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TRANSPORT FUNCTIONS OF NITROGEN UP TO 26,000°K

W. Hermann and E. Schade

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It turns out, that at the highest measured arc current, i.e., 570 A, in the axial region of the arc about 95% of the supplied energy is carried off by radiation.

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Transport Functions of Nitrogen up to 26,000°K * ** /333 ***

W. Hermann and E. Schade

Elektrophysikalisches Institut der Technischen Hochschule, Muenchen

Submitted 19 October 1969

Abstract

The E(I)-characteristic and a large number of T(r,I)-distributions measured in a 5 mm Ø N_2 cascade arc at normal pressure are used to evaluate the transport properties of nitrogen up to 26,000°K. The electrical conductivity σ(T) and from this the cross-section for atom-electron collision and the Coulomb-cross-section are determined directly from the E(I)- and several T(r,I) curves. For the evaluation of the thermal conductivity K(T) three temperature regions are discerned: Up to about 10,000°K K(T) is derived derived directly from the energy equation since here the energy transport by radiation does not play an important role. Between 10,000 and 15,000°K the radiative energy flux for different arc currents, the thermal conductivity and from this the charge exchange cross-section are determined in a good approximation utilizing the large number of measured temperature distributions. Above 15,000°K the already evaluated collision cross-sections are used to compute K(T). With K(T) known the radial distribution of the balance

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*** Numbers in margin indicate foreign pagination between emission and absorption of radiative power per unit volume is evaluated for different arc currents. It turns out, that at the highest measured arc current, i.e. 570 A, in the axial region of the arc about 95% of the supplied energy is carried off by radiation.

I. Introduction

The cylindrical cascade arc is an especially suited arrangement for experimental determinations of transport coefficients of hot gases and plasmas. Its simple geometry makes possible the evaluation of the desired material functions with the aid of the Elenbaas-Heller energy equation from the measured current-field strength characteristic $E(I)$ and various radial temperature distributions $T(r,I)$. In the cascade arc described by Maecker (1) and especially in the improved version of Maecker and Steinberger (2) it is possible to generate plasmas up to maximum temperatures of 26,000°K corresponding to a degree of ionization of above 100% in a stationary manner.

Earlier investigations of this type with the nitrogen arc /334 (Burhorn, ref. 3 to 13,000°K; Maecker, ref. 4, to 16,000°K) produced results, for the electrical conductivity $\sigma(T)$ over the entire temperature range considered, for the thermal conductivity $K(T)$ to about 10,000 - 12,000°K, which agreed well with theory. Above this limit the experimental $K(T)$ values were found, to an increasing degree, to lie above the theoretical curve. Above all there was not, as shown by Maecker (5), any flattening of the curve of the $K(T)$ results corresponding to a maximum which was to be expected at 14,500°K as a result of ionization. Even a refinement of this evaluation method by Uhlenbusch (6) and Monterde (7) did not change this discrepancy between theory and measurements. Rather the reason for this deviation lies in the fact that in all these evaluation methods, beside the thermal heat conduction, only the energy loss by transparent radiation was taken into account. In reality, however, above 10,000°K the energy transport resulting from the radiation emitted in the arc and then again absorbed becomes important to an increasing degree. Thus, in this temperature range, there appears in the energy equation, in addition to the pure temperature function $\sigma(T)$, $K(T)$ and the emission of radiation power per unit volume $e(T)$ also the absorption of radiation power per unit volume $a(T, T_A)$ which depends not only on the temperature at the point of incidence, but also on the radiation field in the entire arc, i.e. on the axis temperature T_A and thus on the electric data.

If the temperature- and frequency dependence of the absorption coefficients $\alpha(v,T)$ were completely known, one could, for measured temperature distributions, calculate (8-10) the emission and absorption of radiation as a function of the location on the arc with the aid of the radiation transport equation. Then by means of the integrated Ohm's law and the energy equation the electrical and thermal conductivities $\sigma(T)$, resp. $K(T)$ could be determined from the measurements. However, since $\alpha(v,T)$ is not known with sufficient accuracy, we shall present and carry out in the following a method which allows the evaluation of arc measurements without assumptions of emission - and absorption coefficients, i.e. without using the radiation transport equation. /335

The radial temperature distributions $T(r,I)$ on which this work is based, were measured by Schade (11) in an N_2 cascade arc of 5 mm diameter at atmospheric pressure in very narrow current steps up to a maximum axis temperature of 26,000°K corresponding to a degree of ionization of 115%. The associated $E(I)$ characteristic was determined by Plantikow (12) up to a current strength of 570 A.

Since the radial temperature distributions are given for many current strengths, the electrical conductivity $\sigma(T)$ can be evaluated directly over the entire temperature range considered with the aid of the temperature distributions and the $E(I)$ characteristic. Up to temperatures of about 10,000°K the radiation contribution to the energy transport can be disregarded so that even $K(T)$ can be determined directly. Between 10,000 and 15,000°K it is possible, with the aid of the current values determined in very close intervals the balance $u(T,I)$ between the radiation power emitted, $e(T)$, and absorbed, $a(T,I)$, per unit volume can be determined with good approximation and thus $K(T)$ can again be determined.

Above 15,000°K the energy transport by heat conduction compared to that by radiation plays an increasingly minor role. In this range the thermal conductivity is calculated from the directly evaluated electrical conductivity using euqations from kinetic gas theory. With the aid of these two functions the radiation behavior of the arc can then be evaluated. Thus we obtain by evaluation not only $\sigma(T)$ and $K(T)$ to 26,000°K but also the radial dependence of the radiation balance $u(T,I) = e(T) - a(T,I)$ of the arc investigated for all measured current strengths.

