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SUBPROGRAMS FOR INTEGRATING THE EQUATIONS OF MOTION OF SATELLITES.

FORTRAN FOUR.

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SUBPROGRAMS FOR INTEGRATING THE EQUATIONS OF MOTION OF SATELLITES, FORTRAN FOUR.

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In this work is contained a description of the subprograms intended for the formation of the right members of the equations of motion of artificial earth satellites (ISZ), integration of systems of differential equations by Adams' method, and the calculation of the values of various functions from the AES parameters of motion.

These subprograms are written in the Fortran IV language and constitute an essential part of the package of applied programs for the calculation of navigational parameters AES.
PREFACE

The present preprint is a continuation of the description, begun in (5), of the description of the library of subprograms arising in the process of creation of a packet of applied programs for the calculation of navigational parameters of artificial Earth satellites of the Earth (AES), wherein are contained the subprograms having indexes from F to I. In the subprograms for the calculation of the right members of the AES equations of motion, presented in Chapter I, the subprograms described in (5) are utilized. Here there operate also the principles of scaling the dimensional quantities, determined in (1). The constants and scale factors for the subprograms of chapters 1, 3, and 4 are routed to the domain COMMON by conversion to the subprogram CONST. The system of coordinates determined in (5) is employed.

The subprogram for integration of the system of differential equations by the method of Adams, described in Chapter 2, on the other hand, is sufficiently autonomous and may be used for integration of any system of ordinary differential equations.

In Chapter 3 are presented subprograms ensuring the fixation of the attainment in the process of integration of the assigned values of different functions from the solution of systems of differential equations; the minimum and maximum of an arbitrary continuous function from the solution as a function of an independent variable; the exit of the AES at the ascending node of the orbit; the minimal and maximal altitudes of the AES above the surface of the terrestrial ellipsoid.

Chapter 4 contains subprograms for the computation of the values
of various functions from the parameters of motion of the AES.

The subprogram ROOT4(Ill), used for the computation of the moments of entry of the AES into the umbra of the earth, is intended for the calculation of the roots of the algebraic equations of the fourth, third and second degree, has a significantly independent character and may be used for the solution of other problems.

The subprogram FA GRAV (F 03) was written by E. E. Ryazanov. For the computation of the geomagnetic parameters B, L the subprograms BL, INVAR, LINES, STAR, CARMEL, INTEG, NEWMAG, ASIN (I 04) are utilized, submitted through the courtesy of Yu. N. Gal'perin and V. M. Sinitsyn. The author of the remaining subprograms is the author of the preprint.

The author wishes to express his thanks to E. A. Chistyakova for assistance in the editing of the texts of the subprograms for publication, to L. V. Zaytseva and V. F. Smirnova for help in the preparation of the manuscript.
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Chapter I. Right members of the system of equations of motion of the AES
(index F)

I.1. Equation of motion of the AES. Various models.

We will write the differential equations of motion of the AES in an absolute system of coordinates in the general case in the following form:

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \Delta_N v_x + \Delta_A v_x + \Delta_D v_x + \Delta_g v_x + \Delta_L v_x, \\
\dot{v}_y &= \Delta_N v_y + \Delta_A v_y + \Delta_D v_y + \Delta_g v_y + \Delta_L v_y, \\
\dot{v}_z &= \Delta_N v_z + \Delta_A v_z + \Delta_D v_z + \Delta_g v_z + \Delta_L v_z.
\end{align*}
\]

(1.1)

In the Greenwich relative system of coordinates these equations have the form:

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \omega_x^2 x + 2\omega_x v_y + \Delta_N \dot{v}_x + \Delta_A \dot{v}_x + \Delta_D \dot{v}_x + \Delta_g \dot{v}_x + \Delta_L \dot{v}_x, \\
\dot{v}_y &= \omega_y^2 y + 2\omega_y v_x + \Delta_N \dot{v}_y + \Delta_A \dot{v}_y + \Delta_D \dot{v}_y + \Delta_g \dot{v}_y + \Delta_L \dot{v}_y, \\
\dot{v}_z &= \Delta_N \dot{v}_z + \Delta_A \dot{v}_z + \Delta_D \dot{v}_z + \Delta_g \dot{v}_z + \Delta_L \dot{v}_z.
\end{align*}
\]

(1.2)

where the projections of acceleration, determined by the influence of the corresponding forces, are:

*Numbers in the margin indicate pagination in the foreign text.*
\[ \Delta \mu, \Delta \nu, \Delta \tilde{v}, \Delta \tilde{v} \] -- normal gravitational field of the earth

\[ \Delta \mu, \Delta \nu, \Delta \tilde{v}, \Delta \tilde{v} \] -- resistance of the earth's atmosphere

\[ \Delta \mu, \Delta \nu, \Delta \tilde{v}, \Delta \tilde{v} \] -- gravitational anomalies

\[ \Delta \mu, \Delta \nu, \Delta \tilde{v}, \Delta \tilde{v} \] -- gravitational perturbations of the moon and sun

\[ \omega_1, \omega_2 \] -- centrifugal force

\[ 2\omega_3 \nu, 2\omega_3 \nu \] -- force of Coriolis

\[ \omega_3 \] -- angular velocity of the earth's rotation

For the calculation of the right members of the system of equations of motion of \( \text{AES} \) one has the collection of subprograms: FNGRAV, FAGRAV, FATM, FGRL, FLIGHT, each of which allows for its component in the right members of the equations of motion of the \( \text{AES} \).

These subprograms have a subsidiary nature; from them it is possible to construct the subprogram for calculation of the right-hand sides for a system of equations, with this or that degree of completion for the described motion of the \( \text{AES} \). In (4) were introduced the indexes KC, KG, KA, K3, KL and table 2.1, permitting the regulation of the variants of the system of forces, acting on the \( \text{AES} \) (and of the system of coordinates, in which the motion of the \( \text{AES} \) is analyzed).

Table 1.1 is a repetition of table 2.1 from work (4).

We recall also the designation of indexes.

The index KS characterizes the system of coordinates (possible values 1, 2).

To each of the forces acting is attached its index:

KG--force of the earth's attraction (possible values 0, 1, 2, 3, 4);
KA--atmospheric resistance (possible values 0, 1, 2, 3, 4);
KS--gravitational perturbation by the moon and sun (possible values 0, 1, 2, 3);
KL--pressure of light (possible values 0, 1, 2).

Giving to each of the above enumerated indexes the value determined, we give the determined model of forces, characterized by the five-valued index, consisting of the values of the indexes.
There is a subprogram FORCE, realizing all possible variants of the models of force, specified by table 1.1. However for concrete models it is possible to recommend to the user the creation of "truncated" subprograms, which may be obtained from the subprogram FORCE by means of discarding conversions to those subprograms, which are not utilized in the given model.

Table 1.1  System of forces, considered in the equations of motion of the AM. Indexes KC, KG, KA, KS, KL

<table>
<thead>
<tr>
<th>Systems of Coord.</th>
<th>Grav. Field</th>
<th>Atmosphere</th>
<th>Grav. Perturbation</th>
<th>Pressure of Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>KC 1</td>
<td>KO 2</td>
<td>KA 3</td>
<td>KS 4</td>
<td>KL 5</td>
</tr>
<tr>
<td>0 osculating ects.</td>
<td>central</td>
<td>not considered</td>
<td>not considered</td>
<td>not considered</td>
</tr>
<tr>
<td>1 Greenwich relative</td>
<td>normal</td>
<td>without calc. var.</td>
<td>from moon</td>
<td>without calculation of the earth's umbra</td>
</tr>
<tr>
<td>2 absolute monal harmonics</td>
<td>with calc. of long period var.</td>
<td>from sun</td>
<td>with calculation of the earth's umbra</td>
<td></td>
</tr>
<tr>
<td>3 full anomalies</td>
<td>with calc. of long and short period var.</td>
<td>from moon and sun</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 harmonics 22,30,40</td>
<td>CIRA-72</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The subprogram FC1100(F06), may serve as an example of such a subprogram; in this subprogram is realized the model of the motion of the Am, corresponding to the following values of the indexes: KC equals 1 or 2, KG equals 0 or 1, KA equals 1, KS equals 0, KL equals 0.

In order that the designations of concrete subprograms of the right-hand sides, created by the user, reflect the model, realized in them, of the motion of the Am, it is suggested that one construct identifiers of these subprograms according to this sort of principle: the first letter—F, after it a five-valued index, corresponding to the chosen variant of model from table 1.1 (as this was done in the case of FC1100).
1. Calculation of the normal gravitational field of the Earth and (for the case of the Greenwich system of coordinates) translational and Coriolis accelerations (FOI-FNGRAV).

1. Designation. The terms $\Delta_N \dot{v}_x, \Delta_N \dot{v}_y, \Delta_N \dot{v}_z$ are determined; they are dependent on the influence of the normal gravitational field of the Earth in the right members of the equations of motion of the AES (1.1) in the absolute system of coordinates. If the motion of the AES is calculated in the Greenwich system of coordinates (system of equations 1.2), then these sums are determined:

$$\Delta_N \dot{v}_x + \omega_y x + 2 \omega_z v_y, \Delta_N \dot{v}_y + \omega_z y - 2 \omega_x v_x, \Delta_N \dot{v}_z$$

2. Structure. The subprogram FNGRAV. General units: $/CA00/ , /CRZ/ , /CAE/ , /CAEL/ , /ComZP/ .

3. Conversion: CALL FNGRAV (KC, KG, Y, HC, RC, F)

4. Initial data: KC--index, characterizing the system of coordinates (KC = 1 for Greenwich system of coordinates, KC = 2 for absolute system of coordinates); KG--index, characterizing the gravitational field of the Earth (for KG equal to zero there is considered only the central field)

$V_0$--main part, containing $x, y, z, V_x, V_y, V_z$, or $x, y, z, v_x, v_y, v_z$

5. Results:

HC--elevation of AES above the surface of the Earth; RC--modulus of the radius-vector of the AES; $F_6$--main part of the right members of the equation of motion of the AES, containing $x, y, z, V_x, V_y, V_z$, or $i, j, z, \dot{x}, \dot{y}, \dot{z}$.

6. Utilization of the groups COMMON: The constants of the groups are utilized: $/CAOC/ , /CRZ/ , /CAE/ , /CAEL/ , /ComZP/ (see in § 3), Nos. 2, 10, 11, 12, 15 table 2.1)

7. Algorithm:

a) absolute system of coordinates;
\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \Delta_N v_x + \omega_z^2 x + 2 \omega_y v_y, \\
\dot{v}_y &= \Delta_N v_y + \omega_z^2 y - 2 \omega_x v_x, \\
\dot{v}_z &= \Delta_N v_z,
\end{align*}
\]

where \(C = 3/2 \omega_0^2 (R/r)^2\), \(D = 2/3 \omega_0^2 (R/r)^2\), \(r = (x^2 + y^2 + z^2)^{1/2}\), \(R\) - average radius of the earth, \(\alpha_0\), \(\alpha_{20}\) - parameters of the normal gravitational field of the earth; for \(\alpha_{20} = 0\) there is considered only the influence of the central gravitational field of the earth.

b) Greenwich system of coordinates:

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \\
\dot{v}_x &= \Delta_N v_x + \omega_z^2 x + 2 \omega_y v_y, \\
\dot{v}_y &= \Delta_N v_y + \omega_z^2 y - 2 \omega_x v_x, \\
\dot{v}_z &= \Delta_N v_z,
\end{align*}
\]

c) elevation \(h_c\) of the AES above the earth's surface ellipsoid is computed according to the formula \(h_c = r - (a_e - \alpha_{20} z^2 / r^2)\), where \(a_e\), \(\alpha_{20}\) - semimajor axis and constriction of the general terrestrial ellipsoid.

S. Text
1.3 Calculation of the influence of atmospheric resistance (FATM).

1. Designation. The terms \( \Delta A \dot{V}_X, \Delta A \dot{V}_Y, \Delta A \dot{V}_Z \) are determined, depending on the influence of the atmospheric resistance in the equations of (1.1) or (1.2).

2. Structure. Subprogram FATM.

General groups: /BSB/1, /COMMON/.


4. Initial data: KC--index, characterizing the system of coordinates; \( Y_0 \)--main part, containing \( X, Y, \dot{X}, V_X, V_Y, \dot{V}_X, \dot{V}_Y, x, y, z, v_x, v_y, v_z \); \( \dot{\rho} \)--density of the atmosphere; \( F_0 \)--main part of right members of equations of motion of system of units \( X, Y, \dot{X}, V_X, V_Y, \dot{V}_X, \dot{V}_Y, x, y, z, v_x, v_y, v_z \) with the computation of the influence of atmospheric resistance.

