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PILOT-OPTIMAL MULTIVARIABLE CONTROL SYNTHESIS BY OUTPUT FEEDBACK

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INTRODUCTION

If all the flight vehicle dynamic characteristics that yield good handling qualities and good pilot-vehicle performance were known, flight control engineers could synthesize controllers that produce this dynamic response.

In the case of flight vehicles exibiting non-conventional dynamics, sufficient data and handling qualities are seldom available. Based on the fact that the behavior of the pilot during a task is influenced by the plant he is controlling and that the optimal control model structure for the human operator[1] has been found to be related to opinion rating[2 3], a simultaneous optimal control solution has been proposed by Schmidt[4]. Such a method is based on closed -loop design philosophy and leads to a controller that has full-state feedback structure.

The present paper introduces two basic innovations with the intent of describing a more practical stability augmentation system design. In the previous work[4] the pilot's " state estimator " was neglected in the augmentation synthesis. It will be shown that this assumption has a strong influence on the stability augmentation (SAS) gains. Secondly, the SAS structure will be assumed herein to be a linear combination of selected system outputs rather than full-state feedback.

SYSTEM STRUCTURE AND PILOT MODEL

The complete plant-controller dynamics are represented by the block diagram in Fig.1. In state variable form we have the following linear time-invariant system,

$$\begin{cases} \dot{x} = Ax + Bu + D\omega \\ y_p = C_p x(t-\tau) + v_p \\ y_s = C_s x, \end{cases} \qquad (1)$$

with $x \epsilon R_n$, $u \epsilon R_m$, $y_p \epsilon R_p$, $y_s \epsilon R_s$, $\omega \epsilon R_k$ as dimensions of the time varying variables and,

$$u = u_p + u_s$$

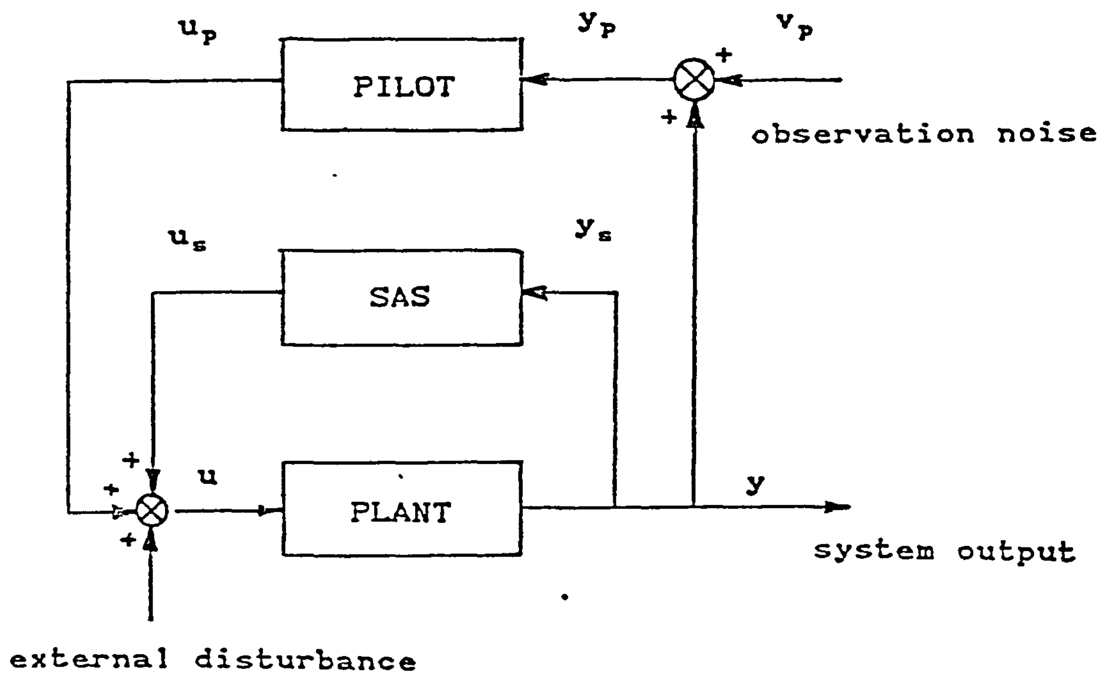


FIG 1. BLOCK DIAGRAM OF THE SYSTEM DYNAMICS

$$u_s = Ey_s$$

In the above, note that u is the total control vector which includes the contributions of both pilot input u_p and augmentation input u_s; $v_p(t)$ is the pilot's observation noise and is characteristic of the pilot's imperfect perception, assumed to be described by a white noise process with statistics: $N(0, V_p)$* . Also ω is the system driving noise that models the effect of external disturbances (or commands in the case of a tracking task) and it is also described as a white noise process with statistics: N(0, W).

The following assumptions are made:

i) The system described by (1) is completely controllable (stabilizable) and observable (detectable).[5]

ii) The output vector to the SAS, y_s, is noise free. This implies that errors in the measurements are negligible with respect to the pilot's observation noise.

At this point. the characteristics of the chosen pilot model will be briefly reviewed.

The optimal control model is based on the hypothesis that a well-trained, well motivated pilot chooses his control policy u_p, subject to human limitations and understanding of the task he is performing, to minimize a quadratic index of performance given by:

$$J_p = E\{ \lim_{T\to\infty} 1/T\int_0^T (x^TQx + u^T{}_pRu_p + \dot{u}^T{}_pG\dot{u}_p)dt\} \qquad (2)$$

where $E\{\cdot\}$ denotes the expected value and $(\cdot)^T$ denotes vector transpose.

The weighting matrices $Q\geq0$, $R\geq0$, $G>0$ quantify the pilot's strategy and workload/performance trade off. Also, the control rate weight G in (2) can be related to neuromuscular lag in the pilot.