II. Basic Equations

The energy balance in a volume element of the arc is described by the Elenbaas-Heller differential equation

$$\sigma E^2 - \operatorname{div} q_T - \operatorname{div} q_S = 0.$$

The product of the electrical conductivity σ and the square of the electrical field strength E^2 gives the energy added, per unit time and volume, through ohmic heating. This is carried off partially by heat conduction and partially by radiation. The vector $q_T = -\kappa \nabla T$ is the heating current density; thus $\operatorname{div} q_T$ represents the difference between the power conducted in and out per unit volume by heat conduction. The balance u between the radiation power e, carried off per unit volume, and that added by absorption a, equals the divergence of the vector q_S of the radiation current density /336

$$\operatorname{div} q_S = u = e - a.$$

For cylindrical symmetry the energy equation becomes, as a function of $\rho^2 = r^2/R^2$:

$$\sigma E^2 - u + \frac{4}{R^2} \frac{d}{d\rho^2}\left(\kappa \rho^2 \frac{dT}{d\rho^2}\right) = 0.$$

The total current I is given by the integral over the current density $j=\sigma E$ in an arc cross section:

$$I=\int_F j\,df=\pi R^2 E\int_0^1 \sigma\,d\rho^2.$$

The radiation power U emitted by the arc per unit length is then obtained by integration of the radiation balance u = e - a over the arc cross section:

$$U=\int_F u\,df=\pi R^2\int_0^1 u\,d\rho^2.$$

While σ, K and e, as pure material functions at constant pressure depend only on the temperature, the absorption a is determined not only by the temperature, but also by the radiation intensity at the particular location, i.e. by emission and absorption in the entire vicinity of the incident point. Thus a and also u are not pure temperature functions, but they also depend on the current strength I.

III. Electrical Conductivity σ(T)

As long as the absorption of radiation can be neglected, only material properties appear in the energy equation which are unique functions of temperature. If we introduce the heating current potential $S=\int_0^T \kappa\,dT=S(T)$ into the energy equation, then solely from the E(I)- and the U(I)- characteristic of the σ(S)- and e(S)- curves, as well as for each current strength I_n, the radial distribution of $S(r, I_n)$ can be determined. If we now compare it with the measured temperature distribution $T(r, I_n)$ we obtain the curve S(T) and thus the desired functions σ(T) and e(T) independent of current strength I_n (Maecker, ref. 13 and Plantikow, ref. 14). The differentiation of S(T) finally gives /337
the heat conductivity K(T). However, this evaluation mode is possible only to about 10,000°K for nitrogen because above it the absorption of radiation takes on increasing importance and the characteristic U(I) no longer gives information as to the

course of the radiation balance within the arc. In this case one must first determine the electrical conductivity $\sigma(T)$ from the $E(I)$ characteristic and from a number of $T(r,I)$ distributions over the following integral equation of the Volterra type which results from the transformation of (2a):

$$I/E = G(I) = \pi R^2 \int_0^1 \sigma \, d\rho^2 = -\pi R^2 \int_0^{T_A} [\sigma(T)/(dT/d\rho^2)] \, dT.$$

Here $T_A(I)$ denotes the axis temperature for a current strength I. The nucleus of this integral equation is known for all current strengths for which the temperature distribution was measured.

If $\sigma(T)$ is known for a given current strength I_{n-1} up to the corresponding axis temperature $T_{A,n-1}$, that is for $0 \leqq T \leqq T_{A,n-1}$, then for the next highest current strength I_n the integral in (2b) can be divided into two parts

$$G(I_n) = \pi R^2 \int_0^{T_{A,n-1}} \sigma(-d\rho^2/dT)\, dT + \pi R^2 \int_{T_{A,n-1}}^{T_{A,n}} \sigma(-d\rho^2/dT)\, dT,$$

of which the first can be calculated with the T distribution $T(r,I_n)$. In the small interval between $T_{A,n-1}$ and $T_{A,n}$ $\sigma(T)$ is developed into a series about $T_{A,n-1}$ with few terms for which all coeffieicnts, but one, can be obtained from known $\sigma(T)$ curve at $T_{A,n-1}$. The still missing coefficient is determined from the second interval in (2c) whose magnitude is known since $G(I_n) = I_n/E_n$ was measured. Thus $\sigma(T)$ is known up to $T_{A,n}$ and one can write equation (2c) for I_{n+1} in order to calculate $\sigma(T)$ for the next interval $T_{A,n} \leqq T \leqq T_{A,n+1}$. In this way one can proceed from lower current strengths to higher ones if an initial section of the $\sigma(T)$ curve is already known. As such $\sigma(T)$ was taken from Plantikow (14) from the radiation-free range where it can be calculated by the method described at the beginning of this section. The simplest two possibilities for continuation of the $\sigma(T)$ curve consist of replacing $\sigma(T)$ between $T_{A,n-1}$ and $T_{A,n}$ each time by a constant value or by a straight line which thus produces a stepped curve resp. a polygon shape for $\sigma(T)$. If during the evaluation very small current steps are used. Then the second integral in (2c) is considerably smaller than the first and a relatively small error in the first integral or in $G(I_n)$ leads to a large error in the to be determined coefficient of the $\sigma(T)$ series because of the difference between two large numbers. /338 However, one can avoid this by using very large current steps; but only a very rough $\sigma(T)$ curve results. In the present measurements the $T(r,I)$ distributions are given in very narrow I steps. If in the evaluation one uses a straight line for $\sigma(T)$, then a $\sigma(T)$ curve with rapidly increasing oscillations results. Therefore, for the series of $\sigma(T)$ in the innermost range $T_{A,n-1} \leqq T \leqq T_{A,n}$ a three-term parabolic curve is always used. Since the errors in the evaluation values become noticeable especially at the end of the parabola at $T_{A,n}$ then each time only the first half of the $\sigma(T)$ parabola is used for the next evaluation step. Thus a non-oscillating $\sigma(T)$ curve is obtained from which the individual parabola ends are dropped. The magnitude and the direction of the parabola ends is used as a criterion for determining how an optimal average curve for $\sigma(T)$ is to be established. The quality of this curve can be checked by repeating the described evaluation; however, one must use the values from the average curve for the part of the $\sigma(T)$ function assumed to be known. The $\sigma(T)$ curve resulting from this method is shown in figure 1. Its initial section agrees very well with a $\sigma(T)$ curve (15) determined previously for the temperature range to 15,000°K from $T(r,I)$ distributions according to Maecker (4).

