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Semi-Actuator Disk Theory for Compressor Choke Flutter

J. Micklow and J. Jeffers

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Semi-Actuator Disk Theory for Compressor Choke Flutter

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SUMMARY

Utilizing semi-actuator disk theory, a mathematical analysis was developed to predict the unsteady aerodynamic environment for a cascade of airfoils harmonically oscillating in choked flow. In the model, a normal shock is located in the blade passage, its position depending on the time dependent geometry and pressure perturbations of the system. In addition to shock dynamics, the model includes the effect of compressibility, interblade phase lag, and an unsteady flow field upstream and downstream of the cascade.

Calculated unsteady aerodynamic forces using the semiactuator disk model were compared to experimental data from isolated airfoil wind tunnel tests. The wind tunnel data simulate the special cascade condition of 180 deg interblade phase. Agreement between experimental and theory was reasonable. The semiactuator theory was also evaluated using compressor airfoil choke flutter data from single-spool tests of the F100 turbofan engine. The model was incorporated into a flutter prediction program in which calculated aerodynamic damping is correlated to construct flutter onset boundaries. The calculated flutter boundaries compared well with the measured flutter boundaries. Based on these evaluations, it was concluded that a conservative choke flutter design system could be established based on the semiactuator disk model.

INTRODUCTION

Compressor airfoil flutter remains a continuing problem in the design and development of advanced aircraft gas turbine engines. Flutter occurs over a wide range of operating conditions, but can be categorized into four regions: (1) subsonic/transonic stall, (2) subsonic/transonic choke, (3) supersonic unstalled, and (4) supersonic stalled, as shown in Figure 1.

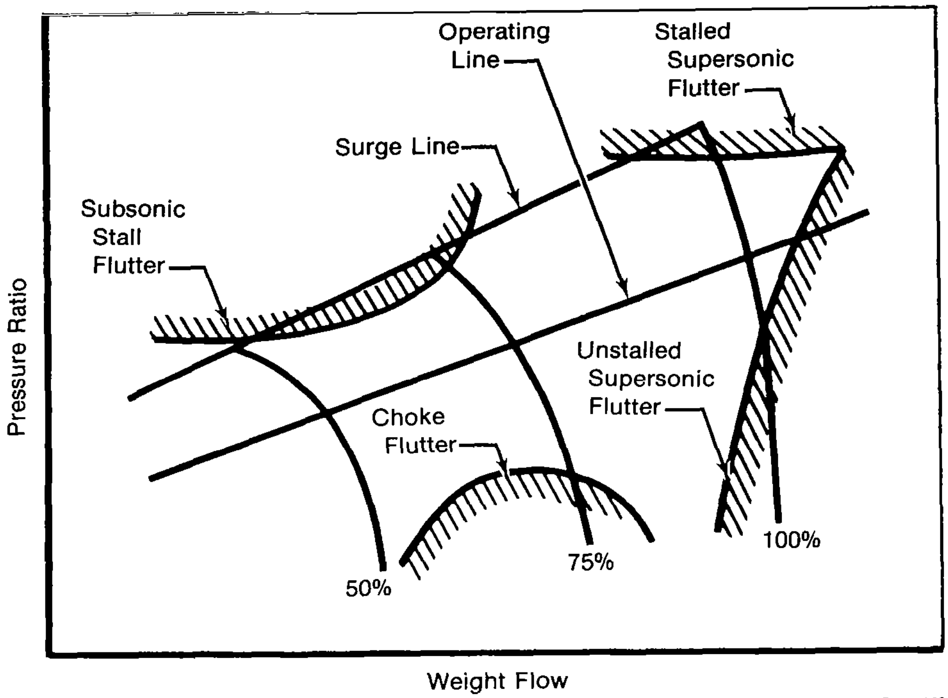

Figure 1. Possible Flutter Boundaries

The subsonic stall flutter problem has been investigated by a number of authors. Jeffers (Reference 1) devised a semi-empirical unsteady aerodynamic theory based on combining the unsteady unstalled aerodynamic forces from Smith's theory (Reference 2) with correction from theory and experimental data of isolated airfoils operating at high incidence in incompressible flows. Sisto (Reference 3) used steady aerodynamic data to treat the unsteady flow problem in a quasi-steady manner. Perumal (Reference 4) developed an essentially "Helmholtz flow" model, while Yashima and Tanaka (Reference 5) adapted a rigid wake model to obtain reasonably good correlation with linear cascade experimental data. Most recently, Chi (Reference 6) used a small perturbation technique to model flow separation.

The supersonic flow region has also been discussed by a number of authors. For the unstalled regime, a finite difference method was first used by Verdon (Reference 7) and Brix and Platzer (Reference 8) to model the unsteady supersonic aerodynamics. Other approaches by Kurosaka (Reference 9) and Verdon and McCune (Reference 10) extend a velocity potential method first developed by Miles (References 11 and 12) for simple supersonic cascade configurations.

Recently, an unsteady actuator disk model was developed by Adamczyk (Reference 13) with encouraging results for supersonic stall bending flutter. The supersonic stalled region was also investigated by Goldstein, Braun and Adamczyk (Reference 14) in which the small perturbation analysis included the presence of a strong in passage shock.

The choke flutter problem that has arisen in advanced gas turbine engines with variable inlet guide vanes poses a very serious problem and no analytical model exists at present to predict the unsteady aerodynamic environment. The complex nature of this environment has thus far resisted rigorous mathematical formulation, but a "simplified" model has been undertaken herein based on a modified semi-actuator disk approach with one-dimensional channel flow. The channel flow approach originally used by NASA-NACA to analyze inlet diffusers of ramjet and turbojet engines was selected because airfoil cascades can exhibit flow characteristics similar to those of inlet diffusers. The flow in an inlet diffuser and a choked blade passage both contain a shock wave whose position strongly affects the pressure forces on the channel walls or blade surfaces. The position of the shock depends upon channel geometry and, therefore, in the case of the airfoil cascade, can be related to the vibratory motion of the airfoils in flutter. A preliminary analysis was completed in the initial phase of this effort which produced promising results when used in a stability prediction of a compressor rotor that experienced choke flutter at off-schedule operating conditions. However, concern for certain aspects of the preliminary model led to the present approach which includes a modified semi-actuator disk method to describe the upstream and downstream flow fields.

The section following contains the analytical derivation and definition of the mathematical model, including a steady-state interblade analysis and a linearized small perturbation analysis. The next section details the results obtained using this channel flow model. In order to rid the text of this report of complex, cumbersome, and lengthy mathematical manipulations and assumptions, numerous appendices are included herewith which allow model development in a straight forward manner.

ANALYTICAL MODEL

Model Definition

The semi-actuator disk model consists of two solutions: a steady-state intrablade analysis and an unsteady linearized small perturbation analysis. The steady-state analysis utilizes steady isentropic one-dimensional relations to define the intrablade conditions. An iterative procedure, ending in the match of the known static pressure ratio across the blade, locates the steady-state normal shock position. The procedure appears in Appendix H. The flow entering and leaving the cascade is defined externally by a streamline analysis. The unsteady solution consists of three basic flow fields: (1) upstream flow field, (2) intrablade flow analysis, and (3) a downstream flow field, as shown in Figure 2.

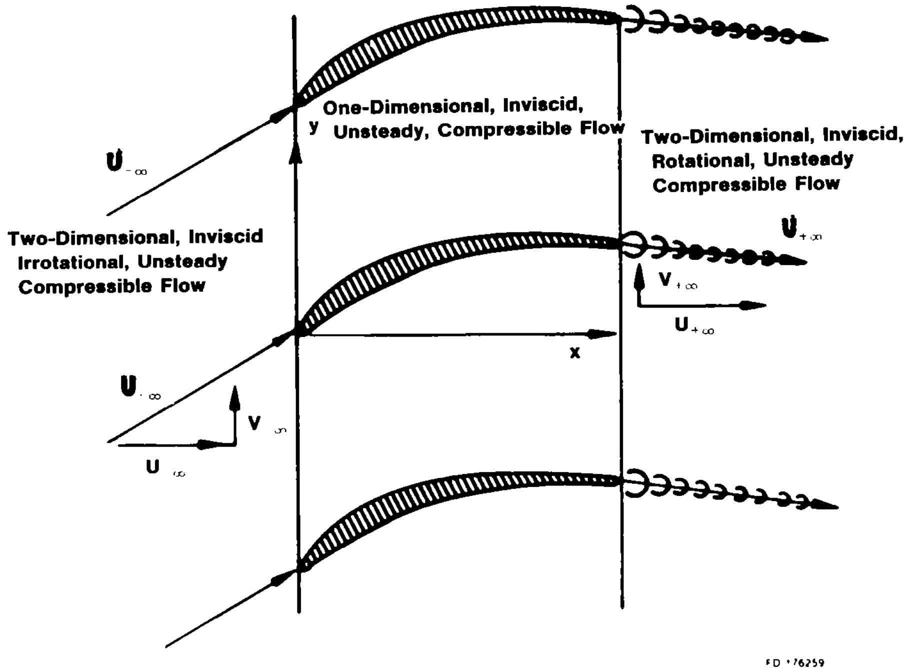

Figure 2. Unsteady Flow Field Description

Assumptions and Boundary Conditions

Assumptions

The following assumptions were made relative to the flow within the three basic flow fields of the unsteady solution:

1. *Upstream Flow Field* — The flow is assumed to be two-dimensional, inviscid, irrotational, unsteady, and compressible.

2. *Intrablade Flow Analysis* — The flow is assumed to be one-dimensional, inviscid, unsteady, and compressible. Figure 3 details the division of the flow field into three sections: a subsonic section from blade leading edge to the blade throat or M = 1, a supersonic section from blade throat to shock location, and a subsonic section from shock location to blade trailing edge.

3. *Downstream Flow Field* — The flow is assumed to be two-dimensional, rotational, inviscid, compressible, and constructed of the sum of two basic solutions: an irrotational part similar to the upstream flow field and a rotational part due to the vortices being shed off the blade trailing edge.

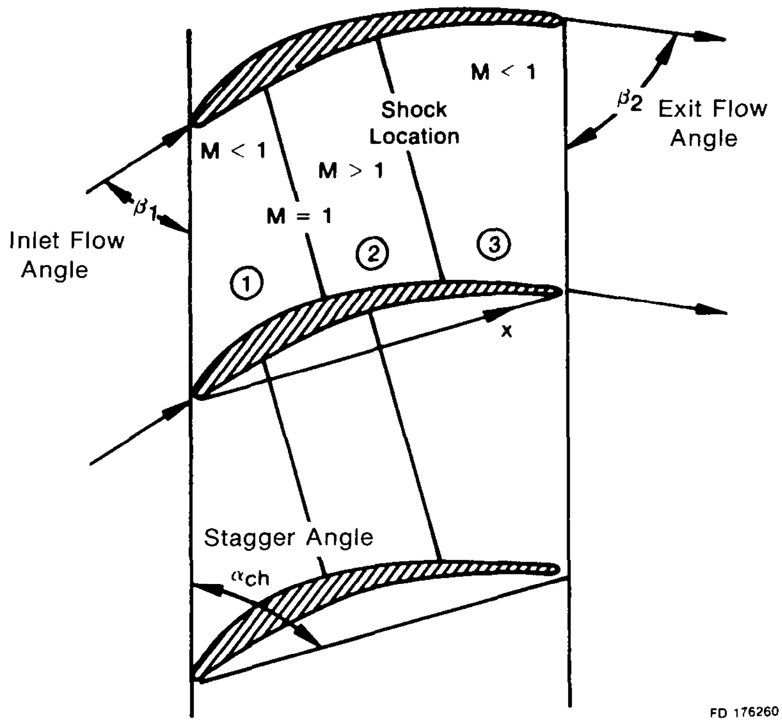

Figure 3. Intrablade Flow Field

Boundary Conditions

Boundary conditions for the unsteady solutions consist of the following:

1. The mass flow is continuous at the leading- and trailing-edge lines.

2. Conservation of mass, energy, and momentum was observed within each section of the blade channel.

3. The Kutta condition at the trailing edge is satisfied by specifying the exit air angle.

Derivation of the Unsteady Model

Upstream and Downstream Irrotational Flow Equations

Continuity Equation

Establishing a coordinate system, as shown in Figure 4 produces the following form for the continuity equation:

$$\frac{\partial(\rho u A)_{\pm\infty}}{\partial x} + \frac{\partial(\rho v A)_{\pm\infty}}{\partial y} + \frac{\partial(\rho A)_{\pm\infty}}{\partial t} = 0 \quad (1)$$

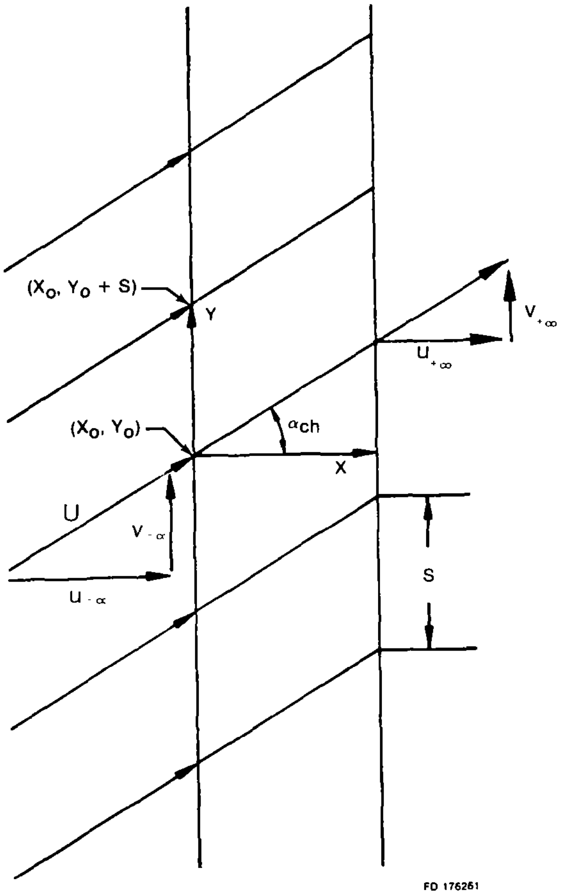

Figure 4. Cascade Geometry

Small perturbations of the flow variables are assumed as follows:

$$\rho = \bar{\rho} + \rho' \qquad u = \bar{u} + u' \qquad v = \bar{v} + v'$$

Substituting the above relationships into Equation 1, subtracting out the steady-state equation, and neglecting higher order terms gives the small perturbation form of the continuity equation, as follows:

$$\frac{\partial \rho'_{\pm\infty}}{\partial t} + \bar{u}_{\pm\infty}\left(\frac{\partial \rho'_{\pm\infty}}{\partial x}\right) + \bar{v}_{\pm\infty}\left(\frac{\partial \rho'_{\pm\infty}}{\partial y}\right) + \bar{\rho}_{\pm\infty}\left(\frac{\partial u'_{\pm\infty}}{\partial x} + \frac{\partial v'_{\pm\infty}}{\partial y}\right) = 0 \quad (2)$$

Momentum Equation

In tensor notation, the two-dimensional form of the momentum equation is as follows:

$$\frac{\partial (\rho u_i u_j)}{\partial x_j} + \frac{\partial (\rho u_i)}{\partial t} = -\frac{\partial p}{\partial x_i} + \rho f_i$$

which becomes

x-direction

$$\frac{\partial (\rho u_{\pm\infty} u_{\pm\infty})}{\partial x} + \frac{\partial (\rho u_{\pm\infty} v_{\pm\infty})}{\partial y} + \frac{\partial (\rho u_{\pm\infty})}{\partial t} = -\frac{\partial p_{\pm\infty}}{\partial x} \quad (3a)$$

y-direction

$$\frac{\partial (\rho u_{\pm\infty} v_{\pm\infty})}{\partial x} + \frac{\partial (\rho v_{\pm\infty} v_{\pm\infty})}{\partial y} + \frac{\partial (\rho v_{\pm\infty})}{\partial t} = -\frac{\partial p_{\pm\infty}}{\partial y} \quad (3b)$$

Assuming small perturbations of the flow variables in Equations 3a and 3b, neglecting higher order terms and subtracting out the mean flow equation, gives the small perturbation forms of the momentum equation, as shown below:

x-direction

$$\frac{\partial u'_{\pm\infty}}{\partial t} + \bar{u}_{\pm\infty}\frac{\partial u'_{\pm\infty}}{\partial x} + \bar{v}_{\pm\infty}\frac{\partial u'_{\pm\infty}}{\partial y} = \frac{-1}{\bar{\rho}_{\pm\infty}}\left(\frac{\partial p'_{\pm\infty}}{\partial x}\right) \quad (4a)$$

y-direction

$$\frac{\partial v'_{\pm\infty}}{\partial t} + \bar{u}_{\pm\infty}\frac{\partial v'_{\pm\infty}}{\partial x} + \bar{v}_{\pm\infty}\frac{\partial v'_{\pm\infty}}{\partial y} = \frac{-1}{\bar{\rho}_{\pm\infty}}\left(\frac{\partial p'_{\pm\infty}}{\partial y}\right) \quad (4b)$$

Nondimensionalized Wave Equation

Flow entering the cascade is assumed to be irrotational and can therefore be represented by a potential function as can the irrotational portion of the flow leaving the cascade. The velocity perturbations can then be represented in the following forms:

$$u'_{\pm\infty} = \frac{\partial \Phi'_{\pm\infty}}{\partial x} \qquad v'_{\pm\infty} = \frac{\partial \Phi'_{\pm\infty}}{\partial y} \tag{5}$$

Substituting these relationships into the continuity and momentum equations and combining these equations gives the small perturbation form of the nondimensionalized wave equation (after some manipulation), as shown:

$$\bar{M}^2_{\pm\infty} \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial t^{*2}} \right) + 2\bar{M}_{\pm\infty}\bar{M}_{x\pm\infty} \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial x^* \partial t^*} \right) + 2\bar{M}_{\pm\infty}\bar{M}_{y\pm\infty} \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial y^* \partial t^*} \right) +$$

$$2\bar{M}_{x\pm\infty}\bar{M}_{y\pm\infty} \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial x^* \partial y^*} \right) + (\bar{M}^2_{x\pm\infty} - 1) \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial x^{*2}} \right) +$$

$$(\bar{M}^2_{y\pm\infty} - 1) \left(\frac{\partial^2 \Phi'_{\pm\infty}}{\partial y^{*2}} \right) = 0 \tag{6}$$

where x and y are nondimensionalized by semichord b, and time is nondimensionalized by the quantity U/b. The derivation of the wave equation appears in Reference 15.

Because Equation 6 is linear, a solution can be obtained by superposition of fundamental solutions, taking the following form:

$$\Phi'_{\pm\infty} = A_{\pm\infty} \exp i(Bx + Cy + kt) \tag{7}$$

where A, B, and C are unknown constants and k represents the reduced frequency based on semichord $k = b\omega/U$. Assuming the blades vibrate with a constant interblade phase angle σ, the tangential wave constant C is controlled by an unsteady periodicity condition. Any perturbation velocity potential at $(x_o, y_o + s)$ leads or lags the same potential at (x_o, y_o) by σ at all times. This may be expressed as follows:

$$\Phi'(x_o, y_o + s, t) = \Phi'(x_o, y_o, t)\, e^{i\sigma} \tag{8}$$

where s defines the blade gap-to-semichord ratio. Substituting Equation 8 into Equation 7 gives

$$C = \frac{\sigma}{s}$$

Substituting Equation 7 into Equation 6, dividing by $A_{\pm\infty} \exp i(Bx + Cx + kt)$ and solving for B yields

$$B_{1,2} = \frac{-D_1 \pm \sqrt{D_1^2 + \beta_x^2 D_2}}{-\beta_x^2} \tag{9}$$

where,

$$D_1 = \bar{M}_{x\pm\infty}\bar{M}_{\pm\infty}k + \bar{M}_{x\pm\infty}\bar{M}_{y\pm\infty}C$$

$$D_2 = k^2\bar{M}^2_{\pm\infty} + 2\bar{M}_{\pm\infty}\bar{M}_{y\pm\infty}Ck - C^2\beta_y^2$$

$$\beta_x^2 = 1 - \bar{M}^2_{x\pm\infty}$$

$$\beta_y^2 = 1 - \bar{M}^2_{y\pm\infty}$$

Thus, the solution takes the form:

$$\Phi'_{\pm\infty}(x, y, t) = [A_{1\pm\infty}e^{iB_1x} + A_{2\pm\infty}e^{iB_2x}][e^{i(Cy + kt)}] \quad (10)$$

The velocity perturbation must approach zero in the far field. Now if the quantity $D_1^2 + \beta_x^2 D_2 < 0$, B_1 and B_2 are complex conjugates. In order to satisfy the far field condition, the solution must take the following form:

$$\Phi'_{\pm\infty}(x, y, t) = A_{1\pm\infty} \exp(iB_1x) \exp i(Cy + kt) \quad (11)$$

Now consider the case where $D_1^2 + \beta_x^2 D_2 > 0$. Here B_1 and B_2 will be real numbers and the boundedness condition cannot be applied in the far field. This problem can be solved by representing the flow field as containing a transient part. The assumed solution becomes form

$$\Phi'_{\pm\infty}(x, y, t) = A_{\pm\infty} \exp i\left[(Bx + Cy + kt) + k_1t\right] \quad (12)$$

Substituting Equation 12 into Equation 6 and dividing by $A_{\pm\infty}\exp[i(Bx + Cy + kt) + K_it]$ gives

$$B_{1,2\,real} = \frac{D_1^s \pm \sqrt{D_1^{s2} + \beta_x^2 D_2^s}}{\beta_x^2} \quad (13)$$

and

$$B_{imag} = \frac{-i(\bar{M}_{y\pm\infty}C + \bar{M}_{\pm\infty}k)}{\bar{M}_{x\pm\infty}} \quad (14)$$

Equation 12 physically implies an unsteady solution under the influence of a slowly divergent, oscillating source with the earlier emitted wave being stronger than the one following it. Since a finite period of time is required for a disturbance to propagate to the far field, the amplitude of the response should decrease with increasing distance from the source. Therefore, the proper upstream solution can be chosen. By letting k_1 approach zero, the correct wave solutions on either side of the harmonically vibrating source can be recovered, as shown in Equation 15.

$$\Phi'_{\pm\infty} = A_{\pm\infty} \exp i[x (B_1 + B_{imag}) + Cy + kt] \tag{15}$$

A summary of the correct solutions for the irrotational upstream and downstream flow fields is presented in Table 1.

Intrablade Flow Equations and Solutions

Perturbation Equations and Solutions for Region 1

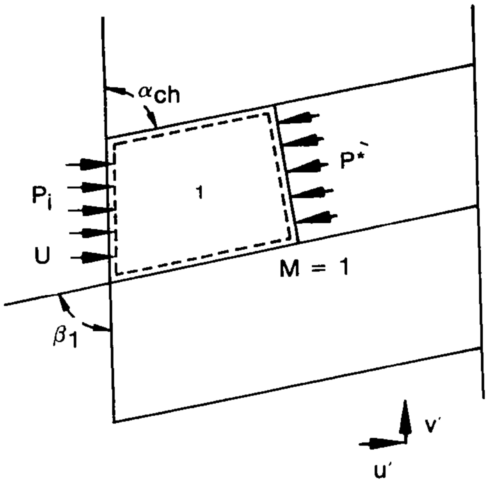

Figure 5. Region 1 from Blade Leading-Edge Line to the Channel Throat, or M = 1

The first region, as shown in Figure 5, defines a control volume from the blade leading-edge line to the channel throat, or M = 1, with the coordinate system aligned with the chord line. In order to define the unsteady flow field, three unknowns must be obtained: (1) the complex constant describing the upstream flow field, (2) the density perturbation at the throat, and (3) the velocity perturbation at the throat. This requires the use of the unsteady form of the mass, momentum, and energy equations.

Equation 16 expresses the conservation of mass for a control volume (Reference 16):

$$\frac{dm}{dt} = 0 = \iint_{cs} \rho U dA + \frac{\partial}{\partial t} \iiint_{vol} \rho \partial (vol) \tag{16}$$

For the control volume shown in the above figure, this can be expressed as follows:

$$\dot{w}^* - \dot{w}_{inlet} = - \frac{\partial(\rho_1 V_1)}{\partial t} \tag{17}$$

Assuming small perturbations on the mean flow variables, neglecting higher order terms and subtracting out the mean flow equation, the following result is obtained after expanding Equation 17:

TABLE 1. IRROTATIONAL FLOW FIELD SOLUTIONS

Upstream Flow Fields

$D_{1-\infty}^{2} + \beta_{x-\infty}^{2} D_{2-\infty} < 0$	$D_{1-\infty}^{2} + \beta_{x-\infty}^{2} D_{2-\infty} > 0$	
$B_{1R} = D_{1-\infty}/\beta_{x-\infty}^{2}$	$\bar{M}_{-\infty}k + \bar{M}_{y-\infty} C > 0$	$\bar{M}_{-\infty}k + \bar{M}_{y-\infty} C < 0$
$B_{1I} = -\sqrt{D_{1-\infty}^{2} + \beta_{x-\infty}^{2} D_{2-\infty}} \,/\beta_{x}^{2}{}_{+\infty}$	$B_{1R} = \dfrac{D_{1-\infty} + \sqrt{D_{1-\infty}^{2} + \beta_{x}^{2} D_{2-\infty}}}{\beta_{x}^{2}}$	$B_{1R} = \dfrac{D_{1-\infty} - \sqrt{D_{1-\infty}^{2} + \beta_{x}^{2} D_{2-\infty}}}{\beta_{x}^{2}}$
	$B_{1I} = \dfrac{-(\bar{M}_{y-\infty} C + M_{-\infty} k)}{\bar{M}_{x-\infty}}$	$B_{1I} = \dfrac{+(\bar{M}_{y-\infty} C + \bar{M}_{-\infty} k)}{\bar{M}_{x-\infty}}$

Downstream Flow Fields

$D_{1+\infty}^{2} + \beta_{x+\infty}^{2} D_{2+\infty} < 0$	$D_{1+\infty}^{2} + \beta_{x+\infty}^{2} D_{2+\infty} > 0$	
$B_{1R} = D_{1+\infty}/\beta_{x+\infty}^{2}$	$\bar{M}_{+\infty}k + \bar{M}_{y+\infty} C > 0$	$\bar{M}_{+\infty}k + \bar{M}_{y+\infty} C < 0$
$B_{1I} = +\sqrt{D_{1+\infty}^{2} + \beta_{x+\infty}^{2} D_{2+\infty}} \,/\beta_{x+\infty}^{2}$	$B_{2R} = \dfrac{D_{1+\infty} - \sqrt{D_{1+\infty}^{2} + \beta_{x+\infty}^{2} D_{2+\infty}}}{\beta_{x+\infty}^{2}}$	$B_{2R} = \dfrac{D_{1+\infty} + \sqrt{D_{1+\infty}^{2} + \beta_{x+\infty}^{2} D_{2+\infty}}}{\beta_{x+\infty}^{2}}$
	$B_{2I} = \dfrac{+(\bar{M}_{y+\infty} C + M_{+\infty} k)}{\bar{M}_{x+\infty}}$	$B_{2I} = \dfrac{-(\bar{M}_{y+\infty} C + \bar{M}_{+\infty} k)}{\bar{M}_{x+\infty}}$

where,

$D_{1\pm\infty} = (\bar{M}_{x\pm\infty} \bar{M}_{\pm\infty} k + \bar{M}_{y\pm\infty} M_{\pm\infty} C)$

$D_{2\pm\infty} = (k^{2} \bar{M}_{\pm\infty}^{2} + 2 \bar{M}_{\pm\infty} My_{\pm\infty} Ck - \beta_{y\pm\infty}^{2} C^{2})$

$\beta_{x\pm\infty}^{2} = 1 - \bar{M}_{x\pm\infty}^{2}$

$\beta_{y\pm\infty}^{2} = 1 - \bar{M}_{y\pm\infty}^{2}$

$$[\rho'^*\bar{A}^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + \bar{\rho}^*\bar{A}^*a'^*] - [\bar{\rho}_1\bar{A}_1 (u_1' \sin \alpha_{ch} + v' \cos \alpha_{ch}) +$$

$$\bar{\rho}_1 A_1' \bar{U}_1 \cos(\alpha_{ch} - \beta_1) + \rho_1' \bar{A}_1 \bar{U}_1 \cos(\alpha_{ch} - \beta_1)]$$

$$= - \left(\bar{\rho}_1 \frac{\partial V_1'}{\partial t} + \bar{V}_1 \frac{\partial \rho_1'}{\partial t} \right) \tag{18}$$

The derivation of the small perturbation forms of the mass, momentum, and energy equations is shown in Appendix A.

The next step involves the derivation of the unsteady control volume form of the momentum equation. The equation takes the form:

$$F = \frac{d(mU)}{dt} = \iint_{cs} U(\rho U)dA + \frac{\partial}{\partial t} \iiint_{Vol} U\rho d\,(Vol) \tag{19}$$

Expanding Equation 19 and assuming small perturbations yields for Region 1:

$$p'\bar{A}_1 + \bar{p}A'_1 - \bar{p}^*A^{*'} - p^{*'}\overline{A}^* = [a'^*\bar{w}^* + \bar{a}^*w^{*'}] -$$

$$[\bar{w}_1 (u'_1 \sin \alpha_{ch} + v'_1 \cos \alpha_{ch}) + \bar{U}_1 w'_1 \cos(\alpha_{ch} - \beta_1)] +$$

$$U_{ave}\bar{V}_1 \frac{\partial \rho'_{ave}}{\partial t} + \bar{\rho}_{ave}\bar{U}_{ave} \frac{\partial V'_1}{\partial t} + \rho_{ave}\bar{V}_1 \frac{\partial U'_{ave}}{\partial t} \tag{20}$$

The next step requires the derivation of the small perturbation form of the energy equation for as inviscid fluid with no external heat addition. The control volume for Region 1 is therefore (Reference 16):

$$\frac{\partial}{\partial t} \iiint_{Vol} \rho e d\,(Vol) = - \iint_{cs} (\rho U \cdot n)\, e ds \tag{21}$$

where, n = Unit vector normal to the surface and,

$$e = \frac{a^2}{\gamma(\gamma - 1)} + \frac{U^2}{2}$$

Substituting the value for e into Equation 21 gives:

$$\frac{\partial}{\partial t} \left[\rho_1 V_1 \left(\frac{a^2}{\gamma(\gamma - 1)} + \frac{U^2}{2} \right)_1\right]$$

$$= - \left[w^* \left(\frac{a^2 (\gamma^2 - \gamma + 2)}{2\gamma(\gamma - 1)} \right) - w_1 \left(\frac{a_1^2}{\gamma(\gamma - 1)} + \frac{U_1^2}{2} \right)\right] \tag{22}$$

Expanding Equation 22 by assuming small perturbations and subtracting out the mean flow equation produces the small perturbation form of the energy equation, as shown below:

$$\frac{1}{\gamma(\gamma-1)}\left[2\bar{a}_1 a'_1 \bar{w}_1 + \bar{a}_1^2 w'_1\right] + \frac{1}{2}\left[\bar{w}_1 \left(2\bar{U}_1 \cos(\alpha_{ch} - \beta_1)\right)\left(u'_1 \sin\alpha_{ch} + \right.\right.$$

$$\left.\left. v'_1 \cos\alpha_{ch}\right) + w'_1 \left(\bar{U}_1 \cos(\alpha_{ch} - \beta_1)\right)^2\right] - \frac{(\gamma^2 - \gamma + 2)}{2\gamma(\gamma-1)}\left[2\bar{a}^* a'^* \bar{w}^* + \bar{a}^{*2} w'^*\right]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\bar{a}^2 \bar{V}_1 \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{a}^2 \frac{\partial V'_1}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}_1 \frac{\partial a'}{\partial t}\right]_1 +$$

$$\frac{1}{2}\left[\bar{U}^2 \bar{V}_1 \frac{\partial \rho'}{\partial t} + \bar{\rho}\,\bar{U}^2 \frac{\partial V'_1}{\partial t} + 2\,\bar{U}\bar{\rho}\,\bar{V}_1 \frac{\partial U'}{\partial t}\right]_1 \qquad (23)$$

Before solving Equations 18, 20, and 23, densities are nondimensionalized by ρ_i, pressures by $\rho_i U_i^2$, velocities by U_{IRE}, lengths by semichord b, time by b/U_{IRE}, areas by A^*, and volumes by A^*b. The nondimensionalized form of the equations of motion for Region 1 are as follows:

Continuity Equation

$$\left[\rho'^*\bar{a}^* + \bar{\rho}^*\bar{a}^* A'^* + \bar{\rho}^* a'^*\right] - \left[\bar{A}_1 (u'_1 \sin\alpha_{ch} + v'_1 \cos\alpha_{ch}) + \right.$$

$$\left. A'_1 \cos(\alpha_{ch} - \beta_1) + \rho'_1 \bar{A}_1 \cos(\alpha_{ch} - \beta_1)\right]$$

$$= \left[\bar{\rho}\frac{\partial V'}{\partial t} + \bar{V}\frac{\partial \rho'}{\partial t}\right]_1 \qquad (24)$$

Momentum Equation

$$\left[p'_1 \bar{A}_1 + \bar{p}_1 A'_1 - p'^* - \bar{p}^* A'^*\right] = \bar{a}^*\left[2\bar{\rho}^* a'^* + \bar{\rho}^* A'^* \bar{a}^* + \rho'^* \bar{a}^*\right] -$$

$$\cos(\alpha_{ch} - \beta_1)\left[\rho'_1 \bar{A}_1 \cos(\alpha_{ch} - \beta_1) + A'_1 \cos(\alpha_{ch} - \beta_1) + 2\bar{A}_1 (u'_1 \sin\alpha_{ch} + \right.$$

$$\left. v'_1 \cos\alpha_{ch})\right] + \bar{U}_1 \bar{V}_1 \frac{\partial \rho'_1}{\partial t} + \bar{\rho}_1 \bar{U}_1 \frac{\partial V'_1}{\partial t} + \bar{\rho}_1 \bar{V}_1 \frac{\partial U'_1}{\partial t} \qquad (25)$$

Energy Equation

$$\left[\frac{\bar{a}_1^2}{\gamma(\gamma-1)} + \frac{\cos^2(\alpha_{ch}-\beta_1)}{2}\right]\left[\rho'_1\bar{A}_1\cos(\alpha_{ch}-\beta_1) + A'_1\cos(\alpha_{ch}-\beta_1) + \right.$$

$$\left.\bar{A}_1(u'_1\sin\alpha_{ch} + v'_1\cos\alpha_{ch})\right] + \frac{2\bar{A}_1\cos(\alpha_{ch}-\beta_1)\,\bar{a}_1 a'_1}{\gamma(\gamma-1)} +$$

$$\bar{A}_1\cos^2(\alpha_{ch}-\beta_1)(u'_1\sin\alpha_{ch} + v'_1\cos\alpha_{ch}) -$$

$$\frac{(\gamma^2-\gamma+2)}{2\gamma(\gamma-1)}\left[\bar{a}^{*2}(\rho'^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + 3\bar{\rho}^*a'^*)\right]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\bar{V}\bar{a}^2\frac{\partial\rho'}{\partial t} + \bar{\rho}\bar{a}^2\frac{\partial V'}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}\frac{\partial a'}{\partial t}\right]_1 +$$

$$\frac{1}{2}\left[\bar{V}\bar{U}^2\frac{\partial\rho'}{\partial t} + \bar{\rho}\bar{U}^2\frac{\partial V'}{\partial t} + 2\bar{U}\bar{V}\bar{\rho}\frac{\partial U'}{\partial t}\right]_1 \qquad (26)$$

In Equations 24, 25, and 26, all quantities are nondimensionalized. The solution to these equations will next be presented.

After separating the equations into real and imaginary parts, a method of substitution is used to solve for the unsteady, complex flow field. All flow parameters within the blade channel are assumed to vary harmonically with time, as follows:

$$f' = \bar{f}'e^{ikt} \qquad (27)$$

where f is any flow parameter within the blade channel. The two-dimensional upstream flow field is converted to a one-dimensional flow field utilizing a technique commonly known in analyzing the turbulent channel flows (Reference 17):

$$\bar{f}' = \frac{1}{\bar{y}}\int_0^{\bar{y}} f'\,dy \qquad (28)$$

The energy equation is utilized to solve for the complex constant ($A_{-\infty}$) describing the upstream flowfield. The result is as follows:

$$A_{-\infty R} = E_{16}\bar{A}'_{1R} + E_{17}\bar{A}'_{1I} + E_{18}\bar{\rho}'_R{}^* + E_{19}\bar{\rho}'_I{}^* + E_{20}\bar{A}'_R{}^* + E_{21}\bar{A}'_I{}^* +$$
$$E_{22}\bar{a}'_R{}^* + E_{23}\bar{a}'_I{}^* + E_{24}\bar{V}'_{1I} + E_{25}\bar{V}'_{1R} \qquad (29)$$

$$A_{\infty I} = E_{26}\bar{A}'_{1R} + E_{27}\bar{A}'_{1I} + E_{28}\bar{\rho}'_R{}^* + E_{29}\bar{\rho}'_I{}^* + E_{30}\bar{A}'_R{}^* + E_{31}\bar{A}'_I{}^* +$$
$$E_{32}\bar{a}'_R{}^* + E_{33}\bar{a}'_I{}^* + E_{34}\bar{V}'_{1I} + E_{35}\bar{V}'_{1R} \qquad (30)$$

The derivations of Equations 29 and 30 are presented in Appendix B. The E constants in the above equations are a function of the steady flow parameter and are presented in detail in Appendix E. Calculation of the steady flow parameters appears in Appendix F. Area and volume perturbations are a function of the known vibrational mode shapes. The calculation procedure appears in Appendix G. The two complex unknowns in Equations 29 and 30, $\bar{\rho}'^*$ and $\bar{a}'^*$, are obtained through the use of the momentum and continuity equations.

Using the momentum equation to solve for the complex density perturbation at the throat gives:

$$\bar{\rho}'_R{}^* = M_{32}\bar{A}'_{iR} + M_{33}\bar{A}'_{iI} + M_{34}\bar{A}'_R{}^* + M_{35}\bar{A}'_I{}^* + M_{36}\bar{a}'_R{}^* + M_{37}\bar{a}'_I{}^* + M_{38}\bar{V}'_{1I} + M_{39}\bar{V}'_{1R} \tag{31}$$

$$\bar{\rho}'_I{}^* = M_{40}\bar{A}'_{iR} + M_{41}\bar{A}'_{iI} + M_{42}\bar{A}'_R{}^* + M_{43}\bar{A}'_I{}^* + M_{44}\bar{a}'_R{}^* + M_{45}\bar{a}'_I{}^* + M_{46}\bar{V}'_{1I} + M_{47}\bar{V}'_{1R} \tag{32}$$

The M constants represent a function of the steady flow parameters. The derivation of Equations 31 and 32 appear in Appendix B. The final unknown, $\bar{a}^{*\prime}$, is obtained through the use of the continuity Equation (24). The result is:

$$\bar{a}'_R{}^* = C_{46}\bar{A}'_{iR} + C_{47}\bar{A}'_{iI} + C_{48}\bar{A}'_R{}^* + C_{49}\bar{A}'_I{}^* + C_{50}\bar{V}'_{1I} + C_{51}\bar{V}'_{1R} \tag{33}$$

$$\bar{a}'_I{}^* = C_{52}\bar{A}'_{iR} + C_{53}\bar{A}'_{iI} + C_{54}\bar{A}'_R{}^* + C_{55}\bar{A}'_I{}^* + C_{56}\bar{V}'_{1I} + C_{57}\bar{V}'_{1R} \tag{34}$$

The C constants also represent functions of the steady flow parameters and are presented in Appendix E. The derivations of Equations 33 and 34 also appear in Appendix B. This completes the analysis for Region 1.

Perturbation Equations for Region 2

Region 2 in Figure 6 represents the supersonic region from the blade throat (M=1) to the steady-state shock position. Since all the inlet flow parameters are known, only two flow parameters need to be found: (1) perturbation velocity (U_{US}) and (2) density (ρ'_{US}) on the upstream side of the shock. Thus, only the continuity and momentum equations are needed to solve for the density and velocity perturbations. Dealing first with the continuity equation, recall that:

$$\frac{dm}{dt} = 0 = \iint \rho U dA + \frac{\partial}{\partial t}\iiint \rho d\,(Vol) \tag{16}$$

which becomes for Region 2:

$$\rho_{us}U_{us}A_s - \rho^*A^*a^* = \frac{-\partial \rho V_2}{\partial t} \tag{35}$$

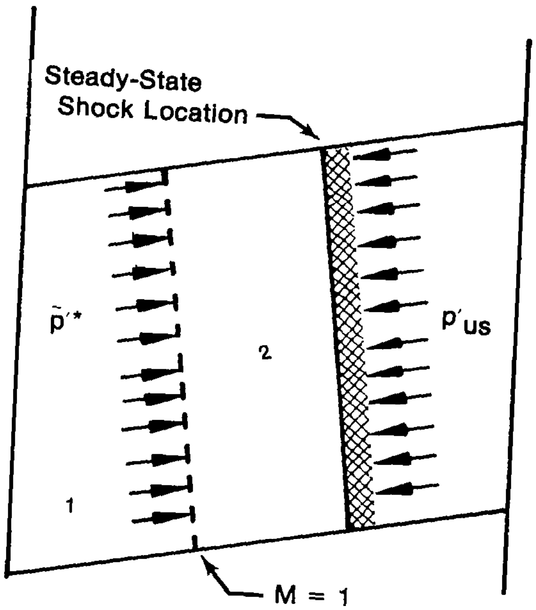

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Figure 6. Region 2 from Channel Throat (M = 1) to the Steady-State Shock Location

Assuming small perturbations produces:

$$(\rho'_{us}\bar{U}_{us}A_s + \bar{\rho}_{us}U'_{us}\bar{A}_s + \bar{\rho}_{us}\bar{U}_{us}A'_s) - (\rho'^*\bar{A}^*\bar{a}^* + \bar{\rho}^*\bar{A}^*a'^* + \bar{\rho}^*A'^*\bar{a}^*) = -\left(\bar{V}_2 \frac{\partial \rho'}{\partial t} + \bar{\rho}\frac{\partial V'_2}{\partial t}\right)_2 \tag{36}$$

Following similar lines to the derivation of the momentum equation in Region 1, the resultant equation becomes:

$$(-p'_{us}\bar{A}_s - \bar{p}_{us}A'_s + p'^*\bar{A}^* + \bar{p}^*A'^*)$$
$$= (U'_{us}\bar{w}_{us} + \bar{U}_{us}w'_{us}) - (a'^*\bar{w}^* + \bar{a}^*w'^*) +$$
$$\bar{U}_2\ \bar{V}_2 \frac{\partial \rho'_2}{\partial t} + \bar{\rho}_2\bar{U}_2 \frac{\partial V'_2}{\partial t} + \bar{\rho}_2\bar{V}_2\frac{\partial U'_2}{\partial t} \tag{37}$$

These equations are nondimensionalized in the same manner as in Region 1 to yield the following equations for continuity and momentum:

Continuity Equation

$$(\rho'_{us}\bar{U}_{us}\bar{A}_s + \bar{\rho}_{us}U'_{us}\bar{A}_s + \bar{\rho}_{us}\bar{U}_{us}A'_s) - (\rho'^*\bar{a}^* + \bar{\rho}^*a'^* + \bar{\rho}^*A'^*\bar{a}^*)$$
$$= -\left(\bar{V}_2\frac{\partial \rho'}{\partial t} + \bar{\rho}\frac{\partial V'_2}{\partial t}\right)_2 \tag{38}$$

Momentum Equation

$$(-p'_{us}\bar{A}_s - \bar{p}_{us}A'_s + p'^* + \bar{p}^*A'^*)$$
$$= (U'_{us}\bar{w}_{us} + \bar{U}_{us}w'_{us}) - [\bar{\rho}^*a'^*\bar{a}^* + \bar{a}^*(\rho'^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + \bar{\rho}^*a'^*)] +$$
$$\bar{U}_2\bar{V}_2\frac{\partial \rho'_2}{\partial t} + \bar{\rho}_2\bar{U}_2\frac{\partial V'_2}{\partial t} + \bar{\rho}_2\bar{V}_2\frac{\partial U'_2}{\partial t} \tag{39}$$

The momentum equation is used to obtain the expression for the density perturbation upstream of the shock to yield:

$$\bar{\rho}'_{USI} = M_{52}\bar{A}'_{SR} + M_{53}\bar{A}'_{SI} + M_{54}\bar{p}'^*_R + M_{55}\bar{p}'^*_I + M_{56}\bar{A}'^*_R +$$
$$M_{57}\bar{A}'^*_I + M_{58}\bar{U}'_{USR} + M_{59}\bar{U}'_{USI} + M_{60}\bar{a}'^*_I + M_{61}\bar{a}'^*_R +$$
$$M_{62}\bar{\rho}'^*_I + M_{63}\bar{\rho}'^*_R + M_{64}\bar{V}'_{2I} + M_{65}\bar{V}'_{2R} \tag{40}$$

$$\bar{\rho}'_{USR} = M_{66}\bar{A}'_{SR} + M_{67}\bar{A}'_{SI} + M_{68}\bar{p}'^*_R + M_{69}\bar{p}'^*_I + M_{70}\bar{A}'^*_R +$$
$$M_{71}\bar{A}'^*_I + M_{72}\bar{U}'_{USR} + M_{73}\bar{U}'_{USI} + M_{74}\bar{a}'^*_I + M_{75}\bar{a}'^*_R +$$
$$M_{76}\bar{\rho}'^*_I + M_{77}\bar{\rho}'^*_R + M_{78}\bar{V}'_I + M_{79}\bar{V}'_R \tag{41}$$

The derivation of Equations 40 and 41 appears in Appendix C. The expressions for the density perturbations contain one unknown: the complex velocity perturbation upstream of the shock ($\bar{U}'_{US}$). The continuity equation (37) gives this relationship:

$$\bar{U}'_{USR} = C_{73}\bar{A}'_{SR} + C_{74}\bar{A}'_{SI} + C_{75}\bar{\rho}'^{*}_{R} + C_{76}\bar{\rho}'^{*}_{I} + C_{77}\bar{a}'^{*}_{R} + C_{78}\bar{a}'^{*}_{I} + C_{79}\bar{p}'^{*}_{R} + C_{80}\bar{p}'^{*}_{I} + C_{81}\bar{A}'^{*}_{R} + C_{82}\bar{A}'^{*}_{I} + C_{83}\bar{V}'_{I} + C_{84}\bar{V}'_{R} \quad (42)$$

$$\bar{U}'_{USI} = C_{85}\bar{A}'_{SR} + C_{86}\bar{A}'_{SI} + C_{87}\bar{\rho}'^{*}_{R} + C_{88}\bar{\rho}'^{*}_{I} + C_{89}\bar{a}'^{*}_{R} + C_{90}\bar{a}'^{*}_{I} + C_{91}\bar{p}'^{*}_{R} + C_{92}\bar{p}'^{*}_{I} + C_{93}\bar{A}'^{*}_{R} + C_{94}\bar{A}'^{*}_{I} + C_{95}\bar{V}'_{R} + C_{96}\bar{V}'_{I} \quad (43)$$

The derivation of Equations 42 and 43 is presented in Appendix C. The M and C coefficients are found in Appendix E.

This completes the analysis for Region 2. The unsteady shock wave movements are next defined along with equations describing flow discontinuities across the shock.

Unsteady Shock Movement

For a normal shock wave moving at a velocity, Us with respect to the channel, the pressures on the two sides of the wave relate in the following manner (Reference 18):

$$p_{ds} = p_{us}\left[\frac{2\gamma\left(M_{US} - \frac{U_{US}}{a_{us}}\right) - (\gamma - 1)}{\gamma + 1}\right] \quad (44)$$

Assuming small perturbations and nondimensionalizing Equation 44 yields:

$$\frac{2\gamma}{\gamma + 1}\bar{p}_{us}U'_{s} = \left[\bar{p}_{us}\bar{M}_{us} - \bar{p}_{ds} - \left(\frac{\gamma-1}{\gamma+1}\right)\bar{p}_{us}\right]a'_{us} + \left[\frac{2\gamma}{\gamma+1}\bar{a}_{us}\bar{M}_{us} - U_{s} - \left(\frac{\gamma-1}{\gamma+1}\right)\bar{a}_{us}\right]p'_{us} + \left[\frac{2\gamma}{\gamma+1}\bar{a}_{us}\bar{p}_{us}M'_{us} - \bar{a}_{us}p'_{ds.}\right] \quad (45)$$

Again, all flow parameters are assumed to vary harmonically with time. The change in shock position can be related to the shock velocity perturbation by:

$$U'_{s} = \bar{U}'_{s}\exp ikt = \frac{\partial x'_{s}}{\partial t} \quad (46)$$

Integrating Equation 46 gives:

$$\bar{x}'_{SR} = \frac{\bar{U}'_{SI}}{k}$$

$$\bar{x}'_{SI} = -\frac{\bar{U}'_{SR}}{k} \quad (47)$$

Substituting Equation 46 into Equation 45, dividing by e^{ikt}, and solving for $\bar{U}'_S$ produces:

$$\bar{U}'_{SR} = \frac{1}{S_1}\left[S_2\bar{a}'_{usR} + S_3\bar{p}'_{usR} + S_4 \frac{U'_{usR}}{\bar{a}_{us}} - \bar{a}_{us}\bar{p}'_{dsR} \right]$$

$$\bar{U}'_{SI} = \frac{1}{S_1}\left[S_2\bar{a}'_{usI} + S_3\bar{p}'_{usI} + S_4 \frac{U'_{usI}}{\bar{a}_{us}} - \bar{a}_{us}\bar{p}'_{dsI} \right] \qquad (48)$$

where the S coefficients are presented in Appendix E.

The pressure perturbation upstream of the shock can now be related to the density perturbation in the following manner:

$$\bar{p}'_{US} = \bar{a}^2_{US}\bar{\rho}'_{US}$$

and to the speed of sound perturbation as

$$\bar{a}'_{US} = \left(\frac{\gamma - 1}{2}\right)\left(\frac{\bar{a}_{US}}{\bar{\rho}_{US}}\right)\rho'_{US}$$

Thus, there exists only one unknown in Equation 48: $\bar{p}'_{ds}$. The downstream pressure perturbation is found by expanding the following equation in small perturbation form. The density discontinuity across the shock is

$$\rho_{ds}\,[(\gamma - 1)\,M_{US}^2 + 2] = \rho_{US}\,[(\gamma + 1)\,M_{US}^2\,] \qquad (49)$$

This equation in small perturbation form can be expressed as

$$\bar{\rho}'_{dsR} = S_8\,\bar{U}'_{usR} + S_9\,\bar{\rho}'_{usR}$$

$$\bar{\rho}'_{dsI} = S_8\,\bar{U}'_{usI} + S_9\,\bar{\rho}'_{usI} \qquad (50)$$

Substituting the relationships of Equation 50 into Equation 48 produces the shock perturbation velocity.

The velocity perturbation downstream of the shock is found by satisfying the continuity across the shock, as noted in the following equation:

$$\rho_{us}U_{us}A_s = \rho_{ds}U_{ds}A_s \qquad (51)$$

Expanding Equation 51 in small perturbation form and solving for $\bar{U}'_{ds}$ gives

$$\bar{U}'_{dsR} = \frac{1}{\bar{\rho}_{ds}}\left[\bar{\rho}'_{usR}\,\bar{U}_{us} + \bar{\rho}_{us}\,\bar{U}'_{usR} - \bar{\rho}'_{dsR}\,\bar{U}_{ds}\right] \qquad (52)$$

$$\bar{U}'_{dsI} = \frac{1}{\bar{\rho}_{ds}}\left[\bar{\rho}'_{usI}\,\bar{U}_{us} + \bar{\rho}_{us}\,\bar{U}'_{usI} - \bar{\rho}'_{dsI}\,\bar{U}_{us}\right] \qquad (53)$$

This completes the relationships describing the flow perturbations across the normal shock.

Perturbation Equations for Region 3

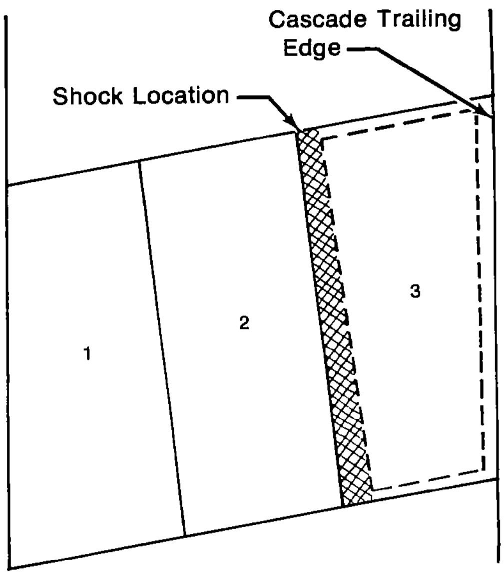

Figure 7. Region 3 from the Steady-State Shock Location to the Cascade Trailing Edge

Region 3 is the subsonic intrablade region from the shock location to the cascade trailing edge, as shown in Figure 7. The unsteady flow field is derived using the perturbation relationships for the flow entering the region across the normal shock and exiting the cascade into the downstream flow field. The unsteady flow field exiting Region 3 is represented as the sum of two flow fields: (1) the irrotational part derived previously in the section on upstream and downstream irrotational flow, and (2) a rotational part related to the vorticities being shed off the trailing edge of the blades due to the unsteady vibratory motion.

Rotational Downstream Flowfield

Lacking viscosity, the equation of motion in vorticity form for two-dimensional flow is (Reference 19):

$$\rho \frac{D\zeta}{Dt} - \zeta \frac{D\rho}{Dt} = 0 \tag{54}$$

In small perturbation form this equation becomes

$$\frac{D\zeta'}{Dt} - \frac{\bar{\zeta}}{\bar{\rho}} \frac{D\rho'}{Dt} = 0 \tag{55}$$

where, D/Dt represents the substantial derivative.

The vorticity of a nonvicous field remains zero, if at the beginning the vorticity was equal to zero and the fluid is only subjected to the forces which have a potential associated with them. A shock wave does not fit into this category. However, according to Crocco's Theorem (Reference 20), if the fluid passes through a stationary shock wave, the flow can conserve its irrotational character only if the entropy rise is uniform across the shock. This is the case for the normal shock in a one-dimensional channel flow. Thus, the mean flow vorticity is zero. Using this fact in Equation 55 produces

$$\frac{D\zeta'}{Dt} = 0 \tag{56}$$

A solution to Equation 56 is assumed.

$$\zeta\,(x,y,t) = Z_{+\nu} \exp i\,(Rx + Cy + kt) \tag{57}$$

where C is defined in Equation 9 and k is the reduced frequency. Substituting Equation 57 into Equation 56 and solving for R results in

$$R = \frac{-\,(k + \bar{v}_E\, C)}{\bar{u}_E} \tag{58}$$

The vorticity can be related to the stream function as follows (Reference 21):

$$\nabla^2 \psi' = -\zeta'$$

where,

$$u' = \frac{\partial \psi'}{\partial y}$$

$$v' = -\frac{\partial \psi'}{\partial x}$$

A solution for the stream function ψ exists in the following form:

$$\psi' = \frac{Z_{+\infty}}{R^2 + C^2} \exp i\,(Rx + Cy + kt) \tag{59}$$

which will give the rotational velocity perturbation at the exit.

Continuity and Momentum Equations for Region 3

Because there are only two unknowns in the downstream flow field, the complex irrotational and rotational constants, the momentum and continuity equations are all that are required. The nondimensionalized small perturbation forms of these equations are

Continuity

$$[(\rho'_E \bar{A}_E \bar{U}_E + \bar{\rho}_E A'_E \bar{U}_E + \bar{\rho}_E \bar{A}_E U'_E) - (\bar{\rho}_{ds} \bar{A}_s U'_{ds} + \bar{\rho}_{ds} A'_s \bar{U}_{ds} + \rho'_{ds} \bar{A}_s \bar{U}_{ds})] = \left[\bar{\rho} \frac{\partial V'}{\partial t} + \bar{V} \frac{\partial \rho'}{\partial t}\right]_3 \tag{60}$$

Momentum

$$(p'_{ds} \bar{A}_s + \bar{p}_{ds} A'_s - p'_E \bar{A}_E - \bar{p}_E A'_E) = (U'_E \bar{w}_E + \bar{U}_E w'_E) - (U'_{ds} \bar{w}_{ds} + \bar{U}_{ds} w'_{ds}) + \bar{U}_{ave} \bar{V}_3 \frac{\partial \rho'_3}{\partial t} + \bar{\rho}_3 \bar{U}_3 \frac{\partial V'_3}{\partial t} + \bar{\rho}_3 \bar{V}_3 \frac{\partial U'_3}{\partial t} \tag{61}$$

The momentum equation is used to solve for the complex constant $A_{+\infty}$ to produce

$$A_{+\infty R} = Z_{+\infty} M_{82} + Z_{+\infty I} M_{83} + RC\, M_{84} + IC\, M_{85} \tag{62}$$

$$A_{+\infty I} = Z_{+\infty R} M_{86} + Z_{+\infty I} M_{87} + RC\, M_{88} + IC\, M_{89} \tag{63}$$

The M constants, along with RC and IC, appear in Appendix E.

The continuity equation is used to solve for the complex constant describing the rotational downstream flow field resulting in

$$Z_{+\infty R} = RC\, C_{112} + IC\, C_{113} + C_{114} \tag{64}$$

$$Z_{+\infty I} = RC\, C_{115} + IC\, C_{116} + C_{117} \tag{65}$$

where the C constants are found in Appendix E.

Equations 64 and 65 can then be substituted into Equations 62 and 63 to solve for $A_{+\infty}$ giving a complete description of the flow field in Region 3. The derivation of Equations 62 through 65 is presented in Appendix D.

Thus, with the solution of the flow field in Region 3, a complete description of the cascade flow field is obtained. A compilation of the computer code for the semiactuator disk model is presented in Appendix J.

RESULTS

The semi-actuator disk theory was evaluated in two ways. The first method consisted of a comparison with test data presented by Tanida and Saito (Reference 22) for an isolated airfoil oscillating in choked flow in a wind tunnel. The second method involved a flutter analysis of the F100(3) 6th stage of the high-pressure compressor of the F100(3) turbofan engine, which encountered choke flutter while operating at off-design conditions in a core engine (no low rotor) at the Arnold Engineering Development Center (AEDC).

Wind Tunnel Test Data

Tanida and Saito oscillated an airfoil in a wind tunnel at constant amplitude for various combinations of inlet Mach number, back pressure, reduced frequency twist axis location, and tunnel wall separation, and recorded both steady and unsteady aerodynamic characteristics. Because reflections from the tunnel walls create a special case in a cascade in which adjacent blades are exactly out of phase, the experimental test conditions were simulated with an interblade phase angle of 180 deg. After calculating unsteady pressures through the blade channel, the unsteady lift and momentum coefficients were calculated in the manner presented in Appendix I.

General agreement was achieved for the imaginary moment due to pitch for back-pressure ratios less than 0.7, as shown in Figure 8. At higher back-pressure ratios, flow is not fully transonic during the full cycle of operation; i.e., a weak shock appears and disappears on the airfoil surface. A basic assumption of the analytical model stipulates that flow is fully transonic, and a strong normal shock exists in the blade passage throughout the oscillatory cycle. Because this was not the case at high back-pressure ratios, good correlation was not anticipated.

Flutter Analysis

Computational Method

Investigations also involved an assessment of the semi-actuator disk unsteady aerodynamics by combining the model with existing P&WA cyclic work and aerodynamic damping calculations, and then performing a flutter analysis for the F100(3) sixth compressor stage, which experienced choke flutter at off-design conditions. The P&WA approach to flutter prediction stems from a cyclic energy method in which total system damping is calculated. The system becomes unstable when total damping, which is comprised of aerodynamic and mechanical damping components, is less than zero; i.e.,

$$\delta_{TOT} = (\delta_{aero} + \delta_{mech}) < 0 \Longrightarrow \text{flutter}$$

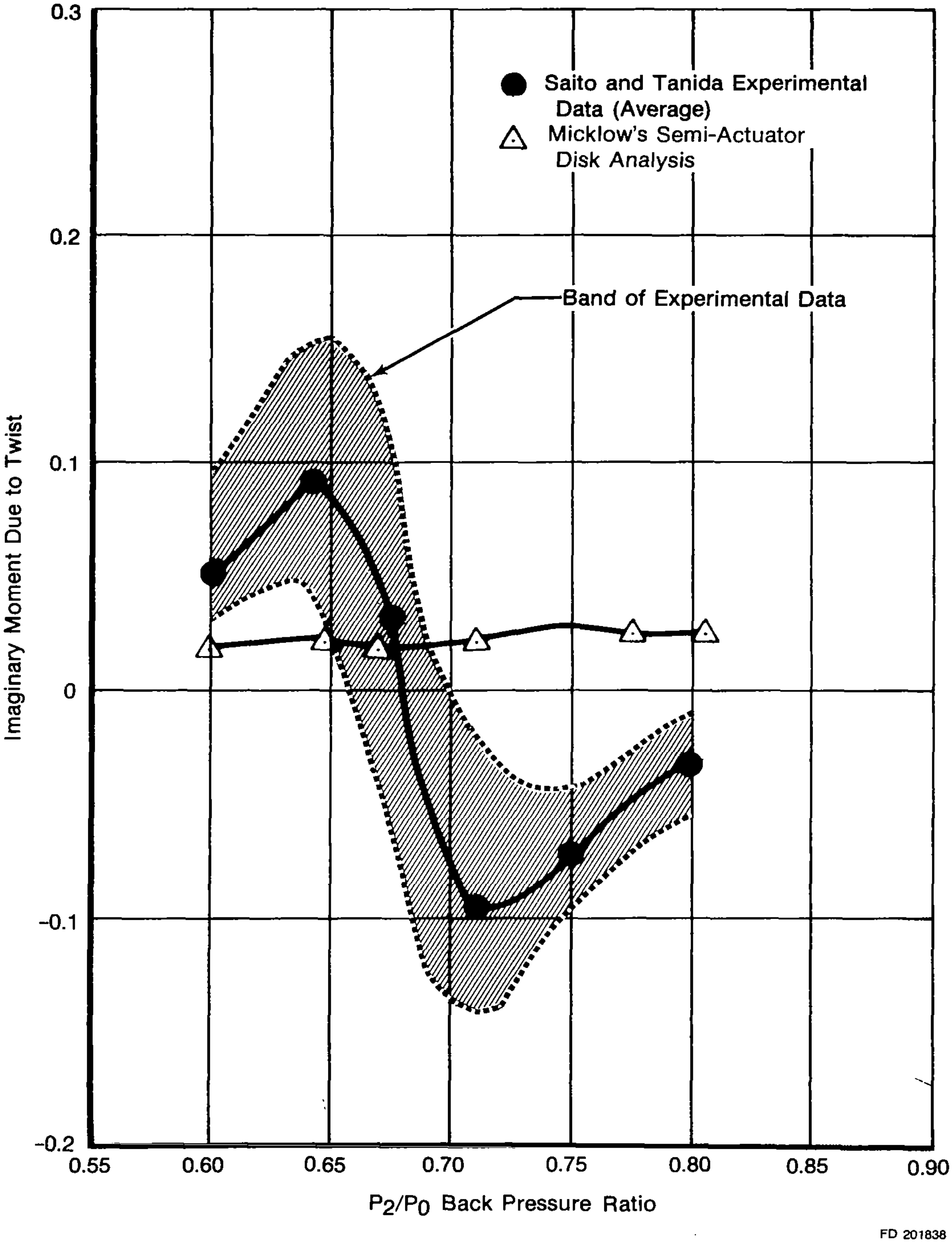

Figure 8. Comparison Between Wind Tunnel Data for Isolated Airfoil and Semi-Actuator Disk Analysis

No method currently exists to determine mechanical damping. Thus, stability predictions are based on correlations of aerodynamic damping for an assumed level of mechanical damping. To calculate δ_{aero}, a quasi-three-dimensional analysis is used in which two-dimensional unsteady aerodynamic work (W) is calculated for strips along the airfoil span. As shown by Carta (Reference 23), the two-dimensional work can be expressed as

$$W = \rho \pi b^2 U^2 k^2 \left\{ L_{HI} \bar{h}^2 + \bar{\alpha} \bar{h} \left[(L_{\alpha R} - M_{HR}) \sin \theta + (L_{\alpha I} + M_{HI}) \cos \theta \right] + M_{\alpha I} \bar{\alpha}^2 \right\}$$

where,

L	=	unsteady lift coefficient
M	=	unsteady moment coefficient
$\bar{\alpha}$	=	normalized mode shape deflection of maximum twist
$\bar{h}$	=	normalized mode shape deflection of maximum bending
θ	=	phase relationship between α and h.

Numerically integrating the two-dimensional work along the span produces total unsteady cyclic work for one blade (W_{TOT}). The logarithmic decrement, or aerodynamic damping, is then calculated as

$$\delta_{aero} = - \frac{n W_{TOT}}{4KE}$$

where,

n	=	number of blades in the system
KE	=	normalized average kinetic energy of the system vibrating in the normalized mode.

Flutter Prediction Results

A flutter analysis of the F100 sixth stage compressor operating at sea level conditions was performed, and a summary of the data points, operating conditions and predicted damping values is presented in Table 2. The model predicted the first bending mode to be least stable, which is consistent with the results observed by Lubomski (Ref. 24). However, test data showed Rotor 6 encountered negative incidence flutter in the second coupled mode, with some secondary first-mode response.

For both the first bending and second coupled modes, the model shows a distinct difference in damping level between flutter and non-flutter points, with all of the flutter points analyzed having a negative damping value, indicating an unstable condition as seen in Figure 9 and 10. In general, the model predicts the correct trend with increasing speed, i.e., a decrease in stability with increasing speed.

