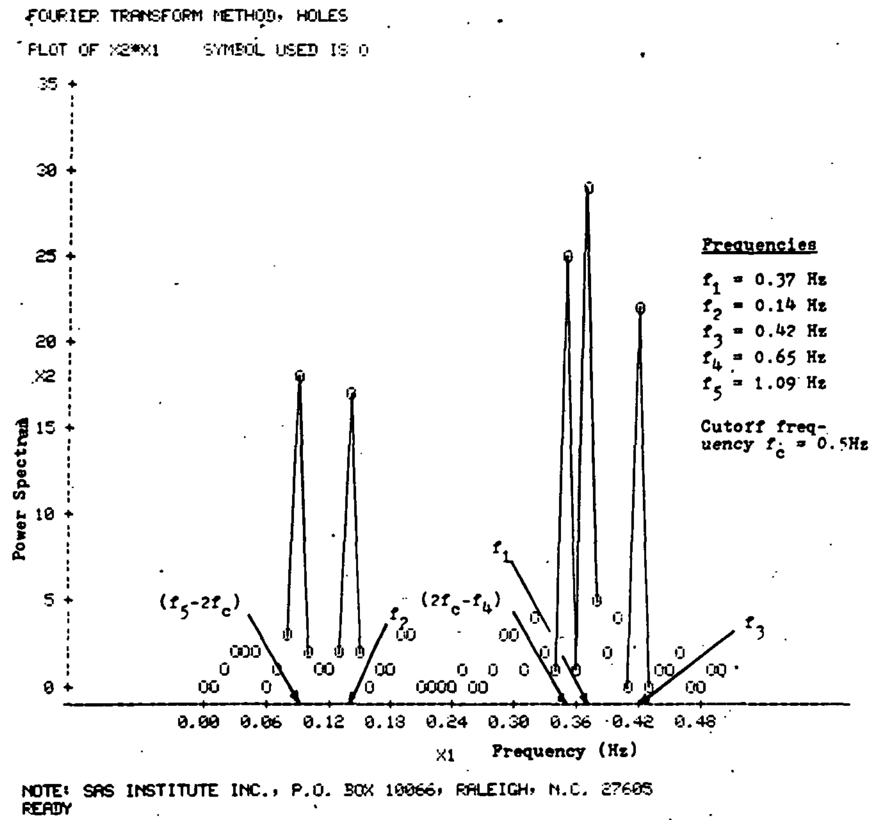

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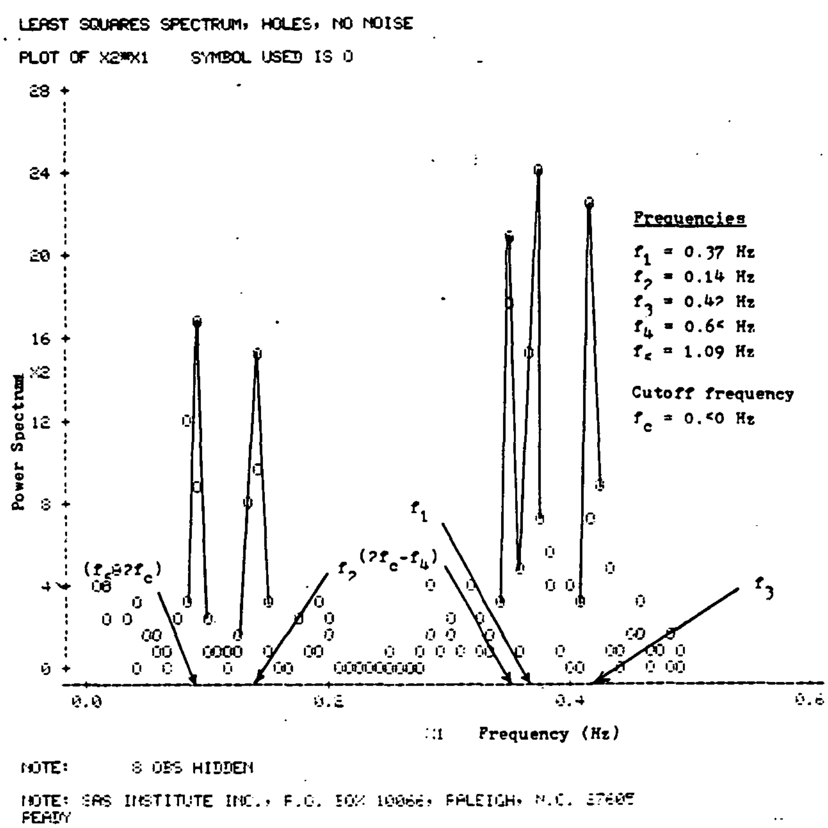

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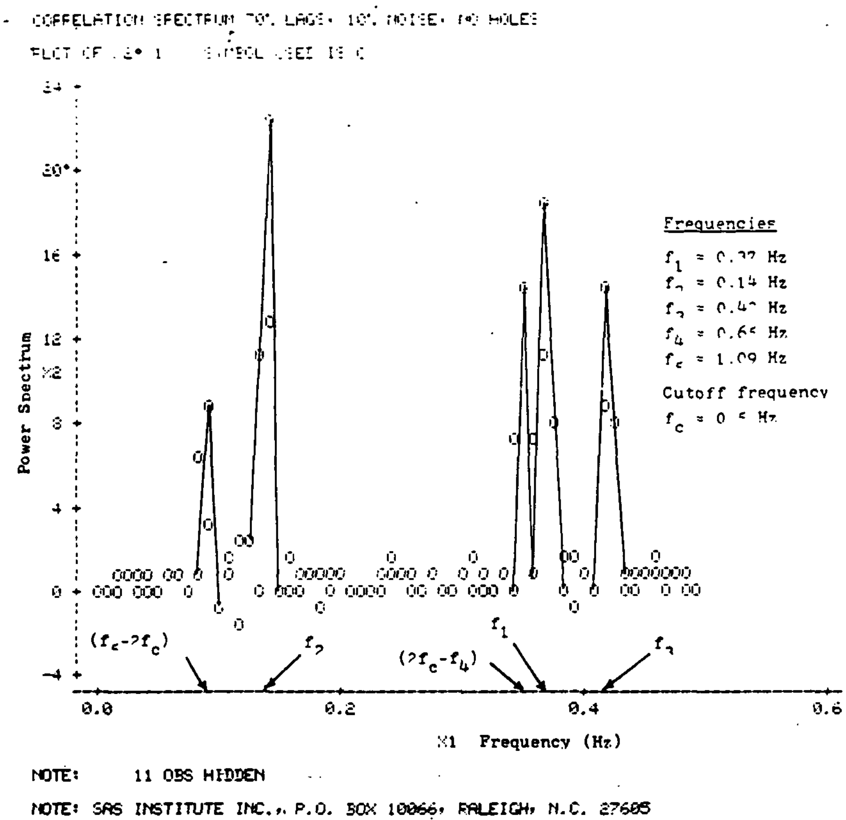
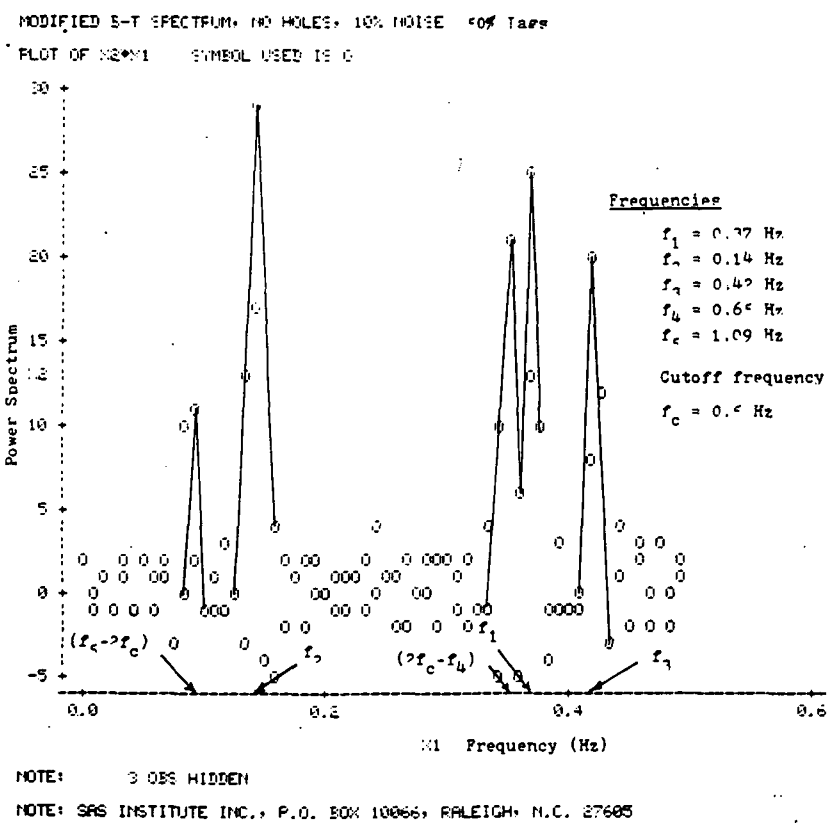


Figure 9



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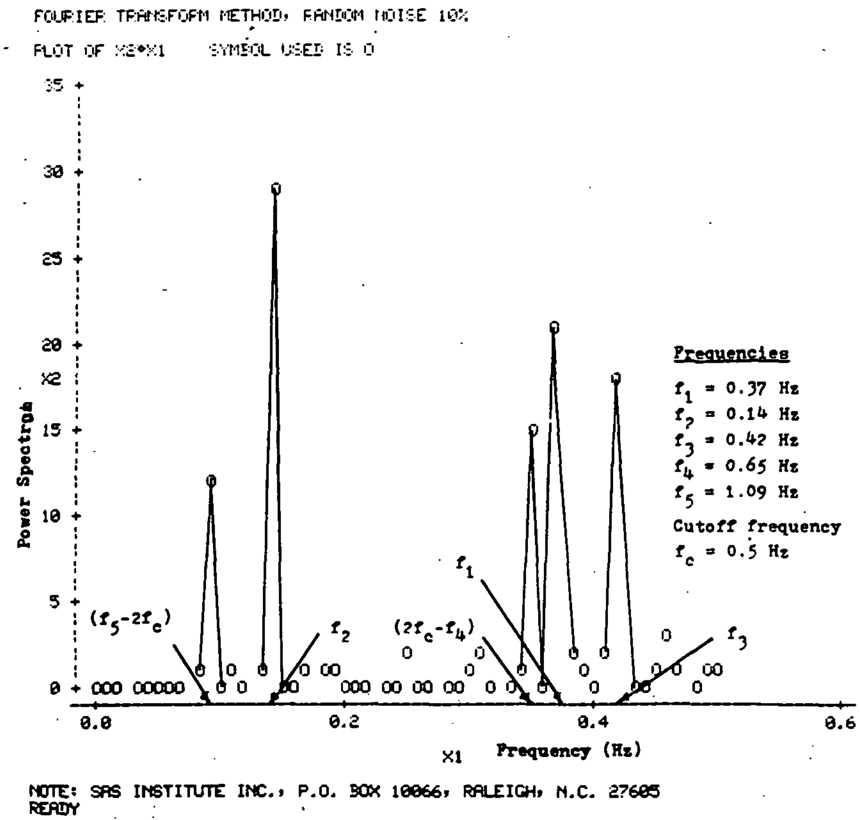

Figure 11

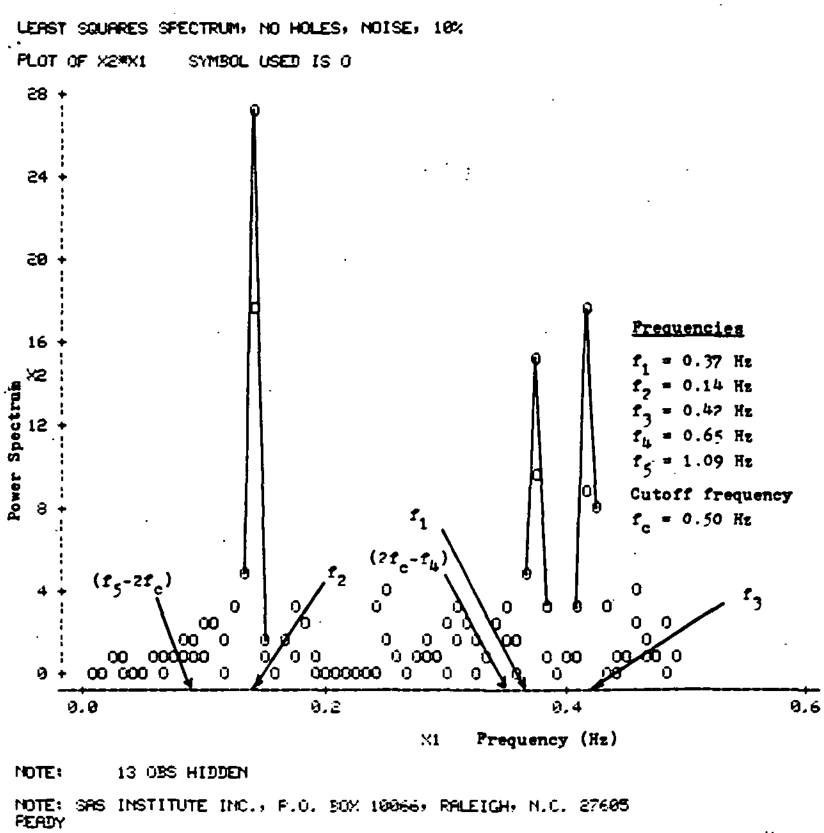

Figure 12

CORRELATION SPECTRUM, 70% LAGS, HOLES PLUS 10% NOISE

PLOT OF X2*X1   SYMBOL USED IS 0

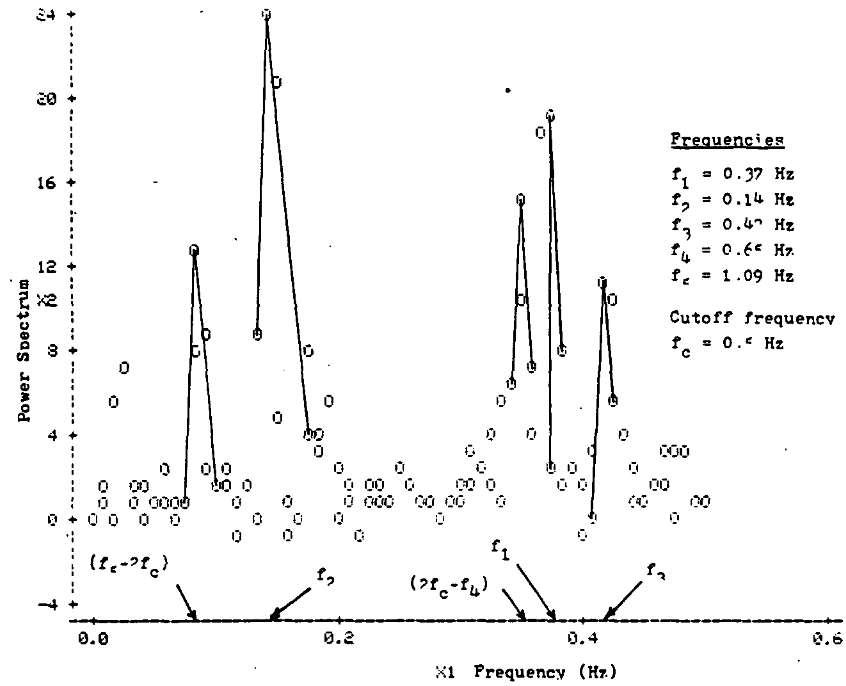

Figure 13

NOTE: 6 OBS HIDDEN

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MODIFIED B-K SPECTRUM 60% LAGS, HOLES PLUS 10% NOISE

PLOT OF X2*X1   SYMBOL USED IS 0

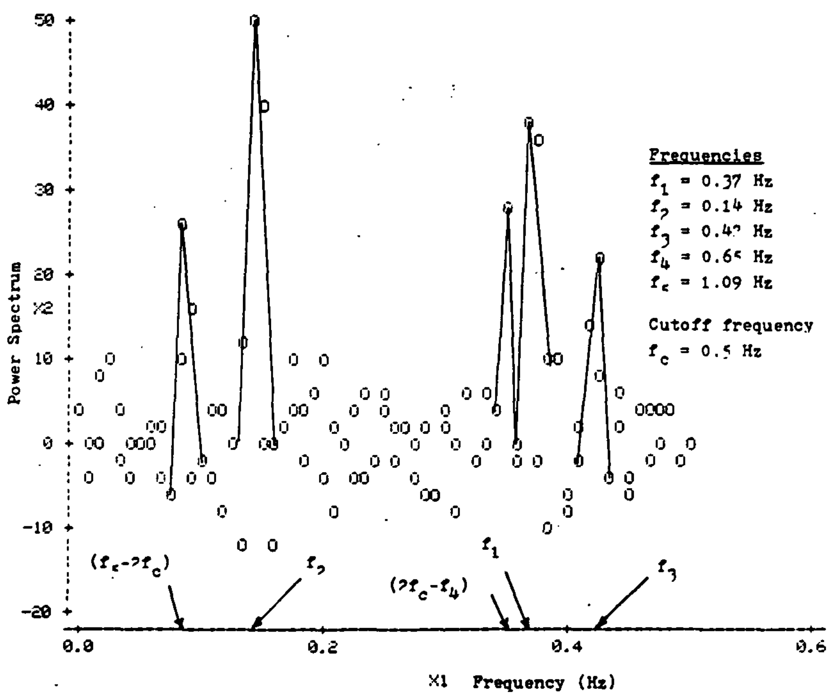

Figure 14

NOTE: 3 OBS HIDDEN

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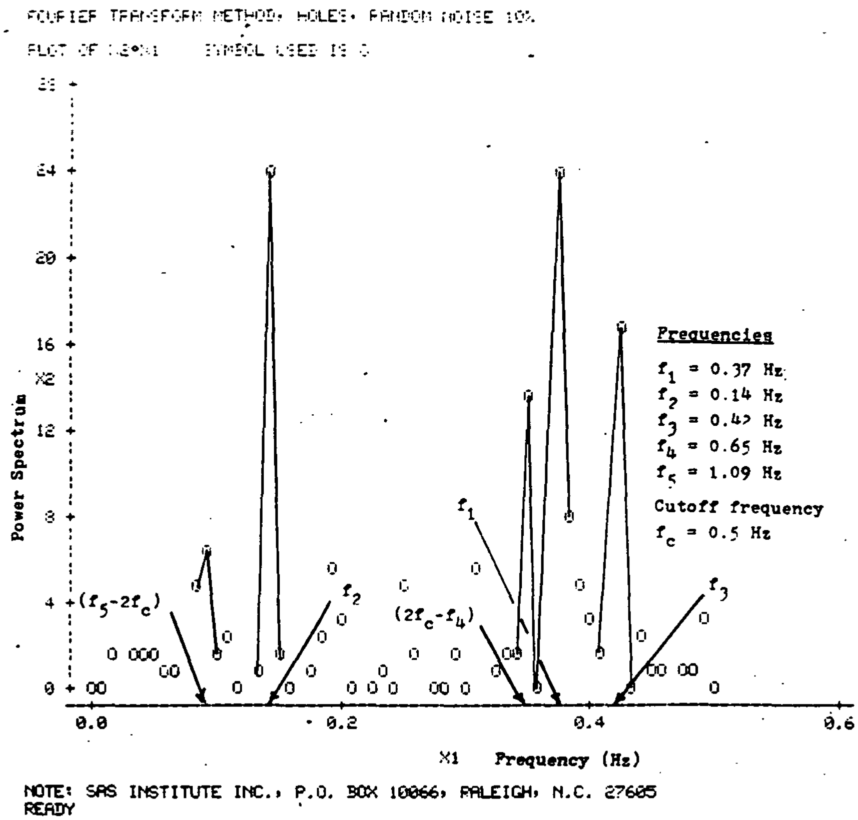