With the aid of kinetic gas theory $\sigma(T)$ as well as other material functions can be traced back to the various types of effective cross sections $Z_{ab}^{(st)}$ of the associated plasma partners. The definition used here for the effective cross sections is related to the Chapman-Cowling Ω integrals as follows:

$$Z_{ab}^{(st)} = 8\sqrt{\frac{m_a m_b}{2kT(m_a + m_b)}}\,\Omega_{ab(t)}^{(s)}.$$

Here the indices a and b denote the particle types of the impact partners while s and t denote the ranks of the cross sections.

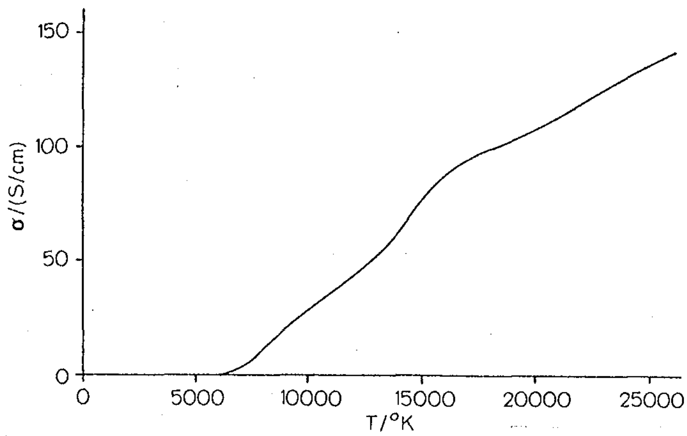

Figure 1 Electrical conductivity $\sigma_{N_2}(T)$ for p = 1 atm.

The relation between transport coefficients such as electrical conductivity and effective cross sections is given in general determinant form in basic books (such as Chapman and Cowling (16), Hirschfelder et al. (17)) as well as somewhat more simply in various publications (e.g. Ahyte (18), Devoto (19)). If one knows this theoretical relation between the electrical conductivity /339
and the cross sections, then one can obtain information with regard to the various impact cross sections from the given function $\sigma(T)$. If one also especially adopts the relations of impact cross sections of different orders $Z_{ab}^{(st)}/Z_{ab}^{(ik)}$ from the theory, then from the $\sigma(T)$ curve the cross section for the impact between electron and atom $Z_{ae}(T)=Z_{ae}^{(11)}(T)$ and the coulomb cross section $Z_c(T)=Z_{ee}^{(11)}(T)$ can be determined directly. This method for evaluation of effective cross sections has already been treated in detail in a previous publication (15) and was carried out for temperatures to about 15,000°K. Just as in this earlier publication, the formula with two terms in the Sonin polynomial series is used for $\sigma(T)$ which represents a sufficiently good approximation. For calculating the relations of impact cross sections of different orders we used the

model of hard elastic spheres (17) for impact of electrons with neutral particles, and the modified Coulomb model (20) for impact of charged particles. The latter is also used for calculating the relations of cross sections for impacts between multiple-charge particles to those between particles with a single charge. These relations are given by the explicitly written expressions in Devoto (21) for the effective cross sections of different orders for the impact between charged particles. For the impact cross section of molecules against electrons Z_{me} which is of importance for $\sigma(T)$ only below 7000°K, we can use $Z_{me}=2\ Z_{ae}$ as a rough approximation. The Coulomb cross section obtained by this evaluation is plotted in figure 2 along with its theoretical curve in accordance with /340
with Devoto (21). To compare the resulting cross sections Z_{ae}

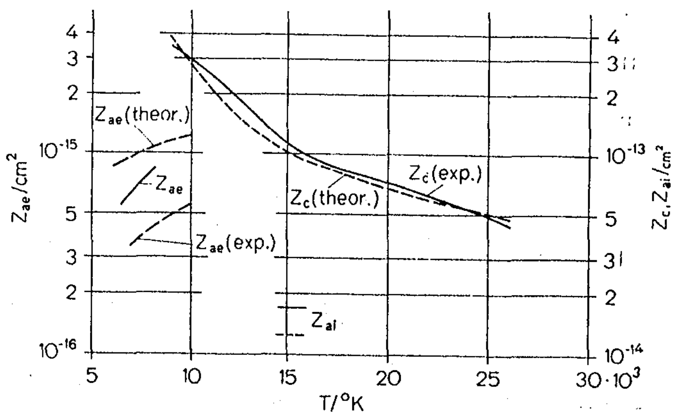

Figure 2: Various effective cross sections of nitrogen.
———— evaluated; ------values taken from the literature

with those from the literature the experimental and the theoretical cross sections, still velocity dependent, were integrated in accordance with Neynaber et al. (22) over the velocity volume and the comparison curves thus obtained were plotted in figure 2 along with Z_{ae}. The cross section Z_{ae} thus determined lies exactly between the two values taken from the literature and has a temperature dependence similar to those two.

Thus, in addition to a $\sigma(T)$ curve for temperatures to 26,000°K the curves of effective cross sections in relation to earlier results (15) are also known; these can again be used for calculating other material functions.

