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Automated Dynamic Analytical Model Improvement

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SUMMARY

This report describes a method of improving a dynamic analytical model of a structure using natural frequencies and normal modes measured in a test of the structure. The report includes a discussion of the problem, the derivation of the algorithms and a description of the computer implementation. In addition, applications of the method to a large structure are presented. The results of these applications confirm the expectations that the method can become a practical and useful tool in the development of valid dynamic analytical models.

INTRODUCTION

Structural dynamic analytical models of aerospace structures are required for analyses which are sensitive to the frequency response characteristics of the model. The prediction of internal loads, aeroelastic stability analyses, and dynamic optimization studies fall in this class.

Ground vibration tests are often performed to validate an analytical model. These tests provide direct information which may lead to the correction of major deficiencies in the analysis such as insufficient modeling detail in certain areas of the structure. In practice, the acceptance of an an analytical model is usually based on a subjective evaluation of the predicted and measured natural frequencies and mode shapes. No generally accepted method is presently employed which uses the test data objectively to improve the analytical model.

There are numerous sources of error in both the analysis and the test which are related to theoretical and physical assumptions and mechanical and equipment limitations. The method described in this report does not deal with the errors in the test data or the sources of the errors in the analysis. What is determined are minimum changes in the analytical model required to make it exactly predict the measured mode shapes and natural frequencies.

This approach does not eliminate the need for engineering judgment. It does, however, change the character of the judgment required. In the conventional and the suggested method the engineer must first accept the test data as a valid representation of the structure which is modeled. Conventionally, the analytical predictions are then compared to the test results. If the correlation is not "acceptable", methods of improving the model must be decided upon. This process may be repeated until acceptable correlation is achieved. This method can be very expensive and time consuming with no prior assurance of success.

In the suggested method a cycle of subjective model improvement may be carried out if obvious modeling deficiencies are apparent. Then the analytical model improvements are computed which will result in a model which will exactly predict the measured data. The judgment required here is an engineering evaluation of the predicted minimum changes. If they fall within acceptable limits the model may be taken as a valid baseline for further analysis. If the changes are judged to be excessive or unreasonable, it must be concluded that other major deficiencies exist in the test or in the original model. This is information which cannot be determined by the conventional procedure. Given good test data and a reasonable, but not precise, analytical model, this method will yield an improved model which is exactly compatible with the test.

As stated above, the subject of this report is a method of improving analytical models of structures to make them agree with measured natural frequencies and normal mode shapes. The technology related to ground vibration testing, the extraction of mode shapes and frequencies, and the validation and improvement of this data are important but are outside the scope of this presentation. Similarly, the particular method used to obtain the analytical model is not relevant.

The starting point for this analysis is the following data: a set of normal mode shapes and natural frequencies which the analyst would like the model to predict and a mass and stiffness matrix analytical model which is representative of the structure such that small changes in the elements will yield the desired characteristics. It should be noted that the requirement for "small" changes is not an assumption of the analysis but is specified so that the resulting model is representative of the physical structure. The criteria for "small" is left to engineering judgment.

Publications relevant to this area are all quite recent. In 1967, Rodden[1] used ground vibration test data to derive structural influence coefficients. This was a use of measured mode shapes and frequencies for other than direct verification of model predictions. A method which corrected a mass matrix and developed an "incomplete" stiffness matrix was published in 1971[2], a statistical approach to the problem was presented in 1974 by Collins, et al.[3], and a perturbation method by Chen and Garba was published in 1979[4]. More comprehensive lists of publications are found in References 5,6. Publications by Baruch and Bar Itzhack[7,8] are very important and the method they developed for correcting the stiffness matrix has been adapted for this analysis.

The method of "incomplete models" by Berman and Flannelly[2] developed the conceptual basis of the method given here. A successful application to a practical problem was presented in 1975[9]. That method, however, has several features which limits its application to large systems. The formulation of the mass correction method results in a set of underdetermined simultaneous equations which are solved by the "pseudo inverse" method. For structures with a large number of degrees of freedom and a relatively large number of test modes the size of this problem can become quite large. A method was recently developed and published[10] which overcomes this problem. The method of Reference 2 also does not provide for the correction of the analytical stiffness matrix, but synthesizes an "incomplete" stiffness matrix which may not be completely adequate for a baseline analytical model. The method of Baruch[7,8] has been adapted for this purpose as part of the complete process described here. (see References 11,12 for comments on the general approach and the derivation of Reference 7.)