Figure 15

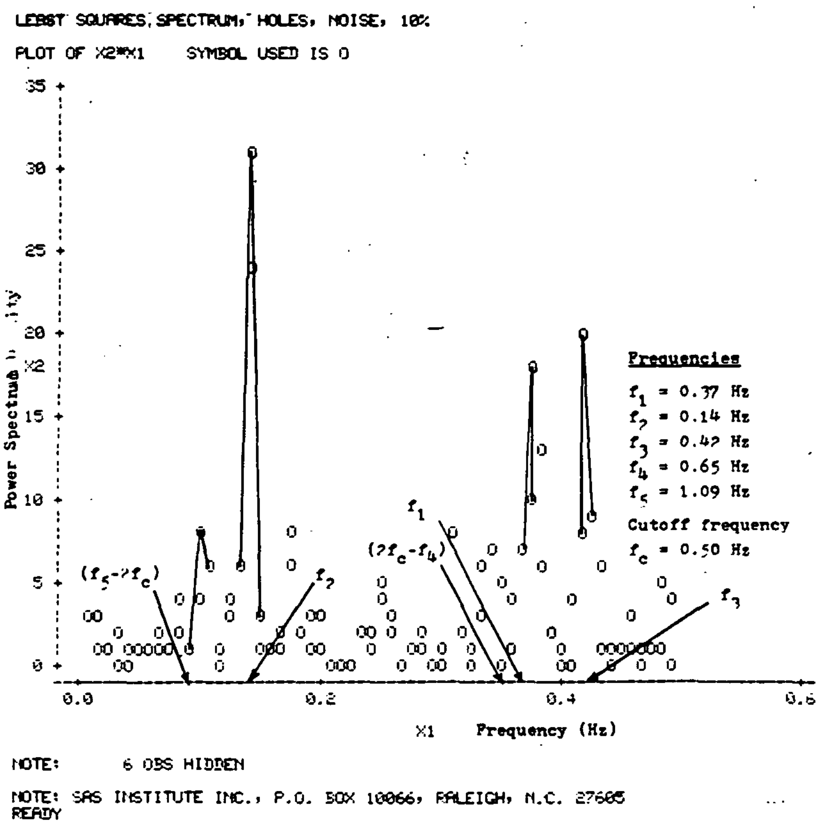

Figure 16

SECOND PRINCIPAL COMPONENT TIME SERIES

PLOT OF X2*X1   SYMBOL USED IS 0

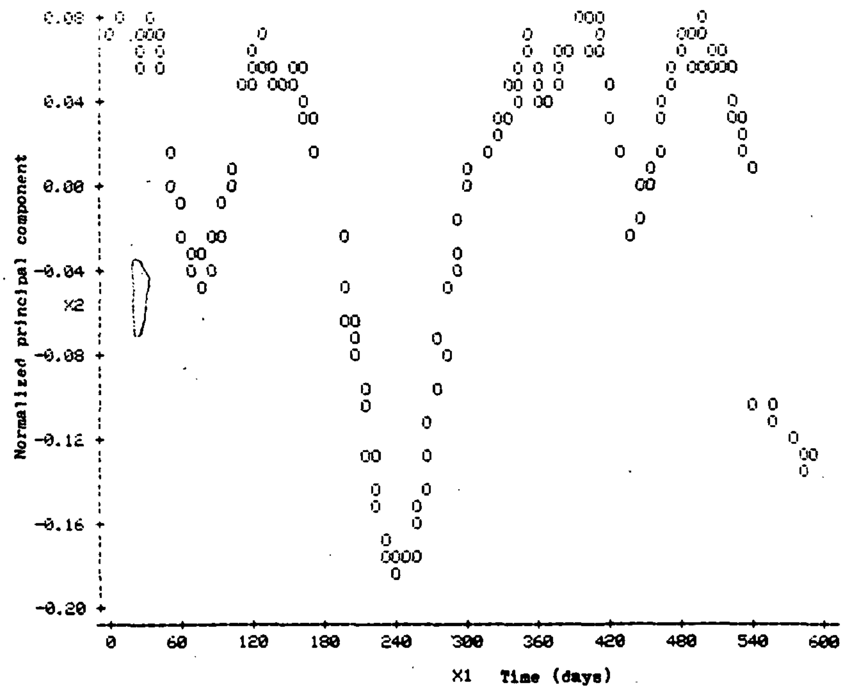

Figure 17

NOTE: 36 OBS HIDDEN

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SECOND PRINCIPAL COMPONENT POWER SPECTRUM - LEAST SQUARES

PLOT OF X2*X1   SYMBOL USED IS 0

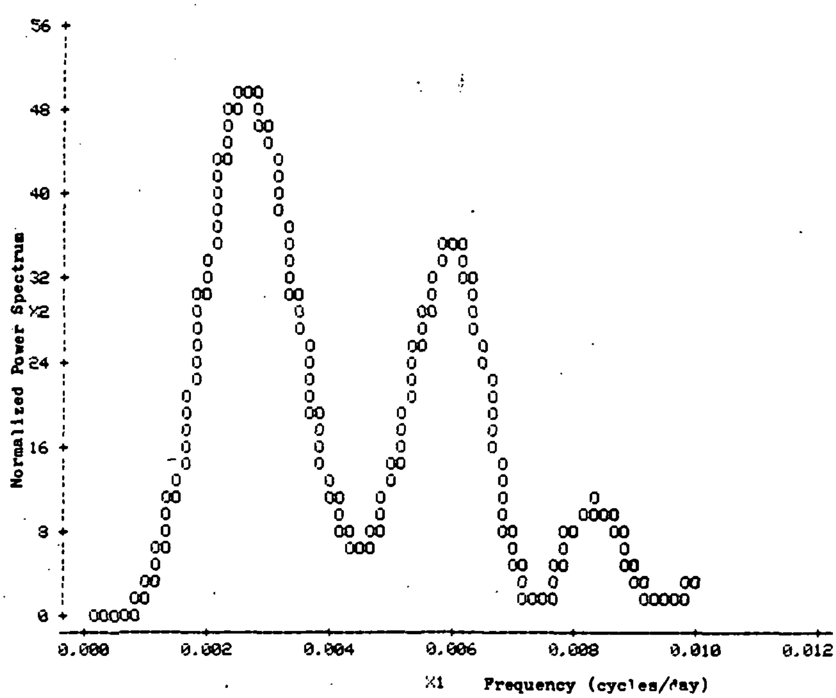

Figure 18

NOTE: 244 OBS HIDDEN

NOTE: SAS INSTITUTE INC., P.O. BOX 10066, RALEIGH, N.C. 27605

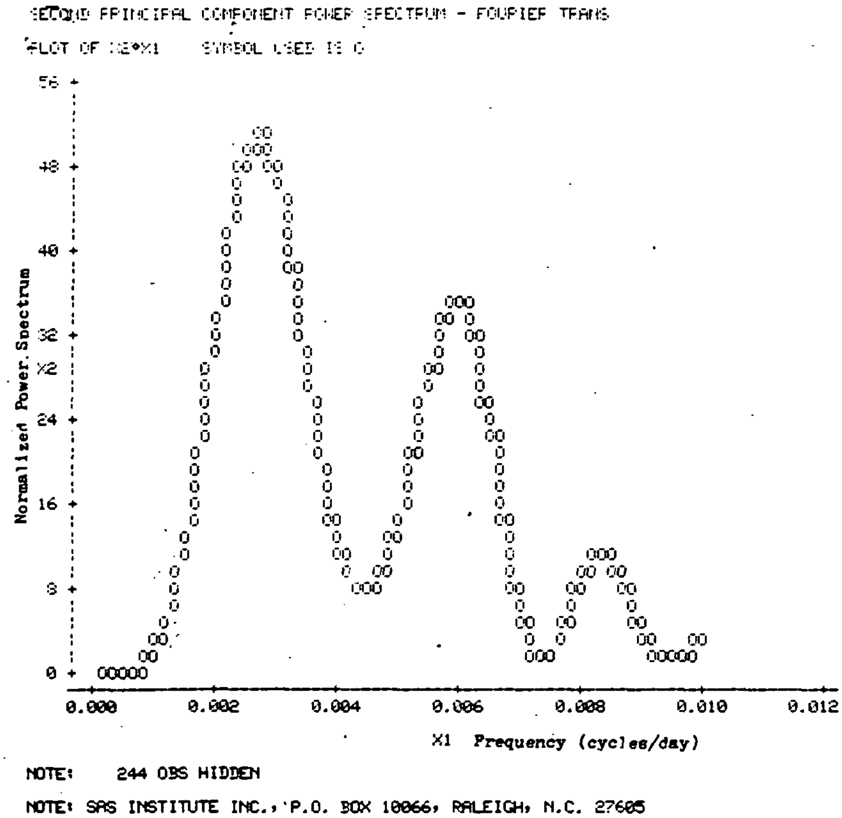

Figure 19

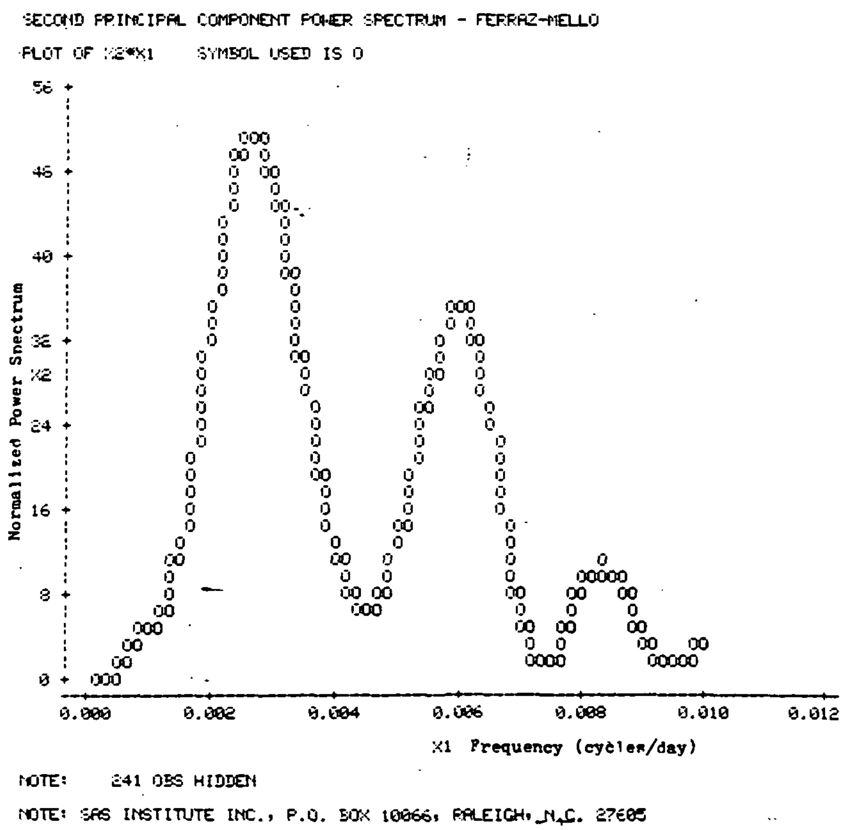

Figure 20

SECOND PRINCIPAL COMP SPECTRUM - CORRELATION 100% LAGS

PLOT OF X2*X1   SYMBOL USED IS O

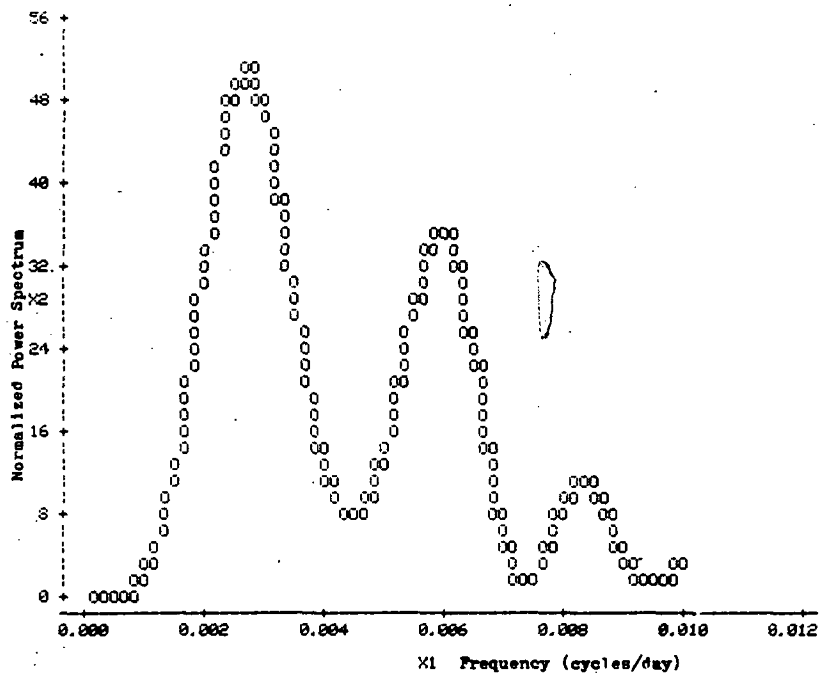

Figure 21

NOTE: 244 OBS HIDDEN

NOTE: SAS INSTITUTE INC., P.O. BOX 10066, RALEIGH, N.C. 27605
READY

SECOND PRINCIPAL COMP SPECTRUM - CORRELATION 80% LAGS

PLOT OF X2*X1   SYMBOL USED IS O

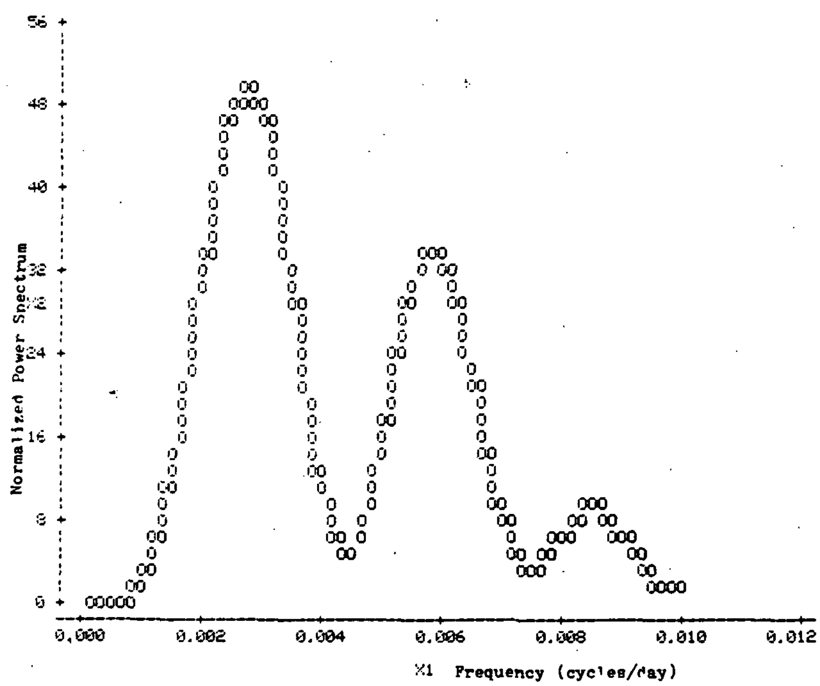

Figure 22

NOTE: 247 OBS HIDDEN

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READY

SECOND PRINCIPAL COMP SPECTRUM - CORRELATION 70% LAGS

PLOT OF X2*X1   SYMBOL USED IS O

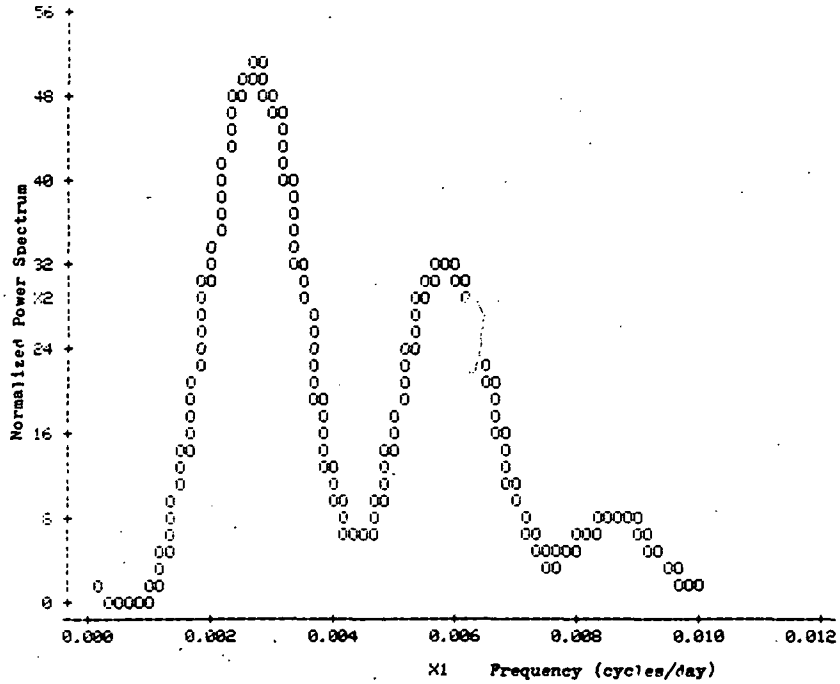


Figure 23

NOTE: 248 OBS HIDDEN

NOTE: SAS INSTITUTE INC., P.O. BOX 10066, RALEIGH, N.C. 27605

SECOND PRIN COMP 60% CORRELATION METHOD POWER SPECTRUM

PLOT OF X2*X1   SYMBOL USED IS O

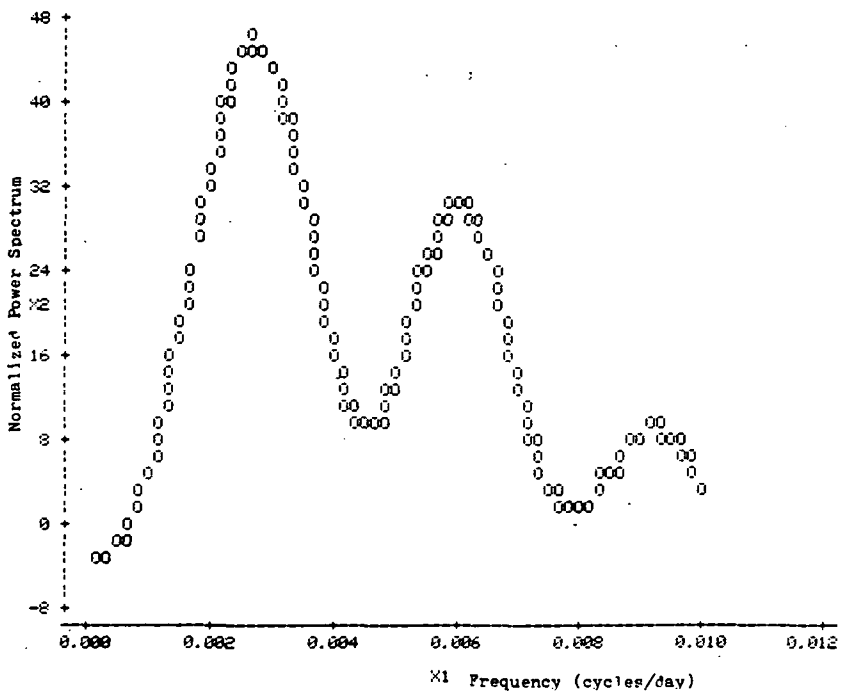


Figure 24

NOTE: 78 OBS HIDDEN

NOTE: SAS INSTITUTE INC., P.O. BOX 10066, RALEIGH, N.C. 27605

SECOND PRIN COMP 60% MAX LAG. MODIFIED B-T SPECTRUM

PLOT OF X2*X1   SYMBOL USED IS O

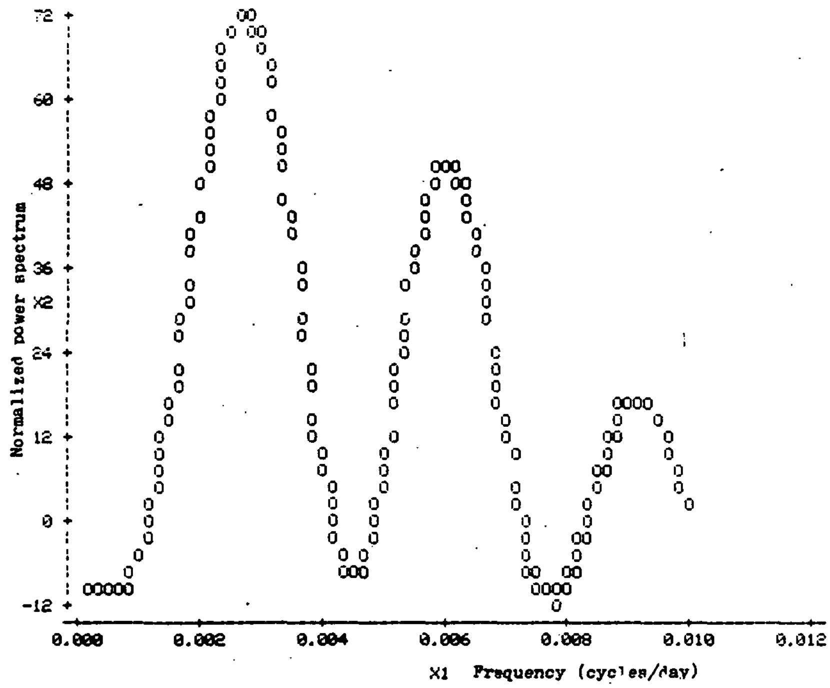


Figure 25

NOTE: 56 OBS HIDDEN

NOTE: SAS INSTITUTE INC., P.O. BOX 10066, RALEIGH, N.C. 27605

APPENDIX A

Definitions

A real-valued function x(t) is periodic with period T if for all t

$$x(t + T) = x(t) \quad \text{A.1}$$

The Fourier transform X(f) of x(t) is defined as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt \quad \text{A.2}$$

if the integral exists for all f, where f is frequency and t is time.

If x(t) is periodic and limited over some interval T_o, then x(t) can be represented by its Fourier Series

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2\pi n f_o t} \quad \text{A.3}$$

where $f_o = 1/T_o$ and

$$a_n = (1/T_o)\int_{-T_o/2}^{T_o/2} x(t)e^{-j2\pi n f_o t}\,dt \quad \text{A.4}$$

It should be noted that in taking the Fourier transform of a periodic function x(t), the Fourier coefficients a_n which are obtained, are the same as the Fourier transform evaluated at n/T_o.

Another useful relationship is the correlation integral given by

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t + \tau)d\tau \quad \text{A.5}$$

It can be shown (Brigham 1974) that the Fourier transform Z(f) is given by

$$Z(f) = H(f) \cdot X^*(f) \quad \text{A.6}$$

where X*(f) is the complex conjugate of X(f).

The discrete analogue of A.5 and A.6 is that if

$$z(k) = \sum_{i=0}^{N-1} x(i) \cdot h(k+i) \qquad \text{A.7}$$

$$(k = 0, 1, 2, \ldots N-1)$$

is the correlation function for x(i) and h(i), then the Fourier transform Z(n) of z(k) is given by

$$Z(n) = X^*(n) \cdot H(n) \qquad \text{A.8}$$

where the discrete Fourier transform X(n) of x(k) is defined as

$$X(n) = \sum_{k=0}^{N-1} x(k) e^{-j2\pi nk/N} \qquad \text{A.9}$$

$$(n = 0, 1, \ldots, N-1)$$

If x(t) and h(t) are the same function in A.5, we have the autocorrelation function of x(t)

$$z(t) = \int_{-\infty}^{\infty} x(\tau) x(t+\tau) d\tau \qquad \text{A.10}$$

or if x(i) is the same as h(i) in A.7, the discrete autocorrelation function of x(i) is

$$z(k) = \sum_{i=0}^{N-1} x(i) x(i+k) \qquad \text{A.11}$$

$$(k = 0, 1, \ldots, N-1)$$

Then, using A.6 and A.8, the Fourier transform of the autocorrelation function Z(f) is given by

$$Z(f) = X^*(f) \cdot X(f) = |X(f)|^2 \qquad \text{A.12}$$

in the continuous case and by

$$Z(n) = X^*(n) \cdot X(n) = |X(n)|^2 \qquad \text{A.13}$$

in the discrete case.

Equations A.12 and A.13 represent the contribution of the frequency f (or n) to the total power in the waveform.

The total power in the time domain is equivalent to the total power in the frequency domain by Parseval's equation which is given by

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} |X(f)|^2 df \qquad \text{A.14}$$

in the continuous case and by

$$\sum_{k=0}^{N-1} x^2(k) = (1/N) \sum_{n=0}^{N-1} X^2(n) \qquad \text{A.15}$$

in the discrete case.

The relationship in A.12 (or equivalently A.13) is an important one in that it provides us with another tool for obtaining the power spectrum of a time series. Note that the power spectrum can also be obtained by taking the square of the magnitude of X(f) in A.2 or of X(n) in A.9.

APPENDIX B

Definition of Impulse Function and Properties

The impulse function $\delta(t)$ is a distribution which assigns to a test function $x(t)$ the value of $x(0)$

$$\int_{-\infty}^{\infty} \delta(t)x(t)dt = x(0) \qquad \text{B.1}$$

The function $\delta(t - t_o)$ is defined as

$$\int_{-\infty}^{\infty} \delta(t - t_o)x(t)dt = x(t_o) \qquad \text{B.2}$$

From the above we obtain the following

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \qquad \text{B.3}$$

The product of a $\delta(t)$ function by an ordinary function $y(t)$ is defined by

$$\int_{-\infty}^{\infty} [\delta(t)y(t)]\, x(t)dt = \int_{-\infty}^{\infty} \delta(t)\,[y(t)x(t)]\; dt \qquad \text{B.4}$$

and if $x(t)$ is continuous at $t = t_o$