6. Use of the domain COMMON. Before conversion to the subprogram FATM in the group COMMON/BSB/SB it is necessary to address the value of the ballistic coefficient in the system of units, obtained from the calculations, for this it is sufficient that the value of the ballistic coefficient, given originally in the system of units kg, m, s, be divided by the scale factor from the group COMMON/BSB/SB (see in (5) No. 12 table 2.3).
From the group COMMON/COM: there is utilized the constant
(see in (5) No. 14 table 7.1)

7. Algorithm

$$\Delta \dot{V}_x = -3B \cdot P \cdot V_{an} \cdot V_x$$
$$\Delta \dot{V}_y = -3B \cdot P \cdot V_{an} \cdot V_y$$
$$\Delta \dot{V}_z = -3B \cdot P \cdot V_{an} \cdot V_z$$

$$\dot{V}_x = F_x + \Delta \dot{V}_x$$
$$\dot{V}_y = F_y + \Delta \dot{V}_y$$
$$\dot{V}_z = F_z + \Delta \dot{V}_z$$

where $SB =$ ballistic coefficient $SB = C_x \cdot F_m / 2m$

$C_x =$ dimensionless coefficient of air resistance

$F_m =$ square of the mid section, $m =$ mass of AES,
$I =$ air density

$V_{an}, V_{an}, V_{an} =$ components of the vector of flight speed relative
to the air

$$V_{an} = (V_{an} + V_{an} + V_{an})$$

In the Greenwich system of coordinates

$$V_{an} - V_x, V_{an} - V_y, V_{an} - V_z$$

In the absolute system of coordinates

$$V_{an} - V_x + \omega_3 Y, V_{an} - V_y - \omega_3 X, V_{an} - V_z$$

$F_x, F_y, F_z =$ other terms in the right members of equations (1.1)
or (1.7).

c. Text

```
SUBROUTINE FATHM(KC,Y,P,F)
COMMON /BBB/BB
COMMON /COMZ/OMZ
DIMENSION V(3),F(3),V(3)
DO 1 J=1,3
   1 V(J)=Y(J)+3
   GOTO(2,3,KC)
   2 V(2)=V(2)+OMZ=V(1)
   3 V(1)=V(1)+COMZ=V(2)
   4 W=W+V(J)+V(J)
   5 W=BB*POSRT(W)
   DO 9 J=1,3
   9 F(J+3)=F(J+3)+W/W
   RETURN
END
```

1.4. Calculation of anomalies of gravitational field of the earth
(F03-PAKAI)

1. Designation. The terms $\delta_{C} \dot{V}_x, \delta_{C} \dot{V}_y,$ $\delta_{C} \dot{V}_z,$ in
the right members of the system of equations (1.1) or (1.7), derived
from the influence of anomalies of the earth's gravitational field.
2. Structure. Subprogram FACRAV.

There are utilized the exterior subprograms: DEG2, DEG3, DEG4 (EU1).

General group: /RAD/.


4. Initial data.

The main part Y6, containing X, Y, Z, VX, VY, VZ or x, y, z,
VX, VY, VZ; the main part XG3, containing x, y, z;
F6--main part of right members of equations (1.1) or (1.2),
containing the terms, determined by the influence of other factors;
KG--index, determining the nature of the considered anomalies of the gravitational field of the earth (possible values: 2, 3, 4);

For KG=2 one considers only the zonal harmonics in the breakdown of the gravitational field of the earth, for KG=3 there are considered the zonal, tesseral, and sectorial harmonics, for KG=4 one considers only the harmonics 2°, 3°, 4°;
NM--number of considered harmonics (NM less than 22).

Note 1. For KG=2 or 3 it is necessary first to turn to the subprogram CONGR(A02) for the addresses in the group COMMCN/BCONGR/36 coefficients of the gravitational field of the earth. For KG=4 one uses the coefficients from the block COMMCN/CA4/4 (see No. 4 table 2.1), value NM is not used here.

Note 2. The main part XG is used only for KG=3 or 4.

5. Results: Main part F6 containing new values X, Y, Z, VX,
VY, VZ; or x, y, z, VX, VY, VZ with calculation of the influence of the anomalies of the earth's gravitational field.

6. Use of the domain COMMON. In the block COMMCN/RAD/RC, RL one addresses the values RC=r and RL=r1.

7. Algorithm:

\[
\begin{align*}
\Delta_0 \dot{V}_x &= -\Delta g_r X/r - \Delta g_m ZX/r_1 - \Delta g_c Y/r_1, \\
\Delta_0 \dot{V}_y &= -\Delta g_r Y/r - \Delta g_m ZY/r_1 + \Delta g_c X/r_1, \\
\Delta_0 \dot{V}_z &= -\Delta g_r Z/r + \Delta g_m r_1/r.
\end{align*}
\]
\[ V_X = F_X + \delta g \cdot V_X, \quad V_Y = F_Y + \delta g \cdot V_Y, \quad V_\gamma = F_\gamma + \delta g \cdot V_\gamma, \quad \text{where } r = (x^2 + y^2 + z^2)^{\frac{1}{2}}, \]

\[ r_\perp = (x^2 + y^2)^{\frac{1}{2}}, \quad F_X, \quad F_Y, \quad F_\gamma \quad \text{-- components of other terms in the right members of equations (1.1) or (1.7)}, \]

\[ \delta g_r \quad \text{-- radial component of vector } \delta g, \quad \text{acceleration due to the influence of anomalies of the earth's gravitational field}, \]

\[ \delta g_m \quad \text{-- meridional component of the vector } \delta g, \]

\[ \delta g_\perp \quad \text{-- projection of the vector } \delta g \text{ on the normal to the plane of the meridian.} \]

Note. The components of the vector \( \delta g \), for some or some other recommended anomalies of the earth's gravitational field, respectively, are calculated with the subprograms with index 501: \( DEG \, 1, \quad DEG \, 3 \) or \( DEG \, 4 \) see in (5) p. 6.1 (there is introduced also the algorithm for the computation of the components of vector \( \delta g \)).

3. Text.

```
SUBROUTINE GRAV(XG,FX,FY,FX,ANG,NA)
DIMENSION XG(6),FX(6),FY(6)
COMMON/RAD/R1,R
X2=X(1)*X(1)
Y2=Y(2)*Y(2)
Z2=X(3)*X(3)
R12=X2+Y2
R1=SQRT(R12)
R2=R12-Z2
R=SQRT(R2)
NV=XV-1
W=R*R1
GO TO (1,2,3,4)
1 CALL DEG2(XG,FX,FY,ANG)
GO TO 4
2 CALL DEG3(XG,FX,FY,ANG)
GO TO 4
3 CALL DEG4(XG,FX,FY,ANG)
4 F(1)=FX(1)-(CG(1)*X(1))\,\,R-(DG(2)*X(1)*X(3))/W-(DG(3)*X(2))/R1
F(2)=FX(2)-(CG(1)*X(2))\,\,R-(DG(2)*X(2)*X(3))/W+(DG(3)*X(2))/R1
F(3)=FX(3)-(CG(1)*X(3))\,\,R-(DG(2)*R1)/R
RETURN
END
```
1. Calculation of the gravitational perturbations connected with the influence of the moon or sun (F04-FGRSS).

1. Designation. The terms deltaS VX, deltaS VY, deltaS VZ in the right members of the equations of motion (1.1) or (1.2), dependent on the gravitational perturbation of the moon or sun.

2. Structure. Subprogram FGRSS.


4. Initial data: GDR3 = μSR2, where μS -- product of the gravitational constant and the mass of the moon (or sun), R -- modulus of the radius vector of the moon (or sun).

The main part Y6, containing the values X, Y, Z, Vx, Vy, Vz; main part XS3, containing the values X0, Y0, Z0 or X0, Y0, ZS; main part F6, containing the values X, Y, Z, Vx, Vy, Vz, or x, y, z, vx, vy, vz, considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In the main part F6 the new values of the right members of the equations of motion of the AES are addressed with the computation of the influence of gravitational perturbations, produced by the moon (or sun).

6. Algorithm:

\[ \Delta_S V_X = \mu_S r_S^2 \left( \frac{(x_S^0 - x/r_S)^3}{|r_S - r|^3 - X_S^0} \right) \]
\[ \Delta_S V_Y = \mu_S r_S^2 \left( \frac{(y_S^0 - y/r_S)^3}{|r_S - r|^3 - Y_S^0} \right) \]
\[ \Delta_S V_Z = \mu_S r_S^2 \left( \frac{(z_S^0 - z/r_S)^3}{|r_S - r|^3 - Z_S^0} \right) \]
\[ \dot{V}_X = F_X + \Delta_S \dot{V}_X, \quad \dot{V}_Y = F_Y + \Delta_S \dot{V}_Y, \quad \dot{V}_Z = F_Z + \Delta_S \dot{V}_Z \]
\[ |r_S - r|/r_S = \left( (x_S^0 - x/r_S)^2 + (y_S^0 - y/r_S)^2 + (z_S^0 - z/r_S)^2 \right)^{1/2} \]
Subroutine FORSS (GDRS, V, XP, RP, F)

DIMENSION WGT, XP(3), F(8), XPR(9), WR(9)
R=0
DO 1 J=1,3
XPR(J)=V(J)/RP
W(J)=XP(J)-XPR(J)
1 R=W(J)*W(J)+R
R=1./R*SQRT(R)
DO 2 J=1,3
F(J+3)=F(J+3)*GDRS*(W(J)*R-XP(J))
RETURN
END

1.6 Computation of the influence of the pressure of light (FO5-FLIGHT)

1. Designation. The terms \( \Delta V_X, \Delta V_Y, \Delta V_Z \) are determined, considering the influence of light pressure on the right members of equations (1.1) or (1.2).

2. Structure. Subprogram FLIGHT.

General blocks: /EB/1, /CR2/1.


4. Initial data: KL--index, governing the computation of the umbra of the earth (for KL=1 is produced the calculation of the influence of light pressure independently of the shadow, for KL=2 the shadow is considered);

   The main part \( Y_6 \), containing the values \( X, Y, Z, V_X, V_Y, V_Z \) or \( x, y, z, v_x, v_y, v_z \); main part \( XS_\varphi \), containing the directed cosines of the radius-vector of the sun: \( X_\varphi, Y_\varphi, Z_\varphi \) or \( x_\varphi, y_\varphi, z_\varphi \); considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In main part \( F_6 \) are addressed the new values of right members of equations of motion of the ABS with calculation of the influence of the pressure of light.

6. Use of the domain COMECN. Before conversion to the
subprogram FLIGHT in the block COMMON/BB/B it is necessary to address the value of the coefficient B (see par. 7). Starting from this, that in the kg/s, a system of units the coefficient has the dimensions m/s², it is necessary to make it agree with the system of units used in the computations, dividing by the scale factor from the block COMMON/CELB/ELB (see table 2.3, N. 13).

From the block COMMON/CEI/1 one uses the constant (No. 10 table 2.1 (5)).

7. Algorithm:

\[
\begin{align*}
\dot{v}_x &= -B(x^o - x/r_o) r_o^3/|r_o - r|^3 + F_x, \\
\dot{v}_y &= -B(y^o - y/r_o) r_o^3/|r_o - r|^3 + F_y, \\
\dot{v}_z &= -B(z^o - z/r_o) r_o^3/|r_o - r|^3 + F_z,
\end{align*}
\]

gde: \(|r_o - r|/r_o = \left( (x^o - x/r_o)^2 + (y^o - y/r_o)^2 + (z^o - z/r_o)^2 \right)^{1/2},

\[ B = \frac{F_M}{m} q_0 \]

where \(F_M \) --square of the mid section, \(m \) --mass of the satellite, 
\(q_0 = 4.5 \times 10^{-7} \text{kg/m}^2\) --pressure of light in the region of the earth's orbit,
\(k = 1\) for full optical reflection, \(k = 1.44\) for full diffused reflection.

In the case of calculation of the influence of light pressure with calculation of the shadow of the earth the entry condition of the Able into the earth's shadow is verified:

\[ \cos \gamma < 0 \text{ и } r \sin \gamma < R; \]

где \( \cos \gamma = (X X_c + Y Y_c + Z Z_c)/r \), \( r = (X^2 + Y^2 + Z^2)^{1/2}, \)

R--average radius of the earth.
1.7 Right members of the system of equations of motion of the \( \text{AES} \), considering the normal gravitational field of the earth and standard five-layered atmosphere (F06–FC1100)

1. Designation. The subprogram FC 1100 is intended for use in the capacity of a subprogram for the right members for integration of a system of differential equations of motion of the \( \text{AES} \) models, characterized by the indexes 11100, 21100, 10100, 20100 (see table 1.1).

2. Structure. Subprogram FC1100
   External subprograms: FNGRAV (F01), RO(D01), FAT:(F02).
   General blocks: /BHC/1, /BK/5, /BEB/1.


4. Initial data: \( T \)--independent variable (time), \( Y \)--main part, containing \( X, Y, Z, V_X, V_Y, V_Z \) or \( x, y, z, v_x, v_y, v_z \).

5. Results: \( F \)--main part, containing \( \dot{X}, \dot{Y}, \dot{Z}, \ddot{V}_X, \ddot{V}_Y, \ddot{V}_Z \) or \( \dot{x}, \dot{y}, \dot{z}, \ddot{v}_x, \ddot{v}_y, \ddot{v}_z \).

6. Use of the domain COMMON. In the block COMMON /BEB/SB is necessary to give the value of the ballistic coefficient, relative to the scale factor from the block COMMON/CE3B/ESB (see (5) ), table 2.3, No. 12).

   In the block COMMON/BK/KC, KG,KA, KS, KL there must be given the values of the indexes of the model of forces (see par. 1.1). In the given case for the index KC the possible values are 1, 2; for KG = 0, 1; the values of the remaining indexes are not used.

   In the block COMMON/BHC/KC, as a result of the work of the subprogram FC1100 is addressed the value HC—elevation of the \( \text{AES} \) above the surface of the terrestrial ellipsoid.
1. Designation. Subprogram FORCE is intended for use as a sub-
program of right-hand sides for integration of the system of equations 
of motion of the ABS for any models of forces, described in table 1.1. The subprogram LOGMOD for the given model of forces determines 
the value of the logical parameters, governing the computation of 
sidereal time, the positions of the moon and sun.

2. Structure. The package of subprograms.

Inputs ... for the users: FORCE, LOGMOD.

Internal inputs RODENS

Utilized external subprograms:

\texttt{FNORAV(F01), FATM(F02), FAORAV(F03), FORSS(F04),}
\texttt{FLIGHT(F05), AGIOAC(B06), RO(D01), DENS(D02),}
\texttt{SELENA(C01), STT(B05), GCLTN(B10), SUN(C02), ADEN,}
\texttt{AMBAR, GRAV, TLOCAL(D03), DEG2, DEG3, DEG4(E01).}

\textbf{General blocks}
\texttt{COMMON BK/5, /BLOQ/4, /BSB/4, /BB/4, /BMA/4, /BRO/4,}
\texttt{/BDT/4, /BSO/4, /BDYEAR/4, /BCONGR/60, /QOREF/60, /SYEAR/60,}
\texttt{/CAED/4, /CA0/2, /CA2/4, /CRE/4, /CRZ/4, /CAE/2, /CGR/2,}
\texttt{/GDS/4, /COMZ/4, /COMZP/2, /CAEL/2.}
3. Conversion C.I.I. FORCE (T, Y, F)

4. Initial data: T— independent variable (time), Y — main part of the functions sought; \( X, Y, Z, V_X, V_Y, V_Z \) or \( x, y, z, v_x, v_y, v_z \)

5. Results: \( F \) — main part of functions derived from those sought: \( X, Y, Z, V_X, V_Y, V_Z \) or \( x, y, z, v_x, v_y, v_z \) — right members of the system of differential equations (1.1) or (1.7).