Another deficiency of all previous methods is that the analytical model must have degrees of freedom which are limited to the points at which test measurements are made. This is undesirable since a larger baseline model is normally desired. This limitation is also eliminated in the procedure presented and makes use of an observation of Kidder[13]. A related use of this relationship for orthogonality checks has also been published[14].

The method described has been implemented in a computer program and has been applied to the LDEF[15] structure. The results are presented in this report.

SYMBOLS

I	Unit matrix
K,M	Full improved stiffness and mass matrices
K_A,M_A	Full approximate analytical stiffness and mass matrices
K_1,M_1	Partitions of K_A,M_A corresponding to test coordinates
K_2,M_2	Partitions of K_A,M_A corresponding to coupling coordinates
K_4,M_4	Partitions of K_A,M_A corresponding to unmeasured coordinates
$m_A =$	$\Phi^T M_A \Phi$, $m_{A_{ii}} = 1$
m	number of modes
n	Number of degrees of freedom
ε	Minimization function
λ	Lagrangian multiplier
$\Lambda,\Lambda_K,\Lambda_o,\Lambda_s$	Matrices of Lagrangian multipliers
Φ	Modal matrix
ϕ_i	ith mode, ith column of Φ
ϕ_{1_i},ϕ_{2_i}	Measured and unmeasured partitions of ϕ_i
ψ	Lagrangian function
Ω	Diagonal matrix of measured natural frequencies
ω_i	ith natural frequency, Ω_{ii}
T	Superscript, transpose of matrix
$\|\|(\ )\|\|$	Sum of the squares of the elements of the included matrix.

THEORETICAL DEVELOPMENT

Basic Relationships

This analysis makes use of two theoretical relationships which apply to linear, undamped structures represented as finite element models. These are: the orthogonality of the normalized modes,

$$\Phi^T M \Phi = I \tag{1}$$

and; the eigenvalue equation for the ith mode,

$$[K - \omega_i^2 M]\Phi_i = 0 \tag{2}$$

All of the analysis that follows is based on these two equations.

Full Mode Computation

It is desired to correct an analytical model which is not limited to the degrees of freedom measured in a test. There are several possible methods of achieving this goal. One method was first investigated and then was superseded by the method presented. This method consisted of: reducing the model to the test coordinates; correcting the reduced model; then performing an "inverse Guyan reduction." Another approach would use a geometric interpolation method to estimate the modal displacements at the unmeasured coordinates. For complex three-dimensional structures with relatively few measurements such methods may not be appropriate.

The method presented here may be thought of as an interpolation method based on the physics of the structure, rather than the geometry. The relationship between two subsets of a mode shape has been given by Kidder[13] using a partitioned form of equation (2):

$$\left\{ \begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} - \omega_i^2 \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix} \right\} \begin{Bmatrix} \phi_{1_i} \\ \phi_{2_i} \end{Bmatrix} = 0 \qquad (3)$$

where the ϕ_{1_i} represents the measured and ϕ_{2_i} represents the unknown elements of the ith mode shape, ω_i is measured and K and M reasonably represent the structure. From equation (3) it is apparent that

$$\phi_{2_i} = -(K_4 - \omega_i^2 M_4)^{-1} (K_2^T - \omega_i^2 M_2^T)\phi_{1_i} \qquad (4)$$

Equation (4) may be solved at three levels of approximation. If ω_i is considered to be very small and ignorable, we have, in effect, the reduction of Guyan[16]. If it is small, the series approximation of Kidder[13] may be used. Otherwise an exact solution is most efficiently performed by solving the simultaneous equations (e.g., LU decomposition) rather than inverting the matrix. This is the procedure presently implemented.

At the start of the process for improving the analytical model one has only approximate mass and stiffness matrices. If these matrices are reasonable approximations, ϕ_2 can be expected to be reasonably accurate. In any case, however, the analysis that follows will result in a corrected mass and stiffness model which will predict the measured ω_i and ϕ_{1_i}. The predicted changes are left to the engineer, as previously discussed, for his evaluation as being physically representative of the structure.

When the corrected matrices are obtained, an iteration through the above procedure is an option which can be expected to improve the results and converge rapidly if the changes are "small".

Given a means of transforming the measured mode shapes to the modes over the full coordinate system, it is now possible to work with the full matrices in equations (1), (2).