$$\delta(t_o)x(t) = \delta(t_o)x(t_o) \qquad \text{B.5}$$

Derivation of Discrete Fourier Transform for an arbitrary spacing

Let $x(t)$ be the function to be sampled and define the sampling function $s(t)$ as a series of impulse functions to be applied at the times t_k

(1)The symbol $\delta(t)$ for the impulse function is not to be confused with the symbol δ used in the text. The δ there is the Kronecker delta function.

$$s(t) = \sum_{k=1}^{N} \delta(t - t_k) \tag{B.6}$$

Then the sampled function is

$$\hat{x}(t) = x(t) \sum_{k=1}^{N} \delta(t - t_k) \tag{B.7}$$

The discrete Fourier transform of $\hat{x}(t)$ is then

$$X_N(f) = \int_{-\infty}^{\infty} \hat{x}(t) e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} x(t) \sum_{k=1}^{N} \delta(t - t_k) e^{-j2\pi ft}\, dt \tag{B.8}$$

Interchanging the integral and summation in B.8 and using B.2 and B.5

$$X_N(f) = \sum_{k=1}^{N} \int_{-\infty}^{\infty} \delta(t - t_k) x(t) e^{-j2\pi ft}\, dt$$

$$= \sum_{k=1}^{N} x(t_k) e^{-j2\pi ft_k} \tag{B.9}$$

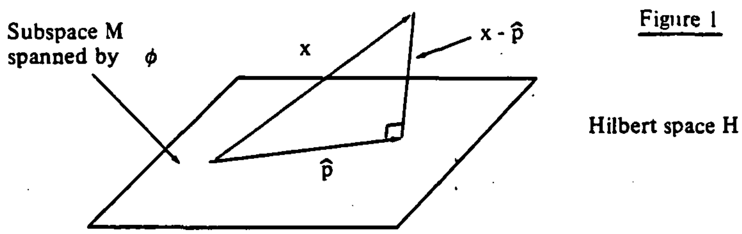
APPENDIX C

MATHEMATICAL DEVELOPMENT OF LEAST SQUARES POWER SPECTRUM

The standard least squares problem of determining a best fit of the data to an assumed mathematical model is equivalent to a minimum norm problem in Hilbert space. The latter approach is taken by Vanicek (1969, 1971) and Wells and Vanicek (1978) in their treatment of the least squares power spectrum.

In this appendix we will develop the optimum least squares estimate of the power spectrum from both viewpoints. A geometric interpretation of the problem can be seen in Figure 1.

Figure 1



Let x be any (measurement) vector in a Hibert space H where x is given by

$$x^T = (x_1, x_2, \ldots, x_N) \qquad \text{C.1}$$

and where x_i is the value of the time series at the time t_i.

Let M be a closed subspace spanned by the (Nx1) vectors ϕ_1 and ϕ_2

$$\phi = (\phi_1, \phi_2) = \begin{pmatrix} \cos\omega t_1 & \sin\omega t_1 \\ \cos\omega t_2 & \sin\omega t_2 \\ \cdots\cdots & \\ \cos\omega t_N & \sin\omega t_N \end{pmatrix} \qquad \text{C.2}$$

Then, the *projection theorem* assures us that there is a unique N vector $\hat{p} = \phi\hat{c}$ such that(1)

$$\|x-\hat{p}\| \leqslant \|x-p\| \text{ for all } p \in M \qquad \text{C.3}$$

where the norm $\|x\|$ is defined in terms of the inner product of x with itself

$$\|x\|^2 = x^T x \qquad \text{C.4}$$

(1)The use of the caret in $\hat{p}$ and $\hat{c}$ refers to the least squares estimate and is not to be confused with Wells and Vanicek's (1978) use of this symbol which is used to refer to the space $\hat{\phi}$ of "constituents" (bias term, linear trend, quadratic trend, etc.). We are assuming no constituents and therefore Vanicek's $\hat{p} = 0$.

The necessary and sufficient condition that $\hat{p}$ exists is that $(x-\hat{p})$ be orthogonal to M, or, in particular ϕ. That is, the inner product of $(x-\hat{p})$ and ϕ must be equal to zero.

$$\phi^T(x-\phi\hat{c}) = 0 \qquad \text{C.5}$$

Equation (C.5) is the vector representation of the usual normal equations. Note that since ϕ_1 and ϕ_2 span the space M, they are linearly independent, and therefore, the Gram matrix $\phi^T\phi$ is non-singular. Hence, we can solve for $\hat{c}$ uniquely.

$$\hat{c} = (\phi^T\phi)^{-1}\phi^T x \qquad \text{C.6}$$

Then, in the least squares sense, the best approximating vector to x (which is contained in M) is the projection of x onto the subspace M. See Figure 1. Therefore,

$$\hat{p} = \phi\hat{c} \qquad \text{C.7}$$

This particular vector gives the minimum norm $\|x-\hat{p}\|$ or the distance from x to the subspace M.

An optimum normalized least squares spectrum $S_L(\omega)$ can now be defined [Wells and Vanicek (1978)] as

$$S_L(\omega) = 1 - \frac{\|x-\hat{p}\|^2}{\|x\|^2} \qquad \text{C.8}$$

A "nice" feature of this particular choice for the spectrum is that it has a minimum of zero when the vector x is orthogonal to M (or when the projection of x onto M, $\hat{p}$, is equal to zero), and a maximum when x is contained in M (that is, when the projection of x onto M is equal to x and therefore $(x-\hat{p}) = 0$).

Using the fact that

$$\hat{p}^T(x-\hat{p}) = 0 \qquad \text{C.9}$$

and the definition of the norm in (4) we can write

$$S_L(\omega) = \frac{x^T\hat{p}}{x^T x} \qquad \text{C.10}$$

or using C.7

$$S_L(\omega) = \frac{x^T\phi(\phi^T\phi)^{-1}\phi^T x}{x^T x} \qquad \text{C.11}$$

In the usual least squares terminology we wish to fit data points (t_i, x_i) to some assumed mathematical model

$$p(t) = \cos(\omega t)\,a + \sin(\omega t)b \qquad \text{C.12}$$

determining the least squares estimates $\hat{a}$ and $\hat{b}$ for a and b such that

$$Q = \sum_{i=1}^{N} \left\{x_i - [(\cos\omega t_i)a + (\sin\omega t_i)b]\right\}^2 \qquad \text{C.13}$$

is minimized. This is equivalent to finding $\hat{p}$ to minimize $\|x\text{-}p\|$ in C.3.

The standard procedure is to differentiate C.13 with respect to both a and b, then set these expressions equal to zero. Two equations in two unknowns result

$$\begin{aligned}
\left(\sum_{i=1}^{N} \cos^2\omega t_i\right)a + \left(\sum_{i=1}^{N} (\sin\omega t_i)(\cos\omega t_i)\right)b &= \sum_{i=1}^{N} x_i\cos\omega t_i \\
\left(\sum_{i=1}^{N} (\cos\omega t_i)(\sin\omega t_i)\right)a + \left(\sum_{i=1}^{N} \sin^2\omega t_i\right)b &= \sum_{i=1}^{N} x_i\sin\omega t_i
\end{aligned} \qquad \text{C.14}$$

which are the normal equations or in matrix notation equation C.5.