6. Use of the domain COMMON. Before conversion to the subprogram FORCE it is necessary to ensure in the domain COMMON the values of the parameters utilized. In the block /BK/KC, KA, KS, KL it is necessary to address the values of the indexes of the chosen model of forces (see table 1.1). In the block COMMON/BLOG/LSUN, LSUN, LSEL, LST by means of conversion to the subprogram LCOMMON (described below, in par. 9), it is necessary to address the values of the logical variables, used for regulation of the computation of sidereal time (for LST-TRUE), positions of the moon (for LSEL-TRUE), and sun (for LSUN-TRUE).

The remaining blocks COMMON are used only for the determined values of the indexes of the model of forces, for which in each case there is an indication. For KA greater than or equal to 1 in the block COMMON/TSR/BR it is necessary to address the value of the ballistic coefficient, relative to the scale factor from the block COMMON/USRB/1 (see (5), table 2.3, No. 17).

For KL greater than \( C \) (computation of the pressure of light) in the block /BII/B it is necessary to replace the value of the coefficient \( B \) (see par. 1.6) relative to the scale multiplier from block /CELB/1 (see (1) ), table 2.3, No. 13) For KG=2 or 3 it is necessary in the block COMMON/BNM/1 to address the value of the number of harmonics considered in the gravitational field of the earth, in the block COMMON/BCONGR/1246 — value of the coefficients of the breakdown of the gravitational field of the earth (by conversion to the sub-program CCONGR(AO2) (5) ). For KG=4 in the block COMMON/CA00/2 it is necessary to transfer the values of the corresponding variables from the block /CA00A/2 (see (5), table 2.1, Nos. 2, 3).
or KL greater than 0, and for KC=9, if KG is greater than 0, in the block COMMON/RSC/SC, Tj, N3 it is necessary to address the value T3 — sidereal time at midnight Greenwich of the date DT, and TS and NS to set equal to zero.

For KA=2 or 3 it is necessary in the blocks COMMON/COEF/50 and SYEAR to address the values of the coefficients of the model of the atmosphere and set of numerical corrections for the semi-annual effect, which is accomplished by conversion to the subprogram VMIA(EOP); in the block /B/Y/AR-1 one addresses the date and time in the form of the number of 4-hr. periods from the beginning of the year; in the block COMMON/BRO/K107, FO, AI, D, I one addresses the values F10.7 of the intensity of solar radio-radiation F10.7 with computation of the time of retardation (for KA=1 one sets F10.7 less than 0.79), FC=—of the average level of solar radio-radiation (possible values: 75, 100, 125, 150), Al — trihourly index of geomagnetic perturbation with computation of the time lag (for KA=2 set AI less than 0.79), D—parameter, regulating the calculation of the semiannual effect (for D less than 0 the semiannual effect is not considered), 1— parameter, regulating the calculation of the diurnal effect (CE) (for KC CE is considered without the term with the coefficient C, for I CE is not considered, for I greater than 0 CE is considered fully) For KA=4 (atmosphere CIRA-72) in the block COMMON/RSC/F107, FO, AKI, D, I it is sufficient to address the values F10.7—index F10.7 (time of retardation 1.17 days), FC—values F10.7, averaged for 48-hr. solar rotations, AKI—geomagnetic index Kp, considering 7=2.662, 30=3.303, 31=3.833 and so on (time of retardation ~0.79 days), values of D and I without importance.

In the block COMMON/BHC/HC/RAD/RC, R1/B1/CEL/CE, SE/BFT/TM, BS, RS, AE, B3, NL, XS(3), XL(3), GS, GL in the process of operation of the subprogram FORCE one addresses as the values HC—elevation of the AEB above the surface of the terrestrial ellipsoid, RC, RL—radius-vector of AEB and projections of the radius vector of the AEB on the plane of the earth's equator (in the subprogram AGRAV); CE, SE—cosine and sine of the angle of inclination of the plane of the earth's equator to the plane of the ecliptic (in the subprogram SUN); TM=T—current time (Moscow); ST—sidereal time; AS, BS—right ascension and declination of the sun; RS, XS(3), RL, XL(3)—are, respectively, for the modules and directed cosines (in the system of coordinates, determined by the index KC) radius vector of the sun and moon; GS, GL—mu_0 / R_0 and mu_moon / R_moon. In the remaining blocks COMMON, enumerated in para. 2, it is necessary to address the values of the constants and sale factors in conjunction with the tables 1.1-2.4 (!) by means of conversion to the subprogram CONST(AO1).

7. The subsidiary subprogram RODENS, intended for switching into the subprogram FORCE of the subprograms of computation of the density of the earth's atmosphere according to various models. For KA=2 or 3 one uses the model, realized in the subprogram DENS(DO2), for KG=4 one uses the model CIRA-72, subprogram ADEN (DO3).

Structure. Subprogram RODENS.
External subprograms AGIGAC(B06), CGLTLN(B10), DEN3(3C7), *DEN*, AMBAR, GRAV, TLEC6L(D03).

Conversion: CALL KOBERS (T, Y, HC, RC, ST, AS, BS, I).
Initial data: T=ti; Y=X, Y, V, V, V, V, RC, HC--modulus of radius vector and elevation of ABS above the surface of the terrestrial ellipsoid; ST--sidereal time; AS, BS--right ascension and declination of the sun.

Result: P--density of the atmosphere (in the system of units used for the computation). Concerning the use of blocks COMMON see para. 6.

8. Subprogram LOGMOD, starting from the values of the indexes of the model of forces, given in the block COMMON/BK/KC, KG, KA, KD, KL, addresses in the domain COMMON/ALCG/LSUN, LSUN, LSEL, LST the values of the logical parameters, which may be used for the regulation of the calculation of sidereal time (LST) and positions of the moon (LSE3) and sun (LSUN). Below are introduced the conditions, under which each of these parameters takes on the value TRUE.

LSUN=TRUE for KG greater than 1 or KL greater than 0 or KA greater than 1;
LSUN=TRUE for KG greater than 1 or KL greater than 0;
LSEL=TRUE for KG=1 or KD=1;
LST=TRUE for KG greater than 1 or KL greater than 0 or KL greater than 0, if KG=1 and for KG greater than 1, if KC =2.

Conversion: CALL LOGMOD.
SUBROUTINE FORCE(X,V,F)
DIMENSION V(6),F(6),VG(3)
LOGICAL LSUN, LBSUN, LSEL, LST
COMMON/BT/T, ST, RS, AS, BS, RL, XS(5), XL(3), GG, OL
COMMON/BLOG/LSUN, LBSUN, LSEL, LST
COMMON/BK/KC, KG, KA, KS, KL
COMMON/BS/GRS, GG, GRS
COMMON/BHC/HC
COMMON/BM/NMO
COMMON/CQ/GRL/GRL
COMMON/BDT/DT

CALL SUN(DT,X,RS,AS,BS,XS)
GG=GRS/RS/RS*GRR
GOTO(20,10),KC
20 IF(LSUN)
   CALL AGIGAC(ST,XS,1,XS)
10 IF(.NOT.LSEL)
   GOTO 6
   CALL SELENA(DT,X,XL,RL)
   GL=GRAL/RL/RL
   GOTO(5,6),KC
   T=X
   IF(LST)
   ST=STT(X)
   IF(.NOT.LSUN)
   GOTO 10

RETURN
END
SUBROUTINE DODENS(T,V,HC,RST,AS,DS,P)  
DIMENSION V(6),R0(5),SU(2),SAT,2),X(3),TEMP(2),ALOG(6)  
COMMON /RK/KC,KG,KA,KS,KL  
COMMON /BRK/Geo(3),D,1  
COMMON /BDY/DT  
COMMON /BDVEAR/DVE  
COMMON /CEM/EM  
COMMON /CIT3/T3  
COMMON /CSAD/SDAV  
COMMON /CE3/ES  
COMMON /REM/EM,ES  
COMMON /CER/ERO  
DO 1 J=1,3  
1 X(J)=V(J)/R  
GOTO (3,2),KC  
CALL AG16AC(ST,X*2,X)  
SU(1)=AS  
SU(2)=BS  
SAT(3)=HC/ES=EM  
IF(KE,GEQ,4)  
GOTO 7  
IF(0(.4,5,9)  
D=NTN*T/SDAY  
CALL DODENS(SAT(3),X(1),X(2),X(3),SU,CEO(3),CEO(1),D,1,RO)  
P=EO(4)=ERO  
GOTO 10  
CALL AG160N(X,SAT(2),SAT(1))  
AMJD=DT+(T-13)/SDAY+19019,9  
CALL ADEN(AMJ0D,;SU,SAT,CEO,TEMX,ALOG,AMMN,P)  
P=P/CM=ERO  
10 CONTINUE  
RETURN  
END  

SUBROUTINE LOGOO  
COMMON/BLG/LSUN,LSUNS,LSL,LS  
LOGICAL LSUN,LSUNS,LSL,LS  
COMMON /RK/KC,KG,KA,KS,KL  
LOGICAL LSUN,LSL,LSU,LSK  
COMMON /BFT/T,ST(13)  
T=1000  
LSUN=KA.GT.1  
LSUNS=KS.GT.1.OR,KS.GT.U  
LSL=KS,GEQ,1.OR,KS,GEQ,3  
L=KG.GT,2  
LSUN=LSUN.OR,LSUNS  
LSL=LSL.OR,LSL  
LSK=KC,GEQ,1  
LS=LSL AND,LSK. AND, NOT,LSK  
RETURN  
END
Chapter 2. Integration of systems of differential equations
(index G).

2.1. Integration of a system of differential equations by the method of Adams (GCL-ADAM)

1. Designation. In the subprogram the method of Adams is realized with the use of 8 differences, with automatic choice of step for integration of a system of differential equations:

\[ \dot{Y} = F(t, Y) \]

with the initial conditions \( Y(t_0) = Y_0 \), where \( Y, \dot{Y}, Y_0, F \)-\( n \)-dimensional vectors, \( t \)-independent variable. The start is produced by the Runge-Kutta method of the fourth order.

2. Structure. Subprogram ADAMS.

One uses the external subprograms: IPR and BKK—composed by the user.

3. Conversion: CALL ADAMS (KII, H, T, Y, IRI, BKK, NKY, MKY, EL, EP, N, AF, FS, F1, TL, Y1, Y2, Y3).

4. Initial data: KII—variable, ensuring the possibility of integrating with constant pace or with automatic choice of pace (in the first case it is necessary to take KII=7, in the second KII=2); H—initial step of integration; T—first value of the independent variable;

\( Y \)-\( n \)-dimensional block of dimension \( N \), containing the initial values of the desired functions;

IRI—name of the subprogram, computing the right members of equations, set up by the user;

BKK—name of the subprogram, prepared by the user, intended for control of the end of integration. In this subprogram there occurs conversion from the subprogram ADAMS after each step of the integration;

NKY, MKY—number of the first and last function in the block \( Y \), respectively, according to which proceeds the control of the accuracy of the integration with automatic choice of step;

EL, EP—blocks of dimension \( N \), containing lower and upper bounds of the permissible errors of the controlled functions;

These values are used: EL(NKY0, EP(NKY), EL(NKY+1), EP(NKY+1),...

EL(MKY), EP(MKY);

\( N \)-\( n \)-dimension of the system of equations

AF, FS, F1—auxiliary blocks of dimension \( N \);  

F0—two-dimensional block of dimension \( N, 2 \);  

F4—two-dimensional block of dimension \( N, 4 \).

Note 1. The designations of subprograms IPR and BKK must be written in EXTERNAL subprogram, used for subprogram ADAMS.

Subprogram IRI must have the form SUBROUTINE IRI(T, Y, F), where \( T \)-independent variable

\( Y \)-block of current values of the desired functions

F—block of right members, length of blocks \( Y \) and \( F \) equal to \( N \).

Subprogram BKK: SUBROUTINE BKK(T, Y, F0, IBKK, H), where \( T \)-current value of independent variable

\( Y \)—current value of desired functions, block of dimension \( N \), \( H \)—step of integration,

\( F0 \)—block of dimension \( N, 2 \), containing values of the right members.
in current points of integration, IBKK-integral variable, which, within BKK, must be ascribed the value \( \varepsilon \) for fulfillment of the conditions, according to which is determined the moment of exit from the integration (before the start of integration the subprogram AAXMSI ascribes to IBKK the value \( \varepsilon \)).

In subprograms RRI and BKK the values \( \varepsilon, Y, F, \eta \) are established by subprogram AAXMSI in the process of integration.

Note 2. Some times in the starting portion there arises a need to break down the step, not connected with ensuring the prescribed accuracy of integration. Ascribing to index IBKK the value \( \varepsilon \) leads to interruption of the process of starting that was begun, disconnecting the influence of KII on the choice of step, return to the starting point and to the start with a halved step.

Giving the index KBKK the value \( \varepsilon \) leads to a new interruption of the process of integration, establishment of the initial step, reestablishment of the influence of KII on the choice of step, return to the initial point and to the transfer to the standard process of integration.

The subprogram AAXMSI(HC4) may serve as an illustration of the application of this possibility see par. 7...

5. Results: TI and the block \( Y_1 \) — values of the independent variable and the desired functions at the final point of the integration.

6. Method. We consider the system (7.1). According to the known values \( Y_k \) in the Kth point and the approximate values \( Y_{k+1} \) produced at the Kth and \( 7 \) preceding points, one determines by the extrapolation formula of Adams

\[
Y_{k+1} = Y_k + h_t \sum_{i=0}^{\gamma} \alpha_i \beta_{i+1} F_{k+1-i}
\]

where \( h_t \) — step of integration.

