Mechanical Engineering Department
College of Engineering
Clemson University
Clemson, South Carolina

(NASA-CR-164536) OPTIMIZATION OF FIBER REINFORCED STRUCTURES TO SATISFY AEROELASTIC REQUIREMENTS Final Technical Report (Clemson Univ.) 8 p HC A02/MF A01 CSCL 11D
N81-27200
Unclas
G3/24 26789

CLEMSON UNIVERSITY
SOUTH CAROLINA
1889

FINAL TECHNICAL REPORT

NASA GRANT No. NSG 1411

OPTIMIZATION OF FIBER REINFORCED STRUCTURES TO SATISFY AEROELASTIC REQUIREMENTS

by

Carl S. Rudisill

ABSTRACT

A numerical procedure was developed for minimizing the structural mass of an aircraft structure which must have a specified minimum flutter velocity or divergence velocity. During the optimization process the arrangement of the structural members remains fixed, while the stiffness parameters of the structure are varied.

I. Recurrence Relation

Let w be the total weight of an aircraft structure and let ϕ be an aeroelastic response parameter such as the flutter velocity or the divergence velocity. It is desired to minimize the total weight of the structure in such a way that the flutter velocity and the divergence velocity are greater than or equal to some specified values. The problem can be stated mathematically as

$$\text{Minimize:} \quad w = f(P)$$

subject to the restriction $V(P) - V_0 \geq 0$, $U(P) \geq U_0$

where P is the vector of variable design parameters P_i, V is the flutter velocity and U is the divergence velocity. V_0 and U_0 are the specified flutter velocity and divergence velocity constraints respectively.

Since it is seldom possible or to optimize a structure such that $V = V_0$ and $U = U_0$ simultaneously, the problem will be restated as follows:

$$\text{Minimize:} \quad w = f(P) \tag{1}$$

subject to the restriction $\phi = \phi_0$ (2)

where ϕ is either the flutter or the divergence velocity.

An expression for the total weight of the structure can be formulated such that w is a linear function of the design parameters,

$$W = W_0 + \sum_{i=1}^{n} w_i P_i$$

where W_0 is the non-variable structural weight. Using the method of Lagrangian multipliers, the parameters P_1, P_2, ... P_n must be found which will cause the function

$$F(P) = W - \frac{1}{\lambda} \varphi \tag{3}$$

to be an extremal subject to the condition

$$\varphi - \varphi_0 = 0 \tag{4}$$

Since F(P) must be an extremal

$$\frac{\partial F}{\partial P_i} = w_i - \frac{1}{\lambda}\frac{\partial \varphi}{\partial P_i} = 0 \qquad i = 1, 2, \cdots, n \tag{5}$$

where w_i are constand derivatives of w with respect to P_i.

Let $\phi(P)$ be approximated by the expression

$$\varphi(P) = a + \sum b_i P_i + \frac{1}{2}\sum_{i=1}^{n} C_i P_i^2 \tag{6}$$

Substituting equation (6) into equation (5) yields

$$w_i - \frac{1}{\lambda}\left(b_i + C_i P_i\right) = 0$$

or

$$P_i = (\lambda w_i - b_i)/C_i \tag{7}$$

The parameters b_i and c_i are computed from the following relations

$$\frac{\partial \varphi_1}{\partial P_{i1}} = b_i + C_i P_{i1} \tag{8}$$

$$\frac{\partial \varphi_2}{\partial P_{i2}} = b_i + C_i P_{i2} \tag{9}$$

where P_{i_1} and $\frac{\partial\phi_1}{\partial P_{1_1}}$ are values of P_i and $\frac{\partial\phi_2}{\partial P_i}$ respectively at point 1 in the design space while P_{i_2} and $\frac{\partial\phi_2}{\partial P_i}$ are values of P_i and $\frac{\partial\phi}{\partial P_i}$ respectively at point 2 in the design space.

From equations (7) and (8) it is seen that

$$b_i = \left(P_{i2}\frac{\partial \varphi_1}{\partial P_{i1}} - P_{i1}\frac{\partial \varphi_2}{\partial P_{i2}}\right) \Big/ (P_{i2} - P_{i1}) \tag{10}$$

$$C_i = \left(\frac{\partial \varphi_2}{\partial P_{i2}} - \frac{\partial \varphi_1}{\partial P_{i1}}\right) \Big/ (P_{i2} - P_{i1}) \tag{11}$$

and

$$a = \varphi_2 - \sum_{i=1}^{n} b_i P_{i2} - \frac{1}{2}\sum_{i=1}^{n} C_i P_i^2 \tag{12}$$

where the values of a, b_i and c_i will change with each redesign step. Substituting equations (6) and (7) into equation (4) yields the equation

$$a + \sum_{i=1}^{n} b_i \left(\frac{\lambda w_i - b_i}{C_i}\right) + \frac{1}{2}\sum_{i=1}^{n} C_i \left(\frac{\lambda w_i - b_i}{C_i}\right)^2 - \varphi_o = 0$$

or

$$\frac{1}{2}\sum_{i=1}^{n} \frac{w_i^2}{C_i}\lambda + \frac{1}{2}\sum_{i=1}^{n} \frac{b_i^2}{C_i} + a - \varphi_o = 0 \tag{13}$$

solving for λ

$$\lambda = \sqrt{\frac{\sum_{i=1}^{n} \frac{b_i^2}{C_i} + 2a - 2\varphi_o}{\sum \frac{w_i^2}{C_i}}} \tag{14}$$

Substituting equation (14) into equation (7) yields the following recurrence equation for P_i:

$$P_i = \left(w_i \sqrt{\frac{\sum_{i=1}^{n} \frac{b_i^2}{C_i} + 2a - 2\varphi_o}{\sum \frac{w_i^2}{C_i}}} - b_i\right) \Big/ C_i \tag{15}$$

where b_i, C_i and a are given by equations (10), (11) and (12).