TABLE 2. SUMMARY OF LEAST-STABLE NODAL DIAMETERS AND DAMPING VALUES FOR SEA LEVEL CONDITIONS

Test Data				Stability Calculations		
					Aerodynamic Damping δ_{aero}	
Data Point	*Test Stability*	*% N_{cor}*	*Rear Compressor Variable Vane, RCVV*	*Least Stable Nodal-Dia.*	*Smith*	*Semi-Actuator Disk Theory*
F100(3) Rotor 6 1st Bending Mode						
AA02PT3	NF	80.0	−30.0	2	0.0164	0.0102
AC06PT8	NF	88.7	−30.0	2	0.02978	−0.00104
AC06PT9	FB	89.75	−33.0	4	0.0558	−0.01394
AC06PT10	F	91.4	−37.2	3	0.04605	−0.0110
AC06PT11	FB	90.21	−29.2	3	0.05793	−0.01313
AC06PT12	F	92.4	−29.0	3	0.05516	−0.0106
AC05PT29	NF	84.76	−20.0	2	0.03718	0.03718
AC05PT30	NB	99.68	−20.8	4	0.0795	−0.0356
AC05PT31	F	102.00	−21.0	4	0.06492	−0.03990
F100(3) Rotor 6 2nd Coupled Mode						
AA02PT3	NF	80.0	−30.0	3	0.000125	0.000101
AC06PT8	NF	88.7	−30.0	3	0.000275	0.000097
AC06PT9	FB	89.75	−33.0	6	0.01894	−0.00101
AC06PT10	F	91.4	−37.2	6	0.01334	−0.00053
AC06PT11	FB	90.21	−29.2	6	0.0182	−0.00097
AC06PT12	F	92.4	−29.0	4	0.00170	−0.00077
AC05PT29	NF	84.76	−20.0	2	0.000354	0.000354
AC05PT30	FB	99.68	−20.8	6	0.0299	−0.00130
AC05PT31	F	102.0	−21.0	6	0.0243	−0.00166

where,
NF = No Flutter
FB = Flutter Boundary
F = Flutter

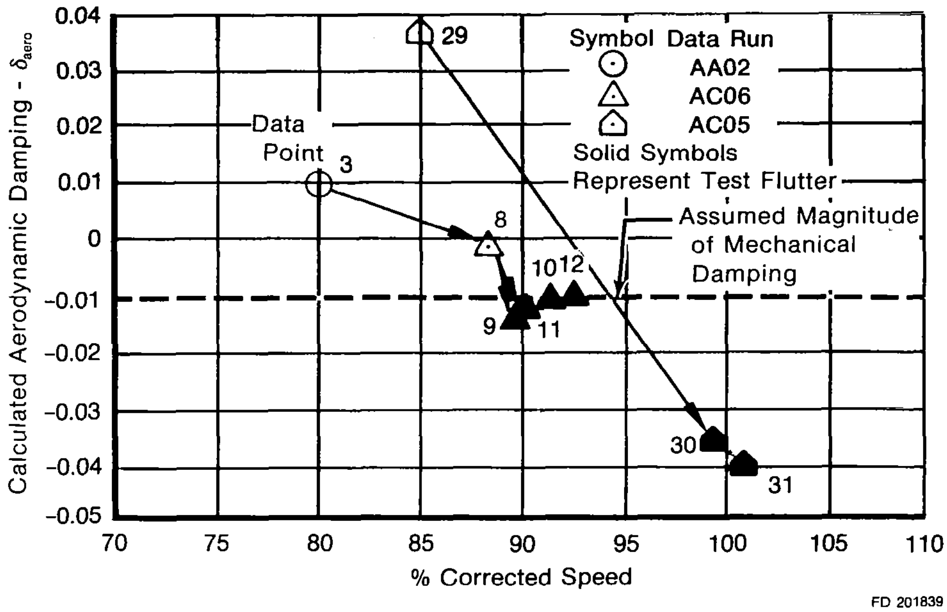

Figure 9. Comparison of Calculated Aerodynamic Damping and Observed Stability for the First Vibratory Mode of the F100 6th-Stage Compressor

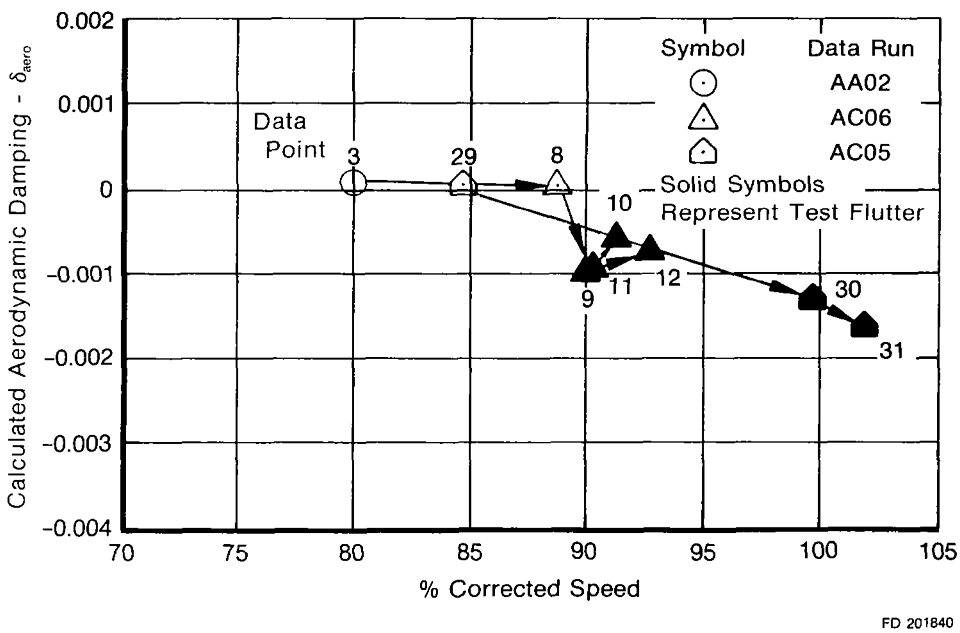

Figure 10. Comparison of Calculated Aerodynamic Damping and Observed Stability for the Second Vibratory Mode of the F100 6th-Stage Compressor

The slight discrepancy in this trend exhibited by the calculated damping values for flutter data points AC06-10 and -12 may be attributed to inaccuracies in the steady streamline aerodynamic input. The inaccuracies arise due to the difficulty in predicting the steady aerodynamic environment while the compressor is operating well off the design condition in flutter. The model is very sensitive to inlet air angle, relative inlet and exit Mach numbers, and static pressure ratio across the stage. The inlet Mach number and air angle are used to check for choked flow in the blade passage, and the static pressure ratio is used to locate the steady-state shock location. For data points AC06-9 and -11, the static pressure ratio across the stage was matched within 2 percent, while for AC06-10, the error was as high as 15 percent. This magnitude of discrepancy in matching the stage static pressure ratio could lead to inaccuracies in locating the steady-state normal shock. In the analysis, the chordwise location of the shock greatly affects the magnitude and sign of the unsteady aerodynamic coefficients, thus greatly affecting the damping calculation. It was noted that data points AC06-9 and -11 were calculated to be choked along the entire span, while AC06-10 and -12, operating deeper into the flutter boundary, were not. This has a large effect on the stability calculation because if the flow is determined to be unchoked, the unsteady, zero-incidence Smith coefficients (Reference 2) are used in place of the semi-actuator disk coefficients. The Smith coefficients always predicted the rotor to be stable.

The magnitude of the inlet and exit Mach numbers proved to be important parameters in describing the upstream and downstream flow fields. With increasing inlet Mach number, the unsteady lift due to flap increased in a destabilizing manner and with increasing exit Mach number. The unsteady moment due to twist showed a similar trend. Data points AC05-30 and -31 are approximately double the value of the negative damping of the other flutter points. This is due in part to the greater inlet and exit Mach numbers. The magnitude of the inlet air angles are related to the magnitude of the inlet Mach number through the continuity equation in the steady streamline analysis. It is noted that the streamline analysis indicated data point AC06-9 was operating above β_{max} at the tip. The air angle is related to the flow area; therefore, to satisfy continuity the inlet Mach number must decrease at this station. This is particularly significant because the maximum unsteady work occurs at the tip. A similar trend was noted at other sections. AC05-30 had similar trends, but not as severe, giving a more accurate inlet Mach number. It is speculated that a more accurate representation of AC06-9 would give a larger inlet Mach number and increase the damping in a negative sense, bringing the flutter points closer to the same damping level.

By plotting the aerodynamic damping versus corrected speed, as in Figures 9 and 10, or versus variable inlet guide vane setting, as in Figure 11, a predicted flutter boundary can be obtained by assuming a value for mechanical damping. As shown in Figure 12, the results of the flutter analysis prove to be conservative, particularly at higher corrected speeds. This conservatism is believed to be due to the inlet Mach number and air angle discrepancy discussed above resulting in two negative damping levels. A summary of the least stable nodal diameters and damping values is presented in Table 2 for the 1st bending and 2nd coupled modes of vibration. All the predicted least stable nodal diameters are relatively small because the model encountered numerical difficulties at large interblade phase angles as shown in Figure 13.

The General Electric annular cascade data (Reference 25), which was originally to be analyzed, had to be abandoned because the published data gave an inadequate description of the steady flow field. No exit data were published and attempts to calculate aerodynamic damping using exit conditions based on PWA 2-dimensional cascade test data produced no meaningful results.

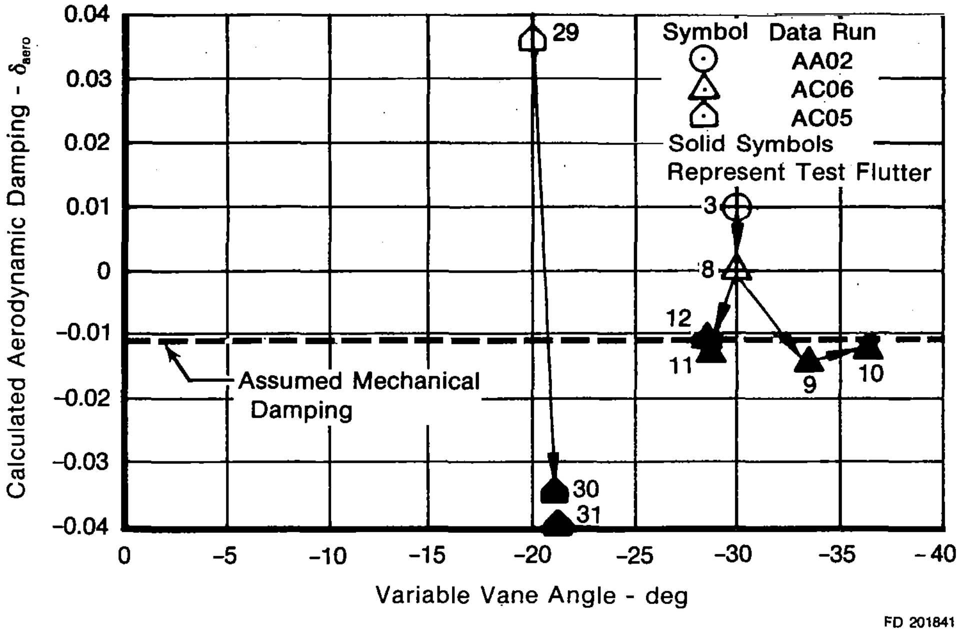

Figure 11. Calculated Aerodynamic Damping Versus Variable Vane Angle With 1st Mode of Vibration, Backward Traveling Wave

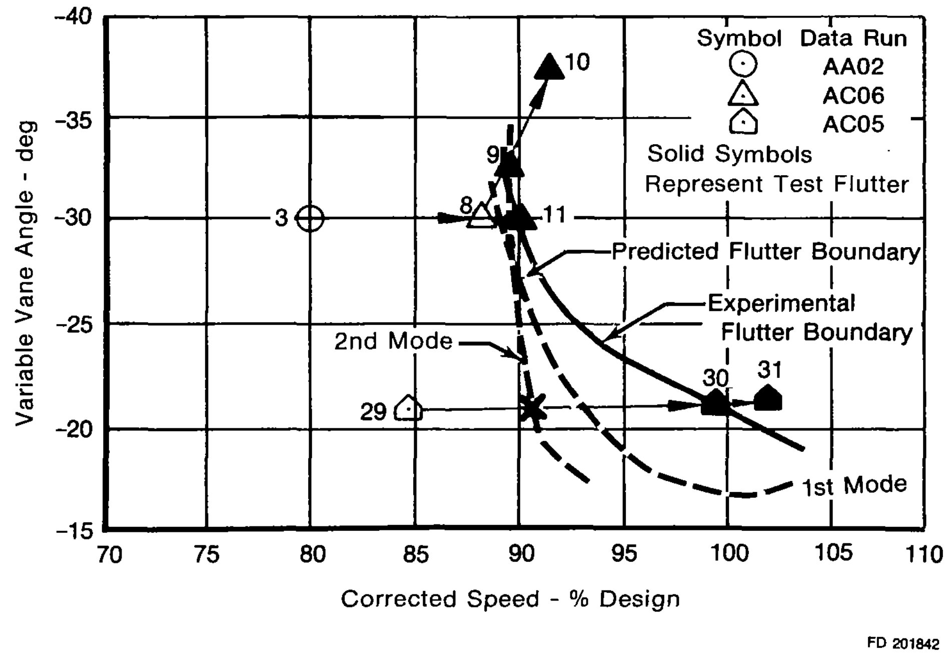

Figure 12. Comparison of Predicted and Experimental Flutter Boundaries

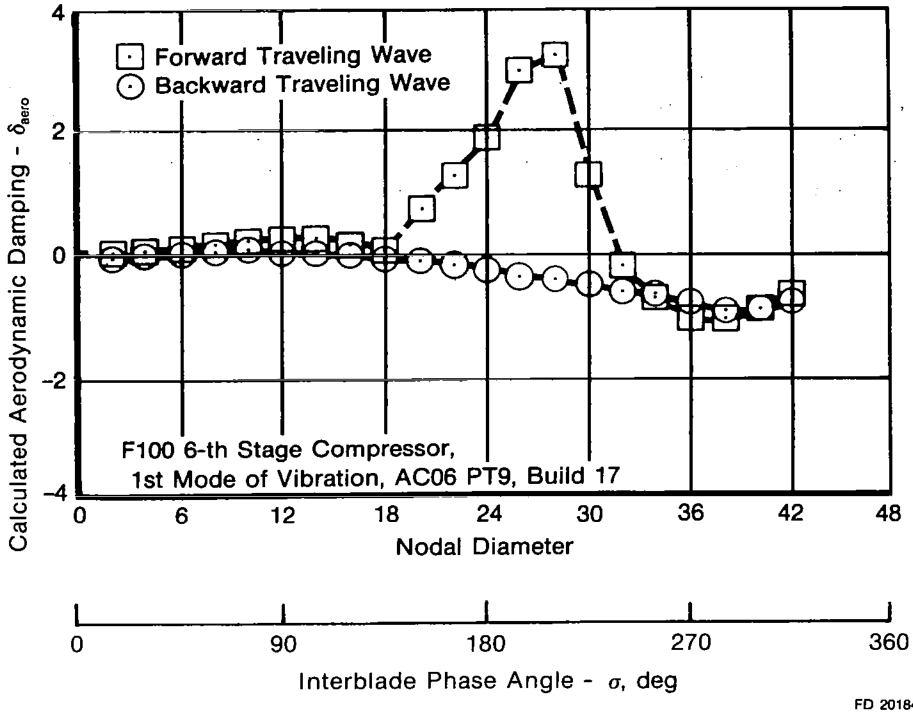

Figure 13. Effect of Nodal Diameter and Interblade Phase Angle on Aerodynamic Damping

Summary of Results

The following is a summarization of the results of the flutter prediction study.

1. The model showed a distinct difference in damping level between flutter and non-flutter points indicating the importance of the unsteady shock in the blade passage.

2. The model shows the correct trend with increasing corrected engine speed and vane angle; i.e., with increasing choked conditions, the model predicts the aerodynamic damping to become less stable.

3. The model was very sensitive to the steady aerodynamic input. In order to perform an accurate flutter analysis, an accurate steady aerodynamic description of the flow field must be obtained.

4. The model was conservative in predicting the flutter boundary at high corrected engine speeds.

5. Due to limitations on interblade phase angle inherent in semiactuator disk theory, the model is limited to interblade phase angles of ±90 deg.

Conclusions

Based on the results of the F100(3) 6th-stage compressor flutter analysis, it is concluded that the model is useful as a conservative choke flutter design system. The model is sensitive to the steady-state aerodynamic input, particularly inlet and exit relative Mach number, inlet air angle, and static pressure ratio across the stage. In order to perform an accurate flutter analysis, an accurate description of the steady flowfield must be obtained.

APPENDIX A

DERIVATION OF THE SMALL PERTURBATION FORM OF THE EQUATIONS OF MOTION IN REGION 1

CONTINUITY EQUATION

The continuity equation for Region 1 takes the form

$$\dot{w}^* - \dot{w}_{inlet} = - \frac{\partial(\rho_1 V_1)}{\partial t} \qquad (A1)$$

Mean Flow

Assuming small perturbations about the mean flow gives the following expressions for the flow rates, neglecting higher order terms.

$$\dot{w}_{inlet} = \bar{\dot{w}}_{inlet} + \dot{w}'_{inlet} = (\bar{\rho}_I + \rho_I')(\bar{A}_I + A_I')(\bar{U}_{inlet} + U')$$

$$= \bar{\rho}_I\bar{A}_I\bar{U}_I + \rho'_I\bar{A}_I\bar{U}_I + \bar{\rho}_I A'_I\bar{U}_I + \bar{\rho}_I\bar{A}_I U'_I$$

Let U'_i be the perturbation velocity aligned with chord line. Then

$$U'_i = u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch}$$

The inlet velocity is $U_{IRE} \cos (\alpha_{ch} - \beta_1)$. The inlet flowrate is then defined as

$$\dot{w}_{inlet} = \bar{\dot{w}}_{inlet} + \dot{w}'_{inlet} = \bar{\rho}_i\bar{A}_i\bar{U}_i \cos (\alpha_{ch} - \beta_1) +$$

$$\bar{\rho}_i\bar{A}_i (v'_i \cos \alpha_{ch} + u'_i \sin \alpha_{ch}) + \bar{\rho}_i\bar{U}_i \cos (\alpha_{ch} - \beta_1) A'_i +$$

$$\rho'_i\bar{A}_i\bar{U}_i \cos (\alpha_{ch} - \beta_1) \qquad (A2)$$

The flowrate at the throat is represented by (A3)

$$\dot{w}^* = \bar{\dot{w}}^* + \dot{w}'^* = \bar{\rho}^*\bar{A}^*\bar{a}^* + \rho'^*\bar{A}^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + \bar{\rho}^*\bar{A}^*a'^*$$

Next, expand the term $\frac{\partial(\rho_1 V_1)}{\partial t}$

$$\frac{\partial(\rho_1 V_1)}{\partial t} = \rho_1 \frac{\partial V_1}{\partial t} + V_1 \frac{\partial \rho_1}{\partial t}$$

By assuming small perturbations, the following relationships emerge:

$$\rho_1 \frac{\partial V_1}{\partial t} = (\bar{\rho}_1 + \rho_1')\left[\underbrace{\frac{\partial \bar{V}_1}{\partial t}}_{0} + \frac{\partial V'_1}{\partial t}\right] = \bar{\rho}_1 \frac{\partial V_1'}{\partial t}$$

$$V_1 \frac{\partial \rho}{\partial t} = (\bar{V}_1 + V'_1)\left[\underbrace{\frac{\partial \bar{\rho}}{\partial t}}_{0} + \frac{\partial \rho'}{\partial t}\right] = \bar{V}_1 \frac{\partial \rho'_1}{\partial t}$$

$$\frac{\partial(\rho V)}{\partial t} = \bar{\rho}_1 \frac{\partial V_1'}{\partial t} + \bar{V}_1 \frac{\partial \rho_1'}{\partial t} \qquad (A4)$$

The small perturbation form of the continuity equation for Region 1 is then:

$$[\rho'^*\bar{A}^*\bar{a}^* + \bar{\rho}^*\bar{A}'^*\bar{a}^* + \bar{\rho}^*\bar{A}^*a'^*] - [\bar{\rho}_i\bar{A}_i\,(u'_i \sin\alpha_{ch} + v'_i \cos\alpha_{ch}) + \bar{\rho}_i A'_i \bar{U}_i \cos(\alpha_{ch} - \beta_1) + \rho'_i \bar{A}_i \bar{U}_i \cos(\alpha_{ch} - \beta_1)]$$

$$= -\left(\bar{\rho}_1 \frac{\partial V'_1}{\partial t} + \bar{V}_1 \frac{\partial \rho'_1}{\partial t}\right) \qquad \text{(A5)}$$

Unsteady Control Volume Form of Momentum Equation

The next step involves the derivation of the unsteady control volume form of the momentum equation. The equation has the form

$$F = \frac{d(mU)}{dt} = \iint_{cs} U(\rho U)dA + \frac{\delta}{\delta t}\iiint_{Vol} U\rho d\,(Vol) \qquad \text{(A6)}$$

with

$$\sum F = p_iA_i - p^*A^* = (\bar{p}_i + p_i')(\bar{A}_i + A_i') - (\bar{p}^* + p'^*)(\bar{A}^* + A'^*)$$

Neglecting higher order terms and subtracting out the mean flow forces gives

$$\sum F' = (p_i'\bar{A}_i + \bar{p}_iA_i') - (\bar{p}^*A'^* + p^{*\prime}\bar{A}^*) \qquad \text{(A7)}$$

Now to evaluate the first part of Equation A6,

$$\iint_{cs} U\,(\rho U \cdot dA)$$

which is essentially

$$a^*\dot{w}^* - U_i\,(\cos(\alpha_{ch} - \beta_1))\,\dot{w}_i$$

In terms of small perturbations, this expands to

$$(\bar{a}^*\bar{w}^* + a'^*\bar{w}^* + \bar{a}^*w'^*) - (\bar{U}_i\bar{w}_i \cos(\alpha_{ch} - \beta_1) + U'_i\bar{w}_i + \bar{U}_i \cos(\alpha_{ch} - \beta_1)\,w_i')$$

Subtracting out the mean flow quantities gives

$$(a'^*\bar{w}^* + \bar{a}^*w'^*) - (u'_i \sin\alpha_{ch} + v'_i \cos\alpha_{ch})\,\bar{w}_i + \bar{U}_i \cos(\alpha_{ch} - \beta_1)\,w_i') \qquad \text{(A8)}$$

To evaluate the second term of Equation A6

$$\frac{\partial}{\partial t}\iiint U\rho\partial\,(Vol) = \frac{\partial}{\partial t}(\rho_1 U_1 V_1)$$

$$= U_1V_1\frac{\partial \rho_1}{\partial t} + \rho_1 U_1 \frac{\partial V_1}{\partial t} + \rho_1 V_1 \frac{\partial U_1}{\partial t}$$

Working with the first term and neglecting higher order terms produces

$$U_1V_1 \quad \frac{\partial \rho_1}{\partial t} = (\bar{U}_1\bar{V}_1 + U'_1\bar{V}_1 + \bar{U}_1V'_1)\left(\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \rho'}{\partial t}\right)$$

$$= \bar{U}_1\bar{V}_1 \quad \frac{\partial \rho'_1}{\partial t}$$

Similarly, for the second and third terms

$$\rho_1 U_1 \frac{\partial V_1}{\partial t} = \bar{\rho}_1 \bar{U}_1 \frac{\partial V'_1}{\partial t}$$

$$\rho_1 V_1 \frac{\partial U_1}{\partial t} = \bar{\rho}_1 \bar{V}_1 \frac{\partial U'_1}{\partial t} \tag{A9}$$

Combining Equations A7, A8, and A9 gives the small perturbation form of the momentum equation, as shown in Equation A10.

$$p'\bar{A}_1 + \bar{p}A'_1 - \bar{p}^*A'^* - p'^*\bar{A}^* = [a'^*\bar{w}^* + \bar{a}^*w'^*] -$$
$$[\bar{w}_1 (u'_1 \sin \alpha_{ch} + v'_1 \cos \alpha_{ch}) + \bar{U}_1 w'_1 \cos (\alpha_{ch} - \beta_1)] +$$
$$\bar{U}_1\bar{V}_1 \frac{\partial \rho'_1}{\partial t} + \bar{\rho}_1\bar{U}_1 \frac{\partial V'_1}{\partial t} + \bar{\rho}_1\bar{V}_1 \frac{\partial U'_1}{\partial t} \tag{A10}$$

Small Perturbation Form of Energy Equation

The next step requires the derivation of the small perturbation form of the energy equation for an inviscid fluid with no external heat addition. The control volume form for Region 1 is therefore

$$\frac{\partial}{\partial t} \iiint_{Vol} \rho e \, d\,(Vol) = - \iint_{cs} (\rho U \cdot n)\, e ds \tag{A11}$$

where,

n = unit vector normal to surface

$$e = \frac{a^2}{\gamma(\gamma - 1)} + \frac{U^2}{2}$$

Substituting this relationship into Equation A11 produces

$$\frac{\partial}{\partial t} \iiint_{vol} \rho \left(\frac{a^2}{\gamma(\gamma - 1)} + \frac{U^2}{2}\right) d\ (Vol)$$

$$= - \iint (\rho U \cdot n) \left(\frac{a^2}{\gamma(\gamma - 1)} + \frac{U^2}{2}\right) ds$$

which becomes

$$\underbrace{\frac{\partial}{\partial t}\left[\rho V\left(\frac{a^2}{\gamma(\gamma-1)}+\frac{U^2}{2}\right)\right]}_{3}$$

$$= -\left[\underbrace{w^*\left(\frac{a^{*2}(\gamma^2-\gamma+2)}{2\gamma(\gamma-1)}\right)}_{2} \quad \underbrace{- w_i\left(\frac{a_i^2}{\gamma(\gamma-1)}+\frac{U_i^2}{2}\right)}_{1}\right] \qquad \text{(A12)}$$

Now, in order to expand by assuming small perturbations.

Term 1 (neglecting higher order terms)

$$-\frac{\gamma^2-\gamma+2}{2\gamma(\gamma-1)}\left[\bar{a}^{*2}\bar{w}^* + 2\bar{a}^*a'^*\bar{w}^* + \bar{a}^{*2}w'^*\right] \qquad \text{(A13)}$$

Term 2 (neglecting higher order terms)

$$(\bar{w}_i + w'_i)\left[\frac{\bar{a}_i^2 + 2\bar{a}_i a_i'}{\gamma(\gamma-1)} + \frac{1}{2}\left[(\bar{U}_i\cos(\alpha_{ch}-\beta_1))^2 + 2\bar{U}_i\cos(\alpha_{ch}-\beta_1)\cdot\right.\right.$$

$$\left.\left.(u'_i\sin\alpha_{ch} + v'_i\cos\alpha_{ch})\right]\right]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\bar{a}_i^2\bar{w}_i + 2\bar{a}_i a_i'\bar{w}_i + \bar{a}_i^2 w_i'\right] + \frac{1}{2}\left[\bar{w}_i(\bar{U}_i\cos(\alpha_{ch}-\beta_1))^2 + \right.$$

$$\left. 2\bar{U}_i\cos(\alpha_{ch}-\beta_1)(u'_i\sin\alpha_{ch} + v'_i\cos\alpha_{ch})\bar{w}_i + w_i'(\bar{U}_i\cos(\alpha_{ch}-\beta_1))^2\right] \qquad \text{(A14)}$$

Term 3

$$\frac{\partial}{\partial t}\left[\rho_1 V_1\left(\frac{a^2_1}{\gamma(\gamma-1)}+\frac{U^2_1}{2}\right)\right] = \underbrace{\frac{1}{\gamma(\gamma-1)}\frac{\partial}{\partial t}(\rho_1 V_1 a^2_1)}_{a} +$$

$$\underbrace{\frac{1}{2}\frac{\partial}{\partial t}(\rho_1 V_1 U^2_1)}_{b}$$

Term 3a

$$\frac{1}{\gamma(\gamma-1)}\left[\frac{\partial(\bar{\rho}_1\bar{V}_1\bar{a}^2)}{\partial t}\to 0 + \frac{\partial(\rho'_1\bar{V}_1\bar{a}_1^2)}{\partial t} + \frac{\partial(\bar{\rho}'_1 V'_1\bar{a}_1^2)}{\partial t} + \frac{2\partial(a'_1\bar{a}_1\bar{\rho}_1\bar{V}_1)}{\partial t}\right]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\bar{V}_1\bar{a}^2\frac{\partial\rho'}{\partial t} + \bar{\rho}\bar{a}^2\frac{\partial V'_1}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}_1\frac{\partial a'}{\partial t}\right], \qquad \text{(A15)}$$

Term 3b

$$\frac{1}{2}\ \frac{\partial(\rho \bar{V} U_1^2)}{\partial t} = \frac{1}{2}\left[\ \bar{V}_1 \bar{U}^2\ \frac{\partial \rho'}{\partial t} + \rho \bar{U}^2\ \frac{\partial V'_1}{\partial t} + 2\bar{U}\bar{\rho}\bar{V}_1\ \frac{\partial U'}{\partial t}\ \right]_1 \tag{A16}$$

Combining Equations A13 through A16 and subtracting out the mean flow equation yields the small perturbation form of the energy equation:

$$\frac{1}{\gamma(\gamma-1)}\ [2\bar{a}_i a'_i \bar{w}_i + \bar{a}_i^2 w'_i] + \frac{1}{2}\ [\bar{w}_i\ (2\bar{U}_i \cos(\alpha_{ch} - \beta_1))\ (u'_i \sin\alpha_{ch} +$$

$$v'_i \cos\alpha_{ch}) + w'_i\ (\bar{U}_i \cos(\alpha_{ch} - \beta_1))^2\ ] - \frac{(\gamma^2 - \gamma + 2)}{2\gamma(\gamma-1)}\ [2\bar{a}^* a'^* \bar{w}^* + \bar{a}^{*2} w'^*]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\ \bar{a}^2\bar{V}_1\ \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{a}^2\ \frac{\partial V'_1}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}_1\ \frac{\partial a'}{\partial t}\ \right]_1 +$$

$$\frac{1}{2}\left[\ \bar{U}^2\bar{V}_1\ \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{U}^2\ \frac{\partial V'_1}{\partial t} + 2\bar{U}\bar{\rho}\bar{V}_1\ \frac{\partial U'}{\partial t}\ \right]_1 \tag{A17}$$

In order to nondimensionalize Equations A14, A15, and A17, densities will be nondimensionalized by $\bar{\rho}_{inlet}$, pressure by $\bar{\rho}_i\bar{U}_i^2$, velocities by $\bar{U}_{IRE}$, lengths by semichord, time by b/U_i, areas by A^*, and volumes by A^*b. Starting with the continuity equation:

$$\left[\bar{\rho}_1\left(\frac{\rho'^*}{\rho_1}\right)\ \bar{A}^*\left(\frac{\bar{A}^*}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{\bar{a}^*}{U_1}\right) + \bar{\rho}_1\left(\frac{\bar{\rho}^*}{\bar{\rho}_1}\right)\bar{A}^*\left(\frac{A'^*}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{\bar{a}^*}{\bar{U}_1}\right) +\right.$$

$$\left.\bar{\rho}_1\left(\frac{\bar{\rho}^*}{\bar{\rho}_1}\right)\bar{A}^*\left(\frac{\bar{A}^*}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{a'^*}{\bar{U}_1}\right)\right] - \left[\ \bar{\rho}_1\left(\frac{\bar{\rho}_1}{\bar{\rho}_1}\right)\ \bar{A}^*\left(\frac{\bar{A}_1}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{u'_1}{\bar{U}_1}\sin\alpha_{ch} +\right.\right.$$

$$\left.\frac{v'_i}{\bar{U}_1}\cos\alpha_{ch}\right) + \bar{\rho}_1\left(\frac{\bar{\rho}_1}{\bar{\rho}_1}\right)\bar{A}^*\left(\frac{A'_1}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{\bar{U}_1}{\bar{U}_1}\right)\ \cdot$$

$$\left.\cos(\alpha_{ch} - \beta_1) + \bar{\rho}_1\left(\frac{\rho'_1}{\bar{\rho}_1}\right)\ \bar{A}^*\left(\frac{\bar{A}_1}{\bar{A}^*}\right)\ \bar{U}_1\left(\frac{\bar{U}_1}{\bar{U}_1}\right)\ \cos(\alpha_{ch} - \beta_1)\right]$$

$$= -\left(\ \bar{\rho}_1\left(\frac{\bar{\rho}_{ave}}{\bar{\rho}_1}\right)\ \left(\bar{A}^*\ b\ \frac{\bar{U}_1}{b}\right)\ \frac{b_1\partial V'_1}{A^* b U_i \partial t} + \frac{\bar{A}^* b \bar{V}_1}{\bar{A}^* b}\ \bar{\rho}_1\ \frac{\bar{U}_i}{b}\ \frac{b_1 \partial\rho' V}{U_i \partial t b \bar{\rho}_i}\ \right)$$

Dividing through by $\bar{\rho}_1 \bar{A}^* \bar{U}_1$ gives

$$[\rho'^* \bar{a}^* + \bar{\rho}^* A'^* \bar{a}^* + \bar{\rho}^* a'^*] - [\bar{A}_i (u'_i \sin \alpha_{ch} + v_i \cos \alpha_{ch}) + \bar{A}_i \cos (\alpha_{ch} - \beta_1) + \rho'_i \bar{A}_i \cos (\alpha_{ch} - \beta_1)]$$

$$= - \left[\bar{\rho} \frac{\partial V'_1}{\partial t} + \bar{V}_1 \frac{\partial \rho'}{\partial t} \right]_1 \quad \text{(A18)}$$

Consider next the momentum equation (Equation A10). The result is, after dividing by $\bar{\rho}_1 \bar{U}_i^2 \bar{A}_1^*$

$$(p'_i \bar{A}_i + \bar{p}_i A'_i - p'^* - \bar{p}^* A'^*)$$

$$= [a'^* \bar{\rho}^* \bar{a}^* + \bar{a}^* (\bar{\rho}^* a'^* + \bar{\rho}^* A'^* \bar{a}^* + \rho'^* \bar{a}^*)] - [(u'_1 \sin \alpha_{ch} + v'_1 \cos \alpha_{ch}) \bar{A}_1 \cos (\alpha_{ch} - \beta_1) + \cos (\alpha_{ch} - \beta_1) (\rho'_1 \bar{A}_1 \cos (\alpha_{ch} - \beta_1) + A'_i \cos (\alpha_{ch} - \beta_1) + \bar{A}_i (u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch}))] + \left(\bar{U}\bar{V}_1 \frac{\partial \rho'}{\partial t} \right)_1 + \left(\bar{\rho}\bar{U} \frac{\partial V'_1}{\partial t} \right)_1 + \left(\bar{\rho}\bar{V}_1 \frac{\partial U'}{\partial t} \right)_1 \quad \text{(A19)}$$

Proceeding next to the energy equation (Equation A17):

$$\frac{\bar{\rho}_1 \bar{A}^* \bar{U}_1^3}{\gamma(\gamma-1)} [2\bar{A}_1 \cos (\alpha_{ch} - \beta_1) \bar{a}_1 a'_1 + (\rho'_i \bar{A}_1 \cos (\alpha_{ch} - \beta_1) + A'_i \cos (\alpha_{ch} - \beta_1) + \bar{A}_1 (u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch})) \bar{a}_i^2] + \frac{\bar{\rho}_1 \bar{A}^* \bar{U}_1^3}{2} [2\bar{A}_1 \cos^2 (\alpha_{ch} - \beta_1) (u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch}) + \cos^2 (\alpha_{ch} - \beta_1) (\rho'_i \bar{A}_1 \cos (\alpha_{ch} - \beta_1) + A'_i \cos (\alpha_{ch} - \beta_1) + \bar{A}_1 (u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch}))] - \frac{(\gamma^2 - \gamma + 2)}{2\gamma(\gamma - 1)} \bar{\rho}_1 \bar{A}^* \bar{U}_1^3 [2\bar{a}^{*2} a'^* \bar{\rho}^* + \bar{a}^{*2} (\rho'^* \bar{a}^* + \bar{\rho}^* A'^* \bar{a}^* + \bar{\rho}^* a'^*)]$$

$$= \frac{\bar{\rho}_1 \bar{A}^* \bar{U}_1^3}{\gamma(\gamma-1)} \left[\bar{V}\bar{a}^2 \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{a}^2 \frac{\partial V'_1}{\partial t} + 2\bar{\rho}\bar{a}\bar{V}_1 \frac{\partial a'}{\partial t} \right]_1 + \frac{\rho_1 \bar{A}^* \bar{U}_1^3}{2} \left[\bar{V}_1 \bar{U}^2 \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{U}^2 \frac{\partial V'_1}{\partial t} + 2\bar{U}\bar{\rho}\bar{V}_1 \frac{\partial U'}{\partial t} \right]_1$$

Dividing the above equation through by $\bar{\rho}_1 \bar{A}^* \bar{U}_1^3$ and rearranging the terms yields

$$\left[\frac{\bar{a}_1^2}{\gamma(\gamma - 1)} + \frac{\cos^2(\alpha_{ch} - \beta_1)}{2} \right] \left[\rho'_1 \bar{A}_1 \cos(\alpha_{ch} - \beta_1) + A'_1 \cos(\alpha_{ch} - \beta_1) + \right.$$

$$\left. \bar{A}_1 (u'_1 \sin \alpha_{ch} + v'_1 \cos\alpha_{ch}) \right] + \frac{2\bar{A}_1 \cos(\alpha_{ch} - \beta_1)\, \bar{a}_1 a'_1}{\gamma(\gamma - 1)} +$$

$$\bar{A}_1 \cos^2(\alpha_{ch} - \beta_1)(u'_1 \sin \alpha_{ch} + v'_1 \cos \alpha_{ch}) -$$

$$\frac{(\gamma^2 - \gamma + 2)}{2\gamma(\gamma - 1)} [\, \bar{a}^{*2} (\rho'^* \bar{a}^* + \bar{\rho}^* A'^* \bar{a}^* + 3\bar{\rho}^* a'^*) \,]$$

$$= \frac{1}{\gamma(\gamma - 1)} \left[\bar{V}_1 \bar{a}^2 \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{a}^2 \frac{\partial V'_1}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}_1 \frac{\partial a'}{\partial t} \right]_1 +$$

$$\frac{1}{2} \left[\bar{V}_1 \bar{U}^2 \frac{\partial \rho'}{\partial t} + \bar{\rho}\bar{U}^2 \frac{\partial V'}{\partial t} + 2\bar{U}\bar{V}\bar{\rho} \frac{\partial U'}{\partial t} \right]_1 \qquad \text{(A20)}$$

The nondimensional equations of motion are as follows

Continuity

$$[\rho'^* \bar{a}^* + \bar{\rho}^* A'^* \bar{a}^* + \bar{\rho}^* a'^*] - [\bar{A}_i (u'_i \sin \alpha_{ch} + v'_i \cos \alpha_{ch}) +$$

$$A'_i \cos(\alpha_{ch} - \beta_1) + \rho'_i \bar{A}_i \cos(\alpha_{ch} - \beta_1)]$$

$$= - \left[\bar{\rho} \frac{\partial V'_1}{\partial t} + \bar{V}_1 \frac{\partial \rho'}{\partial t} \right]_1 \qquad \text{(A18)}$$

Momentum

$$[p'_1 \bar{A}_1 + \bar{p}_1 A'_1 - p'^* - \bar{p}^* A'^*] = \bar{a}^* [2\bar{\rho}^* a'^* + \bar{\rho}^* A'^* \bar{a}^* + \rho'^* \bar{a}^*] -$$

$$\cos(\alpha_{ch} - \beta_1) [\rho'_1 \bar{A}_1 \cos(\alpha_{ch} - \beta_1) + A'_1 \cos(\alpha_{ch} - \beta_1) + 2\bar{A}_1 (u'_1 \sin \alpha_{ch} +$$

$$v'_1 \cos \alpha_{ch})] + \bar{U}_1 \bar{V}_1 \frac{\partial \rho'_1}{\partial t} + \bar{\rho}_1 \bar{U}_1 \frac{\partial V'_1}{\partial t} + \bar{\rho}_1 \bar{V}_1 \frac{\partial U'_1}{\partial t} \qquad \text{(A19)}$$

Energy

$$\left[\frac{\bar{a}_1^2}{\gamma(\gamma-1)} + \frac{\cos^2(\alpha_{ch}-\beta_1)}{2}\right]\left[\rho'_1\bar{A}_1\cos(\alpha_{ch}-\beta_1) + A'_1\cos(\alpha_{ch}-\beta_1) + \right.$$

$$\left.\bar{A}_1(u'_1\sin\alpha_{ch} + v'_1\cos\alpha_{ch})\right] + \frac{2\bar{A}_1\cos(\alpha_{ch}-\beta_1)\,\bar{a}_1 a'_1}{\gamma(\gamma-1)} +$$

$$\bar{A}_1\cos^2(\alpha_{ch}-\beta_1)(u'_1\sin\alpha_{ch} + v'_1\cos\alpha_{ch}) -$$

$$\frac{(\gamma^2-\gamma+2)}{2\gamma(\gamma-1)}\left[\bar{a}^{*2}(\rho'^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + 3\bar{\rho}^*a'^*)\right]$$

$$= \frac{1}{\gamma(\gamma-1)}\left[\bar{V}_1\bar{a}^2\frac{\partial\rho'}{\partial t} + \bar{\rho}\bar{a}^2\frac{\partial V'_1}{\partial t} + 2\bar{a}\bar{\rho}\bar{V}_1\frac{\partial a'}{\partial t}\right]_1 +$$

$$\frac{1}{2}\left[\bar{V}_1\bar{U}^2\frac{\partial\rho'}{\partial t} + \bar{\rho}\bar{U}^2\frac{\partial V'}{\partial t} + 2\bar{U}\bar{V}\bar{\rho}\frac{\partial U'}{\partial t}\right]_1 \qquad \text{(A20)}$$

The small perturbation forms of the equations of motion for Region 2 and 3 are derived in a similar manner.

APPENDIX B
SOLUTIONS TO THE EQUATIONS OF MOTION FOR REGION 1

The nondimensionalized linear equations of motion for Region 1 are represented by Equations 24, 25, and 26, and are solved to obtain the three unknowns: $A_{-\infty}$ $\rho^{*\prime}$, and $a^{*\prime}$. Starting with the inlet flow parameters

$$u'_i = \frac{\partial \Phi'}{\partial x^*} \quad v'_i = \frac{\partial \Phi'}{\partial y^*}$$

The expression for Φ is given in Equation 11. The inlet velocity perturbations become

$$u'_i = \frac{i\, B_1\, A_{-\infty}}{b\, U_1} \exp i(B_1 x + Cy + kt) \qquad \text{(nondimensional)} \tag{B1}$$

$$v'_i = \frac{i\, C\, A_{-\infty}}{b\, U_1} \exp i(B_1 x + Cy + kt) \qquad \text{(nondimensional)} \tag{B2}$$

The inlet pressure perturbation can be obtained from Bernoulli's relationship in the following manner:

$$\frac{\partial p_i}{\partial t} = -\bar{\rho}_i \left[\underbrace{\frac{\partial^2 \Phi'_{-\infty}}{\partial t^2}}_{1} + \bar{u}_i \underbrace{\frac{\partial u'_i}{\partial t}}_{2} + \bar{v}_i \underbrace{\frac{\partial v'_i}{\partial t}}_{3} \right]$$

or

$$\int \partial p'_i = -\bar{\rho}_i \int \left(\frac{\partial^2 \Phi'_{-\infty}}{\partial t^2} + \bar{u}_i \frac{\partial u'_i}{\partial t} + \bar{v}_i \frac{\partial v'_i}{\partial t} \right) \partial t \tag{B3}$$

Substituting for $\Phi'_{-\infty}$, u'_i, and v'_i, and integrating produces

$$p'_i = -i\, [A_{-\infty} \exp i(B_1 x + Cy + kt)]\, [k + \bar{u}_i B_1 + \bar{v}_i C] \tag{B4}$$

where all quantities are nondimensional. The inlet density perturbation can be related to the inlet pressure for isentropic flow

$$\partial \rho'_i = \frac{1}{\bar{a}^2}\, p'_i \tag{B5}$$

Substituting Equation B4 into Equation B5 gives

$$\rho'_i = \frac{-i}{\bar{a}_i^{\,2}}\, [A_{-\infty} \exp i(B_1 x + Cy + kt)]\, [k + \bar{u}_i B_1 + \bar{v}_i C] \tag{B6}$$

The speed of sound perturbation at the inlet is related to p_i by

$$a'_i = \frac{\gamma - 1}{2} \left(\frac{p'_i}{\bar{\rho}_i \bar{a}_i} \right) \tag{B7}$$

Substituting Equation B4 into Equation B7 gives

$$a'_i = \frac{i(\gamma - 1)}{2\bar{a}_i} \quad [A_{-\infty} \exp i(B_1x + Cy + kt)] \; [k + \bar{u}_iB_i + \bar{v}_iC]$$

The inlet flow parameters represented by a two-dimensional flow field which is converted to a one-dimensional flow field by a technique commonly used in turbulent channel flows. Here the relationship holds.

$$\bar{f}'(x) = \frac{1}{\bar{y}_i}\int_0^{\bar{y}_i} f'(x, y)\, dy \qquad \text{(B8)}$$

where f is any flow parameter. Starting with the velocities

$$u'(x, y) = i\, B_1A_{-\infty} \exp i(B_1x + Cy + kt)$$

$$\bar{u}'(x) = \frac{1}{\bar{y}_i}\int_0^{\bar{y}_i} i\, B_1A_{-\infty} \exp i(B_1x + Cy + kt)\, dy$$

$$\Rightarrow \bar{u}'(x=0) = \frac{B_1A_{-\infty}}{C\bar{y}_i} e^{ikt} (e^{iC\bar{y}_i} - 1) \qquad \text{(B9)}$$

The following steps are used to evaluate the y component of the velocity:

$$v'(x, y) = i\, CA_{-\infty} \exp i(B_1x + Cy + kt)$$

$$\bar{v}'(x=0) = \frac{1}{\bar{y}_i}\int_0^{\bar{y}_i} i\, CA_{-\infty} \exp i(B_1x + Cy + kt)\, dy$$

$$\Rightarrow \bar{v}'_i = \frac{A_{-\infty}\, e^{ikt}\, (e^{iC\bar{y}_i} - 1)}{\bar{y}_i} \qquad \text{(B10)}$$

Returning to the inlet pressure expression noted in Equation B4, the following results:

$$\bar{p}'_i = -\frac{(k + \bar{u}_iB_1 + \bar{v}_iC]}{C\bar{y}_i} \quad [A_{-\infty}\, e^{ikt}\, (e^{iC\bar{y}_i} - 1)] \qquad \text{(B11)}$$

Returning to the inlet density expression, the following results:

$$\bar{\rho}'_i = -\frac{(k + \bar{u}_iB_1 + \bar{v}_iC)}{C\bar{y}_i\bar{a}_i^2} [A_{-\infty}\, e^{ikt}\, (e^{iC\bar{y}_i} - 1)] \qquad \text{(B12)}$$

Finally, the speed of sound expression becomes

$$\bar{a}'_i = -\left(\frac{\gamma - 1}{2\bar{a}_i}\right)\left(\frac{1}{C\bar{y}_i}\right) [A_{-\infty}\, e^{ikt} \; (e^{iC\bar{y}_i} - 1)] \quad [k + \bar{u}_iB_1 + \bar{v}_iC] \qquad \text{(B13)}$$

The next step assumes that all flow parameters within the blade channel are harmonically varying with time. As such, the following equations apply:

$$p'^* = \bar{p}'^* \exp ikt$$

$$\rho'^* = \bar{\rho}'^* \exp ikt$$

$$a'^* = \bar{a}'^* \exp ikt$$

$$A'^* = \bar{A}'^* \exp ikt \qquad \text{(B14)}$$

$$V' = \bar{V}' \exp ikt$$

In addition to these expressions, the time rate of change of density, velocity, and speed of sound are also needed. Starting with the velocity at the inlet from Equation B9, the sequence progresses as follows:

$$\bar{u}'_1 = \frac{B_1 A_{-\infty}}{C\bar{y}_1} e^{ikt} (e^{iC\bar{y}_1} - 1) \qquad \text{(B9)}$$

$$\frac{\partial \bar{u}'_1}{\partial t} = \frac{ikB_1 A_{-\infty}}{C\bar{y}_1} e^{ikt} (e^{iC\bar{y}_1} - 1) \qquad \text{(B14)}$$

$$\bar{v}'_1 = \frac{A_{-\infty} e^{ikt} (e^{iC\bar{y}_1} - 1)}{\bar{y}_1} \qquad \text{(B10)}$$

$$\frac{\partial \bar{v}'_1}{\partial t} = \frac{ikA_{-\infty}}{\bar{y}_1} e^{ikt} (e^{iC\bar{y}_1} - 1) \qquad \text{(B15)}$$

The time rate of change of density at the inlet equals

$$\frac{\partial \bar{\rho}'_1}{\partial t} = \frac{-ik\,(k + \bar{u}_1 B_1 + \bar{v}_1 C)}{C\bar{y}_1 \bar{a}_1^2} \left[A_{-\infty} e^{ikt} (e^{iC\bar{y}_1} - 1)\right] \qquad \text{(B16)}$$

and at the throat

$$\frac{\partial \rho'^*}{\partial t} = ik\bar{\rho}'^* \exp ikt \qquad \text{(B17)}$$

The time rate of change of the speed of sound at the inlet equals

$$\frac{\partial \bar{a}_1}{\partial t} = -ik \left(\frac{\gamma - 1}{2\bar{a}_1}\right) \left(\frac{1}{C\bar{y}_1}\right) \left[A_{-\infty} e^{ikt} (e^{iC\bar{y}_1} - 1)\right] \left[k + \bar{u}_1 B_1 + \bar{v}_1 C\right] \qquad \text{(B18)}$$

and at the throat

$$\frac{\partial a'^*}{\partial t} = ik\bar{a}'^* \exp ikt \qquad \text{(B19)}$$

Working first with the energy equation, let

$$E_1 = \left[\frac{\bar{a}_1^2}{\gamma(\gamma-1)} + \frac{\cos^2(\alpha_{ch} - \beta_1)}{2} \right]$$

$$I = \cos(\alpha_{ch} - \beta_1)$$

$$E_2 = \frac{\bar{a}^{*2}(\gamma^2-\gamma+2)}{2\gamma(\gamma-1)}$$

Inserting Equations B9 through B19 into Equation 26 and dividing through by e^{ikt} produces the following expression:

$$E_1 \left[-I\bar{A}_1 \left(\frac{k + \bar{u}_i B_1 + \bar{v}_i C}{C\bar{y}_1 \bar{a}_1^2} \right) A_{-\infty} (e^{iC\bar{y}_1} - 1) + \bar{A}'_1 I + \right.$$

$$\left. \bar{A}_1 \left(\sin\alpha_{ch} \left(\frac{B_1 A_{-\infty}}{C\bar{y}_1} \right) (e^{iC\bar{y}_1} - 1) + \left(\frac{A_{-\infty}}{\bar{y}_1} \right) \cos\alpha_{ch}(e^{iC\bar{y}_1} - 1) \right) \right] -$$

$$\frac{2\bar{A}_1 I\bar{a}_1}{\gamma(\gamma-1)} \left(\frac{\gamma-1}{2\bar{a}_1} \right) \left(\frac{1}{C\bar{y}_1} \right) (k + \bar{u}_i B_1 + \bar{v}_i C) A_{-\infty} (e^{iC\bar{y}_1} - 1) +$$

$$\bar{A}_1 I^2 \left[\sin\alpha_{ch} \left(\frac{B_1 A_{-\infty}}{C\bar{y}_1} \right) (e^{iC\bar{y}_1} - 1) + \left(\frac{A_{-\infty}}{\bar{y}_1} \right) \cos\alpha_{ch}(e^{iC\bar{y}_1} - 1) \right] -$$

$$E_2[\bar{\rho}'^* \bar{a}^* + \bar{\rho}^* \bar{A}'^* \bar{a}^* + 3\bar{\rho}^* \bar{a}'^*]$$

$$= \frac{1}{\gamma(\gamma-1)} \left\{ \frac{\bar{V}_1 \bar{a}^2}{2} \left[ik \left(\bar{\rho}'^* - \left(\frac{k + \bar{u}_i B_1 + \bar{v}_i C}{C\bar{y}_1 \bar{a}_1^2} \right) A_{-\infty}(e^{iC\bar{y}_1} - 1) \right] + \right.$$

$$ik\bar{\rho}\bar{a}^2 \bar{V}'_1 + \bar{\rho}\bar{a}\bar{V}_1 \left[ik \left(\bar{a}'^* - \left(\frac{\gamma-1}{2\bar{a}_1} \right) \left(\frac{1}{C\bar{y}_1} \right) A_{-\infty}(e^{iC\bar{y}_1} - 1) \right. \right.$$

$$\left. \left. \left. (k + \bar{u}_i B_1 + \bar{v}_i C) \right) \right] \right\}_1 + \tfrac{1}{2} \left\{ \frac{\bar{U}^2 \bar{V}_1}{2} \left[ik (\bar{\rho}'^* - (k + \bar{u}_i B_1 + \bar{v}_i C) \right. \right.$$

$$\left(\frac{A_{-\infty}}{C\bar{y}_1 \bar{a}_1^2} \right) (e^{iC\bar{y}_1} - 1) \Big) \Big] + ik\bar{\rho}\bar{U}^2 \bar{V}'_1 + \bar{\rho}\bar{U}\bar{V}_1 \left[ik \left(\bar{a}'^* + \right. \right.$$

$$\left. \left. \left. \left(\frac{B A_{-\infty}}{C\bar{y}_1} \right) (e^{iC\bar{y}_1} - 1) \sin\alpha_{ch} + \left(\frac{A_{-\infty}}{\bar{y}_1} \right) (e^{iC\bar{y}_1} - 1) \cos\alpha_{ch} \right) \right] \right\}_1 \qquad \text{(B20)}$$

The E, M, and C constants presented in this section are found in Appendix E.

The next step requires that Equation 220 be broken down into its real and imaginary parts. The solution deals first with the four terms containing the unknown complex constant $A_{-\infty}$. These are broken into real and imaginary parts as follows:

1. $B_1 A_{-\infty} (e^{iC\bar{y}_i} - 1)$

$$B_1 = B_{1R} + iB_{1I}$$

$$A_{-\infty} = A_{-\infty R} + iA_{-\infty I}$$

$$e^{iC\bar{y}_i} = \cos(C\bar{y}_i) + i \sin(C\bar{y}_i)$$

Thus, the real part of the expression is

$$B_{1R}A_{-\infty R} (\cos(C\bar{y}_i) - 1) - B_{1I}A_{-\infty I} (\cos(C\bar{y}_i) - 1) -$$

$$B_{1R}A_{-\infty I} \sin(C\bar{y}_i) - B_{1I}A_{-\infty R} \sin(C\bar{y}_i)$$

and the imaginary part of the expression is

$$B_{1R}A_{-\infty I} (\cos(C\bar{y}_i) - 1) + B_{1I}A_{-\infty R} (\cos(C\bar{y}_i) - 1) +$$

$$B_{1R}A_{-\infty R} \sin(C\bar{y}_i) - B_{1I}A_{-\infty I} \sin(C\bar{y}_i)$$

2. $A_{-\infty} (e^{iC\bar{y}_i} - 1)$

The real part of the expression is

$$A_{-\infty R} (\cos(C\bar{y}_i) - 1) - A_{-\infty I} \sin(C\bar{y}_i)$$

The imaginary part of the expression is

$$A_{-\infty I} (\cos(C\bar{y}_i) - 1) + A_{-\infty R} \sin(C\bar{y}_i)$$

3. $ikB_1 A_{-\infty} (e^{iC\bar{y}_i} - 1)$

The real part of the expression is

$$-k[B_{1R}A_{-\infty I} (\cos(C\bar{y}_i) - 1) + B_{1I}A_{-\infty R} (\cos(C\bar{y}_i) - 1) +$$

$$B_{1R}A_{-\infty R} \sin(C\bar{y}_i) - B_{1I}A_{-\infty I} \sin(C\bar{y}_i)]$$

The imaginary part of the expression is

$$k[B_{1R}A_{-\infty R} (\cos(C\bar{y}_i) - 1) - B_{1I}A_{-\infty I} (\cos(C\bar{y}_i) - 1) -$$

$$B_{1R}A_{-\infty I} \sin(C\bar{y}_i) - B_{1I}A_{-\infty R} \sin(C\bar{y}_i)]$$

4. $ikA_{-\infty} (e^{iC\bar{y}_i} - 1)$

The real part of the expression is

$$-k[A_{-\infty I}(\cos(C\bar{y}_i)-1)+A_{-\infty R}\sin(C\bar{y}_i)]$$

The imaginary part of the expression is

$$k[A_{-\infty R}(\cos(C\bar{y}_i)-1)-A_{-\infty I}\sin(C\bar{y}_i)]$$

Now let,

$$E_3 = -\frac{E_1 I\bar{A}_i\bar{u}_i}{C\bar{y}_i\bar{a}_i^2} + \frac{E_1\bar{A}_i\sin\alpha_{ch}}{C\bar{y}_i} - \frac{2\bar{A}_i I\bar{u}_i}{2\gamma C\bar{y}_i} + \frac{\bar{A}_i I^2\sin\alpha_{ch}}{C\bar{y}_i} \quad \text{(B21)}$$

$$E_4 = -\frac{E_1 I\bar{A}_i\,(k+\bar{v}_i C)}{C\bar{y}_i\bar{a}_i^2} + \frac{E_1\bar{A}_i\cos\alpha_{ch}}{\bar{y}_i} - \frac{2\bar{A}_i I\,(k+\bar{v}_i C)}{2\gamma C\bar{y}_i} + \frac{\bar{A}_i I^2\cos\alpha_{ch}}{\bar{y}_i} \quad \text{(B22)}$$

$$E_5 = \frac{-\bar{a}_i^2\bar{V}_1\bar{u}_i}{2\gamma(\gamma-1)C\bar{y}_i\bar{a}_i^2} - \frac{\bar{\rho}_i\bar{a}_i\bar{V}_1\bar{u}_i}{\gamma(\gamma-1)}\left(\frac{\gamma-1}{2\bar{a}_i C\bar{y}_i}\right) - \frac{\bar{U}_i^2\bar{V}_1\bar{u}_i}{4C\bar{y}_i\bar{a}_i^2} + \frac{\bar{\rho}_i\bar{U}_i\bar{V}_1\sin\alpha_{ch}}{2C\bar{y}_i} \quad \text{(B23)}$$

$$E_6 = \frac{-\bar{a}_i^2\bar{V}_1(k+\bar{v}_i C)}{2\gamma(\gamma-1)C\bar{y}_i\bar{a}_i^2} - \bar{\rho}_i\bar{a}_i\bar{V}_1\left(\frac{\gamma-1}{2C\bar{y}_i\bar{a}_i}\right)\left(\frac{k+\bar{v}_i C}{\gamma(\gamma-1)}\right) + \frac{\bar{U}_i^2\bar{V}_1(k+\bar{v}_i C)}{4C\bar{y}_i\bar{a}_i^2} + \frac{\bar{\rho}_i\bar{U}_i\bar{V}_1\cos\alpha_{ch}}{2\bar{y}_i} \quad \text{(B24)}$$

Substituting the relationships for the real part of the $A_{+\infty}$ terms, along with Equations B21 through B24 gives

$$\begin{aligned}
&E_3\,[B_{1R}A_{\infty R}(\cos(C\bar{y}_i)-1) - B_{1I}A_{-\infty I}(\cos(C\bar{y}_i)-1) - \\
&\quad B_{1R}A_{-\infty I}\sin(C\bar{y}_i) - B_{1I}A_{-\infty R}\sin(C\bar{y}_i)] + \\
&\quad E_4\,[A_{-\infty R}(\cos(C\bar{y}_i)-1) - A_{-\infty I}\sin(C\bar{y}_i)] + \\
&\quad E_1\bar{A}'_{iR}I - E_2\,(\bar{\rho}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}'^* + 3\bar{\rho}^*\bar{a}'_R{}^*) \\
&= -kE_5\,[B_{1R}A_{-\infty I}(\cos(C\bar{y}_i)-1) + B_{1I}A_{-\infty R}(\cos(C\bar{y}_i)-1) + \\
&\quad B_{1R}A_{-\infty R}\sin(C\bar{y}_i) - B_{1I}A_{-\infty I}\sin(C\bar{y}_i)] + \\
&\quad E_6\,[-k(A_{-\infty I}(\cos(C\bar{y}_i)-1) + A_{-\infty R}\sin(C\bar{y}_i))] \\
&- \left(\frac{\bar{a}^2\bar{V}}{2\gamma(\gamma-1)}\right)_1 (k\bar{\rho}'_I{}^*) - \bar{V}'_{1I}k\left(\frac{\bar{\rho}\bar{a}^2}{\gamma(\gamma-1)}\right)_1 - k\bar{a}'_I{}^*\left(\frac{\bar{\rho}\bar{a}\bar{V}}{\gamma(\gamma-1)}\right)_1 \\
&- k\bar{\rho}'_I{}^*\left(\frac{\bar{U}^2\bar{V}}{4}\right)_1 - k\bar{V}'_{1I}\left(\frac{\bar{\rho}\bar{U}^2}{2}\right)_1 - k\bar{a}'_I{}^*\left(\frac{\bar{\rho}\bar{U}\bar{V}}{2}\right)_1
\end{aligned} \quad \text{(B25)}$$

Collecting terms and rearranging Equation B25 produces:

$$
\begin{aligned}
E_8 A_{-\infty R} + E_9 A_{-\infty I} = & -E_1 \bar{A}'_{1R} I + \\
& \left(\frac{\bar{a}^2 \bar{V}_1}{2\gamma(\gamma - 1)} + \frac{\bar{U}^2 \bar{V}_1}{4} \right)_1 k \bar{\rho}'^{*}_{I} - \\
& k \bar{a}'^{*}_{I} \left(\frac{\bar{\rho} \bar{a} \bar{V}_1}{\gamma(\gamma - 1)} + \frac{\rho \bar{U} \bar{V}_1}{2} \right)_1 - \\
& k \bar{V}'_{1I} \left(\frac{\bar{\rho} \bar{U}^2}{2} + \frac{\bar{\rho} \bar{a}^2}{\gamma(\gamma - 1)} \right)_1 \\
& + E_2 \left(\bar{a}^* \bar{\rho}'_R{}^* + \bar{\rho}^* \bar{A}'_R{}^* \bar{a}^* + 3\bar{a}'_R{}^* \bar{\rho}^* \right)
\end{aligned}
\qquad \text{(B26)}
$$

with

$$
\begin{aligned}
E_7 = & \cos(C\bar{y}_1) - 1 \\
E_8 = & E_3 \left(E_7 B_{1R} - B_{1I} \sin(C\bar{y}_1) \right) + E_4 E_7 + E_5 k \left(B_{1I} E_7 + \right. \\
& \left. B_{1R} \sin(C\bar{y}_1) \right) + E_6 k \sin(C\bar{y}_1) \\
E_9 = & -E_3 \left(B_{1I} E_7 + B_{1R} \sin(C\bar{y}_1) \right) - E_4 \sin(C\bar{y}_1) + \\
& E_5 k \left(B_{1R} E_7 - B_{1I} \sin(C\bar{y}_1) \right) + E_6 k E_7
\end{aligned}
$$

Equation B26 represents the real component of Equation B20. The imaginary component is

$$
\begin{aligned}
& E_3 \left[B_{1R} A_{-\infty I} E_7 + B_{1I} A_{-\infty R} E_7 + B_{1R} A_{-\infty R} \sin(C\bar{y}_1) - \right. \\
& \left. B_{1I} A_{-\infty I} \sin(C\bar{y}_1) \right] + E_4 \left[A_{-\infty I} E_7 + A_{-\infty R} \sin(C\bar{y}_1) \right] + \\
& E_1 \bar{A}'_{1I} I - E_2 \left[\bar{\rho}'_1{}^* \bar{a}^* + \bar{\rho}^* \bar{A}'_1{}^* \bar{a}^* + 3\bar{\rho}^* \bar{a}'_1{}^* \right] \\
= \; & E_5 k \left[B_{1R} A_{-\infty R} E_7 - B_{1I} A_{-\infty I} E_7 - B_{1R} A_{-\infty I} \sin(C\bar{y}_1) - \right. \\
& \left. B_{1I} A_{-\infty R} \sin(Cy) \right] + E_6 k \left[A_{-\infty R} E_7 - A_{-\infty I} \sin(C\bar{y}_1) \right] + \\
& k \bar{\rho}'_R{}^* \left(\frac{\bar{a}^2 \bar{V}_1}{2\gamma(\gamma - 1)} + \frac{\bar{U}^2 \bar{V}_1}{4} \right)_1 + \\
& k \bar{a}'_R{}^* \left(\frac{\bar{\rho} \bar{a} \bar{V}_1}{\gamma(\gamma - 1)} + \frac{\bar{\rho} \bar{U} \bar{V}_1}{2} \right)_1 \\
& + k \left[\frac{\bar{\rho} \bar{a}^2}{\gamma(\gamma - 1)} + \frac{\bar{\rho} \bar{U}^2}{2} \right]_1 \bar{V}'_{1R}
\end{aligned}
\qquad \text{(B27)}
$$