The two elements of the vector $\hat{c}$ in C.6 are the estimates $\hat{a}$ and $\hat{b}$ and we have

$$\hat{c}^T = (\hat{a}, \hat{b}) \qquad \text{C.15}$$

We therefore obtain, as above, the best estimate for p (or $\hat{p}$) in M as equation C.7.

It is worthwhile at this point to compare the expression obtained for the least squares spectrum with the Discrete Fourier Transform spectrum, $S_F(\omega)$.

Recall that the Discrete Fourier Transform $X_N(\omega)$ of x_i (Appendix B) is given by

$$X_N(\omega) = \sum_{i=1}^{N} x_i e^{-j\omega t_i}$$

$$= \sum_{i=1}^{N} x_i \cos(\omega t_i) - j \sum_{i=1}^{N} x_i \sin(\omega t_i)$$

In matrix notation since ϕ_1 and ϕ_2 are defined in C.2 and x in C.1

$$\begin{aligned} X_N(\omega) &= \phi_1^T x - j\phi_2^T x \\ &= (\phi_1^T - j\phi_2^T)x \end{aligned} \qquad \text{C.17}$$

Then the amplitude or power spectrum will be given by

$$\begin{aligned} \|X_N(\omega)\|^2 &= X_N^*(\omega)^T X_N(\omega) \\ &= x^T(\phi_1 + j\phi_2)(\phi_1^T - j\phi_2^T)x \\ &= x^T \phi\phi^T x \end{aligned} \qquad \text{C.18}$$

where $X_N^*(\omega)$ is the complex conjugate of $X_N(\omega)$.

We can now write a normalized Fourier Transform Spectrum as

$$S_F(\omega) = \frac{2x^T \phi\phi^T x}{Nx^T x} \qquad \text{C.19}$$

where this latter expression is obtained by noting that in the case of equally spaced data we will have

$$\phi^T \phi = (N/2)I \qquad \text{C.20}$$

where I is the 2 X 2 identity matrix.

In the case where the data is equally spaced the least squares estimate $S_L(\omega)$ in C.11 will be equivalent to C.19, the Discrete Fourier Transform power spectrum.

Finally, it should be noted that if the spacing is not too irregular (within 20% of equal sampling) the two spectrums will be about the same.

APPENDIX D: PROGRAM LISTINGS AND EXAMPLES

MODIFIED BLACKMAN-TUKEY AND CORRELATION FUNCTION POWER SPECTRUM PROGRAM FOR UNEQUALLY SPACED DATA POINTS

```
C     MAIN DRIVER FOR PONCOR
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(500),T(500),RN1(500)
      DIMENSION NTIM(500)
      DIMENSION TM(500),XM(500),RN(500)
      DIMENSION DEL(500)
C     SETTING UP SIMULATED DATA (T(I),X(I)), I = 1, NT  WHERE
C     T(I) CONTAINS RANDOM NOISE RN(I), ZERO MEAN AND STANDARD
C     DEVIATION OF 0.10 FROM A GAUSSIAN DISTRIBUTION
      NT = 139
      PI = 3.14159267D0
      ARG1 = 2.0D0*PI*0.37D0
      ARG2 = 2.0D0*PI*0.14D0
      ARG3 = 2.0D0*PI*0.42D0
      ARG4 = 2.0D0*PI*0.65D0
      ARG5 = 2.0D0*PI*1.09D0
      DO 1 I = 1,NT
      T(I) = 0.72D0*(I-1)
      RN(I) = BARN1(-1,0,12787,0.0D0,0.10D0)
      T(I) = T(I) +  RN(I)
      X(I) = DSIN(ARG1*T(I)) + DCOS(ARG2*T(I)) + DSIN(ARG3*T(I)) +
     1DCOS(ARG4*T(I))+DSIN(ARG5*T(I))
      STDEV = 0.10D0*X(I)
      RN1(I) = BARN1(-1,0,13787,0.0D0,STDEV)
C     ADDING GAUSSIAN NOISE WITH ZERO MEAN AND SIGMA EQUAL TO 20% OF
C     THE AMPLITUDE OF THE FUNCTION.
      X(I) = X(I) + RN1(I)
    1 CONTINUE
      DO 2 I = 1,NT
    2 NTIM(I) = 1
C     PUTTING 'HOLES' IN THE DATA
      DO 4 I = 1,8
      NTIM(I+10) = 0
      NTIM(I+40) = 0
      NTIM(I+70) = 0
    4 NTIM(I+90) = 0
      L = 0
      DO 3 I = 1,NT
      IF(NTIM(I).EQ.1) L = L + 1
      IF(NTIM(I).EQ.1) TM(L) = T(I)
    3 IF(NTIM(I).EQ.1) XM(L) = X(I)
      NT = L
      CALL PONCOR(0.0D0,NT,0.60D0,0.0001D0,100,XM,TM)
      STOP
      END
      SUBROUTINE PONCOR(SW,NT,FRLAG,FST,NW,F,T)
C     SW IS A SWITCH PARAMETER. IF SW = 0 MODIFIED BLACKMAN-TUKEY
C     SPECTRUM IS CALCULATED. IF SW = 1, THE CORRELATION FUNCTION
C     POWER SPECTRUM IS CALCULATED.
C     NT IS THE TOTAL NUMBER OF POINTS
C     T(I) ARE THE TIME POINTS AND F(I) ARE THE VALUES OF THE TIME
C     SERIES (I = 1,....NT).
C     M IS THE MAXIMUM LAG USED FOR COMPUTATION PURPOSES AND IS
C     EQUAL TO A FRACTION FRLAG OF THE MAXIMUM LAG IN THE DATA MAXLAG.
C     NW IS THE NUMBER OF FREQUENCIES CALCULATED BETWEEN THE STARTING
```

```
C         FREQUENCY FST AND THE CUTOFF FREQUENCY  1/(2*DELT)
          IMPLICIT REAL*8 (A-H,O-Z)
C         NT POINTS, M IS MAXIMUM LAG INDEX USED FOR COMPUTATIONAL PURPOSES.
          DIMENSION F(500),T(500),C(500)
          DIMENSION SM1(500),SM2(500),TM(500),FM(500)
C         OBTAINING AVERAGE TIME INTERVAL DELT
          CALL DELTAT(T,NT,4,1.0D0,DELT)
          WRITE (4,999) DELT
      999 FORMAT(1X,' DELT = ', F16.9)
          MM1 = M - 1
          PI = 3.14159265359OD0
C         SETTING COUNTER EQUAL TO ZERO.
          DO   3 I = 1,500
          SM1(I) = 0
          SM2(I) = 0
        3 C(I) = 0
          NM1 = NT - 1
C         REFERENCING TIME POINTS TO THE ORIGIN
          TM(1) = 0.0D0
          DO   4 I = 1,NM1
          DELA = T(I+1)-T(I)
        4 TM(I+1) = TM(I) + DELA
          DO 5 I = 1,NT
        5 T(I) = TM(I)
C         ROUNDING TIME POINTS TO THE NEAREST  MULTIPLE OF AVERAGE TIME
C         DIFFERENCE DELT OBTAINED FROM DELTAT SUBROUTINE
          DO   6 I = 1,NT
          MTM = TM(I)/DELT + 0.5D0
        6 TM(I) = MTM
C         ELIMINATING DUPLICATE TIME POINTS.
          L = 0
          DO   7 I = 1,NM1
          IF(TM(I).EQ.TM(I+1)) GO TO   7
          L = L + 1
          T(L) = TM(I)
          FM(L) = F(I)
        7 CONTINUE
          T(L+1) = TM(NT)
          FM(L+1) = F(NT)
          NT = L
          NM1 = NT - 1
          MM1 = M-1
C         CALCULATING THE MAXIMUM LAG 'MAXLAG', THE NUMBER OF LAGS TO
C         USE M = FRLAG*MAXLAG, LAGS OF A GIVEN SIZE AND CORRESPONDING
C         SUMS OF LAGGED PRODUCTS.
          MAXLAG = T(NT) - T(1) + 1.0D0
          M = FRLAG*MAXLAG
          DO 10 I = 1, NT
          DO 9 J = I, NT
          ILAG = T(J) - T(I) + 1.0D0
          SM1(ILAG) = SM1(ILAG) + 1.0D0
        9 SM2(ILAG) = SM2(ILAG) + FM(I)*FM(J)
       10 CONTINUE
C
C         CALCULATING LAGGED PRODUCTS OR MEAN LAGGED PRODUCTS FROM
C         COUNTERS SM1(I) AND SM2(I) AABOVE.
C
          IF(SW.EQ.1.0D0) GO TO 12
          DO  11 I = 1,M
          IF(SM1(I) .LE. 1.0D0) C(I) = 0.0D0
          IF(SM1(I) .LE. 1.0D0) GO TO  11
          C(I) = SM2(I)/SM1(I)
```

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```
   11 CONTINUE
      GO TO 14
   12 DO 13 I = 1,M
   13 C(I) = SM2(I)
   14 FNORM = 0.0D0
      DO 15 I = 1, NT
   15 FNORM = FNORM + FM(I)**2
      FEND = 1.0D0/(2.0D0*DELT)
      DELF = (FEND - FST)/DFLOAT(NN)
      FREQ = FST
      DO 19 I = 1, NN
      SUMR = 0.0D0
      DO 16 J = 1, M
      FJ = J - 1
      ARG = 2.0D0*PI*FREQ*FJ*DELT
   16 SUMR = SUMR + C(J)*DCOS(ARG)
      IF (SN.EQ.1.0D0) GO TO 17
      SPCOR = 200.0D0*(2.0D0*SUMR - FNORM/DFLOAT(NT))/FNORM
      GO TO 18
   17 SPCOR = 200.0D0*(2.0D0*SUMR-FNORM)/(DFLOAT(NT)*FNORM)
   18 WRITE (4, 20) FREQ, SPCOR
   19 FREQ = FREQ + DELF
   20 FORMAT(2F16.9)
 9999 RETURN
      END
      REAL FUNCTION BARN1*8(I,IKEY,IFRN,AMEAN,SD)
      IMPLICIT REAL*8(A-H,O-Z,$)
C     SD....THE DESIRED STANDARD DEV
C     AMEAN...THE DESIRED MEAN.
C     H...THE POP SIZE.
      DATA IHERE/12787/
      DATA H/36.0D0/
      IF (IKEY) 5,4,4
    4 IHERE = IFRN
    5 IF (I) 6,7,7
    6 CALL GAUSS (IHERE,SD,AMEAN,VAL,H)
      IFRN = IHERE
      GO TO 8
    7 CALL RANDU (IHERE,IFRN,VAL)
      IHERE = IFRN
    8 BARN1 = VAL
      RETURN
       END
      SUBROUTINE GAUSS(IX,S,AM,V,H)
      IMPLICIT REAL*8 (A-H,O-Z,$)
      K = H
      A = 0.0D0
      DO 50 I = 1,K
      CALL RANDU(IX,IY,Y)
      IX = IY
   50 A = A + Y
      HO = H/12.0D0
      H2 = H/2.0D0
      V = (S*(A-H2))/DSQRT(HO) + AM
      RETURN
      END
      SUBROUTINE RANDU(IX,IY,YFL)
      IMPLICIT REAL*8(A-H,O-Z,$)
      DATA JJJ5/1027/
      IY = IX*JJJ5
      IF(IY) 5,6,6
    5 IY = IY + 2147483647+1
```

```
    6 YFL = IY
      YFL = YFL*.4656613D-9
      RETURN
      END
      SUBROUTINE DELTAT(T,N,KLOOP,TOL,DELT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(500), DEL(500), DELM(500)
C     T(I) = TIME POINTS FROM I = 1, N
C     KLOOP = A PARAMETER DEFINING THE NUMBER OF TIMES THE
C       SUBROUTINE GOES THROUGH A LOOP TO THROW OUT TIME DIFFERENCES
C       GREATER THAN TOL*SD
C     SD = STANDARD DEVIATION OF THE TIME DIFFERENCES
C     DELT = AVERAGE TIME DIFFERENCE CALCULATED FROM SUBROUTINE
C     XM = AVERAGE TIME DIFFERENCE. DELT TAKES THIS
C     VALUE UPON EXIT FROM THE SUBROUTINE.
      NM1 = N - 1
      KL = 1
      DO 1 I = 1,NM1
    1 DEL(I) = T(I+1) - T(I)
   99 SUM1 = 0
      SUM2 = 0
      DO 2 I = 1,NM1
      SUM1 = SUM1 + DEL(I)
    2 SUM2 = SUM2 + DEL(I)**2
      XM = SUM1/DFLOAT(NM1)
      ARG = (SUM2/DFLOAT(NM1)) - XM**2
      IF(ARG.LE.0.0D0) SD = 0
      IF(SD.EQ.0.0D0) GO TO 999
      SD = DSQRT(ARG)
      IF(SD.LE.XM) GO TO 999
      L = 0
      DO 3 I = 1, NM1
      IF(DEL(I).LE.TOL*SD) L = L + 1
    3 IF(DEL(I).LE.TOL*SD) DELM(L) = DEL(I)
      IF(KL .GT. KLOOP) GO TO 999
      KL = KL + 1
      NM1 = L
      DO 4 I = 1,NM1
    4 DEL(I) = DELM(I)
      GO TO 99
  999 DELT = XM
      RETURN
      END
END OF DATA
INPUT
QED
```

COMPUTATION OF THE MODIFIED BLACKMAN-TUKEY POWER SPECTRUM (SN = 0) FROM THE POWCOR PROGRAM. NOISE ON TIME POINTS AND THE FUNCTION PLUS HOLES IN THE DATA. 60% OF THE MAXIMUM LAG USED FOR M.

```
DELT =        0.716926883
   0.000100000     -1.542176726
   0.007073212     -0.150789825
   0.014046424     -0.569139551
   0.021019636     -0.754129357
   0.027992848     -0.071476469
   0.034966060     -1.253636824
```

0.041939271	-0.700170678
0.048912483	2.756354618
0.055885695	2.758760965
0.062858907	-1.988390526
0.069832119	1.632227636
0.076805331	2.626718432
0.083778543	-1.047745857
0.090751755	1.107620138
0.097724967	-0.189538279
0.104698179	0.688879172
0.111671390	0.409942943
0.118644602	1.228373473
0.125617814	0.911728260
0.132591026	1.896405508
0.139564238	17.457179640
0.146537450	7.938622769
0.153510662	-3.471830306
0.160483874	4.215563718
0.167457086	-3.623829737
0.174430298	-0.361802086
0.181403510	-0.179820478
0.188376721	-1.382386927
0.195349933	4.793682891
0.202323145	-3.223365853
0.209296357	3.377895992
0.216269569	-1.284619378
0.223242781	-3.889691259
0.230215993	8.042041039
0.237189205	-2.041714553
0.244162417	1.196875771
0.251135629	0.149844388
0.258108841	-0.866045029
0.265082052	3.288623278
0.272055264	-5.486837804
0.279028476	4.622737647
0.286001688	-8.669465699
0.292974900	1.088096458
0.299948112	38.965659688
0.306921324	12.829537300
0.313894536	-4.218917966
0.320867748	-1.688957381
0.327840960	-2.366650455
0.334814171	2.882904671
0.341787383	-5.167401777
0.348760595	5.590805527
0.355733807	-1.983645083
0.362707019	5.099041451
0.369680231	46.535006004
0.376653443	23.142117579
0.383626655	-10.729409462
0.390599867	-2.204022813
0.397573079	7.885270909
0.404546291	-3.769650657
0.411519502	0.141104102
0.418492714	36.778866377
0.425465926	15.255659188
0.432439138	-11.040259686
0.439412350	6.893443000
0.446385562	-1.712196398
0.453358774	1.449938204
0.460331986	5.745924092
0.467305198	-2.048029330

0.474278410	5.960421200
0.481251622	1.638838335
0.488224833	2.379054125
0.495198045	1.365755033
0.502171257	-4.429314707
0.509144469	0.280715652
0.516117681	-1.321523319
0.523090893	0.869202236
0.530064105	-1.165751987
0.537037317	-0.807237399
0.544010529	3.407178821
0.550983741	-2.527616220
0.557956952	0.759171349
0.564930164	-0.309190605
0.571903376	-2.168750319
0.578876588	1.791152921
0.585849800	-1.768993475
0.592823012	-0.019009763
0.599796224	-0.180850333
0.606769436	0.485932476
0.613742648	0.979295680
0.620715860	-2.078632462
0.627689072	1.981811540
0.634662283	-2.485895741
0.641635495	0.998141407
0.648608707	12.165057662
0.655581919	3.804976281
0.662555131	-2.087106152
0.669528343	0.689841105
0.676501555	1.855003671
0.683474767	2.603868543
0.690447979	1.673795283

END OF DATA

COMPUTATION OF THE CORRELATION FUNCTION POWER SPECTRUM (SN = 1) FROM THE PONCOR PROGRAM. NOISE ON TIME POINTS AND THE FUNCTION PLUS HOLES IN THE DATA. 70% OF THE MAXIMUM LAG USED FOR M.

DELT = 0.716926883

0.000100000	-0.345560677
0.007073212	0.540255097
0.014046424	-0.362687738
0.021019636	0.695394888
0.027992848	-0.421677236
0.034966060	0.483970455
0.041939271	-0.238465377
0.048912483	2.526149713
0.055885695	1.551455083
0.062858907	0.113976701
0.069832119	1.178969943
0.076805331	1.552425450
0.083778543	0.823391820
0.090751755	1.417395157
0.097724967	1.040920636
0.104698179	0.797709866
0.111671390	1.173196238
0.118644602	0.584598515
0.125617814	2.300821346
0.132591026	1.727363267

0.139564238	10.176666832
0.146537450	3.889439332
0.153510662	1.425407500
0.160483874	1.464490486
0.167457086	-0.384136815
0.174430298	0.531557907
0.181403510	-0.300573396
0.188376721	1.766209922
0.195349933	1.567189266
0.202323145	0.687719004
0.209296357	1.764325963
0.216269569	0.368885954
0.223242781	0.460426377
0.230215993	2.553320647
0.237189205	1.557248741
0.244162417	0.592495241
0.251135629	2.426166158
0.258108841	0.587155337
0.265082052	0.596352564
0.272055264	0.040828099
0.279028476	1.011154596
0.286001688	0.676407792
0.292974900	2.337354015
0.299948112	18.409046128
0.306921324	8.780617689
0.313894536	0.815599416
0.320867748	2.373113740
0.327840960	-0.554626704
0.334814171	2.494545994
0.341787383	-1.129107705
0.348760595	3.165783654
0.355733807	3.781719032
0.362707019	5.935293015
0.369680231	24.775661369
0.376653443	11.739464182
0.383626655	-0.304713013
0.390599867	1.203760666
0.397573079	2.885475348
0.404546291	2.370993875
0.411519502	1.505807399
0.418492714	20.133406527
0.425465926	9.017891033
0.432439138	-0.510718677
0.439412350	3.145133523
0.446385562	-0.148181663
0.453358774	3.556310916
0.460331986	2.364341999
0.467305198	2.857835519
0.474278410	3.167749053
0.481251622	2.252322577
0.488224833	2.187511233
0.495198045	0.163112881
0.502171257	0.168894982
0.509144469	0.370628052
0.516117681	0.735494975
0.523090893	0.515012859
0.530064105	-0.006442272
0.537037317	0.923724235
0.544010529	1.045559132
0.550983741	0.187313995
0.557956952	0.784870916
0.564930164	0.513389836

0.571903376	0.126901111
0.578876588	0.339612943
0.585849800	0.538331249
0.592823012	0.006942076
0.599796224	1.148327725
0.606769436	0.715353602
0.613742648	0.362155934
0.620715860	0.361367736
0.627689072	0.669707526
0.634662283	0.545068484
0.641635495	1.148651415
0.648608707	6.213838616
0.655581919	3.149508091
0.66255513[illegible]	0.277665275
0.66952834	0.901379757
0.67650155	1.285836394
0.68347476[illegible]	2.080647764
0.690447979	1.759819507

END OF DATA

INPUT

LEAST SQUARES POWER SPECTRUM PROGRAM FOR UNEQUALLY SPACED DATA POINTS (VANICEK)