Interpolation formula of Adams

\[
Y_{k+1} = Y_k + h_t \sum_{i=0}^{\gamma} \alpha_i \beta_{i+1} F_{k+1-i}
\]

allows the obtained values of \( Y_{k+1} \) to be made more precise.

The coefficients of the interpolation and extrapolation formulas of Adams have the following values:

\[
\begin{align*}
\alpha_0 &= 3,58995535, \\
\alpha_1 &= 9,52520668, \\
\alpha_2 &= 18,0545837, \\
\alpha_3 &= 22,327753, \\
\alpha_4 &= 17,9796544, \\
\alpha_5 &= 8,61212797, \\
\alpha_6 &= 2,44516359, \\
\beta_0 &= 0,304234537, \\
\beta_1 &= 1,15615906, \\
\beta_2 &= -1,00691364, \\
\beta_3 &= 1,01793461, \\
\beta_4 &= 0,732085838, \\
\beta_5 &= 0,349092257, \\
\beta_6 &= 0,0938408392, \\
\beta_7 &= 0,0113673942.
\end{align*}
\]
For automatic choice of step of integration one computes the differences $\Delta Y_{k+1} = Y_{k+1} - Y_k$.

We give the allowable bounds of error for integration: 0 less than or equal to $\varepsilon_{1,j}$ less than or equal to $\varepsilon_{2,j}$.

If there are fulfilled the conditions $\varepsilon_{1,j} < \varepsilon_{2,j}$ and

$$\Delta Y_{j,k+1} < \varepsilon_{1,j},$$

where $N_1$, $N_2$—numbers of the first and last controlled functions, then the step of integration $h_t$ does not change.

If the absolute value of $\Delta Y_{j,k+1}$ is less than $\varepsilon_{1,j}$ for all $j = N_1, ..., N_2$ then there results a doubling of the step.

For this integration continues with step $h_t$ until the accumulation of the necessary number of points, for which after an interval of time, the multiple $2h_t$ the known values of the function $F$, after which the step is doubled.

If the absolute value of $\Delta Y_{j,k+1}$ is greater than $\varepsilon_{2,j}$, even if for one value of $j$: $N_1$ less than or equal to $j$ less than or equal to $N_2$, then a breakdown of the step is produced. For this it is necessary to have the values of $F$ in the seven preceding points with the interval $h_t/2$. It is possible to obtain them by means of the interpolation according to the formula of Lagrange for known values of $F$ with step $h_t$.

We introduce the variable $xi = (t_k - t)/h_t$, then the interpolation formula of Lagrange for the determination of $F(t)$ is written in the form:

$$F(t) = F_{k-7}(\xi^7 - 21 \xi^6 + 175 \xi^5 - 735 \xi^4 + 1624 \xi^3 - 1764 \xi^2 + 720 \xi)/9040 -$$

$$- F_{k-6}(\xi^7 - 22 \xi^6 + 190 \xi^5 - 820 \xi^4 + 1849 \xi^3 - 2038 \xi^2 + 840 \xi)/720 +$$

$$+ F_{k-5}(\xi^7 - 23 \xi^6 + 270 \xi^5 - 925 \xi^4 + 2144 \xi^3 - 2412 \xi^2 + 1008 \xi)/240 -$$

$$- F_{k-4}(\xi^7 - 24 \xi^6 + 226 \xi^5 - 1056 \xi^4 + 2545 \xi^3 - 2952 \xi^2 + 1260 \xi)/144 +$$

$$+ F_{k-3}(\xi^7 - 25 \xi^6 + 247 \xi^5 - 1219 \xi^4 + 3112 \xi^3 - 3796 \xi^2 + 1680 \xi)/144 -$$

$$- F_{k-2}(\xi^7 - 26 \xi^6 + 270 \xi^5 - 1420 \xi^4 + 3929 \xi^3 - 5274 \xi^2 + 2520 \xi)/240 +$$

$$+ F_{k-1}(\xi^7 - 27 \xi^6 + 295 \xi^5 - 1665 \xi^4 + 5104 \xi^3 - 8028 \xi^2 + 5040 \xi)/720 -$$

$$- F_k(\xi^7 - 28 \xi^6 + 322 \xi^5 - 1960 \xi^4 + 6769 \xi^3 - 13132 \xi^2 + 13068 \xi - 5040)/5040.$$
To obtain the values of $F$ in the first 7 points one uses the Runge-Kutta method of the fourth order.

$$Y_{h+1} = Y_k + \frac{1}{6}(e_1 + 2e_2 + 2e_3 + e_4),$$

$$e_1 = h_t F(t_k, Y_k),$$

$$e_2 = h_t F(t_k + \frac{1}{2}h_t, Y_k + \frac{1}{2}e_1),$$

$$e_3 = h_t F(t_k + \frac{1}{2}h_t, Y_k + \frac{1}{2}e_2),$$

$$e = h_t F(t_k + h_t, Y_k + e).$$

Literature: (7), p. 333.
DO 34 J=1,N
26 V2(J)=V2(J)+A(K)*F(J,K)
   CALL PRINT(X1,V2,AF)
   DO 26 K=1,7
   DO 26 J=1,N
26 V1(J)=V1(J)+B(K)*F(J,K+1)
   DO 29 J=1,N
29 V1(J)=V1(J)+B(K)*AF(J)
   GOTO(106,107,101)
107 CONTINUE
   DO 29 J=1,N
   IF(ABS(V1(J)-V2(J))<E2(J))20,20,108
20 CONTINUE
   DO 29 J=1,N
   IF(ABS(V1(J)-V2(J))<E1(J))30,30,116
116 I=0
   GOTO 106
20 CONTINUE
C M=M+2
   I=IA-1
   IF (IA-1) 31,31,106
31 DO 33 K=1,4
   L=K-1
   DO 33 J=1,N
33 F(J,K)=F(J,K+1)
   GOTO 104
C M=M/2
104 M=K1-N
293.5
   DO 37 K=1,4
     R(K)=-(111/2*[Z-28.]*Z+322.)1*[Z-19.60.]*Z+679.]1*Z
     -3132.1*Z+13065.)/12040.)*Z+1.
     R(7)=111/2*[Z-27.]*Z+295.1*[Z-16.69.]*Z+319.]1*Z
     -8026.1*Z+5040.1*Z+720.1*Z
     R(6)=111/2*[Z-26.]*Z+270.1*[Z-14.20.]*Z+392.]1*Z
     -9274.1*Z+2520.1*Z+240.1*Z
     R(5)=111/2*[Z-25.]*Z+247.1*[Z-12.10.]*Z+311.]1*Z
     -3796.1*Z+1480.1*Z+144.1*Z
     R(4)=111/2*[Z-24.]*Z+226.1*[Z-10.56.]*Z+254.]1*Z
     -2932.1*Z+1260.1*Z+144.1*Z
     R(3)=111/2*[Z-23.]*Z+207.1*[Z-8.92.]*Z+244.]1*Z
     -2412.1*Z+1000.1*Z+240.1*Z
     R(2)=111/2*[Z-22.]*Z+198.1*[Z-7.35.]*Z+184.]1*Z
     -2058.1*Z+840.1*Z+720.1*Z
     R(1)=111/2*[Z-21.]*Z+179.1*[Z-6.75.]*Z+162.]1*Z
     -1764.1*Z+720.1*Z+3040.1*Z
   DO 38 J=1,N
     F(J,K)=0
   DO 30 L=1,6
38 F(J,K)=F(J,K)+R(L)+F(J,L)
37 Z=Z-1.
   DO 39 K=5,7
   L=K-8
   DO 39 J=1,N
39 F(J,L)=F(J,K)
   DO 40 K=1,4
   L=Z+K-1
40 F(J,L)=F(J,K)
   DO 41 J=1,N
   V2(J)=V3(J)
41 V1(J)=V3(J)
   DO 51 J=1,8
   A(J)=A(J)+3
51 B(J)=B(J)+3
   H=M+0.5
   GOTO 119
1000 RETURN
END
2.2. Interpolation according to Adams (GO2-ADINT).

1. Designation. The subprogram allows in the process of integration by the method of Adams of the system of differential equations $\dot{Y}=F(t,Y)$, where $Y$, $\dot{Y}$, $F$—$N$-dimensional vectors, the computation of the values of the desired functions at the moments of time $t_{nu}$, not multiples for the step of integration.

2. Structure. Subprogram ADINT.


4. Initial data:
   - Value of the variable $\xi=(t_k-t_{nu})/h_t$, where $t_{nu}$—given moment of time, $h_t$—step of integration, $t_k$—current value of the variable of integration, corresponding to the condition: $t_k-h_t$ less than $t_{nu}$ less than $t_k$;
   - $P8$—block of dimension $(N, 8)$, containing those produced from the desired functions in the eight last points: $t_k-7$, $t_k-6$, $\ldots$, $t_k$;
   - $Y$—block of dimension $N$, containing the values of the desired functions at the point $t_k$;
   - $M$—number of interpolated functions ($M$ less than or equal to $N$);
   - $H$—step of integration ($h_t$);
   - $N$—order of the system of equations.

5. Results: $YR$—block of dimension $N$, containing $y_j(t_{nu})$, $j=1, \ldots, M$.

6. Algorithm: We introduce the variable $\xi=(t_k-t_{nu})/h_t$, ($t_{nu}$, $h_t$ and $t_k$ determined above).

The value of the desired functions at the point $t_{nu}$ is computed by the formula
\[
y_j(t_{nu})=y_j(t_k)-h_t \sum_{i=k-7}^{k} f_{i,j} \psi_{i}/120960,
\]
where $j=1, \ldots, M$ less than or equal to $N$; $f_{i,j}$—values of the right members on the points $t_i$ ($i=k-7, \ldots, k$); $y_j(t_k)$—values of the desired functions at the point $t_k$;
\[
\psi_{K-7} = (3e^6 - 72e^7 + 700e^6 - 3528e^5 + 9744e^4 - 1412e^3 + 8640e^2),
\]
\[
\psi_{K-6} = -21e^8 - 528e^7 + 5320e^6 - 27552e^5 + 77658e^4 - 11412e^3 + 70560e^2,\]
\[
\psi_{K-5} = (63e^8 - 1656e^7 + 17388e^6 - 93240e^5 + 270144e^4 - 405216e^3 + 254016e^2),
\]
\[
\psi_{K-4} = -(105e^8 - 2880e^7 + 31640e^6 - 177408e^5 + 534450e^4 - 826560e^3 +
+ 529200e^2),
\]
\[
\psi_{K-3} = -(105e^8 - 3000e^7 + 34580e^6 - 204792e^5 + 653520e^4 - 1062280e^3 +
+ 705600e^2),\]
\[
\psi_{K-2} = -(63e^8 - 1872e^7 + 22680e^6 - 143136e^5 + 495054e^4 - 886032e^3 +
+ 635040e^2),\]
\[
\psi_{K-1} = (21e^8 - 648e^7 + 8260e^6 - 55944e^5 + 214368e^4 - 449560e^3 + 423360e^2),
\]
\[
\psi_{K} = -(3e^8 - 96e^7 + 128e^6 - 9408e^5 + 40614e^4 - 105056e^3 + 156816e^2 -
- 120960e).\]

Дополнительно: [2].

SUBROUTINE ADINT(Z,F,V,VR,M,N)
DIMENSION F(N,6),V(N),VR(N),R(6)
R(1) = ( ( ( ( ( (3,-72) =2+700) =2+3528) ) =2+9744) ) =2+1412) =2+8640).
R(2) = ( ( ( ( ( (1,-21) =2+528) =2+3320) ) =2+27552) ) =2+77658) ) =2+114124).
R(3) = ( ( ( ( ( (1,0) =2+1656) ) =2+17388) ) =2+93240) ) =2+270144) ) =2+534450).
R(4) = ( ( ( ( ( (1,-10) =2+2880) ) =2+31640) ) =2+177408) ) =2+495054) ) =2+990108).
R(5) = ( ( ( ( ( (1,10) =2+3000) ) =2+34580) ) =2+204792) ) =2+553520) ) =2+1062280).
R(6) = ( ( ( ( ( (1,-63) =2+1872) ) =2+22680) ) =2+143136) ) =2+495054) ) =2+990108).
R(7) = ( ( ( ( ( (1,63) =2+648) ) =2+8260) ) =2+55944) ) =2+214368) ) =2+449560).
R(8) = ( ( ( ( ( (1,-3) =2+96) ) =2+1268) ) =2+7944) ) =2+156816) ) =2+120960).
DO 1 J=1,M
1 VR(J)=V(J)
DO 2 K=1,N
R(K)=R(K)+Z/120960.
DO 2 J=1,M
2 VR(J)=VR(J)-F(J,K)*R(K)
RETURN
END

3.1. Arriving at the prescribed value of an arbitrary function from the solution (HOI-REACH) in the process of integration.

1. Designation. In the process of integration of the system of differential equations (2,1) the subprogram REACH allows one to determine the value of the independent variable TR, corresponding to reaching the given value FRI of an arbitrary continuous function phi(T)=FREACH(T,Y), Y--N-dimensional vector.

2. Structure. Subprogram REACH. Utilized external subprograms: ADINT(G02), subprogram--function, FREACH (composed by the user).


4. Initial data: T--independent variable; Y--block of current values of the vector Y; FO--two-dimensional block (N, 8)--values of right members in 8 points with step H; H--value of the step of integration; FREACH--designation of the subprogram--function, computed phi(T)-- FR--current value of the function FREACH;

*For the subprograms of this chapter the conversion must take place in the process of integration of the system of differential equations, i.e. from the subprograms BKK, to which is transferred the regulation from the subprogram ADAMSF (see par. 2.1) after each step of integration.
FRK—value of FREACH at the previous step of integration (at the point T−1);
FR1—given value of the function FREACH;
N—order of system of equations;
ε—allowable relative error, ensuring the choice of value of the independent variable TR, satisfying the relationship

\[
\left| \frac{FR1 - FREACH(TR, YR)}{\Delta F} \right| \leq \epsilon
\]

where δF—increment in the function FREACH from the step of integration.