Mass Matrix Improvement

When there are fewer modes than degrees of freedom there are an infinite number of mass matrices which will make the modes orthogonal (equation (1)). If some measure of the change in the mass matrix is minimized a unique solution will result.

It is physically reasonable and mathematically convenient to minimize the function

$$\varepsilon = ||M_A^{-\frac{1}{2}}(M - M_A)M_A^{-\frac{1}{2}}|| \tag{5}$$

where M is the unknown improved matrix and M_A is the given analytical matrix. This function tends to minimize the relative changes in the elements of the matrix. (Note that it is not necessary to compute $M_A^{-\frac{1}{2}}$ since only M_A appears in the final result.)

Defining a Lagrangian multiplier, λ_{ij}, for each element in equation (1) the following Lagrangian function may be written:

$$\psi = \varepsilon + \sum_{i=1}^{m} \sum_{j=1}^{m} \lambda_{ij} \left(\Phi^T (M-M_A)\Phi - I + m_A\right)_{ij} \tag{6}$$

where $m_A = \Phi^T M_A \Phi$, the measured generalized mass matrix having unit diagonals, and $\Phi^T(M - M_A)\Phi - I + m_A = 0$ (equation (1)). Differentiating equation (6) with respect to each element of M and equating these results to 0 will satisfy the minimization of equation (5) if the constraint of equation (1) is also satisfied. This process results in the matrix equation

$$2\, M_A^{-1}(M - M_A)M_A^{-1} + \Phi\Lambda\Phi^T = 0$$

or

$$M - M_A = -\tfrac{1}{2}M_A\Phi\Lambda\Phi^T M_A \tag{7}$$

where Λ is a square (m x m) matrix of λ_{ij}.

Substituting equation (7) into equation (1) allows the solution for Λ:

$$\Lambda = -2\, m_A^{-1}\, (I - m_A)\, m_A^{-1} \tag{8}$$

which is then substituted into equation (6) to obtain

$$M = M_A + M_A\, \Phi m_A^{-1}\, (I - m_A)\, m_A^{-1}\, \Phi^T M_A \tag{9}$$

Equation (9) is an easily evaluated expression for the corrected mass matrix to make it consistent with the measured modes. Note that M is symmetrical as is theoretically necessary. The only inversion required is that of m_A which is of the order of the number of measured modes. This process is considered an improvement on the method of Reference 2 when applied to large matrices.

Stiffness Matrix Improvement

In a similar but considerably more complicated manner, it is possible to correct the stiffness matrix using Eq.(2) and the now known frequencies, modes, and improved mass matrix. The most convenient formulation* includes three equations of constraint: the eigenvalue equation for all the measured modes, the orthogonality relationship for the stiffness matrix, and the requirement for symmetry which is not automatically satisfied as in the case of the mass matrix:

$$K\Phi - M\Phi\Omega^2 = 0 \tag{10}$$

$$\Phi^T K\Phi - \Omega^2 = 0 \tag{11}$$

$$K - K^T = 0 \tag{12}$$

The number of elements in each of these equations are (nxm), (mxm), and (nxn) respectively, where n is the order of the M and K matrices

*This analysis is equivalent to that of References 7 and 8 but the introduction of equation (11) improves the derivation and the final form used here is somewhat more convenient (see Reference 12).

and m is the number of modes.

The function to be minimized is analogous to that of equation (5)

$$\varepsilon = ||M^{-\frac{1}{2}}(K - K_A)M^{-\frac{1}{2}}|| \tag{13}$$

Defining a Lagrangian multiplier matrix for each of the constraint equations $(\Lambda_K, \Lambda_o, \Lambda_s)$ results in the Lagrangian function

$$\psi = \varepsilon + \sum_{i=1}^{n} \sum_{j=1}^{m} \Lambda_{K_{ij}} (K\Phi - M\Phi\,\Omega^2)_{ij}$$
$$+ \sum_{i=1}^{m} \sum_{j=1}^{m} \Lambda_{o_{ij}} (\Phi^T K\Phi - \Omega^2)_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} \Lambda_{s_{ij}} (K-K^T)_{ij} \tag{14}$$

Note that the K matrix is the only physical parameter which is unknown.