Defining the augmented state vector:

$$\chi = \begin{bmatrix} x \\ u_p \end{bmatrix} \qquad (3)$$

* N(a, b) denotes normal probability density with mean a and variance b

(1) becomes:

$$\dot{X} = \begin{bmatrix} A_s & B \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ I \end{bmatrix} \dot{u}_p + \begin{bmatrix} D \\ 0 \end{bmatrix} \omega \quad (4)$$

$$y_p = \begin{bmatrix} C_p & 0 \end{bmatrix} X(t-\tau) + v_p$$

with: $A_s = A + BEC_s$ (plant plus any augmentation) and,

$$J_p = E\{ \lim_{T\to\infty} 1/T \int_0^T (X^T Q_1 X + \dot{u}^T{}_p G \dot{u}_p) dt \} \quad (5)$$

$$Q_1 = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \geq 0$$

In the optimal control model, the pilot estimates all the states of the system from his vector of observations y_p. This activity is modeled as a Kalman-Bucy filter along with a least mean-square predictor to compensate for his time delay.

It can be shown that under this structure the optimal controller is

$$\dot{u}_p = K\hat{X} + v_m = K_x\hat{x} + K_u\hat{u}_p + v_m \quad (6)$$

where v_m is a white noise process which represents motor noise contaminating the pilot s commanded control. It (v_m) is assumed to have zero mean and intensity V_m. The gain matrix is:

$$K = -G^{-1}\begin{bmatrix} 0 & I \end{bmatrix} P \quad (7)$$

where P is the solution of the algebraic Riccati equation:

$$0 = \begin{bmatrix} A^T{}_s & 0 \\ B^T & 0 \end{bmatrix} P + P\begin{bmatrix} A_s & B \\ 0 & 0 \end{bmatrix} + Q_1 - P\begin{bmatrix} 0 \\ I \end{bmatrix} G^{-1} \begin{bmatrix} 0 & I \end{bmatrix} P \quad (8)$$

The estimate $\hat{X}$ of the state vector X is derived from a least mean-square predictor in series with the Kalman Filter estimating the delayed state $\hat{x}(t-\tau)$. Now, in our work, we will consider only the Kalman Filter dynamics and increase the noise variances slightly to account for delay and prediction. So we have then:

$$\dot{\hat{X}} = \begin{bmatrix} A_s & B \\ 0 & 0 \end{bmatrix} \hat{X} + \begin{bmatrix} 0 \\ I \end{bmatrix} \dot{u}_p + \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} (y_p - \begin{bmatrix} C_p & 0 \end{bmatrix} \hat{X}) \tag{9}$$

The Kalman Filter gain matrix M is given by:

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \Sigma^T \begin{bmatrix} C^T{}_p \\ 0 \end{bmatrix} V_p^{-1} \tag{10}$$

with Σ solution of:

$$\begin{bmatrix} A_s & B \\ 0 & 0 \end{bmatrix} \Sigma + \Sigma \begin{bmatrix} A^T{}_s & 0 \\ B^T & 0 \end{bmatrix} + \overline{W} - \Sigma \begin{bmatrix} C^T{}_p \\ 0 \end{bmatrix} V_p^{-1} \begin{bmatrix} C_p & 0 \end{bmatrix} \Sigma = 0 \tag{11}$$

and:

$$\overline{W} = \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & V_m \end{bmatrix} \begin{bmatrix} D^T & 0 \\ 0 & I \end{bmatrix}$$

DETERMINATION OF THE OPTIMAL CONTROLLER

In the original approach[4], the pilot's estimation process was assumed to have little influence on the augmentation system, heuristically invoking a "separation principle". This was done by defining:

$$\hat{X} = X + \epsilon$$

and the error ϵ was treated as an additional white disturbance to the system (and thereby treated as uncorrelated with X).

In the present paper we will show instead that the presence of additional dynamics due to the state estimator affects substantially the augmentation gains and can not be neglected in a design synthesis procedure. Further, we will derive the expressions for the optimal SAS gains considering the control to be of the form:

$$u_s = Ey_s \tag{12}$$

or a linear combination of measurable outputs.

The dynamics of the plant with the augmentation and pilot input is given by:

$$\dot{X} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ K \end{bmatrix} \hat{X} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_s + \begin{bmatrix} \omega \\ v_m \end{bmatrix}$$

$$y_s = \begin{bmatrix} C_s & 0 \end{bmatrix} X \qquad (13)$$

plus the state estimator dynamics inherent in the pilot model:

$$\dot{\hat{X}} = \begin{bmatrix} A_s & B \\ K_x & K_u \end{bmatrix} \hat{X} + \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} (y_p - \begin{bmatrix} C_p & 0 \end{bmatrix} \hat{X}) + \begin{bmatrix} 0 \\ v_m \end{bmatrix} \qquad (13')$$

where as defined previously:

$$A_s = A + BEC_s$$

We can now state the optimal control problem for the SAS; we wish to find the optimal controller $u_s = Ey_s$ which minimizes the index of performance:

$$J_s = J_p + E \{ \lim_{T \to \infty} 1/T \int_0^T u^T{}_s F u_s \} \qquad (14)$$

with $F>0$ and J_p as in (5) subject to constraints (13) and(13'), expressable as:

$$\dot{q} = \begin{bmatrix} \dot{X} \\ \hline \dot{\hat{X}} \end{bmatrix} = \left[\begin{array}{cc|cc} A & B & 0 & 0 \\ 0 & 0 & K_x & K_u \\ \hline M_1 C_p & 0 & A_s - M_1 C_p & B \\ M_2 C_p & 0 & K_x - M_2 C_p & K_u \end{array}\right] q + \begin{bmatrix} B \\ 0 \\ \hline 0 \\ 0 \end{bmatrix} u_s + \left[\begin{array}{ccc} D & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & 0 & M_1 \\ 0 & I & M_2 \end{array}\right] \begin{bmatrix} \omega \\ v_m \\ v_p \end{bmatrix} \qquad (15)$$

$$y_s = \left[\begin{array}{cc|cc} C_s & 0 & 0 & 0 \end{array}\right] q$$

Also noting that:

$$J_s = E \{ \lim_{T \to \infty} 1/T \int_0^T (q^T \bar{Q} q + u^T{}_s F u_s) dt \} \qquad (15')$$

with:

$$\bar{Q} = \left[\begin{array}{cc|cc} Q & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ \hline 0 & 0 & K^T{}_x G K_x & K^T{}_u G K_x \\ 0 & 0 & K^T{}_x G K_u & K^T{}_u G K_u \end{array}\right]$$

Rewriting (15) and (15') in a more compact form we have:

$$\dot{q} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} q + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u_s + \tilde{D}\omega' \tag{16}$$

$$y_s = \begin{bmatrix} C_2 & 0 \end{bmatrix} q \qquad q \epsilon R_{2n}$$

and:

$$J_s = E \{ \lim_{T \to \infty} 1/T \int_0^T q^T \bar{Q} q + u^T{}_s F u_s) dt \} \tag{17}$$

The newly introduced matrices have obvious structure from (15) and (15').