Note 1. Subprogram —function FREACH, composed by the user, must have the form: FUNCTION FREACH(T, Y), where T—dependent variable;
Y_N—block of current values of the vector Y.

Note 2. Function FREACH must be described in the subroutine, using the subroutine FREACH.

Note 3. Before use of the subprogram FREACH it is necessary to find that moment of time in the process of integration, for which is satisfied one of the relationships
FRK less than or =FR1 less than or =FR or FRK greater than or =FR1 greater than or =FR.

1. Results: TR—value of independent variable, corresponding to reaching the function FREACH of the value FR1;
Y_N—block of values of the desired functions Y at the point TR.

2. Algorithm. Let us consider, in the process of integration of the system of equations (2.1) a continuous function \( \phi(T) = FREACH(T,Y) \), where Y—N-dimensional vector, taking on the values FR at the current moment of time T, FRK—at the moment of time \( T_k = T - H \),
where H—step of integration.

It is necessary to find the moment of time, for which the function \( \phi(T) \) takes on the value FR1.

Under the condition, that there is fulfilled one of the inequalities FRK less than or =FR1 less than or =FR or FRK greater than or =FR1 greater than or =FR, we determine F=FR−FRK, \( x_1 = (FR - FR1)/\delta F \).

We set some epsilon greater than 0. If the absolute value of \( x_1 \) is less than or =epsilon, that moment of time TR=T−x_1H gives the solution of the problem proposed. In the opposite case, using the subroutine of interpolation by Adams' method (par. 2.2), we determine the value YR at the moment of time TR and the value P=\( \phi(TR) = FREACH(TR, YR) \).
For the new iteration we determine $x_i' = (P - FR1)/\delta F$, we verify the condition absolute value of $x_i'\leq \epsilon$; then or $=\epsilon$. If this condition is satisfied, then $TR = T - x_iH - x_i'H$ gives the solution to the proposed problem, otherwise we go on to a new iteration and so on.

```
SUBROUTINE REACH(T,V,F,H,E,FR,FR1,PK,T,H,FREACH,N,VR)
DIMENSION V(N),F(N+1),VR(N)
TR=1
IF(ABS(F(1,1)+F(2,1)+F(3,1))=1.0-15)1,1,4
4 DP=FR-FR1
CO=FR-FRK
W=0.
3 W1=DP/0Z
W=W+H1
TR=T-W0H
IF(ABS(W1)-E11,1,2
2 CALL ADINT(W,F,V,VR,M,N)
P=FREACH(TR,VR)
4 DP=FR-FR1
GOTO 3
1 RETURN
END
```
3.2 Determination, in the process of integration, of the minimum and maximum of an arbitrary continuous function from the solution as functions of an independent variable 
(H02-EXTRM, PARAB)

1. Designation. Subprogram EXTRM determines in the process of integration of the system of equations (2.1) the minimum and maximum of an arbitrary continuous function \( \phi(T) = \text{FREACH}(T,Y) \), where \( Y \) is a \( N \)-dimensional vector, and the value of the independent variable, corresponding to the minimal and maximal values of this function.

2. Structure. Subprogram EXTRM.

Internal inputs: \( \text{F} \), \( \text{AR} \), \( \text{B} \).

Utilized external subprograms: ADINT(102), FREACH (composed by the user).

General blocks: \( /\text{BMIN}/2, /\text{BFC}/1, /\text{BSM}/1 \).

3. Conversion:

CALL EXTRM (\( T \), \( Y \), \( F \), \( H \), \( i \), \( 1 \), \( 2 \), \( \text{MIN} \), \( \text{MIN} \), \( \text{FMAX} \), \( \text{FMAX} \), \( \text{FREACH} \), \( N \), \( YR \))

4. Initial data: \( T \) -- independent variable;
\( Y \) \_ \_ block of current values of vector \( Y \);
\( H \) -- step of integration;
\( F \) -- two-dimensional block \( (N, \delta) \), containing the values of the right-hand sides at 8 points with step \( H \);
\( P \), \( 1 \), \( 2 \) -- permissible relative errors, ensuring the choice of extremum, satisfying the relationships

\[
\left| \frac{FIE - \phi(TE)}{\Delta F} \right| < p_1 \quad \text{or} \quad \left| \frac{TIE - TE}{H} \right| < p_2,
\]

where \( FIE \), \( TIE \) -- value of the function and of the argument corresponding to the actual extremum; \( \phi(TE), TE \) -- approximate values of the extremum of the function and the corresponding argument; \( \Delta F \) -- increment in the function \( \phi(T) \) for the step of integration;
\( H \) -- order of the system of equations;
FREACH -- designation of the subprogram -- function, computed \( \phi(T) \).

This subprogram is composed by the user and has the following construction:

\( \text{FUNCTION FREACH}(T,Y) \)

where \( T \) -- independent variable;
\( Y \) -- block of values of the vector \( Y \).

Note. The name of the subprogram -- function FREACH must be described in EXTRM subprogram, used for subprogram EXTRM.

5. Results: \( \text{MIN} \) -- last local minimum of function FREACH for the interval of time from the beginning of integration to the current moment; \( \text{MIN} \) -- moment of time, corresponding to the value FREACH = \( \text{MIN} \); \( \text{FMAX} \) -- last local maximum of function FREACH in the same time interval, \( \text{FMAX} \) -- moment of time, corresponding to the value FREACH = \( \text{FMAX} \); \( Y_{N} \) -- block of values of vector \( Y \) at the point of the extremum.

6. Use of the domain COMMON: After the beginning of integration
in the block COMMON/BEGIN/1, H2 one must substitute these values: H1=0, H2=0.

In the block COMMON/BEGIN/1 at each step of integration it is necessary to address the current value of the function FREACH.

In the block COMMON/BSM/ the subprogram EXTR addresses the value of the step of integration.

7. Algorithm.

Let HC, H2, H1 be the values of the function FREACH at three consecutive moments of time \( t_i \): \( t_1=t \), \( t_2=t-h \), \( t_3=t-h-h \) where \( h \) is the step of integration, \( h_1 \) for integration with constant step (for integration with automatic choice of step \( h_1 \) may be different from \( h \)).

If the inequality \( (H2-H1)(HC-H2) \) less than 0 is satisfied, then the extremum (for \( H2-H1 \) greater than 0--maximum, for \( H2-H1 \) less than 0--minimum) is found in the time interval \( (t_2, t_1) \).

We will term such a situation extremal. It remains to determine more precisely the moment of time, corresponding to the extremum, and the value of the extremum.

We proceed from the moments of time \( t_1 \) to the moments \( \tau_1=(t-t_1)/h \); then \( \tau_1=0 \), \( \tau_2=1 \), \( \tau_3=1+h_1/h \) correspond to the values of the function HC, H2, H1.

For convenience in further discussions, we shall rename the moments of time and the corresponding values of the function, starting from the following considerations.

**Eqn** IF \( |HC-H2|<|H2-H1| \) THEN \( B_1=H2 \), \( x_1=\tau_2=1 \); \( B_2=HC \), \( x_2=\tau_1=0 \); \( B_3=H1 \), \( x_3=1+h_1/h \).

**Eqn** IF \( |HC-H2|>|H2-H1| \) THEN \( B_1=H2 \), \( x_1=\tau_2=1 \); \( B_2=H1 \), \( x_2=\tau_3=1+h_1/h \); \( B_3=HC \), \( x_3=\tau_1=0 \).

Thus, we have 3 values of the function \( B_1 \) for three values of the arguments \( x_1 \). We construct a parabola, passing through these points, and find the value of \( W \), the argument of the extremum guaranteed by this parabola.

Turning to the subprogram of interpolation following Adams (par. 2.2) with the obtained value of \( W \), we determine the value \( YR \) of the vector \( Y \) at the moment in time \( TR=t-WH \), then the value \( HR \) of the function FKREACH at this moment in time.

If the absolute value of \( (HR-B_1)/\delta F \) is less than \( \epsilon_1 \), or the absolute value of \( W-x_1 \) is less than \( \epsilon_2 \), where \( \epsilon_1, \epsilon_2 \)--given values of the permissible errors, \( \delta F =HC-H2 \), then the problem is solved, otherwise we go on over to a new iteration, using the values \( B_3=B_2 \), \( B_2=B_1 \), \( B_1=HR \) at the moments in time \( x_3=x_2 \), \( x_2=x_1 \), \( x_1=W \) and so forth.
SUBROUTINE EXTREME(V, F, N, P1, P2, PMIN, PMAX, TMIN, TMAX, FRAC, VR)

DIMENSION V(N), F(N-1), VR(N)

DIMENSION X(3), B(3)

COMMON /SM/ S

COMMON /MIN/N1, H2

COMMON /OPT/ HC, H1

IF(H2) 1, 2, 1

IF(H1) 3, 5, 3

1 H1=H2+1

H2=HC-H1

IF(H1) 4, 5, 5

4 IF(F(1, 1)+F(2, 1)+F(3, 1)) 30, 3, 30

5 K=SIGN(1, W1)

6 B(1)=H2

7 X(1)=1.

8 X(3)=1. + S2/H

9 IF(HC-H1) 19, 0, 0

8 B(2)=HC

9 B(3)=H1

10 GOTO 10

9 B(2)=H1

11 X(2)=X(3)

12 B(3)=HC

13 X(3)=0

10 CALL PARAB(X, B, XM)

11 X=XM

12 CALL ACINT(XM, F, V, VR, N, H, H1)

13 TMIN=H

14 HR=FRAC(TR, VR)

15 IF((HR-B(1))<K) 25, 21, 20

20 IF(ABS((HR-B(1))/V2)-P1) 21, 21, 23

23 -IF(ABS(XM-X(1))=P2) 21, 21, 24

24 DO 26 J=1, 2

25 L=J

26 B(L)=B(L-1)

27 B(1)=HR

28 X(1)=XM

29 GOTO 10

30 H2=HC

31 H1=H2

32 FMIN=HR

33 FMAX=HR

30 TMIN=TR

31 GOTO 5

33 FMAX=TR

35 H1=H2

36 H2=HC

37 S2=H

RETURN

END
9. The subsidiary subprogram FARAB, intended for the determination of the extremum of the parabola, passing through three points: \( \phi(\xi_1) \), \( \phi(\xi) \), \( \phi(y_2) \) of some function \( \phi(x) \).

Conversion: CALL FARAB (X, B, XM).

Initial data: \( x_i \)--block of values of the argument, \( B_i \)--block of values of the function.

Results: \( XM \)--value of the argument, giving the extremum of the parabola, passing through the given 3 points.

Algorithm. Let the values \( B_i \) of some function \( \phi(x) \) be known for three values of the argument \( x_i \): \( B_i = \phi(x_i) \).

It is necessary to determine \( XM \)--the value of the argument, giving the extremum of the parabola, passing through the indicated values \( B_1 \).

\[
XM = \frac{1}{2} \frac{(B_2 - B_1)(x_2^2 - x_1^2) - (B_3 - B_1)(x_3^2 - x_1^2)}{(B_2 - B_1)(x_3 - x_1) - (B_3 - B_1)(x_3 - x_2)}.
\]

TEXT.

```
SUBROUTINE PARAB(X, B, XM)
DIMENSION X(3), B(3), W(2), T(2), T2(2)
V = X(1) + X(1)
DO 1, J = 2, 3
W(J-1) = B(J) - B(1)
T(J-1) = X(J) - X(1)
1 T2(J-1) = X(J) + X(1)
XM = 9.0*(W(1)*T2(2) - W(2)*T2(1))/4*(W(1)*T(2) - W(2)*T(1))
RETURN
END
```
3.3. Exit of the A.E.S at the ascending node of the orbit (NO3-MODE)

1. Designation. The subprogram permits one to determine in the process of integration of the systems of equations (1.1 or (1.2)) the moment of time, corresponding to the A.E.S crossing the ascending node, and also the coordinates and components of the velocity vector of the A.E.S at this moment in time.

2. Structure. Subprogram NO3.

Utilized external subprograms: ADINT, INK

General domain: /B, K/"K, /BIN/1

3. Conversion:

CALL MODE(T, Y, H, F8, TV, VV, NR).

4. Initial data: T- time; Y- block of current values X, Y, Z, VX, VY, Vz; h- step of integration; F0- two-dimensional block of dimension (1,0), containing the values of X, Y, Z, VX, VY, Vz produced at 8 points: T=7H, ..., T-H, T

NB--loop number:

5. Results: If TV in greater than 0, then TV, VV- moment of time and block of values X, Y, Z, VX, VY, Vz corresponding to the A.E.S crossing the ascending node.

Note. If the condition of exit of A.E.S at the ascending node is fulfilled at the take-off stage by the method of Runge-Kutta (i.e. initial conditions are given in the vicinity of the ascending node), then the procedure of determining more precisely the moment of exit at the node is disconnected, and the loop number (NR) is increased by 1.

6. Use of the domain COMMON.

Before the beginning of integration in the block COMMON it is necessary to address the value k(t) to the place K and KX; in the block COMMON/SIN/INOD one sets K(0)=1.

7. Algorithm.

In the process of integration of equations of motion of the A.E.S one determines the moment of time t, for which is fulfilled the condition: 3(t-h) k(t) less than 0, k(t) greater than 0.

Starting from the value of the parameter xi, k(t), delta i, where delta k(k(t)-k(t-h)), using the subprogram of integration according to Adams (par. 2.3), we determine the values X, Y, Z, VX, VY, Vz at the moment of time t = t-xi, where h- step of integration.

If the absolute value of Z/delta 3 is less than epsilon as given, then VV, VX, VY, VX, VX, VY, Vz give the solution to the proposed problem. If the condition is not fulfilled, then a new iteration is carried out, starting from the value xi = xi +
\[ Z_f / \Delta \delta \text{ and so forth.} \]

Note. The chosen value \( \epsilon = 10^{-7} \) (variable EN, value of which is given in the operator DATA) ensures exit at the ascending node with an error, not exceeding 0.31 m., for the \( \text{ABS} \) with apogee on the order of 500 km.