IV. Thermal Conductivity K(T)

Since now $\sigma(T)$ is known, one can calculate the power introduced within a cylinder with radius r per unit length, namely $\pi R^2 E^2 \int_0^{\rho^2} \sigma \, d\rho^2$. If we divide this expression according to the once integrated energy equation by $-4\pi\rho^2 \partial T/\partial\rho^2$, then we obtain an effective heat conductivity as a function of I and ρ^2, in which the energy transport by radiation is contained formally. /341

$$\kappa_{eff}(\rho^2, I) = \frac{R^2 E^2}{4} \int_0^{\rho^2} \sigma \, d\rho^2 / (-\rho^2 \partial T/\partial\rho^2).$$

If we plot this $\kappa_{eff}(\rho^2, I)$ not against ρ^2 but against T with I as parameter using the measured $T(\rho^2, I)$- distributions, then we see that to about 10,000°K there results no single-valued function of the temperature independent of current strength, i.e. that K_{eff} is identical, up to this temperature, with the thermal conductivity K(T) with the addition of a radiation diffusion coefficient. However, an estimate using the absorption coefficients given by Wilson and Nicolet (23) shows that the radiation diffusion portion below 10,000°K as well as in the following temperature range of 10,000 to 15,000°K, considered here, can be neglected. Thus to 10,000°K the thermal conductivity K(T) = K_{eff} can be evaluated directly (figure 3). The value of K(T) thus determined exhibits at about 7000°K the maximum corresponding to transport of energy of dissociation. The K(T) function agrees extraordinarily well with earlier experimental results (Burhorn (3), Maecker (5), Hermann and Monterde (15), Plantikow (12)) and with the theoretical K(T) curves (Yos, ref. 24).

For temperatures above 10,000°K the K_{eff} (I,T) curves calculated for various current values I deviate from each other to an increasing degree and in a systematic fashion which leads one to conclude that there are strong contributions of radiation to energy transport. In this case the heat conductivity K(T) can be evaluated from the transformed energy equation:

$$\kappa(T) = \frac{R^2 E^2}{2} \int_0^{\rho^2} \sigma \, d\rho^2 / (-2\rho^2 \partial T/\partial \rho^2) - \frac{R^2}{2} \int_0^{\rho^2} u \, d\rho^2 / (-2\rho^2 \partial T/\partial \rho^2)$$
$$= \kappa_{eff}(I, T) \qquad - Q_S(I, T)/(-2\rho^2 \partial T/\partial \rho^2).$$

Here according to equation (1a), $2\pi Q_S(I, T) = 2\pi R \rho q_S$ is the radiation current per unit length through the cylinder surface with radius r and q_s is the radiation current density for radius r, which for symmetry reasons has only a radial component.

In equation (5) we find, in addition to the desired thermal conductivity K(T) an additional unknown function, namely the radiation balance u resp. the radiation current Q_s. As long as the absorption term a is neglected in the radiation balance u, and thus u = e depends only on temperature, u can be determined with the aid of equation (3). However, if radiation is absorbed in /342 the arc, then u becomes a function of current strength I and of temperature T, and K(T) as well as u must be determined from the energy equation. Since radial temperature distributions T(r,I) are known for very many current values, then one can, with certain assumptions as to the stability of the u(I,T) and K(T) curves, obtain from the given values information not only about K(T) but also about u(I,T) as long as the radiation portion of the energy transport does not overbalance too greatly the portion attributable to thermal conduction. For this purpose we shall first investigate which statements can be made the field of the radiation current $Q_S(\rho^2, I)$ with the aid of the evaluated functions. If K(T) is already known up to a certain temperature T_{in}, then the radiation current $Q_S(\rho^2, T)$ can be determined from equation (5b) for the outer regions of arcs of higher current strength, for which $T \leqq T_m$, i.e. for all values of $\rho^2(T \leqq T_m)$. This means that, especially for current strengths whose axis temperatures are not much higher than T_m, the $Q_S(\rho^2, I)$-curve is known for an additional outer range, namely for $\rho^2(T_m) \leqq \rho^2 \leqq 1$. In the vicinity of the arc axis the radiation current Q_s varies approximately porportional to

ρ^2: $Q_S(\rho^2 \to 0, I) \approx u(\rho^2 = 0, I) R^2 \rho^2 / 2$. Thus one can extrapolate the radiation current $Q_S(\rho^2, I)$) without any large errors to the missing range in the interior, namely to $0 \leqq \rho^2 \leqq \rho^2(T_m)$, as long as $\rho^2(T_m) \ll 1$ ist.

If we write equation (5b) twice for a fixed temperature, but for two different current strengths I_1 and I_2 and then subtract the two equations, we obtain the conductivity K(T) depending only on T and we obtain the relation between the values for the radiation current Q_S for different current strengths, however always for radial positions with the same temperature T.

$$\kappa_{eff}(I_1, T) - \kappa_{eff}(I_2, T) = Q_S(I_1, T)/(-2\rho^2 \partial T/\partial \rho^2)_{I_1, T} - Q_S(I_2, T)/(-2\rho^2 \partial T/\partial \rho^2)_{I_2, T}.$$

If for a given current strength I_n the radiation current per unit length $Q_S(I_n, T)$ for a given temperature T is already known, then $Q_S(I,T)$ can be calculated with equation (6) for this temperature for all current strengths. If, for a relatively low current strength I_n, we then obtain by extrapolation the $Q_S(\rho^2, I_n)$- curve even in the region near the axis, then even for higher current strengths the region for which $Q_S(\rho^2)$ is known, is expanded, and for current strengths a little above I_n extrapolation to the axis is also possible. In this way one can, starting with low current strengths where radiation absorption is not important and where K(T) can be evaluated directly, determine the field $Q_S(I,T)$ for higher current strengths and thus the thermal conductivity /343 K(T) by means of the difference formation described in equation (5b) as a good approximation.