Eqs. (16) and (17) with the constraint $u_s = Ey_s$ define the suboptimal output feedback problem[6,7]. The necessary conditions for optimality, based on first variation principles give the solution as

$$E = -F^{-1} \begin{bmatrix} B^T{}_2 & 0 \end{bmatrix} HL \begin{bmatrix} C^T{}_2 \\ 0 \end{bmatrix} \left(\begin{bmatrix} C_2 & 0 \end{bmatrix} L \begin{bmatrix} C^T{}_2 \\ 0 \end{bmatrix} \right)^{-1} \tag{18}$$

with H and L positive definite, unique solutions respectively of

$$A^T{}_c H + HA_c + \bar{Q} + \begin{bmatrix} C^T{}_2 E^T F E C_2 & 0 \\ 0 & 0 \end{bmatrix} = 0 \tag{19}$$

$$A_c L + LA^T{}_c + \bar{D}\bar{\bar{W}}\bar{D}^T = 0 \tag{20}$$

with:

$$A_c = \begin{bmatrix} A_{11} + B_2 E C_2 & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{21}$$

$$\bar{\bar{W}} = \text{diag.} \left[W, V_m, V_p \right]$$

The matrix L is the covariance matrix for the system and is given by

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \qquad L_{11} = E\{XX^T\},\ L_{22} = E\{\hat{X}\hat{X}^T\}.\ L_{12} = E\{X\hat{X}^T\} = L_{21}{}^T = L_{22}$$

rewriting (18) in partitioned form

$$E = -F^{-1}\begin{bmatrix} B^T{}_2H_{11} & B^T{}_2H_{12}\end{bmatrix}\begin{bmatrix} L_{11} & C^T{}_2 \\ L_{12} & C^T{}_2\end{bmatrix}(C_2L_{11}C^T{}_2)^{-1}$$

$$E = -F^{-1}B^T{}_2H_{11}L_{11}C^T{}_2(C_2L_{11}C^T{}_2)^{-1} - F^{-1}B^T{}_2H_{12}L_{12}C^T{}_2(C_2L_{11}C^T{}_2)^{-1}$$

or:

$$E = E_1 + E_2 \tag{22}$$

As we can see from (22), two terms contribute to the total SAS gain matrix, one is due to the plant dynamics, while the second is due to the estimator and plant dynamics. This shows tnat we can not necessarily neglect the presence of the estimator when we proceed to the synthesis of a stability augmentation system with the pilot included in the control loop.

SPECIAL NOTES

From the general result of (22) we can make two particular notes:

1) First of all if we assume that SAS has a full-state feedback structure or:

$$C_2 = I$$

the gain matrix E becomes

$$E = -F^{-1}B^T{}_2H_{11} \;\; -F^{-1}B^T{}_2H_{12}L_{12}L_{11}^{-1} \tag{23}$$

The first term on the RHS of (23) is now the familiar result for the linear regulator problem and the second term represents the contribution, to the total gain ,of the estimator in the pilot model.

2) If now we neglect the estimator as in Ref. 4, the second term on the RHS of (23) disappears and we are left with gains identical to those in the Reference.

NUMERICAL EXAMPLE

To demonstrate the methodology described in the previous sections we will use the example of Ref.4. A second order plant in a tracking task is considered. Its dynamics are:

$$\frac{\theta(s)}{\delta(s)} = \frac{11.7}{s^2}$$

The commanded signal $\theta_c(t)$ is obtained from a white noise process $\omega(t)$ driving the linear system:

$$\frac{\theta(s)}{\omega(s)} = \frac{3.67}{s^2 + 3s + 2.25}$$

$$\omega(t) \ , \ N(0,1).$$

In state variable form, defining $x = \left[\theta_c , \dot{\theta}_c , \theta, \dot{\theta} \right]^T$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.25 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 11.7 \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 3.67 \\ 0 \\ 0 \end{bmatrix} \omega$$

where $\delta = \delta_p + \delta_s$, the sum of pilot's and augmentation controls.

The indices of performance that δ_p and δ_s are to minimize. are respectively:

$$J_p = E \{ \lim_{T \to \infty} 1/T \int_0^T [(\theta - \theta_c)^2 + 0.01\delta^2{}_p + g\dot{\delta}^2{}_p] dt \}$$

and:

$$J_s = J_p + E \{ \lim_{T \to \infty} 1/T \int_0^T f\delta^2{}_s dt \}$$

The parameter g is adjusted in such a way that a neuromuscular time constant $\tau_n = .1$ secs. is obtained. The weighting f on the augmentation control is a free parameter which is varied to yield a family of controllers.

The pilot is assumed to be able to perceive both position error and error rate, that is:

$$y_p = \begin{bmatrix} \theta_c - \theta \\ \dot{\theta}_c - \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x + v_p$$

Three different combinations for output vectors y_s are analyzed. In all the following results, the simplified pilot model(with out predictor but including the Kalman Filter) was used in the solution, unless noted. However, the complete pilot model as presented in Ref. 1 was used for performance evaluations and comparisons.

The first table (Table 1) gives the system performance in the absence of augmentation. Based on Refs. 2 and 3, the term PR indicates the pilot rating on a Cooper-Harper scale predicted with the index of performance J_p. Tables 2 and 3 furnish the performance of the augmented system dynamics and the values of SAS gains (in deg/deg and sec. respectively) in the case where only θ and $\dot{\theta}$ are fedback. The results are given as a function of the scalar f, weighting the augmentation energy introduced into the system.