Again, collecting terms for $A_{-\infty I}$ and $A_{-\infty R}$, let

$$E_{10} = E_3 \,[B_{1I}E_7 + B_{1R} \sin (C\bar{y}_i)] + E_4 \sin (C\bar{y}_i) -$$

$$E_5 k \,[B_{1R}E_7 - B_{1I} \sin (C\bar{y}_i)] - E_6 k E_7$$

$$E_{11} = E_3 \,[B_{1R}E_7 - B_{1I} \sin (C\bar{y}_i)] + E_4 E_7 +$$

$$E_5 k \,[B_{1I}E_7 + B_{1R} \sin (C\bar{y}_i)] + E_6 k \sin (C\bar{y}_i)$$

Substituting the above relationships into Equation B27 gives

$$E_{10}A_{-\infty R} + E_{11}A_{-\infty I} = -E_1\bar{A}'_{iI}I + E_2\,[\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^* + 3\bar{\rho}^*\bar{a}'_I{}^*] +$$

$$k\bar{\rho}'_R{}^* \left(\frac{\bar{a}^2\bar{V}_1}{2\gamma(\gamma - 1)} + \frac{\bar{U}^2\bar{V}_1}{4} \right)_1 + k\bar{V}'_{1R} \left(\frac{\bar{\rho}\bar{a}^2}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}^2}{2} \right)_1 +$$

$$k\bar{a}'_R{}^* \left(\frac{\bar{\rho}\bar{a}\bar{V}_1}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}\bar{V}_1}{2} \right)_1 \qquad \text{(B28)}$$

The next step involves the combination of Equation B26 with Equation B28 and solving for $A_{-\infty R}$ and $A_{-\infty I}$. The sequence requires that Equation B26 be divided by E_8 and Equation B28 be divided by E_{10}, as shown in Equations B29 and B30.

$$A_{-\infty R} + A_{-\infty I} \left(\frac{E_9}{E_8} \right) = -\bar{A}'_{iR}I \left(\frac{E_1}{E_8} \right) + (\bar{\rho}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}^* +$$

$$3\bar{\rho}^*\bar{a}'_R{}^*) \left(\frac{E_2}{E_8} \right) - \bar{\rho}'_I{}^* \left(\frac{E_{12}}{E_8} \right) - \bar{V}'_{1I} \left(\frac{E_{13}}{E_8} \right) - \bar{a}'_I{}^* \left(\frac{E_{14}}{E_8} \right) \qquad \text{(B29)}$$

$$A_{-\infty R} + A_{-\infty I} \left(\frac{E_1}{E_{10}} \right) = -\bar{A}'_{iI}I \left(\frac{E_1}{E_{10}} \right) +$$

$$(\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^* + 3\bar{\rho}^*\bar{a}_I{}^*) \left(\frac{E_2}{E_{10}} \right) +$$

$$\bar{\rho}'_R{}^* \left(\frac{E_{12}}{E_{10}} \right) + \bar{V}'_{1R} \left(\frac{E_{13}}{E_{10}} \right) + \bar{a}'_R{}^* \left(\frac{E_{14}}{E_{10}} \right) \qquad \text{(B30)}$$

where,

$$E_{12} = \left(\frac{\bar{a}^2\bar{V}k}{2\gamma(\gamma - 1)} + \frac{\bar{U}^2\bar{V}k}{4} \right)_1$$

$$E_{13} = \left(\frac{\bar{\rho}\bar{a}^2 k}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}^2 k}{2} \right)_1$$

$$E_{14} = \left(\frac{\bar{\rho}\bar{a}\bar{V}k}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}\bar{V}k}{2} \right)_1$$

Subtracting Equation B30 from Equation B29 yields

$$A_{-\infty I} = \Big[-E_1 I \left(\frac{\bar{A}'_{iR}}{E_8} - \frac{\bar{A}'_{iI}}{E_{10}} \right) + E_2 \Big[\bar{a}^* \left(\frac{\bar{\rho}'_R{}^*}{E_8} - \frac{\bar{\rho}'_I{}^*}{E_{10}} \right) +$$

$$\bar{\rho}^* \bar{a}^* \left(\frac{\bar{A}'_R{}^*}{E_8} - \frac{\bar{A}'_I{}^*}{E_{10}} \right) + 3\bar{\rho}^* \left(\frac{\bar{a}'_R{}^*}{E_8} - \frac{\bar{a}'_I{}^*}{E_{10}} \right) \Big] -$$

$$E_{12} \left(\frac{\bar{\rho}'_I{}^*}{E_8} - \frac{\bar{\rho}'_R{}^*}{E_{10}} \right) - E_{13} \left(\frac{\bar{V}'_{iI}}{E_8} - \frac{\bar{V}'_{iR}}{E_{10}} \right) -$$

$$E_{14} \left(\frac{\bar{a}'_I{}^*}{E_8} - \frac{\bar{a}'_R{}^*}{E_{10}} \right) \Big] \div \left[\frac{E_9}{E_8} - \frac{E_{11}}{E_{10}} \right] \qquad \text{(B31)}$$

Substituting Equation B31 into Equation B29 and solving for $A_{-\infty R}$ produces

$$A_{-\infty R} = -\bar{A}'_{iR} I \left(\frac{E_1}{E_8} \right) + (\bar{\rho}'_R{}^* \bar{a}^* + \bar{\rho}^* \bar{A}'_R{}^* \bar{a}^* + 3\bar{\rho}^* \bar{a}'_R{}^*) \left(\frac{E_2}{E_8} \right) -$$

$$\bar{\rho}'_I{}^* \left(\frac{E_{12}}{E_8} \right) - \bar{V}'_{iI} \left(\frac{E_{13}}{E_8} \right) - \bar{a}'_I{}^* \left(\frac{E_{14}}{E_8} \right) - A_{-\infty I} \left(\frac{E_9}{E_8} \right) \qquad \text{(B32)}$$

Before proceeding further, the following relationships must be given:

$$E_{15} = \left(\frac{E_9}{E_8} - \frac{E_{11}}{E_{10}} \right)$$

$$E_{16} = I \left(\frac{E_1}{E_8} \right) \left(\frac{E_9}{E_{15} E_8} - 1 \right)$$

$$E_{17} = I \left(\frac{-E_1}{E_{10} E_{15}} \right) \left(\frac{E_8}{E_9} \right)$$

$$E_{18} = \left(\frac{E_2}{E_8} \right) \left(\bar{a}^* - \frac{\bar{a}^* E_9}{E_{15} E_8} \right) - \left(\frac{E_{12} E_9}{E_8 E_{10} E_{15}} \right)$$

$$E_{19} = -\frac{E_{12}}{E_8} + \left(\frac{\bar{a}^* E_2}{E_{10} E_{15}} \right) \left(\frac{E_9}{E_8} \right) + \left(\frac{E_{12}}{E_8 E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{20} = \bar{\rho}^* \bar{a}^* \left[\frac{E_2}{E_8} - \left(\frac{E_2}{E_8 E_{15}} \right) \left(\frac{E_9}{E_8} \right) \right]$$

$$E_{21} = \bar{\rho}^* \bar{a}^* \left(\frac{E_2}{E_{10} E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{22} = 3\bar{\rho}^* \left(\frac{E_2}{E_8} \right) \left(1 - \frac{E_9}{E_8 E_{15}} \right) - \left(\frac{E_9 E_{14}}{E_8 E_{10} E_{15}} \right)$$

$$E_{23} = -\frac{E_{14}}{E_8} + 3\bar{\rho}^* \left(\frac{E_2 E_9}{E_8 E_{10} E_{15}} \right) + \left(\frac{E_{14}}{E_8 E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{24} = -\frac{E_{13}}{E_8} + \left(\frac{E_{13}}{E_8 E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{25} = -\left(\frac{E_{13}}{E_{10} E_{15}} \right) \left(\frac{E_9}{E_8} \right) \qquad \text{(B33)}$$

Substituting Equation B31 into Equation B32 and using the relationships detailed in Equation B33 yields

$$A_{-\infty R} = (E_{16}\bar{A}'_{iR} + E_{17}\bar{A}'_{iI}) + (E_{18}\bar{\rho}'_R{}^* + E_{19}\bar{\rho}'_I{}^*) +$$
$$(E_{20}\bar{A}'_R{}^* + E_{21}\bar{A}'_I{}^*) + (E_{22}\bar{a}'_R{}^* + E_{23}\bar{a}'_I{}^*) +$$
$$(E_{24}\bar{V}'_{1I} + E_{25}\bar{V}'_{1R}) \qquad \text{(B34)}$$

The nondimensional momentum equation has the form

$$\bar{p}'_i\bar{A}_i + \bar{p}_i\bar{A}'_i - \bar{p}^*\bar{A}'^* - \bar{p}'^* = \bar{a}^*(2\bar{\rho}^*\bar{a}'^* + \bar{\rho}^*\bar{A}'^*\bar{a}^* + \bar{\rho}'^*\bar{a}^*) -$$
$$I[\bar{\rho}'_i\bar{A}_iI + \bar{A}'_iI + 2\bar{A}_i(\bar{u}'_i \sin\alpha_{ch} + \bar{v}'_i\cos\alpha_{ch})] +$$
$$\bar{U}_1\bar{V}_1 \frac{\partial\bar{\rho}'_1}{\partial t} + \bar{\rho}_1\bar{U}_1\frac{\partial\bar{V}'_1}{\partial t} + \bar{\rho}_1\bar{V}_1\frac{\partial\bar{U}'_1}{\partial t}$$

Inserting the relationships given in Equations B9 through B19 into Equation B25 and dividing through by e^{ikt} produces

$$\left[-\left(\frac{k + \bar{u}_iB_1 + \bar{v}_iC}{C\bar{y}_i}\right)\right.$$
$$\left.(A_{-\infty}(e^{iC\bar{y}_i} - 1))\bar{A}_i + \bar{p}_i\bar{A}'_i - \bar{p}^*\bar{A}'^* - \rho'^*\bar{a}^{*2}\right]$$
$$= \bar{a}^*(2\bar{\rho}^*\bar{a}'^* + \bar{\rho}^*\bar{A}'^*\bar{a}^* + \bar{\rho}'^*\bar{a}^*) - I\left\{-\right.$$
$$\left(\frac{k + \bar{u}_iB_1 + \bar{v}_iC}{C\bar{y}_i\bar{a}_i^2}\right)\bar{A}_iIA_{-\infty}(e^{iC\bar{y}_i} - 1) + \bar{A}'_iI +$$
$$2\bar{A}_i\left[\left(\frac{B_1A_{-\infty}}{C\bar{y}_i}\sin\alpha_{ch}\right)(e^{iC\bar{y}_i} - 1) +\right.$$
$$\left.\left.\left(\frac{A_{-\infty}}{\bar{y}_i}\cos\alpha_{ch}\right)(e^{iC\bar{y}_i} - 1)\right]\right\} +$$
$$\frac{\bar{U}_1\bar{V}_1}{2}\left\{ik\left[\bar{\rho}'^* - \left(\frac{k + \bar{u}_iB_1 + \bar{v}_iC}{C\bar{y}_i\bar{a}_i^2}\right)\right.\right.$$
$$\left.\left.A_{-\infty}(e^{iC\bar{y}_i} - 1)\right]\right\} + \bar{\rho}_1\bar{U}_1ik\bar{V}'_1 + \frac{\bar{\rho}_1\bar{V}_1}{2}\left\{ik\bar{a}'^* +\right.$$
$$\left.ik\left[\frac{B_1A_{-\infty}}{C\bar{y}_i}(e^{iC\bar{y}_i} - 1)\sin\alpha_{ch} + \frac{A_{-\infty}}{\bar{y}_i}(e^{iC\bar{y}_i} - 1)\cos\alpha_{ch}\right]\right\} \qquad \text{(B35)}$$

Rearrange the Equation B35 to collect $A_{-\infty}$ on right side of expression. Assume the following relationships:

$$M_1 = -\frac{\bar{u}_i\bar{A}_i}{C\bar{y}_i} - \frac{\bar{u}_i\bar{A}_iI^2}{C\bar{y}_i\bar{a}_i^2} + \frac{2\bar{A}_iI}{C\bar{y}_i} \sin \alpha_{ch}$$

$$M_2 = -\frac{(k + \bar{v}_iC)\,\bar{A}_i}{C\bar{y}_i} - \frac{(k + \bar{v}_iC)\,\bar{A}_iI^2}{C\bar{y}_i\bar{a}_i^2} + \frac{2\bar{A}_iI^2}{\bar{y}_i} \cos \alpha_{ch}$$

$$M_3 = \frac{\bar{U}_1\bar{V}_1}{2C\bar{y}_i\bar{a}_i^2} - \frac{\bar{\rho}_1\bar{V}_1 \sin \alpha_{ch}}{2C\bar{y}_i}$$

$$M_4 = \left(\frac{k + \bar{v}_iC}{C\bar{y}_i\bar{a}_i^2}\right)\left(\frac{\bar{U}_1\bar{V}_1}{2}\right) - \frac{\bar{\rho}_1\bar{V}_1 \cos \alpha_{cl}}{2\bar{y}_i} \qquad \text{(B36)}$$

Substitute the relationships of Equation B36 into Equation B35.

$$M_1\,[B_1A_{-\infty}\,(e^{iC\bar{y}_i} - 1)] + M_2\,[A_{-\infty}\,(e^{iC\bar{y}_i} - 1)] +$$

$$M_3\,[ikB_1A_{-\infty}\,(e^{iC\bar{y}_i} - 1)] + M_4\,[ikA_{-\infty}\,(e^{iC\bar{y}_i} - 1)]$$

$$= (-\bar{p}_i - I^2)\,\bar{A}'_1 + (\bar{p}^* + \bar{a}^2\bar{\rho}^*)\,\bar{A}'^* + 2\bar{a}^{*2}\bar{\rho}'^* + \frac{ik\bar{\rho}'^*\bar{U}_1\bar{V}_1}{2} +$$

$$ik\bar{\rho}_1\bar{U}_1\bar{V}'_1 + \frac{ik\bar{\rho}_1\bar{a}'^*\bar{V}_1}{2} + 2\bar{\rho}^*\bar{a}^*\bar{a}'^* \qquad \text{(B37)}$$

The next step in the sequence involves the separation of Equation B37 into real and imaginary parts. First, assume

$$M_5 = -\bar{p}_i - I^2$$

$$M_6 = \bar{p}^* + \bar{\rho}^*\bar{a}^{*2} \qquad \text{(B38)}$$

and

$$M_7 = M_1\,[B_{1R}E_7 - B_{iI} \sin (C\bar{y}_i)] + M_2E_7 -$$

$$M_3\,[kB_{1I}E_7 - kB_{1R} \sin (C\bar{y}_i)] - M_4k \sin (C\bar{y}_i)$$

$$M_8 = -M_1\,[B_{1I}E_7 + B_{1R} \sin (C\bar{y}_i)] - M_2 \sin (C\bar{y}_i) +$$

$$kM_3\,[-B_{1R}E_7 + B_{1I} \sin (C\bar{y}_i)] - kM_4E_7 \qquad \text{(B39)}$$

Real Part

$$M_1 [B_{1R}A_{-\infty R}E_7 - B_{1I}A_{-\infty I}E_7 - B_{1R}A_{-\infty I} \sin (C\bar{y}_i) -$$
$$B_{1I}A_{-\infty R} \sin (C\bar{y}_i)] + M_2 [A_{-\infty R}E_7 - A_{-\infty I} \sin (C\bar{y}_i)] -$$
$$kM_3 [B_{1R}A_{-\infty I}E_7 + B_{1I}A_{-\infty R}E_7 + B_{1R}A_{-\infty R} \sin (C\bar{y}_i) -$$
$$B_{1I}A_{-\infty I} \sin (C\bar{y}_i)] - kM_4 [A_{-\infty I}E_7 + A_{-\infty R} \sin (C\bar{y}_i)]$$
$$= M_5\bar{A}'_{iR} + M_6\bar{A}'_R{}^* + 2\bar{\rho}'_R{}^*\bar{a}^{*2} - \frac{k\bar{\rho}'_I{}^*\bar{U}_1\bar{V}_1}{2} -$$
$$k\bar{\rho}_1\bar{U}_1\bar{V}'_{1I} - \frac{k\bar{\rho}_1\bar{a}'_I{}^*\bar{V}_1}{2} + 2\bar{\rho}^*\bar{a}^*\bar{a}'_R{}^* \qquad \text{(B40)}$$

Substituting Equation B39 into Equation B40 yields

$$M_7A_{-\infty R} + M_8A_{-\infty I} = M_5\bar{A}'_{iR} + M_6\bar{A}'_R{}^* + 2\bar{\rho}'_R{}^*\bar{a}^{*2} -$$
$$\frac{k\bar{\rho}'_I{}^*\bar{U}_1\bar{V}_1}{2} - k\bar{\rho}_1\bar{U}_1\bar{V}'_I - \frac{k\bar{\rho}_1\bar{a}'_I{}^*\bar{V}}{2} + 2\bar{\rho}^*\bar{a}^*\bar{a}'_R{}^* \qquad \text{(B41)}$$

Imaginary Part

$$M_1 [B_{1R}A_{-\infty I}E_7 + B_{1I}A_{-\infty R}E_7 + B_{1R}A_{-\infty R} \sin (C\bar{y}i) - B_{1I}A_{-\infty I} \sin (C\bar{y}i)] +$$
$$M_2 [A_{-\infty I}E_7 + A_{-\infty R} \sin (C\bar{y}i)] + kM_3 [B_{1R}A_{-\infty R}E_7 - B_{1I}A_{-\infty I}E_7 -$$
$$B_{1R}A_{-\infty I} \sin (C\bar{y}i) - B_{1I}A_{-\infty R} \sin (C\bar{y}i)] + kM_4 [A_{-\infty R}E_7 - A_{-\infty I} \sin (C\bar{y}i)]$$
$$= (-\bar{\rho}_i - I^2)\ \bar{A}'_{iI} + (\bar{p}^* + \bar{\rho}^*\bar{a}^{*2})\ \bar{A}'_I{}^* + 2\bar{\rho}'_I{}^*\bar{a}^{*2} + 2\bar{\rho}^*\bar{a}^*\bar{a}'_I{}^* +$$
$$\frac{k\bar{\rho}'_R{}^*\bar{U}_1\bar{V}_1}{2} + k\bar{\rho}_1\bar{U}_1\bar{V}'_{R1} + \frac{k\bar{\rho}_1\bar{a}'_R{}^*\bar{V}_1}{2} \qquad \text{(B42)}$$

Assume

$$M_9 = M_1 [B_{1I}E_7 + B_{1R} \sin (C\bar{y}i)] + M_2 \sin (C\bar{y}i) +$$
$$kM_3 [B_{1R}E_7 - B_{1I} \sin (C\bar{y}i)] + M_4E_7k$$
$$M_{10} = M_1 [B_{1R}E_7 - B_{1I} \sin (C\bar{y}i)] + M_2E_7 -$$
$$kM_3 [B_{1I}E_7 + B_{1R} \sin (C\bar{y}i)] - kM_4 \sin (C\bar{y}i) \qquad \text{(B43)}$$

Substituting the relationships of Equation B43 into Equation B42 yields

$$M_9A_{-\infty R} + M_{10}A_{-\infty I} = M_5\bar{A}'_{iI} + M_6\bar{A}'_I{}^* + 2\bar{\rho}'_I{}^*\bar{a}^{*2} +$$
$$2\bar{\rho}^*\bar{a}^*\bar{a}'_I{}^* + \frac{k\bar{\rho}'_R{}^*\bar{U}_1\bar{V}_1}{2} + k\bar{\rho}_1\bar{U}_1\bar{V}'_{1R} + \frac{k\bar{\rho}_1\bar{a}'_R{}^*\bar{V}_1}{2} \qquad \text{(B44)}$$

In order to solve for $A_{-\infty R}$ and $A_{-\infty I}$, make the following assumptions.

$$E_{26} = -I \left(\frac{E_1}{E_8 E_{15}} \right)$$

$$E_{27} = \frac{E_1}{E_{10} E_{15}}$$

$$E_{28} = \frac{\bar{a}^* E_2}{E_8 E_{15}} + \frac{E_{12}}{E_{10} E_{15}}$$

$$E_{29} = - \frac{\bar{a}^* E_2}{E_{10} E_{15}} - \frac{E_{12}}{E_8 E_{15}}$$

$$E_{30} = \frac{\bar{\rho}^* \bar{a}^* E_2}{E_8 E_{15}}$$

$$E_{31} = - \frac{\bar{\rho}^* \bar{a}^* E_2}{E_{10} E_{15}}$$

$$E_{32} = \frac{3\bar{\rho}^* E_2}{E_8 E_{15}} + \frac{E_{14}}{E_{10} E_{15}}$$

$$E_{33} = - \frac{3\bar{\rho}^* E_2}{E_{10} E_{15}} - \frac{E_{14}}{E_8 E_{15}}$$

$$E_{34} = - \frac{E_{13}}{E_8 E_{15}}$$

$$E_{35} = \frac{E_{13}}{E_{10} E_{15}} \qquad \text{(B45)}$$

Substitute the relationships of Equation B45 into Equations B41 and B44 to solve for $A_{-\infty R}$ and $A_{-\infty I}$, such that

$$A_{-\infty R} = E_{16}\overline{A}'_{iR} + E_{17}\overline{A}'_{iI} + E_{18}\overline{\rho}'_R{}^* + E_{19}\overline{\rho}'_I{}^* + E_{20}\overline{A}'_R{}^* + E_{21}\overline{A}'_I{}^* +$$

$$E_{22}\overline{a}'_R{}^* + E_{23}\overline{a}'_I{}^* + E_{24}\overline{V}'_{II} + E_{25}\overline{V}'_{IR} \quad \text{(B46)}$$

$$A_{-\infty I} = E_{26}\overline{A}'_{iR} + E_{27}\overline{A}'_{iI} + E_{28}\overline{\rho}'_R{}^* + E_{29}\overline{\rho}'_I{}^* + E_{30}\overline{A}'_R{}^* + E_{31}\overline{A}'_I{}^* +$$

$$E_{32}\overline{a}'_R{}^* + E_{33}\overline{a}'_I{}^* + E_{34}\overline{V}'_{II} + E_{35}\overline{V}'_{IR} \quad \text{(B47)}$$

Substitute Equations B46 and B47 into Equation B41 to give

$$M_7 (E_{16}\overline{A}'_{iR} + E_{17}\overline{A}'_{iI} + E_{18}\overline{\rho}'_R{}^* + E_{19}\overline{\rho}'_I{}^* + E_{20}\overline{A}'_R{}^* + E_{21}\overline{A}'_I{}^* +$$

$$E_{22}\overline{a}'_R{}^* + E_{23}\overline{a}'_I{}^* + E_{24}\overline{V}'_{II} + E_{25}\overline{V}'_{IR}) + M_8 (E_{26}\overline{A}'_{iR} +$$

$$E_{27}\overline{A}'_{iI} + E_{28}\overline{\rho}'_R{}^* + E_{29}\overline{\rho}'_I{}^* + E_{30}\overline{A}'_R{}^* + E_{31}\overline{A}'_I{}^* + E_{32}\overline{a}'_R{}^* +$$

$$E_{33}\overline{a}'_I{}^* + E_{34}\overline{V}'_{II} + E_{35}\overline{V}'_{IR})$$

$$= M_5\overline{A}'_{iR} + M_6\overline{A}'_R{}^* + 2\overline{\rho}'_R{}^*\overline{a}'^{*2} - \frac{k\overline{\rho}'_I{}^*\overline{U}_I\overline{V}_I}{2} -$$

$$k\overline{\rho}_I\overline{U}_I\overline{V}'_{II} - \frac{k\overline{\rho}_I\overline{a}'_I{}^*\overline{V}_I}{2} + 2\overline{\rho}^*\overline{a}^*\overline{a}'_R{}^* \quad \text{(B48)}$$

Let;

$$M_{11} = E_{18}M_7 + E_{28}M_8 - 2\overline{a}^{*2}$$

$$M_{12} = E_{19}M_7 + E_{29}M_8 + \frac{k\overline{U}_I\overline{V}_I}{2}$$

$$M_{13} = -E_{16}M_7 - E_{26}M_8 + M_5$$

$$M_{14} = -E_{17}M_7 - E_{27}M_8$$

$$M_{15} = -E_{20}M_7 - E_{30}M_8 + M_6$$

$$M_{16} = -E_{21}M_7 - E_{31}M_8$$

$$M_{17} = -E_{22}M_7 - E_{32}M_8 + 2\overline{\rho}^*\overline{a}^*$$

$$M_{18} = E_{23}M_7 - E_{33}M_8 - \frac{k\overline{\rho}_I\overline{V}_I}{2}$$

$$M_{19} = -E_{24}M_7 - E_{34}M_8 - k\overline{\rho}_I\overline{U}_I$$

$$M_{20} = -E_{25}M_7 - E_{35}M_8 \quad \text{(B49)}$$

Rearrange Equation B48 to collect $\overline{\rho}'^*_R$ and $\overline{\rho}'^*_I$ on the left side of the equation and all other terms on the right side. Substitute the relationships of Equation B49 into Equation B48.

$$M_{11}\overline{\rho}'_R{}^* + M_{12}\overline{\rho}'^*_I = M_{13}\overline{A}'_{iR} + M_{14}\overline{A}'_{iI} + M_{15}\overline{A}'^*_R + M_{16}\overline{A}'^*_I +$$

$$M_{17}\overline{a}'^*_R + M_{18}\overline{a}'^*_I + M_{19}\overline{V}'_{II} + M_{20}\overline{V}'_{IR} \quad \text{(B50)}$$

Working with the imaginary part of the momentum equation, substitute Equation B46 and B47 into Equation B44 to give

$$M_9\,[E_{16}\overline{A}'_{iR} + E_{17}A'_{iI} + E_{18}\overline{\rho}'_R{}^* + E_{19}\overline{\rho}'_I{}^* + E_{20}\overline{A}'_R{}^* + E_{21}\overline{A}'_I{}^* +$$

$$E_{22}\overline{a}'_R{}^* + E_{23}\overline{a}'_I{}^* + E_{24}\overline{V}'_{1I} + E_{25}\overline{V}'_{1R}] + M_{10}\,[E_{26}\overline{A}'_{iR} +$$

$$E_{27}\overline{A}'_{iI} + E_{28}\overline{\rho}'_R{}^* + E_{29}\overline{\rho}'_I{}^* + E_{30}A'_R{}^* + E_{31}\overline{A}'_I{}^* + E_{32}\overline{a}'_R{}^* + E_{33}\overline{a}'_I{}^* +$$

$$E_{34}\overline{V}'_{1I} + E_{35}\overline{V}'_{1R}\,] = M_5\overline{A}'_{iI} + M_6\overline{A}'_I{}^* + 2\overline{\rho}'_I{}^*\overline{a}^{*2} + 2\overline{\rho}^*\overline{a}^*\overline{a}'_I{}^* +$$

$$\frac{k\overline{\rho}'^*{}_R\overline{U}_1\overline{V}_1}{2} + k\overline{\rho}_1\overline{U}_1\overline{V}'_{1R} + \frac{k\overline{\rho}_1\overline{a}'^*{}_R\overline{V}_1}{2} \qquad \text{(B51)}$$

Further, assume the following relationships.

$$M_{21} = E_{18}M_9 + E_{28}M_{10} - \frac{k\overline{U}_1\overline{V}_1}{2}$$

$$M_{22} = E_{19}M_9 + E_{29}M_{10} - 2\overline{a}^{*2}$$

$$M_{23} = -E_{16}M_9 - E_{26}M_{10} + M_5$$

$$M_{24} = -E_{17}M_9 - E_{27}M_{10} + M_6$$

$$M_{25} = -E_{20}M_9 - E_{30}M_{10}$$

$$M_{26} = -E_{21}M_9 - E_{31}M_{10}$$

$$M_{27} = -E_{22}M_9 - E_{32}M_{10} + \frac{k\overline{\rho}_1\overline{V}_1}{2}$$

$$M_{28} = -E_{23}M_9 - E_{33}M_{10} + 2\overline{\rho}^*\overline{a}^*$$

$$M_{29} = -E_{24}M_9 - E_{34}M_{10}$$

$$M_{30} = -E_{25}M_9 - E_{35}M_{10} + k\overline{\rho}_1\overline{U}_1 \qquad \text{(B52)}$$

Substituting the relationships of Equation B52 into Equation B51 and rearranging the resultant to isolate $\overline{\rho}'^*{}_R$ and $\overline{\rho}'_I{}^*$ gives

$$M_{21}\overline{\rho}'_R{}^* + M_{22}\overline{\rho}'_I{}^* = M_{23}\overline{A}'_{iR} + M_{24}\overline{A}_{iI} + M_{25}\overline{A}'_R{}^* + M_{26}\overline{A}'_I{}^* +$$

$$M_{27}\overline{a}'_R{}^* + M_{28}\overline{a}'_I{}^* + M_{29}\overline{V}'_{1I} + M_{30}\overline{V}'_{1R} \qquad \text{(B53)}$$

Solve for $\overline{\rho}'_R{}^*$ using Equations B50 and B53. First multiply Equation B53 by M_{12}/M_{22} to give

$$\left(\frac{M_{21}M_{12}}{M_{22}}\right)\overline{\rho}'_R{}^* + M_{12}\overline{\rho}'_I{}^* = \frac{M_{12}}{M_{22}}\,[M_{23}\overline{A}'_R +$$

$$M_{24}\overline{A}'_{iI} + M_{25}\overline{A}'_R{}^* + M_{26}\overline{A}'_I{}^* +$$

$$M_{27}\overline{a}'_R{}^* + M_{28}\overline{a}'_I{}^* +$$

$$M_{29}\overline{V}'_{1I} + M_{30}\overline{V}'_{1R}]$$

Next, subtract Equation B53 from Equation B50.

$$\bar{\rho}_R'^* = \left\{ \left(M_{13} - \frac{M_{12}M_{23}}{M_{22}} \right) \bar{A}'_{iR} + \left(M_{14} - \frac{M_{12}M_{24}}{M_{22}} \right) \bar{A}'_{iI} + \left(M_{15} - \frac{M_{12}M_{25}}{M_{22}} \right) \bar{A}'_R{}^* + \left(M_{16} - \frac{M_{12}M_{26}}{M_{22}} \right) \bar{A}'_I{}^* + \left(M_{17} - \frac{M_{12}M_{27}}{M_{22}} \right) \bar{a}'_R{}^* + \left(M_{18} - \frac{M_{12}M_{28}}{M_{22}} \right) \bar{a}'_I{}^* + \left(M_{19} - \frac{M_{12}M_{29}}{M_{22}} \right) \bar{V}'_{iI} + \left(M_{20} - \frac{M_{12}M_{30}}{M_{22}} \right) \bar{V}'_{iR} \right\} \div \left\{ M_{11} - \frac{M_{12}M_{21}}{M_{22}} \right\} \quad \text{(B54)}$$

Now, let

$$M_{31} = M_{11} - \frac{M_{12}M_{21}}{M_{22}}$$

$$M_{32} = \left(M_{13} - \frac{M_{12}M_{23}}{M_{22}} \right) \div M_{31}$$

$$M_{33} = \left(M_{14} - \frac{M_{12}M_{24}}{M_{22}} \right) \div M_{31}$$

$$M_{34} = \left(M_{15} - \frac{M_{12}M_{25}}{M_{22}} \right) \div M_{31}$$

$$M_{35} = \left(M_{16} - \frac{M_{12}M_{26}}{M_{22}} \right) \div M_{31}$$

$$M_{36} = \left(M_{17} - \frac{M_{12}M_{27}}{M_{22}} \right) \div M_{31}$$

$$M_{37} = \left(M_{18} - \frac{M_{12}M_{28}}{M_{22}} \right) \div M_{31}$$

$$M_{38} = \left(M_{19} - \frac{M_{12}M_{29}}{M_{22}} \right) \div M_{31}$$

$$M_{39} = \left(M_{20} - \frac{M_{12}M_{30}}{M_{22}} \right) \div M_{31} \quad \text{(B55)}$$

Substituting the relationships in Equation B55 into Equation B54 results in the following:

$$\bar{\rho}'_R{}^* = M_{32}\bar{A}'_{iR} + M_{33}\bar{A}'_{iI} + M_{34}\bar{A}'_R{}^* + M_{35}\bar{A}'_I{}^* + M_{36}\bar{a}'_R{}^* + M_{37}\bar{a}'_I{}^* + M_{38}\bar{V}'_{iI} + M_{39}\bar{V}'_{iR} \quad \text{(B56)}$$

$$\bar{\rho}'_I{}^* = \frac{1}{M_{12}} [(M_{13} - M_{32}M_{11}) \bar{A}'_{iR} + (M_{14} - M_{33}M_{11}) \bar{A}'_{iI} +$$

$$(M_{15} - M_{34}M_{11}) \bar{A}'_R{}^* + (M_{16} - M_{35}M_{11}) \bar{A}'_I{}^* +$$

$$(M_{17} - M_{36}M_{11}) \bar{a}'_R{}^* + (M_{18} - M_{37}M_{11}) \bar{a}'_I{}^* +$$

$$(M_{19} - M_{38}M_{11}) \bar{V}'_{1I} + (M_{20} - M_{39}M_{11}) \bar{V}'_{1R}]$$

Assume the following relationships:

$$M_{40} = \frac{M_{13} - M_{32}M_{11}}{M_{12}}$$

$$M_{41} = \frac{M_{14} - M_{33}M_{11}}{M_{12}}$$

$$M_{42} = \frac{M_{15} - M_{34}M_{11}}{M_{12}}$$

$$M_{43} = \frac{M_{16} - M_{35}M_{11}}{M_{12}}$$

$$M_{44} = \frac{M_{17} - M_{36}M_{11}}{M_{12}}$$

$$M_{45} = \frac{M_{18} - M_{37}M_{11}}{M_{12}}$$

$$M_{46} = \frac{M_{19} - M_{38}M_{11}}{M_{12}}$$

$$M_{47} = \frac{M_{20} - M_{39}M_{11}}{M_{12}}$$

Substituting the relationships in Equation B51 into Equation B57 results in the value of $\bar{\rho}'_I{}^*$ in the momentum equation.

$$\bar{\rho}'_I{}^* = M_{40}\bar{A}'_{iR} + M_{41}\bar{A}'_{iI} + M_{42}\bar{A}'_R{}^* +$$

$$M_{43}\bar{A}'_I{}^* + M_{44}\bar{a}'_R{}^* + M_{45}\bar{a}'_I{}^* +$$

$$M_{46}\bar{V}'_{1I} + M_{47}\bar{V}'_{1R} \tag{B59}$$

Working with the continuity equation, recall that

$$[\rho'^*\bar{a}^* + \bar{\rho}^*A'^*\bar{a}^* + \bar{\rho}^*a'^*] - [\bar{A}_i (u_i' \sin \alpha_{ch} + v'_i \cos \alpha_{ch}) +$$

$$A'_i I + \rho'_i\bar{A}_i I] = - \left[\bar{\rho}_1 \frac{\partial V'_1}{\partial t} + \bar{V}_1 \frac{\partial \rho'_1}{\partial t}\right] \tag{B26}$$

Substituting for the perturbation quantities and dividing through by e^{ikt} gives

$$(\bar{\rho}'^*\bar{a}^* + \bar{\rho}^*\bar{A}'^*\bar{a}^* + \bar{\rho}^*\bar{a}'^*) - \left\{\bar{A}_i \left[\frac{B_1 A_{-\infty}}{C\bar{y}_i}(e^{iC\bar{y}_i} - 1)\sin\alpha_{ch} + \frac{A_{-\infty}}{\bar{y}_i}(e^{iC\bar{y}_i} - 1)\cos\alpha_{ch}\right] + \bar{A}'_i I - \left(\frac{k + \bar{u}_i B_1 + \bar{v}_i C}{C\bar{y}_i\bar{a}_i^2}\right)\bar{A}_i A_{-\infty} I (e^{iC\bar{y}_i} - 1)\right\}$$

$$= - \left[ik\bar{\rho}_1\bar{V}'_1 + \frac{ik\bar{V}_1}{2}\left(\bar{\rho}'^* - \left(\frac{k + \bar{u}_i B_1 + v_i C}{C\bar{y}_i\bar{a}_i^2}\right) A_{-\infty}(e^{iC\bar{y}_i} - 1)\right)\right] \qquad \text{(B60)}$$

Assume the following relationships:

$$C_1 = -\frac{\bar{A}_i}{C\bar{y}_i}\sin\alpha_{ch} + \frac{\bar{u}_i\bar{A}_i I}{C\bar{y}_i\bar{a}_i^2}$$

$$C_2 = -\frac{\bar{A}_i}{\bar{y}_i}\cos\alpha_{ch} + \left(\frac{k + \bar{v}_i C}{C\bar{y}_i\bar{a}_i^2}\right)\bar{A}_i I$$

$$C_3 = \frac{\bar{u}_i\bar{V}_1}{2C\bar{y}_i\bar{a}_i^2}$$

$$C_4 = \left(\frac{k + \bar{v}_i C}{2C\bar{y}_i\bar{a}_i^2}\right) V_1 \qquad \text{(B61)}$$

Substituting the relationships in Equation B61 into Equation B60 and separating the real parts of the resultant equation yields

$$C_1 [B_{1R}A_{-\infty R}E_7 - B_{1I}A_{-\infty I}E_7 - B_{1R}A_{-\infty I}\sin(C\bar{y}_i) - B_{1I}A_{-\infty R}\sin(C\bar{y}_i)] + C_2 [A_{-\infty R}E_7 - A_{-\infty I}\sin(C\bar{y}_i)] + [\bar{\rho}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_R{}^* - \bar{A}'_{iR}I]$$

$$= -kC_3 [B_{1R}A_{-\infty I}E_7 + B_{1I}A_{-\infty R}E_7 + B_{1R}A_{-\infty R}\sin(C\bar{y}_i) - B_{1I}A_{-\infty I}\sin(C\bar{y}_i)] - kC_4 [A_{-\infty I}E_7 + A_{-\infty R}\sin(C\bar{y}_i)] + k\bar{\rho}_1\bar{V}'_{1I} + \frac{k\bar{\rho}'_I{}^*\bar{V}_1}{2} \qquad \text{(B62)}$$

Again, assume

$$C_5 = C_1 B_{1R}E_7 - C_1 B_{1I}\sin(C\bar{y}_i) + C_2 E_7 + C_3 k B_{1I}E_7 + C_3 k B_{1R}\sin(C\bar{y}_i) + C_4 k\sin(C\bar{y}_i)$$

$$C_6 = -C_1 B_{1I}E_7 - C_1 B_{1R}\sin(C\bar{y}_i) - C_2\sin(C\bar{y}_i) + C_3 k B_{1R}E_7 - C_3 k B_{1I}\sin(C\bar{y}_i) + C_4 k E_7 \qquad \text{(B63)}$$

Substituting the relationships in Equation B63 into Equation B62 yields

$$C_5A_{-\infty R} + C_6A_{-\infty I} + (\bar{\rho}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_R{}^* - I\bar{A}'_{iI}) -$$

$$k\bar{\rho}_1\bar{V}'_{1I} - \frac{k\bar{\rho}'^*\bar{V}_1}{2} = 0 \qquad \text{(B64)}$$

Next, substitute the values for $A_{-\infty R}$ and $A_{-\infty I}$ into Equation B64

$$C_5(E_{16}\bar{A}'_{iR} + E_{17}\bar{A}'_{iI} + E_{18}\bar{\rho}'_R{}^* + E_{19}\bar{\rho}'_I{}^* + E_{20}\bar{A}'_R{}^* + E_{21}\bar{A}'_I{}^* +$$

$$E_{22}\bar{a}'_R{}^* + E_{23}\bar{a}'_I{}^* + E_{24}\bar{V}'_{1I} \quad E_{25}\bar{V}'_{1R})$$

Combining like terms produces

$$(C_5E_{16} + C_6E_{26} - I)\,\bar{A}'_{iR} + (C_5E_{17} + C_6E_{27})\,\bar{A}'_{iI} +$$

$$(C_5E_{18} + C_6E_{28} + \bar{a}^*)\,\bar{\rho}'_R{}^* + (C_5E_{19} + C_6E_{29} - k\bar{V}_1/2)\,\bar{\rho}'_I{}^* +$$

$$(C_5E_{20} + C_6E_{30} + \bar{\rho}^*\bar{a}^*)\,\bar{A}'_R{}^* + (C_5E_{21} + C_6E_{31})\,\bar{A}'_I{}^* +$$

$$(C_5E_{22} + C_6E_{32} + \bar{\rho}^*)\,\bar{a}'_R{}^* + (C_5E_{23} + C_6E_{33})\bar{a}'_I{}^* +$$

$$(C_5E_{24} + C_6E_{34} - k\bar{\rho}_1)\,\bar{V}'_{1I} + (C_5E_{25} + C_6E_{35})\,\bar{V}'_{1R} = 0$$

Again, assume

$$C_7 = C_5E_{16} + C_6E_{26} - I$$

$$C_8 = C_5E_{17} + C_6E_{27}$$

$$C_9 = C_5E_{18} + C_6E_{28} + \bar{a}^*$$

$$C_{10} = C_5E_{19} + C_6E_{29} - k\bar{V}_1/2$$

$$C_{11} = C_5E_{20} + C_6E_{30} + \bar{\rho}^*\bar{a}^*$$

$$C_{12} = C_5E_{21} + C_6E_{31}$$

$$C_{13} = C_5E_{22} + C_6E_{32} + \bar{\rho}^*$$

$$C_{14} = C_5E_{23} + C_6E_{33}$$

$$C_{15} = C_5E_{24} + C_6E_{34} - k\bar{\rho}_1$$

$$C_{16} = C_5E_{25} + C_6E_{35} \qquad \text{(B65)}$$

Substituting Equation B65 into Equation B64 gives

$$C_7\bar{A}'_{iR} + C_8\bar{A}'_{iI} + C_9\bar{\rho}'_R{}^* + C_{10}\bar{\rho}'_I{}^* + C_{11}\bar{A}'_R{}^* + C_{12}\bar{A}'_I{}^* +$$

$$C_{13}\bar{a}'_R{}^* + C_{14}\bar{a}'_I{}^* + C_{15}\bar{V}'_{1I} + C_{16}\bar{V}'_{1R} = 0 \qquad \text{(B66)}$$

Substituting for $\bar{\rho}'_R{}^*$ and $\bar{\rho}'_I{}^*$ results in

$$C_9\,(M_{32}\bar{A}'_{iR} + M_{33}\bar{A}'_{iI} + M_{34}\bar{A}'_R{}^* + M_{35}\bar{A}'_I{}^* + M_{36}\bar{a}'_R{}^* + M_{37}\bar{a}'_I{}^* +$$
$$M_{38}\bar{V}'_I + M_{39}\bar{V}'_R) + C_{10}\,(M_{40}\bar{A}'_{iR} + M_{41}\bar{A}'_{iI} + M_{42}\bar{A}'_R{}^* + M_{43}\bar{A}'_I{}^* +$$
$$M_{44}\bar{a}'_R{}^* + M_{45}\bar{a}'_I{}^* + M_{46}\bar{V}'_{1I} + M_{47}\bar{V}'_{1R}) \quad C_7\bar{A}'_{iR} + C_8\bar{A}'_{iI} +$$
$$C_{11}\bar{A}'_R{}^* + C_{12}\bar{A}'_I{}^* + C_{13}\bar{a}'_R{}^* + C_{14}\bar{a}'_I{}^* + C_{15}\bar{V}'_{1I} + C_{16}\bar{V}'_{1R} = 0$$

Combining like terms gives

$$(C_9M_{32} + C_{10}M_{40} + C_7)\,\bar{A}'_{iR} + (C_9M_{33} + C_{10}M_{41} + C_8)\,\bar{A}'_{iI} +$$
$$(C_9M_{34} + C_{10}M_{42} + C_{11})\,\bar{A}'_R{}^* + (C_9M_{35} + C_{10}M_{43} + C_{12})\,\bar{A}'_I{}^* +$$
$$(C_9M_{36} + C_{10}M_{44} + C_{13})\,\bar{a}'_R{}^* + (C_9M_{37} + C_{10}M_{45} + C_{14})\,\bar{a}'_I{}^* +$$
$$(C_9M_{38} + C_{10}M_{46} + C_{15})\,\bar{V}'_{1I} + (C_9M_{39} + C_{10}M_{47} + C_{16})\,\bar{V}'_{1R} = 0 \qquad \text{(B67)}$$

Assume

$$C_{17} = C_9M_{32} + C_{10}M_{40} + C_7$$
$$C_{18} = C_9M_{33} + C_{10}M_{41} + C_8$$
$$C_{19} = C_9M_{34} + C_{10}M_{42} + C_{11}$$
$$C_{20} = C_9M_{35} + C_{10}M_{43} + C_{12}$$
$$C_{21} = -\,(C_9M_{36} + C_{10}M_{44} + C_{13})$$
$$C_{22} = -\,(C_9M_{37} + C_{10}\,M_{45} + C_{14})$$
$$C_{23} = C_9M_{38} + C_{10}\,M_{46} + C_{15}$$
$$C_{24} = C_9M_{39} + C_{10}\,M_{47} + C_{16} \qquad \text{(B68)}$$

Substituting the relationships in Equation B68 into Equation B67 and solving for $a'_R{}^*$ and $a'_I{}^*$ gives

$$C_{21}\bar{a}'_R{}^* + C_{22}\bar{a}'_I{}^* = C_{17}\bar{A}'_{iR} + C_{18}\bar{A}'_{iI} + C_{19}\bar{A}'_R{}^* + C_{20}\bar{A}'_I{}^* + C_{23}\bar{V}'_{1I} + C_{24}\bar{V}'_{1R} \qquad \text{(B69)}$$

The imaginary part of Equation 24 is

$$(\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}\bar{A}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_I{}^*) - I\bar{A}'_{iI} + C_1\,[B_{1R}A_{-\infty I}E_7 +$$
$$B_{1I}A_{-\infty R}E_7 + B_{1R}A_{-\infty R}\sin(C\bar{y}_i) - B_{1I}A_{-\infty I}\sin(Cy_i)] +$$
$$C_2\,[A_{-\infty I}E_7 + A_{-\infty R}\sin(C\bar{y}_i)] - kC_3\,[B_{1R}A_{-\infty R}E_7 -$$
$$B_{1I}A_{-\infty I}E_7 - B_{1R}A_{-\infty I}\sin(C\bar{y}_i) - B_{1I}A_{-\infty R}\sin(C\bar{y}_i)] -$$
$$kC_4\,[A_{-\infty R}E_7 - A_{-\infty I}\sin(C\bar{y}_i)] + k\bar{\rho}_1\,\bar{V}'_{1R} + \frac{k\bar{\rho}'_R{}^*\bar{V}_1}{2} = 0 \qquad \text{(B70)}$$

Again, let

$$C_{25} = C_1 [B_{1I}E_7 + B_{1R} \sin (C\bar{y}_i)] + C_2 \sin (C\bar{y}_i) - kC_3 [B_{1R}FE_7 - B_{1I} \sin (C\bar{y}_i)] - kC_4E_7$$

$$C_{26} = C_1 [B_{1R}E_7 - B_{1I} \sin (C\bar{y}_i)] + C_2E_7 + kC_3 [B_{1I}E_7 + B_{1R} \sin (C\bar{y}_i)] + kC_4 \sin (C\bar{y}_i) \qquad \text{(B71)}$$

Substituting the relationships of Equation B71 into Equation B70 yields

$$C_{25}A_{-\infty R} + C_{26}A_{-\infty I} + (\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_I{}^*) - I\bar{A}'_{iI} + k\bar{\rho}_1 \bar{V}'_{1R} + \frac{k\bar{\rho}'_R{}^*\bar{V}_1}{2} = 0 \qquad \text{(B72)}$$

Substituting for $A_{-\infty R}$ and $A_{-\infty I}$ into Equation B72 gives

$$C_{26} [E_{26}\bar{A}'_{1R} + E_{27}\bar{A}'_{iI} + E_{28}\bar{\rho}'_R{}^* + E_{29}\bar{\rho}'_I{}^* + E_{30}\bar{A}'_R{}^* + E_{31}\bar{A}'_I{}^* + E_{32}\bar{a}'_R{}^* + E_{33}\bar{a}'_I{}^* + E_{34}\bar{V}'_{1I} + E_{35}\bar{V}'_{1R}] + C_{25} [E_{16}\bar{A}'_{1R} + E_{17}\bar{A}'_{iI} + E_{18}\bar{\rho}'_R{}^* + E_{19}\bar{\rho}'_I{}^* + E_{20}\bar{A}'_R{}^* + E_{21}\bar{A}'_I{}^* + E_{22}\bar{a}'_R{}^* + E_{23}\bar{a}'_I{}^* + E_{24}\bar{V}'_{1I} + E_{25}\bar{V}'_{1R}] + [\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_I{}^*] - I\bar{A}'_{iI} + k\rho_1\bar{V}'_{1R} + \frac{k\bar{V}_1}{2}\bar{\rho}'_R{}^* = 0$$

Collecting like terms, as before, produces

$$(C_{26} E_{26} + C_{25} E_{16}) \bar{A}'_{iR} + (C_{26} E_{27} + C_{25} E_{17} - I) \bar{A}'_{iI} + (C_{26} E_{28} + C_{25} E_{18} + \frac{k\bar{V}_1}{2}) \bar{\rho}'_R{}^* + (C_{26} E_{29} + C_{25} E_{19} + \bar{a}^*) \bar{\rho}'_I{}^* + (C_{26} E_{30} + C_{25} E_{20}) \bar{A}'_R{}^* + (C_{26} E_{31} + C_{25} E_{21} + \bar{\rho}^* \bar{a}^*) \bar{A}'_I{}^* + (C_{26} E_{32} + C_{25} E_{22}) \bar{a}'_R{}^* + (C_{26} E_{33} + C_{25} E_{23} + \bar{\rho}^*) \bar{a}'_I{}^* + (C_{26} E_{34} + C_{25} E_{24}) \bar{V}'_{1I} + (C_{26} E_{35} + C_{25} E_{25} + k\bar{\rho}_1)\bar{V}'_{1R} = 0 \qquad \text{(B73)}$$

Again, assume

$$C_{27} = C_{26} E_{26} + C_{25} E_{16}$$

$$C_{28} = C_{26} E_{27} + C_{25} E_{17} - I$$

$$C_{29} = C_{26} E_{28} + C_{25} E_{18} + \frac{k\bar{V}_1}{2}$$

$$C_{30} = C_{26} E_{29} + C_{25} E_{19} + \bar{a}^*$$

$$C_{31} = C_{26} E_{30} + C_{25} E_{20}$$

$$C_{32} = C_{26} E_{31} + C_{25} E_{21} + \bar{\rho}^* \bar{a}^*$$

$$C_{33} = C_{26}\,E_{32} + C_{25}\,E_{22}$$

$$C_{34} = C_{26}\,E_{33} + C_{25}\,E_{23} + \bar{\rho}^*$$

$$C_{35} = C_{26}\,E_{34} + C_{25}\,E_{24}$$

$$C_{36} = C_{26}\,E_{35} + C_{25}\,E_{25} + k\,\bar{\rho}_1 \qquad \text{(B74)}$$

Substituting the relationships of Equation B74 and the relationships for $\bar{\rho}'_R{}^*$ and $\bar{\rho}'_I{}^*$, and collecting terms gives

$$(C_{29}M_{32} + C_{30}M_{40} + C_{27})\,\bar{A}'_{iR} + (C_{29}M_{33} + C_{30}M_{41} + C_{28})\,\bar{A}'_{iI} +$$
$$(C_{29}M_{34} + C_{30}M_{42} + C_{31})\,\bar{A}'_R{}^* + (C_{29}M_{35} + C_{30}M_{43} + C_{32})\,\bar{A}'_I{}^* +$$
$$(C_{29}M_{36} + C_{30}M_{44} + C_{33})\,\bar{a}'_R{}^* + (C_{29}M_{37} + C_{30}M_{45} + C_{34})\,\bar{a}'_I{}^* +$$
$$(C_{29}M_{38} + C_{30}M_{46} + C_{35})\,\bar{V}'_{1I} + (C_{29}M_{39} + C_{30}M_{47} + C_{36})\,\bar{V}'_{1R} = 0 \qquad \text{(B75)}$$

Further, assume

$$C_{37} = C_{29}M_{32} + C_{30}M_{40} + C_{27}$$

$$C_{38} = C_{29}M_{33} + C_{30}M_{41} + C_{28}$$

$$C_{39} = C_{29}M_{34} + C_{30}M_{42} + C_{31}$$

$$C_{40} = C_{29}M_{35} + C_{30}M_{43} + C_{32}$$

$$C_{41} = -\,(C_{29}M_{36} + C_{30}M_{44} + C_{33})$$

$$C_{42} = -\,(C_{29}M_{37} + C_{30}M_{45} + C_{34})$$

$$C_{43} = C_{29}M_{38} + C_{30}M_{46} + C_{35}$$

$$C_{44} = C_{29}M_{39} + C_{30}M_{47} + C_{36} \qquad \text{(B76)}$$

Substituting the relationships of Equation B76 into Equation B75 gives

$$C_{41}\,\bar{a}'_R{}^* + C_{42}\,\bar{a}'_I{}^* = C_{37}\,\bar{A}'_{iR} + C_{38}\,\bar{A}'_{iI} +$$
$$C_{39}\,\bar{A}'_R{}^* + C_{40}\bar{A}'_I + C_{43}\,\bar{V}'_{1I} + C_{44}\,\bar{V}'_{1R} \qquad \text{(B77)}$$

Recall that

$$C_{21}\bar{a}'_R{}^* + C_{22}\bar{a}'_I{}^* = C_{17}\bar{A}'_{iR} + C_{18}\bar{A}'_{iI} + C_{19}\bar{A}'_R{}^* +$$
$$C_{20}\bar{A}'_I{}^* + C_{23}\bar{V}'_{1I} + C_{24}\bar{V}'_{1R} \qquad \text{(B69)}$$

Multiplying Equation B78 by C_{42}/C_{22} and subtracting the resultant from Equation B77 yields

$$\bar{a}'_R{}^* = \Bigg[\left(C_{37} - \frac{C_{17}C_{42}}{C_{22}}\right)\bar{A}'_{iR} + \left(C_{38} - \frac{C_{18}C_{42}}{C_{22}}\right)\bar{A}'_{iI} +$$

$$\left(C_{39} - \frac{C_{19}C_{42}}{C_{22}}\right)\bar{A}'_R{}^* + \left(C_{40} - \frac{C_{20}C_{42}}{C_{22}}\right)\bar{A}'_I{}^* +$$

$$\left(C_{43} - \frac{C_{23}C_{42}}{C_{22}}\right)\bar{V}'_{1I} + \left(C_{44} - \frac{C_{24}C_{42}}{C_{22}}\right)\bar{V}'_{1R}\Bigg] \div$$

$$\left[C_{41} - \frac{C_{21}C_{42}}{C_{22}}\right]$$

Further, assure

$$C_{45} = C_{41} - \frac{C_{21}C_{42}}{C_{22}}$$

$$C_{46} = \frac{1}{C_{45}}\left(C_{37} - \frac{C_{17}C_{42}}{C_{22}}\right)$$

$$C_{47} = \frac{1}{C_{45}}\left(C_{38} - \frac{C_{18}C_{42}}{C_{22}}\right)$$

$$C_{48} = \frac{1}{C_{45}}\left(C_{39} - \frac{C_{19}C_{42}}{C_{22}}\right)$$

$$C_{49} = \frac{1}{C_{45}}\left(C_{40} - \frac{C_{20}C_{42}}{C_{22}}\right)$$

$$C_{50} = \frac{1}{C_{45}}\left(C_{43} - \frac{C_{23}C_{42}}{C_{22}}\right)$$

$$C_{51} = \frac{1}{C_{45}}\left(C_{44} - \frac{C_{24}C_{42}}{C_{22}}\right) \qquad \text{(B78)}$$

Thus,

$$\Rightarrow \bar{a}'_R{}^* = C_{46}\bar{A}'_{iR} + C_{47}\bar{A}'_{iI} + C_{48}\bar{A}'_R{}^* + C_{49}\bar{A}'_I{}^* +$$

$$C_{50}\bar{V}'_{1I} + C_{51}\bar{V}'_{1R} \qquad \text{(B79)}$$

Substituting Equation B79 into Equation B77 and solving for $a'_I{}^*$ and then combining like terms gives

$$\bar{a}'_I{}^* = \frac{1}{C_{42}}\left[(C_{37} - C_{41}C_{46})\,\bar{A}'_{iR} + (C_{38} - C_{41}C_{47})\,\bar{A}'_{iI} +\right.$$

$$(C_{39} - C_{41}C_{48})\,\bar{A}'_R{}^* + (C_{40} - C_{41}C_{49})\,\bar{A}'_I{}^* +$$

$$\left.(C_{43} - C_{41}C_{50})\,\bar{V}'_{1I} + (C_{44} - C_{41}C_{51})\,\bar{V}'_{1R}\right]$$

Further, assume

$$C_{52} = (C_{37} - C_{41}C_{46}) \div C_{42}$$

$$C_{53} = (C_{38} - C_{41}C_{47}) \div C_{42}$$

$$C_{54} = (C_{39} - C_{41}C_{48}) \div C_{42}$$

$$C_{55} = (C_{40} - C_{41}C_{49}) \div C_{42}$$

$$C_{56} = (C_{43} - C_{41}C_{50}) \div C_{42}$$

$$C_{57} = (C_{44} - C_{41}C_{51}) \div C_{42} \qquad \text{(B80)}$$

Substituting the relationships of Equation 136 into the relationship for $a'_I{}^*$ gives

$$\Rightarrow \bar{a}'_I{}^* = C_{52}\overline{A}'_{iR} + C_{53}\overline{A}'_{iI} + C_{54}\overline{A}'_R{}^* + C_{55}\overline{A}'_I{}^* +$$

$$C_{56}\overline{V}'_{II} + C_{57}\overline{V}'_{IR} \qquad \text{(B81)}$$

This completes the analysis for Region 1.

