SOURCE STRUCTURE ERRORS IN RADIO-INTERFEROMETRIC CLOCK SYNCHRONIZATION FOR TEN MEASURED DISTRIBUTIONS

J. B. Thomas
Jet Propulsion Laboratory
Pasadena, California

ABSTRACT

Source structure can introduce errors in radio interferometry measurements whenever natural radio sources are used. To increase our understanding of this problem, an analysis previously applied to analytical cases has been extended to the brightness distributions measured for ten extragalactic sources. The results of this analysis are presented along with an approach for avoiding the largest structure errors.

INTRODUCTION

As the accuracy of the radio interferometry technique improves, the extended structure of natural radio sources will become an increasingly important source of error. To further increase our understanding of structure effects, this paper extends an analysis that was presented at last year's PTTI conference (1). That earlier presentation included a brief introduction to the theory of structure corrections and applied the theory to the analytical cases of a double-point source and a triple-point source. Although analytical examples are instructive, only a thorough study of many actual source distributions can give a complete picture of source structure effects. To begin such an investigation, ten brightness distributions measured by the Caltech VLBI group (2, 3) have been analyzed. This paper includes a summary of the results of this more recent analysis and presents a visibility-dependent limit formula that, if verified, would be valuable in both estimating and reducing the overall structure effect in bandwidth-synthesis (BWS) delay. Since BWS delay is currently the primary observable in geophysical and clock-synchronization applications, the analysis will focus on that observable.

STRUCTURE THEORY

In order to tie in with the earlier presentation, the theory of structure corrections will be briefly discussed before proceeding to the results. The effect of structure on the cross-correlation signal, the interferometer fringes, can be obtained by evaluating the Fourier transform of the brightness distribution:

$$R(u, v) = \iint D(\beta, \gamma) e^{-2\pi i [u(\beta-\beta_o)+v(\gamma-\gamma_o)]} d\beta d\gamma$$

in which

$$(u, v) = \left(\frac{x_s}{\lambda}, \frac{y_s}{\lambda}\right)$$

where (β, γ) are plane-of-the-sky coordinates, (β_o, γ_o) is an assigned reference position, $D(\beta, \gamma)$ is the brightness distribution, (x_s, y_s) is the sky-projected baseline vector and λ is the observing wavelength. The modulus of this complex quantity gives the effect of structure on the amplitude of the fringes while its phase gives the shift in fringe phase. As one can see, the transform is generally a complicated function of the instantaneous baseline vector between the two antennas. The dependence on baseline enters as the "sky-projected" vector (x_s, y_s), the two-dimensional vector obtained by projecting the baseline vector onto the plane perpendicular to the source direction. Thus, to obtain a general picture of the effect of a given brightness distribution, one must compute and plot structure effects as a function of (x_s, y_s). As suggested by the transform, it is convenient to express the two baseline components in units of λ, the observing wavelength at RF. When expressed in this form, the two components are usually designated (u, v). To obtain the effect of structure on amplitude and BWS delay, rewrite the brightness transform in the form

$$R(u, v) = |R(u, v)| e^{i2\pi\phi_B(u,v)}$$

where $|R|$ is fringe amplitude, and ϕ_B is structure phase. The effect of structure on BWS delay, which will be referred to as structure delay, is approximately computed by taking the partial of structure phase with respect to frequency:

$$\Delta\tau \approx \frac{\partial\phi_B}{\delta f}$$

(The computation of structure delay is actually carried out through an intermediate calculation of effective position (4). For brevity, that procedure will not be discussed here.) For the amplitude effect, it is convenient to define a quantity called fringe visibility that gives fringe amplitude relative to maximum amplitude:

$$v_s \equiv \left| \frac{R(u, v)}{R(o, o)} \right|$$

Thus, once the brightness distribution for a source has been supplied, one can compute visibility, structure phase and structure delay as a function of the sky-projected baseline vector (u, v).