Table 1. Unaugmented System Performance

$(\theta-\theta_c)$ RMS*	δ_p RMS*	J_p	PR
1.170	1.00	1.86	7.7

Table 2. Augmented System Performance. $v_s = [\theta, \dot{\theta}]$

f	$(\theta_c-\theta)$ RMS*	δ_p RMS*	J_p	PR
100	1.140	1.054	1.743	7.4
10	0.786	0.657	0.768	5.4
1	0.628	0.489	0.463	4.1
0.1	0.618	0.607	0.449	4.0
0.01	0.608	0.795	0.438	3.9

Tables 4 and 5 show a comparison of results between the cases in which the state estimator dynamics are included in the SAS design (KF) versus neglected (NKF). The results are for a weighting on augmentation energy f = 1. and three different possible sets of feedback variables for the SAS are considered. Note the significant difference in gains and performance between cases.

*All angles in degrees

Table 3. SAS Gains. $y_s = [\theta, \dot{\theta}]$

f	E^{θ}	$E^{\dot{\theta}}$
100	-0.0064	-0.0029
10	-0.0625	-0.0271
1	-0.2242	-0.0818
0.1	-0.5823	-0.1074
0.01	-0.7877	-0.1428

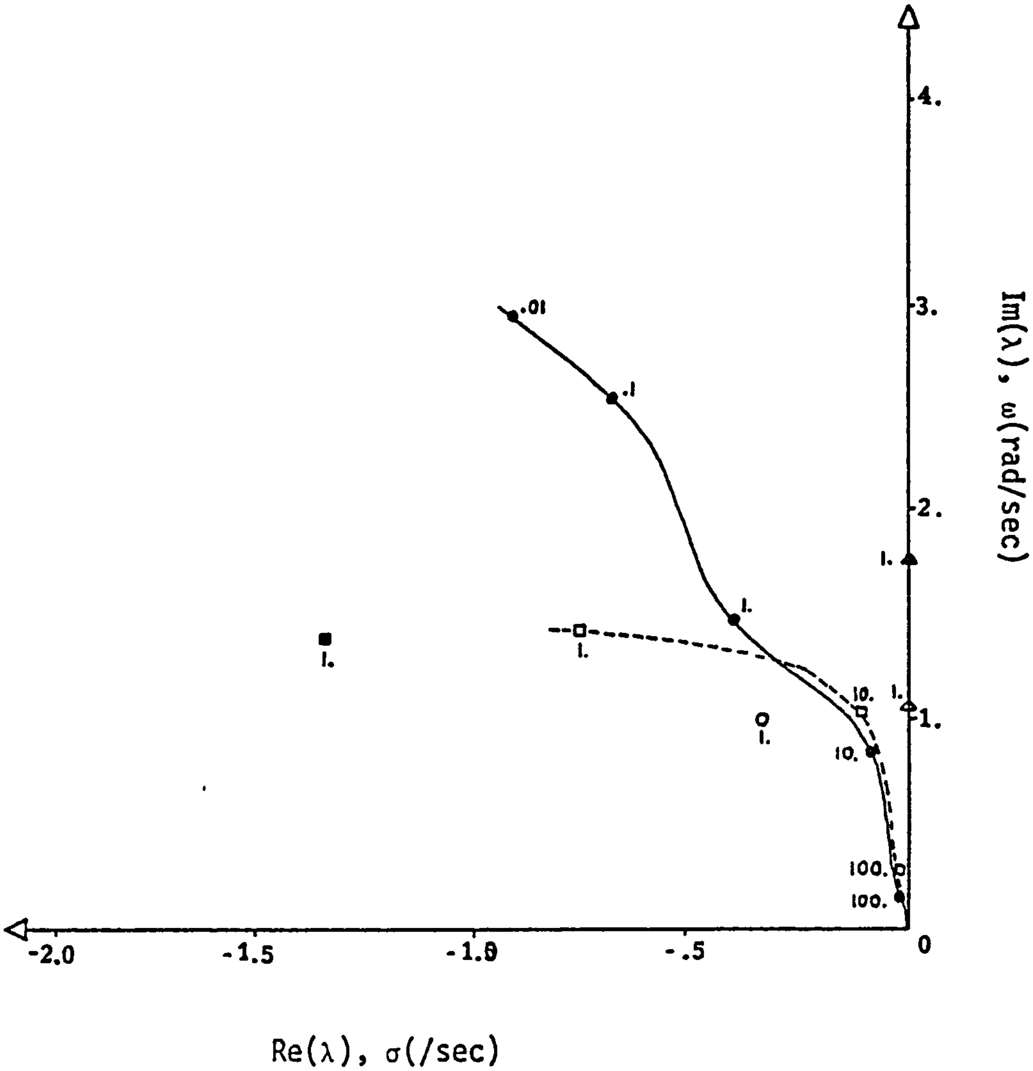
Now consider Figure 2 that shows the trend of the augmented plant eigenvalues ($y_s = [\theta, \dot{\theta}]$) as a function of the parameter f(continuous line), and a comparison with the eigenvalue pattern from Ref.4 (dashed line). Also for a value f=1, the effect of the presence of the state estimator in the system dynamics (solid vs. open symbols) is shown for the same cases in Tables 4 and 5.

The pilot's index of performance J_p and rating PR is represented in Figure 3 with $y_s = [\theta, \dot{\theta}]$; furthermore, for f =1, a comparison between augmentation feedback with and without estimator dynamics is carried out as before.

Finally, the tracking performance (σ_ϵ) and workload ($\sigma_{\dot{\delta}_p}$) are shown in Figures 4 and 5 for these same cases.