APPENDIX C
SOLUTIONS TO THE EQUATIONS OF MOTION FOR REGION 2

The nondimensionalized equations of motion are represented by Equations 38 and 39. Only two equations are needed because all the inlet parameters to Region 2 are known. This leaves two unknowns: the density and velocity perturbations just upstream of the shock. To obtain a solution to these equations, again make the assumption that all flow parameters vary harmonically in time. Thus,

$$p'_{us} = \bar{p}'_{us} \exp ikt$$

$$A'_s = \bar{A}'_s \exp ikt$$

$$\rho'_{us} = \bar{\rho}'_{us} \exp ikt$$

$$U'_{us} = \bar{U}'_{us} \exp ikt \qquad \text{(C1)}$$

making these substitutions into the momentum equation (39) and dividing by e^{ikt} gives

$$(\bar{p}^*\bar{A}'^* + \bar{p}'^* - \bar{p}_{us}\bar{A}'_{us} - \bar{p}'_{us}\bar{A}_s)$$
$$= [\bar{U}'_{us}\bar{w}_{us} + \bar{U}_{us}(\bar{\rho}'_{us}\bar{U}_{us}\bar{A}_s + \bar{\rho}_{us}\bar{U}'_{us}\bar{A}_s + \bar{\rho}_{us}\bar{U}_{us}\bar{A}'_s)] -$$
$$[\bar{\rho}^*\bar{a}'^*\bar{a}^* + \bar{a}^*(\rho'^*\bar{a}^* + \bar{\rho}^*\bar{A}'^*\bar{a}^* + \bar{\rho}^*\bar{a}'^*)] +$$
$$\frac{ik\bar{U}_2\bar{V}_2}{2}(\bar{\rho}'^* + \bar{\rho}'_{us}) + ik\bar{\rho}_2\bar{U}_2\bar{V}'_2 + ik\frac{\bar{\rho}_2\bar{V}_2}{2}(\bar{a}'^* + \bar{U}'_{us}) \qquad \text{(C2)}$$

Making the substitution $p' = \bar{a}^2\rho'$ and collecting terms produces

$$(\bar{p}^*\bar{A}'^* + \bar{p}'^* - \bar{p}_{US}\bar{A}'_S - \bar{\rho}'_{US}\bar{A}_S\bar{a}^2_{US})$$
$$= (2\bar{U}'_{US}\bar{w}_{US} + \bar{\rho}'_{US}\bar{U}^2_{US}\bar{A}_S + \bar{\rho}_{US}\bar{U}^2_{US}\bar{A}'_S) - (2\bar{\rho}^*\bar{a}'^*\bar{a}^* + \rho'^*\bar{a}^{*2} + \bar{\rho}^*\bar{A}'^*\bar{a}^{*2}) +$$
$$\frac{ik\bar{U}_2\bar{V}_2}{2}(\bar{\rho}'^* + \bar{\rho}'_{US}) + ik\bar{\rho}_2\bar{U}_2\bar{V}'_2 + ik\bar{\rho}_2\bar{V}_2(\bar{a}'^* + \bar{U}'_{US}) \qquad \text{(C3)}$$

Real Part

$$\bar{p}'^*_R + \bar{p}^*\bar{A}'^*_R - \bar{\rho}'_{USR}\bar{A}_S\bar{a}^2_{US} - \bar{p}_{US}\bar{A}'_{SR}$$
$$= \bar{\rho}'_{USR}\bar{U}^2_{US}\bar{A}_S - \frac{k\bar{\rho}'_{USI}\bar{U}_2\bar{V}_2}{2} + 2\bar{U}'_{USR}\bar{w}_{US} - \frac{k\bar{\rho}_2\bar{U}'_{USI}\bar{V}_2}{2} -$$
$$2\bar{\rho}^*\bar{a}'^*_R\bar{a}^* \frac{-k\bar{\rho}_2\bar{a}'^*_I\bar{V}_2}{2} - \bar{\rho}'^*_R\bar{a}^{*2} - \frac{k\bar{\rho}'^*_I\bar{U}_2\bar{V}_2}{2} - k\bar{\rho}_2\bar{U}_2\bar{V}'_{2I} +$$
$$\bar{\rho}_{US}\bar{U}^2_{US}\bar{A}'_{SR} - \bar{\rho}^*\bar{A}'^*_R\bar{a}^{*2} \qquad \text{(C4)}$$

Rearranging terms produces

$$(-\bar{a}^2_{US}\bar{A}_S - \bar{U}^2_{US}\bar{A}_S)\,\bar{\rho}'_{USR} + \left(\frac{k\bar{U}_2\bar{V}_2}{2}\right)\bar{\rho}'_{USI}$$

$$= (\bar{\rho}_{US}\bar{U}^2_{US} + \bar{p}_{US})\,\bar{A}'_{SR} - 2\bar{p}'^*_R - (\bar{p}^* + \bar{\rho}^*\bar{a}^{*2})\,\bar{A}'^*_R + 2\bar{U}'_{USR}\bar{w}_{US} -$$

$$\frac{k\bar{\rho}_2\bar{U}'_{USI}\bar{V}_2}{2} - \frac{k\bar{\rho}'_I{}^*\bar{U}_2\bar{V}_2}{2} - k\bar{\rho}_2\bar{U}_2\bar{V}'_{2I} - 2\bar{\rho}^*\bar{a}'^*_R\bar{a}^* - \frac{k\bar{\rho}_2\bar{a}'_I{}^*\bar{V}_2}{2} \qquad \text{(C5)}$$

Next, assume

$$M_{48} = (-\bar{a}^2_{US} - \bar{U}^2_{US})\,\bar{A}_S$$

$$M_{49} = \frac{k\bar{U}_2\bar{V}_2}{2} \qquad \text{(C6)}$$

$$M_{50} = \bar{p}_{US} + \bar{\rho}_{US}\bar{U}^2_{US}$$

$$M_{50A} = \bar{p}^* + \bar{\rho}^*\bar{a}^{*2}$$

Substituting Equation C6 into Equation C5 gives

$$M_{48}\bar{\rho}'_{USR} + M_{49}\bar{\rho}'_{USI} = M_{50}\bar{A}'_{SR} - 2\bar{p}'_R{}^* + M_{50A}\bar{A}'_R{}^* + 2\bar{U}'_{USI}\bar{w}_{US} - \qquad \text{(C7)}$$

$$\frac{k\bar{\rho}_2\bar{U}'_{USI}\bar{V}_2}{2} - \frac{k\bar{\rho}'^*_I\bar{U}_2\bar{V}_2}{2} - k\bar{\rho}_2\bar{U}_2\bar{V}'_{2I} - 2\bar{\rho}^*\bar{a}'^*_R\bar{a}^* - \frac{k\bar{\rho}_2\bar{a}'^*_I\bar{V}_2}{2}$$

Imaginary Part

$$M_{48}\bar{\rho}'_{USI} - M_{49}\bar{\rho}'_{USR} = M_{50}\bar{A}'_{SI} - 2\bar{p}'_I{}^* - M_{50A}\bar{A}'_I{}^* + 2\bar{U}'_{USI}\bar{w}_{US} +$$

$$\frac{k\bar{\rho}_2\bar{U}'_{USR}\bar{V}_2}{2} + \frac{k\bar{\rho}'^*_R\bar{U}_2\bar{V}_2}{2} + k\bar{\rho}_2\bar{U}_2\bar{V}'_{2R} - 2\bar{\rho}'^*\bar{a}'^*_I\bar{a}^* + \frac{k\bar{\rho}_2\bar{a}'^*_R\bar{V}_2}{2} \qquad \text{(C8)}$$

Multiply Equation C8 by M_{48}/M_{49} and add to Equation C7 to produce

$$\left(M_{49} + \frac{M_{48}^2}{M_{49}}\right) \bar{\rho}'_{USI} = M_{50}\bar{A}'_{SR} + \left(\frac{M_{48}\,M_{50}}{M_{49}}\right) \bar{A}'_{SI} - 2\bar{p}'^*_R - 2\left(\frac{M_{48}}{M_{49}}\right) \bar{p}'_I{}^* +$$

$$M_{50A}\bar{A}'_R{}^* + \left(\frac{M_{48}\,M_{50A}}{M_{49}}\right) \bar{A}'_I{}^* + 2\bar{U}'_{USR}\bar{w}_{US} + 2\left(\frac{M_{48}\,\bar{w}_{US}}{M_{49}}\right) \bar{U}'_{USI} -$$

$$\frac{k\bar{\rho}_2\bar{U}'_{USI}\bar{V}_2}{2} + \frac{k\bar{\rho}_2\bar{U}'_{USR}\bar{V}_2}{2}\left(\frac{M_{48}}{M_{49}}\right) - \frac{k\bar{\rho}'^*_I\bar{U}_2\bar{V}_2}{2} + \frac{k\bar{\rho}'^*_R\bar{U}_2\bar{V}_2}{2}\left(\frac{M_{48}}{M_{49}}\right)$$

$$- k\bar{\rho}_2\bar{U}_2V'_{2I} + \frac{k\bar{\rho}_2\bar{U}_2\bar{V}'_{2R}}{2}\left(\frac{M_{48}}{M_{49}}\right) - 2\bar{\rho}^*\bar{a}'^*_R\bar{a}^* - 2\bar{\rho}^*\bar{a}'^*_I\left(\frac{M_{48}}{M_{49}}\right)\bar{a}^* -$$

$$\frac{k\bar{\rho}_2\bar{a}'^*_I\bar{V}_2}{2} + \frac{k\bar{\rho}_2\bar{a}'^*_R\bar{V}_2}{2}\left(\frac{M_{48}}{M_{49}}\right) \tag{C9}$$

Assume the following relationships

$$M_{51} = M_{49} + \frac{M_{48}^2}{M_{49}}$$

$$M_{52} = \frac{M_{50}}{M_{51}}$$

$$M_{53} = \frac{M_{48}\, M_{50}}{M_{49}\, M_{51}}$$

$$M_{54} = \frac{-2}{M_{51}}$$

$$M_{55} = \frac{-2\, M_{48}}{M_{49}\, M_{51}}$$

$$M_{56} = \frac{M_{50A}}{M_{51}}$$

$$M_{57} = \frac{M_{48}\, M_{50A}}{M_{49}\, M_{51}}$$

$$M_{58} = \left[\, 2\bar{w}_{US} + \frac{k\bar{\rho}_2\bar{V}_2}{2} \left(\frac{M_{48}}{M_{49}}\right)\right] \div M_{51}$$

$$M_{59} = \frac{1}{M_{51}} \left(\frac{2\bar{w}_{US}M_{48}}{M_{49}} - \frac{k\bar{\rho}_2\bar{V}_2}{2}\right)$$

$$M_{60} = \frac{1}{M_{51}} \left(- \frac{\bar{a}^*2\bar{\rho}^*\, M_{48}}{M_{49}} - \frac{k\bar{\rho}_2\bar{V}_2}{2}\right)$$

$$M_{61} = \frac{1}{M_{51}} \left(- 2\bar{\rho}^*\bar{a}^* + \frac{k\bar{\rho}_2\bar{V}_2\, M_{48}}{2M_{49}}\right)$$

$$M_{62} = \frac{1}{M_{51}} \left(\frac{-k\bar{U}_2\bar{V}_2}{2}\right)$$

$$M_{63} = \frac{1}{M_{51}} \left(\frac{k\bar{U}_2\bar{V}_2\, M_{48}}{2M_{49}}\right)$$

$$M_{64} = \frac{-k(\bar{\rho}\,\bar{U})_2}{M_{51}}$$

$$M_{65} = k(\bar{\rho}\,\bar{U})_2 \left(\frac{M_{48}}{M_{49}\, M_{51}}\right) \qquad \text{(C10)}$$

Substituting these values into Equation 3.7 and solving for ρ'_{USI} gives

$$\bar{\rho}'_{USI} = M_{52}\bar{A}'_{SR} + M_{53}\bar{A}'_{SI} + M_{54}\bar{\rho}'_R{}^* + M_{55}\bar{\rho}'_I{}^* + M_{56}\bar{A}'_R{}^* +$$

$$M_{57}\bar{A}'_I{}^* + M_{58}\bar{U}'_{USR} + M_{59}\bar{U}'_{USI} + M_{60}\bar{a}'_I{}^* + M_{61}\bar{a}'_R{}^* +$$

$$M_{62}\bar{\rho}'_I{}^* + M_{63}\bar{\rho}'_R{}^* + M_{64}\bar{V}'_{2I} + M_{65}\bar{V}'_{2R} \qquad \text{(C11)}$$

Substituting the relationships of Equation C11 into C7 and solving for $\bar{\rho}'_{USR}$ gives

$$\bar{\rho}'_{USR} = \frac{1}{M_{48}}\Bigg[(M_{50} - M_{49}M_{52})\ \bar{A}'_{SR} + (M_{50A} - M_{49}M_{53})\ \bar{A}'_{SI} +$$

$$(-2 - M_{49}M_{54})\ \bar{p}'_R{}^* + (-M_{49}M_{55})\ \bar{p}'_I{}^* +$$

$$(M_{50A} - M_{49}M_{56})\ A'_R{}^* + (-M_{49}M_{57})\ \bar{A}'_I{}^* + (2\bar{w}_{US} - M_{49}M_{58})\ \bar{U}'_{USR} +$$

$$\left(\frac{-k\bar{\rho}_2\bar{V}_2}{2} - M_{49}M_{59}\right)\bar{U}'_{USI} + \left(\frac{-k\bar{\rho}_2\bar{V}'_2}{2} - M_{49}M_{60}\right)\bar{a}'_I{}^* +$$

$$(-2\bar{\rho}^* - M_{49}M_{61})\ \bar{a}'_R{}^* + \left(\frac{-k\bar{U}_2\bar{V}_2}{2} - M_{49}M_{62}\right)\ \bar{\rho}'_I{}^* + (-M_{49}M_{63})\ \bar{\rho}'_R{}^* +$$

$$(-k\bar{\rho}_2\bar{U}_2 - M_{49}M_{64})\ \bar{V}'_{2I} + (-M_{49}M_{65})\ \bar{V}'_{2R}\Bigg] \qquad \text{(C12)}$$

Further, let

$$M_{66} = \frac{M_{50} - M_{49}\,M_{52}}{M_{48}}$$

$$M_{67} = -\frac{M_{49}\,M_{54}}{M_{48}}$$

$$M_{68} = \frac{-2 - M_{49}\,M_{54}}{M_{48}}$$

$$M_{69} = \frac{-M_{49}\,M_{55}}{M_{48}}$$

$$M_{70} = (M_{50A} - M_{49}\,M_{56})\,/M_{48}$$

$$M_{71} = -M_{49}\,M_{57}/M_{48}$$

$$M_{72} = (2\bar{w}_{US} - M_{49}\,M_{58})/M_{48}$$

$$M_{73} = (-\tfrac{1}{2}k\bar{\rho}_2\bar{V}_2 - M_{49}\,M_{59})/M_{48}$$

$$M_{74} = (-\tfrac{1}{2}k\bar{\rho}_2\bar{V}_2 - M_{49}\,M_{60})/M_{48}$$

$$M_{75} = (-2\bar{\rho}^*\bar{a}^* - M_{49}\,M_{61})/M_{48}$$

$$M_{76} = (-\tfrac{1}{2}k\bar{U}_2\bar{V}_2 - M_{49}M_{62})/M_{48}$$

$$M_{77} = -M_{49}M_{63}/M_{48}$$

$$M_{78} = (-k(\bar{\rho}\bar{U})_2 - M_{49}M_{63})/M_{48}$$

$$M_{79} = -M_{49}M_{64}/M_{48} \qquad \text{(C13)}$$

Substituting Equation C13 into Equation C12 gives

$$\rho'_{USR} = M_{76}\bar{A}'_{SR} + M_{77}\bar{A}'_{SI} + M_{78}\bar{P}'^*_R + M_{79}\bar{P}'^*_I + M_{80}\bar{A}'^*_R + M_{81}\bar{A}'^*_I +$$
$$M_{82}\bar{U}'_{USR} + M_{83}\bar{U}'_{USI} + M_{84}\bar{a}'^*_I + M_{85}\bar{a}'^*_R + M_{86}\bar{\rho}'^*_I + M_{87}\bar{\rho}'_R{}^* +$$
$$M_{88}\bar{V}'_{2I} + M_{89}\bar{V}'_{2R} \qquad \text{(C14)}$$

Next, the continuity equation is used to solve for the velocity perturbation upstream of the shock. Separating Equation 38 into real and imaginary parts gives

Real Part

$$\bar{\rho}_{US}\bar{U}'_{USR}\bar{A}_s = -(\bar{\rho}'_{USR}\bar{U}_{US}\bar{A}_s + \bar{\rho}_{US}\bar{U}_{US}\bar{A}'_{SR}) + (\bar{\rho}'_R{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_R{}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}^*) +$$
$$k\left[\frac{V_2}{2}(\bar{\rho}'_I{}^* + \bar{\rho}'_{USI}) + \bar{\rho}_2\bar{V}'_{2I}\right] \qquad \text{(C15)}$$

Imaginary Part

$$\bar{\rho}_{US}\bar{U}'_{USI}\bar{A}_S = -(\bar{\rho}'_{USI}\bar{U}_{US}\bar{A}_S + \bar{\rho}_{US}\bar{U}_{US}\bar{A}'_{SI}) + (\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_I{}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^*) -$$
$$k\left[\frac{\bar{V}_2}{2}(\bar{\rho}'_R{}^* + \bar{\rho}'_{USR}) + \bar{\rho}_2\bar{V}'_{2R}\right] \qquad \text{(C16)}$$

Substituting for $\bar{\rho}'_{USR}$ and $\bar{\rho}'_{USI}$ in Equation C15 produces

$$\bar{\rho}_{US}\bar{U}'_{USR}\bar{A}_S = -\bar{U}_{US}\bar{A}_S(M_{66}\bar{A}'_{SR} + M_{67}\bar{A}'_{SI} + M_{68}\bar{p}'_R{}^* + M_{69}\bar{p}'_R{}^* + M_{70}\bar{A}'_R{}^* +$$
$$M_{71}\bar{A}'_I{}^* + M_{72}\bar{U}'_{USR} + M_{73}\bar{U}'_{USI} + M_{74}\bar{a}'_I{}^* + M_{75}\bar{a}'_R{}^* + M_{76}\bar{\rho}'_I{}^* + M_{77}\bar{\rho}'_R{}^* +$$
$$M_{78}\bar{V}'_{2I} + M_{79}\bar{V}'_{2R}) - \bar{\rho}_{US}\bar{U}_{US}\bar{A}'_{SR} + (\bar{\rho}'_R{}^*a^* + \bar{\rho}^*\bar{a}'_R{}^* + \bar{\rho}^*\bar{A}'_R{}^*\bar{a}'^*) +$$
$$\frac{k\bar{V}_2}{2}\bar{\rho}'_I{}^* + \frac{k\bar{V}_2}{2}(M_{52}\bar{A}'_{SR} + M_{53}\bar{A}'_{SI} + M_{54}\bar{p}'_R{}^* + M_{55}\bar{p}'_I{}^* + M_{56}\bar{A}'_R{}^* +$$
$$M_{57}\bar{A}'_R{}^* + M_{58}\bar{U}'_{USR} + M_{59}\bar{U}'_{USI} + M_{60}\bar{a}'_I{}^* + M_{61}\bar{a}'^* + M_{62}\bar{\rho}'_I{}^* + M_{63}\bar{\rho}'_R{}^* +$$
$$M_{64}\bar{V}'_{2I} + M_{65}\bar{V}'_{2R}) + k\bar{\rho}_2\bar{V}'_{2I}$$

Next, combine like terms and rearrange so that ($\overline{U}'_{USR}$ and $\overline{U}'_{USI}$) is on the left-hand side of the equation and let:

$$M_{80} = \bar{\rho}_{US}\bar{A}_S + \bar{U}_{US}\bar{A}_S M_{72} - \frac{k\bar{V}_2}{2} M_{58}$$

$$M_{81} = \bar{U}_{US}\bar{A}_S M_{73} - \frac{k\bar{V}_2}{2} M_{59}$$

$$M_{82} = - \bar{U}_{US}\bar{A}_S M_{66} - \bar{\rho}_{US}\bar{U}_{US} + \frac{k\bar{V}_2}{2} M_{52}$$

$$M_{83} = - \bar{U}_{US}\bar{A}_S M_{67} + \frac{k\bar{V}_2}{2} M_{53}$$

$$M_{84} = - \bar{U}_{US}\bar{A}_S M_{77} + \bar{a}^* + \frac{k\bar{V}_2}{2} M_{63}$$

$$M_{85} = - \bar{U}_{US}\bar{A}_S M_{76} + \frac{k\bar{V}_2}{2} M_{62}$$

$$M_{86} = - \bar{U}_{US}\bar{A}_S M_{68} + \frac{k\bar{V}_2}{2} M_{54}$$

$$M_{87} = - \bar{U}_{US}\bar{A}_S M_{69} + \frac{k\bar{V}_2}{2} M_{55}$$

$$M_{88} = - \bar{U}_{US}\bar{A}_S M_{70} + \bar{\rho}^*\bar{a}^* + \frac{k\bar{V}_2}{2} M_{56}$$

$$M_{89} = - \bar{U}_{US}\bar{A}_S M_{71} + \frac{k\bar{V}_2}{2} M_{57}$$

$$M_{90} = - \bar{U}_{US}\bar{A}_S M_{74} + \frac{k\bar{V}_2}{2} M_{60}$$

$$M_{91} = - \bar{U}_{US}\bar{A}_S M_{75} + \bar{\rho}^* + \frac{k\bar{V}_2}{2} M_{61}$$

$$M_{92} = - \bar{U}_{US}\bar{A}_S M_{78} + \frac{k\bar{V}_2}{2} M_{64} + k\bar{\rho}_2$$

$$M_{93} = - \bar{U}_{US}\bar{A}_S M_{79} + \frac{k\bar{V}_2}{2} M_{65} \qquad \text{(C17)}$$

Using these relationships, the real part becomes

$$M_{80}\bar{U}'_{USR} + M_{81}\bar{U}'_{USI} = M_{82}\bar{A}'_{SR} + M_{83}\bar{A}'_{SI} + M_{84}\bar{\rho}'_R{}^* + M_{85}\bar{\rho}'_I{}^* + M_{86}\bar{\rho}'_R{}^* +$$

$$M_{87}\bar{\rho}'_I{}^* + M_{86}\bar{A}'_R{}^* + M_{89}\bar{A}'_I{}^* + M_{90}\bar{a}'_I{}^* + M_{91}\bar{a}'_R{}^* + M_{92}\bar{V}'_{2I} + M_{93}\bar{V}'_{2R} \quad (C18)$$

Dealing next with the imaginary part, substituting for $\bar{\rho}'_{USI}$ and $\bar{\rho}'_{USR}$ gives

$$\bar{\rho}_{US}\bar{U}'_{USI}\bar{A}_S = -\bar{\rho}_{US}\bar{U}_{US}\bar{A}_{SI} + (\bar{\rho}'_I{}^*\bar{a}^* + \bar{\rho}^*\bar{a}'_I{}^* + \bar{\rho}^*\bar{A}'_I{}^*\bar{a}^*) -$$

$$k\left(\frac{V_2}{2}\bar{\rho}'_R{}^* + \bar{\rho}\bar{V}'_{2R}\right) - \bar{U}_{US}\bar{A}_S\,(M_{52}\bar{A}'_{SR} + M_{53}\bar{A}'_{SI} + M_{54}\bar{p}'_R{}^* + M_{55}\bar{p}'_I{}^* +$$

$$M_{56}\bar{A}'_R{}^* + M_{57}\bar{A}'_I{}^* + M_{58}\bar{U}'_{USR} + M_{59}\bar{U}'_{USI} + M_{60}\bar{a}'_I{}^* + M_{61}\bar{a}'_R{}^* +$$

$$M_{62}\bar{\rho}'_I{}^* + M_{63}\bar{\rho}'_R{}^* + M_{64}\bar{V}'_{2I} + M_{65}\bar{V}'_{2R}) - \frac{k\bar{V}_2}{2}\;(M_{66}\bar{A}'_{SR} + M_{67}\bar{A}'_{SI} +$$

$$M_{68}\bar{p}'_R{}^* + M_{69}\bar{p}'_I{}^* + M_{70}\bar{A}'_R{}^* + M_{71}\bar{A}'_I{}^* + M_{72}\bar{U}'_{USR} + M_{73}\bar{U}_{USI} + M_{74}\bar{a}'_I{}^* +$$

$$M_{75}\bar{a}'_R{}^* + M_{76}\bar{\rho}'_I{}^* + M_{77}\bar{\rho}'_R{}^* + M_{78}\bar{V}'_{2I} + M_{79}\bar{V}'_{2R})$$

Now, let

$$C_{58} = \bar{\rho}_{US}\bar{A}_S + \bar{U}_{US}\bar{A}_S M_{59} + \frac{k\bar{V}_2}{2}M_{73}$$

$$C_{59} = \bar{U}_{US}\bar{A}_S M_{58} + \frac{k\bar{V}_2}{2}M_{72}$$

$$C_{60} = -\bar{U}_{US}\bar{A}_S M_{52} + \frac{k\bar{V}_2}{2}M_{66}$$

$$C_{61} = -\bar{\rho}_{US}\bar{U}_{US} - \bar{U}_{US}\bar{A}_S M_{53} - \frac{k\bar{V}_2}{2}M_{67}$$

$$C_{62} = -\bar{U}_{US}\bar{A}_S M_{63} - \frac{k\bar{V}_2}{2}(1 + M_{77})$$

$$C_{63} = \bar{a}^* - \bar{U}_{US}\bar{A}_S M_{62} - \frac{k\bar{V}_2}{2}M_{76}$$

$$C_{64} = -\bar{U}_{US}\bar{A}_{US}M_{61} - \frac{k\bar{V}_2}{2}M_{75}$$

$$C_{65} = \bar{\rho}^* - \bar{U}_{US}\bar{A}_S M_{60} - \frac{k\bar{V}_2}{2}M_{74}$$

$$C_{66} = -\bar{U}_{US}\bar{A}_S M_{54} - \frac{k\bar{V}_2}{2}M_{68}$$

$$C_{67} = -\bar{U}_{US}\bar{A}_S M_{55} - \frac{k\bar{V}_2}{2}M_{69}$$

$$C_{68} = -\bar{U}_{US}\bar{A}_S M_{56} - \frac{k\bar{V}_2}{2} M_{70}$$

$$C_{69} = \bar{\rho}^*\bar{a}^* - \bar{U}_{US}\bar{A}_S M_{57} - \frac{k\bar{V}_2}{2} M_{71}$$

$$C_{70} = -\bar{U}_{US}\bar{A}_S M_{65} - \frac{k\bar{V}_2}{2} (2\bar{\rho}_2 + M_{79})$$

$$C_{71} = -\bar{U}_{US}\bar{A}_S M_{64} - \frac{k\bar{V}_2}{2} M_{78} \qquad \text{(C19)}$$

Substituting these relationships into the imaginary part of the continuity equation yields

$$C_{59}\bar{U}'_{USR} + C_{58}\bar{U}'_{USI} = C_{60}\bar{A}'_{SR} + C_{61}\bar{A}'_{SI} + C_{62}\bar{\rho}'_R{}^* + C_{63}\bar{\rho}'_I{}^* + C_{64}\bar{a}'_R{}^* + C_{65}\bar{a}'_I{}^* +$$

$$C_{66}\bar{\rho}'_R{}^* + C_{67}\bar{\rho}'_I{}^* + C_{68}\bar{A}'_R{}^* + C_{69}\bar{A}'_I{}^* + C_{70}\bar{V}'_{2I} + C_{71}\bar{V}'_{2R} \qquad \text{(C20)}$$

Multiplying Equation C18 by C_{58}/M_{81} and subtracting the resultant from Equation C20 produces, after solving for $\bar{U}'_{USR}$:

$$\bar{U}'_{USR} = C_{73}\bar{A}'_{SR} + C_{74}\bar{A}'_{SI} + C_{75}\bar{\rho}'_R{}^* + C_{76}\bar{\rho}'_I{}^* + C_{77}\bar{a}'_R{}^* + C_{78}\bar{a}'_I{}^* +$$

$$C_{79}\bar{p}'_R{}^* + C_{80}\bar{p}'_I{}^* + C_{81}\bar{A}'_R{}^* + C_{82}\bar{A}'_I{}^* + C_{83}\bar{V}'_{2I} + C_{84}\bar{V}'_{2R} \qquad \text{(C21)}$$

where,

$$C_{72} = C_{59} - \frac{C_{58}M_{80}}{M_{81}}$$

$$C_{73} = \frac{1}{C_{72}}\left(C_{60} - \frac{C_{58}M_{82}}{M_{81}}\right)$$

$$C_{74} = \frac{1}{C_{72}}\left(C_{61} - \frac{C_{58}M_{83}}{M_{81}}\right)$$

$$C_{75} = \frac{1}{C_{72}}\left(C_{62} - \frac{C_{58}M_{84}}{M_{81}}\right)$$

$$C_{76} = \frac{1}{C_{72}}\left(C_{63} - \frac{C_{58}M_{85}}{M_{81}}\right)$$

$$C_{77} = \frac{1}{C_{72}}\left(C_{64} - \frac{C_{58}M_{91}}{M_{81}}\right)$$

$$C_{78} = \frac{1}{C_{72}}\left(C_{65} - \frac{C_{58}M_{90}}{M_{81}}\right)$$

$$C_{79} = \frac{1}{C_{72}}\left(C_{66} - \frac{C_{58}M_{86}}{M_{81}}\right)$$

$$C_{80} = \frac{1}{C_{72}}\left(C_{67} - \frac{C_{58}M_{87}}{M_{81}}\right)$$

$$C_{81} = \frac{1}{C_{72}} \left(C_{68} - \frac{C_{58}M_{88}}{M_{81}} \right)$$

$$C_{82} = \frac{1}{C_{72}} \left(C_{69} - \frac{C_{58}M_{89}}{M_{81}} \right)$$

$$C_{83} = \frac{1}{C_{72}} \left(C_{70} - \frac{C_{58}M_{93}}{M_{81}} \right)$$

$$C_{84} = \frac{1}{C_{72}} \left(C_{71} - \frac{C_{58}M_{92}}{M_{81}} \right)$$

Substituting Equation C21 into C20 and solving for $\bar{U}'_{USI}$ produces

$$\bar{U}'_{USI} = C_{85}\bar{A}'_{SR} + C_{86}\bar{A}'_{SI} + C_{87}\bar{\rho}_R{}^{*\prime} + C_{88}\bar{\rho}_I{}^{*\prime} + C_{89}\bar{a}_R{}^{*\prime} + C_{90}\bar{a}_I{}^{*\prime} + C_{91}\bar{p}_R{}^{*\prime} + C_{92}\bar{p}_I{}^{*\prime} + C_{93}\bar{A}_R{}^{*\prime} + C_{94}\bar{A}_I{}^{*\prime} + C_{95}\bar{V}_{2I}{}' + C_{96}\bar{V}_{2R}{}' \qquad (C22)$$

where,

$$C_{85} = \frac{1}{C_{58}} (C_{60} - C_{59}C_{73})$$

$$C_{86} = \frac{1}{C_{58}} (C_{61} - C_{59}C_{74})$$

$$C_{87} = \frac{1}{C_{58}} (C_{62} - C_{59}C_{75})$$

$$C_{88} = \frac{1}{C_{58}} (C_{63} - C_{59}C_{76})$$

$$C_{89} = \frac{1}{C_{58}} (C_{64} - C_{59}C_{77})$$

$$C_{90} = \frac{1}{C_{58}} (C_{65} - C_{59}C_{78})$$

$$C_{91} = \frac{1}{C_{58}} (C_{66} - C_{59}C_{79})$$

$$C_{92} = \frac{1}{C_{58}} (C_{67} - C_{59}C_{80})$$

$$C_{93} = \frac{1}{C_{58}} (C_{68} - C_{59}C_{81})$$

$$C_{94} = \frac{1}{C_{58}} (C_{69} - C_{59}C_{82})$$

$$C_{95} = \frac{1}{C_{58}} (C_{70} - C_{59}C_{83})$$

$$C_{96} = \frac{1}{C_{58}} (C_{71} - C_{59}C_{84}) \qquad (C23)$$

The value for the velocity perturbation upstream of the shock given in Equations C21 and C22 can then be inserted into Equations C11 and C14, given the density perturbation upstream of the shock, for a complete description of the flowfield in Region 2.

APPENDIX D
SOLUTION TO THE EQUATIONS OF MOTION FOR REGION 3

The nondimensionalized equations of motion appear as Equations 60 and 61. Again, only the momentum and continuity equations are needed because these are only two unknowns, $A_{+\infty}$ and $Z_{+\infty}$. These parameters describe the irrotational and rotational flowfields.

Exit flow perturbations are defined first. The velocity is represented as the sum of the rotational and irrotational fields, while pressure is a function of the irrotational flowfield only[1]. Conversion of the two-dimensional flowfield into a one-dimensional field proceeds as follows:

$$p'_E = \frac{1}{\bar{y}_E} \int_o^{y_E} p'_E \, dy$$

which produces

$$p'_E = -\bar{\rho}_E \left[\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E} \right] \left[A_{+\infty} e^{ikt} (e^{iC\bar{y}_E} - 1) \right] \tag{D1}$$

The density perturbation can be related through isentropic form relationships to the pressure perturbations as

$$\bar{\rho}'_E = -\bar{\rho}_E \left[\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E \;\; \bar{a}_E^2} \right] \left[A_{+\infty} e^{ikt} (e^{iC\bar{y}_E} - 1) \right] \tag{D2}$$

The expressions for the velocity component are listed below:

Irrotational Field in the x-direction

$$\bar{u}'_{IE} = \frac{B_2 A_{+\infty}}{C\bar{y}_E} e^{ikt} (e^{iC\bar{y}_E} - 1) \tag{D3}$$

$$\bar{v}'_{IE} = \frac{A_{+\infty}}{\bar{y}_E} e^{ikt} (e^{iC\bar{y}_E} - 1) \tag{D4}$$

Rotational Field

$$\bar{u}'_{RE} = \frac{Z_{+\infty}}{(R^2 + C^2)\, \bar{y}_E} e^{ikt} (e^{iC\bar{y}_E} - 1) \tag{D5}$$

$$\bar{v}'_{RE} = \frac{-R\, Z_{+\infty}}{C\,(R^2 + C^2)\, \bar{y}_E} e^{ikt} (e^{iC\bar{y}_E} - 1) \tag{D6}$$

Next, expressions for the time derivatives of the exit density and velocity perturbations are obtained in the following manner:

Density

$$\frac{\partial \rho'_E}{\partial t} = -\bar{\rho}_E \left[\frac{i\, k\, (k + \bar{u}_E B_2 + \bar{v}_E C}{C\, \bar{a}_E^2\, \bar{y}_E} \right] \left[A_{+\infty} e^{ikt} (e^{iC\bar{y}_E} - 1) \right] \tag{D7}$$

[1]Goldstein, M. E., *Aeroacoustics*, pp. 220, 221.

Velocity

Irrotational Component

$$\frac{\partial \bar{u}'_{IE}}{\partial t} = \left(\frac{i\,k\,B_2\,A_{+\infty}}{C\bar{y}_E} \right) e^{ikt} \left(e^{iC\bar{y}_E} - 1 \right) \quad \text{(D8)}$$

$$\frac{\partial \bar{v}'_{IE}}{\partial t} = \left(\frac{i\,k\,A_{+\infty}}{\bar{y}_E} \right) e^{ikt} \left(e^{iC\bar{y}_E} - 1 \right) \quad \text{(D9)}$$

Rotational Component

$$\frac{\partial \bar{u}'_{RE}}{\partial t} = \left[\frac{i\,k\,Z_{+\infty}}{(R^2 + C^2)\,\bar{y}_E}\, e^{ikt} \left(e^{iC\bar{y}_E} - 1 \right) \right] \quad \text{(D10)}$$

$$\frac{\partial \bar{v}'_{RE}}{\partial t} = \left[\frac{-i\,k\,R\,Z_{+\infty}}{C\,(C^2 + R^2)\bar{y}_E}\, e^{ikt} \left(e^{iC\bar{y}_E} - 1 \right) \right] \quad \text{(D11)}$$

Assuming the flow parameters vary harmonically with time, substituting the relationships of Equations D3 through D11 into the momentum equation, and dividing through by e^{ikt} produces

$$\bar{p}'_{ds}\bar{A}_s + \bar{p}_{ds}\bar{A}'_s + \bar{\rho}_E\left[\left(\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E} \right) \bar{A}_E A_{+\infty} \left(e^{iC\bar{y}_E} - 1 \right) \right] - \bar{p}_E \bar{A}'_E$$

$$= \left\{ \left[\left(\frac{B_2 A_{+\infty}}{C\bar{y}_E} + \frac{Z_{+\infty}}{(R^2 + C^2)\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \sin \alpha_{ch} + \right.\right.$$

$$\left. \left(\frac{A_{+\infty}}{\bar{y}_E} - \frac{RZ_{+\infty}}{C(R^2 + C^2)\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \cos \alpha_{ch} \right] \bar{w}_E +$$

$$U_{2REL} \cos (\alpha_{ch} - \beta_2) \left[-\bar{\rho}_E \left(\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E \bar{a}_E^2} \right) \bar{A}_E A_{+\infty} \left(e^{iC\bar{y}_E} - \right.\right.$$

$$\left. 1 \right) \cos (\alpha_{ch} - \beta_2)\, \bar{U}_{2REL} + \bar{\rho}_E \bar{A}_E \left(\left(\frac{B_2 A_{+\infty}}{C\bar{y}_E} + \frac{Z_{+\infty}}{(R^2 + C^2)\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \sin \alpha_{ch} + \right.$$

$$\left. \left(\frac{A_{+\infty}}{\bar{y}_E} - \frac{RZ_{+\infty}}{C(R^2 + C^2)\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \cos \alpha_{ch} \right) +$$

$$\left. \left. \bar{\rho}_E \bar{U}_E \bar{A}'_E \cos (\alpha_{ch} - \beta_2) \right] \right\} - \left\{ \bar{U}'_{ds} \bar{w}_{ds} + \bar{U}_{ds} \bar{w}'_{ds} \right\} +$$

$$\frac{\bar{U}_3 \bar{V}_3}{2} \left\{ ik\bar{\rho}'_{ds} - ik\,\bar{\rho}_E \left(\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E \bar{a}_E^2} \right) A_{+\infty} \left(e^{iC\bar{y}_E} - 1 \right) \right\} +$$

$$+\, ik\,(\overline{\rho \bar{U} V'})_3 + \frac{ik\rho_3 \bar{V}_3}{2} \left\{ \left(\frac{B_2 A_{+\infty}}{C\bar{y}_E} + \frac{Z_{+\infty}}{(R^2 + C^2)\,\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \sin \alpha_{ch} + \right.$$

$$\left. \left(\frac{A_{+\infty}}{\bar{y}_E} - \frac{RZ_{+\infty}}{C\,(R^2 + C^2)\,\bar{y}_E} \right) \left(e^{iC\bar{y}_E} - 1 \right) \cos \alpha_{ch} + \bar{U}'_{ds} \right\} \quad \text{(D12)}$$

The next step requires that Equation D12 be divided into its real and imaginary parts. The momentum equation is used to solve for $Z_{+\infty R}$, $Z_{+\infty I}$ and the continuity equation is used to solve for $A_{+\infty R}$, $A_{+\infty I}$. Combine like terms and make the following assumptions:

$$M_{94} = \frac{\bar{u}_E \bar{A}_E}{C\bar{y}_E}\bar{\rho}_E - \frac{\bar{w}_E \sin \alpha_{ch}}{C\bar{y}_E} +$$

$$\left[U_{2REL} \cos (\alpha_{ch} - \beta_2) \right]^2 \left[\frac{\bar{U}_E \bar{A}_E}{C\bar{y}_E \bar{a}_E^2}\bar{\rho}_E \right] -$$

$$\left[\left(\frac{\bar{\rho}_E U_{2REL} \bar{A}_E}{C\bar{y}_E} \right) (\sin \alpha_{ch}) \right] \cos (\alpha_{ch} - \beta_2)$$

$$M_{95} = \bar{\rho}_E \left(\frac{k + \bar{v}_E C}{C\bar{y}_E} \right) \bar{A}_E - \frac{\bar{w}_E \cos \alpha_{ch}}{\bar{y}_E} +$$

$$[U_{2REL} \cos (\alpha_{ch} - \beta_2)]^2 \left[\left(\frac{k + \bar{v}_E C}{C\bar{y}_E \bar{a}_E^2} \right) \bar{A}_E \bar{\rho}_E \right] -$$

$$\bar{\rho}_E \bar{U}_{2REL} \bar{A}_E \cos \alpha_{ch} \cos (\alpha_{ch} - \beta_2)$$

$$M_{96} = \bar{\rho}_E \frac{\bar{u}_E \bar{U}_3 \bar{V}_3}{2C\bar{y}_E \bar{a}_E^2} - \frac{\rho_3 \bar{\bar{V}}_3 \sin \alpha_{ch}}{2C\bar{y}_E}$$

$$M_{97} = \bar{\rho}_E \frac{(k + \bar{v}_E C) \bar{\bar{U}}_3 \bar{\bar{V}}_3}{2C\bar{y}_E \bar{a}_E^2} - \frac{\rho_3 \bar{\bar{V}}_3 \cos \alpha_{ch}}{2\bar{y}_E}$$

$$M_{98} = \left[\frac{\bar{w}_E \sin \alpha_{ch}}{(R^2 + C^2)\, \bar{y}_E} - \frac{R\bar{w}_E \cos \alpha_{ch}}{C(R^2 + C^2)\bar{y}_E} \right] +$$

$$\left[\frac{\bar{\rho}_E U_{2REL} \bar{A}_E \cos (\alpha_{ch} - \beta_2)}{(R^2 + C^2)\bar{y}_e} \right] \left[\sin \alpha_{ch} - \frac{R \cos \alpha_{ch}}{C} \right]$$

$$M_{99} = \frac{\bar{\rho}_3 \bar{V}_3}{2\, (R^2 + C^2)\, \bar{y}_E} \left(\sin \alpha_{ch} - \frac{R \cos \alpha_{ch}}{C} \right)$$

$$M_{100} = \cos (C\bar{y}_E) - 1 \qquad \text{(D13)}$$

Substituting these relationships into equation D12 and extracting the real part yields

$$
\begin{aligned}
&M_{94}\,[\,B_{2R}A_{+\infty R}M_{100} - B_{21}A_{+\infty}M_{100} - B_{2R}A_{+\infty}\sin(C\bar{y}_E) - \\
&\quad B_{21}A_{+\infty R}\sin(C\bar{y}_E)\,] + M_{95}\,[\,A_{+\infty R}M_{100} - A_{+\infty I}\sin(C\bar{y}_E)\,] - \\
&\quad kM_{96}\,[\,B_{2R}A_{+\infty I}M_{100} + B_{21}A_{+\infty R}M_{100} + B_{2R}A_{+\infty R}\sin(C\bar{y}_E) - \\
&\quad B_{21}A_{+\infty I}\sin(C\bar{y}_E)\,] - kM_{97}\,[\,A_{+\infty I}M_{100} + A_{+\infty R}\sin(C\bar{y}_E)\,] \\
&= -kM_{99}\,[\,Z_{+\infty R}M_{100} + Z_{+\infty I}\sin(C\bar{y}_E)\,] + M_{98}\,[\,Z_{+\infty I}M_{100} - \\
&\quad Z_{+\infty R}\sin(C\bar{y}_E)\,] - \bar{p}'_{dsR}\bar{A}_s - \bar{p}_{ds}\bar{A}'_{sR} + [\,\bar{p}_E + \bar{\rho}_E(U_{2RE}\cos(\alpha_{ch} - \beta_2))^2\,]\,\bar{A}'_{ER} - \\
&\quad \bar{U}'_{dsR}\bar{w}_{ds} - \bar{U}_{ds}\bar{w}'_{dsR} - \frac{k\bar{\rho}'_{dsI}\bar{U}_3\bar{V}_3}{2} - k\bar{\rho}_3\bar{U}_3\bar{V}'_{3I} - \frac{k\bar{\rho}_3\bar{U}'_{dsI}\bar{V}_3}{2}
\end{aligned}
\qquad (D14)
$$

Collect like terms in the following manner and let

$$
\begin{aligned}
M_{101} &= M_{94}(M_{100}B_{2R} - B_{2I}\sin(C\bar{y}_E)) + M_{95}M_{100} \\
&\quad - kM_{96}(B_{2I}M_{100} + B_{2R}\sin(C\bar{y}_E)) - kM_{97}\sin(C\bar{y}_E) \\
M_{102} &= -M_{94}(M_{100}B_{2I} + B_{2R}\sin(C\bar{y}_E)) - M_{95}\sin(C\bar{y}_E) \\
&\quad - kM_{96}(M_{100}B_{2R} - B_{2I}\sin(C\bar{y}_E)) - kM_{97}M_{100} \\
M_{103} &= -kM_{99}\sin(C\bar{y}_E) + M_{98}M_{100} \\
M_{104} &= -kM_{99}M_{100} - M_{98}\sin(C\bar{y}_E)
\end{aligned}
\qquad (D15)
$$

Substituting Equation D15 into D14 yields

$$
M_{101}A_{+\infty R} + M_{102}A_{+\infty I} = M_{103}Z_{+\infty R} + M_{104}Z_{+\infty I} + RC \qquad (D16)
$$

where,

$$
\begin{aligned}
RC &= -\bar{p}'_{dsR}\bar{A}_S - \bar{p}_{ds}\bar{A}'_{SR} + [\,\bar{p}_E + \rho_E(U_{2RE}\cos(\alpha_{ch} - \beta_2))^2]\,\bar{A}'_{ER} - \\
&\quad \bar{U}'_{dsR}\bar{w}_{ds} - \bar{U}_{ds}\bar{w}'_{dsR} - \frac{k\rho'_{dsI}\bar{U}_3\bar{V}_3}{2} - k\bar{\rho}_3\bar{U}_3\bar{V}'_{3I} - \frac{k\bar{\rho}_3\bar{U}'_{dsI}\bar{V}_3}{2}
\end{aligned}
\qquad (D17)
$$

with

$$
\bar{w}'_{dsR} = (\bar{\rho}'_{dsR}\bar{U}_{ds}\bar{A}_S + \bar{\rho}_{ds}\bar{U}'_{dsR}\bar{A}_S + \bar{\rho}_{ds}\bar{U}_{ds}\bar{A}'_{SR}) \qquad (D18)
$$

The imaginary part of the momentum equation is expressed as

$$M_{105}A_{+\infty R} + M_{106}A_{+\infty I} = M_{107}Z_{+\infty R} + M_{108}Z_{+\infty I} + IC \qquad \text{(D19)}$$

where,

$$M_{105} = M_{94}[B_{2I}M_{100} + B_{2R}\sin(C\bar{y}_E)] + M_{95}\sin(C\bar{y}_E) + kM_{96}[B_{2R}M_{100} - B_{2I}\sin(C\bar{y}_E)] + kM_{97}M_{100}$$

$$M_{106} = M_{94}[B_{2R}M_{100} - B_{2I}\sin(C\bar{y}_E)] + M_{95}M_{100} - kM_{96}[B_{2I}M_{100} + B_{2R}\sin(C\bar{y}_E)] - kM_{97}\sin(C\bar{y}_E)$$

$$M_{107} = M_{98}\sin(C\bar{y}_E) + kM_{99}M_{100}$$

$$M_{108} = M_{70}M_{72} - kM_{71}\sin(C\bar{y}_E) \qquad \text{(D20)}$$

and

$$IC = -p_{dsI}\bar{A}_S - \bar{p}_{ds}\bar{A}'_{SI} + [\bar{p}_E + \bar{\rho}_E(\bar{U}_{2REL}\cos(\alpha_{ch} - \beta_2))^2]\bar{A}'_{EI} - \bar{U}'_{dsI}\bar{w}_{ds} - \bar{U}_{ds}\bar{w}'_{dsI} + \frac{k\bar{\rho}'_{dsR}\bar{U}_3\bar{V}_3}{2} + k\bar{\rho}_3\bar{U}_{3L}\bar{V}'_{3R} + \frac{k\bar{\rho}_3\bar{U}'_{dsR}\bar{V}_3}{2} \qquad \text{(D21)}$$

with

$$\bar{w}'_{dsI} = (\bar{\rho}'_{dsI}\bar{U}_{ds}\bar{A}_S + \bar{\rho}_{ds}\bar{U}'_{dsI}\bar{A}_S + \bar{\rho}_{ds}\bar{U}_{ds}\bar{A}'_{SI})$$

In order to solve for $A_{+\infty R}$, multiply Equation D16 by M_{78}/M_{74} and subtract the resultant from Equation D19 to produce

$$A_{+\infty R} = M_{110}Z_{+\infty R} + M_{111}Z_{+\infty I} + M_{112}IC + M_{113}RC \qquad \text{(D22)}$$

where,

$$M_{109} = M_{105} - M_{101}M_{106}/M_{102}$$

$$M_{110} = (M_{107} - M_{103}M_{106}/M_{102})/M_{109}$$

$$M_{111} = (M_{108} - M_{104}M_{106}/M_{102})/M_{109}$$

$$M_{112} = 1/M_{109}$$

$$M_{113} = -M_{106}/M_{102}M_{109}$$

Substitute Equation D22 into Equation D19 and solve for $A_{+\infty I}$ to give

$$A_{+\infty I} = M_{114}Z_{+\infty R} + M_{115}Z_{+\infty I} + M_{116}RC + M_{117}IC \qquad \text{(D23)}$$

where,

$$M_{114} = (M_{107} - M_{105}M_{110})/M_{106}$$

$$M_{115} = (M_{108} - M_{105}M_{109})/M_{106}$$

$$M_{116} = - M_{105}M_{113}/M_{106}$$

$$M_{117} = (1 - M_{105}M_{113})/M_{106} \qquad \text{(D24)}$$

The continuity equation is used to obtain $Z_{+\infty}$. The equation takes the following form after substituting Equation D2 through D7 into Equation 38 and dividing through by e^{ikt}.

$$\left[-\bar{\rho}_E \left(\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E \bar{a}^2_E} \right) \bar{U}_E \bar{A}_E \bar{A}_{+\infty} \left(e^{iC\bar{y}_E} - 1\right) \cos(\alpha_{ch} - \beta_2) \right] +$$

$$\bar{\rho}_E \bar{U}_E \bar{A}'_E + \bar{\rho}_E \bar{A}_E \left(e^{iC\bar{y}_E} - 1\right) \left[\left(\frac{B_2 A_{+\infty}}{C\bar{y}_E} + \frac{Z_{+\infty}}{(R^2 + C^2)\,\bar{y}_E} \right) \sin \alpha_{ch} + \right.$$

$$\left. \left(\frac{A_{+\infty}}{\bar{y}_E} - \frac{RZ_{+\infty}}{C\,(R^2 + C^2)\,\bar{y}_E} \right) \cos \alpha_{ch} \right] - \bar{w}'_{ds}$$

$$= i\bar{\rho}_3 \bar{V}'_3 - \frac{ik\bar{V}_3}{2} \left[-\bar{\rho}_E \left(\frac{k + \bar{u}_E B_2 + \bar{v}_E C}{C\bar{y}_E \bar{a}^2_E} \right) A_{+\infty} \left(e^{iC\bar{y}_E} - 1\right) + \bar{\rho}'_{ds} \right] \qquad \text{(D25)}$$

The following steps separate out and solve for the real and imaginary components of $A_{+\infty}$ and $Z_{+\infty}$. First, make the assumptions noted in Equation D26 below.

$$C_{97} = - \frac{\bar{\rho}_E \bar{u}_E \bar{U}_{2REL} \bar{A}_E \cos(\alpha_{ch} - \beta_2)}{C\bar{y}_E \bar{a}^2_E} + \frac{\bar{\rho}_E \bar{A}_E \sin \alpha_{ch}}{C\bar{y}_E}$$

$$C_{98} = - \frac{\bar{\rho}_E (k + \bar{v}_E C)\, \bar{U}_{2REL} \bar{A}_E \cos(\alpha_{ch} - \beta_2)}{C\bar{y}_E \bar{a}^2_E} + \frac{\bar{\rho}_E \bar{A}_E \cos \alpha_{ch}}{\bar{y}_E}$$

$$C_{99} = \frac{\bar{\rho}_E \bar{A}_E}{(R^2 + C^2)\bar{y}_E} \left(\sin\alpha_{ch} - \frac{R}{C} \cos \alpha_{ch} \right)$$

$$C_{100} = \frac{\bar{u}_E \bar{V}_3 \bar{\rho}_E}{2C\bar{y}_E \bar{a}^2_E}$$

$$C_{101} = \frac{(k + \bar{v}_E C)\bar{V}_3 \bar{\rho}_E}{2C\bar{y}_E \bar{a}^2_E} \qquad \text{(D26)}$$

Substituting Equation D26 into Equation D25 and rearranging term yields

$$C_{102}A_{+\infty R} + C_{103}A_{+\infty I} + C_{104}Z_{+\infty R} + C_{105}Z_{+\infty I} = CR \tag{D27}$$

where,

$$C_{102} = C_{97}B_{2R}M_{100} - C_{97}B_{21} \sin (C\bar{y}_E) + C_{98}M_{100}$$

$$+ kC_{100} (B_{21}M_{100} + B_{2R} \sin (C\bar{y}_E)) + kC_{101} \sin (C\bar{y}_E)$$

$$C_{103} = -C_{97} (B_{2I}M_{100} + B_{2R} \sin (C\bar{y}_E)) - C_{98} \sin (C\bar{y}_E)$$

$$kC_{100}(B_{2R}M_{100} - B_{2I} \sin (C\bar{y}_E)) + kC_{101}M_{100}$$

$$C_{104} = C_{99}M_{100}$$

$$C_{105} = -C_{99} \sin (C\bar{y}_E) \tag{D28}$$

and

$$CR = - \bar{\rho}_E\bar{A}'_{ER}\bar{U}_E + w'_{dsR} + \bar{\rho}_3k\bar{V}'_{3I} + ½k\bar{V}'_3\bar{\rho}'_{dsI} \tag{D29}$$

Substituting Equation D22 and D23 into Equation D27 and combining the terms gives

$$C_{106}Z_{+\infty R} + C_{107}Z_{+\infty I} + C_{108}\, RC + C_{109}\, IC = CR \tag{D30}$$

where,

$$C_{106} = C_{102}M_{110} + C_{103}M_{114} + C_{104}$$

$$C_{107} = C_{102}M_{111} + C_{103}M_{115} + C_{105}$$

$$C_{108} = C_{102}M_{113} + C_{103}M_{116}$$

$$C_{109} = C_{102}M_{112} + C_{103}M_{117} \tag{D31}$$

Next, the imaginary part of the continuity equation is dealt with. Let,

$$C_{110} = C_{97} [B_{21}M_{100} + B_{2R} \sin (C\bar{y}_E)] + C_{98} \sin C\bar{y}_E) -$$

$$kC_{100} [B_{2R}M_{100} - B_{21} \sin (C\bar{y}_E)] - k\, C_{101}M_{100}$$

$$C_{111} = C_{97} [B_{2R}M_{100} - B_{21} \sin (C\bar{y}_E)] + C_{98}M_{100} +$$

$$kC_{100} [B_{21}M_{100} + B_{2R} \sin (C\bar{y}_E)] + k\, C_{101} \sin (C\bar{y}_E)$$

$$C_{112} = C_{99} \sin (C\bar{y}_E)$$

$$C_{113} = C_{99}M_{100} \tag{D32}$$

This yields

$$C_{110}A_{+\infty R} + C_{111}A_{+\infty I} + C_{112}Z_{+\infty R} + C_{113}Z_{+\infty I} = CI \qquad \text{(D33)}$$

where,

$$CI = -\bar{\rho}_E\bar{U}_{2REL}\cos(\alpha_{ch} - \beta_2)\,\bar{A}'_{EI} + \bar{\bar{w}}'_{dsI}$$

$$-\bar{\rho}_3 k\bar{V}'_{3R} - \frac{1}{2}\ k\bar{V}_3\bar{\rho}'_{dsR} \qquad \text{(D34)}$$

Next, substitute the relationships for $A_{+\infty}$ presented in Equations D22 and D23 into D33. Let,

$$C_{114} = C_{110}M_{110} + C_{111}M_{114} + C_{112}$$

$$C_{115} = C_{110}M_{111} + C_{111}M_{115} + C_{113}$$

$$C_{116} = C_{110}M_{113} + C_{111}M_{116}$$

$$C_{117} = C_{110}M_{112} + C_{C111}M_{117} \qquad \text{(D35)}$$

This produces

$$C_{114}Z_{+\infty R} + C_{115}Z_{+\infty I} + C_{116}\ RC + C_{117}\ IC = CI \qquad \text{(D36)}$$

Now, multiply Equation D30 by C_{115}/C_{107} and subtract the resultant from Equation D30.

Let,

$$C_{118} = C_{114} - C_{115}C_{106}/C_{107}$$

$$C_{119} = -(C_{116} - C_{115}C_{108}/C_{107})/C_{118}$$

$$C_{120} = -(C_{117} - C_{115}C_{109}/C_{107})/C_{118}$$

$$C_{121} = (CI - C_{115}\ CR/C_{107})/C_{118} \qquad \text{(D37)}$$

This gives

$$Z_{+\infty R} = C_{119}\ RC + C_{120}\ IC + C_{121} \qquad \text{(D38)}$$

Substituting Equation D38 into Equation D36 and solving for $Z_{+\infty I}$ gives

$$Z_{+\infty I} = C_{122}\ RC + C_{123}\ IC + C_{124} \qquad \text{(D39)}$$

where,

$$C_{122} = (-C_{116} - C_{114}C_{119})/C_{115}$$

$$C_{123} = (-C_{117} - C_{114}C_{120})/C_{115}$$

$$C_{124} = (CI - C_{121}C_{114})/C_{115} \qquad \text{(D40)}$$

The relationships given for $Z_{+\infty}$ in Equations D38 and D39 are then substituted into Equations D22 and D23 to yield $A_{+\infty}$. This, then, produces a complete description of the flowfield for Region 3.

APPENDIX E
STEADY-STATE FLOW COEFFICIENTS FOR REGIONS 1, 2, AND 3

REGION 1

Energy

$$E_1 = \left[\frac{\bar{a}_I^2}{\gamma(\gamma - 1)} + \frac{\cos^2(\alpha_{ch} - \beta_1)}{2} \right]$$

$$E_2 = \frac{\bar{a}^{*2}(\gamma^2 - \gamma + 2)}{2\gamma(\gamma - 1)}$$

$$E_3 = \frac{-E_1 I \bar{A}_i \bar{u}_i}{C\bar{y}_i \bar{a}_i^2} + \frac{E_1 \bar{A}_i \sin \alpha_{ch}}{C\bar{y}_i} - \frac{2\bar{A}_i I \bar{u}_i}{2\gamma C\bar{y}_i} + \frac{\bar{A}_i I^2 \sin \alpha_{ch}}{C\bar{y}_i}$$

$$E_4 = \frac{-E_1 I \bar{A}_i (k + \bar{v}_i C)}{C\bar{y}_i \bar{a}_i^2} + \frac{E_1 \bar{A}_i \cos \alpha_{ch}}{\bar{y}_i} - \frac{2\bar{A}_i I (k + \bar{v}_i C)}{2\gamma C\bar{y}_i} + \frac{\bar{A}_i I^2 \cos \alpha_{ch}}{\bar{y}_i}$$

$$E_5 = \frac{-\bar{a}_I^2 \bar{V}_I \bar{u}_i}{2\gamma(\gamma - 1) C\bar{y}_i \bar{a}_i^2} - \frac{\bar{\rho}_I \bar{a}_I \bar{V}_I \bar{u}_i}{\gamma(\gamma - 1)} \left(\frac{\gamma - 1}{2\bar{a}_i C\bar{y}_i} \right) - \frac{\bar{U}_I^2 \bar{V}_I \bar{u}_i}{4C\bar{y}_i \bar{a}_i^2} +$$

$$\frac{\bar{\rho}_I \bar{U}_I \bar{V}_I \sin \alpha_{ch}}{2C\bar{y}_i}$$

$$E_6 = \frac{-\bar{a}_I^2 \bar{V}_I (k + \bar{v}_i C)}{2\gamma(\gamma - 1) C\bar{y}_i \bar{a}_i^2} - \bar{\rho}_I \bar{a}_I \bar{V}_I \cdot \left(\frac{\gamma - 1}{2C\bar{y}_i \bar{a}_i} \right) \left(\frac{k + v_i C}{\gamma(\gamma - 1)} \right) +$$

$$\frac{\bar{U}_I^2 \bar{V}_I (k + \bar{v}_i C)}{4C\bar{y}_i \bar{a}_I^2} + \frac{\bar{\rho}_I \bar{U}_I \bar{V}_I \cos \alpha_{ch}}{2\bar{y}_i}$$

$$E_7 = \cos(C\bar{y}_i) - 1$$

$$E_8 = E_3 (E_7 B_{1R} - B_{1I} \sin(C\bar{y}_i)) + E_4 E_7 + E_5 k (B_{1I} E_7 + B_{1R} \sin(C\bar{y}_i)) + E_6 k \sin(C\bar{y}_i)$$

$$E_9 = -E_3 (B_{1I} E_7 + B_{1R} \sin(C\bar{y}_i)) - E_4 \sin(C\bar{y}_i) + E_5 k (B_{1R} E_7 - B_{1I} \sin(C\bar{y}_i)) + E_6 k E_7$$

$$E_{10} = E_3 [B_{1I} E_7 + B_{1R} \sin(C\bar{y}_i)] + E_4 \sin(C\bar{y}_i) - E_5 k [B_{1R} E_7 - B_{1I} \sin(C\bar{y}_i)] - E_6 k E_7$$

$$E_{11} = E_3 [B_{1R} E_7 - B_{1I} \sin(C\bar{y}_i)] + E_4 E_7 + E_5 k [B_{1I} E_7 + B_{1R} \sin(C\bar{y}_i)] + E_6 k \sin(C\bar{y}_i)$$

$$E_{12} = \left(\frac{\bar{a}^2\bar{V}k}{2\gamma(\gamma - 1)} + \frac{\bar{U}^2\bar{V}k}{4} \right)_1$$

$$E_{13} = \left(\frac{\bar{\rho}\bar{a}^2k}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}^2k}{2} \right)_1$$

$$E_{14} = \left(\frac{\bar{\rho}\bar{a}\bar{V}k}{\gamma(\gamma - 1)} + \frac{\bar{\rho}\bar{U}\bar{V}k}{2} \right)_1$$

$$E_{15} = \left(\frac{E_9}{E_8} - \frac{E_{11}}{E_{10}} \right)$$

$$E_{16} = I \left(\frac{E_1}{E_8} \right) \left(\frac{E_9}{E_{15}E_8} - 1 \right)$$

$$E_{17} = I \left(\frac{-E_1}{E_{10}E_{15}} \right) \left(\frac{E_8}{E_9} \right)$$

$$E_{18} = \left(\frac{E_2}{E_8} \right) \left(\bar{a}^* - \frac{\bar{a}^*E_9}{E_{15}E_8} \right) - \left(\frac{E_{12}E_9}{E_8E_{10}E_{15}} \right)$$

$$E_{19} = -\frac{E_{12}}{E_8} + \left(\frac{\bar{a}^*E_2}{E_{10}E_{15}} \right) \left(\frac{E_9}{E_8} \right) + \left(\frac{E_{12}}{E_8E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{20} = \bar{\rho}^*\bar{a}^* \left[\frac{E_2}{E_8} - \left(\frac{E_2}{E_8E_{15}} \right) \left(\frac{E_9}{E_8} \right) \right]$$

$$E_{21} = \bar{\rho}^*\bar{a}^* \left(\frac{E_2}{E_{10}E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{22} = 3\bar{\rho}^* \left(\frac{E_2}{E_8} \right) \left(1 - \frac{E_9}{E_8E_{15}} \right) - \left(\frac{E_9E_{14}}{E_8E_{10}E_{15}} \right)$$

$$E_{23} = -\frac{E_{14}}{E_8} + 3\bar{\rho}^* \left(\frac{E_2E_9}{E_8E_{10}E_{15}} \right) + \left(\frac{E_{14}}{E_8E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{24} = -\frac{E_{13}}{E_8} + \left(\frac{E_{13}}{E_8E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{25} = -\left(\frac{E_{13}}{E_{10}E_{15}} \right) \left(\frac{E_9}{E_8} \right)$$

$$E_{26} = -I \left(\frac{E_1}{E_8E_{15}} \right)$$

$$E_{27} = \frac{E_1}{E_{10}E_{15}}$$

$$E_{28} = \frac{\bar{a}^*E_2}{E_8E_{15}} + \frac{E_{12}}{E_{10}E_{15}}$$

$$E_{29} = -\frac{\bar{a}^*E_2}{E_{10}E_{15}} - \frac{E_{12}}{E_8E_{15}}$$

$$E_{30} = \frac{\bar{\rho}^*\bar{a}^*E_2}{E_8E_{15}}$$

$$E_{31} = -\frac{\bar{\rho}^*\bar{a}^*E_2}{E_{10}E_{15}}$$

$$E_{32} = \frac{3\bar{\rho}^*E_2}{E_8E_{15}} + \frac{E_{14}}{E_{10}E_{15}}$$

$$E_{33} = -\frac{3\bar{\rho}^*E_2}{E_{10}E_{15}} - \frac{E_{14}}{E_8E_{15}}$$

$$E_{34} = -\frac{E_{13}}{E_8E_{15}}$$

$$E_{35} = \frac{E_{13}}{E_{10}E_{15}}$$

Momentum

$$M_1 = -\frac{\bar{u}_i\bar{A}_i}{C\bar{y}_i} - \frac{\bar{u}_i\bar{A}_iI^2}{C\bar{y}_i\bar{a}_i^2} + \frac{2\bar{A}_iI}{C\bar{y}_i}\sin\alpha_{ch}$$

$$M_2 = -\frac{(k+\bar{v}_iC)\bar{A}_i}{C\bar{y}_i} - \frac{(k+\bar{v}_iC)\bar{A}_iI^2}{C\bar{y}_i\bar{a}_i^2} + \frac{2\bar{A}_iI}{\bar{y}_i}\cos\alpha_{ch}$$

$$M_3 = \frac{\bar{U}_1\bar{V}_1}{2C\bar{y}_i\bar{a}_i^2} - \frac{\bar{\rho}_1\bar{V}_1\sin\alpha_{ch}}{2C\bar{y}_i}$$

$$M_4 = \left(\frac{k+\bar{v}_iC}{C\bar{y}_i\bar{a}_i^2}\right)\left(\frac{\bar{U}_1\bar{V}_1}{2}\right) - \frac{\bar{\rho}_1\bar{V}_1\cos\alpha_{ch}}{2\bar{y}_i}$$

$$M_5 = -\bar{p}_i - I^2$$

$$M_6 = \bar{p}^* + \bar{\rho}^*\bar{a}^{*2}$$

$$M_7 = M_1\,[B_{IR}E_7 - B_{II}\sin(C\bar{y}_i)] + M_2E_7 - M_3\,[kB_{II}E_7 - kB_{IR}\sin(C\bar{y}_i)] - M_4k\sin(C\bar{y}_i)$$

$$M_8 = -M_1\,[B_{II}E_7 + B_{IR}\sin(C\bar{y}_i)] - M_2\sin(C\bar{y}_i) + kM_3\,[-B_{IR}E_7 + B_{II}\sin(C\bar{y}_i)] - kM_4E_7$$

$$M_9 = M_1\,[B_{II}E_7 + B_{IR}\sin(C\bar{y}_i)] + M_2\sin(C\bar{y}_i) + kM_3\,[B_{IR}E_7 - B_{II}\sin(C\bar{y}_i)] + M_4E_7k$$

$$M_{10} = M_1\,[B_{IR}E_7 - B_{II}\sin(C\bar{y}_i)] + M_2E_7 - kM_3\,[B_{II}E_7 + B_{IR}\sin(C\bar{y}_i)] - kM_4\sin(C\bar{y}_i)$$

$$M_{11} = E_{18}M_7 + E_{28}M_8 - 2\bar{a}^{*2}$$

$$M_{12} = E_{19}M_7 + E_{29}M_8 + \frac{k\overline{U}_1\overline{V}_1}{2}$$

$$M_{13} = -E_{16}M_7 - E_{26}M_8 + M_5$$

$$M_{14} = -E_{17}M_7 - E_{27}M_8$$

$$M_{15} = -E_{20}M_7 - E_{30}M_8 + M_6$$

$$M_{16} = -E_{21}M_7 - E_{31}M_8$$

$$M_{17} = -E_{22}M_7 - E_{32}M_8 + 2\bar{\rho}^*\bar{a}^*$$

$$M_{18} = E_{23}M_7 - E_{33}M_8 - \frac{k\bar{\rho}_1\overline{V}_1}{2}$$

$$M_{19} = -E_{24}M_7 - E_{34}M_8 - k\bar{\rho}_1\overline{U}_1$$

$$M_{20} = -E_{25}M_7 - E_{35}M_8$$

$$M_{21} = E_{18}M_9 + E_{28}M_{10} - \frac{k\overline{U}_1\overline{V}_1}{2}$$

$$M_{22} = E_{19}M_9 + E_{29}M_{10} - 2\bar{a}^{*2}$$

$$M_{23} = -E_{16}M_9 - E_{26}M_{10} + M_5$$

$$M_{24} = -E_{17}M_9 - E_{27}M_{10} + M_6$$

$$M_{25} = -E_{20}M_9 - E_{30}M_{10}$$

$$M_{26} = -E_{21}M_9 - E_{31}M_{10}$$

$$M_{27} = -E_{22}M_9 - E_{32}M_{10} + \frac{k\bar{\rho}_1\overline{V}_1}{2}$$

$$M_{28} = -E_{23}M_9 - E_{33}M_{10} + 2\bar{\rho}^*\bar{a}^*$$

$$M_{29} = -E_{24}M_9 - E_{34}M_{10}$$

$$M_{30} = -E_{25}M_9 - E_{35}M_{10} + k\bar{\rho}_1\overline{U}_1$$

$$M_{31} = M_{11} - \frac{M_{12}M_{21}}{M_{22}}$$

$$M_{32} = \left(M_{13} - \frac{M_{12}M_{23}}{M_{22}}\right) \div M_{31}$$

$$M_{33} = \left(M_{14} - \frac{M_{12}M_{24}}{M_{22}}\right) \div M_{31}$$

$$M_{34} = \left(M_{15} - \frac{M_{12}M_{25}}{M_{22}}\right) \div M_{31}$$

$$M_{35} = \left(M_{16} - \frac{M_{12}M_{26}}{M_{22}} \right) \div M_{31}$$

$$M_{36} = \left(M_{17} - \frac{M_{12}M_{27}}{M_{22}} \right) \div M_{31}$$

$$M_{37} = \left(M_{18} - \frac{M_{12}M_{28}}{M_{22}} \right) \div M_{31}$$

$$M_{38} = \left(M_{19} - \frac{M_{12}M_{29}}{M_{22}} \right) \div M_{31}$$

$$M_{39} = \left(M_{20} - \frac{M_{12}M_{30}}{M_{22}} \right) \div M_{31}$$

$$M_{40} = \frac{M_{13} - M_{32}M_{11}}{M_{12}}$$

$$M_{41} = \frac{M_{14} - M_{33}M_{11}}{M_{12}}$$

$$M_{42} = \frac{M_{15} - M_{34}M_{11}}{M_{12}}$$

$$M_{43} = \frac{M_{16} - M_{35}M_{11}}{M_{12}}$$

$$M_{44} = \frac{M_{17} - M_{36}M_{11}}{M_{12}}$$

$$M_{45} = \frac{M_{18} - M_{37}M_{11}}{M_{12}}$$

$$M_{46} = \frac{M_{19} - M_{38}M_{11}}{M_{12}}$$

$$M_{47} = \frac{M_{20} - M_{39}M_{11}}{M_{12}}$$

Continuity

$$C_1 = -\frac{\bar{A}_i}{C\bar{y}_i} \sin \alpha_{ch} + \frac{\bar{u}_i\bar{A}_i I}{C\bar{y}_i\bar{a}_i^2}$$

$$C_2 = -\frac{\bar{A}_i}{C\bar{y}_i} \cos \alpha_{ch} + \left(\frac{k + \bar{v}_i C}{C\bar{y}_i a_i^2} \right) \bar{A}_i I$$

$$C_3 = \frac{\bar{u}_i \bar{V}_1}{2C\bar{y}_i\bar{a}_i^2}$$

$$C_4 = \left(\frac{k + \bar{v}_i C}{2C\bar{y}_i\bar{a}_1^2} \right) \bar{V}_1$$

$$C_5 = C_1B_{1R}E_7 - C_1B_{1I} \sin(C\bar{y}_i) + C_2E_7 +$$

$$C_3kB_{1I}E_7 + C_3kB_{1R} \sin(C\bar{y}_i) + C_4k \sin(C\bar{y}_i)$$

$$C_6 = C_1B_{1I}E_7 - C_1B_{1R} \sin(C\bar{y}_i) - C_2 \sin(C\bar{y}_i) +$$

$$C_3kB_{1R}E_7 - C_3kB_{1I} \sin(C\bar{y}_i) + C_4kE_7$$

$$C_7 = C_5E_{16} + C_6E_{26} - I$$

$$C_8 = C_5E_{17} + C_6E_{27}$$

$$C_9 = C_5E_{18} + C_6E_{28} + \bar{a}^*$$

$$C_{10} = C_5E_{19} + C_6E_{29} - k\bar{\bar{V}}_1/2$$

$$C_{11} = C_5E_{20} + C_6E_{30} + \bar{\rho}^*\bar{a}^*$$

$$C_{12} = C_5E_{21} + C_6E_{31}$$

$$C_{13} = C_5E_{22} + C_6E_{32} + \bar{\rho}^*$$

$$C_{14} = C_5E_{23} + C_6E_{33}$$

$$C_{15} = C_5E_{24} + C_6E_{34} - k\tilde{\rho}_1$$

$$C_{16} = C_5E_{25} + C_6E_{35}$$

$$C_{17} = C_9M_{32} + C_{10}M_{40} + C_7$$

$$C_{18} = C_9M_{33} + C_{10}M_{41} + C_8$$

$$C_{19} = C_9M_{34} + C_{10}M_{42} + C_{11}$$

$$C_{20} = C_9M_{35} + C_{10}M_{43} + C_{12}$$

$$C_{21} = -(C_9M_{36} + C_{10}M_{44} + C_{13})$$

$$C_{22} = -(C_9M_{37} + C_{10}M_{45} + C_{14})$$

$$C_{23} = C_9M_{38} + C_{10}M_{46} + C_{15}$$

$$C_{24} = C_9M_{39} + C_{10}M_{47} + C_{16}$$

$$C_{25} = C_1 [B_{1I}E_7 + B_{1R} \sin(C\bar{y}_i)] + C_2 \sin(C\bar{y}_i) - kC_3 [B_{1R}E_7 -$$