RESULTS FOR MEASURED BRIGHTNESS DISTRIBUTIONS

Table 1 summarizes the 10 brightness distributions measured by the Caltech group (2, 3) and lists the source name, date of measurement, observing wavelength, interferometer stations, and the maximum values of (u,v) present in each measurement. In all, there are seven separate sources observed on continental baselines at wavelengths of 2.8 to 18 cm. Figure 1 displays the brightness distribution for one of the sources (Distribution #9, 3C345) and shows contours of constant brightness on the plane of the sky, with east to the left and north along the vertical.

The measured distributions can be passed through the structure equations to obtain for each distribution plots of fringe visibility, structure phase, and structure delay. Figure 2 gives the results for visibility for the particular distribution in Figure 1. Contours of constant visibility are plotted as a function of (u, v), the components of the sky-projected baseline vector. A similar contour plot for structure delay is given in Figure 3. In the plots, u is defined to be positive to the east and v positive to the north. Since the u-v coverage associated with a given distribution should not extend beyond the maximum values allowed by baseline length considerations, the contour plots are marked with an approximate boundary outside of which the output has been discarded. One important point concerning the current analysis is that all of the structure delays have been computed on the basis of an "artificial" frequency of 8.3 GHz (λ = 3.6 cm). In effect, this assignment of f pretends that, for structure delay computation, all distributions were measured for the specified u-v values at 8.3 GHz even though they were not. The reason for this assignment is that most of our work will be carried out at X-band and it is therefore important to compare all structure delay results at one frequency. To obtain the structure delay at the actual frequency, one can easily scale the results by f^{-1} (or by λ). To use the delay plot for a given observation, one would compute the sky-projected baseline vector for that observation and obtain the structure delay for that point on the plot. For this particular source, the structure delay reaches about 150 psec at its worst point but usually falls in the range 0 to 50 psec.

Even though the results for the other nine distributions were just as complex, some general descriptive statements can be made. The magnitudes of the structure delays (computed for X-band relative to the centroid) were as large as a nanosecond but typically fell between 0 and 150 psec. The largest delays ($\sim$ 1 nsec) occurred in very localized regions in the u-v plane where very small fringe visibilities ($\sim$0.03) occurred. On average, structure delay increased as fringe visibility decreased, as one would expect (4).

A LIMIT APPROACH TO STRUCTURE DELAYS

These results indicate that, if subnanosecond clock synchronization becomes a goal, some method must be devised for reducing or calibrating structure effects. In geophysical applications, there is already a need for delay accuracies better than 0.1 nsec. A possible method for removing structure delays is the aforementioned calibration scheme based on measured brightness distributions. The primary difficulty with this approach is that an individual VLBI structure measurement is currently a fairly expensive and time-consuming process. The prospect of working with very large catalogues, possibly containing many time-varying members, makes the calibration approach even less inviting. It is therefore worthwhile to investigate alternate methods for overcoming structure problems.

Another approach is suggested by the general tendency of structure delay to increase with decreasing fringe visibility. Suppose a general formula could be established that, purely on the basis of the value of fringe visibility, places an upper limit on structure delay. Then, if the limit turned out to be sufficiently small for some upper range of visibility values, the larger, unacceptable structure delays could be eliminated by merely deleting observations with the smaller visibility values. Such an approach is attractive since fringe visibility can often be obtained along with each VLBI observation. If the visibility determinations were accurate enough, the experiment would not depend on supplementary measurements of brightness distributions. One important point that should be made is that the proposed upper limit would not have to be an absolute limit, valid for every source. For example, it would be useful to establish, if possible, an approximate 3σ statistical limit so that structure errors could be treated like other errors. Another example would be a limit that was valid for all sources except for infrequent pathological cases. In fits with redundant observations, such pathological cases could be discovered and deleted through residual analysis.