Table 4. Augmented System Performance

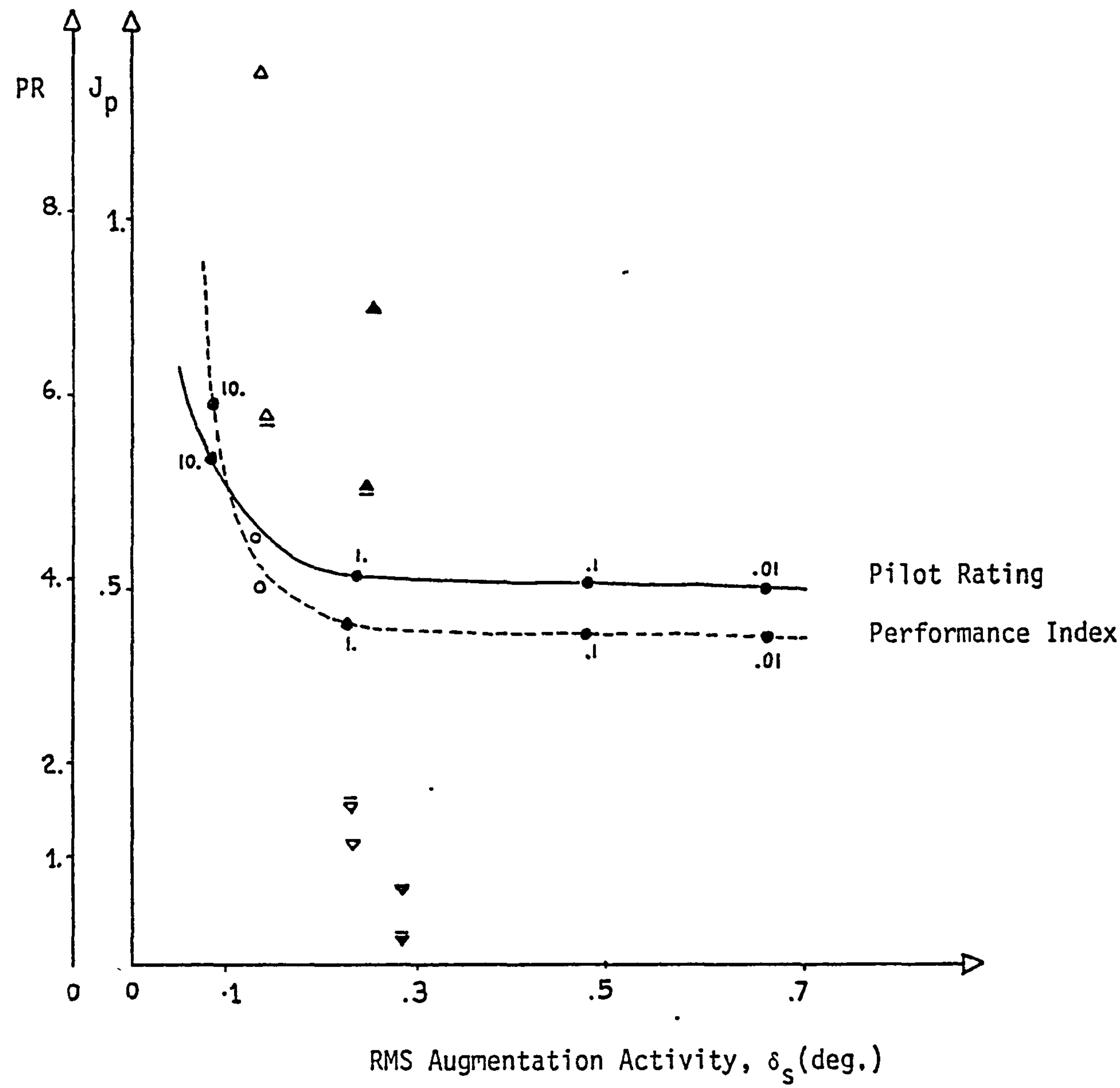
$(\theta_c-\theta)$RMS		δ_pRMS		J_p		PR		y_s
KF	NKF	KF	NKF	KF	NKF	KF	NKF	
0.757	0.856	0.655	0.791	0.708 (▲)	0.930 (△)	5.2 (▲)	5.9 (△)	θ
0.628	0.686	0.489	0.569	0.463 (●)	0.568 (○)	4.1 (●)	4.6 (○)	$\theta, \dot{\theta}$
0.303	0.376	0.300	0.350	0.100 (▼)	0.160 (▽)	<1. (▼)	1.6 (▽)	$\theta_c \theta_c \theta$ θ



NOMENCLATURE

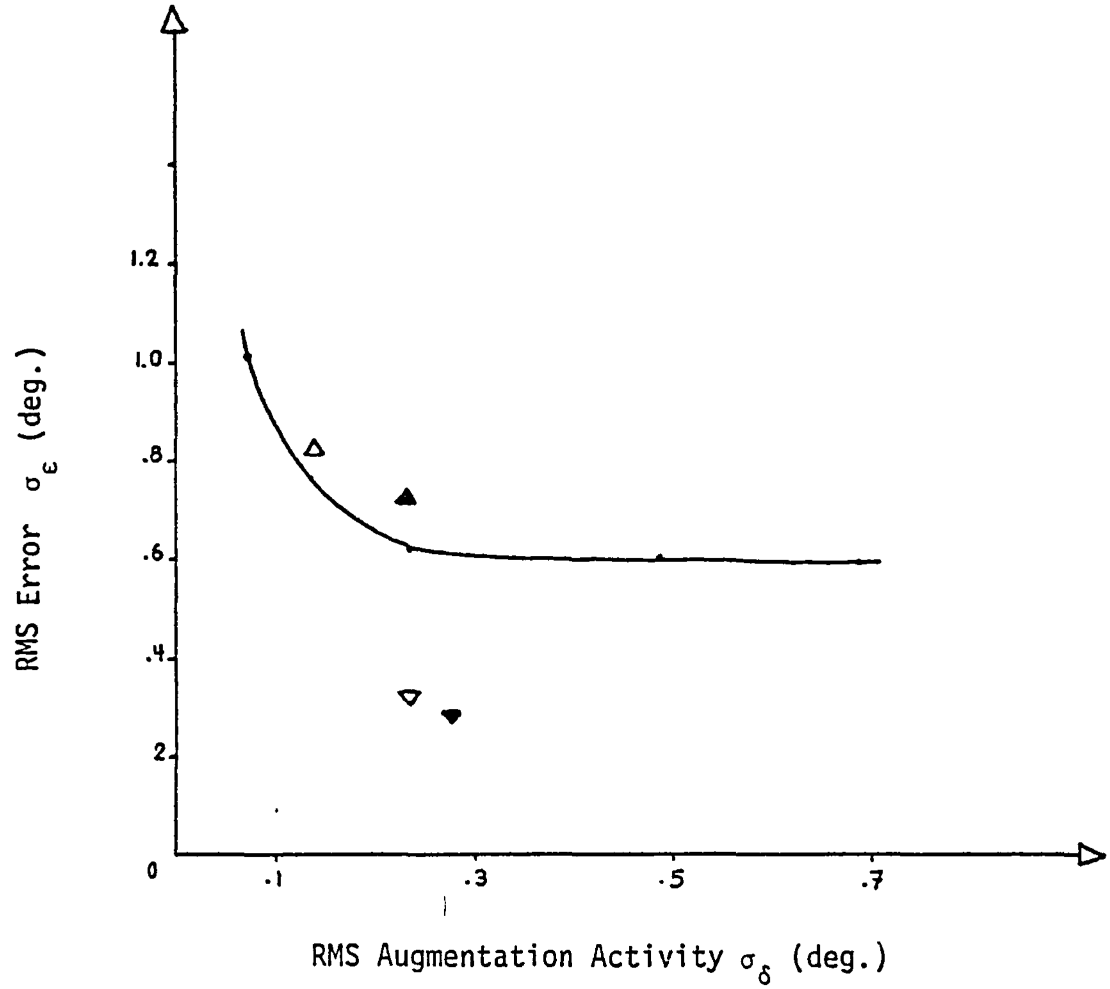
- ● Eigenvalue behavior with f $y_s = [\theta, \dot{\theta}]$
- ■ Eigenvalue behavior f = 1. $y_s = [\theta_c, \dot{\theta}_c, \theta, \dot{\theta}]$
- ▲ Eigenvalue behavior f = 1. $y_s = \theta$
- ○ □ △ Same as above but estimator dynamics is neglected in augmentation design.