$$B_{1I} \sin(C\bar{y}_i)] - kC_4E_7$$

$$C_{26} = C_1 [B_{1R}E_7 - B_{1I} \sin(C\bar{y}_i)] + C_2E_7 + kC_3 [B_{1I}E_7 +$$

$$B_{1R} \sin(C\bar{y}_i)] + kC_4 \sin(C\bar{y}_i)$$

$$C_{27} = C_{26}E_{26} + C_{25}E_{16}$$

$$C_{28} = C_{26}E_{27} + C_{25}E_{17} - I$$

$$C_{29} = C_{26}E_{28} + C_{25}E_{18} + \frac{k\bar{\bar{V}}_1}{2}$$

$$C_{30} = C_{26}E_{29} + C_{25}E_{19} + \bar{a}^*$$

$$C_{31} = C_{26}E_{30} + C_{25}E_{20}$$

$$C_{32} = C_{26}E_{31} + C_{25}E_{21} + \bar{\rho}^*\bar{a}^*$$

$$C_{33} = C_{26}E_{32} + C_{25}E_{22}$$

$$C_{34} = C_{26}E_{33} + C_{25}E_{23} + \bar{\rho}^*$$

$$C_{35} = C_{26}E_{34} + C_{25}E_{24}$$

$$C_{36} = C_{26}E_{35} + C_{25}E_{25} + k\bar{\rho}_1$$

$$C_{37} = C_{29}M_{32} + C_{30}M_{40} + C_{27}$$

$$C_{38} = C_{29}M_{33} + C_{30}M_{41} + C_{28}$$

$$C_{39} = C_{29}M_{34} + C_{30}M_{42} + C_{31}$$

$$C_{40} = C_{29}M_{35} + C_{30}M_{43} + C_{32}$$

$$C_{41} = -(C_{29}M_{36} + C_{30}M_{44} + C_{33})$$

$$C_{42} = -(C_{29}M_{37} + C_{30}M_{45} + C_{34})$$

$$C_{43} = C_{29}M_{38} + C_{30}M_{46} + C_{35}$$

$$C_{44} = C_{29}M_{39} + C_{30}M_{47} + C_{36}$$

$$C_{45} = C_{41} - \frac{C_{21}C_{42}}{C_{22}}$$

$$C_{46} = \frac{1}{C_{45}}\left(C_{37} - \frac{C_{17}C_{42}}{C_{22}}\right)$$

$$C_{47} = \frac{1}{C_{45}}\left(C_{38} - \frac{C_{18}C_{42}}{C_{22}}\right)$$

$$C_{48} = \frac{1}{C_{45}}\left(C_{39} - \frac{C_{19}C_{42}}{C_{22}}\right)$$

$$C_{49} = \frac{1}{C_{45}}\left(C_{40} - \frac{C_{20}C_{42}}{C_{22}}\right)$$

$$C_{50} = \frac{1}{C_{45}}\left(C_{43} - \frac{C_{23}C_{42}}{C_{22}}\right)$$

$$C_{51} = \frac{1}{C_{45}}\left(C_{44} - \frac{C_{24}C_{42}}{C_{22}}\right)$$

$$C_{52} = (C_{37} - C_{41}C_{46}) \div C_{42}$$

$$C_{53} = (C_{38} - C_{41}C_{47}) \div C_{42}$$

$$C_{54} = (C_{39} - C_{41}C_{48}) \div C_{42}$$

$$C_{55} = (C_{40} - C_{41}C_{49}) \div C_{42}$$

$$C_{56} = (C_{43} - C_{41}C_{50}) \div C_{42}$$

$$C_{57} = (C_{44} - C_{41}C_{51}) \div C_{42}$$

REGION 2

Momentum

$$M_{48} = (-\bar{a}^2_{US} - \bar{U}^2_{US})\,\bar{A}_S$$

$$M_{49} = \frac{k\bar{U}_2\bar{V}_2}{2}$$

$$M_{50} = \bar{p}_{US} + \bar{\rho}_{US}\bar{U}^2_{US}$$

$$M_{50A} = \bar{p}^* + \bar{\rho}^*\bar{a}^{*2}$$

$$M_{51} = M_{49} + \frac{M_{48}^{\;2}}{M_{49}}$$

$$M_{52} = \frac{M_{50}}{M_{51}}$$

$$M_{53} = \frac{M_{48}M_{50}}{M_{49}M_{51}}$$

$$M_{54} = \frac{-2}{M_{51}}$$

$$M_{55} = \frac{-2M_{48}}{M_{49}M_{51}}$$

$$M_{56} = \frac{M_{50A}}{M_{51}}$$

$$M_{57} = \frac{M_{48}M_{50A}}{M_{49}M_{51}}$$

$$M_{58} = \left[2\bar{w}_{US} + \frac{k\bar{\rho}_2\bar{V}_2}{2}\left(\frac{M_{48}}{M_{49}}\right)\right] \div M_{51}$$

$$M_{59} = \frac{1}{M_{51}}\left(\frac{2\bar{w}_{US}M_{48}}{M_{49}} - \frac{k\bar{\rho}_2\bar{V}_2}{2}\right)$$

$$M_{60} = \frac{1}{M_{51}} \left(- \frac{\bar{a}^* 2\bar{\rho}^* M_{48}}{M_{49}} - \frac{k\bar{\rho}_2 \bar{V}_2}{2} \right)$$

$$M_{61} = \frac{1}{M_{51}} \left(-2\bar{\rho}^* \bar{a}^* + \frac{k\bar{\rho}_2 \bar{V}_2 M_{48}}{2M_{49}} \right)$$

$$M_{62} = \frac{1}{M_{51}} \left(\frac{-k\bar{U}_2 \bar{V}_2}{2} \right)$$

$$M_{63} = \frac{1}{M_{51}} \left(\frac{k\bar{U}_2 \bar{V}_2 M_{48}}{2M_{49}} \right)$$

$$M_{64} = \frac{-k(\bar{\rho}\bar{U})_2}{M_{51}}$$

$$M_{65} = k(\bar{\rho}\bar{U})_2 \left(\frac{M_{48}}{M_{49} M_{51}} \right)$$

$$M_{66} = \frac{M_{50} - M_{49} M_{52}}{M_{48}}$$

$$M_{67} = - \frac{M_{49} M_{53}}{M_{48}}$$

$$M_{68} = \frac{-2 - M_{49} M_{54}}{M_{48}}$$

$$M_{69} = \frac{-M_{49} M_{55}}{M_{48}}$$

$$M_{70} = (M_{50A} - M_{49} M_{56})/M_{48}$$

$$M_{71} = -M_{49} M_{57}/M_{48}$$

$$M_{72} = (2\bar{w}_{US} - M_{49} M_{58})/M_{48}$$

$$M_{73} = (-\tfrac{1}{2}\, k\bar{\rho}_2 \bar{V}_2 - M_{49} M_{59})/M_{48}$$

$$M_{74} = (-\tfrac{1}{2}\, k\bar{\rho}_2 \bar{V}_2 - M_{49} M_{60})/M_{48}$$

$$M_{75} = (-2\bar{\rho}^* \bar{a}^* - M_{49} M_{61})/M_{48}$$

$$M_{76} = (-\tfrac{1}{2}\, k\bar{U}_2 \bar{V}_2 - M_{49} M_{62})/M_{48}$$

$$M_{77} = -M_{49} M_{63}/M_{48}$$

$$M_{78} = (-k(\bar{\rho}\bar{U})_2 - M_{49} M_{64}/M_{48}$$

$$M_{79} = -M_{49} M_{65}/M_{48}$$

$$M_{80} = \bar{\rho}_{US} \bar{A}_S + \bar{U}_{US} \bar{A}_S M_{72} - \frac{k\bar{V}_2}{2} M_{58}$$

$$M_{81} = \bar{U}_{US} \bar{A}_S M_{73} - \frac{k\bar{V}_2}{2} M_{59}$$

$$M_{82} = -\overline{U}_{US}\overline{A}_S M_{66} - \bar{\rho}_{US}\overline{U}_{US} + \frac{k\overline{V}_2}{2} M_{52}$$

$$M_{83} = -\overline{U}_{US}\overline{A}_S M_{67} + \frac{k\overline{V}_2}{2} M_{53}$$

$$M_{84} = -\overline{U}_{US}\overline{A}_S M_{77} + \bar{a}^* + \frac{k\overline{V}_2}{2} M_{63}$$

$$M_{85} = -\overline{U}_{US}\overline{A}_S M_{76} + \frac{k\overline{V}_2}{2} M_{62}$$

$$M_{86} = -\overline{U}_{US}\overline{A}_S M_{68} + \frac{k\overline{V}_2}{2} M_{54}$$

$$M_{87} = -\overline{U}_{US}\overline{A}_S M_{69} + \frac{k\overline{V}_2}{2} M_{55}$$

$$M_{88} = -\overline{U}_{US}\overline{A}_S M_{70} + \bar{\rho}^*\bar{a}^* + \frac{k\overline{V}_2}{2} M_{56}$$

$$M_{89} = -\overline{U}_{US}\overline{A}_S M_{71} + \frac{k\overline{V}_2}{2} M_{57}$$

$$M_{90} = -\overline{U}_{US}\overline{A}_S M_{74} + \frac{k\overline{V}_2}{2} M_{60}$$

$$M_{91} = -\overline{U}_{US}\overline{A}_S M_{75} + \bar{\rho}^* + \frac{k\overline{V}_2}{2} M_{61}$$

$$M_{92} = -\overline{U}_{US}\overline{A}_S M_{78} + \frac{k\overline{V}_2}{2} M_{64} + k\bar{\rho}_1$$

$$M_{93} = -\overline{U}_{US}\overline{A}_S M_{79} + \frac{k\overline{V}_2}{2} M_{65}$$

Continuity

$$C_{58} = \bar{\rho}_{US}\overline{A}_S + \overline{U}_{US}\overline{A}_S M_{59} + \frac{k\overline{V}_2}{2} M_{73}$$

$$C_{59} = \overline{U}_{US}\overline{A}_S M_{58} + \frac{k\overline{V}_2}{2} M_{72}$$

$$C_{60} = -\overline{U}_{US}\overline{A}_S M_{52} - \frac{k\overline{V}_2}{2} M_{66}$$

$$C_{61} = -\bar{\rho}_{US}\overline{U}_{US} - \overline{U}_{US}\overline{A}_S M_{53} - \frac{k\overline{V}_2}{2} M_{67}$$

$$C_{62} = -\overline{U}_{US}\overline{A}_S M_{63} - \frac{k\overline{V}_2}{2} (1 + M_{77})$$

$$C_{63} = \bar{a}^* - \overline{U}_{US}\overline{A}_S M_{62} - \frac{k\overline{V}_2}{2} M_{76}$$

$$C_{64} = -\bar{U}_{US}\bar{A}_{US}M_{61} - \frac{k\bar{V}_2}{2} M_{75}$$

$$C_{65} = \rho^* - \bar{U}_{US}\bar{A}_S M_{60} - \frac{k\bar{V}_2}{2} M_{74}$$

$$C_{66} = -\bar{U}_{US}\bar{A}_S M_{54} - \frac{k\bar{V}_2}{2} M_{68}$$

$$C_{67} = -\bar{U}_{US}\bar{A}_S M_{55} - \frac{k\bar{V}_2}{2} M_{69}$$

$$C_{68} = -\bar{U}_{US}\bar{A}_S M_{56} - \frac{k\bar{V}_2}{2} M_{70}$$

$$C_{69} = \bar{\rho}^*\bar{a}^* - \bar{U}_{US}\bar{A}_S M_{57} - \frac{k\bar{V}_2}{2} M_{71}$$

$$C_{70} = -\bar{U}_{US}\bar{A}_S M_{65} - \frac{k\bar{V}_2}{2} (2\bar{\rho}_2 + M_{79})$$

$$C_{71} = -\bar{U}_{US}\bar{A}_S M_{64} - \frac{k\bar{V}_2}{2} M_{78}$$

$$C_{72} = C_{59} - \frac{C_{58}M_{80}}{M_{81}}$$

$$C_{73} = \frac{1}{C_{72}} \left(C_{60} - \frac{C_{58}M_{82}}{M_{81}} \right)$$

$$C_{74} = \frac{1}{C_{72}} \left(C_{61} - \frac{C_{58}M_{83}}{M_{81}} \right)$$

$$C_{75} = \frac{1}{C_{72}} \left(C_{62} - \frac{C_{58}M_{84}}{M_{81}} \right)$$

$$C_{76} = \frac{1}{C_{72}} \left(C_{63} - \frac{C_{58}M_{85}}{M_{81}} \right)$$

$$C_{77} = \frac{1}{C_{72}} \left(C_{64} - \frac{C_{58}M_{91}}{M_{81}} \right)$$

$$C_{78} = \frac{1}{C_{72}} \left(C_{65} - \frac{C_{58}M_{90}}{M_{81}} \right)$$

$$C_{79} = \frac{1}{C_{72}} \left(C_{66} - \frac{C_{58}M_{86}}{M_{81}} \right)$$

$$C_{80} = \frac{1}{C_{72}} \left(C_{67} - \frac{C_{58}M_{87}}{M_{81}} \right)$$

$$C_{81} = \frac{1}{C_{72}} \left(C_{68} - \frac{C_{58}M_{88}}{M_{81}} \right)$$

$$C_{82} = \frac{1}{C_{72}} \left(C_{69} - \frac{C_{58}M_{89}}{M_{81}} \right)$$

$$C_{83} = \frac{1}{C_{72}} \left(C_{70} - \frac{C_{58}M_{93}}{M_{81}} \right)$$

$$C_{84} = \frac{1}{C_{72}} \left(C_{71} - \frac{C_{58}M_{92}}{M_{81}} \right)$$

$$C_{85} = \frac{1}{C_{58}} (C_{60} - C_{59}C_{73})$$

$$C_{86} = \frac{1}{C_{58}} (C_{61} - C_{59}C_{74})$$

$$C_{87} = \frac{1}{C_{58}} (C_{62} - C_{59}C_{75})$$

$$C_{88} = \frac{1}{C_{58}} (C_{63} - C_{59}C_{76})$$

$$C_{89} = \frac{1}{C_{58}} (C_{64} - C_{59}C_{77})$$

$$C_{90} = \frac{1}{C_{58}} (C_{65} - C_{59}C_{78})$$

$$C_{91} = \frac{1}{C_{58}} (C_{66} - C_{59}C_{79})$$

$$C_{92} = \frac{1}{C_{58}} (C_{67} - C_{59}C_{80})$$

$$C_{93} = \frac{1}{C_{58}} (C_{68} - C_{59}C_{81})$$

$$C_{94} = \frac{1}{C_{58}} (C_{69} - C_{59}C_{82})$$

$$C_{95} = \frac{1}{C_{58}} (C_{70} - C_{59}C_{83})$$

$$C_{96} = \frac{1}{C_{58}} (C_{71} - C_{59}C_{84})$$

REGION 3

Momentum

$$M_{94} = \frac{\bar{u}_E\bar{A}_E}{C\bar{y}_E} - \frac{\bar{w}_E \sin \alpha_{ch}}{C\bar{y}_E} +$$

$$[U_{2REL} \cos (\alpha_{ch} - \beta_2)]^2 \left[\frac{\bar{u}_E\bar{A}_E}{C\bar{y}_E\bar{a}_E^{\,2}} \bar{\rho}_E \right] -$$

$$\left[\left(\frac{\bar{\rho}_E\bar{U}_{2REL}\bar{A}_E}{C\bar{y}_E} \right) (\sin \alpha_{ch}) \right] \cos (\alpha_{ch} - \beta_2)$$

$$M_{95} = \bar{\rho}_E \left(\frac{k + \bar{v}_E C}{C\bar{y}_E} \right) \bar{A}_E - \frac{\bar{w}_E \cos \alpha_{ch}}{\bar{y}_E} +$$

$$[U_{2REL} \cos (\alpha_{ch} - \beta_2)]^2 \left[\left(\frac{k + \bar{v}_E C}{C\bar{y}_E \bar{a}_E^{\,2}} \right) \bar{A}_E \bar{\rho}_E \right] -$$

$$\bar{\rho}_E \bar{U}_{2REL} \bar{A}_E \cos \alpha_{ch} \cos (\alpha_{ch} - \beta_2)$$

$$M_{96} = \bar{\rho}_E \frac{\bar{u}_E \bar{U}_3 \bar{V}_3}{2C\bar{y}_E \bar{a}_E^{\,2}} - \frac{\rho_3 \bar{V}_3 \sin \alpha_{ch}}{2C\bar{y}_E}$$

$$M_{97} = \bar{\rho}_E \frac{(k + \bar{v}_E C)\, \bar{U}_3 \bar{V}_3}{2C\bar{y}_E \bar{a}_E^{\,2}} - \frac{\rho_3 \bar{V}_3 \cos \alpha_{ch}}{2\bar{y}_E}$$

$$M_{98} = \left[\frac{\bar{w}_E \sin \alpha_{ch}}{(R^2 + C^2)\, \bar{y}_E} - \frac{R\bar{w}_E \cos \alpha_{ch}}{C\,(R^2 + C^2)\, \bar{y}_E} \right] +$$

$$\left[\frac{\bar{\rho}_E U_{2REL} \bar{A}_E \cos (\alpha_{ch} - \beta_2)}{(R^2 + C^2)\, \bar{y}_E} \right] \left[\sin \alpha_{ch} - \frac{R \cos \alpha_{ch}}{C} \right]$$

$$M_{99} = \frac{\bar{\rho}_3 \bar{V}_3}{2\,(R^2 + C^2)\, \bar{y}_E} \left(\sin \alpha_{ch} - \frac{R \cos \alpha_{ch}}{C} \right)$$

$$M_{100} = \cos (C\bar{y}_E) - 1$$

$$M_{101} = M_{94}\, (M_{100} B_{2R} - B_{2I} \sin (C\bar{y}_E)) + M_{95} M_{100} -$$

$$kM_{96}(B_{2I} M_{100} + B_{2R} \sin (C\bar{y}_E)) - kM_{97} \sin (C\bar{y}_E)$$

$$M_{102} = M_{94}\, (M_{100} B_{2I} + B_{2R} \sin (C\bar{y}_E)) - M_{95} \sin (C\bar{y}_E) -$$

$$kM_{96}\, (M_{100} B_{2R} - B_{2I} \sin (C\bar{y}_E)) - kM_{97} M_{100}$$

$$M_{103} = -kM_{99} \sin (C\bar{y}_E) + M_{98} M_{100}$$

$$M_{104} = -kM_{99} M_{100} - M_{98} \sin (C\bar{y}_E)$$

$$M_{105} = M_{94}\, [B_{2I} M_{100} + B_{2R} \sin (C\bar{y}_E)] + M_{95} \sin (C\bar{y}_E) +$$

$$kM_{96}\, [B_{2R} M_{100} - B_{2I} \sin (C\bar{y}_E) + kM_{97} M_{100}$$

$$M_{106} = M_{94}\, [B_{2R} M_{100} - B_{2I} \sin (C\bar{y}_E)] + M_{95} M_{100} -$$

$$kM_{96}\, [B_{2I} M_{100} + B_{2R} \sin (C\bar{y}_E)] - kM_{97} \sin (C\bar{y}_E)$$

$$M_{107} = M_{98} \sin (C\bar{y}_E) + kM_{99} M_{100}$$

$$M_{108} = M_{98} M_{100} - kM_{99} \sin (C\bar{y}_E)$$

$$M_{109} = M_{105} - M_{101} M_{106} / M_{102}$$

$$M_{110} = (M_{107} - M_{103} M_{106} / M_{102}) / M_{109}$$

$$M_{111} = (M_{108} - M_{104} M_{106} / M_{102}) / M_{109}$$

$$M_{112} = 1 / M_{109}$$

$$M_{113} = -M_{106}/M_{102}M_{109}$$

$$M_{114} = (M_{107} - M_{105}M_{110})/M_{106}$$

$$M_{115} = (M_{108} - M_{105}M_{109})/M_{106}$$

$$M_{116} = M_{105}\ M_{113}/M_{106}$$

$$M_{117} = (1 - M_{105}M_{112})/M_{106}$$

Continuity

$$C_{97} = -\frac{\bar{\rho}_E \bar{u}_E \overline{U}_{2REL} \overline{A}_E \cos(\alpha_{ch} - \beta_2)}{C\bar{y}_E \bar{a}_E^2} + \frac{\bar{\rho}_E \overline{A}_E \sin \alpha_{ch}}{C\bar{y}_E}$$

$$C_{98} = -\frac{\bar{\rho}_E (k + \bar{v}_E C)\ \overline{U}_{2REL} \overline{A}_E \cos(\alpha_{ch} - \beta_2)}{C\bar{y}_E \bar{a}_E^2} + \frac{\bar{\rho}_E \overline{A}_E \cos \alpha_{ch}}{\bar{y}_E}$$

$$C_{99} = \frac{\bar{\rho}_E \overline{A}_E}{(R^2 + C^2)\ \bar{y}_E} \left(\sin \alpha_{ch} - \frac{R}{C} \cos \alpha_{ch} \right)$$

$$C_{100} = \frac{\bar{u}_E \overline{V}_3 \bar{\rho}_E}{2C\bar{y}_E \bar{a}_E^2}$$

$$C_{101} = \frac{(k + \bar{v}_E C)\ \overline{V}_3 \bar{\rho}_E}{2C\bar{y}_E \bar{a}_E^2}$$

$$C_{102} = C_{47}B_{2R}M_{100} - C_{97}B_{2I} \sin(C\bar{y}_E) + C_{98}M_{100} +$$
$$kC_{100}\ (B_{2I}M_{100} + B_{2R} \sin(C\bar{y}_E)) + kC_{101} \sin(C\bar{y}_E)$$

$$C_{103} = -C_{97}\ (B_{2I}M_{100} + B_{2R} \sin(C\bar{y}_E)) - C_{98} \sin(C\bar{y}_E) +$$
$$kC_{100}\ (B_{2R}M_{100} - B_{2I} \sin(C\bar{y}_E)) + kC_{101}M_{100}$$

$$C_{104} = C_{99}M_{100}$$

$$C_{105} = -C_{99} \sin(C\bar{y}_E)$$

$$C_{106} = C_{102}M_{110} + C_{103}M_{114} + C_{104}$$

$$C_{107} = C_{102}M_{111} + C_{103}M_{115} + C_{105}$$

$$C_{108} = C_{102}M_{113} + C_{103}M_{116}$$

$$C_{109} = C_{102}M_{112} + C_{103}M_{117}$$

$$C_{110} = C_{97}\ [B_{2I}M_{100} + B_{2R} \sin(C\bar{y}_E)] + C_{98} \sin(C\bar{y}_E) -$$
$$kC_{100}\ [B_{2R}M_{100} - B_{2I} \sin(C\bar{y}_E)] - kC_{101}M_{100}$$

$$C_{111} = C_{97}\ [B_{2R}M_{100} - B_{2I} \sin(C\bar{y}_E)] + C_{98}M_{100} +$$
$$kC_{100}\ [B_{2I}M_{100} + B_{2R} \sin(C\bar{y}_E)] + kC_{101} \sin(C\bar{y}_E)$$

$$C_{112} = C_{99} \sin (C\bar{y}_E)$$

$$C_{113} = C_{99}M_{100}$$

$$C_{114} = C_{110}M_{110} + C_{111}M_{114} + C_{112}$$

$$C_{115} = C_{110}M_{111} + C_{111}M_{115} + C_{113}$$

$$C_{116} = C_{110}M_{113} + C_{111}M_{116}$$

$$C_{117} = C_{110}M_{112} + C_{111}M_{117}$$

$$C_{118} = C_{114} - C_{115}C_{106}/C_{107}$$

$$C_{119} = -(C_{116} - C_{115}C_{108}/C_{107})/C_{118}$$

$$C_{120} = -(C_{117} - C_{115}C_{109}/C_{107})/C_{118}$$

$$C_{121} = (CI - C_{115}CR/C_{107})/C_{118}$$

$$C_{122} = (-C_{116} - C_{114}C_{119})/C_{115}$$

$$C_{123} = (-C_{117} - C_{114}C_{120})/C_{115}$$

$$C_{124} = (CI - C_{121}C_{114})/C_{115}$$

S Coefficients

$$S_1 = \frac{2\gamma}{\gamma + 1} \bar{p}_{us}$$

$$S_2 = \frac{2\gamma}{\gamma + 1} \bar{p}_{us}\bar{M}_{us} - \bar{p}_{ds} - \frac{(\gamma - 1)}{\gamma + 1} \bar{p}_{us}$$

$$S_3 = \frac{2\gamma}{\gamma + 1} \bar{a}_{us}\bar{M}_{us} - \bar{U}_s - \frac{(\gamma - 1)}{\gamma + 1} \bar{a}_{us}$$

$$S_4 = \frac{2\gamma}{\gamma + 1} \bar{a}_{us}\bar{p}_{us}$$

$$S_5 = (\gamma - 1) \bar{M}_{us}^2 + 2$$

$$S_6 = (2 - 2\gamma) \bar{\rho}_{ds} + (2\gamma + 2) \bar{\rho}_{us}$$

$$S_7 = (\gamma + 1) \bar{M}_{us}^2$$

$$S_8 = S_6/(\bar{a}_{us}S_5)$$

$$S_9 = S_7/S_5$$

Miscellaneous

$$RC = -\bar{p}'_{dsR}\bar{A}_S - \bar{p}_{ds}\bar{A}'_{SR} + [\bar{p}_E + \rho_E (U_{2RE} \cos(\alpha_{ch} - \beta_2))^2]\,\bar{A}'_{ER} -$$

$$\bar{U}'_{dsR}\bar{w}_{ds} - \bar{U}_{ds}\;\bar{w}'_{dsR} - \frac{k\rho'_{dsI}\bar{U}_3\bar{V}_3}{2} - k\bar{\rho}_3\bar{U}_3\bar{V}'_{3I} - \frac{k\bar{\rho}_3\bar{U}'_{dsI}\bar{V}_3}{2}$$

with

$$\bar{w}'_{dsR} = (\bar{\rho}'_{dsR}\bar{U}_{ds}\bar{A}_S + \bar{\rho}_{ds}\bar{U}'_{dsR}\bar{A}_S + \bar{\rho}_{ds}\bar{U}_{ds}\bar{A}'_{SR})$$

$$IC = -p_{dsI}\bar{A}_S - \bar{p}_{ds}\bar{A}'_{SI} + [\bar{p}_E + \bar{\rho}_E (\bar{U}_{2REL} \cos(\alpha_{ch} - \beta_2))^2]\,\bar{A}'_{EI} -$$

$$\bar{U}'_{dsI}\bar{w}_{ds} - \bar{U}_{ds}\bar{w}'_{dsI} + \frac{k\bar{\rho}'_{dsR}\bar{U}_3\bar{V}_3}{2} + k\bar{\rho}_3\bar{U}_3\bar{V}'_{3R} + \frac{k\bar{\rho}_3\bar{U}'_{dsR}\bar{V}_3}{2}$$

with

$$\bar{w}'_{dsI} = (\bar{\rho}'_{dsI}\bar{U}_{ds}\bar{A}_S + \bar{\rho}_{ds}\bar{U}'_{dsI}\bar{A}_S + \bar{\rho}_{ds}\bar{U}_{ds}\bar{A}'_{SI})$$

$$CR = -\bar{\rho}_E\bar{A}'_{ER}\bar{U}_E + w'_{dsR} + \bar{\rho}_3 k\bar{V}'_{3I} + \tfrac{1}{2} k\bar{V}'_3\bar{\rho}'_{dsI}$$

$$CI = -\bar{\rho}_E\bar{U}_{2REL} \cos(\alpha_{ch} - \beta_2)\,\bar{A}'_{EI} + \bar{w}'_{dsI} - \bar{\rho}_3 k\bar{V}'_{3R} - \tfrac{1}{2} k\bar{V}_3\bar{\rho}'_{dsR}$$

APPENDIX F
CALCULATION OF MEAN FLOW AERODYNAMICS

Pressure jump across the shock (steady state)

$$\frac{\bar{p}_{ds}}{\bar{p}_{us}} = 1 + \frac{2\,\gamma}{\gamma + 1}\,(\bar{M}^2_{us} - 1) \tag{F1}$$

Steady-state pressure as a function of chordwise location

$$\bar{p}(x) = \bar{p}_{TOT}\left[1 + \frac{\gamma - 1}{2}\,(\bar{M}^2_x)\right]^{\left(\frac{-\gamma}{\gamma-1}\right)} \tag{F2}$$

Steady-state density as a function of chordwise location

$$\bar{\rho}(x) = \bar{\rho}_{TOT}\left[1 + \frac{\gamma - 1}{2}\,(\bar{M}^2_{(x)})\right]^{\left(\frac{-1}{\gamma-1}\right)} \tag{F3}$$

where,

$$\rho_{TOT} = \frac{P_{TOT}}{RT_{TOT}}$$

Steady-state temperature as a function of chordwise position

$$\bar{T}(x) = \bar{T}_{TOT}\left[1 + \frac{\gamma - 1}{2}\,(\bar{M}^2(x))\right]^{-1} \tag{F4}$$

Local steady-state speed of sound as a function of chordwise position.

$$\bar{a}(x) = \left[\frac{\gamma RT_{TOT}}{1 + \frac{\gamma - 1}{2}\,(\bar{M}^2(x))}\right]^{1/2} \tag{F5}$$

Total pressure, temperature, and density downstream of the shock

$$p_{TOT_{ds}} = p_{TOT_{us}}\left[1 + \frac{2\gamma}{\gamma + 1}\,(M^2_{us} - 1)\right]^{\left(\frac{-1}{\gamma-1}\right)}$$

$$\left[\frac{(\gamma + 1)\,\bar{M}_{us}^{\;2}}{(\gamma - 1)\,\bar{M}_{us}^{\;2} + 2}\right]^{\left(\frac{\gamma}{\gamma-1}\right)} \tag{F6}$$

$$T_{TOT_{ds}} = T_{TOT_{us}}$$

Mach number across the shock

$$\bar{M}_{ds}^2 = \frac{\bar{M}_{us}^2 (\gamma - 1) + 2}{2\gamma \bar{M}_{us}^2 - (\gamma - 1)} \tag{F7}$$

APPENDIX G

DEFINITION OF AREA PERTURBATIONS

Bending Mode Perturbations

Figure G1 describes the inlet area to be considered. The blades will be considered to vibrate harmonically with a constant interblade phase lag σ. Assuming this, then the deflections can be represented as follows:

$h = \bar{h}\, e^{ikt}$ (reference blade)

$h' = \bar{h}\, e^{ikt}\, e^{i\sigma}$ (blade adjacent to reference blade)

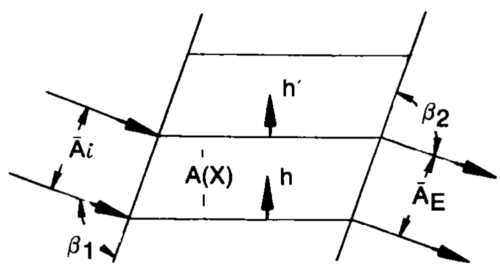

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Figure G1. Bending Mode Area Perturbations

Thus, the arèa perturbation can be represented as shown below:

$$A'_i = (h' - h)\ \frac{\sin \beta_1}{\sin \alpha_{ch}}$$

$$= (\bar{h}^{ikt}\, e^{i\sigma} - \bar{h}\, e^{ikt})\ \frac{\sin \beta_1}{\sin \alpha_{ch}} \qquad \text{(G1)}$$

where all quantities are nondimensional. Simplifying this expression yields

$$\bar{A}'_i = \bar{h}\, e^{ikt}\, (e^{i\sigma} - 1)\ \frac{\sin \beta_1}{\sin \alpha_{ch}} \qquad \text{(G2)}$$

Dividing through be e^{ikt} gives

$$\bar{A}'_i = \bar{h}\, (e^{i\sigma} - 1)\ \frac{\sin \beta_1}{\sin \alpha_{ch}} \qquad \text{(G3)}$$

Separating the expression into its real and imaginary components produces

$$\bar{A}'_{iR} = \bar{h}\, (\cos \sigma - 1)\ \frac{\sin \beta_1}{\sin \alpha_{ch}} \qquad \text{(G4)}$$

$$\bar{A}'_{iI} = \bar{h} \sin \sigma\ \left(\frac{\sin \beta_1}{\sin \alpha_{ch}} \right) \qquad \text{(G5)}$$

Similarly, the other area perturbations for the bending mode will be

$$\bar{A}'^*_R = \bar{h}\,(\cos\sigma - 1) \tag{G6}$$

$$\bar{A}'^*_I = \bar{h}\sin\sigma \tag{G7}$$

$$\bar{A}'_{SR} = \bar{h}\,(\cos\sigma - 1) \tag{G8}$$

$$\bar{A}'_{SI} = \bar{h}\sin\sigma \tag{G9}$$

$$\bar{A}'_{ER} = \bar{h}\,(\cos\sigma - 1)\ \frac{\sin\beta_2}{\sin\alpha_{ch}} \tag{G10}$$

$$\bar{A}'_{EI} = \bar{h}\sin\sigma\ \left(\frac{\sin\beta_2}{\sin\alpha_{ch}}\right) \tag{G11}$$

Torsional Mode Area Perturbations

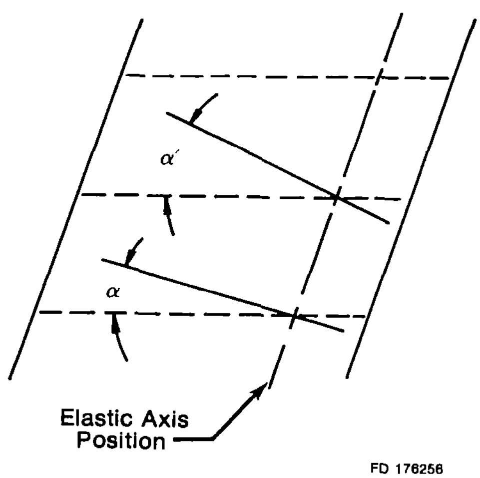

Figure G2. Torsional Mode Area Perturbation

In the torsional mode area perturbations, the airfoils will be assumed to be undergoing rigid body torsional deflections about an arbitrary elastic axis position a. Consider first a cascade of flat plate airfoils oscillating out of phase, as shown in Figure G2. The area perturbation can be represented by

$$A'(x) = c\ \left(Z - \frac{x}{c}\right)(\alpha' - \alpha) \tag{G12}$$

where,

$$Z = \frac{b(1 + a)}{c}$$

a = Elastic axis position referenced to midchord and nondimensionalized by b.

At the blade leading edge

$$A'_i = Zc\,(\alpha' - \alpha)\left(\frac{\sin \beta_1}{\sin \alpha_{ch}}\right) \tag{G13}$$

Next, assume

$$\alpha = \bar{\alpha}\, e^{ikt}$$

$$\alpha' = \bar{\alpha}\, e^{ikt}\, e^{i\sigma} \tag{G14}$$

Thus,

$$\alpha' - \alpha = \bar{\alpha}\, e^{ikt}\,(e^{i\sigma} - 1)$$

Dividing this by e^{ikt} and substituting the resultant into Equation G13 gives

$$\bar{A}'_i = cZ\bar{\alpha}(e^{i\sigma} - 1)\left(\frac{\sin \beta_1}{\sin \alpha_{ch}}\right)$$

Separating this expression into its real and imaginary components yields

$$\bar{A}'_{iR} = cZ\bar{\alpha}(\cos \sigma - 1)\left(\frac{\sin \beta_1}{\sin \alpha_{ch}}\right) \tag{G15}$$

$$\bar{A}'_{iI} = cZ\bar{\alpha}\,\sin \sigma\left(\frac{\sin \beta_1}{\sin \alpha_{ch}}\right) \tag{G16}$$

Similarly, the other area perturbations will be

$$\bar{A}'^{*}_{R} = \left(Z - \frac{x^*}{c}\right) c\bar{\alpha}\,(\cos \sigma - 1) \tag{G17}$$

$$\bar{A}^{*\prime}_{I} = \left(Z - \frac{x^*}{c}\right) c\alpha\,\sin \sigma \tag{G18}$$

At the shock wave, the area perturbations will be

$$\bar{A}'_{SR} = \left(Z - \frac{x_s}{c}\right) c\bar{\alpha}\,(\cos \sigma - 1) \tag{G19}$$

$$\bar{A}'_{SI} = \left(Z - \frac{x_s}{c}\right) c\bar{\alpha}\,\sin \sigma \tag{G20}$$

At the exit, the area perturbations will be

$$\bar{A}'_{ER} = (Z - 1)(\cos\sigma - 1)\, c\bar{\alpha} \left(\frac{\sin \beta_2}{\sin \alpha_{ch}} \right) \qquad \text{(G21)}$$

$$\bar{A}'_{EI} = (Z - 1)\, c\bar{\alpha} \sin\sigma \left(\frac{\sin \beta_2}{\sin \alpha_{ch}} \right)$$

DEFINITION OF THE VOLUME PERTURBATIONS

Bending Mode Perturbations

Figure G3 defines and divides the volumes into three sections. Each section is dealt with separately in this Appendix.

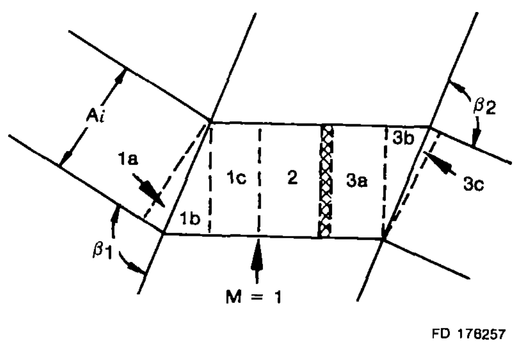

Figure G3. Bending Mode and Torsional Mode Volume Perturbations

Section 1

For ease of determination, Section 1 is divided into three subsections, such that

$$V_1 = V_{1a} + V_{1b} + V_{1c} \qquad \text{(G22)}$$

Subsection 1a

$$V_{1a} = \frac{A_1^2}{2 \text{ cotan } (90 - \beta_1)}$$

Expanding this expression in small perturbations and neglecting higher order terms gives

$$V_{1a} = \frac{(\bar{A}_1^2 + 2\bar{A}_i A'_i)}{2 \text{ cotan } (90 - \beta_1)} \qquad \text{(G23)}$$

which produces for the perturbation volume in terms of real and imaginery components

$$\bar{V}'_{iaR} \quad \frac{\bar{A}_i \bar{A}'_{iR}}{\text{cotan } (90 - \beta_1)}$$

and

$$\bar{V}'_{1aI} \quad \frac{\bar{A}_i \bar{A}'_{iI}}{\text{cotan } (90 - \beta_1)} \tag{G24}$$

where $\bar{A}'_{iR}$ and $\bar{A}'_{1I}$ are given in Equations A4 and A5, respectively, in Appendix A.

Subsection 1b

$$V_{1b} = \frac{1}{2} (\delta \cos \alpha_{ch}) (\delta \sin \alpha_{ch})$$

where δ represents the gap between the blades. Expanding this expression in small perturbations gives

$$V_{1b} = \frac{1}{2} (\bar{\delta}^2 + 2 \bar{\delta}\delta') \sin \alpha_{ch} \cos \alpha_{ch}$$

or

$$V_{1b} = \bar{\delta} \left(\frac{h' - h}{\sin \alpha_{ch}} \right) \sin \alpha_{ch} \cos \alpha_{ch}$$

Breaking this expression down into its real and imaginary components gives

$$\bar{V}'_{1bR} = \bar{\delta}\bar{h} (\cos \sigma - 1) \cos \alpha_{ch} \tag{G25}$$

$$\bar{V}'_{1bI} = \bar{\delta}\bar{h} \sin \sigma \cos \alpha_{ch} \tag{G26}$$

Subsection 1c

$$V_{1c} = A^* (x^* - \delta \cos \alpha_{ch})$$

Expanding this expression in small perturbations and neglecting higher order terms produces

$$V'_{1c} = (\bar{A}^* - A'^*) x^* - (\bar{\delta} \bar{A}^* + \bar{\delta} A'^* + \delta' \bar{A}^*) \cos \alpha_{ch}$$

or

$$V'_{1c} = A'^* (x^* - \bar{\delta} \cos \alpha_{ch}) + \delta'\bar{A}^* \cos \alpha_{ch} \tag{G27}$$

or

$$\bar{V}'_{1cR} = \bar{h} (x^* - \bar{\delta} \cos \alpha_{ch}) (\cos \sigma - 1) + \bar{A}^* \bar{h} (\cos \sigma - 1) \cot \alpha_{ch}$$

$$\bar{V}'_{1cI} = \bar{h} (x^* - \bar{\delta} \sin \alpha_{ch}) \sin \sigma + \bar{A}^* \bar{h} \sin \sigma \cot \alpha_{ch} \tag{G28}$$

Thus, the real and imaginary components for Section 1 represent a summation of the three subsections, as shown in Equation G29.

$$\bar{V}'_{1R} = \bar{V}'_{1aR} + \bar{V}'_{1bR} + \bar{V}'_{1cR}$$

$$\bar{V}'_{1I} = \bar{V}'_{1aI} + \bar{V}'_{1bI} + \bar{V}'_{1cI} \qquad \text{(G29)}$$

Section 2

Since this section represents supersonic flow, downstream disturbances cannot propagate upstream. Therefore, the volume perturbation can be represented as being bounded on the downstream side by a steady-state shock location, as shown in Equation G30.

$$V_2 = \frac{1}{2}(A^* + A_s)(\bar{x}_S - x^*) \qquad \text{(G30)}$$

Expanding in small perturbations produces

$$V'_2 = \frac{1}{2}(A'^* + A'_s)(\bar{x}_S - x^*) \qquad \text{(G31)}$$

Breaking the expression into its real and imaginary components gives

$$\bar{V}'_{2R} = \bar{h}\ (\cos\sigma - 1)(\bar{x}_S - x^*)$$

$$\bar{V}'_{2I} = \bar{h}\sin\sigma\ (\bar{x}_S - x^*) \qquad \text{(G32)}$$

Section 3

Section 3 also has been divided into three subsections. Each subsection is dealt with separately with the volume perturbations of Section 3 equal to the summation of the subsections.

Subsection 3a

$$V_{3a} = \frac{1}{2}(A_S + A_{\frac{x}{c}=1})(c - x_S)$$

$$V'_{3a} = \frac{1}{2}\left[(A'_S + A'_{\frac{x}{c}=1})(c - \bar{x}_S) - (\bar{A}_S + \bar{A}_{\frac{x}{c}=1})\, x'_S\right]$$

$$\bar{V}'_{3aR} = \bar{h}\ (\cos\sigma - 1)(c - \bar{x}_S) - \frac{1}{2}\bar{x}'_{SR}\ (\bar{A}_S + \bar{A}_{\frac{x}{c}=1}) \qquad \text{(G33)}$$

$$\bar{V}'_{3aI} = \bar{h}\sin\ (c - \bar{x}_S) - \frac{1}{2}x'_{SI}\ (\bar{A}_S + \bar{A}_{\frac{x}{c}=1}) \qquad \text{(G34)}$$

Subsection 3b

$$V_{3b} = \frac{1}{2}(\delta \cos \alpha_{ch})(\delta \sin \alpha_{ch})$$

$$V'_{3b} = \delta\delta' \cos \alpha_{ch} \sin \alpha_{ch}$$

$$\overline{V}'_{3bR} = \frac{1}{2}\,\overline{h}\,(\cos \sigma - 1)(\delta \sin 2\alpha_{ch}) \tag{G35}$$

$$\overline{V}'_{3bI} = \frac{1}{2}\,\overline{h} \sin \sigma\,(\delta \sin 2\alpha_{ch}) \tag{G36}$$

Subsection 3c

$$V_{3c} = \frac{A_E^2}{2 \tan \beta_2}$$

$$V'_{3c} = \frac{\overline{A}_E A'_E}{\tan \beta_2}$$

$$\overline{V}'_{3cR} = \frac{\overline{A}_E \overline{h}\,(\cos \sigma - 1) \cos \beta_2}{\sin \alpha_{ch}} \tag{G37}$$

$$V'_{3cI} = \frac{\overline{A}_E \overline{h} \sin \sigma \cos \beta_2}{\sin \alpha_{ch}} \tag{G38}$$

As stated, the real and imaginary components of Section 3 represent a summation of the three subsections, as shown below:

$$\overline{V}'_{3R} = \overline{V}'_{3aR} + \overline{V}'_{3bR} + \overline{V}'_{3cR}$$

$$\overline{V}'_{3I} = \overline{V}'_{3aI} + \overline{V}'_{3bI} + \overline{V}'_{3cI} \tag{G39}$$

Torsional Mode Perturbations

As before, the individual sections will be dealt with separately in the following paragraphs.

Section 1

Section 1 is again divided into three subsections for ease of determination.

Subsection 1a

$$V'_{1a} = \frac{\bar{A}_i\, A'_i}{\text{cotan}\,(90 - \beta_1)}$$

$$\bar{V}'_{1aR} = \frac{\bar{A}_i\, c\, Z\, \bar{\alpha}\, (\cos\sigma - 1)\, \sin\beta_1}{\sin\alpha_{ch}\, \text{cotan}\,(90 - \beta_1)} \tag{G40}$$

$$\bar{V}'_{1aI}\ \frac{\bar{A}_i\, c\, Z\, \bar{\alpha}\, \sin\sigma\, \sin\beta_1}{\sin\alpha\, \text{cotan}\,(90 - \beta_1)} \tag{G41}$$

Subsection 1b

$$V'_{1b} = \frac{1}{2}\, \bar{\delta}\delta'\, \sin\,(2\,\alpha_{ch})$$

$$\bar{V}'_{1bR} = cZ\bar{\alpha}\,\bar{\delta}\,(\cos\sigma - 1)\cos\alpha_{ch} \tag{G42}$$

$$\bar{V}'_{1bI} = cZ\overline{\alpha\delta}\,\sin\sigma\,\cos\alpha_{ch} \tag{G43}$$

Section 1c

$$V'_{1c} = \int_{x_{ENT}}^{x^*} A'(x)dx \qquad A'(x) = c\left(Z - \frac{x}{c}\right)(\alpha' - \alpha)$$

$$= cZ\,(x^* - \bar{\delta}\cos\alpha_{ch})\,(\alpha' - \alpha) - \frac{c[\,x^{*2} - (\bar{\delta}\cos\alpha_{ch})^2\,][\alpha' - \alpha\,]}{2c}$$

$$\bar{V}'_{1cR} = c\bar{\alpha}\,(\cos\sigma - 1)\left[Z\,(x^* - \bar{\delta}\cos\alpha_{ch}) - \frac{x^{*2} - (\bar{\delta}\cos\alpha_{ch})^2}{2c}\right] \tag{G44}$$

$$\bar{V}_{1cI} = c\alpha\,\sin\sigma\left[Z(x^* - \bar{\delta}\cos\alpha_{ch}) - \frac{x^{*2} - (\bar{\delta}\cos\alpha_{ch})^2}{2c}\right] \tag{G45}$$

The summation of the real and imaginary components of the subsections is shown in Equation G46.

$$\bar{V}'_{1R} = \bar{V}'_{1aR} + \bar{V}'_{1bR} + \bar{V}'_{1cR}$$

$$\bar{V}'_{1I} = \bar{V}'_{1aI} + \bar{V}'_{1bI} + \bar{V}'_{1cR} \tag{G46}$$

Section 2

$$V'_2 = \int_{x^*}^{x_a} A'(x)dx$$

$$\bar{V}'_{2R} = c\,\bar{\alpha}\,(\cos\sigma - 1)\left[\,Z\,(x_s - x^*) - \left(\frac{x_s^2 - x^{*2}}{2c}\right)\right] \tag{G47}$$

$$\bar{V}'_{2I} = c\,\bar{\alpha}\,\sin\sigma\left[\,Z\,(x_S - x^*) - \left(\frac{x_S^2 - x^{*2}}{2c}\right)\right] \tag{G48}$$

Section 3

As before, Section 3 will be divided into three subsections.

Subsection 3a

$$V'_{3a} = \int_{x_s}^{x_E} A'(x)dx - \bar{A}_s x'_s$$

$$= c\ (\alpha' - \alpha) \left[Z\ (x_E - x_s) - \left(\frac{x_E^2 - x_s^2}{2c} \right) \right] - \bar{A}_S x'_S$$

$$\bar{V}'_{3aR} = c\ \alpha\ (\cos\sigma - 1) \left[Z\ (c - \bar{x}_S) - \left(\frac{c^2 - \bar{x}_S^2}{2c} \right) \right] - \bar{A}_S \bar{x}'_{SR} \qquad \text{(G49)}$$

$$\bar{V}'_{3aI} = c\ \bar{\alpha} \sin\sigma \left[Z\ (c - \bar{x}_S) - \left(\frac{c^2 - \bar{x}_S^2}{2c} \right) \right] - \bar{A}_S\ \bar{x}'_{SI} \qquad \text{(G50)}$$

Subsection 3b

$$V_{3b} = \frac{1}{2} (\bar{\delta}^2 + 2\ \bar{\delta}\ \delta') \sin\alpha_{ch} \cos\alpha_{ch}$$

$$V'_{3b} = \bar{\delta}\ \delta' \sin\alpha_{ch} \cos\alpha_{ch}$$

where,

$$\delta' = \frac{c(Z - 1)(\bar{\alpha} - \alpha)}{\sin\alpha_{ch}}$$

$$\bar{V}'_{3bR} = c\ \bar{\alpha}\ (Z - 1)\ (\cos\sigma - 1) \cos\alpha_{ch} \cdot \delta \qquad \text{(G51)}$$

$$\bar{V}'_{3bI} = c\ \bar{\alpha}\ (Z - 1) \sin\sigma \cos\alpha_{ch} \cdot \delta \qquad \text{(G52)}$$

Subsection 3c

$$V_{3c} = \frac{A_E^2}{2 \tan\beta_2}$$

$$V'_{3c} = \frac{\bar{A}_E\ A'_E}{\tan\beta_2}$$

$$\bar{V}'_{3cR} = \frac{\bar{A}_E\ c\ \bar{\alpha}\ (Z - 1)\ (\cos\sigma - 1) \cos\beta_2}{\sin\alpha_{ch}} \qquad \text{(G53)}$$

$$\bar{V}'_{3cI} = \frac{\bar{A}_E\ c\ \bar{\alpha}\ (Z - 1) \sin\sigma \cos\beta_2}{\sin\alpha_{ch}} \qquad \text{(G54)}$$

The resulting summations for Section 3 are given in Equation G55.

$$\bar{V}'_{3R} = \bar{V}'_{3aR} + \bar{V}'_{3bR} + \bar{V}'_{3cR}$$

$$\bar{V}'_{3I} = \bar{V}'_{3aI} + \bar{V}'_{3bI} + \bar{V}'_{3cI} \qquad \text{(G55)}$$

APPENDIX H
CALCULATION OF THE STEADY-STATE SHOCK LOCATION AND TEST FOR CHOKED FLOW

The first step in the analysis requires checking for choked flow. Equation H1 is obtained from the isentropic flow relationships.

$$\left(\frac{A_c}{A^*}\right)^2 = \frac{1}{M_c^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_c^2\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)} \qquad \text{(H1)}$$

Rearranging this relationship and solving for A_c gives

$$A_c = \left\{\frac{1}{M_c^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}\right\}^{1/2} A^* \qquad \text{(H2)}$$

For a choked flow condition, A^* will be equal to or greater than the minimum area between the blades (A_{min}). Thus, for choked flow Equation H2 becomes

$$A_c \geq \left\{\frac{1}{M_c^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_c^2\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}\right\}^{1/2} A^* \qquad \text{(H3)}$$

During steady-state conditions

$$A_c = \delta \sin \beta_1 \qquad \text{(H4)}$$

and

$$A_{min} = (\delta \sin \alpha_{ch}) + y_L\, x_2^* - y_u\, x_1^* \qquad \text{(H5)}$$

Substituting the relationships of Equations H4 and H5 into Equation H3 and solving for β_1 yields

$$\beta_1 = \sin^{-1}\left[\frac{1}{\delta}\left\{\frac{1}{M_c^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_c^2\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}\right\}^{1/2}\left\{\delta \sin \alpha_{ch} + y_L(x^*_2) - y_u(x^*_1)\right\}\right] \qquad \text{(H6)}$$

Thus, the flow will be choked if the input β_1 is equal to or greater than the β_1 calculated in Equation H6. If the flow is choked, the areas at various cross-sections will be calculated along with the exit area, as shown.

$$A_{exit} = \delta \sin \beta_2 \qquad \text{(H7)}$$

The next step in the analysis requires that the supersonic Mach numbers downstream of the throat be calculated by solving the following equation for M at the specific chordwise locations:.

$$\frac{A_{(x)}}{A_{min}} = \left\{\frac{1}{M^2(x)}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2(x)\right)\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}\right\}^{1/2} \qquad \text{(H8)}$$

Once the area relationships and Mach numbers as a function of x have been determined, the analysis proceeds with the calculation of the steady-state shock position. The derivation requires only the Mach numbers downstream of the throat since it can be shown that the shock position is stable only in a diverging channel. With the pressure ratio across the stage (PR) input, the capture Mach number fixes the pressure at the entrance, as shown in Equation H9.

$$\frac{P_c}{P_t} = \left(1 + \frac{\gamma-1}{2} M_c^2\right)^{\left(\frac{-\gamma}{\gamma-1}\right)} \tag{H9}$$

The pressure ratio from the throat to the exit can be defined as

$$PR_{corr} = \frac{P_c}{P_t}\left(\frac{2}{\gamma+1}\right)^{\frac{-\gamma}{\gamma+1}} PR \tag{H10}$$

The use of an iteration procedure finds the shock location by first assuming the shock is located at the throat and then incrementally moving downstream until the obtainable pressure ratio (PR_{obt}) at the specific shock location matches the PR_{corr} within some tolerance ϵ. PR_{obt} is calculated in the following manner:

1. Calculate the ratio of pressure entering the shock to the pressure at the throat as

$$\frac{P_{us}}{P^*} = \frac{\left(1 + \frac{\gamma - 1}{2} M_s^2\right)^{\left(\frac{-\gamma}{\gamma-1}\right)}}{\left(\frac{\gamma + 1}{2}\right)^{\left(\frac{-\gamma}{\gamma-1}\right)}} \tag{H11}$$

2. Calculate the pressure rise across the throat as

$$\frac{P_{ds}}{P_{us}} = 1 + \frac{2\gamma}{\gamma + 1}(M_s^2 - 1) \tag{H12}$$

3. Calculate the Mach number exiting the shock to determine the nature of the flow (subsonic or supersonic) as

$$M_{ds}^2 = \frac{1 + \frac{\gamma - 1}{2} M_{us}^2}{\gamma M_{us}^2 - \frac{\gamma - 1}{2}} \tag{H13}$$

where,

$M_{us} = M_s$

Knowing the nature of the flow, M_{exit} can be calculated from the area relationship in Equation H8. Knowing M_{exit}, the pressure ratio from the shock location to the exit can be calculated by

$$\frac{P_{exit}}{P_{ds}} \left(\frac{1 + \frac{\gamma - 1}{2} M_{exit}^2}{1 + \frac{\gamma - 1}{2} M_{ds}^2} \right)^{\left(\frac{-\gamma}{\gamma - 1}\right)} \tag{H14}$$

From this point, PR_{obt} can be defined from Equations H11, H12, and H13 as

$$PR_{obt} = \left(\frac{P_{us}}{P^*} \right) \left(\frac{P_{ds}}{P_{us}} \right) \left(\frac{P_{exit}}{P_{ds}} \right) \tag{H15}$$

Equation H14 is valid for all values for M_s less than or equal to 1.1. If $M_s > 1.1$, there will be a loss in total pressure. Calculate total pressure in this situation in the following manner:

$$\frac{P_{tot_c}}{P_{tot_{exit}}} = \left[1 + \frac{2\gamma}{\gamma + 1} (M_s^2 - 1) \right]^{\left(\frac{1}{\gamma - 1}\right)} \left[\frac{(\gamma - 1) M_s^2 + 2}{(\gamma + 1) M_s^2} \right]^{\left(\frac{\gamma}{\gamma + 1}\right)} \tag{H16}$$

Then, Equation H15 becomes

$$PR_{obt} = \left(\frac{P_{us}}{P^*} \right) \left(\frac{P_{ds}}{P_{us}} \right) \left(\frac{P_{exit}}{P_{ds}} \right) \left(\frac{P_{tot_{exit}}}{P_{tot_c}} \right) \tag{H17}$$

The shock will be located at the point where

$$| PR_{corr} - PR_{obt} | \leq \epsilon \tag{H18}$$

If this relationship is not valid, move to the next discrete section downstream and repeat the calculations. If the shock position is not located in this manner before reaching the end of the channel, the shock location is downstream of the blades.

APPENDIX I
LIFT AND MOMENT COEFFICIENT CALCULATION

With pressure perturbations known at the inlet and outlet of each section, the mean section pressure perturbations can be defined as follows:

Section 1

$$\bar{p}'_{1R} = (\bar{p}'_{iR} + \bar{p}^*{}_R{}') \div 2 \qquad \bar{p}'_{1I} = (\bar{p}'_{iI} + \bar{p}^*{}_I{}') \div 2 \tag{I-1}$$

Section 2

$$\bar{p}'_{2R} = (\bar{p}^*{}_R{}' + \bar{p}'_{USR}) \div 2 \qquad \bar{p}'_{2I} = (\bar{p}^*{}_I{}' + \bar{p}'_{USI}) \div 2 \tag{I-2}$$

Section 3

$$\bar{p}'_{3R} = (\bar{p}'_{dsR} + \bar{p}'_{ER}) \div 2 \qquad \bar{p}'_{3I} = (\bar{p}'_{dsI} + \bar{p}'_{EI}) \div 2 \tag{I-3}$$

Equations I-1 through I-3 give the unsteady pressure distribution on the reference airfoil suction surface. The pressure perturbation for the "channel" below the reference channel can be described in the following manner:

$$p_L = p_u \exp(-i\,\sigma) \tag{I-4}$$

where, L denotes the lower airfoil surface and U denotes the upper airfoil surface,

or

$$p_{LR} + ip_{LI} = (p_{UR} + ip_{UI})(\cos\sigma - i\sin\sigma) \tag{I-5}$$

which gives

$$\begin{aligned} p_{LR} &= p_{UR}\cos\sigma + p_{UI}\sin\sigma \\ p_{LI} &= p_{UI}\cos\sigma - p_{UR}\sin\sigma \end{aligned} \tag{I-6}$$

Then, the pressure perturbations for each section can be calculated as follows:

Section 1

$$\begin{aligned} \bar{p}'_{1RL} &= \bar{p}'_{1RU}\cos\sigma + \bar{p}'_{1IU}\sin\sigma \\ \bar{p}'_{1IL} &= \bar{p}'_{1IU}\cos\sigma - \bar{p}'_{1RU}\sin\sigma \end{aligned} \tag{I-7}$$

Section 2

$$\begin{aligned} \bar{p}'_{2RL} &= \bar{p}'_{2RU}\cos\sigma + \bar{p}'_{2IU}\sin\sigma \\ \bar{p}'_{2IL} &= \bar{p}'_{2IU}\cos\sigma - \bar{p}'_{2RU}\sin\sigma \end{aligned} \tag{I-8}$$

Section 3

$$\begin{aligned} \bar{p}'_{3RL} &= \bar{p}'_{3RU}\cos\sigma + \bar{p}'_{3IU}\sin\sigma \\ \bar{p}'_{3IL} &= \bar{p}'_{3IU}\cos\sigma - \bar{p}'_{3RU}\sin\sigma \end{aligned} \tag{I-9}$$

With the unsteady pressures defined on the suction and pressure sides of the airfoil, the unsteady lift and moment coefficients can be defined as follows for each mode. The real part of the unsteady lift coefficient is

$$C_{LR} = C_{LUR} + C_{LLR} \tag{I-10}$$

where,

$$C_{LUR} = -(\bar{p}'_{1UR}\, x^* + \bar{p}'_{2UR}\,(x_s - x^*) + \bar{p}'_{3UR}\,(c - x_s)) \div b \tag{I-11}$$

$$C_{LLR} = +(\bar{p}'_{1LR}\,(x^* - \delta \cos \alpha_{ch}) + \bar{p}'_{2LR}\,(x_s - x^*) + \bar{p}'_{3LR}\,(c - x_s + \delta \cos \alpha_{ch})) \div b \tag{I-12}$$

The imaginary part of the unsteady lift coefficient is

$$C_{LI} = C_{LUI} + C_{LLI} \tag{I-13}$$

where, C_{LUI} and C_{LLI} are defined in the same manner as in Equations I-11 and I-12 with the imaginary pressure perturbations used. The real part of the moment coefficient can be defined for each mode as

$$C_{MR} = C_{MUR} + C_{MLR} \tag{I-14}$$

where,

$$C_{MUR} = -\left[\bar{p}'_{1UR}\, x^* \left(Zc - \frac{x^*}{2}\right) + \bar{p}'_{2UR}\,(x_s + x^*)\left(Zc - \frac{(x^* + x_s)}{2}\right) + \bar{p}'_{3UR}\,(c - x_s)\left(Zc - \frac{(c + x_s)}{2}\right)\right] \div b^2 \tag{I-15}$$

$$C_{MLR} = +\left[\bar{p}'_{1LR}\,(x^* - \delta \cos \alpha_{ch})\left(Zc\left(\frac{x^* - \delta \cos \alpha_{ch}}{2}\right)\right) + \bar{p}'_{2LR}\,(x_s - x^*)\,(Zc - \tfrac{1}{2}\,(x_s + x^* - 2\delta \cos \alpha_{ch})) + \bar{p}_{3LR}\,(c - x_s + \delta \cos \alpha_{ch})\,(Zc - \tfrac{1}{2}\,(c + x_s - \delta \cos \alpha_{ch}))\right] \div b^2 \tag{I-16}$$

The imaginary part of the moment coefficient for each mode is

$$C_{MI} = C_{MUI} + C_{MLI} \tag{I-17}$$

where, C_{MUI} and C_{MLI} are defined as in Equations I-15 and I-16 with the imaginary pressure perturbations used.