```
            IMPLICIT REAL*8 (A-H,O-Z)
      C     POWER SPECTRUM BY LEAST SQUARES METHOD (VANICEK)
      C     NT = NUMBER OF TIME POINTS
      C     T(I) = TIME POINTS...I = 1, NT
      C     X(I) = VALUES OF TIME SERIES AT POINTS T(I)...I = 1, NT
      C     FST = STARTING VALUE OF FREQUENCY
      C     FEND = ENDING VALUE OF FREQUENCY
      C     FNN = NN = TOTAL NUMBER OF FREQUENCY POINTS IN SPECTRUM
      C     DELF = FREQUENCY INCREMENT
      C     FREQ = CALCULATED FREQUENCY
      C     SPLSQ = CALCULATED LEAST SQUARES POWER SPECTRUM
            DIMENSION T(500), X(500)
            PI = 3.14159267D0
            READ (3,1) NT
          1 FORMAT(I10)
            DO 2 I = 1, NT
          2 READ (3,3) T(I), X(I)
          3 FORMAT(2F16.9)
            XNORM = 0
            DO 4 I = 1, NT
          4 XNORM = XNORM + X(I)**2
            READ (3,5) FST, FEND, FNN
          5 FORMAT(3D15.8)
            DELF = (FEND - FST)/FNN
            FREQ = FST
            NN = FNN
            WRITE (4,6) FST,FEND,FNN,DELF,XNORM
          6 FORMAT(5D15.8)
            DO 9 I = 1, NN
            S1 = 0
            S2 = 0
            S3 = 0
            S4 = 0
            S5 = 0
            DO 7 J = 1, NT
            ARG = 2.0D0*PI*FREQ*T(J)
            CC = DCOS(ARG)
            SS = DSIN(ARG)
            S1 = S1 + CC**2
            S2 = S2 + CC*SS
            S3 = S3 + SS**2
            S4 = S4 + CC*X(J)
          7 S5 = S5 + SS*X(J)
            DET = S1*S3 - S2**2
            A = (S3*S4 - S2*S5)/DET
            B = (-S2*S4 + S1*S5)/DET
            SPLSQ = 100.0D0*(S4*A + S5*B)/XNORM
            WRITE (4,8) FREQ, SPLSQ
          8 FORMAT(2F20.8)
          9 FREQ = FREQ + DELF
        999 STOP
            END
END OF DATA
```

OUTPUT FROM THE LEAST SQUARES POWER SPECTRUM PROGRAM (VANICEK)

0.495049510-02 0.504950500 00 0.101000000 03 0.495049510-02 0.163800570 03

0.00495050	0.04997305
0.00990099	0.03051585
0.01485149	0.44400311
0.01980198	1.31714298
0.02475248	2.13955975
0.02970297	1.63098323
0.03465347	0.10135427
0.03960396	1.80859705
0.04455446	0.23162469
0.04950495	1.28491303
0.05445545	2.11973626
0.05940594	0.50440354
0.06435644	0.05837910
0.06930693	0.34417895
0.07425743	1.17334088
0.07920792	2.35620947
0.08415842	9.96929637
0.08910891	16.05026410
0.09405941	8.96340748
0.09900990	2.30814413
0.10396040	1.08940226
0.10891089	0.65786654
0.11386139	0.29421926
0.11881188	0.59331941
0.12376238	0.63078281
0.12871287	1.03838314
0.13366337	6.46224938
0.13861386	15.04579567
0.14356436	10.32980558
0.14851485	2.86773144
0.15346535	1.05342949
0.15841584	0.23233667
0.16336634	0.01022102
0.16831683	0.33904552
0.17326733	2.08093053
0.17821782	2.18344881
0.18316832	0.46248843
0.18811881	2.94330418
0.19306931	0.77934555
0.19801980	1.23183637
0.20297030	2.54779222
0.20792079	0.33687070
0.21287129	0.10467851
0.21782178	0.03330150
0.22277228	0.05187237
0.22772277	0.07039458
0.23267327	0.15911101
0.23762376	0.20226252
0.24257426	0.10826315
0.24752475	0.87366325
0.25247525	0.39933267
0.25742575	0.06363446
0.26237624	0.02951225
0.26732674	0.16092237
0.27227723	0.27536029
0.27722773	0.39282580
0.28217822	1.49959306
0.28712872	4.08516080

0.29207921	0.67260815
0.29702971	1.43587477
0.30198020	2.38684932
0.30693070	0.96704030
0.31188119	1.10876838
0.31683169	4.33145590
0.32178218	2.20160063
0.32673268	0.49719433
0.33168317	1.53442408
0.33663367	0.40928499
0.34158416	2.72419628
0.34653466	17.20938932
0.35148515	21.17850843
0.35643565	4.89160669
0.36138614	0.93567389
0.36633664	14.97567969
0.37128713	24.18459074
0.37623763	6.71230568
0.38118812	5.60321791
0.38613862	3.69699104
0.39108911	1.11461963
0.39603961	0.23974462
0.40099010	4.06886947
0.40594060	2.79730328
0.41089109	0.14278981
0.41584159	6.74823350
0.42079208	22.06619043
0.42574258	8.58463327
0.43069307	0.82476587
0.43564357	5.02176888
0.44059406	0.29688520
0.44554456	0.51736868
0.45049505	1.37287334
0.45544555	3.05225336
0.46039604	1.24985137
0.46534654	0.86564622
0.47029703	0.09692107
0.47524753	0.84054058
0.48019802	0.12096984
0.48514852	1.88183370
0.49009901	0.71376860
0.49504951	0.20393343
0.50000000	0.73493289

END OF DATA

INPUT FOR LEAST SQUARES POWER SPECTRUM (VANICEK) AND DISCRETE FOURIER TRANSFORM POWER SPECTRUM WHERE THE SPACING IS 1 SECOND EXCEPT FOR HOLES IN THE DATA

69

0.0	2.000000000
1.000000000	1.796187857
2.000000000	-1.433925916
3.000000000	2.702315281
4.000000000	-1.747774162
5.000000000	-0.221231906
6.000000000	1.953107742
7.000000000	-1.591861893

8.000000000	0.577521509
9.000000000	-0.510762121
18.000000000	-3.198131721
19.000000000	-2.030147064
20.000000000	1.533530872
21.000000000	-2.192373378
22.000000000	2.210496898
23.000000000	0.657099021
24.000000000	-0.804905312
25.000000000	0.999999122
26.000000000	-0.150379846
27.000000000	0.443641050
28.000000000	0.832475263
29.000000000	0.792849878
30.000000000	-1.642040024
31.000000000	-0.603910481
32.000000000	-1.843846922
33.000000000	-1.669335104
34.000000000	1.740466054
35.000000000	0.357962310
36.000000000	2.746966817
37.000000000	1.074674420
38.000000000	-0.133607588
39.000000000	-0.508835466
48.000000000	0.872762832
49.000000000	2.008250345
50.000000000	-0.000001707
51.000000000	0.442164700
52.000000000	-0.629492118
53.000000000	-2.458875930
54.000000000	-1.921432041
55.000000000	0.778771764
56.000000000	-1.132120234
57.000000000	2.839844455
58.000000000	2.421442205
59.000000000	-1.579394785
60.000000000	2.680879873
61.000000000	-2.603906630
62.000000000	-1.335986042
63.000000000	1.678993970
64.000000000	-2.427833388
65.000000000	1.260073789
66.000000000	0.003151232
67.000000000	-1.690710371
68.000000000	0.477652776
69.000000000	0.707829263
78.000000000	-1.075911857
79.000000000	2.876358242
80.000000000	1.084501863
81.000000000	-0.217074530
82.000000000	0.595862455
83.000000000	-1.329628830
84.000000000	0.349689219
85.000000000	-0.642044237
86.000000000	0.559295576
87.000000000	-0.720496367
88.000000000	-1.215329622
89.000000000	-0.059234879
98.000000000	0.441133865
99.000000000	-1.696910746
100.000000000	2.000006170

0.49504951D-02 0.50495050D 00 0.10100000D 03
END OF DATA

DISCRETE FOURIER TRANSFORM POWER SPECTRUM PROGRAM FOR UNEQUALLY SPACED DATA

```
           IMPLICIT REAL*8 (A-H,O-Z)
C          POWER SPECTRUM BY DISCRETE FOURIER TRANSFORM METHOD
C          NT = NUMBER OF TIME POINTS
C          T(I) = TIME POINTS...I = 1, NT
C          X(I) = VALUES OF TIME SERIES AT POINTS T(I)...I = 1, NT
C          FST = STARTING VALUE OF FREQUENCY
C          FEND = ENDING VALUE OF FREQUENCY
C          FNW = NW = TOTAL NUMBER OF FREQUENCY POINTS IN SPECTRUM
C          DELF = FREQUENCY INCREMENT
C          FREQ = CALCULATED FREQUENCY
C          SPFOUR = CALCULATED POWER SPECTRUM
           DIMENSION T(500), X(500)
           PI = 3.14159267D0
           READ (3,1) NT
         1 FORMAT(I10)
           FNT = NT
           DO 2 I = 1, NT
         2 READ (3,3) T(I), X(I)
         3 FORMAT(2F16.9)
           XNORM = 0
           DO 4 I = 1, NT
         4 XNORM = XNORM + X(I)**2
           READ (3,5) FST, FEND, FNW
         5 FORMAT(3D15.8)
           DELF = (FEND - FST)/FNW
           FREQ = FST
           NW = FNW
           WRITE (4,6) FST,FEND,FNW,DELF,XNORM
         6 FORMAT(5D15.8)
           DO 9 I = 1, NW
           S1 = 0
           S2 = 0
           DO 7 J = 1, NT
           ARG = 2.0D0*PI*FREQ*T(J)
           CC = DCOS(ARG)
           SS = DSIN(ARG)
           S1 = S1 + CC*X(J)
         7 S2 = S2 + SS*X(J)
           SPFOUR = 200.0D0*(S1**2 + S2**2)/(FNT*XNORM)
           WRITE (4,8) FREQ, SPFOUR
         8 FORMAT(2F20.8)
         9 FREQ = FREQ + DELF
      9999 STOP
           END
END OF DATA
```

OUTPUT FROM DISCRETE FOURIER TRANSFORM POWER SPECTRUM PROGRAM

0.49504951D-02 0.50495050D 00 0.10100000D 03 0.49504951D-02 0.16380357D 03

0.00495050	0.05091136
0.00990099	0.03127586
0.01485149	0.45515950
0.01980198	0.97002374
0.02475248	2.10084275
0.02970297	1.80270439
0,03465347	0.09096589
0.03960396	1.80361200
0.04455446	0.23318094
0.04950495	1.31720030
0.05445545	2.13164406
0.05940594	0.50340859
0.06435644	0.05860604
0.06930693	0.33322186
0.07425743	1.17118928
0.07920792	2.44393129
0.08415842	9.56591945
0.08910891	15.95573054
0.09405941	8.98452125
0.09900990	2.41890379
0.10396040	1.09462632
0.10891089	0.65890843
0.11386139	0.29891170
0.11881188	0.58176274
0.12376238	0.63207940
0.12871287	1.03826824
0.13366337	6.44894353
0.13861386	14.84871381
0.14356436	10.50766744
0.14851485	2.90558788
0.15346535	1.06355585
0.15841584	0.23285471
0.16336634	0.01005980
0.16831683	0.33561158
0.17326733	2.09358050
0.17821782	2.16817546
0.18316832	0.46207088
0.18811881	2.94337991
0.19306931	0.77588650
0.19801980	1.22765234
0.20297030	2.51867874
0.20792079	0.34206607
0.21287129	0.10678977
0.21782178	0.03252864
0.22277228	0.05195729
0.22772277	0.06954987
0.23267327	0.15416479
0.23762376	0.20465991
0.24257426	0.10879932
0.24752475	0.87015305
0.25247525	0.39991951
0.25742575	0.06354450
0.26237624	0.02927865
0.26732674	0.15732281
0.27227723	0.27181631
0.27722773	0.38755105
0.28217822	1.51631602

0.28712872	4.12235802
0.29207921	0.67693104
0.29702971	1.44126665
0.30198020	2.44079023
0.30693070	0.97104549
0.31188119	1.10936584
0.31683169	4.29123459
0.32178218	2.19138203
0.32673268	0.49138329
0.33168317	1.50443386
0.33663367	0.40031486
0.34158416	2.75129005
0.34653466	16.93447854
0.35148515	20.03493041
0.35643565	4.88869606
0.36138614	0.92387466
0.36633664	14.85925091
0.37128713	24.24672833
0.37623763	6.70302788
0.38118812	5.53453300
0.38613862	3.63702013
0.39108911	1.12160418
0.39603961	0.24057666
0.40099010	4.12341794
0.40594060	2.84591403
0.41089109	0.14599285
0.41584159	6.43894989
0.42079208	21.11847138
0.42574258	8.86204152
0.43069307	0.80457456
0.43564357	5.01262218
0.44059406	0.29754738
0.44554456	0.51879288
0.45049505	1.51168026
0.45544555	2.93800988
0.46039604	1.27195229
0.46534654	0.84952559
0.47029703	0.08248712
0.47524753	0.80673755
0.48019802	0.11528606
0.48514852	2.17736784
0.49009901	0.73315529
0.49504951	0.19083616
0.50000000	0.49667494

END OF DATA

COMPARISON PROGRAM FOR LOOKING AT DIFFERENT POWER SPECTRUM ESTIMATES - MODIFIED BLACKMAN-TUKEY, CORRELATION FUNCTION, FOURIER TRANSFORM, AND LEAST SQUARES (ONE BY VANICEK AND THE OTHER BY FERRAZ-MELLO)

```
C      COMPARISON PROGRAM FOR LOOKING AT VARIOUS POWER SPECTRUM
C      COMPUTATIONAL METHODS
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION T(200), X(200),TM(200),XM(200)
       DOUBLE PRECISION N(200),S(200)
       PI = 3.14159267D0
C      READ IN TOTAL NUMBER OF POINTS NT AND TIME SERIES (T(I),X(I))
       READ (3,1) NT
     1 FORMAT(I10)
       FNT = NT
       DO 2 I = 1, NT
     2 READ (3,3) T(I), X(I)
     3 FORMAT(2F16.9)
C      READ IN AVERAGE SPACING BETWEEN DATA POINTS DELT, STARTING
C      FREQUENCY FST, ENDING FREQUENCY FEND, TOTAL NUMBER OF FREQUENCY
C      POINTS FNN, FRACTIONS OF THE MAXIMUM LAG NUMBER IN THE DATA
C      (MAXLAG) FRACT1, FRACT2, FRACT3, AND FRACT4 FOR COMPARISON OF
C      THE MODIFIED BLACKMAN-TUKEY SPECTRUM AND CORRELATION FUNCTION
C      SPECTRUM WITH THE FOURIER, SPFOUR, AND TWO LEAST SQUARES ESTI-
C      MATES SMELLO (FERRAZ-MELLO) AND SPLSQ (VANICEK).
       READ (3,4) DELT,FST,FEND,FNN
       READ (3,5) FRACT1,FRACT2,FRACT3,FRACT4
     4 FORMAT(4D15.8)
     5 FORMAT(4F10.5)
       NN = FNN
       NM1 = NT - 1
C      REFERENCING THE TIME POINTS TO THE ORIGIN.
       TM(1) = 0.0D0
       DO 6 I = 1, NM1
       DELA = T(I+1) -T(I)
     6 TM(I+1) = TM(I) + DELA
C      ROUNDING TIME POINTS TM(I) TO THE NEAREST MULTIPLE OF THE
C      AVERAGE TIME INTERVAL DELT
       DO  7 I = 1,NT
       MTM = (TM(I)/DELT) + 0.5D0
     7 TM(I) = MTM
C      OBTAINING THE NORM OF THE MEASUREMENT VECTOR X(I)
       XNORM = 0.0D0
       DO  8 I = 1,NT
     8 XNORM = XNORM + X(I)**2
       DO    9 I = 1,200
       N(I) = 0.0D0
     9 S(I) = 0.0D0
C      CALCULATING SUMS OF LAGGED PRODUCTS S(I)
       MAXLAG = TM(NT) - TM(1) + 1.0D0
       DO 11 I = 1, NT
       DO 10 J = I, NT
       ILAG = TM(J) - TM(I) + 1.0D0
       N(ILAG) = N(ILAG) + 1.0D0
    10 S(ILAG) = S(ILAG) + X(I)*X(J)
    11 CONTINUE
```