Differentiating equation (14) with respect to the elements of K and setting this result to 0 results in:

$$2\,M^{-1}(K-K_A)M^{-1} + \Lambda_K\Phi^T + \Phi\,\Lambda_o\,\Phi^T + \Lambda_s - \Lambda_s^T = 0$$

or

$$K = K_A - 1/2\,M\,[\Lambda_K\Phi^T + \Phi\Lambda_o\,\Phi^T + \Lambda_s - \Lambda_s^T]\,M \tag{15}$$

Substituting this result into equation (12) to solve for $\Lambda_s - \Lambda_s^T$ and subsituting into (15) eliminates Λ_s and gives:

$$K = K_A - 1/4\,M\,(\Phi\Lambda_K^T + \Lambda_K\Phi^T)M - 1/4\,M\Phi(\Lambda_o + \Lambda_o^T)\Phi^T M \tag{16}$$

Substitute (16) into constraint equation (11) and using $\Phi^T M\Phi = I$ gives:

$$1/4\,(\Lambda_o + \Lambda_o^T) = \Phi^T K_A\Phi - \Omega^2 - 1/4\,(\Lambda_K^T M\Phi + \Phi^T{}_M\,\Lambda_K) \tag{17}$$

and then substituting equation (17) into (16) results in

$$K = K_A - M\Phi(\Phi^T K_A \Phi - \Omega^2)\Phi^T M - 1/4\ M\Phi\Lambda_K{}^T(I - M\Phi\Phi^T)\ M$$

$$- 1/4\ M(I - \Phi\Phi^T M)\Lambda_K \Phi^T M \qquad (18)$$

This equation is substituted in the final constraint, equation (10) giving

$$1/4\ M(I - \Phi\Phi^T M)\ \Lambda_K = (I - M\Phi\Phi^T)K_A\Phi \qquad (19)$$

where the relationship $(I - M\Phi\Phi^T)M\Phi = M\Phi - M\Phi = 0$ is used.

Note that equation (19) may not be solved for Λ_K since $I - \Phi\Phi^T M$ is singular*. The quantity on the left hand side of equation (19) and its transpose appear in equation (18) and these substitutions give

$$K = K_A - (K_A\Phi\Phi^T M + M\Phi\Phi^T K_A) + M\Phi(\Phi^T K_A \Phi + \Omega^2)\Phi^T M \qquad (20)$$

which is precisely the form of Reference 7.

A more convenient formulation of this relationship may be written

$$K = K_A + (\Delta + \Delta^T) \qquad (21)$$

where

$$\Delta = (I - 1/2\ M\Phi\Phi^T)(M\Phi\Omega^2 - K_A\Phi)\Phi^T M \qquad (22)$$

Note that no matrix inversions are required for the evaluation of the corrected stiffness matrix.

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*If $(I - \Phi\Phi^T M)$ is not singular, then $\Phi = (I - \Phi\Phi^T M)^{-1}\ (I - \Phi\Phi^T M)\Phi =$
$(I - \Phi\Phi^T M)^{-1}(\Phi - \Phi) = 0$ which is contradictory, therefore
$(I - \Phi\Phi^T M)$ must be singular.

COMPUTER IMPLEMENTATION

General Description

A computer program was developed and implemented on an IBM 360 at the Kaman Aerospace Corporation. This program was used for all the applications described in this report.

The program performs the following functions:

(1) Mass and stiffness matrices on sequential files are reordered to place test degrees of freedom in the upper left of each matrix. (A separate program, AMID, performs this function)

(2) Test data (frequencies and mode shapes) are read. If there are fewer test points than degrees of freedom, equation (4) is solved to compute the full modes. A user option is available to ignore the frequency terms which makes the computation more efficient since the equations are solved only once for all the modes.

(3) The mass matrix correction is computed using equation (9). Prior to this computation the mode shapes are normalized to make the diagonal elements of m_A equal to 1.

(4) The stiffness matrix is corrected using equations (21), (22).

(5) The improved mass and stiffness matrices are placed on sequential files and statistical data is computed and listed. This data consists of: RMS of original matrix, RMS of changes, the ratio of the proceeding, the mean absolute ratio of the diagonal changes to the original diagonals, the RMS of the changes divided by the corresponding diagonal elements, i.e., the square root of the mean of $\left[\sum_{i,j} (M - M_A)^2_{ij}/(M_{A_{ii}} \times M_{A_{jj}})\right]$. In addition, the 50 largest changes are printed.

(6) If desired, the process may be iterated from step (2) to improve the computed full mode shapes.

Computer Requirements

The program was written to operate in a moderate partition of core (e.g., 100K decimal words). The program is easily redimensioned, if necessary, to treat problems of different sizes.