To begin an assessment of the limit approach, a particular formula associated with a double-point source has been tested. The candidate limit equation is given by

$$\Delta\tau \lesssim \frac{1}{4f} \frac{1-v_s^2}{v_s}$$

which can be rewritten in units of length in the form

$$c\Delta\tau \lesssim \frac{\lambda}{4} \frac{1-v_s^2}{v_s}$$

where f and λ are the observing frequency and wavelength respectively; where v_s is the fringe visibility for the observation at hand; and where $\Delta\tau$ is the structure delay relative to the ordinary centroid. A plot of the function is shown in Figure 4 for X-band. Several arguments can be advanced to support this formula as a general upper limit. First, for small (u, v), one can prove analytically that the formula sets a valid (but loose) upper limit for any source. Second, for a number of special cases, one can prove that no greater structure delay can be obtained for a given visibility (4). Finally, with regard to form, one would expect structure delay to increase, on average, in inverse proportion to visibility for small visibilities (4). Although there are these supporting arguments, one cannot prove analytically that the equation sets a valid upper limit for any arbitrary distribution. Thus, the next step would be comparisons with a large number of measured distributions. When the limit was tested with the ten measured distributions, it was found that, for those cases, the limit was approximately valid for all v_s.

If the same result is obtained for nearly all real sources, then the limit equation would be of great value in both reducing and estimating structure effects in BWS delays. For example, if one set of a lower limit of 0.2 on fringe visibility, then the maximum structure delay predicted by the limit formula would be about 150 psec at X-band (8.3 GHz). Many observations would have visibilities larger than 0.2 and therefore would have smaller limits on structure delay. Further, for the cases analyzed here, actual structure delays were almost always much less than the limit, with the RMS value being about 1/3 of the limit or less at a given value of visibility. If we hypothesize on the basis of this information that the aforementioned maximum delay of 150 psec is an approximate estimate of the 3 σ delay for all observations, then the 1 σ error in delay for all observations would be about 50 psec. This calculation suggests one might be able to reduce the 1 σ structure delay to about 50 psec simply by deleting observations with fringe visibilities less than 0.2.

SUMMARY AND CONCLUSIONS

To begin a study of structure effects for natural sources, 10 brightness distributions measured by the Caltech group have been analyzed. The magnitude of structure delay (the BWS delay effect computed for X-band relative to the ordinary centroid) was as large as a nanosecond but typically fell between 0 and 150 psec. On average, structure delay tended to increase as fringe visibility decreased. These features suggest the possibility of a limit approach to structure delay. To begin an investigation of this approach, a visibility-dependent limit formula has been suggested that appears to correctly set an approximate upper limit on structure delay for 10 particular brightness distributions. The results for these sources suggest that source structure effects in BWS delay might be reduced to about 50 psec (1 σ) simply by deleting observations with fringe visibilities less than 0.2. If this technique or some refinement proves successful in reducing the 1 σ structure delay to 50 psec or below, then it would not be necessary to make supplementary brightness distribution measurements for the purpose of calibrating a source catalogue until the goal for total delay error (1 σ) falls below about 100-150 psec. In the near future, these high accuracies will probably be in greater demand in geophysical and astrometric applications than in clock-synchronization applications.

A final word of caution would be that the limit approach in its present form has not yet been verified. It may turn out that, at the large u-v values associated with intercontinental baselines, too few natural sources possess sufficiently large fringe visibilities in the required u-v regions. Further, the measured distribitions considered here are too few in number and too limited in u-v coverage and observing frequency to provide a general verification. The u-v coverage is too limited because (a) the absence of short baselines ($\lesssim$ 300 km) can allow important large scale structure to be resolved out and missed and (b) the absence of intercontinental baselines can allow important small scale structure to remain unresolved. One example of a source distribution that would violate the limit formula would be a large, strong diffuse component displaced by a considerable distance from a weaker compact component. In this case, the effective position would move from the centroid of both components to the center of the compact component as baseline length increased. If the separation of components were great enough, large changes in structure delay could occur. More survey data is required to determine whether the percentage of sources that fall in this category, or other disallowed categories, is too large for the limit equation in its present form to be valid. Even if the present form is unsuitable, it should be possible to construct another visibility-dependent limit that is valid for nearly all sources. How useful the the final form will be in overcoming source structure problems remains to be determined.

In future work, we plan to investigate more sources with more detailed analysis with the hope that the present limit approach, or some refined version, can be generally established.

ACKNOWLEDGEMENTS

The author wishes to thank S. G. Finley for her enthusiastic and skillful support in generating the software for the contour plots. This paper presents results from one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract number NAS7-100, sponsored by the National Aeronautics and Space Administration.