```
      DO 12 I = 1, MAXLAG
      IF(N(I).EQ.0.0D0) GO TO  12
      R(I) = S(I)/N(I)
   12 CONTINUE
      DELF = (FEND - FST)/FNW
      FST = DELF
      FREQ = FST
      DO 20 IFREQ = 1, NW
      S1 = 0
      S2 = 0
      S3 = 0
      S4 = 0
      S5 = 0
      S6 = 0
      S7 = 0
      S8 = 0
      SM4 = 0
      SM5 = 0
      XMEAN = 0
      DO 21 J = 1,NT
   21 XMEAN = XMEAN + X(J)
      XMEAN = XMEAN/DFLOAT(NT)
      DO 13 J = 1,NT
      FJ = J
      ARG = 2.0D0*PI*FREQ*T(J)
      CC = DCOS(ARG)
      SS = DSIN(ARG)
      S1 = S1 + CC**2
      S2 = S2 + CC*SS
      S3 = S3 + SS**2
      S4 = S4 + CC*X(J)
      S5 = S5 + SS*X(J)
      SM4 = SM4 + CC*(X(J)-XMEAN)
      SM5 = SM5 + SS*(X(J)-XMEAN)
      S6 = S6 + CC
      S7 = S7 + SS
   13 S8 = S8 + X(J)
      A0M2 = DFLOAT(NT)
      A1M2 = S1 - (S6**2/A0M2)
      FACT2 = (S6*S7)/A0M2
      FACT1 = (S2 - FACT2)/A1M2
      A2M2 = S3 - (S7**2/A0M2) - FACT1
      A0 = DSQRT(1.0D0/A0M2)
      A1 = DSQRT(1.0D0/A1M2)
      A2 = DSQRT(1.0D0/A2M2)
   14 C1 = A1*SM4
      FACT2 = S2 - ((S6*S7)/A0M2)
      FACT1 = A1*A2*C1*FACT2
      C2 = A2*SM5 - FACT1
      FACT = C1**2 + C2**2
C     FERRAZ-MELLO SPECTRUM ESTIMATE SMELLO
      SMELLO = 100.0D0*FACT/XNORM
      DET = S1*S3 - S2**2
      A = (S3*S4 - S2*S5)/DET
      B = (-S2*S4 + S1*S5)/DET
C     VANICEK'S LEAST SQUARE ESTIMATE SPLSQ
      SPLSQ = (100.0D0*(S4*A + S5*B))/XNORM
C     DISCRETE FOURIER TRANSFORM POWER SPECTRUM ESTIMATE SPFOUR
      SPFOUR = 200.0D0*((S4**2 + S5**2)/(FNT*XNORM))
      SUM1 = 0.0D0
      SUM2 = 0.0D0
      SUM3 = 0.0D0
      SUM4 = 0.0D0
C     SETTING NUMBER OF LAG PRODUCTS EQUAL TO FRACTIONS OF MAXIMUM LAG
```

```
            NLAG1 = FRACT1*MAXLAG
            NLAG2 = FRACT2*MAXLAG
            NLAG3 = FRACT3*MAXLAG
            NLAG4 = FRACT4*MAXLAG
            DO  18 I = 1, NLAG4
            FI = I - 1
            ARG = 2.0D0*PI*FREQ*FI*DELT
            IF(I.GT.NLAG1) GO TO 15
            SUM1 = SUM1 + N(I)*DCOS(ARG)
         15 IF(I.GT.NLAG2) GO TO 16
            SUM2 = SUM2 + S(I)*DCOS(ARG)
         16 IF(I.GT.NLAG3) GO TO  17
            SUM3 = SUM3 + S(I)*DCOS(ARG)
         17 SUM4 = SUM4 + S(I)*DCOS(ARG)
         18 CONTINUE
C           MODIFIED BLACKMAN-TUKEY SPECTRUM WITH LAGS = FRACT1*MAXLAG
            SPCOR1 = 200.0D0*(2.0D0*SUM1 - XNORM/FNT)/XNORM
C           CORRELATION METHOD SPECTRUM WITH LAGS = FRACT2*MAXLAG
            SPCOR2 = 200.0D0*(2.0D0*SUM2 - XNORM)/(FNT*XNORM)
C           CORRELATION METHOD SPECTRUM WITH LAGS = FRACT3*MAXLAG
            SPCOR3 = 200.0D0*(2.0D0*SUM3 - XNORM)/(FNT*XNORM)
C           CORRELATION METHOD SPECTRUM WITH LAGS = FRACT4*MAXLAG
            SPCOR4 = 200.0D0*(2.0D0*SUM4 - XNORM)/(FNT*XNORM)
            WRITE (4,19) FREQ,SPCOR1,SPCOR2,SPCOR3,SPCOR4,SMELLO,SPFOUR,
           1SPLSQ
         19 FORMAT(F7.4,8F9.4)
         20 FREQ = FREQ + DELF
       9999 STOP
            END
END OF DATA
```

COMPARISON OF NORMALIZED POWER SPECTRUMS (% POWER) FOR THE MODIFIED BLACKMAN-TUKEY METHOD, THE CORRELATION FUNCTION METHOD, FOURIER TRANSFORM AND LEAST SQUARES TECHNIQUES

NOTE: THE SIMULATED DATA IS A TIME SERIES SAMPLED AT EQUAL INTERVALS OF TIME (0.72 SECONDS APART) OVER A TIME SPAN OF 100 SECONDS. THE SERIES CAN BE REPRESENTED BY THE FOLLOWING EXPRESSION.

X(I) = SIN(ARG1*I) + COS(ARG2*I) + SIN(ARG3*I) + COS(ARG4*I) + SIN(ARG5*I)

```
WHERE     ARG1 = 2.0*PI*0.37
          ARG2 = 2.0*PI*0.14
          ARG3 = 2.0*PI*0.42
          ARG4 = 2.0*PI*0.65
          ARG5 = 2.0*PI*1.09
```

AND WHERE I RANGES FROM 0 TO 99.36 SECONDS.   PI = 3.14159267

THE FIRST COLUMN IS FREQUENCY IN HERTZ

THE SECOND COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY THE MODIFIED BLACKMAN-TUKEY POWER SPECTRUM METHOD USING MEAN LAGGED PRODUCTS AND SUMMING UP TO 50% OF THE MAXIMUM LAG IN THE DATA. THE FOURIER TRANSFORM OF THIS MODIFIED AUTOCORRELATION FUNCTION IS THEN THE POWER SPECTRUM.

THE THIRD COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 60% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE FOURTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 70% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE FIFTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 100% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE SIXTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY THE METHOD OF S. FERRAZ-MELLO CALLED A 'DATE COMPENSATE DISCRETE FOURIER TRANSFORM' WHICH WILL BE PUBLISHED SOON.

THE SEVENTH COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY THE FOURIER TRANSFORM METHOD WITH THE OBSERVED TIME POINTS.

THE EIGHTH COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY AN OPTIMUM LEAST SQUARES PROCEDURE (THE VANICEK METHOD WITHOUT 'CONSTITUENTS', THAT IS, AVERAGES, LINEAR TRENDS, ETC.).

0.0050	-0.0487	0.0208	0.2059	0.0320	0.0322	0.0320	0.0320
0.0100	0.8346	0.0224	0.1261	0.0006	0.0006	0.0006	0.0006
0.0150	0.1408	0.0080	-0.3022	0.0326	0.0327	0.0326	0.0326
0.0200	-0.8366	0.0210	0.2770	0.0006	0.0006	0.0006	0.0006
0.0250	-0.2427	0.0191	0.0384	0.0340	0.0340	0.0340	0.0340
0.0300	0.8309	0.0085	-0.2733	0.0006	0.0006	0.0006	0.0006
0.0350	0.3433	0.0322	0.3385	0.0361	0.0362	0.0361	0.0361
0.0400	-0.8314	0.0183	-0.0566	0.0006	0.0006	0.0006	0.0006
0.0450	-0.4604	-0.0066	-0.2236	0.0394	0.0395	0.0394	0.0394
0.0500	0.8231	0.0499	0.3894	0.0006	0.0006	0.0006	0.0006
0.0550	0.5826	0.0419	-0.1592	0.0443	0.0444	0.0443	0.0443
0.0600	-0.8220	-0.0400	-0.1496	0.0006	0.0006	0.0006	0.0006
0.0649	-0.7334	0.0477	0.4292	0.0516	0.0517	0.0516	0.0516
0.0699	0.8100	0.1113	-0.2710	0.0006	0.0006	0.0006	0.0006
0.0749	0.9038	-0.0650	-0.0432	0.0630	0.0630	0.0630	0.0630
0.0799	-0.8068	-0.0156	0.4577	0.0006	0.0006	0.0006	0.0006
0.0849	-1.1324	0.2250	-0.3965	0.0813	0.0814	0.0813	0.0813
0.0899	0.7885	-0.0129	0.1160	0.0006	0.0006	0.0006	0.0006
0.0949	1.4262	-0.1749	0.4753	0.1137	0.1139	0.1137	0.1137
0.0999	-0.7803	0.3382	-0.5469	0.0006	0.0006	0.0006	0.0006
0.1049	-1.8816	0.2446	0.3920	0.1783	0.1785	0.1783	0.1783
0.1099	0.7473	-0.4314	0.4851	0.0005	0.0005	0.0005	0.0005
0.1149	2.6399	0.3595	-0.7426	0.3358	0.3361	0.3358	0.3358
0.1199	-0.7163	1.0714	1.0965	0.0004	0.0004	0.0004	0.0004
0.1249	-4.3800	-0.7236	0.5036	0.9068	0.9078	0.9068	0.9068
0.1299	0.5951	0.5070	-0.6772	0.0007	0.0007	0.0007	0.0007
0.1349	12.7156	9.6717	9.4250	7.8674	7.8754	7.8674	7.8674
0.1399	19.3298	16.8103	18.3007	20.0523	20.0523	20.0523	20.0523
0.1449	12.9058	10.9055	9.8447	8.6640	8.6721	8.6640	8.6640
0.1499	1.0793	0.4222	-0.1924	0.0077	0.0077	0.0077	0.0077
0.1549	-3.9980	-1.2915	0.3026	0.9518	0.9526	0.9518	0.9518
0.1599	-0.9253	1.7009	0.9845	0.0043	0.0043	0.0043	0.0043
0.1649	2.2201	0.5539	-0.4133	0.3504	0.3506	0.3504	0.3504
0.1699	0.8680	-1.1662	0.2821	0.0036	0.0036	0.0036	0.0036
0.1749	-1.4377	0.5170	0.3055	0.1844	0.1845	0.1844	0.1844
0.1799	-0.8497	0.9332	-0.2347	0.0035	0.0035	0.0035	0.0035
0.1849	0.9552	-0.8006	0.2273	0.1153	0.1153	0.1153	0.1153
0.1898	0.8106	-0.2423	0.0817	0.0036	0.0036	0.0036	0.0036
0.1948	-0.6502	1.0067	-0.0975	0.0793	0.0793	0.0793	0.0793
0.1998	-0.8167	-0.2420	0.1548	0.0039	0.0039	0.0039	0.0039
0.2048	0.3687	-0.7382	-0.0098	0.0575	0.0575	0.0575	0.0575

0.2098	0.7706	0.7432	0.0065	0.0044	0.0044	0.0044	0.0044
0.2148	-0.1854	0.3676	0.0675	0.0424	0.0424	0.0424	0.0424
0.2198	-0.7977	-0.8980	-0.0339	0.0052	0.0052	0.0052	0.0052
0.2248	-0.0665	0.2105	0.0843	0.0308	0.0308	0.0308	0.0308
0.2298	0.7330	0.8436	-0.0356	0.0063	0.0063	0.0063	0.0063
0.2348	0.2165	-0.6794	-0.0062	0.0208	0.0208	0.0208	0.0208
0.2398	-0.7942	-0.4387	0.1364	0.0080	0.0080	0.0080	0.0080
0.2448	-0.5368	1.0208	-0.1594	0.0115	0.0115	0.0115	0.0115
0.2498	0.6925	-0.1278	0.0798	0.0107	0.0107	0.0107	0.0107
0.2548	0.7225	-1.0120	0.1563	0.0034	0.0034	0.0034	0.0034
0.2598	-0.8309	0.8105	-0.3164	0.0155	0.0155	0.0155	0.0155
0.2648	-1.3168	0.6489	0.2698	0.0014	0.0014	0.0014	0.0014
0.2698	0.6462	-1.3558	0.1206	0.0256	0.0256	0.0256	0.0256
0.2748	1.7910	0.2328	-0.5287	0.0333	0.0333	0.0333	0.0333
0.2798	-1.0640	1.6968	0.8174	0.0537	0.0537	0.0537	0.0537
0.2848	-3.9666	-1.4996	-0.0902	0.3204	0.3205	0.3204	0.3204
0.2898	0.7447	-0.0038	-0.2413	0.2083	0.2083	0.2083	0.2083
0.2948	12.3405	10.9463	9.8420	8.4607	8.4623	8.4607	8.4607
0.2998	19.1635	16.7517	17.7545	19.5500	19.5500	19.5500	19.5500
0.3048	12.9070	9.1288	9.3594	8.0171	8.0184	8.0171	8.0171
0.3098	0.6508	0.8691	-0.0843	0.1022	0.1022	0.1022	0.1022
0.3147	-4.1561	0.4884	0.8993	1.9533	1.9536	1.9533	1.9533
0.3197	0.2957	1.2732	1.7162	0.0204	0.0204	0.0204	0.0204
0.3247	3.7116	0.2827	-0.1740	1.2788	1.2790	1.2788	1.2788
0.3297	0.5835	0.7924	0.6485	0.0056	0.0056	0.0056	0.0056
0.3347	-2.4933	0.9249	1.2131	1.1835	1.1836	1.1835	1.1835
0.3397	0.3299	-0.2097	0.0213	0.0010	0.0010	0.0010	0.0010
0.3447	3.6142	1.0395	0.5894	1.4055	1.4056	1.4055	1.4055
0.3497	0.4671	1.9780	1.7664	0.0001	0.0001	0.0001	0.0001
0.3547	-3.8524	-0.6026	0.2786	2.3400	2.3403	2.3400	2.3400
0.3597	1.4061	1.6469	1.2626	0.0083	0.0083	0.0083	0.0083
0.3647	14.4623	12.1511	11.2465	10.8667	10.8677	10.8667	10.8667
0.3697	20.5071	16.8273	18.1003	20.0739	20.0739	20.0739	20.0739
0.3747	11.5616	8.3966	8.4386	5.8282	5.8287	5.8282	5.8282
0.3797	-2.0926	-0.3387	-1.8999	0.0220	0.0220	0.0220	0.0220
0.3847	-5.7771	-0.8580	0.4144	0.0794	0.0794	0.0794	0.0794
0.3897	0.4573	0.5166	1.1997	0.0038	0.0038	0.0038	0.0038
0.3947	4.1225	0.2352	-1.7205	0.0694	0.0694	0.0694	0.0694
0.3997	-0.7964	0.2869	1.1693	0.0002	0.0002	0.0002	0.0002
0.4047	-5.6389	-0.4845	0.8274	0.6122	0.6122	0.6122	0.6122
0.4097	-0.1128	0.4377	-1.5103	0.0062	0.0062	0.0062	0.0062
0.4147	12.5571	8.5994	8.8409	6.8052	6.8058	6.8052	6.8052
0.4197	19.0903	16.4427	18.1446	19.7534	19.7534	19.7534	19.7534
0.4247	12.8758	11.6386	10.1039	9.4863	9.4871	9.4863	9.4863
0.4297	1.5238	0.8012	0.3750	0.0435	0.0435	0.0435	0.0435
0.4347	-3.6644	-1.5936	0.1155	0.9199	0.9200	0.9199	0.9199
0.4396	-1.1181	1.6109	0.7618	0.0154	0.0154	0.0154	0.0154
0.4446	1.9158	0.7769	-0.1149	0.2931	0.2931	0.2931	0.2931
0.4496	0.8525	-1.2184	0.1063	0.0088	0.0088	0.0088	0.0088
0.4546	-1.3312	0.2701	0.1584	0.1283	0.1283	0.1283	0.1283
0.4596	-0.8403	1.0051	-0.0090	0.0060	0.0060	0.0060	0.0060
0.4646	0.8893	-0.6297	0.0459	0.0649	0.0649	0.0649	0.0649
0.4696	0.6932	-0.4445	0.0071	0.0045	0.0045	0.0045	0.0045
0.4746	-0.7451	0.8415	0.0698	0.0360	0.0360	0.0360	0.0360
0.4796	-0.7126	0.0042	-0.0201	0.0035	0.0035	0.0035	0.0035
0.4846	0.5317	-0.7166	-0.0249	0.0217	0.0217	0.0217	0.0217
0.4896	0.6061	0.4000	0.1158	0.0029	0.0029	0.0029	0.0029
0.4946	-0.4911	0.4726	-0.0821	0.0148	0.0148	0.0148	0.0148
0.4996	-0.6277	-0.5943	-0.0069	0.0024	0.0024	0.0024	0.0024
0.5046	0.3644	-0.1107	0.1345	0.0120	0.0120	0.0120	0.0120
0.5096	0.5443	0.6340	-0.1356	0.0020	0.0020	0.0020	0.0020
0.5146	-0.3627	-0.2101	0.0378	0.0118	0.0118	0.0119	0.0119

0.5196	-0.5627	-0.4774	0.1295	0.0018	0.0018	0.0018	0.0018
0.5246	0.2848	0.4707	-0.1771	0.0135	0.0135	0.0135	0.0135
0.5296	0.4957	0.2332	0.0985	0.0015	0.0015	0.0015	0.0015
0.5346	-0.3050	-0.5610	0.1026	0.0168	0.0168	0.0168	0.0168
0.5396	-0.5096	0.0909	-0.2032	0.0013	0.0013	0.0013	0.0013
0.5446	0.2654	0.5324	0.1691	0.0217	0.0217	0.0217	0.0217
0.5496	0.4568	-0.3446	0.0547	0.0011	0.0011	0.0011	0.0011
0.5546	-0.3052	-0.3170	-0.2095	0.0286	0.0286	0.0286	0.0286
0.5596	-0.4660	0.5566	0.2459	0.0010	0.0010	0.0010	0.0010
0.5645	0.3077	0.0671	-0.0143	0.0379	0.0379	0.0379	0.0379
0.5695	0.4285	-0.5589	-0.1900	0.0008	0.0008	0.0008	0.0008
0.5745	-0.3763	0.2931	0.3263	0.0508	0.0508	0.0508	0.0508
0.5795	-0.4326	0.4931	-0.1061	0.0007	0.0007	0.0007	0.0007
0.5845	0.4434	-0.5055	-0.1339	0.0694	0.0694	0.0694	0.0694
0.5895	0.4166	-0.1541	0.4095	0.0006	0.0006	0.0006	0.0006
0.5945	-0.5729	0.7491	-0.2254	0.0974	0.0974	0.0974	0.0974
0.5995	-0.4155	-0.1274	-0.0178	0.0006	0.0006	0.0006	0.0006
0.6045	0.7799	-0.6101	0.4979	0.1433	0.1433	0.1433	0.1433
0.6095	0.4417	0.7142	-0.3828	0.0008	0.0008	0.0008	0.0008
0.6145	-1.0939	0.5406	0.2258	0.2279	0.2279	0.2279	0.2279
0.6195	-0.4391	-0.8932	0.6041	0.0016	0.0016	0.0016	0.0016
0.6245	1.7891	0.3657	-0.5995	0.4172	0.4172	0.4172	0.4172
0.6295	0.6064	1.6498	0.9119	0.0052	0.0052	0.0052	0.0052
0.6345	-3.2161	-0.6331	0.8005	1.0347	1.0347	1.0347	1.0347
0.6395	-0.7277	0.0180	-0.6627	0.0311	0.0311	0.0311	0.0311
0.6445	11.0200	9.6274	8.6222	7.2163	7.2164	7.2163	7.2163
0.6495	20.0476	16.6566	18.1818	19.7318	19.7318	19.7318	19.7318
0.6545	14.3981	10.6470	10.2486	9.1576	9.1576	9.1576	9.1576
0.6595	0.4394	0.8219	-0.2134	0.0899	0.0899	0.0899	0.0899
0.6645	-5.4631	-1.0182	0.2588	0.6761	0.6761	0.6761	0.6761
0.6695	-0.3325	1.0709	0.8122	0.0317	0.0317	0.0317	0.0317
0.6745	3.5997	0.2858	-0.5650	0.1609	0.1609	0.1609	0.1609
0.6795	-0.0143	-0.6284	0.4118	0.0205	0.0205	0.0205	0.0205
0.6845	-3.2505	0.3851	0.0588	0.0470	0.0470	0.0470	0.0470
0.6894	-0.1167	0.2689	-0.2952	0.0172	0.0172	0.0172	0.0172
0.6944	2.9165	-0.5013	0.4430	0.0222	0.0222	0.0222	0.0219

D OF DATA