The number of words of core required is given approximately as:

$$13{,}000 + 4(\text{NMMAX})^2 + 4(\text{NMAX}) + (\text{NMAX}) * (\text{NMMAX}) + (\text{NEQ}^2)/4$$

where

NMMAX = maximum number of modes

NMAX = maximum number of degrees of freedom

NEQ = the number of degrees of freedom minus the number of test coordinates

For the cases run in this study: NMMAX = 20; NMAX = 510; NEQ = 508-101 = 407. The core requirements are 68252 words.

In addition six sequential files are required, where each must be large enough to hold a complete matrix. Two of these files contain M_A, K_A and two will contain M, K at the completion of the analysis.

LDL^T Decomposition

The solution for the full mode shapes is the portion of the program which most severely taxes the computer resources. In the cases run, for example it is necessary to solve 407 simultaneous equations. By the most direct method, inversion of the matrix in core, $2(407)^2$ = 332K words of storage would be required. For this reason and for efficiency in computation time, a modified LU decomposition algorithm was developed which performed these solutions in 1/8 the storage, or 42K words.

The decomposition algorithm takes advantage of the symmetry of the equation coefficients and forms a lower, diagonal, lower transposed matrix decomposition.

Consider a symmetric matrix, A, which we desire to represent by a product of a lower triangular matrix, L, a diagonal matrix, D, and the transpose of L. Introducing the D matrix, allows one to specify that the diagonal elements of L are unity.

$$A = L\ D\ L^T \tag{23}$$

or

$$a_{ij} = \sum_{k=1}^{n} \sum_{\ell=1}^{n} \ell_{ik}\ d_{k\ell}\ \ell^T_{\ell j}$$

$$= \sum_{k=1}^{n} \ell_{ik}\ \ell_{jk}\ d_{kk} \quad (\text{since } d_{\ell k} = 0,\ \ell \neq k)$$

$$= \sum_{k=1}^{\min(i,j)} \ell_{ik}\ \ell_{jk}\ d_{kk} \tag{24}$$

$$(\text{since } \ell_{ik} = 0,\ k>i \text{ and } \ell_{jk} = 0,\ k>j)$$

Since $a_{ij} = a_{ji}$, one may work only with the lower triangle of A:

$$\underset{(i\geq j)}{a_{ij}} = \sum_{k=1}^{j} \ell_{ik}\ \ell_{jk}\ d_{kk} \tag{25}$$

If it is assumed that all the ℓ's are known up to, but not including column J and all the d's are known up to but not including d_{JJ}, then

$$a_{JJ} = \sum_{k=1}^{J} \ell^2_{Jk}\ d_{kk} = \sum_{k=1}^{J-1} \ell^2_{Jk}\ d_{kk} + d_{JJ} \quad (\text{since } \ell_{JJ} = 1)$$

or

$$d_{JJ} = a_{JJ} - \sum_{k=1}^{J-1} \ell^2_{Jk}\ d_{kk} \tag{26}$$

where all values on the right hand side are known.

Also

$$\underset{i>J}{a_{iJ}} = \sum_{k=1}^{J} \ell_{ik}\, \ell_{Jk}\, d_{kk} = \sum_{k=1}^{J-1} \ell_{ik}\, \ell_{Jk}\, d_{kk} + \ell_{iJ}\, d_{JJ}$$

or

$$\underset{i>J}{\ell_{iJ}} = \left(a_{iJ} - \sum_{k=1}^{J-1} \ell_{ik}\, \ell_{Jk}\, d_{kk} \right) / d_{JJ} \qquad (27)$$

where all the values on the right hand side are known.

Thus, it is possible, using equation (26), (27) to read one column of A at a time and, starting at the diagonal element, compute the corresponding rows and columns of L and L^T. This process requires all the elements of L in the lower rectangle of L which intersects the diagonal. The maximum storage required is then $(n/2)^2$, where n is the order of the matrix.

In the actual algorithm which was implemented, the decomposition is separated into three phases. A work area of variable dimensions is set up. The first phase (LDLT1) loads as many columns of A as will fit in this area and computes the corresponding columns of L, D, L^T. Then the next phase (LDLT2) loads a column of A and solves for one column of L,D,L^T at a time, starting at the diagonal. The third phase (LDLT3) starts when the work area can hold all the remaining rows of A. During this entire process, the work area is continually being redimensioned and the rows of L and the rows of L^T (columns of L) are written on to a sequential file.