This evaluation can still be improved if, by iteration, one modifies the extrapolated $Q_S(\rho^2)$- curves in such a way that a K(T) curve as smooth as possible as well as a smooth $u(\rho^2, I)$- field arises. The radiation balance u(I,T) is obtained by differentiation from $Q_S(\rho^2, I)$:):

$$u(T, I) = u(\rho^2(T), I) = \frac{2}{R^2} (\partial Q_S/\partial \rho^2)_I.$$

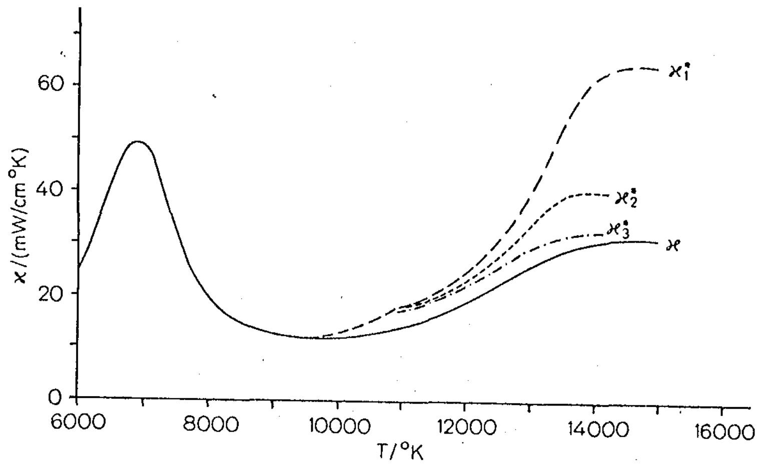

Figure 3 Thermal conductivity $K_{N_2}(T)$ and upper limiting curves $K^*(T)$

The disadvantage of this method is the fact that errors in the evaluation results perpetuate themselves and are added so that the accuracy of the results decreases with increasing distance from a radiation-free starting point. In addition, since the radiation share of the energy transport increases strongly and since, for higher current strengths the percentage contribution of the thermal heat loss is very small and since then the possible error in K(T) becomes too large, this evaluation method should not be used beyond about 15,000°K.

The resulting K(T) is shown in figure 3 as a solid curve. Here we see at about 15,000°K the maximum associated with the ionization peak which was missed in the evaluation when radiation absorption was not taken into account. This curve agrees well with the earlier (15) K(T) function evaluated with the temperature distributions measured by Maecker (4); the maximum is now higher by about a few percent.

The curve thus determined agrees very well with theoretical K(T) curves. In contrast to this result, most previous evaluations of measurements in cascade arcs produced K(T) values for nitrogen (6,7), argon (25), and hydrogen (26) which, above 10,000°K, fell well above the theoretical curve. /344

Since, however, the K(T) curve, evaluated by the above method, is subject to a certain undetermined error, we shall still determine an upper limiting curve for the thermal conductivity in the region of the ionization peak. Here, unlike earlier, absolute value and slope of the K(T) at the starting point of the evaluation are not necessary; a reasonably sure absolute value is sufficient here.

In the following we shall assume that the radiation balance u near the axis continuously increases as the axis is approached. Up to 16,000°K this is surely true since in this temperature range the radiation emission increases greatly with temperature.

With this assumption the envelope of the $K_{eff}(T,I)$ curves already represents an upper limiting curve for the thermal conductivity. It corresponds to a K(T) curve which was evaluated under the assumption of negligible radiation. Its maximum, corresponding to the ionization peak, lies at about 165 mW/cm°K, which is about 5 times the theoretical value.

An appreciably lower lying limiting curve is obtained if one proceeds, in accordance with the described evaluation, from current step to current step and if one introduces a constant radiation balance curve in the as yet unknown channel near the axis:

Here one assumes that for a current strength I_n the thermal conductivity is known in the exterior for $\rho_g^2 \leqq \rho^2 \leqq 1$ while in the interior region $0 \leqq \rho^2 \leqq \rho_g^2$ must still be determined. Then for $\rho^2 = \rho_g^2$ the value of the radiation integral is given by:

$$\int_0^{\rho_g^2} u \, d\rho^2 = \int_0^{\rho_g^2} \sigma E^2 \, d\rho^2 + \frac{4}{R^2} \rho_g^2 (\partial T/\partial \rho^2)_{\rho_g^2} \kappa(\rho_g^2).$$

In the inner region $0 \leqq \rho^2 \leqq \rho_g^2$ we set $u(\rho^2) = \bar{u}_n$ constant where the constant is chosen so that for $\rho^2 = \rho_g^2$ the radiation integral is exactly equal to $\rho_g^2 \bar{u}_n$. From that it follows that for $0 \leqq \rho^2 \leqq \rho_g^2$ the product $\bar{u}_n \rho^2$ is always smaller or at most equal to the radiation integral: /345

$$\int_0^{\rho^2} u \, d\rho^2 \geqq \rho^2 \bar{u}_n = \rho^2 \frac{1}{\rho_g^2} \int_0^{\rho_g^2} u \, d\rho^2; \text{ for } 0 \leqq \rho^2 \leqq \rho_g^2.$$

If in equation (5a) we replace the radiation integral with the product $\bar{u}_n \rho^2$, then we obtain a value of K_1^* which is greater than the real K. If we utilize this K_1^* (T) in order to determine, at the next current step I_{n+1}, the radiation integral for $\rho^2 = \rho_g^2$ corresponding to equation (8), then we obtain a value which is too small. If we again replace the radiation integral with the thus formed $\bar{u}_{n+1}\rho^2$ in the evaluation of K, then the value of K_1^* determined at I_{n+1} is larger everywhere than the real K. In this way we established, starting with T = 11,000°K, a K_1^* curve which produces a maximum corresponding to the top of the ionization peak somewhat below 15,000°K with the value of $K_{1,\,max}^*$ = 65 mW/cm°K (figure 3). The actual K-curve should lie appreciably below K_1^* since the assumption $u(\rho^2)$ = constant for $\rho^2 \leqq \rho_g^2$, from which we get K_1^*, is always a very rough approximation because of the strong temperature dependence of the emission.