REFERENCES

(1) J. B. Thomas, "Source Structure Errors in the Synchronization of Clocks by Radio Interferometry", Proceedings of Eleventh Annual PTTI Meeting, NASA Conference Publication 2129, p. 599, November 1979 .

(2) T. J. Pearson, A. C. S. Readhead, and P. N. Wilkinson, "Maps of the Quasars 3C119, 3C286, 3C345, 3C454.3 and CTA102 with a Resolution of 5 milliarcseconds at 1.67 GHz", Astrophysical Journal 236, 714, March 1980.

(3) A. C. S. Readhead, T. J. Pearson, M. H. Cohen, M. S. Ewing and T. A. Moffet, "Hybrid Maps of the Milliarcsecond Structure of 3C120, 3C273 and 3C345", Astrophysical Journal 231, 299, July 1979.

(4) J. B. Thomas, "An Analysis of Source Structure Effects in Radio Interferometry Measurements", A JPL report to be published.

SUMMARY OF BRIGHTNESS DISTRIBUTION MEASUREMENTS

TABLE 1

DISTRIBUTION NUMBER	SOURCE	MEASUREMENT DATE	OBSERVING WAVELENGTH (cm)	TOTAL FLUX (Jy)	INTERFEROMETER STATIONS	UMAX / VMAX ($10^6\lambda$)
1	3C119	MAY, 1976	18	7.5	1, 2, 3, 4	19 / 12
2	3C286	MAY, 1976	18	13.5	1, 2, 3, 4	19 / 10
3	3C345	MAY, 1976	18	6.5	1, 2, 3, 4	19 / 12
4	3C454.3	MAY, 1976	18	12.0	1, 2, 3, 4	19 / 8
5	CTA102	MAY, 1976	18	6.4	1, 2, 3, 4	19 / 7
6	3C273	JULY, 1977	2.8	41.0	1, 2, 3, 5	135 / 40
7	3C345	JULY, 1977	2.8	7.5	1, 2, 3, 5	140 / 80
8	3C273	DECEMBER, 1977	6	46.2	1, 2, 3, 4	60 / 15
9	3C345	DECEMBER, 1977	6	7.8	1, 2, 3, 4	60 / 40
10	3C120	DECEMBER, 1977	6	6.6	1, 2, 3, 4	60 / 20