When this process is completed, the solution of the matrix equation $Ax = LDL^Tx = b$ is simply performed as follows:

(1) solve for x_2, one element at a time from $Lx_2 = b$, where $x_2 = DL^Tx$ (note L is a lower triangular matrix);

(2) solve for x_1 from $Dx_1 = x_2$, where $x_1 = L^Tx$ (note D is a diagonal matrix);

(3) solve for x from $L^T x = x_1$, where x is the solution of the original equation (note L^T is an upper triangular matrix and the solution must proceed from the bottom up).

APPLICATION TO SIMULATED TEST DATA*

Program Validation

In order to validate the program and the algorithms a 92 degree of freedom NASTRAN† model and the first six analytical modes and frequencies were employed. The odd numbered degrees of freedom were treated as test data. When the analysis was performed using the frequency dependent terms, the "unmeasured" modal displacements were exactly predicted. The procedure was repeated with arbitrary changes in natural frequencies and mode shapes and eigenanalyses were performed on the corrected mass and stiffness matrices. In each case the eigenanalysis yielded the modified natural frequencies and the full modes. These tests validated both the algorithms and the computer code.

Sample Analyses

The model referred to in the previous paragraph was used to perform preliminary analyses with simulated test data. The analytical model has 92 degrees of freedom and represents a full scale, three-dimensional, unconstrained helicopter fuselage. Forty-six of the degrees of freedom of the first six elastic modes were used to simulate measured test data.

The first series of analyses used the exact mode shapes and introduced arbitrary simulated frequency errors. The exact and modified frequencies are shown in Table I and the results of the analyses are shown in Table II. In each case, both frequency options were exercised. That is, in equation (4) ω_i was included or ω_i was set to 0 (equivalent to Guyan transformation). Note that when the frequencies are ignored in equation (4), the mass correction is unrelated to the measured frequencies and in each case the resulting errors in the computed full modes results in a significant change in the mass matrix.

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*The information in this section was previously published in Reference 17.

†NASTRAN: Registered trademark of the National Aeronautics and Space Administration.

TABLE I. - EXACT AND MODIFIED SIMULATED TEST NATURAL FREQUENCIES

Mode	Exact natural frequency, Hz	Arbitrary changed frequency, Hz	Change %
1	4.18	3.98	-4.8
2	5.12	5.57	+8.8
3	6.87	7.16	+4.2
4	8.02	8.75	+9.1
5	12.39	11.94	-3.7
6	15.75	15.12	-4.0

TABLE II. - EFFECT OF NATURAL FREQUENCY ERROR ON IMPROVED MODEL (a)

Case	Modes with changed frequencies (b)	Mass matrix changes, % (c)		Stiffness matrix changes, % (c)	
		With(d)	Without(e)	With(d)	Without(e)
1	All exact	0	7.2	0	.41
2	Mode 2 (+2.6%)	.002	7.2	.0007	.41
3	Mode 2	.005	7.2	.0022	.41
4	Modes 1,2	.014	7.2	.0023	.41
5	Modes 1-6	.43	7.2	.023	.40

[a] All six modes used in each case with exact shapes

[b] Changed frequency given in Table I (except case 2)

[c] Change is rms of changes/rms of original elements

[d] Frequency terms included in equation (4)

[e] Frequency terms omitted from equation (4)

Table III illustrates the effect of mode shape errors while using exact frequencies and Table IV includes the effects of both mode shape and frequency errors.

TABLE III. - EFFECT OF MODE SHAPE ERROR ON IMPROVED MODEL (a)

Case	Description (b)	Mass matrix changes %, (c)		Stiffness matrix changes %, (c)	
		With (d)	Without (e)	With (d)	Without (e)
6	2 figures	1.2	8.0	.14	.45
7	1 figure	1.6	6.9	1.2	1.3

[a]All six modes used in each case with exact frequencies

[b]All mode shapes rounded to specified number of significant figures

[c,d,e] See Table II.

TABLE IV. - EFFECT OF FREQUENCY AND MODE SHAPE ERROR ON IMPROVED MODEL (a)

Case	Description (b)	Mass matrix changes %, (c)		Stiffness matrix changes %, (c)	
		With (d)	Without (e)	With (d)	Without (e)
8	Case 5 + 6	1.6	8.0	.14	.44
9	Case 5 + 7	1.6	6.9	1.2	1.3

[a]Six modes used in each case

[b]Errors in mode shape and frequency as in referenced cases

[c,d,e,] See Table II.