FIG. 2. AUGMENTED EIGENVALUE LOCUS



(symbols defined in parenthesis in Table 4)

FIG. 3 PILOT RATING AND COST vs. SAS ACTIVITY



NOMENCLATURE

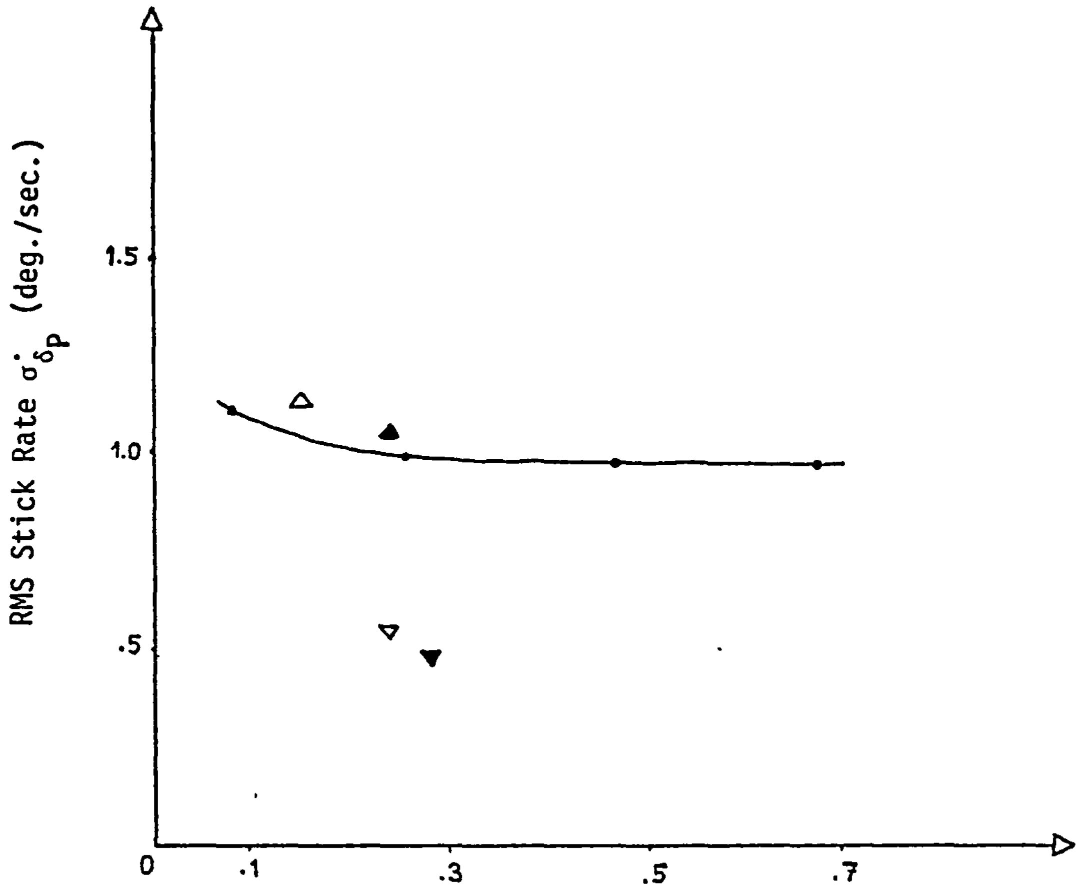
Continuous line behavior with f. $y_s = [\theta, \dot{\theta}]$

▼ behavior for f = 1. $y_s = [\theta_c \cdot \dot{\theta}_c, \ \theta \ \ \dot{\theta}]$

▲ behavior for f = 1. $y_s = \theta$

▽△ Same as above but estimator dynamics is neglected in SAS design.

FIG. 4. TRACKING ERROR vs. SAS ACTIVITY



RMS Augmentation Activity σ_{δ_s} (deg.)

NOMENCLATURE

Continuous line behavior with f, $y_s = [\theta, \dot{\theta}]$

▼ behavior for f = 1. $y_s = [\theta_c, \dot{\theta}_c, \theta, \dot{\theta}]$

▲ behavior for f = 1. $y_s = \theta$

▽ △ Same as above but estimator dynamics is neglected in SAS design.

FIG. 5. STICK RATE vs. SAS ACTIVITY

Table 5. Augmentation Gains

$E^{\theta c}$		$E^{\dot{\theta} c}$		E^{θ}		$E^{\dot{\theta}}$	
KF	NKF	KF	NKF	KF	NKF	KF	NKF
-	-	-	-	-0.264	-0.109	-	-
-	-	-	-	-0.224	-0.107	-0.082	-0.048
0.573	0.487	0.150	0.080	-0.659	-0.536	-0.255	-0.130

EXPERIMENTAL CORRELATION

The methodology is now applied to "predict" the optimal pitch rate feedback gain for a T-33 aircraft in a pitch tracking task. The results will be compared to experimental data available from fixed-base simulations.