3. Text.

```
SUBROUTINE NODE(X,V,W,F,TV,WW,ND)
  DIMENSION V(6),WW(1,F(6,8))
  DATA: ER/1.E-7, EN.
  COMMON/ZK/ZK, ZK.
  COMMON/TH/NOD.
  TV=ABS.
  K$=SIGN(1.,N)
  N=V(18)*EK
  IF(W<11)10,1,1.
  IF|ABS((E(1,8)+F(5,1)+F(5,2)-1.E-14)*10.10,9
  10 GOTO: (10,3), INOD
  16 INOD=2.
  N=EN<1.
  GOTO: 3.
  IF(V(3)) 9,11,7.
  D2=V(3)*ZK
  NNV(3)/D2
  CALL ADINT(W,F,V,WW,9,6)
  WIP(W(3))/D2.
  IF|ABS((W)|EN)>1.111,9
  11 TV=E(K).
  3 ZK=V(3), RETURN.
  END
```

3.4. Determination in the process of integration of the minimal and maximal elevation of the \( \text{ABS} \) above the surface of the terrestrial ellipsoid (MG4--APSIS).

1. Purpose. The subprogram APSIS allows the determination of the minimal and maximal value of the elevation of the \( \text{ABS} \) above the surface of the terrestrial ellipsoid in the process of integration of the equations of motion of the \( \text{ABS} \) (1.1) or (1.2).

2. Structure. Subprogram APSIS.

External subprograms employed: HEIGHT(B10), ADINT(G02), FARAB(H02).

3. Conversion: CALL ADAMS (T, Y, H, Fo, HMIN, HMAX, 1BKK).

4. Initial data: T--time;
Y--block of current values of X, Y, Z, Vx, Vy, Vz;
H--step of integration;
F8--two-dimensional block (X, S), containing the values of
X, Y, Z, Vx, Vy, Vz derived at the moments of time: T=TH, T=6H,
...; T;
BKK--variable, controlling the exit from the integration by the
subprogram "DA39F" (par. 3.1)

5. Results:
HMIN, HMAX--minimal and maximal values of the elevation of the
AES above the surface of the earth's ellipsoid in the time interval
from the beginning of integration (or from the start of the loop)
to the current moment of time.

Remark 1. In order that HMIN, HMAX be determined as minimal and
maximal values of the elevations not on the whole interval of
integration, but on each loop separately, it is necessary to address
the values HC--of the current altitude of AES--at the moments of
exit of the AES at the orbital node for HMIN, HMAX.

Remark 2. If the extremal situation arises in the starting
portion, then the variable 1BKK receives the value 3.

In this case the program of integration ADAMS automatically
reduces the step in half, switches out the influence of KF on the
choice of step and begins the start from the initial point. After
HMIN or HMAX is found, 1BKK receives the value 4, the initial step
of integration and the influence of KF on choice of step are
automatically renewed, and a new start will begin from the initial
point and the integration enters the standard regime.

6. Use of the domain COMMON. Before the beginning of inte-
gration in the blocks COMMON/1JK, ZKN/BA1.3/1HM, H1, H2 it is
necessary to give the value 3 for the variables 1JK, ZKN, to set
1HM=1, H1=0; H2=0.

In the block COMMON/HBN/H at each step of integration it is
necessary to address the value HC, current elevations of AES above
the surface of the earth.

In the block COMMON/BSZ/, the subprogram AFSIS addresses the
value of the step of integration. In the block COMMON/BTAN1-
TMIN/TMAX are addressed the moments of time, corresponding to
HMIN, HMAX.

7. The Algorithm is completely analogous to the algorithm, used
in the subprogram "EXTERN(02)", only in place of the function FPREACH
one uses the altitude of AES above the surface of the earth's ellip-
soild.

For the permissible errors are chosen the values epsilon1=
0.0001, epsilon2=0.01 ( variables P1, F2, values are given to these
by the operator DATA).

4.1. Right ascension of the AES, local time and zenith distance of the sun (IO1--RIGHTA, TLCC, ZDSUNL, ZDSUNC).

1. Purpose. The subprogram TLCC for known geographical longitude (lambda) of AES and sidereal time (ST) determines the right ascension of the AES.

The subprogram TLOC for known geographical longitude of the point (lambda), sidereal time (ST), and right ascension of the sun (alpha) determins local time.

The subprogram ZDSUNL for known geographical latitude (phi) and longitude (lambda) of a point and for the directed cosine of the radius vector of the Sun (x^0, y^0, z^0) in the Greenwich system of coordinates determines the zenith distance of the sun.

The subprogram ZDSUNC determines the zenith distance of the sun for known directed cosines of the normal to the surface of the earth's ellipsoid (x_N, y_N, z_N) and for directed cosines of the radius vector of the sun (x^0, y^0, z^0) in the Greenwich system of coordinates.

2. Structure. The package of independent subprogram-functions: RIGHTA, TLOC, ZDSUNL, ZDSUNC.

Common domain: /CHI/3, /CD2GR/, /CHRAD/.

Remark 1. Subprogram-function ZDSUNC refers to the subprogram-function ZDSUNL.

3. Conversion: A=RIGHTA (ST, ALN),
TL=TLOC (ST, ALN, AS),
XG=ZDSUNL (ALT, ALN, X3G),
ZG=ZDSUNC (XNG, X3G).

4. Initial data: ST--sidereal time in radians; ALT, ALN--geographical latitude and longitude of the point in radians;
AS--right ascension of the sun in radians.
Block XNG--directed cosines of the normal to the surface of the earth's ellipsoid at the point underneath the satellite.
Block X3G--directed cosines of the radius vector of the sun.

5. Results: A--right ascension of the AES in degrees;
TL--local time in hours;
ZG--zenith distance of the sun in degrees.

6. Use of the domain COMMON. The constants from these blocks are used: COMMON/CHI/3, /CD2GR/, /CHRAD/ (see 9.0, No. 1, 2, 3 table 2.3).

7. Algorithm. Right ascension of the AES--lambda, local time--t and the zenith distance of the sun--ZG are determined according to the following formulas:
\[ \lambda = \lambda + ST, \]
\[ t_\circ = \lambda - \alpha_\circ + \pi, \]
\[ \cos ZS_\circ = x_\circ x_\circ + y_\circ y_\circ + z_\circ z_\circ. \]

3. Texts.

5. FUNCTION\_RIGHTA(ST, ALN)
COMM\_O/CDEGR/DEGR
COMM\_O/CPI/P1, P12
T=ST+ALN
K=T/P12
T=T-K*P12
RIGHTA=T*DEGR
RETURN
END

FUNCTION\_TLOC(ST, ALN, AS)
COMM\_O/CPI/P1, P12
COMM\_O/CHRAD/HRAD
T=ST+ALN-AS+PI
IF(T<14,5,9)
T=T+P12
GOTO 6
K=T/P12
T=T-K*P12
TLOC=T+HRAD
RETURN
END

FUNCTION\_ZDSUNC(XE, XSG)
DIMENSION XE(3), XSG(3)
COMM\_O/CPI/P1, P12
COMM\_O/CDEGR/DEGR
C=0
DO J=1,3
3 C=XR(J)*XSG(J)+C
S=SQR(1-C*C)
IF(C<7,6,7)
ZS=P12
GOTO 11
T ZS=ATAN(S/C),
IF(ZS)10,11,11
ZS=ZS+P1
ZDSUNC=ZS*DEGR
RETURN
END

FUNCTION\_ZDSUNL(ALT, ALN, XSG)
DIMENSION XSG(3)*XR(3)
COMM\_O/CD\_O/CPI/P1, P12
COMM\_O/CDEGR/DEGR
C=0
DO J=1,3
3 C=XR(J)*XSG(J)+C
S=SQR(1-C*C)
IF(C<7,6,7)
ZS=P12
GOTO 11
T ZS=ATAN(S/C),
IF(ZS)10,11,11
ZS=ZS+P1
ZDSUNL=ZDSUNC(XR, XSG)
RETURN
END

4.2. Geomagnetic coordinates of the AES, geomagnetic local time
1. Purpose. For the point, given by directed cosines \((x_N, y_N, z_N)\) in the Greenwich system of coordinates for the normal to the earth's ellipsoid, one determines the geomagnetic latitude and longitude (subprogram GMLL) and geomagnetic time (subprogram TGEOM).

2. Structure. Subprogram GMLL and subprogram-function TGEOM.

General domain: /CHI/3, /CDEGR/1, /BGLL/2, /BGS/2.

3. Conversion: C.Li: GMLL (XSG, GLT, GLN),

T= TGEOM (XSG, XSG).

4. Initial data: Block XSG2—directed cosines of the normal to the surface of the earth's ellipsoid in the Greenwich system of coordinates; the block XSG2—directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

5. Results: GLT, GLN—geomagnetic latitude and longitude in degrees;

TM—local geomagnetic time in hours.

6. Use of the domain COMMON. From the blocks COMMON/CHI/3, /CDEGR/1, the constants are used (see Table 2, table 2.3)

In the block COMMON/BGLL/GLT, GLN the subprogram TGEOM gives the values of the geomagnetic latitude and longitude of the point, in the block COMMON/BGL, GLT, GLN—the values of the geomagnetic latitude and longitude of the sun (in degrees).

7. Algorithm. The geomagnetic latitude \(\phi_m\) and longitude \(\lambda_m\) are computed by the formulas:

\[
\sin \phi_m = x_N^0 x_N^0 + y_N^0 y_N^0 + z_N^0 z_N^0, \\
\cos \phi_m = x_N^0 y_N^0 - y_N^0 z_N^0 \\
\cos \lambda_m = \frac{y_N^0 z_N^0 - x_N^0 y_N^0}{z_N^0 \sin \phi_m - z_N^0},
\]

\(x_N^0 = 0.714707, y_N^0 = -0.1861260, z_N^0 = 0.979924705 -
\)

directed cosines (in the Greenwich system of coordinates) of the radius vector of the geomagnetic pole (\(\phi_G = 73.5^\circ, \lambda_G = -69^\circ\)).

Geomagnetic time \(TM = \lambda_m - \lambda_m^0\), where \(\lambda_m^0\) —geomagnetic longitude of the sun, calculated by the same formulas, as \(\lambda_m\), if instead of \(x_N^0, y_N^0, z_N^0\) one substitutes the values \(x_\odot^0, y_\odot^0, z_\odot^0\), directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

8. Texts.
SUBROUTINE GMLL(XE, GLT, SLN)

DIMENSION XE(3)
COMMON/DEGR/DEGR
COMMON/P1/P1, P12, P12
SFPXE(1) = 0.071447878, 1.66120001 • XE(2) = 0.07924309 • XE(3)
CF = SQRT(1 - SF*SF)

IF(CF) 1, 2, 1

2 GLT = SIGN(P1D2, SF) • DEGR

GOTO 3

1 GLT = ATAN(SF/CF) • DEGR

W = W + 0.07924309 • XE(3)

CF = XE(2) = 0.071447878, 1.66120001 • XE(1)

IF(W) 13, 4, 3

4 GLN = SIGN(P1D2, CF)

GOTO 9

3 GLN = ATAN(CF/W)

IF(W) 7, 8, 6

7 GLN = GLN • PI

GOTO 6

8 IF(GLN) 9, 6, 6

9 GLN = GLN • P12

6 GLN = GLN • DEGR

RETURN

END

FUNCTION TGEOM(XE, XSG)

COMMON /BGS/ SLT, SLN
COMMON/BLT/GLT, GLN
DIMENSION XE(3), XSG(3)

CALL GMLL(XE, GLT, GLN)
CALL GMLL(XSG, SLT, SLN)

W = (GLN - SLN)/15.0 + 12.

IF(W) 10, 2, 2

1 W = 24.0 + W

2 TGEOM = W

RETURN

END
4.3. Auroral longitude, auroral time
(RO3--AULONG, TAUR)

1. Purpose. For the point, given by directed cosines of the normal to the surface of the earth's ellipsoid \((x^O_N, y^O_N, z^O_N)\) in the Greenwich system of coordinates, the auroral longitude (subprogram AULONG) and auroral time (subprogram TAUR) are determined.

2. Structure. Subprograms--functions AULONG, TAUR.

   Common blocks: /CHI/5, /CHRAD/1, /BAU/1.

3. Conversion: AU=AULONG (XNG),
   TA=TAUR (XNG, XSG).

   Remark 1. Subprogram--TAUR refers to the subprogram AULONG.

4. Initial data: The block XNG--directed cosines of the normal to the surface of the earth's ellipsoid in the Greenwich system of coordinates; block XSG--directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

5. Result: AU--auroral longitude in hours, TA--auroral time in hours.

6. Use of the domain COMMON. From the blocks COMMON/CHI/5,
   /CHRAD/1 one uses the constants (see (5), Nos. 1, 3 table 2,3).
   In the block COMMON/BAU/AU the subprogram TAUR addresses the value of the auroral longitude.

7. Algorithm. The auroral longitude \(\lambda_\alpha\) according to (6) is computed by the formulas:
   \[ \lambda_\alpha = \arctg \left( \frac{A}{B} \right) - 56\,^\circ, 61, \]
   \[ = 0,930414 \, x_\alpha - 0,331032 \, y_\alpha - 0,157062 \, z_\alpha, \]
   \[ = -0,353629 \, x_\alpha - 0,923585 \, y_\alpha - 0,147963 \, z_\alpha. \]

8. Texts.

1. Purpose. Subprograms intended for the calculation of $B$, $L$, the parameters of the magnetic field of the earth.


3. Conversion: CALL BL (XC, YC, ZC, NG1, TE1, B3, FL).

4. Initial data: XC, YC, ZC--Greenwich coordinates of the point in meters, NG1--number of harmonics considered (NG1 less than or = 9), TE1--period, for which are computed the coefficients of the expansion for the magnetic field of the earth, minus 1960 (for example, if the period 1965 is chosen, then TE1=5).