APPENDIX J
COMPUTER CODE COMPILATION

```
*VERSION 1.3.0 (01 MAY 80)          SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)     DATE 80.353/15.02.47       PAGE   1
REQUESTED OPTIONS: EBCDIC,MAP,NOLIST,NODECK,XREF,OPT=2
OPTIONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(60) SIZE(MAX) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT XREF NOALC NOANSF TERM IBM FLAG(I)

                C          DATA SET 9066MAIN    AT LEVEL 001 AS OF 07/30/80
                C          DATA SET 9066MAIN    AT LEVEL 001 AS OF 04/21/80
                C  DECK 9066--REVISED VERSION OF DECK 8003
                C              NOW TO BE USED AS THE CHANNEL FLOW CHOKE FLUTTER
                C              UNSTEADY AERODYNAMIC MODEL
                C
                C   ANALYST   JEFFREY F. SIMPSON
                C   DATE      JANUARY, 1980
                C   EXT       4315
                C   MAIL LOC  R-47, BLDG 32, PEG
                C
                C  *  ALPBAR - MEAN TORSIONAL DEFLECTION THRU CYCLE (DEG.)
                C  *  ALPCH  - STAGGER OF BLADE ROW (DEG.)
                C  *  BETA1  - INLET AIR ANGLE  (DEG.)
                C  *  BETA2  - EXIT AIR ANGLE  (DEG.)
                C  *  C      - CHORD (IN.)
                C  *  DELTA  - GAP BETWEEN BLADES (IN.)
                C  *  E      - NONDIMENSIONAL ELASTIC AXIS LOCATION REF. TO MIDSPAN
                C  *  EPS    - TOLERANCE FOR PRESSURE RATIO
                C  *  GAM    - SPECIFIC HEAT RATIO
                C  *  HBAR   - MEAN FLAPPING DEFLECTION OF BLADE THRU THE CYCLE (IN.)
                C  *  MI     - INLET MACH NUMBER
                C  *  M1     - L.E. MACH NUMBER
                C  *  NP     - NUMBER OF AIRFOIL COORDINATES
                C  *  NSECT  - NUMBER OF SEGMENTS FOR CHANNEL AREA, MACH NUMBER, ETC.
                C  *  NTIME  - NUMBER OF TIME INCREMENTS
                C  *  OMEGA  - FREQUENCY OF VIBRATION  (RAD. / SEC.)
                C  *  PR     - STATIC PRESSURE RATIO ACROSS STAGE
                C  *  PT     - TOTAL PRESSURE ENTERING STAGE (PSI)
                C  *  RGAS   - GAS CONSTANT (FT-LBF / DEG. R-LB(MOL) )
                C  *  TOLMIT - TOLERANCE FOR MACH NUMBER ITERATION
                C  *  TT     - TOTAL TEMPERATURE ENTERING STAGE (DEG. R)
                C  *  XIN    - ARRAY OF AIRFOIL X COORDINATES (IN.)
                C  *  YLIN   - ARRAY OF AIRFOIL Y LOWER COORDINATES (IN.)
                C  *  YUIN   - ARRAY OF AIRFOIL Y UPPER COORDINATES (IN )
                C
  ISN 0002            COMMON  VOL1,     VOL2,     VOL3,     AE,       ASTAR,    E,
                     ?        AC,       AO,       MO,       MOS,      MOI,      TT,
                     ?        XS,       XSTAR,    P1SS,     P2SS,     P3SS,     C,
                     ?        MI,       PT,       BETA1,    BETA2,
                     ?        ALPCH,    PHIIEB,   PHIIBT,   HBAR,     DELTA,
                     ?        OMEGA,    ALPBAR,   V
                C
  ISN 0003            REAL    LAMBDA,   LUSS,     LLSS,     MI,       M1,       ME,
                     ?        MO,       MSS,      MOS,      MOI,      MC,       MOSS,
                     ?        LSS,      MUSS,     MLSS
                C
  ISN 0004            DIMENSION ASS(51), MSS(51), TITLE(20), XIN(50), YLIN(50),
                     ?          YUIN(50),DMACH(25), X1P(51), XMACH(25),YMACH(25)
  ISN 0005            F1OF(XM) = 1. + .5 * (GAM - 1.) * XM * XM
                C
                C     READ INPUT
                C
  ISN 0006            RGAS = 1716.26
  ISN 0007         10 READ(5,80,END=500) TITLE
```

```
  ISN 0008        80 FORMAT(20A4)
  ISN 0009           READ(5,85) NP,NB,IWAVE,NSECT
  ISN 0010        85 FORMAT(4I5)
  ISN 0011           READ(5,86) ALPCH,BETA1,BETA2,C,DELTA,E,EPS,GAM,MI,M1,OMEGA,PR,PT,
                    ?               TT,V,DIAM,TOLMIT,ALPBAR,HBAR
  ISN 0012        86 FORMAT(8F10.0)
  ISN 0013           READ(5,86) (XIN(I),I=1,NP)
  ISN 0014           READ(5,86) (YLIN(I),I=1,NP)
  ISN 0015           READ(5,86) (YUIN(I),I=1,NP)
  ISN 0016           IF (EPS    .EQ. 0.) EPS    = .05
  ISN 0018           IF (GAM    .EQ. 0.) GAM    = 1.4
  ISN 0020           IF (M1     .EQ. 0.) M1     = MI
  ISN 0022           IF (RGAS   .EQ. 0.) RGAS   = 1716.26
  ISN 0024           IF (TOLMIT .EQ. 0.) TOLMIT = .00001
  ISN 0026           TAU = DELTA
  ISN 0027           PHIIBB = 6.2832 * DIAM/NB
  ISN 0028           IF(IWAVE.LT.0) PHIIBB = -1.0 * PHIIBB
  ISN 0030           PHIIBT = PHIIBB
                C
                C    PRINT INPUT
                C
  ISN 0031           WRITE(6,5) TITLE,
                    ?            ALFBAR,ALPCH,BETA1,BETA2,C,DELTA,DIAM,E,EPS,GAM,HBAR,
                    ?            IWAVE,MI,M1,NB,NP,NSECT,NTIME,OMEGA,PHIIBB,PHIIBT,PR,V
  ISN 0032         5 FORMAT('1CHANNEL FLOW CHOKE FLUTTER ANALYSIS DECK 9066'// 20A4//
                    1' ALFBAR',F10.5,5X,'MEAN TORSIONAL DEFLECTION THRU CYCLE (DEG.)'/
                    2' ALPCH ',F10.3,5X,'STAGGER ANGLE OF BLADE ROW (DEG.)'/
                    3' BETA1 ',F10.3,5X,'INLET AIR ANGLE (DEG.)'/
                    4' BETA2 ',F10.3,5X,'EXIT AIR ANGLE (DEG.)'/
                    5' C     ',F10.5,5X,'CHORD (IN.)'/
                    6' DELTA ',F10.5,5X,'GAP BETWEEN BLADES (IN.)'/
                    7' DIAM  ',F10.3,5X,'NODAL DIAMETER'/
                    8' E     ',F10.6,5X,'ELASTIC AXIS LOCATION REF. TO MIDSPAN'/
                    9' EPS   ',F10.5,5X,'TOLERANCE FOR PRESSURE RATIO'/
                    X' GAM   ',F10.5,5X,'SPECIFIC HEAT RATIO'/
                    1' HBAR  ',F10.6,5X,'MEAN FLAPPING DEFLECTION OF BLADE (IN.)'/
                    2' IWAVE ',I10  ,5X,'WAVE MOTION'/
                    3' MI    ',F10.5,5X,'INLET MACH NUMBER'/
                    4' M1    ',F10.5,5X,'L. E. MACH NUMBER'/
                    5' NB    ',I10  ,5X,'NUMBER OF BLADES'/
                    6' NP    ',I10  ,5X,'NUMBER OF AIRFOIL COORDINATES'/
                    7' NSECT ',I10  ,5X,'NUMBER OF SEGMENTS FROM L.E. TO T.E.'/
                    8' NTIME ',I10  ,5X,'NUMBER OF TIME INCREMENTS'/
                    9' OMEGA ',F10.2,5X,'FREQUENCY OF VIBRATION (CYCLES. / SEC.)'/
                    X' PHIIBB',F10.4,5X,'BENDING MODE INTERBLADE PHASE ANGLE (RAD.)'/
                    1' PHIIBT',F10.4,5X,'TORSIONAL MODE INTERBLADE PHASE ANGLE (RAD.)'/
                    2' PR    ',F10.5,5X,'STATIC PRESSURE RATIO ACROSS STAGE'/
                    3' V     ',F10.3,5X,'RELATIVE INLET VELOCITY (FT/SEC)')
ERROR DETECTED - SCAN POINTER =    0
  ISN 0033           WRITE (6,6) PT,RGAS,TAU,TOLMIT,TT
  ISN 0034         6 FORMAT(' PT    ',F10.4,5X,'TOTAL PRESSURE ENTERING STAGE (PSI)'/
                    1' RGAS  ',F10.2,5X,'GAS CONSTANT (FT**2 /(SEC**2 DEG. R ) )'/
                    2' TAU   ',F10.5,5X,'GAP BETWEEN AIRFOILS WHILE OSCILLATING (IN.)'/
                    3' TOLMIT',F10.6,5X,'TOLERANCE FOR MACH NUMBER ITERATION'/
                    4' TT    ',F10.2,5X,'TOTAL TEMPERATURE ENTERING STAGE (DEG. R)')
  ISN 0035           WRITE (6,7) (XIN(I),YLIN(I),YUIN(I),I=1,NP)
  ISN 0036         7 FORMAT('0AIRFOIL COORDINATES (IN.)'//10X,'X   Y LOWER   Y UPPER'/
                    1(1X,3F10.5))
                C
```

```
ISN 0037          DOR = 57.2957795100
ISN 0038          ALFDAR = ALFDAR / DOR
ISN 0039          ALPCH = ALPCH / DOR
ISN 0040          BETA1 = BETA1 / DOR
ISN 0041          BETA2 = BETA2 / DOR
            C
            C     DEFINE QUANTITIES USED FREQUENTLY IN CALCULATIONS
            C
ISN 0042          PI = 3.14159265358980D0
ISN 0043          OMEG = OMEGA
ISN 0044          OMEGA = OMEGA * 2. * PI
ISN 0045          NSECT1 = NSECT + 1
ISN 0046          NTIME1 = NTIME + 1
            C     SEMICHORD
ISN 0047          B = .5 * C
ISN 0048          BIME = B * (1. - E)
ISN 0049          BIPE = B * (1. + E)
            C     TOTAL DENSITY
ISN 0050          RHOT = PT / (TT * RGAS * 12.)
ISN 0051          ZP = BIPE / C
            C     CASCADE STAGGER ANGLE
ISN 0052          LAMBDA = .5 * PI - ALPCH
ISN 0053          COSLAM = COS(LAMBDA)
ISN 0054          SINLAM = SIN(LAMBDA)
ISN 0055          TSLAM = TAU * SINLAM
ISN 0056          GOIMG = GAM / (1.- GAM)
ISN 0057          GP1GM1 = (GAM + 1.) / (GAM - 1.)
ISN 0058          TWOGP1 = 2. / (GAM + 1.)
ISN 0059          COSACH = COS(ALPCH)
ISN 0060          SINACH = SIN(ALPCH)
ISN 0061          SINB1 = SIN(BETA1)
ISN 0062          COSB2 = COS(BETA2)
ISN 0063          SINB2 = SIN(BETA2)
            C     EXIT STEADY STATE AREA
ISN 0064          ASSE = TAU * SINB2
            C     X INCREMENT FOR CHANNEL AREA, MACH NUMBER, ETC.
ISN 0065          DX1P = (C - TSLAM) / FLOAT(NSECT)
            C
            C     CALCULATE STEADY STATE AREAS THRU CHANNEL & LOCATE THROAT
            C
ISN 0066          ASTAR = 1.E20
ISN 0067          DO 20 NS = 1, NSECT1
            C     X COORDINATE OF REFERENCE BLADE
ISN 0068          X1P(NS) = FLOAT(NS-1) * DX1P + TSLAM
            C     X COORDINATE OF BLADE ABOVE REFERENCE BLADE
ISN 0069          X2P = X1P(NS) - TSLAM
            C     Y COORDINATE OF UPPER SURFACE OF REFERENCE BLADE
ISN 0070          CALL LINE (NP, XIN, YUIN, X1P(NS), YU, DUM, IDUM)
            C     Y COORDINATE OF LOWER SURFACE OF UPPER BLADE
ISN 0071          CALL LINE (NP, XIN, YLIN, X2P, YL, DUM, IDUM)
            C     ARRAY OF STEADY STATE AREAS THRU CHANNEL ABOVE REFERENCE BLADE
ISN 0072          ASS(NS) = TAU * COSLAM - YU + YL
ISN 0073          WRITE(6,16) NS,ASS(NS)
ISN 0074     16   FORMAT(2X,I2,4X,E15.6)
ISN 0075          IF (ASS(NS) .GT. ASTAR) GO TO 20
            C     THROAT AREA
ISN 0077          ASTAR = ASS(NS)
            C     DISTANCE FROM REFERENCE BLADE L.E. TO THROAT
ISN 0078          XSTAR = X1P(NS)
```

```
      ISN 0079              YUSTAR = YU                                                          00001730
      ISN 0080              YLSTAR = YL                                                          00001740
                      C     SUBSCRIPT OF THROAT LOCATION                                         00001750
      ISN 0081              IT = NS                                                              00001760
      ISN 0082           20 CONTINUE                                                             00001770
                      C                                                                          00001780
                      C     CALCULATE SUPERSONIC MACH NUMBERS FROM THROAT TO EXIT                00001790
                      C                                                                          00001800
      ISN 0083              CALL ZERO (MSS(1), MSS(NSECT1), 1.)                                  00001810
      ISN 0084              DO 30 NS = 1, NSECT1                                                 00001820
      ISN 0085              IF (X1P(NS) .LT. XSTAR) GO TO 30                                     00001830
                      C     RATIO OF AREA TO THROAT AREA                                         00001840
      ISN 0087              AOAMIN = ASS(NS) / ASTAR                                             00001850
      ISN 0088              ISS = 1                                                              00001860
      ISN 0089              CALL MACHIT (AOAMIN, MSS(NS), GAM, TOLMIT, ISS, KILL)                00001870
                      C     IF (KILL .NE. 0) WRITE (6,25) AOAMIN, X1P(NS)                        00001880
      ISN 0090           25 FORMAT('0A/ASTAR = ',1PE13.5,' X = ',E13.5)                          00001890
      ISN 0091           30 CONTINUE                                                             00001900
                      C                                                                          00001910
                      C     TEST FOR CHOKED FLOW AND LOCATE STEADY STATE SHOCK                   00001920
                      C                                                                          00001930
      ISN 0092              BETA1C = ARSIN(SQRT(((TWOSP1 * F1OF(M1))**GP1GM1) / M1**2) / TAU *00001940
                           1          (TAU * COSLAM + YLSTAR - YUSTAR))                          00001950
      ISN 0093              IF (BETA1 .GE. BETA1C)  GO TO 41                                     00001960
      ISN 0095              WRITE (6,40)                                                         00001970
      ISN 0096           40 FORMAT(' ***** FLOW IS NOT CHOKED')                                  00001980
      ISN 0097              ICHOKE = 0                                                           00001990
      ISN 0098              ASTAR = TAU*SINB1*MI*(2./(GAM+1.) * (1.+(GAM-1.)/2. * MI**2)) **    00002000
                           ?         (-(GAM+1.)/(2.*(GAM-1.)))                                   00002010
      ISN 0099              XSTAR = DELTA * COSACH + 0.1                                         00002020
      ISN 0100              IF(C-DELTA*COSACH .GT. 0.1) GOTO 330                                 00002030
      ISN 0102              XSTAR = C - DELTA*COSACH                                             00002040
                      C     WRITE(6,340) XSTAR                                                   00002050
      ISN 0103          340 FORMAT(//' *** XSTAR RESET TO',E15.6)                                00002060
      ISN 0104          330 X0 = XSTAR + 0.15                                                    00002070
      ISN 0105              IF(C-X0 .GT. 0.) GOTO 61                                             00002080
      ISN 0107              X0 = XSTAR + 0.05                                                    00002090
                      C     WRITE(6,360) X0                                                      00002100
      ISN 0108          360 FORMAT(//' *** X0 RESET TO',E15.6)                                   00002110
      ISN 0109              GO TO 61                                                             00002120
                      C     INLET STATIC TO TOTAL PRESSURE RATIO                                 00002130
      ISN 0110           41 AC = TAU * SINB1                                                     00002140
      ISN 0111              AOAMIN = AC / ASTAR                                                  00002150
      ISN 0112              ISS = 0                                                              00002160
      ISN 0113              IF (MI .GT. 1.) ISS = 1                                              00002170
      ISN 0115              CALL MACHIT (AOAMIN, MC, GAM, TOLMIT, ISS, KILL)                     00002180
      ISN 0116              PIPT = F1OF(MC)**GO1MG                                               00002190
      ISN 0117              BETA1  = BETA1C                                                      00002200
      ISN 0118              BETAPR = BETA1 * DCR                                                 00002210
      ISN 0119              WRITE(6,117) BETAPR1                                                 00002220
      ISN 0120          117 FORMAT(//' BETA1'/E15.6)                                             00002230
      ISN 0121              ICHOKE = 1                                                           00002240
                      C     EXIT AREA                                                            00002250
      ISN 0122              AE = DELTA * SINB2                                                   00002260
                      C     RATIO OF EXIT AREA TO THROAT AREA                                    00002270
      ISN 0123              AOAMIN = AE / ASTAR                                                  00002280
      ISN 0124              ISS = 0                                                              00002290
      ISN 0125              IF (AE .LT. ASTAR) ISS = 1                                           00002300
      ISN 0127              CALL MACHIT (AOAMIN, ME, GAM, TOLMIT, ISS, KILL)                     00002310
```

```
                C     IF (KILL .NE. 0) WRITE (6,45) AOAMIN, X1P(NS)
   ISN 0128        45 FORMAT('0AE/ASTAR = ',1PE13.5,' X = ',E13.5)
   ISN 0129           DO 60 NS = IT, NSECT1
                C     SHOCK MACH NUMBER
   ISN 0130           M0 = MSS(NS)
   ISN 0131           IF(M0.GT.0.999 .AND. M0.LT.1.001) M0=1.01
                C     TOTAL INLET TO EXIT PRESSURE RATIO
   ISN 0133           PTIPTE = 1.
   ISN 0134           IF (M0 .GT. 1.1) PTIPTE =
                   1           (1. + GAM * TWOGP1 * (M0**2 - 1.))**(1. / (GAM - 1.)) *
                   2           (2. * F1OF(M0) / ((GAM + 1.) * M0**2))**(-GO1MG)
                C     STATIC PRESSURE RATIO ACROSS STAGE
   ISN 0136           PEPI = PR * PTIPTE
                C     THROAT TO EXIT PRESSURE RATIO
   ISN 0137           PRCORR = PIPT * TWOGP1**GO1MG * PR
                C     SHOCK INLET MACH NUMBER
   ISN 0138           M0I = M0
        .       C     SHOCK EXIT MACH NUMBER
   ISN 0139           M0E = SQRT(F1OF(M0I) / (GAM * M0I**2 - (GAM - 1.) * .5))
                C     SHOCK INLET TO THROAT STATIC PRESSURE RATIO
   ISN 0140        50 POIPTH = F1OF(M0)**GO1MG / ((GAM + 1.) * .5)**GO1MG
                C     SHOCK EXIT TO INLET STATIC PRESSURE RATIO
   ISN 0141           POEPOI = 1. + GAM * TWOGP1 * (M0**2 - 1.)
                C     STAGE EXIT TO SHOCK EXIT STATIC PRESSURE RATIO
   ISN 0142           PEPOE = (F1OF(ME) / F1OF(M0E))**GO1MG / PTIPTE
                C     OBTAINABLE PRESSURE RISE (STAGE EXIT TO THROAT STATIC PRESS. RAT.)
   ISN 0143           PROBT = POIPTH * POEPOI * PEPOE
   ISN 0144           IF (ABS(PRCORR/PROBT-1.).GT. EPS) GO TO 60
                C     DISTANCE FROM REFERENCE BLADE L.E. TO STEADY STATE SHOCK
   ISN 0146           X0 = X1P(NS)
   ISN 0147           XS=X0
   ISN 0148           IF(XSTAR.NE.X0) GOTO 71
   ISN 0150           WRITE(6,75)
   ISN 0151        75 FORMAT(' ','THE SHOCK IS LOCATED AT THE THROAT')
   ISN 0152           GOTO 500
   ISN 0153        71 X2P = X0 - TSLAM
   ISN 0154           CALL LINE (NP, XIN, YUIN, X0, YU0, YU0X0, IDUM)
   ISN 0155           CALL LINE (NP, XIN, YLIN, X2P, YL0, YL0X0, IDUM)
   ISN 0156           E0 = ZP - X0 / C
                C     STEADY STATE SHOCK AREA
   ISN 0157           A0 = ASS(NS)
   ISN 0158           P0 = F1OF(M0)**GO1MG * PT
                C     SUBSCRIPT OF SHOCK LOCATION
   ISN 0159           IS = NS
   ISN 0160           GO TO 65
   ISN 0161        60 CONTINUE
   ISN 0162        61 IS = 0
   ISN 0163           WRITE (6,62)
   ISN 0164        62 FORMAT(' ***** STEADY STATE SHOCK POSITION NOT FOUND')
   ISN 0165           GO TO 500
                C
                C     CALCULATE CAPTURE MACH NUMBER
                C
   ISN 0166        65 AC = TAU * SIN(BETA1C)
                C     RATIO OF CAPTURE AREA TO THROAT AREA
   ISN 0167           AOAMIN = AC / ASTAR
   ISN 0168           ISS = 0
   ISN 0169           IF (MI .GT. 1.) ISS = 1
   ISN 0171           CALL MACHIT (AOAMIN, MC, GAM, TOLMIT, ISS, KILL)
```

```
                C      IF (KILL .NE. 0) WRITE (6,66) AOAMIN
   ISN 0172       66 FORMAT('0AC/ASTAR = ',1PE13.5)
                C
                C      CALCULATE STEADY STATE PRESSURES
                C
   ISN 0173            PCPT = F1OF(MC)**G01MG
   ISN 0174            PC = PCPT * PT
   ISN 0175            IF(ICHOKE.EQ.1)  GOTO 370
   ISN 0177            CALL LINE (NP,XIN,YUIN,XSTAR,YU,DUM,IDUM)
   ISN 0178            CALL LINE (NP,XIN,YLIN,XSTAR-TSLAM,YL,DUM,IDUM)
   ISN 0179            AREA = TAU * COSLAM  - YU + YL
   ISN 0180            AOAMIN = AREA / ASTAR
   ISN 0181            CALL MACHIT (AOAMIN,XMS,GAM,TOLMIT,ISS,KILL)
   ISN 0182            PM1PT = F1OF(XMS) ** G01MG
                C      WRITE(6,201) XSTAR,TSLAM,YU,YL,AREA,AOAMIN,XMS,PM1PT
   ISN 0183      201   FORMAT(/' XSTAR,TSLAM,YU,YL,AREA,AOAMIN,XMS,PM1PT'/8E15.6)
   ISN 0184            GOTO 380
   ISN 0185      370   PM1PT = ((GAM + 1.) * .5)**G01MG
   ISN 0186      380   PM1 = PM1PT * PT
   ISN 0187            PIPT = F1OF(M0)**G01MG
   ISN 0188            PENT = PIPT * PT
   ISN 0189            P0EPI = 1. + GAM * TWOGP1 * (M0**2 - 1.)
                C
   ISN 0190            IF(ICHOKE.EQ.1)  GOTO 375
   ISN 0192            CALL LINE (NP,XIN,YUIN,X0,YU,DUM,IDUM)
   ISN 0193            CALL LINE (NP,XIN,YLIN,X0-TSLAM,YL,DUM,IDUM)
   ISN 0194            AREA = TAU * COSLAM  - YU + YL
   ISN 0195            AOAMIN = AREA / ASTAR
   ISN 0196            CALL MACHIT (AOAMIN,XM0,GAM,TOLMIT,ISS,KILL)
   ISN 0197            P0E = F1OF(XM0) ** G01MG *PT
                C      WRITE(6,202) X0,TSLAM,YU,YL,AREA,AOAMIN,XM0,P0E
   ISN 0198      202   FORMAT(/' X0,TSLAM,YU,YL,AREA,AOAMIN,XM0,P0E'/8E15.6)
   ISN 0199            GOTO 385
   ISN 0200      375   P0E = P0EPI * PENT
   ISN 0201      385   PEPT = F1OF(ME)**G01MG
   ISN 0202            PE = PEPT * PT
   ISN 0203            P1SS = (PM1 + PC) * .5
   ISN 0204            P2SS = (PENT + PM1) * .5
   ISN 0205            IF(ICHOKE.EQ.0) P2SS=(P0E+PM1) * 0.5
   ISN 0207            P3SS = (P0E + PE) * .5
                C
                C      CALCULATE STEADY STATE MOMENT
                C
   ISN 0208            MUSS = -P1SS * XSTAR * (B1PE - XSTAR * .5) - P2SS * (X0 - XSTAR) *
                      1       (B1PE - (XSTAR + (X0 - XSTAR) * .5)) - P3SS * (C - X0) *
                      2       (B1PE - (X0 + (C - X0) * .5))
   ISN 0209            DCACH = DELTA * COSACH
   ISN 0210            MLSS = P1SS * (XSTAR - DCACH) * (B1PE - (XSTAR - DCACH) *
                      1       .5) + P2SS * (X0 - XSTAR) * (B1PE - (XSTAR -
                      2       DCACH + (X0 - XSTAR) * .5)) + P3SS * (C - (X0 -
                      3       DCACH)) * (B1PE - (C + (X0 - DCACH)) *
                      4       .5)
   ISN 0211            MOSS =(MUSS + MLSS)/12.
                C
                C      CALCULATE STEADY STATE LIFT
                C
   ISN 0212            LUSS = -P1SS * XSTAR - P2SS * (X0 - XSTAR) - P3SS * (C - X0)
   ISN 0213            LLSS = P1SS * (XSTAR - DCACH) + P2SS * (X0 - XSTAR) + P3SS * (C -
                      1       (X0 - DCACH))
```

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*VERSION 1.3.0 (01 MAY 80)      MAIN     SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)    DATE 80.353/15.02.47        PAGE    7
   ISN 0214           LSS =(LUSS + LLSS)/12.                                             00003500
   ISN 0215           WRITE (6,100) P1SS,P2SS,P3SS,MOSS,LSS                              00003510
   ISN 0216       100 FORMAT(//' STEADY STATE VALUES'//11X,'P1',10X,'P2',10X,'P3',6X,    00003520
                     1'MOMENT',8X,'LIFT'//1X,3F12.4,F12.5,F12.6)                         00003530
                C                                                                        00003540
   ISN 0217           DUM = PROBT/PRCORR                                                 00003550
   ISN 0218           WRITE(6,200) XSTAR,X0,ASTAR,M0,DUM,PTIPTE                          00003560
   ISN 0219       200 FORMAT(  /T5,'XSTAR',T22,'X0',T36,'ASTAR',T52,'M0',T63,            00003570
                     ?             'PROB/PRCORR',T80,'PRLOSS'/6E15.6)                    00003580
   ISN 0220           WRITE(6,600) MSS                                                   00003590
   ISN 0221       600 FORMAT(//' MSS'/(6E15.6))                                          00003600
                C                                                                        00003610
                C  DEFINE STEADY STATE VOLUMES                                           00003620
                C                                                                        00003630
   ISN 0222           AC     =   DELTA*SINACH                                            00003640
   ISN 0223           AE     =   DELTA*SINACH                                            00003650
   ISN 0224           V1A    =   0.0                                                     00003660
   ISN 0225           XENT   =   DELTA*COSACH                                            00003670
   ISN 0226           CALL LINE (NP, XIN, YUIN, XENT, YUENT, DUM, IDUM)                  00003680
   ISN 0227           V1B    =   0.5*DELTA**2*COSACH*SINACH-.5*DELTA*COSACH*YUENT        00003690
   ISN 0228           X2P    =   0                                                       00003700
   ISN 0229           CALL LINE (NP, XIN, YLIN, X2P, YLENT, DUM, IDUM)                   00003710
   ISN 0230           V1C    =   (DELTA*SINACH-(YUSTAR+YUENT)/2.0+(YLSTAR+YLENT)/2.0)*   00003720
                     ?           (XSTAR-XENT)                                            00003730
   ISN 0231           VOL1   =   V1A+V1B+V1C                                             00003740
   ISN 0232           VOL2   =   (DELTA*SINACH-(YU0+YUSTAR)/2.0+(YL0+YLSTAR)/2.0)*       00003750
                     ?           (X0-XSTAR)                                              00003760
   ISN 0233           YUE    =   YUIN(NP)                                                00003770
   ISN 0234           V3A    =   (DELTA*SINACH-(YUE+YU0)/2.0+(YL0+YL)/2.0)*              00003780
                     ?           (C-X0)                                                  00003790
   ISN 0235           XCD    =   C-DELTA*COSACH                                          00003800
   ISN 0236           CALL LINE (NP, XIN, YLIN ,XCD, YLW, DUM, IDUM)                     00003810
   ISN 0237           V3B    =   0.5*DELTA**2*SINACH*COSACH-.5*DELTA*COSACH*YLW          00003820
   ISN 0238           V3C    =   0.0                                                     00003830
   ISN 0239           VOL3   =   V3A+V3B+V3C                                             00003840
   ISN 0240           WRITE(6,605)                                                       00003850
   ISN 0241       605 FORMAT(' ',T3,'V1A',T15,'V1B',T27,'V1C',T39,'YUENT',T51,'YUSTAR',  00003860
                     ?            T63,'YLSTAR',T75,'YLENT')                              00003870
   ISN 0242           WRITE(6,610) V1A,V1B,V1C,YUENT,YUSTAR,YLSTAR,YLENT                 00003880
   ISN 0243       610 FORMAT(' ',8(E11.4,1X))                                            00003890
                C                                                                        00003900
   ISN 0244           CALL UNST                                                          00003910
   ISN 0245       500 STOP                                                               00003920
   ISN 0246           END                                                                00003930
```

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                 *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
B       0047  0043  0049
C       0002  0011  0031  0047  0051  0065  0100  0102  0105  0156  0208  0208  0210  0210  0212  0213  0234  0235
E       0002  0011  0031  0048  0049
I       0013  0013  0013  0014  0014  0014  0015  0015  0015  0035  0035  0035  0035  0035
V       0002  0011  0031
AC      0002  0110  0111  0166  0167  0222
AE      0002  0122  0123  0125  0223
A0      0002  0157
E0      0156
IS      0159  0162
IT      0081  0129
MC      0003  0115  0116  0171  0173
```

```
                       *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
ME      0003  0127  0142  0201
MI      0002  0003  0011  0020  0031  0098  0098  0113  0169
MO      0002  0003  0130  0131  0131  0131  0134  0134  0134  0134  0138  0140  0141  0158  0187  0189  0218
M1      0003  0011  0020  0020  0031  0092  0092
NB      0009  0027  0031
NP      0009  0013  0014  0015  0031  0035  0070  0071  0154  0155  0177  0178  0192  0193  0226  0229  0233  0236
NS      0067  0068  0068  0069  0070  0072  0073  0073  0075  0077  0078  0081  0084  0085  0087  0089  0129  0130  0146
        0157  0159
PC      0174  0203
PE      0202  0207
PI      0042  0044  0052
PR      0011  0031  0136  0137
PT      0002  0011  0033  0050  0158  0174  0186  0188  0197  0202
PO      0158
TT      0002  0011  0033  0050
XS      0002  0147
XO      0104  0105  0107  0146  0147  0148  0153  0154  0156  0192  0193  0208  0208  0208  0208  0208  0210  0210  0210
        0210  0212  0212  0213  0213  0218  0232  0234
YL      0071  0072  0080  0178  0179  0193  0194  0234
YU      0070  0072  0079  0177  0179  0192  0194
ZP      0051  0156
ABS     0144
ASS     0004  0072  0073  0075  0077  0087  0157
COS     0053  0059  0062
DCR     0037  0038  0039  0040  0041  0113
DUM     0070  0071  0177  0178  0192  0193  0217  0218  0226  0229  0236
EPS     0011  0016  0016  0031  0144
GAM     0005  0011  0018  0018  0031  0056  0056  0057  0057  0058  0069  0098  0098  0098  0098  0115  0127  0134  0134
        0134  0139  0139  0140  0141  0171  0181  0185  0189  0196
ISS     0003  0009  0112  0113  0115  0124  0125  0127  0168  0169  0171  0181  0196
LSS     0003  0214  0215
MSS     0003  0004  0083  0083  0089  0130  0220
MOE     0002  0003  0139  0142
MOI     0002  0003  0138  0139  0139
PM1     0186  0203  0204  0205
POE     0197  0200  0205  0207
SIN     0054  0060  0061  0063  0166
TAU     0026  0033  0055  0064  0072  0092  0092  0098  0110  0166  0179  0194
V1A     0224  0231  0242
V1B     0227  0231  0242
V1C     0230  0231  0242
V3A     0234  0239
V3B     0237  0239
V3C     0233  0239
XCD     0235  0236
XIN     0004  0013  0035  0070  0071  0154  0155  0177  0178  0192  0193  0226  0229  0236
XMS     0181  0182
XMO     0196  0197
X1P     0004  0068  0069  0070  0078  0085  0146
X2P     0069  0071  0153  0155  0228  0229
YLW     0236  0237
YLO     0155  0232  0234
YUE     0233  0234
YUO     0154  0232  0234
AREA    0179  0180  0194  0195
ASSE    0064
```

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*VERSION 1.3.0 (01 MAY 80)      MAIN    SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)    DATE 80.353/15.02.47       PAGE   9

                              *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
BINE    0048
BIFE    0049  0051  0208  0208  0208  0210  0210  0210
DIAM    0011  0027  0031
DX1P    0065  0068
FIOF    0005  0092  0116  0134  0139  0140  0142  0142  0158  0173  0182  0187  0197  0201
HBAR    0002  0011  0031
IDUM    0070  0071  0154  0155  0177  0178  0192  0193  0226  0229  0236
KILL    0009  0115  0127  0171  0181  0196
LINE    0070  0071  0154  0155  0177  0178  0192  0193  0226  0229  0236
LLSS    0003  0213  0214
LUSS    0003  0212  0214
MLSS    0003  0210  0211
NOBS    0003  0211  0215
NUSS    0003  0208  0211
CNEG    0043
FCPT    0173  0174
FENT    0183  0200  0204
FEPI    0136
FEPT    0201  0202
PIPT    0116  0137  0187  0188
P1SS    0002  0203  0208  0210  0212  0213  0215
P2SS    0002  0204  0205  0208  0210  0212  0213  0215
P3SS    0002  0207  0208  0210  0212  0213  0215
RGAS    0006  0022  0022  0033  0050
RHOT    0050
STRT    0092  0139
UNST    0244
VOL1    0002  0231
VOL2    0002  0232
VOL3    0002  0239
XENT    0225  0226  0230
YLIN    0004  0014  0035  0071  0155  0178  0193  0229  0236
YUIN    0004  0015  0035  0070  0154  0177  0192  0226  0233
ZERO    0033
ALFCH   0002  0011  0031  0039  0039  0052  0059  0060
ARSIN   0092
ASTAR   0002  0066  0075  0077  0087  0098  0111  0123  0125  0167  0180  0195  0218
BETA1   0002  0011  0031  0040  0040  0061  0093  0117  0118
BETA2   0002  0011  0031  0041  0041  0062  0063
COSB2   0062
DCACH   0009  0210  0210  0210  0210  0210  0213  0213
DELTA   0002  0011  0026  0031  0099  0100  0102  0122  0209  0222  0223  0225  0227  0227  0230  0232  0234  0235  0237
        0237
DMACH   0004
FLOAT   0035  0068
GOING   0056  0116  0134  0137  0140  0140  0142  0158  0173  0182  0185  0187  0197  0201
INAVE   0009  0028  0031
NSECT   0009  0031  0045  0065
NTIME   0031  0046
OMEGA   0002  0011  0031  0043  0044  0044
PEPOE   0142  0143
PHIPT   0132  0165  0186
PROBT   0143  0144  0217
POEPI   0189  0200
SINB1   0061  0098  0110
SINB2   0063  0064  0122
```

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                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
TITLE   0004  0007  0031
TSLAM   0055  0065  0068  0069  0153  0178  0193
XMACH   0004
XSTAR   0002  0078  0085  0099  0102  0104  0107  0148  0177  0178  0208  0208  0208  0208  0208  0210  0210  0210  0210
        0210  0212  0212  0213  0213  0218  0230  0232
YLENT   0229  0230  0242
YLOXO   0155
YMACH   0004
YUENT   0226  0227  0230  0242
YUOXO   0154
ALFBAR  0002  0011  0031  0038  0038
AOAMIN  0087  0089  0111  0115  0123  0127  0167  0171  0180  0181  0195  0196
BETAFR  0118  0119
BETAIC  0092  0093  0117  0166
COSACH  0059  0099  0100  0102  0209  0225  0227  0227  0235  0237  0237
COSLAM  0053  0072  0092  0179  0194
GP1GM1  0057  0092
ICHCKE  0097  0121  0175  0190  0205
LAMDDA  0003  0052  0053  0054
MACHIT  0089  0115  0127  0171  0181  0196
NSECT1  0045  0067  0083  0084  0129
NTIME1  0046
PHIIDD  0002  0027  0028  0028  0030  0031
PHIIBT  0002  0030  0031
PPCCRR  0137  0144  0217
PTIPTE  0133  0134  0136  0142  0218
POEFOI  0141  0143
POIPTH  0140  0143
SINACH  0060  0222  0223  0227  0230  0232  0234  0237
SINLAM  0054  0055
TOLMIT  0011  0024  0024  0033  0089  0115  0127  0171  0181  0196
TWOSPI  0058  0092  0134  0137  0141  0189
YLSTAR  0080  0092  0230  0232  0242
YUSTAR  0079  0092  0230  0232  0242

                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
 LABEL   DEFINED   REFERENCES
   5     0032      0031
   6     0034      0033
   7     0036      0035
  10     0007
  16     0074      0073
  20     0082      0067  0075
  25     0090
  30     0091      0084  0085
  40     0096      0095
  41     0110      0093
  45     0128
  50     0140
  60     0161      0129  0144
  61     0162      0105  0109
  62     0164      0163
  65     0166      0160
  66     0172
  71     0153      0148
  75     0151      0150
```

*****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

LABEL	DEFINED	REFERENCES
80	0003	0007
85	0010	0009
86	0012	0011 0013 0014 0015
100	0216	0215
117	0120	0119
200	0219	0218
201	0133	
202	0193	
330	0104	0100
340	0103	
360	0108	
370	0185	0175
375	0200	0190
380	0166	0184
385	0201	0199
500	0245	0007 0152 0165
600	0221	0220
605	0241	0240
610	0243	0242

/ MAIN / SIZE OF PROGRAM 00247C HEXADECIMAL BYTES

NAME	TAG		TYPE	ADD.	NAME	TAG		TYPE	ADD.	NAME	TAG		TYPE	ADD.	NAME	TAG		TYPE	ADD.
B	SF		R*4	000B58	C	SF	C	R*4	000044	E	SF	C	R*4	000014	I	F		I*4	000B5C
V	SF	C	R*4	000074	AC	SF	C	R*4	000018	AE	SF	C	R*4	00000C	A0	S	C	R*4	00001C
E0	S		R*4	000B60	IS	S		I*4	000B64	IT	SF		I*4	000B68	MC	SFA		R*4	00CB6C
ME	SFA		R*4	000B70	MI	SF	C	R*4	000048	M0	SFA	C	R*4	000020	M1	SFA		R*4	000B74
MB	SF		I*4	000B78	NP	SFA		I*4	000B7C	NS	SFA		I*4	000B80	PC	SF		R*4	000B84
PE	SF		R*4	000B88	PI	SF		R*4	000B8C	PR	SF		R*4	000B90	PT	SF	C	R*4	00004C
P0	S		R*4	000B94	TT	SF	C	R*4	00002C	XS	S	C	R*4	000030	X0	SFA		R*4	000B98
YL	SFA		R*4	000B9C	YU	SFA		R*4	000BA0	ZP	SF		R*4	000BA4	ASS	SF		R*4	000CF0
COS	F	XF	R*4	000000	DCR	SF		R*4	000BA8	DUM	SFA		R*4	000BAC	EPS	SF		R*4	000BB0
GAM	SFA		R*4	000BB4	ISS	SFA		I*4	000BB8	LSS	SF		R*4	000BBC	MSS	SFA		R*4	000DBC
MOE	SFA	C	R*4	000024	MOI	SFA	C	R*4	000028	PM1	SF		R*4	000BC0	POE	SF		R*4	000BC4
SIN	F	XF	R*4	000000	TAU	SFA		R*4	000BC8	V1A	SF		R*4	000BCC	V1B	SF		R*4	000BD0
V1C	SF		R*4	000BD4	V3A	SF		R*4	000BD8	V3B	SF		R*4	000BDC	V3C	SF		R*4	000BE0
X0D	SFA		R*4	000BE4	XIN	SFA		R*4	000E08	XNS	SFA		R*4	000BE8	XM0	SFA		R*4	000BEC
X1P	SFA		R*4	000F30	X2P	SFA		R*4	000BF0	YLN	SFA		R*4	000BF4	YL0	SFA		R*4	000BF8
YUT	SF		R*4	000BFC	YU0	SFA		R*4	000C00	AREA	SF		R*4	000C04	ASSE	S		R*4	000C08
DIME	S		R*4	000C0C	B1FE	SF		R*4	000C10	DIAM	SF		R*4	000C14	DX1P	SF		R*4	000C18
F10F		AASF	R*4	000000	FLAR	SF	C	R*4	000064	ICUM	SFA		I*4	000C1C	KILL	SFA		I*4	000C20
LINC	SF	XF		000000	LLSS	SF		R*4	000C24	LUSS	SF		R*4	000C28	MLSS	SF		R*4	000C2C
MUSS	SF		R*4	000C30	MUSS	SF		R*4	000C34	OMEG	S		R*4	000C38	FCPT	SF		R*4	000C3C
PINT	SF		R*4	000C40	PEPI	S		R*4	000C44	PEPT	SF		R*4	000C48	PIPT	SF		R*4	000C4C
PISS	SF	C	R*4	000038	POSS	SF	C	R*4	00003C	PSSS	SF	C	R*4	000040	RGAS	SF		R*4	000C50
PHOT	S		R*4	000C54	SQRT	FA	XF	R*4	000000	UNST	SF	XF		000000	VOL1	S	C	R*4	000000
VOL2	S	C	R*4	000004	VOL3	S	C	R*4	000008	XENT	SFA		R*4	000C58	YLIN	SFA		R*4	00101C
YUIN	SFA		R*4	001054	ZERO	SF	XF		000000	ALFCH	SFA	C	R*4	000058	ARSIN	F	XF	R*4	000000
ASTAR	SF	C	R*4	000010	BETA1	SFA	C	R*4	000050	BETA2	SFA	C	R*4	000054	COS32	S		R*4	000C5C
DCACH	SF		R*4	000C60	DELTA	SF	C	R*4	000068	DMACH			R*4	NR	GOING	SF		R*4	000C64
INAVE	SF		I*4	000C68	NSECT	SFA		I*4	000C6C	NTIME	F		I*4	000C70	OMEGA	SF	C	R*4	00006C
PEPOE	SF		R*4	000C74	PM1PT	SF		R*4	000C78	PROBT	SFA		R*4	000C7C	POEPI	SF		R*4	000C80
SING1	SF		R*4	000C84	SING2	SF		R*4	000C88	TITLE	SF		R*4	0011AC	TSLAM	SFA		R*4	000C8C
XMACH			R*4	NR	XSTAR	SFA	C	R*4	000034	YLENT	SFA		R*4	000C90	YLOX0	SFA		R*4	000C94
YMACH			R*4	NR	YUENT	SFA		R*4	000C98	YUOX0	SFA		R*4	000C9C	FRXFR#		XF	R*4	000000
ALPBAR	SF	C	R*4	000070	A0AMIN	SFA		R*4	000CA0	BETAPR	SF		R*4	000CA4	BETA1C	SFA		R*4	000CA8

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*VERSION 1.3.0 (01 MAY 80)      MAIN     SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)     DATE 80.353/15.02.47        PAGE  12
     COSACH SF       R*4   000CAC    COSLAM SFA       R*4   000CB0    GP1GM1 SFA       R*4   000CB4    IBCOM# F  XF    I*4   000000
     ICHOKE S        I*4   000CB3    LAMBDA SFA       R*4   000CBC    MACHIT SF  XF          000000    NSECT1 SFA      I*4   000CC0
     NTIME1 S        I*4   000CC4    PHIIBB SF   C    R*4   00005C    PHIIBT SF   C    R*4   000060    PRCORR SFA      R*4   000CC8
     PTIPTE SF       R*4   000CCC    POEPCI SF        R*4   000CD0    POIPTH SF        R*4   000CD4    SINACH SF       R*4   000CD8
     SINLAM SF       R*4   000CDC    TOLMIT SFA       R*4   000CE0    TWOGP1 SFA       R*4   000CE4    YLSTAR SFA      R*4   000CE8
     YUSTAR SFA      R*4   000CEC

                                   *****  COMMON INFORMATION   *****

          NAME OF COMMON BLOCK  *     *  SIZE OF BLOCK   000078 HEXADECIMAL BYTES

       VAR. NAME  TYPE  REL. ADDR.   VAR. NAME  TYPE  REL. ADDR.   VAR. NAME  TYPE  REL. ADDR.   VAR. NAME  TYPE  REL. ADDR.
            VOL1  R*4   000000            VOL2  R*4   000004            VOL3  R*4   000008              AE  R*4   00000C
           ASTAR  R*4   000010               E  R*4   000014              AC  R*4   000018              A0  R*4   00001C
              M0  R*4   000020             MOS  R*4   000024             MOI  R*4   000028              TT  R*4   00002C
              XS  R*4   000030           XSTAR  R*4   000034            P1SS  R*4   000038            P2SS  R*4   00003C
            P3SS  R*4   000040               C  R*4   000044              MI  R*4   000048              PT  R*4   00004C
           BETA1  R*4   000050           BETA2  R*4   000054           ALPCH  R*4   000058          PHIIBB  R*4   00005C
          PHIIBT  R*4   000060            HBAR  R*4   000064           DELTA  R*4   000068           OMEGA  R*4   00006C
          ALFBAR  R*4   000070               V  R*4   000074

  SOURCE STATEMENT LABELS
    LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR
       10    7  001298 NR          20   82  001CA4             30   91  001924            330  104  001A42
       41  110  001A64             50  140  001CC0 NR          71  153  001D90             60  161  001E0C
       61  162  001E16             65  166  001E38            370  185  001F32            380  186  001F54
      375  200  002036            335  201  002042            500  245  00244C
  COMPILER GENERATED LABELS
    LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR
   100000    1  001284         100001    7  0012AC         200001   17  00142C         100008   17  001436
   100009   18  00143E         100010   19  001448         100011   20  001450         100012   21  00145A
   100013   22  001462         100014   23  00146C         100015   24  001474         100016   25  00147E
   100017   26  001486         100018   29  0014BA         100019   30  0014C4         100020   35  0015FE
   100021   35  00161A         100022   68  0017F4         100023   77  001830         100024   83  0016DC
   100025   85  0018F2         100026   87  0018FE         100027   92  001928         100028   95  001946
   100029  102  001A3E         100030  107  001A54         100031  114  001A08         100032  115  001A8C
   100033  123  001D1E         100034  127  001B22         100035  130  001CDE         200002  132  001D5E
   100036  132  001DF6         100037  133  001DFE         100038  135  001C12         100039  136  001C76
   100040  146  001D52         100041  150  001D76         100043  170  001E66         100044  171  001E6A
   100045  177  001EB6         100046  192  001FB6         100047  206  0020A2         100048  207  0020B2
   200003  217  0021F8         200004  235  00237E
  FORMAT STATEMENT LABELS
    LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR            LABEL  ISN  ADDR
       80    8  000028             85   10  00002E             86   12  000034              5   32  00003B
        6   34  00046F              7   35  000527             16   74  0005CB             25   90  000506 NR
       40   96  0005F4            340  103  000611 NR         360  108  00062D NR         117  120  000646
       45  123  000656 NR          75  151  000675             62  164  00069E             66  172  0006CE NR
      201  183  0006E3 NR         202  198  000716 NR         100  216  000744            200  219  000793
      600  221  0007D3            605  241  0007E6            610  243  000826

NUMBER    LEVEL                          FORTRAN H EXTENDED ERROR MESSAGES

IFE029I   8(E)      ISN    0032      THE NUMBER OF CONTINUATION CARDS EXCEEDS 19. COMPILER PROCESSING OF THE
                                             STATEMENT CONTINUES.
IFE226I   8(E)      ISN    0119      THE STATEMENT HAS A VARIABLE WITH MORE THAN SIX CHARACTERS.
                                             THE RIGHTMOST CHARACTERS ARE TRUNCATED.
*OPTIONS IN EFFECT*NAME(MAIN) OPTIMIZE(2) LINECOUNT(60) SIZE(MAX) AUTODBL(NONE)
*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT XREF NOALC NOANSF TERM IBM FLAG(I)
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*VERSION 1.3.0 (01 MAY 80)      MAIN     SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)     DATE 80.353/15.02.47        PAGE  13
*STATISTICS*     SOURCE STATEMENTS =      245, PROGRAM SIZE =      9340, SUBPROGRAM NAME =  MAIN
*STATISTICS*    2 DIAGNOSTICS GENERATED, HIGHEST SEVERITY CODE IS  8
****** END OF COMPILATION ******                                          508K BYTES OF CORE NOT USED
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*VERSION 1.3.0 (01 MAY 80)          SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)     DATE 80.353/15.02.49        PAGE   1
REQUESTED OPTIONS: EBCDIC,MAP,NOLIST,NODECK,XREF,OPT=2
OPTIONS IN EFFECT: NAME(MAIN) OPTIMIZE(2) LINECOUNT(60) SIZE(MAX) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT XREF NOALC NOANSF TERM IBM FLAG(I)

                    C          DATA SET 9066UNST   AT LEVEL 002 AS OF 08/15/80
                    C          DATA SET 9066UNST   AT LEVEL 001 AS OF 07/30/80
                    C          DATA SET 9066UNST   AT LEVEL 001 AS OF 04/21/80
   ISN 0002               SUBROUTINE UNST
                    C
                    C   DECK 9066  CHANNEL FLOW CHOKE FLUTTER UNSTEADY AERODYNAMIC MODEL
                    C
                    C   ANALYST      J. F. SIMPSON
                    C   DATE         JANUARY, 1980
                    C   EXT          4315
                    C   MAIL LOC     R-47, BLDG 32, P5G
                    C
                    C
                    C -------------------------------------------------------------------
                    C ----------------------------DESCRIPTIONS---------------------------
                    C -------------------------------------------------------------------
                    C   B          SEMICHORD OF THE AIRFOIL
                    C   DI         DENSITY AT THE INLET
                    C   DD         DENSITY DOWNSTREAM
                    C   DTOT       TOTAL DENSITY
                    C   DU         DENSITY UPSTREAM
                    C   E          ELASTIC AXIS POSITION REFERENCED TO MIDCHORD
                    C   FLOW       S.S FLOW RATE ENTERING THE SHOCK
                    C   FRE        S.S FLOW RATE AT THE EXIT
                    C   GAMMA      SPECIFIC HEAT RATIO
                    C   MEXIT      MACH NUMBER AT THE EXIT
                    C   MDS        MACH NUMBER DOWNSTREAM
                    C   MI         MACH NUMBER AT THE INLET
                    C   MSHOCK     MACH NUMBER AT THE SHOCK
                    C   MT         MACH NUMBER AT THE THROAT
                    C   MUS        MACH NUMBER UPSTREAM
                    C   OMEGA      FREQUENCY OF VIBRATIONS
                    C   PE         PRESSURE AT THE EXIT
                    C   PD         PRESSURE DOWNSTREAM
                    C   PIN        PRESSURE AT THE INLET
                    C   PTOT       TOTAL PRESSURE
                    C   PU         PRESSURE UPSTREAM
                    C   PGAS       GAS CONSTANT
                    C   SIGMAB     BENDING MODE INTERBLADE PHASE ANGLE
                    C   SIGMAT     TORSIONAL MODE INTERBLADE PHASE ANGLE
                    C   SOSDS      SPEED OF SOUND DOWNSTREAM
                    C   SOSEX      SPEED OF SOUND AT THE EXIT
                    C   SSI        SPEED OF SOUND AT THE INLET
                    C   SST        SPEED OF SOUND AT THE THROAT
                    C   SSU        SPEED OF SOUND UPSTREAM
                    C   TDS        TEMPERATURE DOWNSTREAM
                    C   TE         TEMPERATURE AT THE EXIT
                    C   TI         TEMPERATURE AT THE INLET
                    C   UE         AXIAL VELOCITY AT THE EXIT
                    C   VE         TANGENTIAL VELOCITY AT THE EXIT
                    C   VU         VELOCITY UPSTREAM
                    C
                    C
                    C
   ISN 0003               DIMENSION FE(35),  FM(47),  FC(57),  FM2(18),  M(28),
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                     ?          CL(39),  S(9),    LM(24),  LC(28),   AAR(2),
                     ?          AAI(2),  BAR(2),  BAI(2),  AHR(2),   AHI(2),
                     ?          BHR(2),  BHI(2)
                C
  ISN 0004           REAL          K,       IC,      LC,      LM,      MU,      MD,
                     ?             MDS,     MUS,     MXD,     MXU,     MYD,     MYU,
                     ?             IMAP,    IMDP,    IMPP,    IMSP,    IRFF,    ISSP,
                     ?             IUDS,    IVFD,    MCPT,    M1RE,    M2RE,    IMAPI,
                     ?             IMAPT,   IMDFU,   IMPPU,   IMVPU,   ISOSU,   MEXIT,
                     ?             MSHOCK,  MT,      MI,      M,       I,       IACE,
                     ?             IVPU,    IDDS,    IFDS,    MXSTAR,
                     ?             MYU,     MYD,     IMPPI,   IMDPI,   IMSSPI,  IITVPI,
                     ?             IIAVPI,  IMPPE,   IMDPE,   IMSSPE,  IIAVPE,  IITVPE,
                     ?             IRAVPE,  IRTVPE,  IP1U,    IP2U,    IP3U,    IP1L,
                     ?             IP2L,    IP3L,    ICLU,    ICLL,    ICL,     ICMU,
                     ?             ICML,    ICM
                C
  ISN 0005           COMMON        VOL1,    VOL2,    VOL3,    ACE,     ASTAR,   E,
                     ?             AC,      AO,      MSHOCK,  MDS,     MUS,     TTOT,
                     ?             XS,      XSTAR,   PRES1,   PRES2,   PRES3,   C,
                     ?             M1RE,    PTOT,    BETA1,   BETA2,
                     ?             ALPCH,   SIGMAB,  SIGMAT,  H,       DELTA,
                     ?             OMEGA,   ALPBAR,  U1IRE
                C
  ISN 0006           READ(5,1000) M2RE
  ISN 0007           MEXIT  =  M2RE*COS(ALPCH - BETA2)
  ISN 0008        1000 FORMAT(F10.0)
                C
                C
  ISN 0009           B      =    C/2.0
                C
                C  UNDIMENSIONALIZE TAU
  ISN 0010           TAU    =    DELTA
  ISN 0011           TAU    =    TAU/B
                C
                C  DEFINE QUANTITIES USED FREQUENTLY IN CALCULATUIONS
                C
  ISN 0012           PI     =    3.14159265358980D0
  ISN 0013           MT     =    1.0
  ISN 0014           MI     =    M1RE
  ISN 0015           GAMMA  =    1.4
  ISN 0016           MXSTAR =    1.0
  ISN 0017           Z      =    B*(1+E)/C
  ISN 0018           RGAS   =    1716.26
  ISN 0019           DTOT   =    PTOT/(RGAS*TTOT)*144.0
  ISN 0020           I      =    COS(ALPCH-BETA1)
  ISN 0021           K      =    C/2*OMEGA/U1IRE/12.0
  ISN 0022           WRITE(6,95) K
  ISN 0023        95 FORMAT(//'  K'/E15.6)
  ISN 0024           PU     =    PTOT*(1+(GAMMA-1)/2*MSHOCK**2)**(-GAMMA/(GAMMA-1))
  ISN 0025           PT     =    PTOT*(1+(GAMMA-1)/2*MT**2)**(-GAMMA/(GAMMA-1))
  ISN 0026           PIN    =    PTOT*(1+(GAMMA-1)/2*MI**2)**(-GAMMA/(GAMMA-1))
  ISN 0027           TIN    =    TTOT/(1+(GAMMA-1)/2*MI**2)
  ISN 0028           DI     =    PIN/(RGAS*TIN)*144
  ISN 0029           PD     =    PU*(1+2*GAMMA/(GAMMA+1)*(MUS**2-1))
  ISN 0030           TDS    =    TTOT/(1+(GAMMA-1)/2*MDS**2)
  ISN 0031           DD     =    PD/(RGAS*TDS)*144
  ISN 0032           DU     =    DTOT*(1+(GAMMA-1)/2*MSHOCK**2)**(-1/(GAMMA-1))
                C
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                  C
  ISN 0033              CO     =   SIGMAB/TAU
  ISN 0034              MD     =   M2RE*COS(ALPCH-BETA2)
  ISN 0035              MU     =   M1RE*COS(ALPCH-BETA1)
  ISN 0036              MXD    =   M2RE*COS(ALPCH-BETA2)*SIN(ALPCH)
  ISN 0037              MXU    =   M1RE*COS(ALPCH-BETA1)*SIN(ALPCH)
  ISN 0038              MYD    =   M2RE*COS(ALPCH-BETA2)*COS(ALPCH)
  ISN 0039              MYU    =   M1RE*COS(ALPCH-BETA2)*COS(ALPCH)
  ISN 0040              BXU    =   1-MXU**2
  ISN 0041              BXD    =   1-MXD**2
  ISN 0042              BYU    =   1-MYU**2
  ISN 0043              BYD    =   1-MYD**2
  ISN 0044              D1U    =   MXU*MU*K+MXU*MYU*CO
  ISN 0045              D1D    =   MXD*MD*K+MXD*MYD*CO
  ISN 0046              D2U    =   K**2*MU**2+2*MU*MYU*CO*K-CO**2*BYU
  ISN 0047              D2D    =   K**2*MD**2+2*MD*MYD*CO*K-CO**2*BYD
                  C
  ISN 0048              QUANT  =   D1U**2+BXU*D2U
  ISN 0049              IF(QUANT.GT.0) GOTO 80
  ISN 0051              WRITE(6,71)
  ISN 0052           71 FORMAT (' ','UPSTREAM SOLUTION NUMBER 1')
  ISN 0053              B1I    =   -SQRT(-(D1U**2+BXU*D2U))/BXU
  ISN 0054              B1R    =   D1U/BXU
  ISN 0055              GOTO 90
  ISN 0056           80 WRITE(6,75)
  ISN 0057           75 FORMAT(' ','UPSTREAM SOLUTION NUMBER 2')
  ISN 0058              QUANT  =   MU*K+MYU*CO
  ISN 0059              IF(QUANT.GT.0) GOTO 84
  ISN 0061              B1R    =   (D1U-SQRT(D1U**2+BXU*D2U))/BXU
  ISN 0062              B1I    =   (MYU*CO+MU*K)/MXU
  ISN 0063              GOTO 90
  ISN 0064           84 B1R    =   (D1U+SQRT(D1U**2+BXU*D2U))/BXU
  ISN 0065              B1I    =   -(MYU*CO+MU*K)/MXU
                  C
  ISN 0066           90 QUANT  =   D1D**2+BXD*D2D
  ISN 0067              IF(QUANT.GT.0) GOTO 91
  ISN 0069              WRITE(6,76)
  ISN 0070           76 FORMAT(' ','DOWNSTREAM SOLUTION NUMBER 1')
  ISN 0071              B2I    =   SQRT(-(D1D**2+BXD*D2D))/BXD
  ISN 0072              B2R    =   D1D/BXD
  ISN 0073              GOTO 99
  ISN 0074           91 QUANT  =   MD*K+MYD*CO
  ISN 0075              WRITE(6,78)
  ISN 0076           78 FORMAT(' ','DOWNSTREAM SOLUTION NUMBER 2')
  ISN 0077              IF(QUANT.GT.0) GOTO 94
  ISN 0079              B2R    =   (D1D+SQRT(D1D**2+BXD*D2D))/BXD
  ISN 0080              B2I    =   -(MYD*CO+MD*K)/MXD
  ISN 0081              GOTO 99
  ISN 0082           94 B2R    =   (D1D-SQRT(D1D**2+BXD*D2D))/BXD
  ISN 0083              B2I    =   (MYD*CO+MD*K)/MXD
                  C
  ISN 0084           99 K      =   K * 12
  ISN 0085              PE     =   PTOT*(1+(GAMMA-1)/2*MEXIT**2)**(-GAMMA/(GAMMA-1))
  ISN 0086              TE     =   TTOT/(1+(GAMMA-1)/2*MEXIT**2)
  ISN 0087              DE     =   PE/(RGAS*TE)*144
  ISN 0088              SOSEX  =   SQRT(GAMMA*RGAS*TTOT/(1+(GAMMA-1)/2*MEXIT**2))
  ISN 0089              REV    =   M2RE * SOSEX
  ISN 0090              VE     =   REV*COS(ALPCH-BETA2)*COS(ALPCH)
  ISN 0091              UE     =   REV*COS(ALPCH-BETA2)*SIN(ALPCH)
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ISN 0092            U2IRE  =  REV
ISN 0093            US     =  0.0
ISN 0094            SSI    =  SQRT(GAMMA*RGAS*TTOT/(1+(GAMMA-1)/2*MI**2))
ISN 0095            SST    =  SQRT(GAMMA*RGAS*TTOT/(1+(GAMMA-1)/2*MXSTAR**2))
ISN 0096            SOSDS  =  SQRT(GAMMA*RGAS*TTOT/(1+(GAMMA-1)/2*MDS**2))
ISN 0097            SSU    =  SQRT(GAMMA*RGAS*TTOT/(1+(GAMMA-1)/2*MUS**2))
ISN 0098            VU     =  MUS*SSU
ISN 0099            VD     =  MDS*SOSDS
ISN 0100            WDS    =  VD*AO*DD/(DI*U1IRE*ASTAR)
ISN 0101            AVGSOS =  (SSI+SST)/2.0
ISN 0102            DT     =  DTOT*(1+(GAMMA-1)/2*MXSTAR**2)**(-1/(GAMMA-1))
ISN 0103            AVGD1  =  (DI+DT)/2.0
ISN 0104            AVGD2  =  (DT+DU)/2.0
ISN 0105            AVGD3  =  (DD+DE)/2.0
ISN 0106            AVGV1  =  (U1IRE*COS(ALPCH-BETA1)+SST)/2.0
ISN 0107            AVGV2  =  (SST+VU)/2.0
ISN 0108            AVGV3  =  (VD+REV*COS(ALPCH-BETA2))/2.0
ISN 0109            REV    =  REV/U1IRE
ISN 0110            VELAX  =  U1IRE*COS(ALPCH-BETA1)*SIN(ALPCH)
ISN 0111            VEL    =  U1IRE*COS(ALPCH-BETA1)*COS(ALPCH)
              C
ISN 0112            WRITE(6,100)
ISN 0113        100 FORMAT('1',T37,'INLET',T51,'THROAT',T66,'UPSTREAM',T81,
                ?          'DOWNSTEAM',T96,'EXIT')
ISN 0114            WRITE(6,105) MI,MT,MUS,MDS,MEXIT
ISN 0115            WRITE(6,115) PIN,PT,PU,PD,PE
ISN 0116            WRITE(6,110) DI,DT,DU,DD,DE
ISN 0117            WRITE(6,120) SSI,SST,SSU,SOSDS,SOSEX
ISN 0118        105 FORMAT('0','MACH NUMBER',T32,G14.7,T47,F14.7,T62,F14.7,T77,F14.7,
                ?          T92,F14.7)
ISN 0119        110 FORMAT('0','DENSITY',T32,F14.7,T47,F14.7,T62,F14.7,T77,F14.7,
                ?          T92,F14.7)
ISN 0120        115 FORMAT('0','PRESSURE',T32,F14.7,T50,F14.7,T62,F14.7,T77,F14.7,
                ?          T92,F14.7)
ISN 0121        120 FORMAT('0','SPEED OF SOUND',T32,F14.7,T47,F14.7,T62,F14.7,T77,
                ?          F14.7,T92,F14.7)
              C
              C  NONDIMENSIONALIZE ALL QUANTITIES
              C
              C  DIVIDE ALL THE DENSITIES BY THE INLET DENSITY
              C
ISN 0122            DU     =  DU/DI
ISN 0123            DD     =  DD/DI
ISN 0124            DE     =  DE/DI
ISN 0125            DT     =  DT/DI
ISN 0126            DTOT   =  DTOT/DI
ISN 0127            AVGD1  =  AVGD1/DI
ISN 0128            AVGD2  =  AVGD2/DI
ISN 0129            AVGD3  =  AVGD3/DI
              C
              C  PUT PRESSURE IN P-S-F UNITS AND DIVIDE BY (INLET DENSITY*INLET
              C  VELOCITY)
              C
ISN 0130            PIN    =  PIN*144.0/(DI*U1IRE**2)
ISN 0131            PU     =  PU*144.0/(DI*U1IRE**2)
ISN 0132            PD     =  PD*144.0/(DI*U1IRE**2)
ISN 0133            PE     =  PE*144.0/(DI*U1IRE**2)
ISN 0134            PT     =  PT*144.0/(DI*U1IRE**2)
ISN 0135            PTOT   =  PTOT*144.0/(DI*U1IRE**2)
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ISN 0136          PRES1  =  PRES1*144.0/(DI*U1IRE**2)
ISN 0137          PRES2  =  PRES2*144.0/(DI*U1IRE**2)
ISN 0138          PRES3  =  PRES3*144.0/(DI*U1IRE**2)
            C
            C  DIVIDE ALL VELOCITIES  AND THE SPEED OF SOUNDS BY THE INLET VELOCITY
            C
ISN 0139          SSI    =  SSI/U1IRE
ISN 0140          SST    =  SST/U1IRE
ISN 0141          SOSEX  =  SOSEX/U1IRE
ISN 0142          SOSDS  =  SOSDS/U1IRE
ISN 0143          SSU    =  SSU/U1IRE
ISN 0144          AVGSOS =  AVGSOS/U1IRE
ISN 0145          VE     =  VE/U1IRE
ISN 0146          UE     =  UE/U1IRE
ISN 0147          VU     =  VU/U1IRE
ISN 0148          VD     =  VD/U1IRE
ISN 0149          AVGV1  =  AVGV1/U1IRE
ISN 0150          AVGV2  =  AVGV2/U1IRE
ISN 0151          AVGV3  =  AVGV3/U1IRE
ISN 0152          VELAX  =  VELAX/U1IRE
ISN 0153          VEL    =  VEL/U1IRE
ISN 0154          U2IRE  =  U2IRE/U1IRE
            C
            C  UNDIMENSIONALIZE THE VOLUMES
            C
ISN 0155          VOL1   =   VOL1/(ASTAR*B)
ISN 0156          VOL2   =   VOL2/(ASTAR*B)
ISN 0157          VOL3   =   VOL3/(ASTAR*B)
            C
            C  UNDIMENSIONALIZE THE AREAS
            C
ISN 0158          ACE    =   ACE/ASTAR
ISN 0159          AC     =   AC/ASTAR
ISN 0160          AO     =   AO/ASTAR
            C
            C
            C  INTERBLADE ANALYSIS OF SECTION 1
            C
            C
ISN 0161          FLOW   =  DU*AO*VU
ISN 0162          FRE    =  DE*ACE*U2IRE
ISN 0163          FE(1)  =  SSI**2/(GAMMA*(GAMMA-1))+((COS(ALPCH-BETA1))**2)/2
            C
ISN 0164          FE(2)  =  (GAMMA**2-GAMMA+2)/(2*(GAMMA*(GAMMA-1)))*SST**2
            C
ISN 0165          FE(3)  =  -FE(1)*I*AC*VELAX/(CO*TAU*SSI**2)+FE(1)*AC
                 ?          *SIN(ALPCH)/(CO*TAU)-2*AC*I*VELAX/(2*GAMMA*CO*
                 ?          TAU)+AC*I**2*SIN(ALPCH)/(CO*TAU)
            C
ISN 0166          FE(4)  =  -FE(1)*I*AC*(K+CO*VEL)/(CO*TAU*SST**2)+(FE(1)*AC
                 ?          *COS(ALPCH))/TAU-2*AC*I*(K+CO*VEL)/(2*GAMMA
                 ?          *CO*TAU)+(AC*I**2*COS(ALPCH))/TAU
            C
ISN 0167          FE(5)  =  (-VOL1*AVGSOS**2*VELAX)/(2*GAMMA*(GAMMA-1)*CO*TAU
                 ?          *SSI**2)-AVGSOS*AVGD1*VOL1/GAMMA/(2*SSI*CO*TAU)
                 ?          *VELAX-(VOL1*AVGV1**2*VELAX)/(4*CO*TAU*SSI**2)
                 ?          +(AVGV1*AVGD1*VOL1*SIN(ALPCH))/(2*CO*TAU)
            C
ISN 0168          FE(6)  =  (-VOL1*AVGSOS**2*(K+CO*VEL))/(2*GAMMA*(GAMMA-1)
```