II. Optimization Procedure

The optimization procedure utilizes the recurrence equation (15) in a repative manner to compute new values of P_i. The procedure begins at some arbitrarily chosen point in the design space P_{i_1} where ϕ_1 and $\frac{\partial\phi_1}{\partial P_{i_1}}$ are computed. One gradient step is then made to find the values of P_{i_2} and ϕ_2 then, equations (10), (11), (12) and (15) are used to compute new values of P_i. Next P_{i_1}, ϕ_1 and $\frac{\partial\phi_1}{\partial P_{i_1}}$ are set equal to values of P_{i_2}, ϕ_2 and $\frac{\partial\phi_2}{\partial P_{i_2}}$, respectively and the design parameters P_{i_2} are set equal to the values of P_i computed from equation (15). New values of ϕ_2 and $\frac{\partial\phi_2}{\partial P_{i_2}}$ are computed from values of P_{i_2} and the process is repeated until all the values of $|P_{i_2}-P_{i_1}|$ approaches a small number ε. When any of the design parameters become less than a minimum gage size, then that parameter P_{j_2} is set equal to the gage size with $P_{i_1} = P_{i_2} - \varepsilon$. When any value of P_i is such that $|P_i - P_{i_2}| < \varepsilon$ then P_{i_2} is set equal to P_i with $P_{i_1} = P_{i_2} - \varepsilon$. Computation is continued until $|P_i-P_{i_2}|$ approach some small number ε.

III. Application

The method was applied to the problem of Ref. (1) which was a lifting surface supported by a simple box beam. The method appeared to be superior to that of Ref. (1); however, the comparison was not conclusive since an error in the program for finding the stiffness matrix was discovered. The error will be corrected. The program used for finding the stiffness matrix was not the same as that used for Ref. (1).

At this writing work is underway to apply the program to a fiber reinforced wing. The wing is straight with a uniform cross-section and zero sweep angle. The fiber directions are fixed in four different directions 0°, 45°, -45° and 90° with respect to the span. There are eight lamina in the laminate which are arranged symmetrically about the midplane of the laminate. Each lamina has one fiber direction. The thickness of the lamina are the design parameters for the structure. A program for the analysis of the structure has been completed.

IV. Conclusions

The optimization method which was presented here is a workable method; however, its computational efficiency and usefulness have not been established by testing it on a structure with many design parameters.

V. Reference

Rudisill, C.S. and Bhatia, K.G., "Optimization of Complex Structures to Satisfy Flutter Requirements," AIAA Journal, Vol. 9, No. 8, Aug. 1971, pp. 1487-1491.

II. Optimization Procedure

The optimization procedure utilizes the recurrence equation (15) in a repative manner to compute new values of P_i. The procedure begins at some arbitrarily chosen point in the design space P_{i_1} where ϕ_1 and $\frac{\partial\phi_1}{\partial P_{i_1}}$ are computed. One gradient step is then made to find the values of P_{i_2} and ϕ_2 then, equations (10), (11), (12) and (15) are used to compute new values of P_i. Next P_{i_1}, ϕ_1 and $\frac{\partial\phi_1}{\partial P_{i_1}}$ are set equal to values of P_{i_2}, ϕ_2 and $\frac{\partial\phi_2}{\partial P_{i_2}}$, respectively and the design parameters P_{i_2} are set equal to the values of P_i computed from equation (15). New values of ϕ_2 and $\frac{\partial\phi_2}{\partial P_{i_2}}$ are computed from values of P_{i_2} and the process is repeated until all the values of $|P_{i_2}-P_{i_1}|$ approaches a small number ε. When any of the design parameters become less than a minimum gage size, then that parameter P_{j_2} is set equal to the gage size with $P_{i_1} = P_{i_2} - \varepsilon$. When any value of P_i is such that $|P_i - P_{i_2}| < \varepsilon$ then P_{i_2} is set equal to P_i with $P_{i_1} = P_{i_2} - \varepsilon$. Computation is continued until $|P_i-P_{i_2}|$ approach some small number ε.

III. Application

The method was applied to the problem of Ref. (1) which was a lifting surface supported by a simple box beam. The method appeared to be superior to that of Ref. (1); however, the comparison was not conclusive since an error in the program for finding the stiffness matrix was discovered. The error will be corrected. The program used for finding the stiffness matrix was not the same as that used for Ref. (1).

At this writing work is underway to apply the program to a fiber reinforced wing. The wing is straight with a uniform cross-section and zero sweep angle. The fiber directions are fixed in four different directions 0°, 45°, -45° and 90° with respect to the span. There are eight lamina in the laminate which are arranged symmetrically about the midplane of the laminate. Each lamina has one fiber direction. The thickness of the lamina are the design parameters for the structure. A program for the analysis of the structure has been completed.

IV. Conclusions

The optimization method which was presented here is a workable method; however, its computational efficiency and usefulness have not been established by testing it on a structure with many design parameters.

V. Reference

Rudisill, C.S. and Bhatia, K.G., "Optimization of Complex Structures to Satisfy Flutter Requirements," AIAA Journal, Vol. 9, No. 8, Aug. 1971, pp. 1487-1491.