The upper limiting curve for K(T) can be displaced still further downward if, instead of a constant value for $u(\rho^2)$ we use a curve which, everywhere within $0 \leq \rho^2 \leqq \rho_g^2$ increases less as it approaches the axis than the actual $u(\rho^2)$ and which, when integrated over ρ^2 at $\rho^2 = \rho_g^2$ agrees with the actual radiation integral.

For, if we establish the radiation integral $\int_0^{\rho^2} u\, d\rho^2$ with a $u(\rho^2)$-curve which satisfies these two conditions, then this integral, within $0 \leqq \rho^2 \leqq \rho_g^2$ is always smaller than or at most equal to the actual value; i.e. a $K^*(T)$ which is evaluated with this radiation integral according to equation (5a) is always greater than the actual K(T). A $u(\rho^2(T))$- curve which satisfies the above conditions can be obtained from the $\bar{u}_n$ values found in the determination of K_1^* if one assumes that the actual curve of the axis values of the radiation balance $u_A(T_A)$ increases with temperature at a lower rate than the actual function $u(I_n, T(\rho^2))$ increases for the individual current strengths in the region near the axis. This one can assume without much reservation as long as the emission e(T) per unit volume increases greatly with temperature. An evaluation using this $u(\rho^2(T))$- curve gives a value of $K_2^*(T)$ which lies appreciably below K_1^*,

but which must lie above the actual K(T). An iteration of this process produces a still lower limiting curve K_3^* (T) which must also lie above K(T). The K^* curves are plotted together with the directly evaluated K(T) curve in figure 3. Since K_3^* (T) lies only a little above the evaluated K(T) curve, one can assume that the directly determined K(T) closely approaches the true curve. /346
For that reason, in the following , we take the originally evaluated K(T) function as the true one in the temperature range $10,000^{\circ}K \leqq T \leqq 15,000^{\circ}K$.

From the K(T) curve around $15,000^{\circ}$K one can determine an additional effective cross section, namely the charge exchange cross section Z_{ai}. In the area of the ionization peak K consists of practically only two parts, namely the electron thermal conductivity K_e and the conductivity K_I which describes the transport of ionization energy:

$$K = K_e + K_I$$

The heat conductivity of the ions can be neglected.

If one utilizes the already determined Coulomb cross sections and assumes, as in chapter III, from theory the relations between impact cross sections of various orders, then $K_e(T)$ can be calculated. Since the total conductivity K(T) is given by the evaluation, K_I and from this the charge exchange cross section $Z_{ai}=Z_{ai}^{(11)}$ can be determined. Here we obtain, in good agreement with previous results (15):

$$Z_{ai}(T = 15,000^{\circ}K) = 1.7 \cdot 10^{-16}\ cm^2 .$$

The theoretical value for this cross section at the same temperature, according to Vanderclice et al. (27) is (see figure 2):

$$Z_{ai}(T = 15,000^{\circ}K)_{theoretical} = 1.24 \cdot 10^{-16} cm^2.$$

With the aid of the previously evaluated cross sections the thermal conductivity in the temperature range $15,000^{\circ}K \leqq T \leqq 26,000^{\circ}K$ can be calculated. In this range K(T) essentially consists of three parts, K_e, K_I and the conductivity K_{2I} which describes the transport of the energy required for dual ionization:

$$K = K_e + K_I + K_{2I}$$

For K_e we used the formula with three terms and for K_I and K_{2I} the formula with two terms each in the polynomial Sonin series. To calculate K_I we assumed the charge exchange cross section Z_{ai} to be constant in the range above 15,000°K. This produces no large error since the number of atoms decreases rapidly for higher temperatures for which a deviation of the Z_{ai} curve from a constant value is much more pronounced. The resulting K(T) curve, together with the one evaluated for $T \leq 15{,}000$°K is plotted in figure 4. In addition, an experimental conductivity determined by Westenberg and De Haas (28) for $0 \leq T \leq 2000$°K is plotted in this figure and /347 is extrapolated until it joins our K(T) curve so that the entire course of the heat conductivity of nitrogen from 0 to 26,000°K is known, based on measurements.

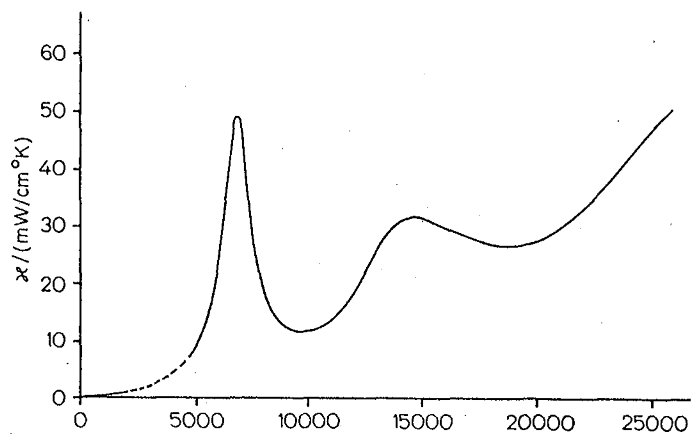

Fig. 4. Wärmeleitfähigkeit $\kappa_{N_2}(T)$ bei $p = 1$ atm

Figure 4 Heat conductivity $K_{N_2}(T)$ at p = 1 atm.