```
                     ?        *CO*TAU*SSI**2)-AVGSOS*AVGD1*VOL1/GAMMA/(2*SSI
                     ?        *CO*TAU)*(K+CO*VEL)+(VOL1*AVGV1**2)/4*(K+CO*VEL)/
                     ?        (CO*TAU*SSI**2)+AVGV1*AVGD1*VOL1*COS(ALPCH)/(2*TAU)
                 C
  ISN 0169                FE(7)  =  COS(CO*TAU)-1
                 C
  ISN 0170                FE(8)  =  FE(3)*(FE(7)*B1R-B1I*SIN(CO*TAU))+FE(4)*FE(7)+FE(5)
                     ?        *K*(B1I*FE(7)+B1R*SIN(CO*TAU))+FE(6)*K*SIN(CO*TAU)
                 C
  ISN 0171                FE(9)  =  -FE(3)*(FE(7)*B1I+B1R*SIN(CO*TAU))-FE(4)*SIN(CO*TAU)+
                     ?        FE(5)*K*(B1R*FE(7)-B1I*SIN(CO*TAU))+FE(6)*K*FE(7)
                 C
  ISN 0172                FE(10) =  FE(3)*(B1I*FE(7)+B1R*SIN(CO*TAU))+FE(4)*SIN(CO*
                     ?        TAU)-FE(5)*(K*B1R*FE(7)-K*B1I*SIN(CO*TAU))
                     ?        -FE(6)*K*FE(7)
                 C
  ISN 0173                FE(11) =  FE(3)*(B1R*FE(7)-B1I*SIN(CO*TAU))+FE(4)*FE(7)
                     ?        +FE(5)*(K*B1I*FE(7)+K*B1R*SIN(CO*TAU))+FE(6)
                     ?        *K*SIN(CO*TAU)
                 C
  ISN 0174                FE(12) =  VOL1 * AVGSOS**2* K/12 / (2*GAMMA*(GAMMA-1)) +
                     ?        VOL1 * AVGV1**2 * K/12 / 4
                 C
  ISN 0175                FE(13) =  K/12 * AVGD1 * AVGSOS**2 / (GAMMA * (GAMMA-1)) +
                     ?        K/12 * AVGD1 * AVGV1**2 / 2
                 C
  ISN 0176                FE(14) =  AVGSOS * AVGD1 * AVGV1 / (GAMMA * (GAMMA - 1)) *
                     ?        K/12 + AVGV1 * AVGD1 * VOL1 * K/12 / 2
  ISN 0177                FE(15) =  FE(9)/FE(8)-FE(11)/FE(10)
                 C
  ISN 0178                FE(16) =  FE(1)/FE(8)*I*(FE(9)/FE(8)/FE(15)-1)
                 C
  ISN 0179                FE(17) =  -FE(8)/FE(9)*FE(1)/FE(10)*I/FE(15)
                 C
  ISN 0180                FE(18) =  FE(2)/FE(8)*(SST-SST/FE(15)*FE(9)/FE(8))-FE(12)/
                     ?        (FE(10)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0181                FE(19) =  -FE(12)/FE(8)+FE(2)*SST/(FE(10)*FE(15))*FE(9)/FE(8)+
                     ?        FE(12)/(FE(8)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0182                FE(20) =  FE(2)/FE(8)*DT*SST-FE(2)*DT*SST/(FE(8)*FE(15))*
                     ?        FE(9)/FE(8)
                 C
  ISN 0183                FE(21) =  FE(2)*DT*SST/(FE(10)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0184                FE(22) =  3*FE(2)/FE(8)*(DT-DT/FE(15)*FE(9)/FE(8))-FE(14)/
                     ?        (FE(10)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0185                FE(23) =  -FE(14)/FE(8)+(FE(2)*3*DT*FE(9)/(FE(10)*FE(15)*FE(8)))+
                     ?        FE(14)*FE(9)/(FE(8)**2*FE(15))
                 C
  ISN 0186                FE(24) =  -FE(13)/FE(8)+FE(13)/(FE(8)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0187                FE(25) =  -FE(13)/(FE(10)*FE(15))*FE(9)/FE(8)
                 C
  ISN 0188                FE(26) =  -FE(1)*I/(FE(8)*FE(15))
                 C
  ISN 0189                FE(27) =  FE(1)/(FE(10)*FE(15))
                 C
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  ISN 0190              FE(28) =   FE(2)*SST/(FE(8)*FE(15))+FE(12)/(FE(10)*FE(15))
                 C
  ISN 0191              FE(29) =   -FE(2)*SST/(FE(10)*FE(15))-FE(12)/(FE(8)*FE(15))
                 C
  ISN 0192              FE(30) =   FE(2)*SST*DT/(FE(8)*FE(15))
                 C
  ISN 0193              FE(31) =   -FE(2)*SST*DT/(FE(10)*FE(15))
                 C
  ISN 0194              FE(32) =   3*DT*FE(2)/(FE(8)*FE(15))+FE(14)/(FE(10)*FE(15))
                 C
  ISN 0195              FE(33) =   -3*DT*FE(2)/(FE(10)*FE(15))-FE(14)/(FE(8)*FE(15))
                 C
  ISN 0196              FE(34) =   -FE(13)/(FE(8)*FE(15))
                 C
  ISN 0197              FE(35) =   FE(13)/(FE(10)*FE(15))
                 C
                 C
  ISN 0198              FM(1)  =   -VELAX*AC/(CO*TAU)-VELAX*AC*I**2/(CO*TAU*SSI**2)+
                      ?            2*AC*SIN(ALPCH)*I/(CO*TAU)
                 C
  ISN 0199              FM(2)  =   -(K+CO*VEL)*AC/(CO*TAU)-(K+CO*VEL)*AC*I**2/(CO*TAU*
                      ?            SSI**2)+2*AC*COS(ALPCH)*I/TAU
                 C
  ISN 0200              FM(3)  =   AVGV1*VOL1*VELAX/(2*CO*TAU*SSI**2)-AVGD1*VOL1*
                      ?            SIN(ALPCH)/(2*CO*TAU)
                 C
  ISN 0201              FM(4)  =   (K+CO*VEL)/(CO*TAU*SSI**2)*AVGV1*VOL1/2-AVGD1*VOL1/2*
                      ?            COS(ALPCH)/TAU
                 C
  ISN 0202              FM(5)  =   -PIN-I**2
                 C
  ISN 0203              FM(6)  =   PT+SST**2*DT
                 C
  ISN 0204              FM(7)  =   FM(1)*(B1R*FE(7)-B1I*SIN(CO*TAU))+FM(2)*FE(7)-FM(3)*
                      ?            (K*B1I*FE(7)-K*B1R*SIN(CO*TAU))-FM(4)*K*SIN(CO*TAU)
                 C
  ISN 0205              FM(8)  =   -FM(1)*(B1I*FE(7)+B1R*SIN(CO*TAU))-FM(2)*SIN(CO*TAU)+
                      ?            FM(3)*K*(-B1R*FE(7)+B1I*SIN(CO*TAU))-FM(4)*K*FE(7)
                 C
  ISN 0206              FM(9)  =   FM(1)*(B1I*FE(7)+B1R*SIN(CO*TAU))+FM(2)*SIN(CO*TAU)+
                      ?            FM(3)*K*(B1R*FE(7)-B1I*SIN(CO*TAU))+FM(4)*FE(7)*K
                 C
  ISN 0207              FM(10) =   FM(1)*(B1R*FE(7)-B1I*SIN(CO*TAU))+FM(2)*FE(7)-
                      ?            FM(3)*K*(B1I*FE(7)+B1R*SIN(CO*TAU))-FM(4)*K*
                      ?            SIN(CO*TAU)
                 C
  ISN 0208              FM(11) =   FE(18)*FM(7)+FM(8)*FE(28)-2*SST**2
                 C
  ISN 0209              FM(12) =   FM(7) * FE(19) + FM(8) * FE(29) + AVGV1 *
                      ?            VOL1 * K/12 /2
                 C
  ISN 0210              FM(13) =   -FM(7)*FE(16)-FM(8)*FE(26)+FM(5)
                 C
  ISN 0211              FM(14) =   -FM(7)*FE(17)-FM(8)*FE(27)
                 C
  ISN 0212              FM(15) =   -FM(7)*FE(20)-FM(8)*FE(30)+FM(6)
                 C
  ISN 0213              FM(16) =   -FM(7)*FE(21)-FM(8)*FE(31)
                 C
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    ISN 0214             FM(17) =   -FM(7)*FE(22)-FM(8)*FE(32)+2*SST*DT
                   C
    ISN 0215             FM(18) =   -FM(7) * FE(23) - FM(8) * FE(33) -AVGD1 *
                        ?           VOL1 * K/12 / 2
                   C
    ISN 0216             FM(19) =   -FM(7) * FE(24) - FM(8) * FE(34) - AVGD1 *
                        ?           AVGV1 * K/12
                   C
    ISN 0217             FM(20) =   -FM(7)*FE(25)-FM(8)*FE(35)
                   C
    ISN 0218             FM(21) =   FM(9) * FE(18) + FM(10) * FE(28) - AVGV1 *
                        ?           VOL1 * K/12 / 2
                   C
    ISN 0219             FM(22) =   FM(9)*FE(19)+FM(10)*FE(29)-2*SST**2
                   C
    ISN 0220             FM(23) =   -FM(9)*FE(16)-FM(10)*FE(26)+FM(5)
                   C
    ISN 0221             FM(24) =   -FE(17)*FM(9)-FM(10)*FE(27)+FM(6)
                   C
    ISN 0222             FM(25) =   -FM(9)*FE(20)-FM(10)*FE(30)
                   C
    ISN 0223             FM(26) =   -FM(9)*FE(21)-FM(10)*FE(31)
                   C
    ISN 0224             FM(27) =   -FM(9) * FE(22) - FM(10) * FE(32) + AVGD1 *
                        ?           VOL1 * K/12 / 2
                   C
    ISN 0225             FM(28) =   -FM(9)*FE(23)-FM(10)*FE(33)+2*SST*DT
                   C
    ISN 0226             FM(29) =   -FM(9)*FE(24)-FM(10)*FE(34)
                   C
    ISN 0227             FM(30) =   -FM(9) * FE(25) - FM(10) * FE(35) + AVGD1 *
                        ?           AVGV1 * K/12
                   C
    ISN 0228             FM(31) =   FM(11)-FM(12)*FM(21)/FM(22)
                   C
    ISN 0229             QUOT   =   FM(12)/FM(22)
    ISN 0230             DO 10      J=32,37
    ISN 0231             FM(J)  =   (FM(J-19)-QUOT*FM(J-9))/FM(31)
    ISN 0232          10 CONTINUE
                   C
    ISN 0233             FM(38) =   (FM(19)-QUOT*FM(29))/FM(31)
                   C
    ISN 0234             FM(39) =   (FM(20)-QUOT*FM(30))/FM(31)
                   C
    ISN 0235             DO 20      J=40,47
    ISN 0236             FM(J)  =   (FM(J-27)-FM(J-8)*FM(11))/FM(12)
    ISN 0237          20 CONTINUE
                   C
                   C
    ISN 0238             FC(1)  =   -AC*SIN(ALPCH)/(CO*TAU)+VELAX*AC*I/(CO*TAU*SSI**2)
                   C
    ISN 0239             FC(2)  =   -AC*COS(ALPCH)/TAU+(K+CO*VEL)*AC*I/(CO*TAU*SSI**2)
                   C
    ISN 0240             FC(3)  =   VOL1/2*VELAX/(CO*TAU*SSI**2)
                   C
    ISN 0241             FC(4)  =   VOL1*(K+CO*VEL)/(CO*TAU*SSI**2)/2
                   C
    ISN 0242             FC(5)  =   FC(1)*B1R*FE(7)-FC(1)*B1I*SIN(CO*TAU)+FC(2)*FE(7)
                        ?           +FC(3)*K*B1I*FE(7)+FC(3)*B1R*SIN(CO*TAU)*K+FC(4)*
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```
                   ?      K*SIN(CO*TAU)
                C
   ISN 0243            FC(6)  =  -FC(1)*B1I*FE(7)-FC(1)*B1R*SIN(CO*TAU)-FC(2)*
                   ?      SIN(CO*TAU)+FC(3)*K*B1R*FE(7)-FC(3)*K*B1I*
                   ?      SIN(CO*TAU)+FC(4)*K*FE(7)
                C
   ISN 0244            FC(7)  =  FC(5)*FE(16)+FC(6)*FE(26)-I
   ISN 0245            FC(8)  =  FC(5)*FE(17)+FC(6)*FE(27)
   ISN 0246            FC(9)  =  FC(5)*FE(18)+FC(6)*FE(28)+SST
   ISN 0247            FC(10) =  FC(5) * FE(19) + FC(6) * FE(29)- K/12 * VOL1 /2
   ISN 0248            FC(11) =  FC(5)*FE(20)+FC(6)*FE(30)+DT*SST
   ISN 0249            FC(12) =  FC(5)*FE(21)+FC(6)*FE(31)
   ISN 0250            FC(13) =  FC(5)*FE(22)+FC(6)*FE(32)+DT
   ISN 0251            FC(14) =  FC(5)*FE(23)+FC(6)*FE(33)
   ISN 0252            FC(15) =  FC(5) * FE(24) + FC(6) * FE(34)- K/12 * AVGD1
   ISN 0253            FC(16) =  FC(5)*FE(25)+FC(6)*FE(35)
   ISN 0254            FC(17) =  FC(9)*FM(32)+FC(10)*FM(40)+FC(7)
   ISN 0255            FC(18) =  FC(9)*FM(33)+FC(10)*FM(41)+FC(8)
                C
   ISN 0256            DO 30 J=  19,24
   ISN 0257            FC(J)  =  FC(9)*FM(J+15)+FC(10)*FM(J+23)+FC(J-8)
   ISN 0258         30 CONTINUE
                C
   ISN 0259            FC(21) =  -FC(21)
   ISN 0260            FC(22) =  -FC(22)
                C
   ISN 0261            FC(25) =  FC(1)*(B1I*FE(7)+B1R*SIN(CO*TAU))+FC(2)*SIN(CO*TAU)-
                   ?      FC(3)*K*(B1R*FE(7)-B1I*SIN(CO*TAU))-FC(4)*K*FE(7)
                C
   ISN 0262            FC(26) =  FC(1)*(B1R*FE(7)-B1I*SIN(CO*TAU))+FC(2)*FE(7)+FC(3)*
                   ?      K*(B1I*FE(7)+B1R*SIN(CO*TAU))+FC(4)*K*SIN(CO*TAU)
                C
   ISN 0263            FC(27) =  FC(26)*FE(26)+FC(25)*FE(16)
   ISN 0264            FC(28) =  FC(26)*FE(27)+FC(25)*FE(17)-I
   ISN 0265            FC(29) =  FC(26) * FE(28) + FC(25) * FE(18) + VOL1 * K/12 / 2
   ISN 0266            FC(30) =  FC(26)*FE(29)+FC(25)*FE(19)+SST
   ISN 0267            FC(31) =  FC(26)*FE(30)+FC(25)*FE(20)
   ISN 0268            FC(32) =  FC(26)*FE(31)+FC(25)*FE(21)+DT*SST
   ISN 0269            FC(33) =  FC(26)*FE(32)+FC(25)*FE(22)
   ISN 0270            FC(34) =  FC(26)*FE(33)+FC(25)*FE(23)+DT
   ISN 0271            FC(35) =  FC(26)*FE(34)+FC(25)*FE(24)
   ISN 0272            FC(36) =  FC(26) * FE(35) + FC(25) * FE(25) + K/12 * AVGD1
   ISN 0273            FC(37) =  FC(29)*FM(32)+FC(30)*FM(40)+FC(27)
   ISN 0274            FC(38) =  FC(29)*FM(33)+FC(30)*FM(41)+FC(28)
                C
   ISN 0275            DO 40 J=  39,44
   ISN 0276            FC(J)  =  FC(29)*FM(J-5)+FC(30)*FM(J+3)+FC(J-8)
   ISN 0277         40 CONTINUE
                C
   ISN 0278            FC(41) =  -FC(41)
   ISN 0279            FC(42) =  -FC(42)
                C
   ISN 0280            QUOT   =  FC(42)/FC(22)
   ISN 0281            FC(45) =  FC(41)-QUOT*FC(21)
   ISN 0282            FC(46) =  1/FC(45)*(FC(37)-QUOT*FC(17))
   ISN 0283            FC(47) =  1/FC(45)*(FC(38)-QUOT*FC(18))
   ISN 0284            FC(48) =  1/FC(45)*(FC(39)-QUOT*FC(19))
   ISN 0285            FC(49) =  1/FC(45)*(FC(40)-QUOT*FC(20))
   ISN 0286            FC(50) =  1/FC(45)*(FC(43)-QUOT*FC(23))
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ISN 0287            FC(51) =   1/FC(45)*(FC(44)-QUOT*FC(24))
ISN 0288            FC(56) =   1/FC(42)*(FC(43)-FC(41)*FC(50))
ISN 0289            FC(57) =   1/FC(42)*(FC(44)-FC(41)*FC(51))
              C
ISN 0290            DO 50 J=   52,55
ISN 0291            FC(J)  =   1/FC(42)*(FC(J-15)-FC(41)*FC(J-6))
ISN 0292         50 CONTINUE
              C
              C
              C
              C  INTERBLADE ANALYSIS OF SECTION 2
              C
              C
ISN 0293            FM2(1) =   AO*(-SSU**2-VU**2)
ISN 0294            FM2(2) =   AVGV2 * VOL2 * K/12 / 2
ISN 0295            FM2(3) =   FU+DU*VU**2
ISN 0296            FM23A  =   PT+DT*SST**2
ISN 0297            FM2(4) =   FM2(2)+FM2(1)**2/FM2(2)
ISN 0298            FM2(5) =   FM2(3)/FM2(4)
ISN 0299            FM2(6) =   FM2(3)*FM2(1)/(FM2(2)*FM2(4))
ISN 0300            FM2(7) =   -2/FM2(4)
ISN 0301            FM2(8) =   -2*FM2(1)/(FM2(2)*FM2(4))
ISN 0302            FM2(9) =   FM23A/FM2(4)
ISN 0303            FM2(10)=   FM2(1)/FM2(2)*FM23A/FM2(4)
ISN 0304            FM2(11)=   1 / FM2(4) * (2 * FLOW + FM2(1) /
                  ?            FM2(2) * AVGD2 * VOL2 * K/12 / 2)
ISN 0305            FM2(12)=   1 / FM2(4) * (FM2(1) / FM2(2) * 2 *
                  ?            FLOW - AVGD2 * VOL2 * K/12 / 2)
ISN 0306            FM2(13)=   1 / FM2(4) * (-2 * FM2(1) / FM2(2) * DT *
                  ?            SST - AVGD2 * VOL2 * K/12 / 2)
ISN 0307            FM2(14)=   1 / FM2(4) * (-2 * DT * SST + K/12 * FM2(1) /
                  ?            2 / FM2(2) * VOL2 * AVGD2)
ISN 0308            FM2(15)=   -AVGV2 * VOL2 * K/12 / (2 * FM2(4))
ISN 0309            FM2(16)=   FM2(1) / FM2(2) * AVGV2 * VOL2 * K/12 / (2 * FM2(4))
ISN 0310            FM2(17)=   -AVGV2 * AVGD2 * K/12 / FM2(4)
ISN 0311            FM2(18)=   AVGD2 * AVGV2 * K/12 * FM2(1) / (FM2(2) * FM2(4))
              C
              C
ISN 0312            M(1)   =   (FM2(3)-FM2(2)*FM2(5))/FM2(1)
ISN 0313            M(2)   =   -FM2(2)*FM2(6)/FM2(1)
ISN 0314            M(3)   =   (-2-FM2(7)*FM2(2))/FM2(1)
ISN 0315            M(4)   =   -FM2(2)*FM2(8)/FM2(1)
ISN 0316            M(5)   =   (FM23A-FM2(2)*FM2(9))/FM2(1)
ISN 0317            M(6)   =   -FM2(2)*FM2(10)/FM2(1)
ISN 0318            M(7)   =   (2*FLOW-FM2(2)*FM2(11))/FM2(1)
ISN 0319            M(8)   =   (-K/12 * AVGD2 * VOL2 / 2 - FM2(2) * FM2(12)) / FM2(1)
ISN 0320            M(9)   =   -(AVGD2 * VOL2 * K/12 / 2 + FM2(2) * FM2(13)) / FM2(1)
ISN 0321            M(10)  =   (-2*DT*SST-FM2(2)*FM2(14))/FM2(1)
ISN 0322            M(11)  =   -(AVGV2 * VOL2 * K/12 / 2 + FM2(2) * FM2(15)) / FM2(1)
ISN 0323            M(12)  =   (-FM2(2)*FM2(16))/FM2(1)
ISN 0324            M(13)  =   (-K/12 * AVGD2 * AVGV2 - FM2(2) * FM2(17)) / FM2(1)
ISN 0325            M(14)  =   -FM2(2)*FM2(18)/FM2(1)
ISN 0326            M(15)  =   DU * AO + VU * AO * M(7) - K/12 * VOL2 / 2 * FM2(11)
ISN 0327            M(16)  =   VU * AO * M(8) - K/12 *VOL2 / 2 * FM2(12)
ISN 0328            M(17)  =   -VU * AO * M(1) - DU * VU + K/12 * VOL2 / 2 * FM2(5)
ISN 0329            M(18)  =   -VU * AO * M(2) + K/12 * VOL2 / 2 * FM2(6)
ISN 0330            M(19)  =   -VU * AO * M(12) + SST + K/12 * VOL2 / 2 * FM2(16)
ISN 0331            M(20)  =   -VU * AO * M(11) + K/12 * VOL2 / 2 * FM2(15)
ISN 0332            M(21)  =   -VU * AO * M(3) + K/12 * VOL2 / 2 * FM2(7)
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ISN 0333           M(22)  =   -VU * AO * M(4) + K/12 * VOL2 / 2 * FM2(8)                00005860
ISN 0334           M(23)  =   -VU * AO * M(5) + DT * SST + K/12 * VOL2 / 2 * FM2(9)     00005870
ISN 0335           M(24)  =   -VU * AO * M(6) + K/12 * VOL2 / 2 * FM2(10)               00005880
ISN 0336           M(25)  =   -VU * AO * M(9) + K/12 * VOL2 / 2 * FM2(13)               00005890
ISN 0337           M(26)  =   -VU * AO * M(10) + K/12 * VOL2 / 2 * FM2(14) + DT         00005900
ISN 0338           M(27)  =   -VU * AO * M(13) + K/12 * VOL2 / 2 *                      00005910
                  ?            FM2(17) + K/12 * AVGD2                                    00005920
ISN 0339           M(28)  =   -VU * AO * M(14) + K/12 * VOL2 * FM2(18) / 2              00005930
             C                                                                           00005940
             C                                                                           00005950
ISN 0340           CL(I)  =   DU * AO + VU * AO * FM2(12) + K/12 * VOL2 / 2 * M(8)      00005960
ISN 0341           CL(2)  =   VU * AO * FM2(11) + K/12 * VOL2 / 2 * M(7)                00005970
ISN 0342           CL(3)  =   -VU * AO * FM2(5) - K/12 * VOL2 / 2 *M(1)                 00005980
ISN 0343           CL(4)  =   -DU * VU - VU * AO * FM2(6) - K/12 * VOL2 / 2 * M(2)      00005990
ISN 0344           CL(5)  =   -K/12 * VOL2 / 2 - VU * AO * FM2(16) -                    00006000
                  ?            K/12 * VOL2 / 2 * M(12)                                   00006010
ISN 0345           CL(6)  =   SST - VU * AO * FM2(15) - K/12 * VOL2 / 2 * M(11)         00006020
ISN 0346           CL(7)  =   -VU * AO * FM2(14) - K/12 * VOL2 / 2 * M(10)              00006030
ISN 0347           CL(8)  =   DT - VU * AO * FM2(13) - K/12 * VOL2 / 2 * M(9)           00006040
ISN 0348           CL(9)  =   -VU * AO * FM2(7) - K/12 * VOL2 /2 * M(3)                 00006050
ISN 0349           CL(10) =   -VU * AO * FM2(8) - K/12 * VOL2 / 2 * M(4)                00006060
ISN 0350           CL(11) =   -VU * AO * FM2(9) - K/12 * VOL2 / 2 * M(5)                00006070
ISN 0351           CL(12) =   -VU * AO * FM2(10) - K/12 * VOL2 / 2 * M(6) + AVGD2*SST00006080
ISN 0352           CL(13) =   -VU * AO * FM2(18) - K/12 * VOL2 / 2 * (2 * DT + M(14))00006090
ISN 0353           CL(14) =   -VU * AO * FM2(17) - K/12 *VOL2 / 2 * M(13)               00006100
ISN 0354           CL(15) =   CL(2)-CL(1)/M(16)*M(15)                                   00006110
             C                                                                           00006120
ISN 0355           DO 60 J=   16,19                                                     00006130
ISN 0356           CL(J)  =   1/CL(15)*(CL(J-13)-CL(1)/M(16)*M(J+1))                    00006140
ISN 0357        60 CONTINUE                                                              00006150
             C                                                                           00006160
ISN 0358           CL(20) =   1/CL(15)*(CL(7)-CL(1)/M(16)*M(26))                        00006170
ISN 0359           CL(21) =   1/CL(15)*(CL(8)-CL(1)/M(16)*M(25))                        00006180
ISN 0360           CL(22) =   1/CL(15)*(CL(9)-CL(1)/M(16)*M(21))                        00006190
ISN 0361           CL(23) =   1/CL(15)*(CL(10)-CL(1)/M(16)*M(22))                       00006200
ISN 0362           CL(24) =   1/CL(15)*(CL(11)-CL(1)/M(16)*M(23))                       00006210
ISN 0363           CL(25) =   1/CL(15)*(CL(12)-CL(1)/M(16)*M(24))                       00006220
ISN 0364           CL(26) =   1/CL(15)*(CL(13)-CL(1)/M(16)*M(28))                       00006230
ISN 0365           CL(27) =   1/CL(15)*(CL(14)-CL(1)/M(16)*M(27))                       00006240
             C                                                                           00006250
ISN 0366           DO 70      J =   28,39                                                00006260
ISN 0367           CL(J)    = (CL(J-25)-CL(2)*CL(J-12))/CL(1)                           00006270
ISN 0368        70 CONTINUE                                                              00006280
             C                                                                           00006290
             C                                                                           00006300
             C  INTERBLADE ANALYSIS OF SECTION 3                                         00006310
             C                                                                           00006320
             C                                                                           00006330
ISN 0369           S(1)   =   2*GAMMA*PU/(GAMMA+1)                                      00006340
ISN 0370           S(2)   =   (2*GAMMA/(GAMMA+1))*PU*MUS-PD-(GAMMA-1)/(GAMMA+1)*PU      00006350
ISN 0371           S(3)   =   2*GAMMA/(GAMMA+1)*SSU*MUS+US-(GAMMA-1)/(GAMMA+1)*         00006360
                  ?            SSU                                                       00006370
ISN 0372           S(4)   =   2*GAMMA/(GAMMA+1)*SSU*PU                                  00006380
ISN 0373           S(5)   =   (GAMMA-1)*MUS**2+2                                        00006390
ISN 0374           S(6)   =   (2-2*GAMMA)*DD+(2*GAMMA+2)*DU                             00006400
ISN 0375           S(7)   =   (GAMMA+1)*MUS**2                                          00006410
ISN 0376           S(8)   =   S(6)/(SSU*S(5))                                           00006420
ISN 0377           S(9)   =   S(7)/S(5)                                                 00006430
             C                                                                           00006440
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              C
 ISN 0378            LM(1)  =   DE*UE*ACE/(CO*TAU)-FRE*SIN(ALPCH)/(CO*TAU)+(REV*
                    ?           COS(ALPCH-BETA2))**2*ACE*UE*DE/(CO*TAU*SOSEX**2)-DE*
                    ?           ACE/(CO*TAU)*SIN(ALPCH)*REV*COS(ALPCH-BETA2)
              C
 ISN 0379            LM(2)  =   DE*(K+CO*VE)*ACE/(CO*TAU)-COS(ALPCH)*FRE/TAU+DE*(REV*
                    ?           COS(ALPCH-BETA2))**2*ACE*(K+CO*VE)/(CO*TAU*SOSEX**2)-
                    ?           DE*ACE*COS(ALPCH)*REV*COS(ALPCH-BETA2)
              C
 ISN 0380            LM(3)  =   AVGV3*VOL3/2*UE*DE/(CO*TAU*SOSEX**2)-AVGD3*VOL3*
                    ?           SIN(ALPCH)/(2*CO*TAU)
              C
 ISN 0381            LM(4)  =   AVGV3*VOL3/(2*CO*TAU*SOSEX**2)*(K+CO*VE)*DE-
                    ?           AVGD3*VOL3*COS(ALPCH)/(2*TAU)
              C
 ISN 0382            R      =   -(K+VE*CO)/UE
 ISN 0383            LM(5)  =   FRE*SIN(ALPCH)/(TAU*(R**2+CO**2))-R*COS(ALPCH)*FRE/(CO*
                    ?           TAU*(R**2+CO**2))+REV*COS(ALPCH-BETA2)*DE*ACE*
                    ?           (SIN(ALPCH)-R/CO*COS(ALPCH))/(TAU*(R**2+CO**2))
              C
 ISN 0384            LM(6)  =   AVGD3*VOL3/(2*TAU*(R**2+CO**2))*(SIN(ALPCH)-R/CO*
                    ?           COS(ALPCH))
              C
 ISN 0385            LM(7)  =   COS(CO*TAU)-1
              C
 ISN 0386            LM(8)  =   LM(1)*(LM(7)*B2R-B2I*SIN(CO*TAU))+LM(2)*LM(7)-K*
                    ?           LM(3)*(B2I*LM(7)+B2R*SIN(CO*TAU))-K*LM(4)*SIN(CO*TAU)
              C
 ISN 0387            LM(9)  =   -LM(1)*(LM(7)*B2I+B2R*SIN(CO*TAU))-LM(2)*SIN(CO*TAU)-
                    ?           K*LM(3)*(B2R*LM(7)-B2I*SIN(CO*TAU))-K*LM(4)*LM(7)
              C
 ISN 0388            LM(10) =   -K*LM(6)*SIN(CO*TAU)+LM(5)*LM(7)
              C
 ISN 0389            LM(11) =   -K*LM(6)*LM(7)-LM(5)*SIN(CO*TAU)
              C
 ISN 0390            LM(12) =   LM(1)*(B2I*LM(7)+B2R*SIN(CO*TAU))+LM(2)*SIN(CO*TAU)+
                    ?           LM(3)*K*(B2R*LM(7)-B2I*SIN(CO*TAU))+LM(4)*K*LM(7)
              C
 ISN 0391            LM(13) =   LM(1)*(B2R*LM(7)-B2I*SIN(CO*TAU))+LM(2)*LM(7)-LM(3)*
                    ?           K*(B2I*LM(7)+B2R*SIN(CO*TAU))-LM(4)*K*SIN(CO*TAU)
              C
 ISN 0392            LM(14) =   LM(5)*SIN(CO*TAU)+LM(6)*K*LM(7)
              C
 ISN 0393            LM(15) =   LM(5)*LM(7)-LM(6)*K*SIN(CO*TAU)
              C
 ISN 0394            LM(16) =   LM(12)-LM(13)/LM(9)*LM(8)
              C
 ISN 0395            LM(17) =   1/LM(16)*(LM(14)-LM(13)*LM(10)/LM(9))
              C
 ISN 0396            LM(18) =   1/LM(16)*(LM(15)-LM(13)*LM(11)/LM(9))
              C
 ISN 0397            LM(19) =   1/LM(16)
              C
 ISN 0398            LM(20) =   -LM(13)/(LM(9)*LM(16))
              C
 ISN 0399            LM(21) =   (LM(14)-LM(12)*LM(17))/LM(13)
 ISN 0400            LM(22) =   (LM(15)-LM(12)*LM(18))/LM(13)
 ISN 0401            LM(23) =   -LM(12)*LM(20)/LM(13)
 ISN 0402            LM(24) =   (1-LM(12)*LM(19))/LM(13)
```

```
  ISN 0403           WRITE(6,333) B1R,B1I
  ISN 0404      333  FORMAT(//' B1'/(2E15.6))
               C
  ISN 0405           WRITE(6,6000) FE
  ISN 0406           WRITE(6,6100) AVGV1
  ISN 0407           WRITE(6,6200) VOL1
  ISN 0408           WRITE(6,6001) FM
  ISN 0409           WRITE(6,6002) FC
  ISN 0410           WRITE(6,6003) FM2
  ISN 0411           WRITE(6,6004) M
  ISN 0412           WRITE(6,6005) CL
  ISN 0413           WRITE(6,6006) S
  ISN 0414           WRITE(6,6007) LM
  ISN 0415           WRITE(6,6008) LC
  ISN 0416      6000 FORMAT('1',' FE',/,9(E12.4,2X))
  ISN 0417      6100 FORMAT('-',' AVGV1',/,1X,E12.4,2X)
  ISN 0418      6200 FORMAT('-',' VOL1',/,1X,E12.4,2X)
  ISN 0419      6001 FORMAT('-',' FM',/,9(E12.4,2X))
  ISN 0420      6002 FORMAT('-',' FC',/,9(E12.4,2X))
  ISN 0421      6003 FORMAT('-',' FM2',/,9(E12.4,2X))
  ISN 0422      6004 FORMAT('-','  M',/,9(E12.4,2X))
  ISN 0423      6005 FORMAT('-',' CL',/,9(E12.4,2X))
  ISN 0424      6006 FORMAT('-','  S',/,9(E12.4,2X))
  ISN 0425      6007 FORMAT('-',' LM',/,9(E12.4,2X))
  ISN 0426      6008 FORMAT('-',' LC',/,9(E12.4,2X))
               C
               C     CALCULATE EITHER IN TORSIONAL OR BENDING MODE
               C
               C
  ISN 0427           DO  15 II =   1,2
  ISN 0428           IF (II.EQ.1) GOTO 13
               C
               C CALCULATES TORSIONAL MODES AREAS
               C
  ISN 0430           RMAPI  =   C*Z*ALFBAR*(COS(SIGMAT)-1)/ASTAR
  ISN 0431           IMAPI  =   C*Z*ALFBAR*(SIN(SIGMAT))/ASTAR
  ISN 0432           RMAPT  =   (Z-XSTAR/C)*(COS(SIGMAT)-1)*C*ALFBAR/ASTAR
  ISN 0433           IMAPT  =   (Z-XSTAR/C)*SIN(SIGMAT)*C*ALFBAR/ASTAR
  ISN 0434           RMAP   =   (Z-XS/C)*(COS(SIGMAT)-1)*C*ALFBAR/ASTAR
  ISN 0435           IMAP   =   (Z-XS/C)*SIN(SIGMAT)*C*ALFBAR/ASTAR
  ISN 0436           RACE   =   (Z-1)*(COS(SIGMAT)-1)*C*ALFBAR/ASTAR
  ISN 0437           IACE   =   (Z-1)*SIN(SIGMAT)*C*ALFBAR/ASTAR
               C
               C  CALCULATES TORSIONAL MODE VOLUMES FOR THE FIRST TWO SECTIONS
               C
  ISN 0438           V1AR   =    0.0
  ISN 0439           V1AI   =    0.0
               C
  ISN 0440           V1BR   =    C*DELTA*Z*ALFBAR*(COS(SIGMAT)-1)*COS(ALFCH)/(ASTAR*B)
  ISN 0441           V1BI   =    C*DELTA*Z*ALFBAR*(SIN(SIGMAT))*COS(ALFCH)/(ASTAR*B)
               C
  ISN 0442           V1CR   =    C*ALFBAR*(COS(SIGMAT)-1)*(Z*(XSTAR-DELTA*COS(ALFCH))-
                    ?            (XSTAR**2-(DELTA*COS(ALFCH))**2)/(2*C))/(ASTAR*B)
  ISN 0443           V1CI   =    C*ALFBAR*(SIN(SIGMAT))*(Z*(XSTAR-DELTA*COS(ALFCH))-
                    ?            (XSTAR**2-(DELTA*COS(ALFCH))**2)/(2*C))/(ASTAR*B)
               C
  ISN 0444           V1R    =    V1AR+V1BR+V1CR
  ISN 0445           V1I    =    V1AI+V1BI+V1CI
               C
```

```
  ISN 0446          V2R    =   C*ALFBARY(COS(SIGMAT)-1)*(Z*(XS-XSTAR)-((XS**2-
                  ?            XSTAR**2)/(2*C)))/(ASTAR*B)
  ISN 0447          V2I    =   C*ALFBAR*(SIN(SIGMAT))*(Z*(XS-XSTAR)-((XS**2-
                  ?            XSTAR**2)/(2*C)))/(ASTAR*B)
              C
  ISN 0448          GOTO 17
              C
              C CALCULTES BENDING MODE AREAS
              C
  ISN 0449     13   RACE   =   H*(COS(SIGMAB)-1)/ASTAR
  ISN 0450          IACE   =   H*SIN(SIGMAB)/ASTAR
  ISN 0451          IMAP   =   H*SIN(SIGMAB)/ASTAR
  ISN 0452          RMAP   =   H*(COS(SIGMAB)-1)/ASTAR
  ISN 0453          RMAPI  =   H*(COS(SIGMAB)-1)/ASTAR
  ISN 0454          IMAPT  =   H*SIN(SIGMAB)/ASTAR
  ISN 0455          IMAPI  =   H*SIN(SIGMAB)/ASTAR
  ISN 0456          RMAPT  =   H*(COS(SIGMAB)-1)/ASTAR
              C
              C CALCULTES BENDING MODE VOLUMES FOR THE FIRST TWO SECTIONS
              C
  ISN 0457          V1AR   =   0.0
  ISN 0458          V1AI   =   0.0
              C
  ISN 0459          V1BR   =   DELTA*H*(COS(SIGMAB)-1)*COS(ALPCH)/(ASTAR*B)
  ISN 0460          V1BI   =   DELTA*H*(SIN(SIGMAB))*COS(ALPCH)/(ASTAR*B)
              C
  ISN 0461          V1CR   =   (H*(XSTAR-DELTA*COS(ALPCH))*(COS(SIGMAB)-1)+ASTAR*H*
                  ?            (COS(SIGMAB)-1)*COTAN(ALPCH))/(ASTAR*B)
  ISN 0462          V1CI   =   (H*(XSTAR-DELTA*COS(ALPCH))*(SIN(SIGMAB))+ASTAR*H*
                  ?            (SIN(SIGMAB))*COTAN(ALPCH))/(ASTAR*B)
              C
  ISN 0463          V1R    =   V1AR+V1BR+V1CR
  ISN 0464          V1I    =   V1AI+V1BI+V1CI
              C
  ISN 0465          V2R    =   H*(COS(SIGMAB)-1)*(XS-XSTAR)/(ASTAR*B)
  ISN 0466          V2I    =   H*(SIN(SIGMAB))*(XS-XSTAR)/(ASTAR*B)
              C
              C
              C
  ISN 0467     17   ISSP   =   FC(52)*RMAPI+FC(53)*IMAPI+FC(54)*RMAPT+FC(55)*IMAPT+
                  ?            FC(56)*V1I+FC(57)*V1R
              C
  ISN 0468          RSSP   =   FC(46)*RMAPI+FC(47)*IMAPI+FC(48)*RMAPT+FC(49)*IMAPT+
                  ?            FC(50)*V1I+FC(51)*V1R
              C
  ISN 0469          IMDP   =   FM(40)*RMAPI+FM(41)*IMAPI+FM(42)*RMAPT+FM(43)*IMAPT+
                  ?            FM(44)*RSSP+FM(45)*ISSP+FM(46)*V1I+FM(47)*V1R
              C
  ISN 0470          RMDP   =   FM(32)*RMAPI+FM(33)*IMAPI+FM(34)*RMAPT+FM(35)*IMAPT+
                  ?            FM(36)*RSSP+FM(37)*ISSP+FM(38)*V1I+FM(39)*V1R
              C
  ISN 0471          IVPU   =   FE(26)*RMAPI+FE(27)*IMAPI+FE(28)*RMDP+FE(29)*IMDP+
                  ?            FE(30)*RMAPT+FE(31)*IMAPT+FE(32)*RSSP+FE(33)*ISSP+
                  ?            FE(34)*V1I+FE(35)*V1R
              C
  ISN 0472          RVPU   =   FE(16)*RMAPI+FE(17)*IMAPI+FE(18)*RMDP+FE(19)*IMDP+
                  ?            FE(20)*RMAPT+FE(21)*IMAPT+FE(22)*RSSP+FE(23)*ISSP+
                  ?            FE(24)*V1I+FE(25)*V1R
              C
```

```
   ISN 0473        IMPP    =   IMDP*SST**2                                              00008220
   ISN 0474        RMPP    =   RMDP*SST**2                                              00008230
               C                                                                        00008240
   ISN 0475        RMVPU   =   CL(16)*RMAP+CL(17)*IMAP+CL(18)*RMDP+CL(19)*IMDP+CL(20)*00008250
                 ?             RSSP+CL(21)*ISSP+CL(22)*RMPP+CL(23)*IMPP+CL(24)*RMAPT+ 00008260
                 ?             CL(25)*IMAPT+CL(26)*V2R+CL(27)*V2I                       00008270
               C                                                                        00008280
   ISN 0476        IMVPU   =   CL(28)*RMAP+CL(29)*IMAP+CL(30)*RMDP+CL(31)*IMDP+         00008290
                 ?             CL(32)*RSSP+CL(33)*ISSP+CL(34)*RMPP+CL(35)*IMPP+         00008300
                 ?             CL(36)*RMAPT+CL(37)*IMAPT+CL(38)*V2R+CL(39)*V2I          00008310
               C                                                                        00008320
   ISN 0477        RMDPU   =   M(1)*RMAP+M(2)*IMAP+M(3)*RMPP+M(4)*IMPP+M(5)*            00008330
                 ?             RMAPT+M(6)*IMAPT+M(7)*RMVPU+M(8)*IMVPU+M(9)*ISSP+        00008340
                 ?             M(10)*RSSP+M(11)*IMDP+M(12)*RMDP+M(13)*V2I+M(14)*V2R     00008350
               C                                                                        00008360
   ISN 0478        IMDPU   =   FM2(5)*RMAP+FM2(6)*IMAP+FM2(7)*RMPP+FM2(8)*IMPP+         00008370
                 ?             FM2(9)*RMAPT+FM2(10)*IMAPT+FM2(11)*RMVPU+FM2(12)*        00008380
                 ?             IMVPU+FM2(13)*ISSP+FM2(14)*RSSP+FM2(15)*IMDP+            00008390
                 ?             FM2(16)*RMDP+FM2(17)*V2I+FM2(18)*V2R                     00008400
               C                                                                        00008410
   ISN 0479        RDDS    =   S(8)*RMVPU+S(9)*RMDPU                                    00008420
   ISN 0480        IDDS    =   S(8)*IMVPU+S(9)*IMDPU                                    00008430
   ISN 0481        RMPPU   =   SSU**2*RMDPU                                             00008440
   ISN 0482        IMPPU   =   SSU**2*IMDPU                                             00008450
   ISN 0483        RSOSU   =   (GAMMA-1)/2*RMPPU/(DU*SSU)                               00008460
   ISN 0484        ISOSU   =   (GAMMA-1)/2*IMPPU/(DU*SSU)                               00008470
   ISN 0485        RPDS    =   SOSDS**2*RDDS                                            00008480
   ISN 0486        IPDS    =   SOSDS**2*IDDS                                            00008490
   ISN 0487        USR     =   1/S(1)*(S(2)*RSOSU+S(3)*RMPPU+S(4)/SSU*RMVPU-SSU*RPDS) 00008500
   ISN 0488        USI     =   1/S(1)*(S(2)*ISOSU+S(3)*IMPPU+S(4)/SSU*IMVPU-SSU*IPDS) 00008510
   ISN 0489        IMSP    =   -USR/K                                                   00008520
   ISN 0490        RMSP    =   USI/K                                                    00008530
               C                                                                        00008540
   ISN 0491        RMPPI   =   -VELAX/(CO*TAU)*(B1R*RVPU*(COS(CO*TAU)-1)-B1I*IVPU*      00008550
                 ?             (COS(CO*TAU)-1)-IVPU*B1R*SIN(CO*TAU)-B1I*RVPU*SIN(CO*    00008560
                 ?             TAU))-(K+CO*VEL)/(CO*TAU)*(RVPU*(COS(CO*TAU)-1)-IVPU*    00008570
                 ?             SIN(CO*TAU))                                             00008580
               C                                                                        00008590
   ISN 0492        IMPPI   =  -VELAX/(CO*TAU)*(IVPU*B1R*(COS(CO*TAU)-1)+B1I*RVPU*       00008600
                 ?             (COS(CO*TAU)-1)+B1R*RVPU*SIN(CO*TAU)-B1I*IVPU*SIN(CO*    00008610
                 ?             TAU))-(K+CO*VEL)/(CO*TAU)*(IVPU*(COS(CO*TAU)-1)+RVPU*    00008620
                 ?             SIN(CO*TAU))                                             00008630
               C                                                                        00008640
   ISN 0493        RMDPI   =   RMPPI/SSI**2                                             00008650
   ISN 0494        IMDPI   =   IMPPI/SSI**2                                             00008660
               C                                                                        00008670
   ISN 0495        RMSSPI  =   (GAMMA-1)/(2*SSI)*RMPPI                                  00008680
   ISN 0496        IMSSPI  =   (GAMMA-1)/(2*SSI)*IMPPI                                  00008690
               C                                                                        00008700
   ISN 0497        RITVPI  =   1/TAU*(RVPU*(COS(CO*TAU)-1)-IVPU*SIN(CO*TAU))            00008710
   ISN 0498        IITVPI  =   1/TAU*(IVPU*(COS(CO*TAU)-1)+RVPU*SIN(CO*TAU))            00008720
               C                                                                        00008730
   ISN 0499        RIAVPI  =   1/(CO*TAU)*(B1R*RVPU*(COS(CO*TAU)-1)-B1I*IVPU*           00008740
                 ?             (COS(CO*TAU)-1)-IVPU*B1R*SIN(CO*TAU)-B1I*RVPU*           00008750
                 ?             SIN(CO*TAU))                                             00008760
               C                                                                        00008770
   ISN 0500        IIAVPI  =   1/(CO*TAU)*(B1R*IVPU*(COS(CO*TAU)-1)+B1I*RVPU*           00008780
                 ?             (COS(CO*TAU)-1)+RVPU*B1R*SIN(CO*TAU)-B1I*IVPU*           00008790
                 ?             SIN(CO*TAU))                                             00008800
```

```
                C
  ISN 0501          IF(II.EQ.2) GOTO 24
                C
                C  CALCULATES THE BENDING MODES VOLUMES  FOR THE THIRD SECTION
                C
  ISN 0503          V3AR   =  (H*(COS(SIGMAB)-1)*(C-XS)-(AO+ACE)*ASTAR/2*RMSP*B)/
                   ?          (ASTAR*B)
  ISN 0504          V3BR   =  DELTA/2.0*SIN(2*ALFCH)*H*(COS(SIGMAB)-1)/(ASTAR*B)
  ISN 0505          V3CR   =  0.0
  ISN 0506          V3AI   =  (H *SIN(SIGMAB) * (C - XS) - (AO + ACE) * ASTAR /
                   ?          2 * IMSP * B) / (ASTAR * B)
  ISN 0507          V3BI   =  DELTA/2.0*SIN(2*ALFCH)*H*(SIN(SIGMAB))/(ASTAR*B)
  ISN 0508          V3CI   =  0.0
  ISN 0509          WRITE(6,410)
  ISN 0510      410 FORMAT('1',' BENDING')
  ISN 0511          GOTO 25
                C
                C  CALCULTES THE TORSIONAL MODE VOLUMES FOR THE THIRD SECTION
                C
  ISN 0512      24  V3AR   =  (C*ALFBAR*(COS(SIGMAT)-1)*(Z*(C-XS)-(C**2-XS**2)/
                   ?          (2*C))-AO*ASTAR*RMSP*B)/(ASTAR*B)
  ISN 0513          V3AI   =  (C*ALFBAR*(SIN(SIGMAT))*(Z*(C-XS)-(C**2-XS**2)/(2*C))-
                   ?          AO*ASTAR*IMSP*B)/(ASTAR*B)
  ISN 0514          V3BR   =  ALFBAR*(Z-1)*(COS(SIGMAT)-1)*C*COS(ALPCH)*
                   ?          DELTA/(ASTAR*B)
  ISN 0515          V3BI   =  ALFBAR*(Z-1)*SIN(SIGMAT)*C*COS(ALPCH)*DELTA/(ASTAR*B)
  ISN 0516          V3CR   =  0.0
  ISN 0517          V3CI   =  0.0
  ISN 0518          WRITE(6,420)
  ISN 0519      420 FORMAT('1',' TORSIONAL')
                C
  ISN 0520      25  V3I    =  V3AI+V3BI+V3CI
  ISN 0521          V3R    =  V3AR+V3BR+V3CR
                C
                C
                C
                C
                C
  ISN 0522          RUDS   =  1/DD*(RMDPU*VU+DU*RMVFU-RDDS*VD)
  ISN 0523          IUDS   =  1/DD*(IMDPU*VU+DU*IMVFU-IDDS*VD)
  ISN 0524          WDSI   =  IDDS*VD*AO+DD*IUDS*AO+DD*VD*IMAP
  ISN 0525          WDSR   =  RDDS*VD*AO+DD*RUDS*AO+DD*VD*RMAP
  ISN 0526          CI     =  -DE*REV*COS(ALPCH-BETA2)*IACE+WDSI-AVGD3 * K/12 * V3R -
                   ?          K/12 *VOL3/2*RDDS
                C
  ISN 0527          CR     =  -DE*RACE*U2IRE+WDSR+AVGD3 * K/12 * V3I+K*VOL3/2*IDDS
                C
  ISN 0528          IC     =  -IFDS*AO-FD*IMAP+IACE*(PE+(REV*COS(ALPCH-BETA2))**2*
                   ?          DE) - WDS * IUDS + AVGV3 * VOL3 * K/12 * RDDS / 2 +
                   ?          AVGD3 * AVGV3 * K/12 * V3R + K/12 *AVGD3 * VOL3 *
                   ?          RUDS / 2 - WDSI * VD
  ISN 0529          RC     =  -RFDS*AO-FD*RMAP+RACE*(PE+(REV*COS(ALPCH-BETA2))**2*
                   ?          DE) - WDS * RUDS - AVGV3 * VOL3 * K/12 * IDDS / 2 -
                   ?          AVGD3 * AVGV3 * K/12 * V3I - K/12 * AVGD3 * VOL3 *
                   ?          IUDS / 2 - WDSR * VD
                C
                C
                C
  ISN 0530          LC(1)  =  -UE*DE*ACE*REV*COS(ALPCH-BETA2)/(CO*TAU*SOSEX**2)+DE*
```

```
                          ?         ACE*SIN(ALPCH)/(CO*TAU)
                       C
    ISN 0531              LC(2)  =  -DE*(K+CO*VE)*ACE*REV*COS(ALPCH-BETA2)/
                          ?         (CO*TAU*SOSEX**2)+DE*ACE*COS(ALPCH)/TAU
                       C
    ISN 0532              LC(3)  =  DE*ACE/(TAU*(R**2+CC**2))*(SIN(ALPCH)-R/CO*COS(ALPCH))
                       C
    ISN 0533              LC(4)  =  VOL3*UE*DE/(2*CO*TAU*SOSEX**2)
                       C
    ISN 0534              LC(5)  =  VOL3/2*(K+CO*VE)*DE/(CO*TAU*SOSEX**2)
                       C
    ISN 0535              LC(6)  =  LC(1)*B2R*LM(7)-LC(1)*B2I*SIN(CO*TAU)+LC(2)*LM(7)+
                          ?         K*LC(4)*(B2I*LM(7)+B2R*SIN(CO*TAU))+K*LC(5)*
                          ?         SIN(CO*TAU)
                       C
    ISN 0536              LC(7)  =  -LC(1)*(B2I*LM(7)+B2R*SIN(CO*TAU))-LC(2)*SIN(CO*TAU)+
                          ?          K*LC(4)*(LM(7)*B2R-B2I*SIN(CO*TAU))+K*LC(5)*LM(7)
                       C
    ISN 0537              LC(8)  =  LC(3)*LM(7)
                       C
    ISN 0538              LC(9)  =  -LC(3)*SIN(CO*TAU)
                       C
    ISN 0539              LC(10) =  LC(6)*LM(17)+LC(7)*LM(21)+LC(8)
                       C
    ISN 0540              LC(11) =  LC(6)*LM(18)+LC(7)*LM(22)+LC(9)
                       C
    ISN 0541              LC(12) =  LC(6)*LM(20)+LC(7)*LM(23)
                       C
    ISN 0542              LC(13) =  LC(6)*LM(19)+LC(7)*LM(24)
                       C
    ISN 0543              LC(14) =  LC(1)*(B2I*LM(7)+B2R*SIN(CO*TAU))+LC(2)*SIN(CO*TAU)-
                          ?         LC(4)*K*(B2R*LM(7)-B2I*SIN(CO*TAU))-LC(5)*K*LM(7)
                       C
    ISN 0544              LC(15) =  LC(1)*(B2R*LM(7)-B2I*SIN(CO*TAU))+LC(2)*LM(7)+
                          ?         LC(4)*K*(B2I*LM(7)+B2R*SIN(CO*TAU))+LC(5)*K*
                          ?         SIN(CO*TAU)
                       C
    ISN 0545              LC(16) =  LC(3)*SIN(CO*TAU)
                       C
    ISN 0546              LC(17) =  LC(3)*LM(7)
                       C
    ISN 0547              LC(18) =  LC(14)*LM(17)+LC(15)*LM(21)+LC(16)
                       C
    ISN 0548              LC(19) =  LC(14)*LM(18)+LC(15)*LM(22)+LC(17)
                       C
    ISN 0549              LC(20) =  LC(14)*LM(20)+LC(15)*LM(23)
                       C
    ISN 0550              LC(21) =  LC(14)*LM(19)+LC(15)*LM(24)
                       C
    ISN 0551              LC(22) =  LC(18)-LC(19)/LC(11)*LC(10)
                       C
    ISN 0552              LC(23) =  -(LC(20)-LC(19)/LC(11)*LC(12))/LC(22)
                       C
    ISN 0553              LC(24) =  -(LC(21)-LC(19)/LC(11)*LC(13))/LC(22)
                       C
    ISN 0554              LC(25) =  (CI-CR*LC(19)/LC(11))/LC(22)
                       C
    ISN 0555              RRFF   =  LC(23)*RC+LC(24)*IC+LC(25)
                       C
```

```
ISN 0556          LC(26) =   -(LC(20)+LC(18)*LC(23))/LC(19)
              C
ISN 0557          LC(27) =   -(LC(21)+LC(18)*LC(24))/LC(19)
              C
ISN 0558          LC(28) =   (CI-LC(25)*LC(18))/LC(19)
              C
ISN 0559          IRFF   =   LC(26)*RC+LC(27)*IC+LC(28)
              C
ISN 0560          IVPD   =   LM(21)*RRFF+LM(22)*IRFF+LM(23)*RC+LM(24)*IC
              C
ISN 0561          RVPD   =   LM(17)*RRFF+LM(18)*IRFF+LM(19)*IC+LM(20)*RC
              C
ISN 0562          RMPPE  =   -UE*DE/(CO*TAU)*(B2R*RVPD*(COS(CO*TAU)-1)-B2I*IVPD*
                ?            (COS(CO*TAU)-1)-IVPD*B2R*SIN(CO*TAU)-B2I*RVPD*SIN(CO*
                ?            TAU))-(K+CO*VE)/(CO*TAU)*(RVPD*(COS(CO*TAU)-1)-IVPD*
                ?            SIN(CO*TAU))
              C
ISN 0563          IMPPE  =  -UE*DE/(CO*TAU)*(IVPD*B2R*(COS(CO*TAU)-1)+B2I*RVPD*
                ?            (COS(CO*TAU)-1)+B2R*RVPD*SIN(CO*TAU)-B2I*IVPD*SIN(CO*
                ?            TAU))-(K+CO*VE)/(CO*TAU)*(IVPD*(COS(CO*TAU)-1)+RVPD*
                ?            SIN(CO*TAU))
              C
ISN 0564          RMDPE  =   RMPPE/SOSEX**2
ISN 0565          IMDPE  =   IMPPE/SOSEX**2
              C
ISN 0566          RMSSPE =   (GAMMA-1)/(2*SOSEX*DE)*RMPPE
ISN 0567          IMSSPE =   (GAMMA-1)/(2*SOSEX*DE)*IMPPE
              C
              C  IRROTATIONAL VELOCITY PERTURBATIONS AT THE EXIT
              C
ISN 0568          RITVPE =   1/TAU*(RVPD*(COS(CO*TAU)-1)-IVPD*SIN(CO*TAU))
ISN 0569          IITVPE =   1/TAU*(IVPD*(COS(CO*TAU)-1)+RVPD*SIN(CO*TAU))
              C
ISN 0570          RIAVPE =   1/(CO*TAU)*(B2R*RVPD*(COS(CO*TAU)-1)-B2I*IVPD*
                ?            (COS(CO*TAU)-1)-IVPD*B2R*SIN(CO*TAU)-B2I*RVPD*
                ?            SIN(CO*TAU))
              C
ISN 0571          IIAVPE =   1/(CO*TAU)*(B2R*IVPD*(COS(CO*TAU)-1)+B2I*RVPD*
                ?            (COS(CO*TAU)-1)+RVPD*B2R*SIN(CO*TAU)-B2I*IVPD*
                ?            SIN(CO*TAU))
              C
              C  ROTATIONAL VELOCITY PERTURBATIONS AT THE  EXIT
              C
ISN 0572          RRAVPE =   1/((R**2+CO**2)*TAU)*(RRFF*(COS(CO*TAU)-1)-IRFF*
                ?            SIN(CO*TAU))
ISN 0573          IRAVPE =   1/((R**2+CO**2)*TAU)*(IRFF*(COS(CO*TAU)-1)-RRFF*
                ?            SIN(CO*TAU))
              C
ISN 0574          RRTVPE =   -R/CO*RRAVPE
ISN 0575          IRTVPE =   -R/CO*IRAVPE
              C
ISN 0576          IP1U   =   (IMPPI+IMPP)/2.0
ISN 0577          RP1U   =   (RMPPI+RMPP)/2.0
ISN 0578          IP2U   =   (IMPP+IMPPU)/2.0
ISN 0579          RP2U   =   (RMPP+RMPPU)/2.0
ISN 0580          IP3U   =   (IPDS+IMPPE)/2.0
ISN 0581          RP3U   =   (RPDS+RMPPE)/2.0
              C
ISN 0582          RP1L   =   RP1U*COS(SIGMAT)+IP1U*SIN(SIGMAT)
```

```
ISN 0583          IP1L   =  IP1U*COS(SIGMAT)-RP1U*SIN(SIGMAT)
ISN 0584          RP2L   =  RP2U*COS(SIGMAT)+IP2U*SIN(SIGMAT)
ISN 0585          IP2L   =  IP2U*COS(SIGMAT)-RP2U*SIN(SIGMAT)
ISN 0586          RP3L   =  RP3U*COS(SIGMAT)+IP3U*SIN(SIGMAT)
ISN 0587          IP3L   =  IP3U*COS(SIGMAT)-RP3U*SIN(SIGMAT)
             C
ISN 0588          RCLU   =  (RP1U*XSTAR+RP2U*(XS-XSTAR)+RP3U*(C-XS))/B
ISN 0589          ICLU   =  (IP1U*XSTAR+IP2U*(XS-XSTAR)+IP3U*(C-XS))/B
             C
ISN 0590          RCLL   =  (RP1L*(XSTAR-DELTA*COS(ALPCH))+RP2L*(XS-XSTAR)+
                 ?          RP3L*(C-XS+DELTA*COS(ALPCH)))/B
ISN 0591          ICLL   =  (IP1L*(XSTAR-DELTA*COS(ALPCH))+IP2L*(XS-XSTAR)+
                 ?          IP3L*(C-XS+DELTA*COS(ALPCH)))/B
             C
ISN 0592          RCL    =  -RCLU+RCLL
ISN 0593          ICL    =  -ICLU+ICLL
             C
ISN 0594          RCMU   =  -(RP1U*XSTAR*(Z*C-XSTAR/2)+RP2U*(XS-XSTAR)*(Z*C-
                 ?          (XSTAR+XS)/2)+RP3U*((C-XS)*(Z*C-(C+XS)/2)))/(B**2)
             C
ISN 0595          ICMU   =  -(IP1U*XSTAR*(Z*C-XSTAR/2)+IP2U*(XS-XSTAR)*(Z*C-
                 ?          (XSTAR+XS)/2)+IP3U*((C-XS)*(Z*C-(C+XS)/2)))/(B**2)
             C
ISN 0596          RCML   =  (RP1L*(XSTAR-DELTA*COS(ALPCH))*(Z*C-(XSTAR-DELTA*
                 ?          COS(ALPCH))/2)+RP2L*(XS-XSTAR)*(Z*C-(XS+XSTAR-2*DELTA*
                 ?          COS(ALPCH))/2)+RP3L*(C-XS+DELTA*COS(ALPCH))*(Z*C-
                 ?          (XS+C-DELTA*COS(ALPCH))/2))/(B**2)
             C
ISN 0597          ICML   =  (IP1L*(XSTAR-DELTA*COS(ALPCH))*(Z*C-(XSTAR-DELTA*
                 ?          COS(ALPCH))/2)+IP2L*(XS-XSTAR)*(Z*C-(XS+XSTAR-2*DELTA*
                 ?          COS(ALPCH))/2)+IP3L*(C-XS+DELTA*COS(ALPCH))*(Z*C-
                 ?          (XS+C-DELTA*COS(ALPCH))/2))/(B**2)
             C
ISN 0598          RCM    =  RCMU+RCML
ISN 0599          ICM    =  ICMU+ICML
             C
ISN 0600          IF (II.EQ.1) GOTO 33
ISN 0602          AAR(2)  =   (-RCL/(PI*K**2*ALPBAR))/12
ISN 0603          AAI(2)  =   (-ICL/(PI*K**2*ALPBAR))/12
ISN 0604          BAR(2)  =   RCM/(PI*K**2*ALPBAR)/12
ISN 0605          BAI(2)  =   ICM/(PI*K**2*ALPBAR)/12
ISN 0606          AARK    =   (AAR(2)*K**2)
ISN 0607          AAIK    =   (AAI(2)*K**2)
ISN 0608          BARK    =   (BAR(2)*K**2)
ISN 0609          BAIK    =   (BAI(2)*K**2)
ISN 0610          AHRK    =   (AHR(2)*K**2)
ISN 0611          AHIK    =   (AHI(2)*K**2)
ISN 0612          BHRK    =   (BHR(2)*K**2)
ISN 0613          BHIK    =   (BHI(2)*K**2)
ISN 0614          GOTO 34
             C
ISN 0615       33 AHR(2)  =   (B*RCL/(PI*K**2*H))/12
ISN 0616          AHI(2)  =   (B*ICL/(PI*K**2*H))/12
ISN 0617          BHR(2)  =   -B*RCM/(PI*K**2*H)/12
ISN 0618          BHI(2)  =   -B*ICM/(PI*K**2*H)/12
             C
ISN 0619       34 WRITE(6,6666) LC
ISN 0620     6666 FORMAT('-',' LC',10(E8.2,3X))
ISN 0621          WRITE(6,440) RMAPI,IMAPI,RMAPT,IMAPT,RMAP,IMAP
```