STATION CODE: 1 = NRAO; 2 = FDVS; 3 = OVRO; 4 = HCRK; 5 = HSTK

AN EXAMPLE BRIGHTNESS DISTRIBUTION

3C 345 DISTRIBUTION 9

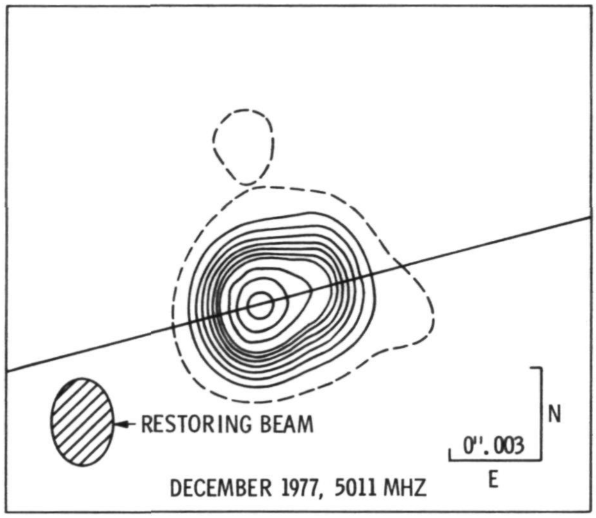


FIGURE 1

FRINGE VISIBILITY

3C 345 DISTRIBUTION 9

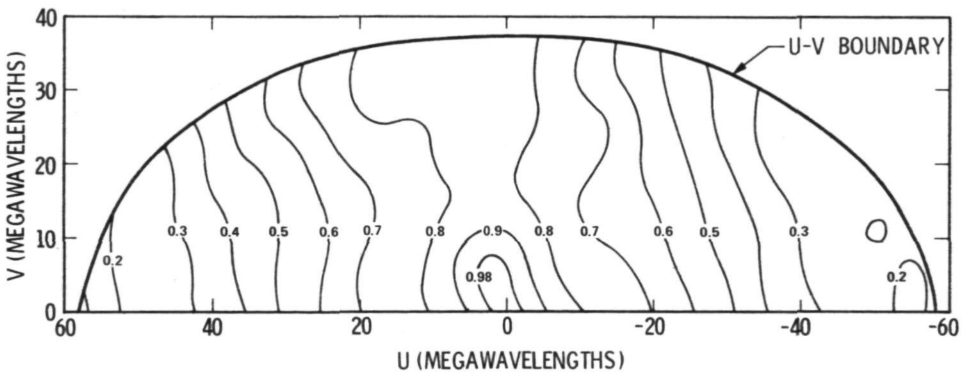
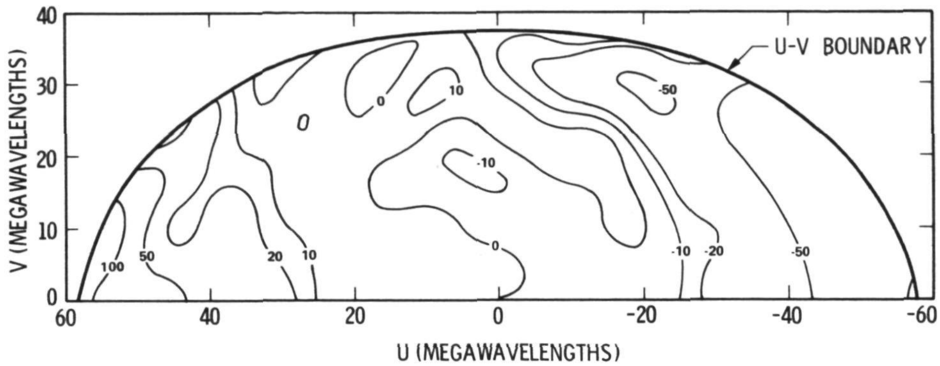


FIGURE 2

STRUCTURE DELAY* RELATIVE TO BRIGHTNESS CENTROID

3C 345 DISTRIBUTION 9



* FOR X-BAND IN PSEC

FIGURE 3

A LIMIT FORMULA FOR STRUCTURE EFFECTS* IN BWS DELAY

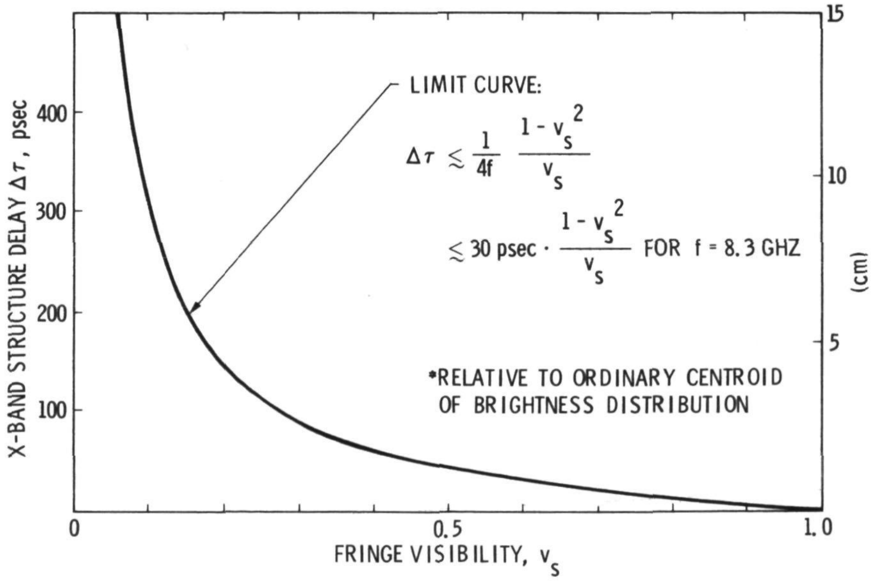


FIGURE 4