5. Results B3--block, containing four actual magnitudes: $B_{phi}$, $B_{theta}$, $B_{r}$, $B$, coordinates of the vector $B$ in the geographical
system of coordinates and the modulus of the vector B in degrees, FL--value of L in radius of the earth.

6. Use of domain COMMON.
In the block COMMON/COF/3 are assigned H_c, phi_c, lambda_c -- geographic coordinates of the magnetic--conjugate point (H_c in km, phi_c, lambda_c-- in degrees).

7. There is a more detailed description in (6).

4.5. Invariant geomagnetic latitude.
(105--AINLAT, OINLAT)

1. Purpose. The subprogram AINLAT determines the invariant geomagnetic latitude for known values H and L, of the geomagnetic parameters. The subprogram OINLAT determines the invariant latitude for the basis of the line of force.

7. Structure. Subprograms--functions AINLAT, OINLAT.
Common block /CDEGR/.

3. Conversions: ALI=AINLAT (B, FL),
ALO=OINLAT (FL).

4. Initial data: B, FL--B, L--parameters of the magnetic field of the earth.

5. Results: ALI--invariant geomagnetic latitude in degrees,
ALO--invariant latitude of the basis of the line of force in degrees.

6. Use of the domain COMMON: One uses the constant from the block /CDEGR/ (see (5) No. 7 table E.3)

7. Algorithm. The invariant geomagnetic longitude Lambda, according to (6), is computed according to the formula:

\[ \Lambda = \arctg \left( \sqrt{1/B_s} - 1 \right), \]

где \( B_s = \sum_{n=0}^{\infty} b_n a^n \),
\[ a = \left( \frac{0.311653}{B} \right)^{1/3}, \]

где \( b_0 = 1.259921, b_4 = -0.00308824, \)
\( b_1 = -0.1984259, b_5 = 0.00082777, \)
\( b_2 = -0.04686632, b_6 = -0.00105977, \)
\( b_3 = -0.01314096, b_7 = 0.0018314. \)

Инвариантная широта у основания силовой линии,

\[ \Lambda_o = \arctg \left( \sqrt{L} - 1 \right), \]

где B, L -- геомагнитные параметры.
8. Texts.

FUNCTION OINLAT(FL)
COMMON /CDEGR/DEGR
OINLAT = 0
TN = FL - 1.
IF(TN) 2, 2, 1
1 OINLAT = ATAN(SQR(TN)) * DEGR
2 RETURN
END

FUNCTION AINLAT(R, FL)
COMMON /CDEGR/DECR
A = (1.311653/R) ** (1.333333333/FL)
B = A = (((((A - 0.00185142 - 0.00374777**2 - 0.00027777)**2 + 0.00308026
A = -0.01014932 + A - 0.00006432 + A - 0.19815501 + A + 1.299021)
AINLAT = 0.
TN = 1, BS - 1.
IF(TN) 2, 2, 1
1 AINLAT = ATAN(SQR(TN)) * DEGR
2 RETURN.
END

4.6. Geographical coordinates of T--point--points of intersection with the plane of the terminator of the line, connecting the AES with the sun (106--FTERM)

1. Purpose. From the known Greenwich coordinates of the AES (x, y, z) and the directed cosines of the radius vector of the sun (x_0, y_0, z_0) one determines the geographical coordinates of T--the point.

2. Structure: Subprogram FTERM. One uses the external sub-program GEOGRC (B10).


4. Initial data: Block XG--Greenwich coordinates of AES; block XSG--directed cosines of radius vector of the sun in Greenwich system of coordinates.

5. Results: HT, TLT, TLN--elevation above the surface of the
earth, latitude and longitude of T-point (angles in radians).

6. Algorithm: \( x_T, y_T, z_T \) -- Greenwich coordinates of the T-point, points of intersection with the plane of the terminator of the line, connecting the AES with the sun (or continuation of this line), are computed with the formulas:

\[
\begin{align*}
x_T &= x - a_x x_0^0, \\
y_T &= y - a_y y_0^0, \\
z_T &= z - a_z z_0^0,
\end{align*}
\]

The geographical coordinates of T-point are computed with the algorithm par. 3.10 (5).

7. Text.

SUBROUTINE PTM (XG, XSG, H, T, X, Y, Z, X0, Y0, Z0, A, B, C, D)

7. Determination of the Greenwich coordinates of the point, given geographical latitude, longitude and elevation above the surface of the earth's ellipsoid, and matrices of the transformation from the Greenwich to the point system of coordinates, connected with this point (I 07-111.1).

1. Purpose. The Greenwich coordinates \((x_G, y_G, z_G)\) are determined for the point \(I\), prescribed geographical latitude, \((\phi_I)\) longitude \((\lambda_I)\) and elevation \((h_I)\) above the surface of the earth's ellipsoid.

The point topocentric system of coordinates, connected with the point \(I\), is determined in the following manner.

The origin of the coordinates coincides with the point \(I\), the axis \(ix_I\) is directed to the north pole of the earth by the tangent to the meridian of the point \(I\), the axis \(iy_I\) -- by the external normal to the surface of the earth's ellipsoid, the axis \(iz_I\) completes the system up to the right side.

The subprogram IGINN determines the matrix of the transformation from the Greenwich system of coordinates to the point system.

2. Structure. Subprogram IGINN.

Common domain: /CAE/.
3. Conversion: CALL P12T (it, ilT, ilN, XG, GI).
4. Initial data: M, ilT, ilN-elevation, geographical latitude, longitude of the point i.
5. Results: Block XIGJ-Greenwich coordinates of the point i; GP=twodimensional i block (5, 5) matrix of the transformation from Greenwich system i coordinates to the point system.
6. Use of the domain COMMON: one uses the constants from the block /CAS/ (see (.), No. 11, Table 2.1).

7. Algorithm: Coordinates of point i in the Greenwich system of coordinates are determined with the formulas:

\[
\begin{align*}
    x_{P0} & = (a_e/A + h_p) \cos \varphi_p \cos \lambda_p, \\
    y_{P0} & = (a_e/A + h_p) \cos \varphi_p \sin \lambda_p, \\
    z_{P0} & = (a_e(1-\alpha)^2/A + h_p) \sin \varphi_p, \\
    A & = (\cos^2 \varphi_p + (1-\alpha)^2 \sin^2 \varphi_p)^{1/2},
\end{align*}
\]

\(a_e, \alpha, h_p\)-semi-major axis and coefficient of compression of the earth's ellipsoid.

Matrix of the transformation \((G_{ij})\) from the Greenwich system of coordinates to the point system (determined in par. 1), which has the following form:

\[
\begin{bmatrix}
    G_{Pij} \\
\end{bmatrix} =
\begin{bmatrix}
    -\sin \varphi_p \cos \lambda_p & -\sin \varphi_p \sin \lambda_p & \cos \varphi_p \\
    \cos \varphi_p \cos \lambda_p & \cos \varphi_p \sin \lambda_p & \sin \varphi_p \\
    -\sin \lambda_p & \cos \lambda_p & 0
\end{bmatrix}
\]
1. Determine the coordinates of the vector of velocity in the point of the Greenwich system of coordinates, the radius vector between the particle and the Earth, and the projection of the vector of velocity in the Greenwich plane perpendicular to the plane perpendicular to the Greenwich axis.

2. Convert the Greenwich system of coordinates to the Earth system of coordinates.

3. Convert the Earth system of coordinates to the vector of velocity in the point of the Greenwich system of coordinates.

4. Calculate the coordinates of the vector of velocity in the point of the Greenwich system of coordinates.

5. Calculate the vector of velocity in the point of the Greenwich system of coordinates.
Results: block $X_2$—coordinates of $\mathbf{AES}$ in topocentric point system of coordinates, DAL, AM, AZ, determined above.

4. Conversion to subprogram TURN.
   CALL TURN ($X$, $Y$, $N$).

   Initial data: $X$—block of dimension $N$; $A$—block of dimension $(N, N)$; $N$—dimension.

Results: $Y$—block of dimension $N$ ($Y = AX$).

6. Algorithm

\[
\begin{bmatrix}
  z_p \\
  y_p \\
  x_p
\end{bmatrix} =
\begin{bmatrix}
  x - x_{PG} \\
  y - y_{PG} \\
  z - z_{PG}
\end{bmatrix} =
\begin{bmatrix}
  v_{x_p} \\
  v_{y_p} \\
  v_{z_p}
\end{bmatrix} =
\begin{bmatrix}
  v_x \\
  v_y \\
  v_z
\end{bmatrix} = [GP_{ij}]
\]

\[
DAL = \left((x - x_{PG})^2 + (y - y_{PG})^2 + (z - z_{PG})^2\right)^{1/2};
\]

\[
AM = \arctg \left( \frac{z_p}{x_p^2 + y_p^2} \right);
\]

\[
AZ = \arctg \left( \frac{y_p}{x_p} \right);
\]

$(G_{ij})$—matrix of transformation from Greenwich system of coordinates to the point form;

7. Text:

```fortran
SUBROUTINE GRPX(XG, XP, GP, XP, CAL, AM, AZ)
DIMENSION XG(3), XP(3), GP(3, 3), XP(3, 3)
DAL = 0.
DO 2 J = 1, 3
  X(J) = XG(J) - XP(J)
  DAL = DAL + X(J) * X(J)
  DAL = SQRT(DAL)
CALL TURN(X, GP, XP, 3)
END
```

```fortran
SUBROUTINE TURN(XG, GP, XP, N)
DIMENSION XG(N), GP(N, N), XP(N)
DO 1 I = 1, N
  XP(I) = 0.
DO 1 J = 1, N
  XP(I) = XP(I) + GP(I, J) * XG(J)
END
```
4. Y. Transformation from the absolute system of coordinates to the solar-ecliptic (109-ASSECL).

1. Purpose. For known coordinates of AES \((X, Y, Z)\) in the absolute system of coordinates one determines the coordinates of the AES in the solar-ecliptic system of coordinates, determined by the following means: the center of the system \(O\) coincides with the center of the earth, axes CX\(_{ce}\), CY\(_{ce}\) lie in the plane of the ecliptic, the axis CX\(_{ce}\) directed towards the sun, axis OZ\(_{ce}\) perpendicular to the plane of the ecliptic and forming an acute angle with the axis of rotation of the earth directed toward the north.

2. Structure. Subprogram ABSECL.
Subprograms utilized: GCLTLN (B10). Common block /BIECL/.


4. Initial data: Block XA\(_2\)--absolute coordinates of AES;
Block XAS\(_2\)--directed cosines of radius vector of the sun in the absolute system of coordinates.

5. Results: Block XSE\(_2\)--coordinates of AES; CLTJ, CLNS--geocentric latitude and longitude of AES in solar-ecliptic system of coordinates.

6. Use of the domain COMMON. From the block COMMON /BIECL/.

one uses cos epsilon and sin epsilon, computed in the subprogram SUN(C02), where epsilon--angle of inclination of the plane of the earth's equator to the plane of the ecliptic.

7. Algorithm:

\[
\begin{bmatrix}
X_{SE} \\
Y_{SE} \\
Z_{SE}
\end{bmatrix}
= 
\begin{bmatrix}
X^0 \\
Y^0 \\
Z^0
\end{bmatrix}
\begin{bmatrix}
-\frac{X^0 Y^0}{r^1} \sin Y^0 \\
\frac{X^0 Z^0}{r^1} \sin Y^0 \\
\frac{Y^0}{r^1} \sin Y^0
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix},
\]

\[r^1 = (Y^0_r + Z^0_r)^{1/2}.
\]

8. Text:
8. Text:

SUBROUTINE ABSEC(L(V,XS,XE,AEC,DEC)
COMMON /BIEC/CEP,SEP
DIMENSION V(3),XS(3),XE(3)
R1=SQRT(XS(2)*XS(2)+XS(3)*XS(3))
(T(R1),2,3,2)
3 XECP
-1=SEP
GOTO 4
2 XECP(S2)/R1
W1=XS(3)/R1
4 XECP(3)
DO 1 J=1,3
1 XECP(1)=XEC(1)+V(1)*X(1)+V(3)*X(3)
RETURN
END

4,10. Moments of time of the entry of AES into the earth's shadow and exit from the shadow (I12--3134'4')

1. Purpose. For known elements of the orbit: a, e, i, Omega, omega, for position in the orbit (u) at a given moment in time t one determines the moments of time of entry of the I33 into the shadow of the earth and exit from the shadow.

2. Structure: Subprogram SHADOW.
One uses external subprograms ROOT 4 (I11), SUN (COP).
Common blocks: /CHR/1, /CRE/1, /CHI/3.

3. Conversion: CALL SHADOW (DT, T, A, T1, T2).

4. Initial data: Block A, containing elements of the orbit and the argument of latitude: a, e, i, Omega, omega, u; DT-data in the form RJD, T--Moscow time, corresponding to the argument of latitude u.

5. Results: T1, T2—are, respectively, the moments of entry of the AES into the shadow of the earth and exit from the shadow (in units given by the scale factor ESEC). If AES does not get into the shadow, then T1=T2=0.

6. Use of the domain COMMON: One uses the constants from the blocks COMMON;/CHR/1, /CRE/1, /CHI/3, (see (5), Nos. 5, 9, table 2.1, No. 1 table 2.3).