V. Radiation Behavior of the N_2 Arc of 5 mm Diam. to 26,000° K

Now that not only $\sigma(T)$ but also K(T) have been established for the entire interesting temperature range, one can calculate, with the aid of equation (1b) the radiation balance u, i.e. the difference between emitted and absorbed radiation power per unit volume $u=e-a=\mathrm{div}\, q_S$. In contrast to the pure temperature function e(T), a and thus also u depend not only on temperature, but also on the field strength over the radiation field at the incident point:

$$a=a(I,T)=a(I,\rho^2);$$
$$u=e(T)-a(I,T)=u(I,T)=u(I,\rho^2).$$

In figure 5a the radiation balance u is plotted against ρ with current strength I as a parameter. In the core of the arc u is a maximum. There the emission outweighs by far the absorption. At the edge of the arc u decreases sharply and finally becomes negative. Thus there the absorption is greater than the emission so that the extreme arc layers are heated by radiation. In figure 5b u is plotted against T with I as a parameter.

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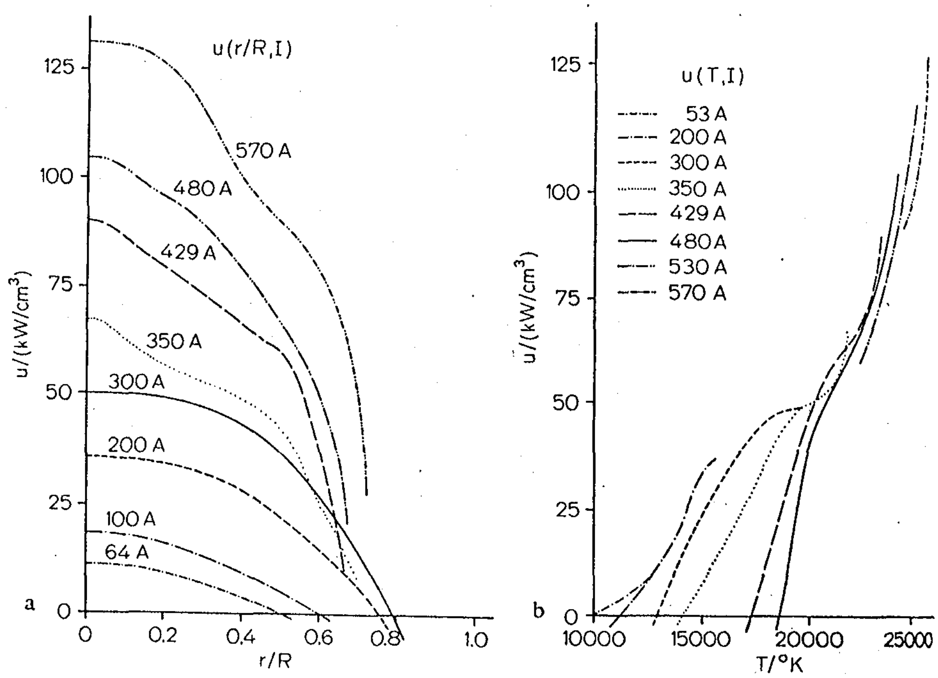

Figures 5a and b. Difference between radiation power per unit volume emitted and absorbed in a 5 mm diam. N_2 arc at p = 1 atm.
a) plotted against relative radius with current as parameter
b) plotted against temperature with current as parameter.

The upper ends of the curves each correspond to the axis region. Here also one can observe a decrease of the u-curves to lower temperatures, i.e. toward the edge of the arc which can be attributed to the decrease in emission and the very rapid rise of absorption in the outer layers of the arc.

Of the radiation emitted across the entire spectrum only a small portion contributes to energy transport because the free optical path length is so short for the most part that only a vanishingly small energy transfer occurs. If one imagines the portion of the total emission per unit volume which makes a substantial contribution to the energy transport, plotted against temperature in figure 5b, then the u(I,T) curves must still lie below this function by the amount of the radiation power absorbed per unit volume and, therefore, must lie far below the total emission of radiation power per unit volume. A comparison of the radiation balance in the arc axis u_A with the material quantity e(T) calculated from data from the literature (23) shows that at 12,000°K the value of u_A is about 7% of the emission. The percentage increases to about 13% at 16,000°K and then drops to about 4% at 24,000°K. /349
Up to about 16,000°K e(T) is determined primarily by the N I-continuum and the N I-lines in VUV. Since the emission maximum for nitrogen atoms lies at 15,000°K, and since the emission of nitrogen ions, i.e. N II-VUV lines sets in only slowly at this point, the total emission e(T) is flattened somewhat in the range of 14,000 to 18,000°K. The contribution of the N II contimuum to the total radiation can be neglected. The u_A(T) curve shows a flattening similar to e(T), but more pronounced and displaced to higher temperatures. This can presumably be explained by the fact that the N II lines which first appear as the temperature increases, soon become optically thick and thus make only a very small contribution to the radiation balance while the weaker lines contribute to energy transport only at higher temperatures and thus bring about a great increase in the radiation balance.