```
ISN 0622            WRITE(6,450) RACE,IACE
ISN 0623            WRITE(6,460) RSSP,ISSP,RSOSU,ISOSU,RMSSPI,IMSSPI
ISN 0624            WRITE(6,470) RMSSPE,IMSSPE
ISN 0625            WRITE(6,480) RMPP,IMPP,RMPPU,IMPPU,RPDS,IPDS
ISN 0626            WRITE(6,490) RMPPI,IMPPI,RMPPE,IMPPE
ISN 0627            WRITE(6,500) RMDP,IMDP,RMDPU,IMDPU,RDDS,IDDS
ISN 0628            WRITE(6,510) RMDPI,IMDPI,RMDPE,IMDPE
ISN 0629            WRITE(6,520) USR,USI,RMSP,IMSP
ISN 0630            WRITE(6,530) V1AR,V1AI,V1BR,V1BI,V1CR,V1CI
ISN 0631            WRITE(6,540) V1R,V1I,V2R,V2I,V3AR,V3AI
ISN 0632            WRITE(6,550) V3BR,V3BI,V3CR,V3CI,V3R,V3I
ISN 0633            WRITE(6,560) RVPU,IVPU,RVPD,IVPD
ISN 0634            WRITE(6,570) RMVPU,IMVPU,RITVPI,IITVPI,RIAVPI,IIAVPI
ISN 0635            WRITE(6,580) RIAVPE,IIAVPE,RITVPE,IITVPE,RRAVPE,IRAVPE
ISN 0636            WRITE(6,590) RRTVPE,IRTVPE
ISN 0637            WRITE(6,600) RUDS,IUDS,WDSR,WDSI
ISN 0638            WRITE(6,610) CI,CR,IC,RC
ISN 0639            WRITE(6,620) RRFF,IRFF
ISN 0640            WRITE(6,630) RP1U,IP1U,RP2U,IP2U,RP3U,IP3U
ISN 0641            WRITE(6,640) RP1L,IP1L,RP2L,IP2L,RP3L,IP3L
ISN 0642            WRITE(6,650) RCLU,ICLU,RCLL,ICLL,RCL,ICL
ISN 0643            WRITE(6,660) RCMU,ICMU,RCML,ICML,RCM,ICM
ISN 0644       440  FORMAT('-',T11,'RMAPI',T28,'IMAPI',T45,'RMAPT',T62,'IMAPT',T79,
                  ?        'RMAP',T96,'IMAP'/4X,6(F15.6,2X))
              C
ISN 0645       450  FORMAT('-',T11,'RACE',T28,'IACE'/4X,2(F15.6,2X))
              C
ISN 0646       460  FORMAT('-',T11,'RSSP',T28,'ISSP',T45,'RSOSU',T62,'ISOSU',T79,
                  ?        'RMSSPI',T96,'IMSSPI'/4X,6(F15.6,2X))
              C
ISN 0647       470  FORMAT('-',T11,'RMSSPE',T28,'IMSSPE'/4X,2(F15.6,2X))
              C
ISN 0648       480  FORMAT('-',T11,'RMPP',T28,'IMPP',T45,'RMPPU',T62,'IMPPU',T79,
                  ?        'RPDS',T96,'IPDS'/4X,6(F15.6,2X))
              C
ISN 0649       490  FORMAT('-',T11,'RMPPI',T28,'IMPPI',T45,'RMPPE',T62,'IMPPE'
                  ?        /4X,4(F15.6,2X))
              C
ISN 0650       500  FORMAT('-',T11,'RMDP',T28,'IMDP',T45,'RMDPU',T62,'IMDPU',
                  ?        T79,'RDDS',T96,'IDDS'/4X,6(F15.6,2X))
              C
ISN 0651       510  FORMAT('-',T11,'RMDPI',T28,'IMDPI',T45,'RMDPE',T62,'IMDPE'
                  ?        /4X,4(F15.6,2X))
              C
ISN 0652       520  FORMAT('-',T11,'USR',T28,'USI',T45,'RMSP',T62,'IMSP'
                  ?        /4X,4(F15.6,2X))
              C
ISN 0653       530  FORMAT('-',T11,'V1AR',T28,'V1AI',T45,'V1BR',T62,'V1BI',T79,
                  ?        'V1CR',T96,'V1CI'/4X,6(F15.6,2X))
              C
ISN 0654       540  FORMAT('-',T11,'V1R',T28,'V1I',T45,'V2R',T62,'V2I',T79,
                  ?        'V3AR',T96,'V3AI'/4X,6(F15.6,2X))
              C
ISN 0655       550  FORMAT('-',T11,'V3BR',T28,'V3BI',T45,'V3CR',T62,'V3CI',T79,
                  ?        'V3R',T96,'V3I'/4X,6(F15.6,2X))
              C
ISN 0656       560  FORMAT('-',T11,'RVPU',T28,'IVPU',T45,'RVPD',T62,'IVPD'
                  ?        /4X,4(F15.6,2X))
              C
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  ISN 0657      570  FORMAT('-',T11,'RMVPU',T28,'IMVPU',T45,'RMITVPI',T62,'IMITVPI',   00011760
                   ?        T79,'RMIAVPI',T96,'IMIAVPI'/4X,6(F15.6,2X))               00011770
                  C                                                                   00011780
  ISN 0658      580  FORMAT('1',T11,'RMIAVPE',T28,'IMIAVPE',T45,'RMITVPE',T62,        00011790
                   ?        'IMITVPE',T79,'RMRAVPE',T96,'IMRAVPE'/4X,6(F15.6,2X))     00011800
                  C                                                                   00011810
  ISN 0659      590  FORMAT('-',T11,'RMRTVPE',T28,'IMRTVPE'/4X,2(F15.6,2X))           00011820
                  C                                                                   00011830
  ISN 0660      600  FORMAT('-',T11,'RUDS',T28,'IUDS',T45,'WDSR',T62,'WDSI'           00011840
                   ?        /4X,4(F15.6,2X))                                          00011850
                  C                                                                   00011860
  ISN 0661      610  FORMAT('-',T11,'CI',T28,'CR',T45,'IC',T62,'RC'                   00011870
                   ?        /4X,4(F15.6,2X))                                          00011880
                  C                                                                   00011890
  ISN 0662      620  FORMAT('-',T11,'REFF',T28,'IRFF'/4X,2(F15.6,2X))                 00011900
                  C                                                                   00011910
  ISN 0663      630  FORMAT('-',T11,'RP1U',T28,'IP1U',T45,'RP2U',T62,'IP2U',          00011920
                   ?        T79,'RP3U',T96,'IP3U'/4X,6(F15.6,2X))                     00011930
                  C                                                                   00011940
  ISN 0664      640  FORMAT('-',T11,'RP1L',T28,'IP1L',T45,'RP2L',T62,'IP2L',          00011950
                   ?        T79,'RP3L',T96,'IP3L'/4X,6(F15.6,2X))                     00011960
                  C                                                                   00011970
  ISN 0665      650  FORMAT('-',T11,'RCLU',T28,'ICLU',T45,'RCLL',T62,'ICLL',          00011980
                   ?        T79,'RCL',T96,'ICL'/4X,6(F15.6,2X))                       00011990
  ISN 0666      660  FORMAT('-',T11,'RCMU',T28,'ICMU',T45,'RCML',T62,'ICML',          00012000
                   ?        T79,'RCM',T96,'ICM'/4X,6(F15.6,2X))                       00012010
                  C                                                                   00012020
  ISN 0667       15  CONTINUE                                                         00012030
  ISN 0668           WRITE(6,670) AAR(2), AAI(2), BAR(2), BAI(2)                      00012040
  ISN 0669           WRITE(6,680) AHR(2), AHI(2), BHR(2), BHI(2)                      00012050
  ISN 0670           WRITE(6,690) AARK, AAIK, BARK, BAIK                              00012060
  ISN 0671           WRITE(6,700) AHRK, AHIK, BHRK, BHIK                              00012070
                  C                                                                   00012080
  ISN 0672      670  FORMAT('-',T11,'AAR',T28,'AAI',T45,'BAR',T62,'BAI'               00012090
                   ?        /4X,4(F15.6,2X))                                          00012100
  ISN 0673      680  FORMAT('-',T11,'AHR',T28,'AHI',T45,'BHR',T62,'BHI'               00012110
                   ?        /4X,4(F15.6,2X))                                          00012120
  ISN 0674      690  FORMAT('-',T11,'AARK',T28,'AAIK',T45,'BARK',T62,'BAIK'           00012130
                   ?        /4X,4(F15.6,2X))                                          00012140
  ISN 0675      700  FORMAT('-',T11,'AHRK',T28,'AHIK',T45,'BHRK',T62,'BHIK'           00012150
                   ?        /4X,4(F15.6,2X))                                          00012160
                  C                                                                   00012170
  ISN 0676           RETURN                                                           00012180
  ISN 0677           END                                                              00012190
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                 *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
 B      0009 0011 0017 0155 0155 0157 0440 0441 0442 0443 0446 0447 0459 0460 0461 0462 0465 0466 0503
        0503 0504 0506 0506 0507 0512 0512 0513 0513 0514 0515 0588 0589 0590 0591 0594 0595 0596 0597
        0615 0616 0617 0618
 C      0005 0009 0017 0021 0430 0431 0432 0432 0433 0433 0434 0434 0435 0435 0436 0437 0440 0441 0442
        0442 0443 0443 0446 0446 0447 0447 0503 0506 0512 0512 0512 0512 0513 0513 0513 0513 0514 0515
        0588 0589 0590 0591 0594 0594 0594 0594 0594 0595 0595 0595 0595 0595 0596 0596 0596 0596 0596
        0597 0597 0597 0597 0597
        0005 0017
 H      0005 0449 0450 0451 0452 0453 0454 0455 0456 0459 0460 0461 0461 0462 0462 0465 0466 0503 0504
        0506 0507 0615 0616 0617 0618
 I      0004 0020 0165 0165 0165 0166 0166 0166 0178 0179 0188 0198 0198 0199 0199 0202 0238 0239 0244
        0264
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                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
J       0230 0231 0231 0231 0235 0236 0236 0236 0256 0257 0257 0257 0257 0275 0276 0276 0276 0276 0290
        0291 0291 0291 0335 0356 0356 0356 0366 0367 0367 0367
        0004 0021 0022 0044 0045 0046 0046 0047 0047 0058 0062 0065 0074 0080 0083 0084 0084 0166 0166
        0168 0168 0168 0170 0170 0171 0171 0172 0172 0172 0173 0173 0173 0174 0174 0175 0175 0176 0176
        0199 0199 0201 0204 0204 0204 0205 0205 0206 0206 0207 0207 0209 0215 0216 0218 0224 0227 0239
        0241 0242 0242 0242 0243 0243 0243 0247 0252 0261 0261 0262 0262 0265 0272 0294 0304 0305 0306
        0307 0308 0309 0310 0311 0319 0320 0322 0324 0326 0327 0328 0329 0330 0331 0332 0333 0334 0335
        0336 0337 0338 0338 0339 0340 0341 0342 0343 0344 0344 0345 0346 0347 0348 0349 0350 0351 0352
        0353 0379 0379 0381 0382 0386 0386 0387 0387 0388 0389 0390 0390 0391 0391 0392 0393 0489 0490
        0491 0492 0526 0526 0527 0527 0528 0528 0528 0529 0529 0529 0531 0534 0535 0535 0536 0536 0543
        0543 0544 0544 0562 0563 0602 0603 0604 0605 0606 0607 0608 0609 0610 0611 0612 0613 0615 0616
        0617 0618
M       0003 0004 0312 0313 0314 0315 0316 0317 0318 0319 0320 0321 0322 0323 0324 0325 0326 0326 0327
        0327 0328 0328 0329 0329 0330 0330 0331 0331 0332 0332 0333 0333 0334 0334 0335 0335 0336 0336
        0337 0337 0338 0338 0339 0339 0340 0341 0342 0343 0344 0345 0346 0347 0348 0349 0350 0351 0352
        0353 0354 0354 0356 0356 0358 0358 0359 0359 0360 0360 0361 0361 0362 0362 0363 0363 0364 0364
        0365 0365 0411 0477 0477 0477 0477 0477 0477 0477 0477 0477 0477 0477 0477 0477 0477
R       0382 0383 0383 0383 0383 0383 0384 0384 0532 0532 0572 0573 0574 0575
S       0003 0369 0370 0371 0372 0373 0374 0375 0376 0376 0376 0377 0377 0377 0413 0479 0479 0480 0480
        0487 0487 0487 0487 0488 0488 0488 0488
Z       0017 0430 0431 0432 0433 0434 0435 0436 0437 0440 0441 0442 0443 0446 0447 0512 0513 0514 0515
        0594 0594 0594 0595 0595 0595 0596 0596 0596 0597 0597 0597
AC      0005 0159 0159 0165 0165 0165 0165 0166 0166 0166 0166 0198 0198 0198 0199 0199 0199 0238 0238
        0239 0239
AO      0005 0100 0160 0160 0161 0293 0326 0326 0327 0328 0329 0330 0331 0332 0333 0334 0335 0336 0337
        0338 0339 0340 0340 0341 0342 0343 0344 0345 0346 0347 0348 0349 0350 0351 0352 0353 0503 0506
        0512 0513 0524 0524 0525 0525 0528 0529
CI      0526 0554 0558 0638
CL      0003 0340 0341 0342 0343 0344 0345 0346 0347 0348 0349 0350 0351 0352 0353 0354 0354 0354 0356
        0356 0356 0356 0358 0358 0358 0358 0359 0359 0359 0359 0360 0360 0360 0360 0361 0361 0361 0361
        0362 0362 0362 0362 0363 0363 0363 0363 0364 0364 0364 0364 0365 0365 0365 0365 0367 0367 0367
        0367 0367 0412 0475 0475 0475 0475 0475 0475 0475 0475 0475 0475 0475 0475 0476 0476 0476 0476
        0476 0476 0476 0476 0476 0476 0476 0476
CO      0003 0044 0045 0046 0046 0047 0047 0058 0062 0065 0074 0080 0083 0165 0165 0165 0165 0166 0166
        0166 0166 0167 0167 0167 0167 0168 0168 0168 0168 0168 0168 0169 0170 0170 0170 0171 0171 0171
        0172 0172 0172 0173 0173 0173 0198 0198 0198 0199 0199 0199 0199 0200 0200 0201 0201 0204 0204
        0204 0205 0205 0205 0206 0206 0206 0207 0207 0207 0238 0238 0239 0239 0240 0241 0241 0242 0242
        0242 0243 0243 0243 0251 0261 0261 0262 0262 0262 0378 0378 0378 0378 0379 0379 0379 0379 0380
        0380 0381 0381 0382 0383 0383 0383 0383 0383 0384 0384 0385 0386 0386 0386 0387 0387 0387 0388
        0389 0390 0390 0390 0391 0391 0391 0392 0393 0491 0491 0491 0491 0491 0491 0491 0491 0491 0492
        0492 0492 0492 0492 0492 0492 0492 0492 0497 0497 0498 0498 0499 0499 0499 0499 0499 0500 0500
        0500 0500 0500 0530 0530 0531 0531 0532 0532 0533 0534 0534 0535 0535 0535 0536 0536 0536 0538
        0543 0543 0543 0544 0544 0544 0545 0562 0562 0562 0562 0562 0562 0562 0562 0562 0563 0563 0563
        0563 0563 0563 0563 0563 0563 0568 0568 0569 0569 0570 0570 0570 0570 0570 0571 0571 0571 0571
        0571 0572 0572 0572 0573 0573 0573 0574 0575
CR      0527 0554 0623
DD      0031 0100 0105 0116 0123 0123 0374 0522 0523 0524 0524 0525 0525
DE      0057 0105 0116 0124 0124 0162 0378 0378 0378 0379 0379 0379 0380 0381 0383 0526 0527 0528 0529
        0530 0530 0531 0531 0532 0533 0534 0562 0563 0566 0567
DI      0028 0100 0103 0116 0122 0123 0124 0125 0126 0127 0128 0129 0130 0131 0132 0133 0134 0135 0136
        0137 0138
DT      0102 0103 0104 0116 0125 0125 0182 0182 0183 0184 0184 0185 0192 0193 0194 0195 0203 0214 0225
        0243 0250 0268 0270 0296 0306 0307 0321 0334 0337 0347 0352
DU      0032 0104 0116 0122 0122 0161 0295 0326 0328 0340 0343 0374 0483 0484 0522 0523
FC      0003 0238 0239 0240 0241 0242 0242 0242 0242 0242 0242 0242 0243 0243 0243 0243 0243 0243 0243
        0244 0244 0244 0245 0245 0246 0246 0246 0247 0247 0247 0248 0248 0248 0249 0249 0249 0250 0250
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                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
        0251 0251 0251 0252 0252 0252 0253 0253 0253 0254 0254 0254 0254 0255 0255 0255 0255 0257 0257
        0257 0257 0259 0259 0260 0260 0261 0261 0261 0261 0261 0262 0262 0262 0262 0262 0263 0263 0263
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        0284 0284 0284 0285 0285 0285 0285 0286 0286 0286 0286 0287 0287 0287 0287 0288 0288 0288 0288
        0288 0289 0289 0289 0289 0289 0291 0291 0291 0291 0291 0409 0467 0467 0467 0467 0467 0467 0468
        0468 0468 0468 0468 0468
FE      0003 0163 0164 0165 0165 0165 0166 0166 0166 0167 0168 0169 0170 0170 0170 0170 0170 0170 0170
        0170 0171 0171 0171 0171 0171 0171 0171 0171 0172 0172 0172 0172 0172 0172 0172 0172 0173 0173
        0173 0173 0173 0173 0173 0173 0174 0175 0176 0177 0177 0177 0177 0177 0178 0178 0178 0178 0178
        0178 0179 0179 0179 0179 0179 0179 0180 0180 0180 0180 0180 0180 0180 0180 0180 0180 0180 0181
        0181 0181 0181 0181 0181 0181 0181 0181 0181 0181 0181 0181 0182 0182 0182 0182 0182 0182 0182
        0182 0183 0183 0183 0183 0183 0183 0184 0184 0184 0184 0184 0184 0184 0184 0184 0184 0184 0185
        0185 0185 0185 0185 0185 0185 0185 0185 0185 0185 0185 0185 0186 0186 0186 0186 0186 0186 0186
        0187 0187 0187 0187 0187 0187 0188 0188 0188 0188 0189 0189 0189 0189 0190 0190 0190 0190 0190
        0190 0190 0191 0191 0191 0191 0191 0191 0191 0192 0192 0192 0192 0193 0193 0193 0193 0194 0194
        0194 0194 0194 0194 0194 0195 0195 0195 0195 0195 0195 0195 0196 0196 0196 0196 0197 0197 0197
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        0211 0211 0212 0212 0213 0213 0214 0214 0215 0215 0216 0216 0217 0217 0218 0218 0219 0219 0220
        0220 0221 0221 0222 0222 0223 0223 0224 0224 0225 0225 0226 0226 0227 0227 0242 0242 0242 0243
        0243 0243 0244 0244 0245 0245 0245 0246 0246 0247 0247 0248 0248 0249 0249 0250 0250 0250 0251
        0251 0252 0252 0253 0253 0261 0261 0261 0262 0262 0262 0263 0263 0264 0264 0265 0265 0266 0266
        0267 0267 0268 0268 0269 0269 0270 0270 0271 0271 0272 0272 0405 0471 0471 0471 0471 0471 0471
        0471 0471 0471 0471 0472 0472 0472 0472 0472 0472 0472 0472 0472 0472
FM      0003 0198 0199 0200 0201 0202 0203 0204 0204 0204 0204 0204 0205 0205 0205 0205 0205 0206 0206
        0206 0206 0206 0207 0207 0207 0207 0207 0208 0208 0208 0209 0209 0209 0210 0210 0210 0210 0211
        0211 0211 0212 0212 0212 0212 0213 0213 0213 0214 0214 0214 0215 0215 0215 0216 0216 0216 0217
        0217 0217 0218 0218 0218 0219 0219 0219 0220 0220 0220 0220 0221 0221 0221 0221 0222 0222 0222
        0223 0223 0223 0224 0224 0224 0225 0225 0225 0226 0226 0226 0227 0227 0227 0228 0228 0228 0228
        0228 0229 0229 0231 0231 0231 0231 0233 0233 0233 0233 0234 0234 0234 0234 0236 0236 0236 0236
        0236 0254 0254 0255 0255 0257 0257 0273 0273 0274 0274 0276 0276 0408 0469 0469 0469 0469 0469
        0469 0469 0469 0470 0470 0470 0470 0470 0470 0470 0470
IC      0004 0528 0555 0559 0560 0561 0638
II      0427 0428 0501 0600
LC      0003 0004 0415 0530 0531 0532 0533 0534 0535 0535 0535 0535 0535 0535 0536 0536 0536 0536 0536
        0537 0537 0538 0538 0539 0539 0539 0539 0540 0540 0540 0540 0541 0541 0541 0542 0542 0542 0543
        0543 0543 0543 0543 0544 0544 0544 0544 0544 0545 0545 0546 0546 0547 0547 0547 0547 0548 0548
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        0553 0553 0553 0553 0553 0553 0554 0554 0554 0554 0555 0555 0555 0556 0556 0556 0556 0556 0557
        0557 0557 0557 0557 0558 0558 0558 0558 0559 0559 0559 0619
LM      0003 0004 0378 0379 0380 0381 0383 0384 0385 0386 0386 0386 0386 0386 0386 0386 0386 0387 0387
        0387 0387 0387 0387 0387 0387 0388 0388 0388 0388 0389 0389 0389 0389 0390 0390 0390 0390 0390
        0390 0390 0390 0391 0391 0391 0391 0391 0391 0391 0391 0392 0392 0392 0392 0393 0393 0393 0393
        0394 0394 0394 0394 0394 0395 0395 0395 0395 0395 0395 0396 0396 0396 0396 0396 0396 0397 0397
        0398 0398 0398 0398 0399 0399 0399 0399 0399 0400 0400 0400 0400 0400 0401 0401 0401 0401 0402
        0402 0402 0402 0414 0535 0535 0535 0536 0536 0536 0537 0539 0539 0540 0540 0541 0541 0542 0542
        0543 0543 0543 0544 0544 0544 0546 0547 0547 0548 0548 0549 0549 0550 0550 0560 0560 0560 0560
        0561 0561 0561 0561
MD      0004 0034 0045 0047 0047 0074 0080 0083
MI      0004 0014 0026 0027 0094 0114
MT      0004 0013 0025 0114
MU      0004 0035 0044 0046 0046 0058 0062 0065
PD      0029 0031 0115 0132 0132 0370 0528 0529
PE      0085 0087 0115 0133 0133 0528 0529
PI      0012 0602 0603 0604 0605 0615 0616 0617 0618
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*VERSION 1.3.0 (01 MAY 80)      UNST    SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)    DATE 80.353/15.02.49        PAGE  24

                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
PT      0025 0115 0134 0134 0203 0296
FU      0024 0029 0115 0131 0131 0295 0369 0370 0370 0372
RC      0529 0555 0559 0560 0561 0633
TE      0036 0057
UE      0091 0146 0146 0378 0378 0380 0382 0530 0533 0562 0563
US      0093 0371
VD      0099 0100 0103 0148 0148 0522 0523 0524 0524 0525 0525 0528 0529
VE      0090 0145 0145 0379 0379 0381 0382 0531 0534 0562 0563
VU      0093 0107 0147 0147 0161 0293 0295 0326 0327 0328 0328 0329 0330 0331 0332 0333 0334 0335 0336
        0337 0338 0339 0340 0341 0342 0343 0343 0344 0345 0346 0347 0348 0349 0350 0351 0352 0353 0522
        0523
XS      0005 0434 0435 0446 0446 0447 0447 0465 0466 0503 0506 0512 0512 0513 0513 0588 0588 0589 0589
        0590 0590 0591 0591 0594 0594 0594 0594 0595 0595 0595 0595 0596 0596 0596 0596 0597 0597 0597
        0597
AAI     0003 0603 0607 0668
AAR     0003 0602 0606 0668
ACE     0005 0158 0158 0162 0378 0378 0378 0379 0379 0379 0383 0503 0506 0530 0530 0531 0531 0532
AHI     0003 0611 0616 0669
AHR     0003 0610 0615 0669
BAI     0003 0605 0609 0668
BAR     0003 0604 0608 0668
BHI     0003 0613 0618 0669
BHR     0003 0612 0617 0669
BXD     0041 0066 0071 0071 0072 0079 0079 0082 0082
BXU     0040 0048 0053 0053 0054 0061 0061 0064 0064
BYD     0043 0047
BYU     0042 0046
B1I     0053 0062 0065 0170 0170 0171 0171 0172 0172 0173 0173 0204 0204 0205 0205 0206 0206 0207 0207
        0242 0242 0243 0243 0261 0261 0262 0262 0403 0491 0491 0492 0492 0499 0499 0500 0500
B1R     0054 0061 0064 0170 0170 0171 0171 0172 0172 0173 0173 0204 0204 0205 0205 0206 0206 0207 0207
        0242 0242 0243 0243 0261 0261 0262 0262 0403 0491 0491 0492 0492 0499 0499 0500 0500
B2I     0071 0080 0083 0306 0306 0307 0307 0390 0390 0391 0391 0535 0535 0536 0536 0543 0543 0544 0544
        0562 0562 0563 0363 0570 0570 0571 0571
B2R     0072 0079 0082 0366 0385 0387 0387 0390 0390 0391 0391 0535 0535 0536 0536 0543 0543 0544 0544
        0562 0562 0563 0363 0570 0570 0571 0571
COS     0007 0020 0034 0035 0036 0037 0038 0038 0039 0039 0090 0090 0091 0106 0108 0110 0111 0111 0163
        0166 0166 0168 0169 0199 0201 0239 0378 0378 0379 0379 0379 0379 0381 0383 0383 0383 0384 0385
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        0460 0461 0461 0461 0462 0465 0491 0491 0491 0492 0492 0492 0497 0498 0499 0499 0500 0500 0503
        0504 0512 0514 0514 0515 0526 0528 0529 0530 0531 0531 0532 0562 0562 0562 0563 0563 0563 0568
        0569 0570 0570 0571 0571 0572 0573 0582 0583 0584 0585 0586 0587 0590 0590 0591 0591 0596 0596
        0596 0596 0596 0597 0597 0597 0597 0597
D1D     0045 0066 0071 0072 0079 0079 0082 0082
D1U     0044 0048 0053 0054 0061 0061 0064 0064
D2D     0047 0066 0071 0079 0082
D2U     0046 0048 0053 0061 0064
FM2     0003 0293 0294 0295 0297 0297 0297 0297 0298 0298 0298 0299 0299 0299 0299 0299 0300 0300 0301
        0301 0301 0301 0302 0302 0303 0303 0303 0303 0304 0304 0304 0304 0305 0305 0305 0305 0306 0306
        0306 0306 0307 0307 0307 0307 0308 0308 0309 0309 0309 0309 0310 0310 0311 0311 0311 0311 0312
        0312 0312 0312 0313 0313 0313 0314 0314 0314 0315 0315 0315 0316 0316 0316 0317 0317 0317 0318
        0318 0318 0319 0319 0319 0320 0320 0320 0321 0321 0321 0322 0322 0322 0323 0323 0323 0324 0324
        0324 0325 0325 0325 0326 0327 0328 0329 0330 0331 0332 0333 0334 0335 0336 0337 0338 0339 0340
        0341 0342 0343 0344 0345 0346 0347 0348 0349 0350 0351 0352 0353 0410 0478 0478 0478 0478 0478
        0478 0478 0478 0478 0478 0478 0478 0478 0478
FRE     0162 0378 0379 0383 0383
ICL     0004 0593 0603 0616 0642
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                   *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
ICM     0004 0599 0605 0618 0643
MDS     0004 0005 0030 0096 0099 0114
MUS     0004 0005 0029 0097 0098 0114 0370 0371 0373 0375
MXO     0004 0036 0041 0045 0045 0080 0083
MXU     0004 0037 0040 0044 0044 0062 0065
MYD     0004 0004 0033 0043 0045 0047 0074 0080 0083
MYU     0004 0004 0039 0042 0044 0046 0058 0062 0065
PIN     0026 0028 0115 0130 0130 0202
RCL     0592 0602 0615 0642
RCM     0598 0604 0617 0643
REV     0089 0090 0091 0092 0108 0109 0109 0378 0378 0379 0379 0383 0526 0528 0529 0530 0531
SIN     0036 0037 0091 0110 0165 0165 0167 0170 0170 0170 0171 0171 0171 0172 0172 0172 0173 0173 0173
        0198 0200 0204 0204 0204 0205 0205 0205 0206 0206 0206 0207 0207 0207 0238 0242 0242 0242 0243
        0243 0243 0261 0261 0261 0262 0262 0262 0378 0378 0380 0383 0383 0384 0386 0386 0386 0387 0387
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        0451 0454 0455 0460 0462 0462 0466 0491 0491 0491 0492 0492 0492 0497 0498 0499 0499 0500 0500
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        0544 0544 0545 0562 0562 0562 0563 0563 0563 0568 0569 0570 0570 0571 0571 0572 0573 0582 0583
        0584 0585 0586 0587
SSI     0094 0101 0117 0139 0139 0163 0165 0167 0167 0167 0168 0168 0168 0198 0199 0200 0201 0238 0239
        0240 0241 0268 0493 0494 0495 0496
SST     0095 0101 0106 0107 0117 0140 0140 0164 0166 0180 0180 0181 0182 0182 0183 0190 0191 0192 0193
        0203 0208 0214 0219 0225 0246 0248 0266 0296 0306 0307 0321 0330 0334 0345 0351 0473 0474
SSU     0097 0098 0117 0143 0143 0293 0371 0371 0372 0373 0481 0482 0483 0484 0487 0487 0488 0488
TAU     0010 0011 0011 0033 0165 0165 0165 0165 0166 0166 0166 0166 0167 0167 0167 0167 0168 0168 0168
        0168 0169 0170 0170 0170 0171 0171 0171 0172 0172 0172 0173 0173 0173 0198 0198 0198 0199 0199
        0199 0200 0200 0201 0201 0204 0204 0204 0205 0205 0205 0206 0206 0206 0207 0207 0207 0238 0238
        0239 0239 0240 0241 0242 0242 0242 0243 0243 0243 0261 0261 0261 0262 0262 0262 0378 0378 0378
        0378 0379 0379 0379 0380 0380 0381 0381 0383 0383 0383 0384 0385 0386 0386 0386 0387 0387 0387
        0388 0389 0390 0390 0390 0391 0391 0391 0392 0393 0491 0491 0491 0491 0491 0491 0491 0491 0492
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        0563 0563 0563 0568 0568 0568 0569 0569 0569 0570 0570 0570 0570 0570 0571 0571 0571 0571 0571
        0572 0572 0572 0573 0573 0573
TDS     0030 0031
TIN     0027 0028
USI     0488 0490 0629
USR     0487 0489 0629
VEL     0111 0153 0153 0166 0166 0168 0168 0168 0199 0199 0201 0239 0241 0491 0492
V1I     0445 0464 0467 0468 0469 0470 0471 0472 0631
V1R     0444 0463 0467 0468 0469 0470 0471 0472 0631
V2I     0447 0466 0475 0476 0477 0478 0631
V2R     0446 0465 0475 0476 0477 0478 0631
V3I     0520 0527 0529 0632
V3R     0521 0526 0528 0632
WDS     0100 0528 0529
AAIK    0607 0670
AARK    0606 0670
AHIK    0611 0671
AHRK    0610 0671
BAIK    0609 0670
BARK    0608 0670
BHIK    0613 0671
BHRK    0612 0671
DTOT    0019 0032 0102 0126 0126
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                              *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
FLCN    0161  0304  0305  0318
IACE    0004  0437  0450  0526  0528  0622
ICLL    0004  0591  0593  0642
ICLU    0004  0589  0593  0642
ICML    0004  0597  0599  0643
ICMU    0004  0595  0599  0643
IDDS    0004  0480  0486  0523  0524  0527  0529  0627
INAP    0004  0435  0451  0475  0476  0477  0478  0524  0528  0621
INDP    0004  0469  0471  0472  0473  0475  0476  0477  0478  0627
INPP    0004  0473  0475  0476  0477  0478  0576  0578  0625
INSP    0004  0489  0506  0513  0629
IPDS    0004  0486  0488  0523  0530  0625
IP1L    0004  0583  0591  0597  0641
IP1U    0004  0576  0582  0583  0589  0595  0640
IP2L    0004  0585  0591  0597  0641
IP2U    0004  0578  0584  0585  0589  0595  0640
IP3L    0004  0587  0591  0597  0641
IP3U    0004  0580  0586  0587  0589  0595  0640
IRFF    0004  0559  0560  0561  0572  0573  0639
ISSP    0004  0467  0469  0470  0471  0472  0475  0476  0477  0478  0623
IUDS    0004  0523  0524  0528  0529  0637
IVPD    0004  0560  0562  0562  0562  0563  0563  0563  0568  0569  0570  0570  0571  0571  0633
IVPU    0004  0471  0491  0491  0491  0492  0492  0492  0497  0498  0499  0499  0500  0500  0633
MCPT    0004
M1RE    0004  0005  0014  0035  0037  0039
M2RE    0004  0006  0007  0034  0036  0038  0039
PTOT    0005  0019  0024  0025  0026  0085  0135  0135
QUOT    0229  0231  0233  0234  0280  0281  0282  0283  0284  0285  0286  0287
RACE    0436  0449  0527  0529  0622
RCLL    0590  0592  0642
RCLU    0588  0592  0642
RCML    0596  0598  0643
RCMU    0594  0598  0643
RDDS    0479  0485  0522  0525  0526  0528  0627
RGAS    0018  0019  0028  0031  0087  0088  0094  0095  0096  0097
RNAP    0434  0452  0475  0476  0477  0478  0525  0529  0621
RNDP    0470  0471  0472  0474  0475  0476  0477  0478  0627
RNPP    0474  0475  0476  0477  0478  0577  0579  0625
RNSP    0490  0503  0512  0629
RPDS    0485  0487  0529  0531  0625
RP1L    0582  0590  0596  0641
RP1U    0577  0582  0583  0588  0594  0640
RP2L    0584  0590  0596  0641
RP2U    0579  0584  0585  0588  0594  0640
RP3L    0586  0590  0596  0641
RP3U    0581  0586  0587  0588  0594  0640
RRFF    0555  0560  0561  0572  0573  0639
RSSP    0468  0469  0470  0471  0472  0475  0476  0477  0478  0623
RUDS    0522  0525  0528  0529  0637
RVPD    0561  0562  0562  0562  0563  0563  0563  0568  0569  0570  0570  0571  0571  0633
RVPU    0472  0491  0491  0491  0492  0492  0492  0497  0498  0499  0499  0500  0500  0633
SQRT    0053  0061  0064  0071  0079  0082  0088  0094  0095  0096  0097
TTOT    0005  0019  0027  0030  0086  0088  0094  0095  0096  0097
UNST    0002
VOL1    0005  0155  0155  0167  0167  0167  0167  0168  0168  0168  0168  0174  0174  0176  0200  0200  0201  0201  0209
        0215  0218  0224  0240  0241  0247  0265  0407
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                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
VOL2    0005 0156 0156 0294 0304 0305 0306 0307 0308 0309 0319 0320 0322 0326 0327 0328 0329 0330 0331
        0332 0333 0334 0335 0336 0337 0338 0339 0340 0341 0342 0343 0344 0344 0345 0346 0347 0348 0349
        0350 0351 0352 0353
VOL3    0005 0157 0157 0380 0380 0381 0381 0384 0526 0527 0528 0528 0529 0529 0533 0534
V1AI    0439 0445 0458 0464 0630
V1AR    0439 0444 0457 0463 0630
V1BI    0441 0445 0460 0464 0630
V1BR    0440 0444 0459 0463 0630
V1CI    0443 0445 0462 0464 0630
V1CR    0442 0444 0461 0463 0630
V3AI    0506 0513 0520 0631
V3AR    0503 0512 0521 0631
V3BI    0507 0515 0520 0632
V3BR    0504 0514 0521 0632
V3CI    0508 0517 0520 0632
V3CR    0505 0516 0521 0632
WDSI    0524 0526 0528 0637
WDSR    0525 0527 0529 0637
ALFCH   0005 0007 0020 0034 0035 0036 0036 0037 0037 0038 0038 0039 0039 0090 0090 0091 0091 0106 0108
        0110 0110 0111 0111 0163 0165 0165 0166 0166 0167 0168 0198 0199 0200 0201 0238 0239 0378 0378
        0378 0378 0379 0379 0379 0379 0380 0381 0383 0383 0383 0383 0383 0384 0384 0440 0441 0442 0442
        0443 0443 0459 0460 0461 0461 0462 0462 0504 0507 0514 0515 0526 0528 0529 0530 0530 0531 0531
        0532 0532 0590 0590 0591 0591 0596 0596 0596 0596 0596 0597 0597 0597 0597 0597
ASTAR   0005 0100 0155 0156 0157 0158 0159 0160 0430 0431 0432 0433 0434 0435 0436 0437 0440 0441 0442
        0443 0446 0447 0449 0450 0451 0452 0453 0454 0455 0456 0459 0460 0461 0461 0462 0462 0465 0466
        0503 0503 0504 0506 0506 0507 0512 0512 0513 0513 0514 0515
AVGD1   0103 0127 0127 0167 0167 0168 0168 0175 0175 0176 0176 0200 0201 0215 0216 0224 0227 0252 0272
AVGD2   0104 0128 0128 0304 0305 0306 0307 0310 0311 0319 0320 0324 0338 0351
AVGD3   0105 0129 0129 0380 0381 0384 0526 0527 0528 0528 0529 0529
AVGV1   0106 0149 0149 0167 0167 0168 0168 0174 0175 0176 0176 0200 0201 0209 0216 0218 0227 0406
AVGV2   0107 0150 0150 0294 0308 0309 0310 0311 0322 0324
AVGV3   0108 0151 0151 0380 0381 0528 0528 0529 0529
BETA1   0005 0020 0035 0037 0106 0110 0111 0163
BETA2   0005 0007 0034 0036 0038 0039 0090 0091 0108 0378 0378 0379 0379 0383 0526 0528 0529 0530 0531
COTAN   0461 0462
DELTA   0005 0010 0440 0441 0442 0442 0443 0443 0459 0460 0461 0462 0504 0507 0514 0515 0590 0590 0591
        0591 0596 0596 0596 0596 0596 0597 0597 0597 0597 0597
FM23A   0296 0302 0303 0316
GAMMA   0015 0024 0024 0024 0025 0025 0025 0026 0026 0026 0027 0029 0029 0030 0032 0032 0065 0085 0085
        0086 0088 0088 0094 0094 0095 0095 0096 0096 0097 0097 0102 0102 0163 0163 0164 0164 0164 0164
        0165 0166 0167 0167 0167 0168 0168 0168 0174 0174 0175 0175 0176 0176 0369 0369 0370 0370 0370
        0370 0371 0371 0371 0371 0372 0372 0373 0374 0374 0375 0483 0484 0495 0496 0566 0567
IMAPI   0004 0431 0455 0467 0468 0469 0470 0471 0472 0621
IMAPT   0004 0433 0434 0467 0468 0469 0470 0471 0472 0475 0476 0477 0478 0621
INDPE   0004 0565 0628
INDPI   0004 0494 0628
INDPU   0004 0478 0480 0482 0523 0627
INFPE   0004 0563 0565 0567 0590 0626
INPPI   0004 0492 0494 0496 0576 0626
INPFU   0004 0482 0484 0488 0578 0625
INVPU   0004 0476 0477 0478 0480 0488 0523 0634
ISOSU   0004 0484 0488 0623
MEXIT   0004 0007 0085 0086 0088 0114
OMEGA   0005 0021
PRES1   0005 0136 0136
PRES2   0005 0137 0137
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                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
SYMBOL  INTERNAL STATEMENT NUMBERS
FRES3   0005 0133 0133
QUANT   0048 0049 0058 0059 0066 0067 0074 0077
RMAPI   0430 0453 0467 0468 0469 0470 0471 0472 0621
RMAPT   0432 0456 0467 0468 0469 0470 0471 0472 0475 0476 0477 0478 0621
RMDPE   0564 0623
RMDPI   0493 0628
RMDPU   0477 0479 0481 0522 0627
RMPPE   0562 0564 0566 0581 0626
RMPPI   0491 0493 0495 0577 0626
RMPPU   0481 0483 0487 0579 0625
RMVPU   0475 0477 0478 0479 0487 0522 0634
RSQSU   0483 0487 0623
SQSDS   0096 0099 0117 0142 0142 0435 0486
SQSEX   0083 0039 0117 0141 0141 0378 0379 0380 0381 0530 0531 0533 0534 0564 0565 0566 0567
U1IRE   0005 0021 0100 0106 0109 0110 0111 0130 0131 0132 0133 0134 0135 0136 0137 0138 0139 0140 0141
        0142 0143 0144 0145 0146 0147 0148 0149 0150 0151 0152 0153 0154
U2IRE   0092 0154 0134 0162 0527
VELAX   0110 0152 0152 0165 0165 0167 0167 0167 0198 0198 0200 0238 0240 0491 0492
XSTAR   0005 0432 0433 0442 0442 0443 0443 0446 0446 0447 0447 0461 0462 0465 0466 0588 0588 0589 0589
        0590 0590 0591 0591 0594 0594 0594 0594 0595 0595 0595 0595 0596 0596 0596 0596 0597 0597 0597
        0597
ALFBAR  0005 0430 0431 0432 0433 0434 0435 0436 0437 0440 0441 0442 0443 0446 0447 0512 0513 0514 0515
        0602 0603 0604 0605
AVGSQS  0101 0144 0144 0167 0167 0168 0168 0174 0175 0176
IIAVPE  0004 0571 0635
IIAVPI  0004 0500 0634
IITVPE  0004 0569 0635
IITVPI  0004 0493 0634
INSQPE  0004 0567 0624
INSQPI  0004 0496 0623
IRAVPE  0004 0573 0575 0635
IRTVPE  0004 0575 0636
MSHOCK  0004 0005 0024 0032
MXSTAR  0004 0016 0095 0102
RIAVPE  0570 0635
RIAVPI  0499 0634
RITVPE  0568 0635
RITVPI  0497 0634
RMSSPE  0566 0624
RMSSPI  0495 0623
RRAVPE  0572 0574 0635
RRTVPE  0574 0636
SIGMAB  0005 0033 0449 0450 0451 0452 0453 0454 0455 0456 0459 0460 0461 0461 0462 0462 0465 0466 0503
        0504 0506 0507
SIGMAT  0005 0430 0431 0432 0433 0434 0435 0436 0437 0440 0441 0442 0443 0446 0447 0512 0513 0514 0515
        0582 0582 0583 0583 0584 0584 0585 0585 0586 0586 0587 0587

                    *****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****
LABEL  DEFINED  REFERENCES
 10    0232     0230
 13    0449     0428
 15    0667     0427
 17    0467     0448
 20    0237     0235
 24    0512     0501
 25    0520     0511
```

*****F O R T R A N   C R O S S   R E F E R E N C E   L I S T I N G*****

LABEL	DEFINED	REFERENCES	
30	0258	0256	
33	0615	0600	
34	0619	0614	
40	0277	0275	
50	0292	0290	
60	0357	0355	
70	0368	0366	
71	0052	0051	
75	0057	0056	
76	0070	0069	
78	0076	0075	
80	0056	0049	
84	0064	0059	
90	0066	0055	0063
91	0074	0067	
94	0032	0077	
95	0023	0022	
99	0034	0073	0081
100	0113	0112	
105	0118	0114	
110	0119	0116	
115	0120	0115	
120	0121	0117	
333	0404	0403	
410	0510	0509	
420	0519	0518	
440	0644	0621	
450	0645	0622	
460	0646	0623	
470	0647	0624	
480	0648	0625	
490	0649	0626	
500	0650	0627	
510	0651	0628	
520	0652	0629	
530	0653	0630	
540	0654	0631	
550	0655	0632	
560	0656	0633	
570	0657	0634	
580	0658	0635	
590	0659	0636	
600	0660	0637	
610	0661	0638	
620	0662	0639	
630	0663	0640	
640	0664	0641	
650	0665	0642	
660	0666	0643	
670	0672	0668	
680	0673	0669	
690	0674	0670	
700	0675	0671	
1000	0008	0006	
6000	0416	0405	
6001	0419	0408	

*****F O R T R A N C R O S S R E F E R E N C E L I S T I N G*****

LABEL	DEFINED	REFERENCES
6002	0420	0409
6003	0421	0410
6004	0422	0411
6005	0423	0412
6006	0424	0413
6007	0425	0414
6008	0426	0415
6100	0417	0406
6200	0418	0407
6666	0620	0619

/ UNST / SIZE OF PROGRAM 006992 HEXADECIMAL BYTES

NAME	TAG	TYPE	ADD.	NAME	TAG	TYPE	ADD.	NAME	TAG	TYPE	ADD.	NAME	TAG	TYPE	ADD.
B	SF	R*4	000D50	C	F C	R*4	000044	E	F C	R*4	000014	H	F C	R*4	000064
I	SF	R*4	000D54	J	SF	I*4	000D58	K	SF	R*4	000D5C	M	SF	R*4	00105C
R	SF	R*4	000D60	S	SF	R*4	0010CC	Z	SF	R*4	000D64	AC	SF C	R*4	000018
AO	SF C	R*4	00001C	CI	SF	R*4	000D68	CL	SF	R*4	0010F0	CO	SFA	R*4	000D6C
CR	SF	R*4	000D70	DD	SF	R*4	000D74	CE	SF	R*4	000D78	DI	SF	R*4	000D7C
DT	SF	R*4	000D80	DU	SF	R*4	000D84	FC	SF	R*4	00118C	FE	SF	R*4	001270
FM	SF	R*4	0012FC	IC	SF	R*4	000D88	II	SF	I*4	000D8C	LC	SF	R*4	0013B8
LM	SF	R*4	001428	ND	SF	R*4	000D90	MI	SFA	R*4	000D94	MT	SF	R*4	000D98
NU	SF	R*4	000D9C	FD	SF	R*4	000DA0	PE	SF	R*4	000DA4	PI	SF	R*4	000DA8
PT	SF	R*4	000DAC	PU	SF	R*4	000DB0	RC	SF	R*4	000DB4	TE	SF	R*4	000DB8
UE	SF	R*4	000DBC	US	SF	R*4	000DC0	VD	SF	R*4	000DC4	VE	SF	R*4	000DC8
VU	SF	R*4	000DCC	XS	F C	R*4	000030	AAI	SF	R*4	001488	AAR	SF	R*4	001490
ACE	SF C	R*4	00000C	AHI	SF	R*4	001498	AHR	SF	R*4	0014A0	BAI	SF	R*4	0014A8
BAR	SF	R*4	0014B0	BHI	SF	R*4	0014B8	BHR	SF	R*4	0014C0	BXD	SFA	R*4	000DD0
BXU	SFA	R*4	000DD4	BYD	SF	R*4	000DD8	BYU	SF	R*4	000DDC	B1I	SF	R*4	000DE0
B1R	SF	R*4	000DE4	B2I	SF	R*4	000DE8	B2R	SF	R*4	000DEC	COS	F XF	R*4	000000
D1D	SFA	R*4	000DF0	D1U	SFA	R*4	000DF4	D2D	SFA	R*4	000DF8	D2U	SFA	R*4	000DFC
FM2	SF	R*4	0014C8	FRE	SF	R*4	000E00	ICL	SF	R*4	000E04	ICM	SF	R*4	000E08
NDS	FA C	R*4	000024	NUS	FA C	R*4	000028	MXD	SF	R*4	000E0C	MXU	SF	R*4	000E10
MYD	SF	R*4	000E14	MYU	SF	R*4	000E18	PIN	SF	R*4	000E1C	RCL	SF	R*4	000E20
PCM	SF	R*4	000E24	REV	SF	R*4	000E28	SIN	F XF	R*4	000000	SSI	SF	R*4	000E2C
SST	SF	R*4	000E30	SSU	SF	R*4	000E34	TAU	SFA	R*4	000E38	TDS	SF	R*4	000E3C
TIN	SF	R*4	000E40	USI	SF	R*4	000E44	USR	SF	R*4	000E48	VEL	SF	R*4	000E4C
V1I	SF	R*4	000E50	V1R	SF	R*4	000E54	V2I	SF	R*4	000E58	V2R	SF	R*4	000E5C
V3I	SF	R*4	000E60	V3R	SF	R*4	000E64	WDS	SF	R*4	000E68	AAIK	SF	R*4	000E6C
AARK	SF	R*4	000E70	AHIK	SF	R*4	000E74	AHRK	SF	R*4	000E78	BAIK	SF	R*4	000E7C
BARK	SF	R*4	000E80	BHIK	SF	R*4	000E84	BHRK	SF	R*4	000E88	DTOT	SF	R*4	000E8C
FLOW	SF	R*4	000E90	IACE	SF	R*4	000E94	ICLL	SF	R*4	000E98	ICLU	SF	R*4	000E9C
ICML	SF	R*4	000EA0	ICMU	SF	R*4	000EA4	IDDS	SF	R*4	000EA8	IMAP	SF	R*4	000EAC
IMDP	SF	R*4	000EB0	IMPP	SF	R*4	000EB4	IMSP	SF	R*4	000EB8	IPDS	SF	R*4	000EBC
IP1L	SF	R*4	000EC0	IP1U	SF	R*4	000EC4	IP2L	SF	R*4	000EC8	IP2U	SF	R*4	000ECC
IP3L	SF	R*4	000ED0	IP3U	SF	R*4	000ED4	IRFF	SF	R*4	000ED8	ISSP	SF	R*4	000EDC
IUDS	SF	R*4	000EE0	IVPD	SF	R*4	000EE4	IVPU	SF	R*4	000EE8	MCPT		R*4	NR
M1RE	F C	R*4	000048	M2RE	SF	R*4	000EEC	PTOT	SF C	R*4	00004C	QUOT	SF	R*4	000EF0
RACE	SF	R*4	000EF4	RCLL	SF	R*4	000EF8	RCLU	SF	R*4	000EFC	RCML	SF	R*4	000F00
RCMU	SF	R*4	000F04	RDDS	SF	R*4	000F08	RGAS	SFA	R*4	000F0C	RMAP	SF	R*4	000F10
RMDP	SF	R*4	000F14	RMPP	SF	R*4	000F18	RMSP	SF	R*4	000F1C	RPDS	SF	R*4	000F20
RP1L	SF	R*4	000F24	RP1U	SF	R*4	000F28	RP2L	SF	R*4	000F2C	RP2U	SF	R*4	000F30
RP3L	SF	R*4	000F34	RP3U	SF	R*4	000F38	RRFF	SF	R*4	000F3C	RSSP	SF	R*4	000F40
RUDS	SF	R*4	000F44	RVPD	SF	R*4	000F48	RVPU	SF	R*4	000F4C	SQRT	F XF	R*4	000000
TTOT	FA C	R*4	00002C	UNST		R*4	000F50	VOL1	SF C	R*4	000000	VOL2	SF C	R*4	000004
VOL3	SF C	R*4	000008	V1AI	SF	R*4	000F54	V1AR	SF	R*4	000F58	V1BI	SF	R*4	000F5C

V1DR	SF	R*4	000F60	V1CI	SF	R*4	000F64	V1CR	SF	R*4	000F68	V3AI	SF	R*4	000F6C
V3AR	SF	R*4	000F70	V3DI	SF	R*4	000F74	V3DR	SF	R*4	000F78	V3CI	SF	R*4	000F7C
V3CR	SF	R*4	000F80	WDSI	SF	R*4	000F84	WDSR	SF	R*4	000F88	ALPCH	FA C	R*4	000058
ASTAR	F C	R*4	000010	AVGD1	SF	R*4	000F8C	AVGD2	SF	R*4	000F90	AVGD3	SF	R*4	000F94
AVGV1	SF	R*4	000F98	AVGV2	SF	R*4	000F9C	AVGV3	SF	R*4	000FA0	BETA1	FA C	R*4	000050
BETA2	FA C	R*4	000054	COTAN	F XF	R*4	000000	DELTA	F C	R*4	000068	FM23A	SF	R*4	000FA4
GAMMA	SFA	R*4	000FA8	IMAPI	SF	R*4	000FAC	IMAPT	SF	R*4	000FB0	INDPE	SF	R*4	000FB4
INDPI	SF	R*4	000FB8	INDPU	SF	R*4	000FBC	INPPE	SF	R*4	000FC0	INPPI	SF	R*4	000FC4
INPPU	SF	R*4	000FC8	INVPU	SF	R*4	000FCC	ISOSU	SF	R*4	000FD0	MEXIT	SFA	R*4	000FD4
OMEGA	F C	R*4	00006C	PRES1	SF C	R*4	000038	PRES2	SF C	R*4	00003C	PRES3	SF C	R*4	000040
QUANT	S	R*4	000FD8	RMAPI	SF	R*4	000FDC	RMAPT	SF	R*4	000FE0	RNDPE	SF	R*4	000FE4
RNDPI	SF	R*4	000FE8	RNDPU	SF	R*4	000FEC	RMPPE	SF	R*4	000FF0	RMPPI	SF	R*4	000FF4
RMPPU	SF	R*4	000FF8	RMVPU	SF	R*4	000FFC	RSOSU	SF	R*4	001000	SOSDS	SF	R*4	001004
SOSEX	SF	R*4	001008	U1IRE	F C	R*4	000074	U2IRE	SF	R*4	00100C	VELAX	SF	R*4	001010
XSTAR	F C	R*4	000034	FRXPR#	XF	R*4	000000	ALPBAR	F C	R*4	000070	AVGSOS	SF	R*4	001014
IBCOM#	F XF	R*4	000000	IIAVPE	SF	R*4	001018	IIAVPI	SF	R*4	00101C	IITVPE	SF	R*4	001020
IITVPI	SF	R*4	001024	IMSSPE	SF	R*4	001028	IMSSPI	SF	R*4	00102C	IRAVPE	SF	R*4	001030
IRTVPE	SF	R*4	001034	MSHOCK	F C	R*4	000020	MXSTAR	SFA	R*4	001038	RIAVPE	SF	R*4	00103C
RIAVPI	SF	R*4	001040	RITVPE	SF	R*4	001044	RITVPI	SF	R*4	001048	RMSSPE	SF	R*4	00104C
RMSSPI	SF	R*4	001050	RRAVPE	SF	R*4	001054	RRTVPE	SF	R*4	001058	SIGMAB	FA C	R*4	00005C
SIGMAT	FA C	R*4	000060												

***** COMMON INFORMATION *****

NAME OF COMMON BLOCK * * SIZE OF BLOCK 000078 HEXADECIMAL BYTES

VAR. NAME	TYPE	REL. ADDR.	VAR. NAME	TYPE	REL. ADDR.	VAR. NAME	TYPE	REL. ADDR.	VAR. NAME	TYPE	REL. ADDR.
VOL1	R*4	000000	VOL2	R*4	000004	VOL3	R*4	000008	ACE	R*4	00000C
ASTAR	R*4	000010	E	R*4	000014	AC	R*4	000018	AO	R*4	00001C
MSHOCK	R*4	000020	MDS	R*4	000024	MUS	R*4	000028	TTOT	R*4	00002C
XS	R*4	000030	XSTAR	R*4	000034	PRES1	R*4	000038	PRES2	R*4	00003C
PRES3	R*4	000040	C	R*4	000044	M1RE	R*4	000048	PTOT	R*4	00004C
BETA1	R*4	000050	BETA2	R*4	000054	ALFCH	R*4	000058	SIGMAB	R*4	00005C
SIGMAT	R*4	000060	H	R*4	000064	DELTA	R*4	000068	OMEGA	R*4	00006C
ALFBAR	R*4	000070	U1IRE	R*4	000074						

SOURCE STATEMENT LABELS

LABEL	ISN	ADDR	LABEL	ISN	ADDR	LABEL	ISN	ADDR	LABEL	ISN	ADDR
80	56	001ED2	84	64	001F2E	90	66	001F6E	91	74	001FCC
94	82	00204E	99	84	0020CE	10	232	0031DE	20	237	00322E
30	258	0034D2	40	277	00368A	50	292	0037C6	60	357	003EE6
70	368	00400A	13	449	005166	17	467	0051F6	24	512	00539A
25	520	005900	33	615	0062CE	34	619	006322	15	667	006830

COMPILER GENERATED LABELS

LABEL	ISN	ADDR	LABEL	ISN	ADDR	LABEL	ISN	ADDR	LABEL	ISN	ADDR
100001	2	001B04	200001	27	001C76	200002	40	001D78	100002	51	001E6A
100003	61	001EEA	100004	69	001F86	100005	79	00200A	200003	93	00218A
200004	112	0022CA	200005	136	002492	200006	166	0026EE	200007	169	00284A
200008	173	002972	200009	177	002A48	200010	180	002D1A	200011	186	002DF2
200012	194	002CE0	200013	200	002DD8	200014	205	002E96	200015	208	002F42
200016	216	003052	200017	224	003130	100006	231	0031C8	100007	233	0031E2
100008	236	003218	100009	238	003232	200018	243	0032FE	200019	248	0033C0
200020	256	00340E	100010	257	003498	100011	259	0034B6	200021	264	003540
200022	272	003618	100012	276	003670	100013	278	00363E	200023	290	00379E
100014	291	0037AC	100015	293	0037CA	200024	305	003BD4	200025	312	0039D6
200026	321	003AF2	200027	329	003C1C	200028	336	003CC2	200029	343	003D74
200030	350	003E2C	100016	356	003EC8	100017	358	003EEA	200031	365	003FCA
100018	367	003FF4	100019	369	00400E	200032	379	0041A8	200033	382	004298

```
*VERSION 1.3.0 (01 MAY 80)      UNST    SYSTEM/370 FORTRAN H EXTENDED (ENHANCED)    DATE 80.353/15.02.49        PAGE  32
   200034  337  0043D6          200035  391  004476          200036  396  004522          200037  408  004634
   100020  423  0050DE          100021  430  0050C6          200038  441  00511A          200039  447  00515A
   200040  462  0051DE          200041  471  00530A          200042  476  005468          200043  479  0055FE
   200044  492  00577C          200045  499  0058OA          100022  503  00582C          200046  529  005A7A
   200047  533  0058CE          200048  537  005DC8          200049  544  005C74          200050  550  005D08
   200051  559  005DDA          200052  563  005ECC          200053  569  005F5C          200054  573  005F92
   200055  587  006096          200056  595  006194          200057  593  006202          100023  602  006224
   200058  628  0064D0          200059  636  0066C0          100024  668  006890
  FORMAT STATEMENT LABELS
    LABEL  ISN   ADDR            LABEL  ISN   ADDR            LABEL  ISN   ADDR            LABEL  ISN   ADDR
     1000    8  000028              95   23  00002D              71   52  00003A              75   57  0000EB
       76   70  00007C              78   76  00009F             100  113  0000C2             105  118  0000FB
      110  119  000126             115  120  00014D             120  121  000175             333  404  0001A3
     6000  416  000135            6100  417  0001C3            6200  418  0001DD            6001  419  0001F1
     6002  420  000204            6003  421  000217            6004  422  00022B            6005  423  00023E
     6006  424  000251            6007  425  000264            6008  426  000277             410  510  00028A
      420  519  000279            6666  620  0002AA             440  644  0002DC             450  645  000300
      460  646  000320             470  647  000366             480  648  00038A             490  649  0003CC
      500  650  000400             510  651  000442             520  652  000476             530  653  0004A4
      540  654  0004E4             550  655  000520             560  656  00055E             570  657  00058E
      580  658  0005DC             590  659  00062E             600  660  000654             610  661  000684
      620  662  0006AC             630  663  0006CC             640  664  00070C             650  665  00074C
      660  666  00070A             670  672  0007C8             680  673  0007F4             690  674  000820
      700  675  000850
*OPTIONS IN EFFECT*NAME(MAIN) OPTIMIZE(2) LINECOUNT(60) SIZE(MAX) AUTODBL(NONE)
*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK OBJECT MAP NOFORMAT GOSTMT XREF NOALC NOANSF TERM IBM FLAG(I)
*STATISTICS*     SOURCE STATEMENTS =      676, PROGRAM SIZE =     27026, SUBPROGRAM NAME =  UNST
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******                                   280K BYTES OF CORE NOT USED

*STATISTICS*     2 DIAGNOSTICS THIS STEP, HIGHEST SEVERITY CODE IS  8
```

FORTRAN H EXTENDED (ENHANCED)

*** FORTRAN H EXTENDED ERROR MESSAGES ***

```
LINE    00000800                  5 FORMAT('1CHANNEL FLOW CHOKE FLUTTER ANALYSIS DECK 9066'// 20A4//
LINE    00000810                   1' ALFBAR',F10.5,5X,'MEAN TORSIONAL DEFLECTION THRU CYCLE (DEG.)'/
LINE    00000820                   2' ALFCH ',F10.3,5X,'STAGGER ANGLE OF BLADE ROW (DEG.)'/
LINE    00000830                   3' BETA1 ',F10.3,5X,'INLET AIR ANGLE (DEG.)'/
LINE    00000840                   4' BETA2 ',F10.3,5X,'EXIT AIR ANGLE (DEG.)'/
LINE    00000850                   5' C     ',F10.5,5X,'CHORD (IN.)'/
LINE    00000860                   6' DELTA ',F10.5,5X,'GAP BETWEEN BLADES (IN.)'/
LINE    00000870                   7' DIAM  ',F10.3,5X,'NODAL DIAMETER'/
LINE    00000880                   8' E     ',F10.6,5X,'ELASTIC AXIS LOCATION REF. TO MIDSPAN'/
LINE    00000890                   9' EPS   ',F10.5,5X,'TOLERANCE FOR PRESSURE RATIO'/
LINE    00000900                   X' GAM   ',F10.5,5X,'SPECIFIC HEAT RATIO'/
LINE    00000910                   1' HBAR  ',F10.6,5X,'MEAN FLAPPING DEFLECTION OF BLADE (IN.)'/
LINE    00000920                   2' INAVE ',I10  ,5X,'WAVE MOTION'/
LINE    00000930                   3' MI    ',F10.5,5X,'INLET MACH NUMBER'/
LINE    00000940                   4' M1    ',F10.5,5X,'L. E. MACH NUMBER'/
LINE    00000950                   5' NB    ',I10  ,5X,'NUMBER OF BLADES'/
LINE    00000960                   6' NP    ',I10  ,5X,'NUMBER OF AIRFOIL COORDINATES'/
LINE    00000970                   7' NSECT ',I10  ,5X,'NUMBER OF SEGMENTS FROM L.E. TO T.E.'/
LINE    00000980                   8' NTIME ',I10  ,5X,'NUMBER OF TIME INCREMENTS'/
LINE    00000990                   9' OMEGA ',F10.2,5X,'FREQUENCY OF VIBRATION (CYCLES. / SEC.)'/
LINE    00001000                   X' PHIIDB',F10.4,5X,'BENDING MODE INTERBLADE PHASE ANGLE (RAD.)'/
LINE    00001010                   1' PHIIDT',F10.4,5X,'TORSIONAL MODE INTERBLADE PHASE ANGLE (RAD.)'/
LINE    00001020                   2' PR    ',F10.5,5X,'STATIC PRESSURE RATIO ACROSS STAGE'/
LINE    00001030                   3' V     ',F10.3,5X,'RELATIVE INLET VELOCITY (FT/SEC)')
IFE029I SEVERITY  8(E)   ISN 0032   THE NUMBER OF CONTINUATION CARDS EXCEEDS 19. COMPILER PROCESSING OF THE
                                    STATEMENT CONTINUES.

LINE    00002220                    WRITE(6,117) BETAPR1
IFE226I SEVERITY  8(E)   ISN 0119   THE STATEMENT HAS A VARIABLE WITH MORE THAN SIX CHARACTERS.
                                    THE RIGHTMOST CHARACTERS ARE TRUNCATED.

SOURCE STATEMENTS =     245, PROGRAM SIZE =      9340, SUBPROGRAM NAME =  MAIN

SOURCE STATEMENTS =     676, PROGRAM SIZE =     27026, SUBPROGRAM NAME =  UNST
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REFERENCES

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NOMENCLATURE

Symbol	*Definition*
A	Area between the blades
$A_{\pm\infty}$	Complex constant describing irrotational flow field
a	Speed of sound
B	Axial wave number
b	Semi-chord of the airfoil
C	Tangential wave number
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
$\frac{D}{Dt}$	Substantial derivative
e	Specific internal energy
F	Force on the control volume
h	Enthalpy
$\bar{h}$	Blade deflection in bending mode
k	Reduced frequency based on semi-chord
M	Mach number
m	Mass
n	Unit vector normal to surface
p	Pressure
R	Universal gas constant
S	Nondimensional interblade spacing
T	Temperature
U	Velocity along airfoil chord
u	Axial velocity component

NOMENCLATURE (Continued)

Symbol	*Definition*
$\tilde{u}$	Specific intrinsic energy
V	Volume
v	Tangential velocity component
$\dot{w}$	Mass flowrate
Z	Nondimensional elastic axis position
$Z_{+\infty}$	Complex constant describing rotational flow field
$\bar{\alpha}$	Mean torsional deflection
α_{ch}	Cascade stagger angle
β_1	Inlet air angle
β_2	Exit air angle
δ	Gap between the blades
γ	Ratio of specific heats
$\Phi'_{\pm\infty}$	Perturbation velocity potential
ρ	Density
τ	Gap between the blades
σ	Interblade phase lag
ψ	Stream function
ζ	Vorticity

Subscripts

$_{-\infty}$	Far upstream of blade row
$_{+\infty}$	Far downstream of blade row
$_x$	Axial component
$_y$	Tangential component
$_{ave}$	Average flow parameter

NOMENCLATURE

Subscripts	*Definition*
IRE	Relative inlet quantity
i	Inlet to blade row
E	Outlet from blade row
1,2,3	Average quantity in the control volume
us	Upstream of the shock
ds	Downstream of the shock
R	Real part
I	Imaginary part
IR	Irrotational component
S	Shock
E	Cascade exit
Superscripts	
$\bar{}$	Steady-state quantity
$'$	Perturbation quantity
$\bar{}'$	Mean perturbation quantity
*	Blade throat

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16. Abstract

Utilizing semi-actuator disk theory, a mathematical analysis was developed to predict the unsteady aerodynamic environment for a cascade of airfoils harmonically oscillating in choked flow. In the model, a normal shock is located in the blade passage, its position depending on the time dependent geometry and pressure perturbations of the system. In addition to shock dynamics, the model includes the effect of compressibility, interblade phase lag, and an unsteady flow field upstream and downstream of the cascade. The theory was evaluated by comparing calculated unsteady aerodynamics with isolated airfoil wind tunnel data and predicted choke flutter onset boundaries with data from testing of an F100 high pressure compressor stage.

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