7. Algorithm: Moments of time (T1) of entry of AES into the shadow of the earth and (T2) of exit from the shadow are determined without computation of the effect of the penumbra; with the assumption that the earth has no compression and is not displaced orbitally. (7). These moments of time correspond to actual anomalies of the AES, satisfying the following relationship:

\[
\cos \psi = \frac{R_\theta r}{R_\theta + r} = \left(\frac{r^2 - a_e^2}{r}\right)^{1/2}
\]  

(1)

where \(R_\theta\) — radius vector of the sun, \(r\) — radius vector of the AES, \(a_e\) — equatorial radius of the earth, \(\psi\) — angle between radius-vector of AES and radius vector of the sun. The relationship (1) leads to the equation of the fourth degree with respect to
the cosines of the actual anomaly:

\[ S^\prime = A_0 \cos^4 \psi + A_1 \cos^3 \psi + A_2 \cos^2 \psi + A_3 \cos \psi + A_4, \]  
(2)

where

\[ A_0 = \left( \frac{\alpha e}{p} \right)^4 e^2 - 2 \left( \frac{\alpha e}{p} \right)^2 (\xi^2 - \beta^2) e^2 + \left( \beta^2 + \xi^2 \right)^2, \]

\[ A_1 = 4 \left( \frac{\alpha e}{p} \right)^4 e^6 - 4 \left( \frac{\alpha e}{p} \right)^2 (\xi^2 - \beta^2) e^6, \]

\[ A_2 = 6 \left( \frac{\alpha e}{p} \right)^4 e^8 - 2 \left( \frac{\alpha e}{p} \right)^2 (\xi^2 - \beta^2) - 2 \left( \frac{\alpha e}{p} \right)^2 (1 - \xi^2) e^8 + 2 (\xi^2 - \beta^2) (1 - \xi^2) - 4 \beta^2 \xi^2, \]

\[ A_3 = 4 \left( \frac{\alpha e}{p} \right)^4 e^4 - 4 \left( \frac{\alpha e}{p} \right)^2 (1 - \xi^2) e^4, \]

\[ A_4 = \left( \frac{\alpha e}{p} \right)^4 - 2 \left( \frac{\alpha e}{p} \right)^2 (1 - \xi^2) + (1 - \xi^2)^2; \]

\[ \rho \] - parameter of orbit = \( \alpha (1 - e^2) \),

\[ \beta = X^0_o P_x + Y^0_o P_y + Z^0_o P_z, \]

\[ \xi = X^0_o Q_x + Y^0_o Q_y + Z^0_o Q_z, \]

\( X^0_o, Y^0_o, Z^0_o \) – направляющие косинусы радиуса-вектора Солнца, \( R^0_o \),

directed cosines of radius vector of the sun

\( P_x = \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i, \)

\( P_y = \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i, \)

\( P_z = \sin \omega \sin i, \)

\( Q_x = -\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i, \)

\( Q_y = -\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i, \)

\( Q_z = \cos \omega \sin i. \)

The solutions to equations (2), satisfying the conditions:

\[ \text{beta} \cos \psi + \xi \sin \psi \text{ less than } 0, \]

\[ S = (1 + \text{ecos} \psi)^2 + (p/a_e)^2 (\text{beta} \cos \psi + \xi \sin \psi)^2 - (p/a_e)^2 = 0. \]

are assumed for further consideration.

The condition:

\[ S^\prime = \rho^2 (\beta \cos \psi + \xi \sin \psi) (\xi \cos \psi - \beta \sin \psi) - a_e^2 e (1 + \text{ecos} \psi) \sin \psi > 0 \]

determines the value of the actual anomaly \( \psi \), in the entry of the ISZ into the shadow of the earth, \( S^\prime \) less than zero corresponds to the value \( \psi_2 \), determining the exit of the ISZ from the shadow.
With the value of $t$, time, for corresponding given position of AES in orbit, we determine $\tau$, the time of passage for AES, through relations:

$$\tau = \frac{L - (E_i - esinE_i)}{\lambda},$$

$$2 \theta \lambda = \sqrt{\mu / a^3}, \quad t_{E_i/2} = \left(\frac{1-e}{1+e}\right)^{1/2} t_{E_i/2}, \quad v_i = u - \omega,$$

mu-produce of gravitational constant times the mass of the earth. Moments of time $T_i$ (i=1, 2) are determined by the known values $t$ and $v_i$:

$$T_i = \tau + \frac{(E_i - esinE_i)}{\lambda},$$

$$t_{E_i/2} = \left(\frac{1-e}{1+e}\right)^{1/2} t_{E_i/2}.$$
4.11. Calculation of the roots of algebraic equations of the fourth, third and second degree (ILL-RCOT4)

1. Purpose. Determination of the roots of the algebraic equation

\[ A_0 x^4 + A_1 x^3 + A_2 x^2 + A_3 x + A_4 = 0, \]

of the fourth \( A_0 \neq 0 \), third \( (A_0 = 0) \), and second \( (A_0 = 0, A_1 = 0) \) degree with real coefficients.


4. Initial data: Block A--coefficients of equation (1) in order: \( A_0, A_1, A_2, A_3, A_4 \).

5. Results: Block \( Y_4 \)--roots of equation (1);
   Block \( IR_2 \)--characteristics of the roots:
   \( 0, \) if \( Y(1), Y(2) \)--real roots
   \( 4, \) if \( Y(1), Y(2) \)--real and imaginary part of a complex root (for complex-conjugate root \( Y(3) \) taken with the opposite sign)
   \( 1, \) if \( A_0 = A_1 = A_2 = 0 \) (equation of first degree)
   \( 0, \) if \( Y(3), Y(4) \)--real roots
   \( 4, \) if \( Y(3), Y(4) \)--real and imaginary part of a complex root (for complex-conjugate root \( Y(4) \) taken with opposite sign)
   \( 3, \) if \( A_0 = 0 \) (equation of third degree)
   \( 2, \) if \( A_0 = A_1 = 0 \) (equation of second degree).

Remark. For \( IR(1) = 1 \), \( Y(1) \)--root of equation of the first degree. For \( IR(1) = 3 \), \( Y(3) \)--real root of an equation of the third degree.

6. Algorithm. For the solution of equation (1) one uses the algorithm, described in [7] (p. 448):
   a) Equation of 4 degree is solved by the method of Descartes: Dividing equation (1) by \( A_0 \) (if \( A_0 = 0 \), then we proceed to part delta), we obtain
   
   \[ y^4 + B_1 y^3 + B_2 y^2 + B_3 y + B_4 = 0. \]

   We transform this equation, excluding from it the term of the third degree:

   \[ k^4 + P k^2 + Q k + R = 0, \]

   
   \[ P = 6 B_1 h^2 + 3 B_2 h + B_4, \]

   \[ Q = 4 B_1 h^4 + 3 B_1 h^2 + 2 B_2 h + B_3, \]

   \[ R = h^4 + B_1 h^3 + B_2 h^2 + B_3 h + B_4, \]

   \[ h = -B_1/4, \quad y = x + h. \]
For \( c = 0 \) we obtain the biquadratic equation, for the solution of which we use the formulas of part v. For \( c \neq 0 \) we consider the cubic resolvent
\[
\begin{align*}
&c_1 = c + 2t^2 + c_3 t + c_4 = 0, \\
&t^2 + c_2 t + c_3 + c_4 = 0,
\end{align*}
\] (3)
We transform equation (3) into the form:
\[
z^3 + \alpha z + \beta = 0,
\]
\[
\alpha/3 = c_3/3 - s^2, \quad b/2 = +s^2 - c_3 s/2 - c_4/2,
\]
\[
s = c_3/3, \quad z = t + s,
\]
\[
\Delta = \frac{a^5}{27} + \frac{b^2}{4}.
\] (4)
If \( \Delta \) is greater than 0, then the solution of equation (4) is found by the formula of Kaplan:
\[
z_3 = (-\beta/2 + (\Delta)^{1/2})^{1/3} + (\beta/2 - (\Delta)^{1/2})^{1/3}
\]
\( z_1, z_2 \) — kompleksnye korny.
If \( \Delta = 0 \), then the roots of equation (4) are determined by the formula:
\[
z = z_s = (\pm \beta/2)^{1/3}
\]
If \( \Delta \) is less than 0, then we use the representation of the roots as resolvents in trigonometric form,
\[
z_1 = E_0 \cos \phi/3, \quad z_2 = E_0 \cos(\phi/3 + 120^\circ), \quad z_3 = E_0 \cos(\phi/3 + 240^\circ),
\]
\[
E_0 = 2 (-\frac{a}{3})^{1/2}, \quad \cos \phi = -\frac{6}{2} / (-\frac{a}{27})^{1/2}, \quad 0 < \phi < \pi.
\]
We select a critical root of equation (3):
\[R' = \max (z_1 - s, z_2 - s, z_3 - s),\]
where \( z_1 - s \) — real root of equation (3).
Knowing \( R' \), it is possible to break down equation (2) with the multipliers:
\[
(x^2 + x \sqrt{R'} + \xi)(x^2 - x \sqrt{R'} + \beta) = 0,
\]
\[
\xi = \frac{1}{2} (P + R' - Q/\sqrt{R'}), \quad \beta = \frac{1}{2} (P + R' + Q/\sqrt{R'})
\]
Using the relationships of division V, we obtain the solutions \( x_i \) 
(i=1,4) of two quadratic equations (1), and consequently--the roots of equation (1): \( y_i = x_i + h \).

6. Cubic equation we write in the form

\[ A_1 y^3 + A_2 y^2 + A_3 y + A_4 = 0. \]

Dividing the left side by \( A_1 \) (if \( A_1 = 0 \), we go on to division b), we obtain

\[ y^3 + c_2 y^2 + c_3 y + c_4 = 0. \]

This equation coincides with equation (3), the solution of which is described in division a.

Remark For Delta greater than zero the complex roots \( y_1, y_2 \)
of equation (4) are obtained by the solution of a quadratic equation

\[ y^2 + (c_2 + x_3 - s) y + c_3 + (c_2 + x_3 - s)(x_3 - s), \]

where \( x_3 \)--real root of equation (4), \( s=c_2/3. \)

b) Dividing the left side of the quadratic equation

\[ A_1 y^2 + A_2 y + A_4 = 0 \]

by \( A_1 \) (if \( A_1 = 0 \), then we go on to division c), we obtain:

\[ y^2 + R_3 y + R_4 = 0. \]

The roots of this equation

\[ y_{1,2} = \frac{-B_3 \pm \sqrt{B_3^2 - 4B_4}}{2} \]

C) Equation of the first degree \( A_3 y + A_4 = 0 \).

Solution \( y = -A_4/A_3 \).
SUBROUTINE ROOT4(A,V,IR)
DIMENSION A(9),V(4),IR(2),IN(3),R16
DATA PI,PI02 PI2/3.14159265,1.37079022,6.66931035/
DATA IN/1.0,2/;
K=2
IR(1)=0
IR(2)=1
IF(A(1))1,100,1
1 N=1
DO 2 J=1,4
V(J)=0.
2 D(J)=A(J+1)/A(1)
M=-B(1)/4.
V(2)=M.
DO 3 J=1,6
3 V(2)=V(2)+H*B(J)
G=4.
W=3.
DO 4 J=1,3
Q=G*B(J)*W
4 N=W-1.
P=(2.*H*B(1)+3.*H*B(2))
V(1)=P
N=1
IF(ABS(Q)=1.E-16)103,103,100
103 DO 7 J=1,N-2
I=IN(J)
D=V(J)+V(J)-A.-V(J)-1.
V(J)=-V(J)/2.
IF(D)5,6,6
6 D=SQR(D)/2.
V(J+1)=V(J)-D.
V(J)=V(J)+D
IR(1)=0
GOTO 7
7 CONTINUE
GOTO(104,105,107),K
104 IF(IR(1))10,11,10
11 V(4)=-V(1)
V(2)=-V(2)
V(3)=0,
V(1)=0.
N=1
K=3
GOTO 103
10 W1=V(1)+V(1)
W=W1+V(2)+V(2)
R1=SQR(W)
W=SQR(W+W1)
R1=SQR(R1)+2.
V(1)=V(2)/R1
V(2)=W/R1
V(3)=M-V(1)
V(1)=V(1)+M
V(4)=V(2)
IR(2)=4.
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100 IF(A(2))=15: GOTO 13
10 DO 16 J=2,4
16 B(I)=A(J-1)/A(2)
17 IR(2)=3
GOTO 109
108 B(2)=V(1)=2.
109 S=B(2)/3.
110 W=W*S:
AR=B(3)/3,-W:
BR=S=B(3)/2,-5*W-B(4)/2:
DEP=AR+BR=BR:
12 IF(D)17,18,19
17 M=2.5*AR=-AR:
18 IF(BR)120,21,20
19 W=W1+P12
GOTO 102
20 W=SQRT(-D):
IF(BR)51,30,91
GOTO 102
21 W=W1+P12
GOTO 23
22 W=ATAN(W1/BR):
IF(BR)22,23,23
23 W=W1+P11
24 DO 25 J=1,3
V(J)=W*COS(W1/3.)
25 W=W1+P12
GOTO 110
19 W=SQRT(D):
V(1)=BR+W:
V(2)=BR-W:
DO 25 J=1,2:
W=ABS(V(J)):
W1=ALOG(W):
W1=EXP(W1/3.)
25 V(J)=SIGN(W1,V(J))
V(3)=V(1)+V(2)-S
V(1)=0.
V(2)=0.
K=IR(2)
H1=V(3)
GOTO (111,205,205)K
18 W=ABS(BR):
W1=ALOG(W):
W1=EXP(W1/3.)
V(1)=SIGN(W1,BR):
V(2)=-V(1):
V(1)=2*V(1):
V(3)=V(2):
110 DC26 J=1,3
26 V(J)=V(J)-S:
GOTO (106,105,205)K
106 R1=AMAX(V(1),V(2),V(3)):
111 V(1)=SQRT(R1)
V(3)=-V(1):
W=W+R1:
W1=W/V(1):
V(2)=W-W1)/2.
V(4)=(W+W1)/2.
N=3
K=3
GOTO 103
107 DO 30 J=1,N,2:
I=IN(J):
V(J)=V(J)+H:
IF(IR(1))30,31,30
31 V(J+1)=V(J)+H:
30 CONTINUE:
GOTO 105
205 V(1)=B(2)+V(3):
V(2)=V(1)+V(3)+B(3):
N=1:
K=2:
GOTO 103:
101 IF(A(3))=36,102,36
36 V(1)=A(4)/A(3):
V(2)=A(5)/A(3):
IR(2)=2:
N=1:
GOTO 103:
102 IF(A(4))=37,105,37
37 V(1)=A(5)/A(4):
GOTO 103:
END
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