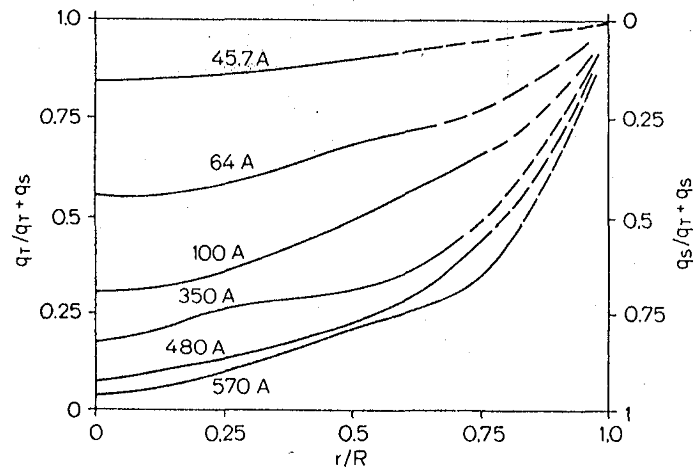

Figure 6 Relative share of heat conductivity and radiation on energy transport for various current strengths, plotted against relative radius.

In order to demonstrate the contribution of radiation to the total transport, the ratio of q_s resp. q_T to the entire energy current density $q = q_s + q_T$ is formed (figure 6). Integration of equation (1b) over the cross sectional area of a concentric cylinder with radius r gives:

$$2\pi\int_0^r \sigma E^2 r\,dr = 2\pi\int_0^r u\,r\,dr + 2\pi\kappa r(-dT/dr)$$
$$= 2\pi r q_S + 2\pi r q_T = 2\pi r q.$$

The term on the left gives the power which is conducted per unit length within the concentric cylinder with radius r by ohmic heating. /350
This energy is passed by radiation and heat conduction across the outer surface to the outside. The magnitude of these two portions is represented by the radiation current $2\pi r q_s$ and the heating current $2\pi r q_r$ per unit length of the cylinder. In figure 6 the ratios q_T/q resp. q_s/q are plotted for various current strengths:

$$q_T/q = q_T/(q_T+q_S) = 2\pi r q_T \Big/ \left(2\pi\int_0^r \sigma E^2 r\,dr\right).$$

The contribution of radiation to the total transport is always highest at the axis. Toward the edge increasing amounts of radiation are absorbed and the energy transport is taken over to an increasing degree by heat conduction. One can see that for the highest current strength, i.e. at 570 A, only about 5% of the added energy is carried off by heat conduction in the area near the axis. The largest part of about 95% is radiated away. Thus for high current strengths $u(I,T)$ is affected only weakly by the value of $K(T)$ used in the evaluation. Even if $K(T)$ is doubled, $u(I,T)$ changes only little in the core region.

We have thus shown that the behavior of the 5 mm diam. N_2 arc is determined to a very considerable extent by radiation energy transport. Especially at high current values the contribution of thermal conduction to the energy transport becomes nearly meaningless in the region near the axis.

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Dr. Ing. W. Hermann
Elektrophysical Institute
of the Technical College
8000 Munich 2, Arcisstr. 21

Dr. Ing. E. Schade
Electrophysical Institute
of the Technical College
8000 Munich 2, Arcisstr. 21

now

Brown Boveri Forschungszentrum
CH-5401 Baden (Switzerland)

References

1. Maecker, H.: Z. Naturforsch, 11a, 457 (1956)
2. Maecker, H., Steinberger, S.: Z. Angew. Phys. 23, 456 (1967).
3. Burhorn, F.: Z. Physik 155, 42 (1959).
4. Maecker, H.: Proc. of the IVth Int. Conf. on Ionisation Phen. in Gases, Uppsala 1959, vol. I, p. 378. Amsterdam: North-Holland Publ. Co.
5. Maecker, H.: An introduction to discharge and plasma physics, Summer School, University of New England, Armidale, Australia (1963).
6. Uhlenbusch, J.: Z. Physik 179, 347 (1964).
7. Monterde-Garcia, A.: Z. Physik 181, 317 (1964).
8. Hermann, W.: Z. Physik 216, 33 (1968).
9. Morris, J.C.: Final Report Contract NSAW-1188.
10. Schramm, K., Uhlenbusch, J.: Bericht HMP 120, TH Aachen Oktober 1968.
11. Schade, E.: Z. Physik 233, 53 (1970).
12. Plantikow, U.: Z. Physik (in Vorbereitung).(in process).
13. Maecker, H.: Z. Physik 158, 392 (1960).
14. Plantikow, U.; Z. Physik 227, 271 (1969).
15. Hermann, W., Monterde, A.: Z. Physik 205, 313 (1967).
16. Chapman, S., Cowling, T.G.: The mathematical theorie of nonuniform gases. Cambridge, England: Cambridge University Press 1958.
17. Hirschfelder, J.O., Curtiss, C.F., Bird, R.B.: Molecular theorie of gases and liquids. New York: John Wiley & Sons 1964.
18. Ahtye, W.F.: NASA Technical Note D-2611 (1965).
19. Devoto, R.S.: Phys. Fluids 9, 1230 (1966).
20. Liboff, R.L.: Phys. Fluids 2, 40 (1959).
21. Devoto, R.S.: Phys. Fluids 10, 354 (1967).
22. Neynaber, R.H., Marino, L.L., Rothe, E.W., Trujillo, S.M.: Phys. Rev. 5, 380 (1962).
23. Wilson, K.H., Nicolet, W.E.: J. Quant. Spectr. Radiative Transfer. 7, 891 (1967).
24. Stewart, B., Knopp, C.F., Yos, J.M.: Techn. Report RAD-TR-65-7 (1965).

References (cont)

24. Z. Physik, vol. 233.
25. Bues. I., Patt, H.J., Richter, J.: Z. Angew. Phys. 22, 345 (1967).
26. Plantikow, U., Steinberger, S.: Z. Physik 231, 109 (1970).
27. Knof, H., Mason, E.A., Vanderslice, J. T.: J. Chem. Phys. 40, 3548 (1964).
28. Westenberg, A.A., DeHaas, N.: Phys. Fluids 5, 266 (1962).