```
C     DATA GENERATOR PROGRAM
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(500),T(500)
      DIMENSION NTIM(500),RN1(300)
      DIMENSION TM(500),XM(500),RN(500)
      DIMENSION DEL(500)
C     SETTING UP SIMULATED DATA (T(I),X(I)), I = 1, NT  WHERE
C     T(I) CONTAINS RANDOM NOISE RN(I), ZERO MEAN AND STANDARD
C     DEVIATION OF 0.20 FROM A GAUSSIAN DISTRIBUTION
      NT = 139
      PI = 3.14159267D0
      ARG1 = 2.0D0*PI*0.37D0
      ARG2 = 2.0D0*PI*0.14D0
      ARG3 = 2.0D0*PI*0.42D0
      ARG4 = 2.0D0*PI*0.65D0
      ARG5 = 2.0D0*PI*1.09D0
      DO 1 I = 1,NT
      T(I) = 0.72D0*(I-1)
      RN(I) = BARN1(-1,0,12787,0.0D0,0.20D0)
      T(I) = T(I) + RN(I)
      X(I) = DSIN(ARG1*T(I)) + DCOS(ARG2*T(I)) + DSIN(ARG3*T(I)) +
     1DCOS(ARG4*T(I))+DSIN(ARG5*T(I))
      STDEV = 0.20D0*X(I)
```

```
      RN1(I) = BARN1(-1,0,13787,0.0D0,STDEV)
C     ADDING GAUSSIAN NOISE WITH ZERO MEAN AND SIGMA EQUAL TO 20% OF
C     THE AMPLITUDE OF THE FUNCTION .
      X(I) = X(I) + RN1(I)
    1 CONTINUE
      DO 2 I = 1,NT
    2 NTIM(I) = 1
C     PUTTING 'HOLES' IN THE DATA
      DO 834 I = 1,8
      NTIM(I+10) = 0
      NTIM(I+40) = 0
      NTIM(I+70) = 0
  834 NTIM(I+90) = 0
      L = 0
      DO 3 I = 1,NT
      IF(NTIM(I).EQ.1) L = L + 1
      IF(NTIM(I).EQ.1) TM(L) = T(I)
    3 IF(NTIM(I).EQ.1) XM(L) = X(I)
      NT = L
      DO 5 I = 1, NT
    5 WRITE (3,6) TM(I), XM(I)
    6 FORMAT(2F16.9)
      STOP
      END
***>
       REAL FUNCTION BARN1*8(I,IKEY,IFRN,AMEAN,SD)
      IMPLICIT REAL*8(A-H,O-Z,$)
C     SD....THE DESIRED STANDARD DEV
C     AMEAN...THE DESIRED MEAN.
C     H...THE POP SIZE.
      DATA IHERE/12787/
      DATA H/36.0D0/
      IF (IKEY) 5,4,4
    4 IHERE = IFRN
    5 IF (I) 6,7,7
    6 CALL GAUSS (IHERE,SD,AMEAN,VAL,H)
      IFRN = IHERE
      GO TO 8
    7 CALL RANDU (IHERE,IFRN,VAL)
      IHERE = IFRN
    8 BARN1 = VAL
      RETURN
       END
      SUBROUTINE GAUSS(IX,S,AM,V,H)
      IMPLICIT REAL*8 (A-H,O-Z,$)
      K = H
      A = 0.0D0
      DO 50 I = 1,K
      CALL RANDU(IX,IY,Y)
      IX = IY
   50 A = A + Y
      HO = H/12.0D0
      H2 = H/2.0D0
      V = (S*(A-H2))/DSQRT(HO) + AM
      RETURN
      END
      SUBROUTINE RANDU(IX,IY,YFL)
      IMPLICIT REAL*8(A-H,O-Z,$)
      DATA JJJ5/1027/
      IY = IX*JJJ5
      IF(IY) 5,6,6
    5 IY = IY + 2147483647+1
```

```
    6 YFL = IY
      YFL = YFL*.4656613D-9
      RETURN
      END
END OF DATA
```

INPUT FOR COMPARISON RUN WHERE THERE
ARE HOLES AND 20% RANDOM NOISE ON TIME POINTS AND 20% AMPLITUDE NOISE

107	
-0.079157614	1.158462259
0.995801464	1.812866086
1.388411039	0.498414629
2.488105237	-3.205363691
2.997525039	2.333194355
3.328048381	0.494922633
4.546576938	-1.709672913
5.222211245	-1.463765533
5.641358839	1.360920456
6.458629013	0.859658146
13.019701019	-0.262391863
13.450376832	-1.088413756
14.466255535	0.520768550
15.185540637	0.796338959
15.677851407	-0.182540222
16.466709307	-1.284497101
17.548493564	-0.279947962
18.198036503	-3.211410844
18.600089159	-0.528001981
19.380675223	1.406833566
20.077563841	1.407727617
20.937723435	-2.441510980
21.690903398	1.431447102
22.264556129	3.132538859
23.477108428	-2.670532276
23.668130265	-3.395948041
24.082577240	-0.116023536
25.226717966	-1.918234136
26.089162237	-0.297399912
26.457930102	-1.129143396
27.137409278	0.033985866
27.949730843	1.533249944
34.754894694	-1.190770707
35.141514026	1.628114026
36.085367321	2.886819038
36.900585064	1.391599708
37.642521910	-2.559712441
38.307134224	0.647067182
38.816997373	1.706898924
39.909432847	-1.854938303
39.961532745	-1.897087577
40.915668131	0.166577117
41.943520713	-1.384453582
42.672478609	-0.734202251
43.503630064	3.613903817
43.714175443	0.660006220
44.734341150	-1.701210978

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45.204268871     -1.489468432
46.286089247      1.309746610
47.254860132     -1.777300554
47.303237286     -1.603746868
47.786374660      0.594762387
48.787141262      1.780230811
49.470199255      1.223692818
56.281192242     -1.166704118
56.651220109     -1.192694774
57.846968851      1.734277406
58.283334607      0.786662494
59.359984810     -3.700379578
59.623354489      0.025252369
60.453997627     -0.469865170
60.762632472     -0.786231510
61.941371290     -0.966216312
62.400994344      0.550697760
63.402638830      1.043751863
63.898181756     -3.298675702
70.530775918      0.572765844
71.264040860     -0.021969185
72.072957599      1.014764312
72.884471097     -0.704129889
73.400590870     -0.293605754
74.217458223     -0.204884508
74.920541358     -2.914847780
75.506811513     -0.564459188
76.515097826      2.498999652
76.940860487      1.794372737
77.671875990     -1.665761412
78.782358726      2.061229107
78.999363022      3.253033123
79.650866773      0.317345243
80.509300443     -3.887773177
81.271458226      0.393115946
82.323938230     -2.388442314
82.600180593     -2.672271758
83.651238056      0.348034844
83.879941038      0.534444548
84.877768074      1.063202509
85.731179684      0.275967620
86.428436106      2.758509429
86.972372291     -0.826682433
87.857162940     -1.208375329
88.214035774     -0.522322975
89.593366947     -1.110142469
90.141111110     -0.466207282
90.877936809      0.859787073
91.564406125     -1.944701843
91.971596358      1.636059130
92.668190844      2.017879417
93.822928167      2.056133472
94.216340853     -1.967563774
94.965115762     -0.448636114
95.834720199      0.224323534
96.842000008     -2.825805208
97.064845663     -3.142959531
98.164480065      2.455402584
98.763961196     -0.189171878
99.543749434     -1.005106092
   0.01000   0.70000 200.0000   0.5000   0.6000   0.7000   1.0000   0.7200
END OF DATA
```

COMPARISON OF NORMALIZED POWER SPECTRUMS (% POWER) FOR THE MODIFIED BLACK-MAN-TUKEY METHOD, THE CORRELATION FUNCTION METHOD, FOURIER TRANSFORM AND LEAST SQUARES TECHNIQUES

THE SIMULATED DATA IS A TIME SERIES WHERE THE TIME POINTS WERE ORIGINALLY SAMPLED AT EQUAL INTERVALS OF TIME (0.72 SECONDS APART) OVER A TIME SPAN OF 100 SECONDS. HOLES WERE THEN PUT IN THE DATA (32 OUT OF A TOTAL OF 139 POINTS MISSING OR ABOUT 23%). RANDOM NOISE WITH A GAUSSIAN DISTRIBUTION HAVING ZERO MEAN AND A STANDARD DEVIATION OF 0.20 UNITS WERE ADDED TO THE REMAINING TIME POINTS AND THE FUNCTION CALCULATED AT THESE PSEUDO RANDOM POINTS. FINALLY, RANDOM NOISE FROM A GAUSSIAN DISTRIBUTION HAVING ZERO MEAN AND STANDARD DEVIATION EQUAL TO 20% OF THE AMPLITUDE OF THE FUNCTION WAS ADDED TO THE FUNCTIONAL VALUES. (RANDOM NOISE ON BOTH AXES). THE FREQUENCIES IN THE DATA ARE 0.37 HZ, 0.14 HZ, 0.42 HZ, 0.65 HZ, AND 1.09 HZ. WITH THIS CHOICE OF FREQUENCIES ALIASING IS EXPECTED AND CAN BE SEEN IN THE PRINTOUT BELOW.

THE FIRST COLUMN IS FREQUENCY IN HERTZ.

THE SECOND COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY THE MODIFIED BLACKMAN-TUKEY POWER SPECTRUM METHOD USING MEAN LAGGED PRODUCTS AND SUMMING UP TO 50% OF THE MAXIMUM LAG IN THE DATA. THE FOURIER TRANSFORM OF THIS MODIFIED AUTOCORRELATION FUNCTION IS THEN THE POWER SPECTRUM.

THE THIRD COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 60% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE FOURTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 70% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE FIFTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY ROUNDING THE TIME DIFFERENCES TO THE NEAREST MULTIPLE OF AN AVERAGE TIME INTERVAL AND USING LAGGED OR CROSSED PRODUCTS UP TO 100% OF THE MAXIMUM LAG IN THE DATA. THEN THE FOURIER TRANSFORM OF THIS FUNCTION IS CALCULATED.

THE SIXTH COLUMN IS RELATIVE POWER IN PERCENT, COMPUTED BY THE METHOD OF S. FERRAZ-MELLO CALLED A 'DATE COMPENSATE DISCRETE FOURIER TRANSFORM' WHICH WILL BE PUBLISHED SOON.

THE SEVENTH COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY THE FOURIER TRANSFORM METHOD WITH THE OBSERVED TIME POINTS.

THE EIGHTH COLUMN IS THE RELATIVE POWER IN PERCENT, COMPUTED BY AN OPTIMUM LEAST SQUARES PROCEDURE (THE VANICEK METHOD WITHOUT 'CONSTITUENTS', THAT IS, AVERAGES, LINEAR TRENDS, ETC.).