The short period dynamics of the aircraft (all angles in degrees) are given by:

$$\begin{cases} \dot{\theta} = q \\ \dot{q} = M_{\alpha}\alpha + M_{q}q + M_{\dot{\alpha}}\dot{\alpha} + M_{\delta}\delta \\ \dot{\alpha} = q + Z_{\alpha}\alpha + Z_{\delta}\delta \end{cases}$$

The commanded pitch angle is given by the relation:

$$\dot{\theta}_c = -0.1\theta_c + \omega$$

$$\omega(t),\ N(0,\ \sigma^2) \quad \sigma^2 \text{ chosen to give } \sigma^2_{\theta_c} = 10 \ \text{deg}^2.$$

For level flight at Mach 0.5 and altitude 4.545 m. (15,000 ft.), we obtain the following system dynamics in state variable form:

$$\dot{x} = \begin{bmatrix} -0.1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1.0524 & -3.01 \\ 0 & 0 & 0 & -1.384 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -14.52 \\ -0.08 \end{bmatrix} \delta + \begin{bmatrix} \omega \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The state vector is $x = [\theta_c,\ \theta,\ \dot{\theta},\ \alpha]^T$ and the total elevator control is the sum$(\delta_p + \delta_s)$ of pilot's and augmentation inputs. The indices of performance to be minimized by δ_p and δ_s are:

$$J_p = E\{ \lim_{T\to\infty} 1/T\int_0^T [(\theta_c-\theta)^2 + 0.002\dot{\delta}^2_p]dt\}$$

$$J_s = J_p + E\{ \lim_{T\to\infty} 1/T\int_0^T 0.01\sigma^2_s dt\}$$

According to Ref.8, we assume that the pilot is able to perceive position error information($\theta_c-\theta$) and that the augmentation measurement is pitch rate θ only, thus:

$$y_p = (\theta_c-\theta) + v_p$$

$$y_s = [0\ 0\ 1.\ \ 0]x$$

$$\delta_s = -K_{\dot{\theta}}\ q$$

The next table contains the augmentation gains predicted with and without inclusion of the pilot's state estimation,or dynamic compensation. (KF and NKF respectively)

Table 6. Predicted Rate Feedback Gain ($K_{\dot{\theta}}$, sec.)

KF	NKF
-0.24	0.09

As it can be seen from Figure 6, the predicted gain and the gain obtained experimentally are in very good agreement in obtaining the minimum pilot rating. In addition, from Table 6 the importance of the effect of the state estimator is underscored. The optimal gain obtained ignoring the estimator actually decreased pitch damping!.

Finally in comparing Figures 6 and 7 we see the strong correlation between pilot ratings obtained experimentally and the sum of the mean-square error and stick rate- or objective function- from simulation.

SUMMARY AND CONCLUSIONS

In the paper we have presented a methodology for the design of stability augmentation systems with the human operator in the loop, using optimal control theory.

The case in which the augmentation has limited-state feedback structure is analyzed and a closed form solution is obtained. The need to consider the estimator in the pilot model is quantified, in contrast to the hypothesis of Ref 4. An example is given to demonstrate these results. In the case of a second-order plant we obtained a significant improvement in performance and large differences in augmentation gains due to the inclusion of a dynamic estimator in the pilot model. It is also shown that, as a function of augmentation activity(σ_{δ_s}), limited state feedback provides much improved performance.

Most significantly, the analytical determination of the optimum rate feedback gain was verified experimentally, for pitch tracking, via fixed-base simulation.