0.0100	-0.7933	-0.3268	0.2249	0.0093	0.0083	0.0089	0.0083
0.0135	-0.9097	-0.0148	-0.5999	0.1983	0.2147	0.1945	0.1932
0.0169	-0.1699	0.7216	0.0179	0.1882	0.1944	0.1786	0.1811
0.0204	0.9001	1.1108	1.4081	0.6097	0.5405	0.5424	0.5332
0.0238	1.5978	1.0684	1.7841	1.7037	1.4866	1.6436	1.4817
0.0272	1.5052	1.0023	0.9806	1.6553	1.3756	1.7427	1.3884
0.0307	0.6804	0.9981	0.4030	0.4699	0.5857	0.5956	0.5862
0.0341	-0.4483	0.6800	0.5062	0.0062	0.0131	0.0124	0.0132

0.0376	-1.3208	-0.0347	0.3495	0.1724	0.1426	0.1472	0.1419
0.0410	-1.4765	-0.4746	-0.2325	0.1673	0.2108	0.1823	0.2080
0.0445	-0.7593	0.0826	-0.0674	0.2593	0.2934	0.2745	0.2942
0.0479	0.5348	1.3596	1.1897	0.7778	0.8590	0.7373	0.8140
0.0514	1.7043	2.1809	2.1487	1.7587	1.7133	1.5781	1.5752
0.0548	2.0026	1.7925	1.7750	2.2145	2.0870	2.0064	1.9853
0.0583	1.1751	0.7683	0.8665	1.2097	1.2169	1.1626	1.1969
0.0617	-0.2475	0.2766	0.5377	0.1822	0.1994	0.1979	0.1989
0.0652	-1.2612	0.6140	0.6016	0.2999	0.3192	0.3136	0.3140
0.0687	-1.1746	0.9340	0.4526	0.6041	0.6506	0.6662	0.6442
0.0721	-0.2243	0.5319	0.3103	0.7102	0.6828	0.7190	0.6822
0.0756	0.5958	-0.0853	0.5123	0.5463	0.6112	0.5990	0.6072
0.0790	0.4382	0.1179	0.6763	0.0422	0.1896	0.1921	0.1896
0.0825	-0.5633	1.1459	0.5972	0.5442	0.3648	0.3599	0.3480
0.0859	-1.3669	1.7523	0.8319	1.6729	1.2866	1.1887	1.2035
0.0894	-1.0349	1.0536	1.3634	1.2800	1.0849	1.0412	1.0584
0.0928	0.2521	-0.1355	1.0782	0.2265	0.1756	0.1774	0.1754
0.0963	1.2413	-0.2167	-0.1182	0.0653	0.0630	0.0612	0.0627
0.0997	0.8122	1.0393	-0.3112	0.3669	0.4443	0.4440	0.4435
0.1031	-0.7591	2.0151	1.4035	1.3609	0.9794	1.0137	0.9788
0.1066	-1.8034	1.3603	2.6310	2.0867	1.3121	1.3481	1.3110
0.1100	-0.7366	0.0502	1.1834	0.8371	0.5821	0.5781	0.5818
0.1135	2.2773	0.4365	-0.5218	0.1899	0.3241	0.3102	0.3230
0.1169	5.1632	3.1308	1.5753	2.3141	2.0807	2.1955	2.0771
0.1204	5.6927	5.7323	6.1803	4.9369	4.5123	4.5757	4.5079
0.1238	3.7141	5.4991	7.2827	6.4766	6.1228	5.6839	6.1235
0.1273	1.7129	3.0098	3.1891	5.1038	4.4361	4.1700	4.4386
0.1307	2.8808	2.0776	0.3195	0.7106	0.6280	0.6070	0.6278
0.1342	8.1527	5.4257	4.6129	2.2600	2.2310	2.3071	2.2288
0.1376	14.8361	11.0240	12.4940	12.8522	11.8335	11.9896	11.8046
0.1411	18.2894	13.6947	15.0296	17.3276	17.2822	16.8823	17.2089
0.1445	15.4891	10.5297	9.5648	8.4817	9.2087	9.2711	9.1813
0.1480	7.5645	4.0303	2.3825	0.6046	0.9680	1.0065	0.9680
0.1514	-0.8894	-0.5958	-0.2612	1.1956	0.8147	0.8321	0.8135
0.1549	-5.2294	-0.7952	0.8998	1.9725	2.1825	2.2137	2.1822
0.1583	-4.1163	1.3662	1.6684	0.2804	0.9059	0.9393	0.9057
0.1618	-0.1076	2.3764	0.9012	0.3912	0.3745	0.3734	0.3743
0.1652	2.8105	1.3307	0.5049	1.5474	1.1397	1.1181	1.1370
0.1687	2.4962	0.1590	1.2017	1.4577	1.2590	1.2814	1.2585
0.1721	-0.0204	0.4845	1.6267	0.9346	0.9446	0.9801	0.9445
0.1756	-2.0162	1.5430	1.0474	0.8009	0.9968	0.9999	0.9958
0.1790	-1.7488	1.6130	0.4085	0.9274	1.2278	1.2687	1.2282
0.1825	0.1397	0.5758	0.5288	0.8134	0.9548	0.9747	0.9545
0.1859	1.5566	0.0015	1.0233	0.5054	0.3639	0.3819	0.3638
0.1894	1.0949	0.8142	1.2862	1.0771	1.0399	1.1058	1.0398
0.1928	-0.7231	1.9169	1.2596	1.8097	2.1170	2.2518	2.1177
0.1963	-2.1425	1.6974	0.9990	1.0010	1.6078	1.5320	1.6059
0.1997	-1.9505	0.2885	0.4998	0.0221	0.2132	0.2117	0.2130
0.2032	-0.5399	-0.5564	0.1387	0.3681	0.1261	0.1242	0.1260
0.2066	0.6858	0.2147	0.4174	0.6898	0.2438	0.2489	0.2436
0.2101	0.8375	1.6310	1.1446	0.7840	0.3482	0.3386	0.3484
0.2135	0.4155	2.1424	1.6562	1.6382	1.2404	1.1782	1.2415
0.2170	0.6892	1.7509	1.8956	2.2445	1.9963	1.9214	1.9963
0.2204	2.2545	1.8426	2.4210	2.2489	1.9708	1.9766	1.9779
0.2239	4.2389	2.9955	3.2253	2.9755	1.9136	1.8499	1.9113
0.2273	4.9704	3.9094	3.4392	3.6819	1.4830	1.4254	1.4799
0.2308	3.4955	3.0709	2.5362	2.6899	0.6680	0.6934	0.6683
0.2342	0.5267	0.9188	1.1229	0.9051	0.3227	0.3210	0.3218
0.2377	-2.0589	-0.5190	0.1727	0.0585	0.4284	0.4065	0.4273
0.2411	-2.7193	-0.1266	0.0120	0.1665	0.3506	0.3610	0.3504
0.2446	-1.4265	1.0037	0.3373	0.4661	0.0579	0.0549	0.0574
0.2480	0.4465	1.1889	0.7278	0.6398	0.1985	0.1894	0.1974

0.2515	1.3437	[illegible]	0.8834	0.7104	0.7334	0.7391	0.7314
0.2549	0.7609	0.0974	0.7744	0.7952	0.7263	0.7251	0.7261
0.2584	-0.4882	0.6917	0.7130	0.9497	0.5251	0.5060	0.5222
0.2618	-1.0747	1.6432	0.9093	0.9592	0.7409	0.7211	0.7404
0.2653	-0.2818	1.1896	1.0545	0.7332	0.6179	0.6201	0.6234
0.2687	1.4344	0.2971	0.9239	0.8088	0.9171	0.9059	0.9172
0.2722	2.8034	0.7478	1.1252	1.5416	2.4274	2.5675	2.4622
[illegible]	2.6920	2.6065	2.2075	2.3817	3.2621	3.3284	3.2584
[illegible]	0.9729	3.6265	3.1430	2.6411	1.9565	2.0485	1.9466
[illegible]	-1.2509	2.1196	2.2478	1.9905	0.9270	0.9299	0.9346
0.2860	-2.2348	-0.3400	0.0916	0.6814	0.5464	0.5477	0.5463
0.2894	-0.7071	-0.3440	-0.2483	0.1522	0.1537	0.1605	0.1539
0.2929	3.1340	3.1113	2.8621	2.1135	0.2283	0.2400	0.2286
0.2963	7.4501	6.9622	6.7645	6.1880	1.2370	1.2418	1.2360
0.2998	9.7366	7.4563	7.4623	8.5145	3.2853	3.0854	3.2845
0.3032	8.4790	4.5549	4.6952	5.3589	3.5078	3.6171	3.5077
0.3067	4.3658	1.8146	2.0210	0.5453	1.1817	1.1750	1.1803
0.3101	0.0111	1.9170	1.9810	1.4577	0.3145	0.3343	0.3149
0.3136	-1.8432	3.6050	3.3044	5.1513	2.9622	3.0552	2.9589
0.3170	-0.3937	3.8456	3.4919	3.6396	2.8258	2.7986	2.8263
0.3205	2.5782	2.0845	2.3047	0.3594	0.1739	0.1797	0.1736
0.3239	4.1655	0.7871	1.4210	1.6815	2.3443	2.2942	2.3434
0.3274	2.8138	1.6782	1.7192	3.4202	5.5094	5.1257	5.5342
0.3308	-0.2994	3.0884	2.2861	1.8880	3.1738	3.2044	3.1771
0.3343	-2.3159	2.3028	1.8685	0.5511	1.2863	1.4097	1.3006
0.3377	-1.3989	-0.2923	0.4859	0.5952	1.6808	1.7049	1.6749
0.3412	1.3263	-1.3178	-0.4524	0.7150	1.5718	1.6221	1.5691
0.3446	2.5613	0.9067	0.3793	0.8175	1.6960	1.8797	1.6960
0.3481	-0.0255	3.5005	2.2829	0.7747	1.3861	1.4878	1.4056
0.3515	-4.9767	2.5815	2.6567	1.8270	0.9414	0.8551	0.9467
0.3550	-7.3071	-0.8355	0.5462	2.7924	2.1834	2.3268	2.1789
0.3584	-2.1772	-0.7252	-0.2299	0.4799	1.3169	1.3315	1.3135
0.3619	10.7187	6.9353	5.6484	2.6766	0.9977	0.9369	0.9950
0.3653	25.7403	18.1830	17.1216	17.0530	14.5740	14.1592	14.5715
0.3688	34.4526	23.9982	24.9202	28.1811	31.3365	30.0995	31.4440
0.3722	31.3162	19.5177	21.0077	20.2688	26.0093	25.1311	25.8968
0.3757	17.7164	9.0255	8.6835	5.6839	7.7578	8.1112	7.7271
0.3791	1.2363	1.0425	-0.6289	0.6418	1.3198	1.4137	1.3095
0.3826	-9.3612	-0.4587	-0.8004	1.6564	3.1035	2.9601	3.0888
0.3860	-9.7797	1.4204	2.9681	1.6298	3.3204	3.0475	3.3283
0.3895	-2.4658	2.0990	3.0850	1.0372	2.1277	2.0976	2.1353
0.3929	5.8356	1.2742	0.1443	1.2702	1.9781	1.8560	1.9781
0.3964	9.1291	1.7952	0.3444	2.3421	3.2884	3.2366	3.2977
0.3998	6.0769	4.3074	4.8053	3.8011	5.2298	5.6012	5.2310
0.4033	0.2421	5.7186	7.3508	5.1647	6.2066	6.1618	6.1883
0.4067	-3.0108	3.7645	3.9816	5.1953	5.2445	4.8199	5.2215
0.4102	-0.4128	1.1552	-0.3336	1.9492	1.6438	1.4949	1.6808
0.4136	7.1976	2.9738	2.1108	0.4237	1.0645	1.0217	1.0643
0.4171	15.7531	10.0352	11.0878	9.0577	16.6655	15.7676	16.6563
0.4205	20.7184	16.7623	18.0645	20.1983	35.7048	36.7529	35.5794
0.4240	19.7513	16.8459	16.4243	17.8545	31.9386	34.7280	31.7504
0.4274	13.7294	10.1702	8.7255	6.4485	11.9396	12.3606	11.8600
0.4309	5.8423	2.7608	2.4977	1.7855	0.4101	0.4437	0.4096
0.4343	-0.4227	0.0237	1.2924	3.3318	3.1188	2.9128	3.1071
0.4378	-3.1315	1.4631	2.3146	2.4618	3.1717	3.2511	3.1659
0.4412	-2.6149	2.6737	1.8491	0.2956	0.4934	0.5340	0.5014
0.4447	-0.8054	1.2624	0.0431	0.1615	0.0958	0.0959	0.0956
0.4481	0.2466	-0.8375	-0.6158	0.4230	0.6420	0.6630	0.6415
0.4516	-0.3122	-0.7979	0.4992	0.3918	1.7344	1.5784	1.7392
0.4550	-1.8177	1.1235	1.5231	0.8559	2.3167	2.0783	2.3208
0.4585	-2.8786	2.1986	1.1143	1.0650	1.1028	1.1670	1.1021
0.4619	-2.6012	1.0565	0.1543	0.6556	0.4329	0.4072	0.4312
0.4654	-1.2760	-0.6440	0.0026	0.2046	0.0273	0.0302	0.0273

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.9109	0.8929
0.4722	0.2775	0.8695	[illegible]	0.5256	4.4149	4.2916	4.2813
[illegible]	-0.3060	[illegible]	0.7205	1.2799	5.4306	5.2450	5.3834
0.4792	-0.8513	[illegible]	0.9955	1.1603	1.8461	1.6641	1.8573
0.4826	-0.5424		1.5066	0.9388	0.0773	0.0730	0.0772
0.4861	0.5178		1.4960	1.4778	1.3874	1.4631	1.3882
0.4895	1.3658	[illegible]	0.8824	1.3514	1.3310	1.1704	1.3320
0.4930	1.0691	1.4400	0.4034	0.3255	0.2519	0.2584	0.2519
0.4964	-0.3671	0.2146	0.3372	0.0294	0.3947	0.3534	0.3962
[illegible]	-1.9209	-0.8312	0.1826	0.2268	0.2246	0.2130	0.2249
[illegible]	[illegible]	[illegible]	-0.1553	0.0540	0.4880	0.4507	0.4879
	[illegible]	0.8086	-0.0013	0.0850	0.5186	0.5626	0.5190
	0.2076	1.3479	0.7855	0.5035	0.0075	0.0063	0.0075
	1.2453	0.7472	1.2514	[illegible]	0.9426	0.9515	0.9429
	0.9992	[illegible]	0.8361	[illegible]	1.8777	1.7633	1.8860
[illegible]	-0.1221	0.6049	0.4140	0.4745	0.4807	0.4837	0.4806
[illegible]	-0.9727	1.4900	0.8769	0.1340	0.9195	0.8279	0.9151
[illegible]	-0.5623	1.6571	1.6093	1.8728	2.4219	2.2175	2.4014
0.5309	1.2539	1.2129	1.6670	2.4550	0.8042	0.7772	0.8009
0.5344	3.7458	1.5476	1.7110	1.0577	0.4034	0.3775	0.4019
0.5378	5.8215	3.3104	3.0430	2.4620	3.4504	3.3953	3.4532
0.5413	6.6224	5.1330	4.9805	5.8766	4.3968	4.6576	4.3341
0.5447	5.8516	5.0220	5.1500	5.3911	3.0807	3.2762	2.9975
0.5482	3.8012	2.7749	2.8279	1.9463	1.8310	1.8496	1.7958
0.5516	1.2142	0.3209	0.2340	0.2518	0.3483	0.3494	0.3481
0.5551	-0.9569	-0.4704	-0.4005	0.2883	0.8417	0.7479	0.8440
0.5585	-1.9156	0.2481	0.4008	0.3456	3.3908	3.3190	3.3896
[illegible]	-1.4371	0.8988	0.7526	0.2382	3.3907	3.2805	3.3903
[illegible]	-0.0684	0.6110	0.3126	0.2291	1.0766	1.1089	1.0734
[illegible]	1.1470	0.0662	0.1903	0.6942	1.0630	1.1364	1.0603
0.5723	1.3497	0.2093	0.6731	[illegible]	1.3865	1.6068	1.3865
0.5758	0.4784	0.8381	0.8521	0.2134	0.4407	0.4578	0.4394
0.5792	-0.7026	0.9958	0.4175	0.1787	0.0870	0.1003	0.0875
[illegible]	-1.2425	0.4355	0.1903	0.9606	0.7061	0.7772	0.7075
0.5861	-0.8016	-0.0141	0.5695	0.6898	0.7392	0.7310	0.7371
0.5896	0.0809	0.3235	0.8365	0.0724	0.2997	0.2996	0.2974
0.5930	0.4948	0.9530	0.5016	0.4960	0.1715	0.1672	0.1714
0.5965	-0.0033	0.8881	0.1440	0.7613	0.2460	0.2265	0.2456
0.5999	-0.9766	0.0889	0.2828	0.3072	0.1699	0.1663	0.1693
0.6034	[illegible]	-0.3857	0.4839	0.0356	0.0857	0.0828	0.0857
0.6068	[illegible]	0.2144	0.3540	0.1640	1.0637	1.0059	1.0627
0.6103	[illegible]	1.2716	0.4254	0.8069	2.2298	2.2134	2.2255
0.6137	1.1206	1.5586	1.0830	1.4830	1.4564	1.5399	1.4547
0.6172	0.9888	0.8158	1.4845	1.0466	0.3560	0.3521	0.3560
0.6206	-0.1518	0.0355	0.7726	0.2320	0.8694	0.9360	0.8648
0.6241	-1.4617	0.0952	-0.2771	0.2525	1.4364	1.4904	1.4333
0.6275	-2.0018	0.6169	-0.2480	0.3431	0.7533	0.7038	0.7654
0.6310	-1.4677	0.5955	0.6183	0.0643	0.4646	0.4531	0.4645
0.6344	[illegible]	-0.1031	0.7312	0.0954	0.8957	0.9094	0.8953
0.6379	[illegible]	-0.4907	-0.1890	0.3070	0.4797	0.5144	0.4794
0.6413	[illegible]	0.1337	-0.5273	0.2494	0.0428	0.0425	0.0427
0.6448	-0.9977	1.0452	[illegible]	0.0666	1.4162	1.4829	1.4207
0.6482	-2.1231	0.9366	1.3326	0.3004	4.0879	4.1130	4.0873
0.6517	-2.2561	-0.1765	0.4508	0.9736	5.1901	4.7751	5.1899
0.6551	-0.7017	-0.5226	-0.6331	0.6756	3.3043	3.0134	3.2837
0.6586	2.4721	1.5126	0.9284	0.1626	2.1518	2.0574	2.1453
0.6620	6.3163	5.2417	5.1168	3.6473	2.5227	2.5079	2.5038
0.6655	9.3061	7.9658	8.4027	9.5005	2.1067	1.8637	2.0362
0.6689	10.0486	7.6177	7.8778	9.3172	0.5200	0.4455	0.4993
0.6724	8.1063	4.8744	4.6289	3.2525	0.0078	0.0076	0.0077
0.6758	4.4242	2.3728	2.0940	0.8316	0.6027	0.6290	0.6028
0.6793	0.9410	1.9444	2.0206	3.5186	2.3475	2.4984	2.3435

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0.6827	-0.5323	2.9683	3.1700	4.2374	4.1234	3.9278	4.1067
0.6862	0.5185	3.5314	3.5492	2.0790	2.4741	2.1030	2.4496
0.6896	2.9484	2.8154	2.7368	1.7659	0.0163	0.0168	0.0163
0.6931	4.7510	1.8668	1.8536	3.2532	1.2400	1.4146	1.2441
0.6965	4.5011	2.0126	1.9844	2.9773	2.2853	2.5079	2.2848

END OF DATA