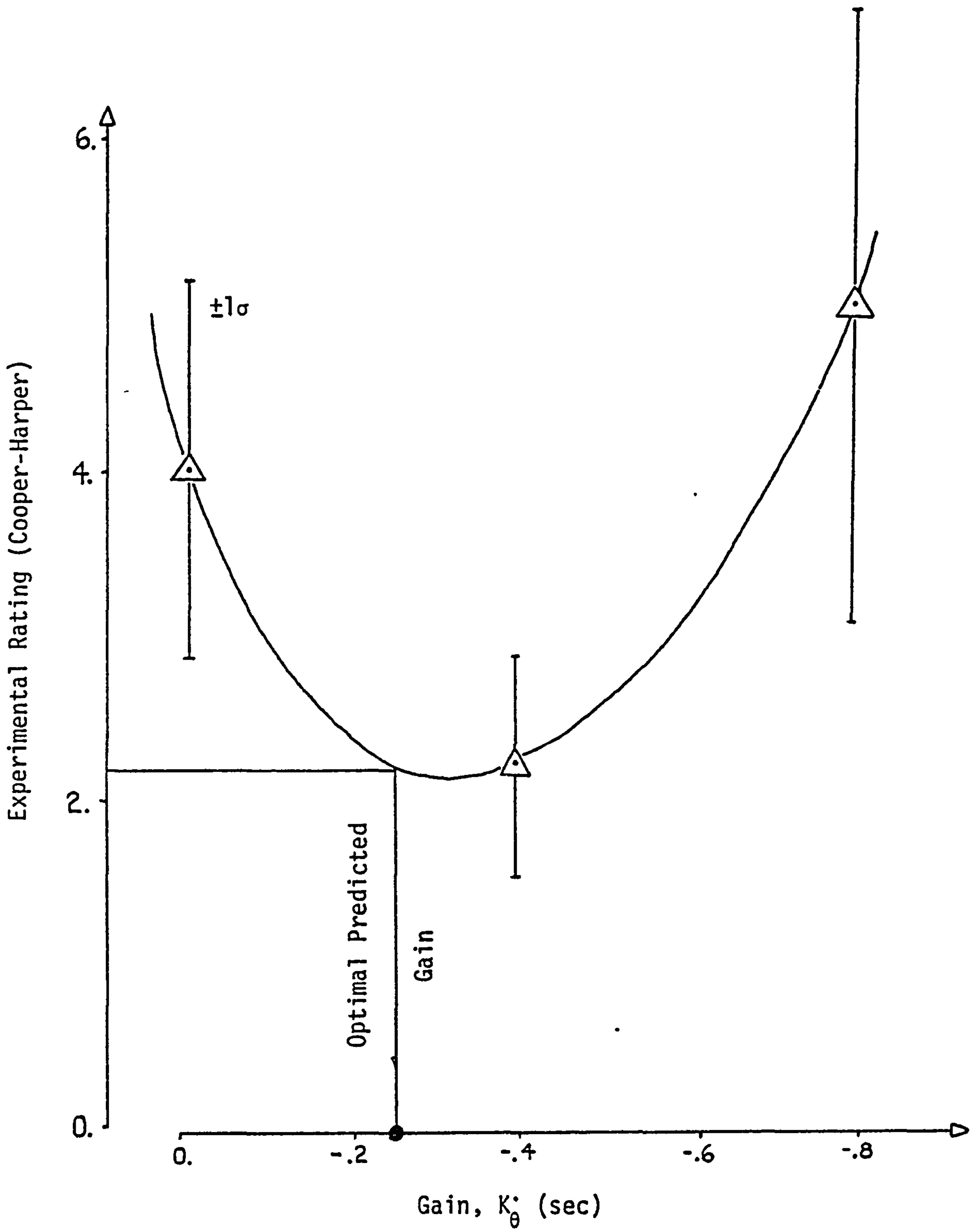


FIG 6. PILOT RATING vs. GAIN

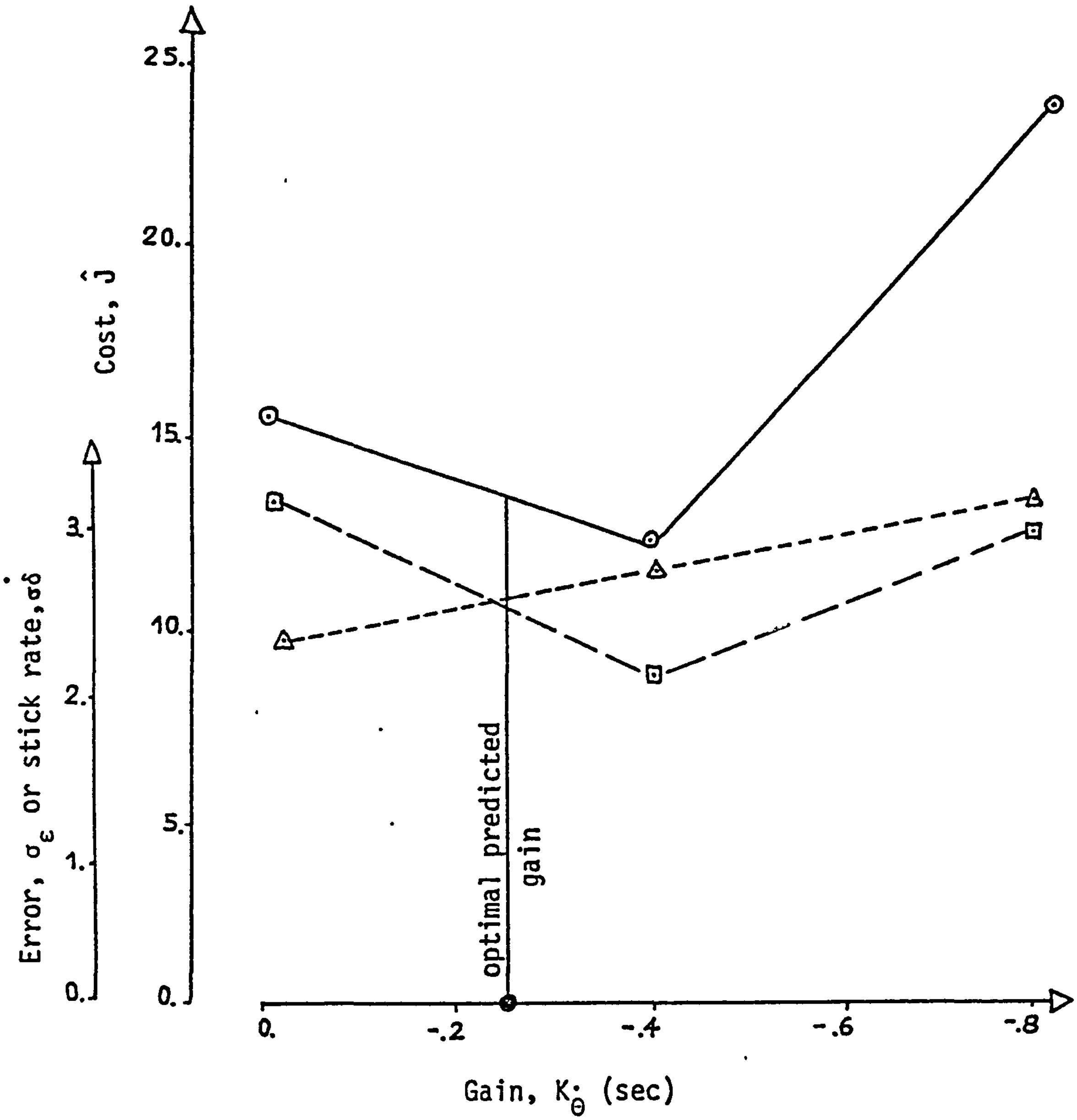


Fig 7: PERFORMANCE vs. GAIN

NOMENCLATURE

⊙	$\hat{J} = \sigma_{\varepsilon}^2 + \sigma_{\dot{\delta}}^2$	cost
⊡	σ_{ε} deg.	RMS error
△	$\sigma_{\dot{\delta}}$ deg. sec.$^{-1}$	RMS pilot stick rate

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16. Abstract

To be presented is a control system design approach for optimal stability augmentation systems using limited state feedback theory, with the specific inclusion of the human pilot in the loop. The methodology is intended to be especially suitable for application to flight vehicles exibiting non-conventional dynamic characteristics and for which quantitative handling qualities specifications might be not available. The design objectives are introduced via the hypothesis of correlation between pilot ratings and the objective function of the optimal control model of the human pilot. Since both human and augmentation controllers must be compatible and operate in parallel, simultaneous optimization for augmentation and pilot gains is required. Finally, the method is experimentally verified for the simple example of pitch-damper gain selection and significantly improved performance obtained.

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