The data presented in the tables suggests that the original premise of this analysis is valid. That is, if a "reasonable" model is available and "reasonable" test data is obtained, relatively small changes in the model can make it consistent with the test data. It is also apparent from this data, that it may not be appropriate to ignore the frequency dependence in equation (4) since this results in relatively large (and unnecessary) changes in the matrices. This is true even where only the lowest frequency modes are used. This effect will become more important when higher frequency modes are used in the analysis.

An important observation may be made based on an inverse interpretation of the data obtained. If one desires to develop an analytical model which will accurately predict the natural frequencies and modes, the accuracy requirements on the mass and especially the stiffness matrices appear to be quite stringent.

The orthogonality of the computed full modes with respect to the analytic mass matrix is worthy of inspection. Consider case 5 in which all the first six frequencies of the analytical model have been assumed to be in error. Tables V and VI are the respective generalized mass matrices, m_A, with and without the frequency dependent terms. Table VI is the orthogonality check matrix which would be obtained if a (Guyan) reduced mass matrix were used. Note that the modes are much better than an analyst would conclude using that conventional procedure. This effect, however, appears to be less important when there are significant errors in the modes and frequencies, as illustrated in Tables VII, and VIII. Note that after the mass matrices are corrected the resulting generalized mass matrices would always be unit matrices.

TABLE V. ORTHOGONALITY CHECK MATRIX FOR CASE 5 WITH FREQUENCY DEPENDENCE.

1.0					
-.00005	1.0				
.00048	-.00007	1.0			
-.00022	.0026	.00033	1.0		
-.012	.00002	.010	.00024	1.0	
.0048	.00044	.00033	.0029	.0025	1.0

TABLE VI. - ORTHOGONALITY CHECK MATRIX FOR EXACT MODES WITHOUT FREQUENCY DEPENDENCE.

1.0					
.0006	1.0				
-.010	.0005	1.0			
.0011	-.013	-.0018	1.0		
-.111	-.0001	.165	.011	1.0	
.017	.011	.0035	.069	.010	1.0

TABLE VII. - ORTHOGONALITY CHECK MATRIX FOR CASE 9 WITH FREQUENCY DEPENDENCE

1.0					
-.011	1.0				
-.016	-.0074	1.0			
-.038	-.185	.011	1.0		
-.0036	-.011	.0055	-.0084	1.0	
.038	.070	-.0091	-.0047	-.0073	1.0

TABLE VIII. - ORTHOGONALITY CHECK MATRIX FOR CASE 7 OR 9 WITHOUT FREQUENCY DEPENDENCE

1.0					
-.011	1.0				
-.027	-.0071	1.0			
-.037	-.192	.0071	1.0		
-.102	-.0090	.161	.0002	1.0	
.042	.062	-.0069	.064	.0036	1.0

APPLICATION TO LDEF DATA

Description of Model and Test Data

The AMI method has been applied to the Long Duration Exposure Facility (LDEF) without trays. (See Ref. 15 for a description of the structure.) The structure was tested at the Langley Research Center and data representing the first 16 measured modes were supplied by the Government and used in these analyses. The tests used up to 130 degrees of freedom, however only 101 were common to all the modes. These 101 were then used as the test degrees of freedom in the analyses.

In addition, the government developed a NASTRAN model of the structure, having 1097 degrees of freedom.* This model was subsequently reduced to 508 degrees of freedom by eliminating the rotational degrees of freedom and certain axial degrees of freedom. It was this 508 degree of freedom model from which the analytical M_A, K_A matrices were used in these analyses. A special program was written by the Government to extract the mass and stiffness matrices from the NASTRAN analysis and format them for use by the AMID program.

As an illustration of the reasonableness of the analytical model, Table IX shows a comparison of the measured natural frequencies and those obtained from analysis using the 1097 degree of freedom model.

Table X gives the relationship between the test points and the degrees of freedom of the reduced model and illustrates the reordering of the matrices which was required before processing of the data.

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*Developed by Thomas C. Jones of the Systems Engineering Divisions at the NASA Langley Research Center.

TABLE IX. - COMPARISON OF MEASURED AND COMPUTED NATURAL FREQUENCIES.[a]

Mode number	Measured natural frequencies, Hz	Computed natural frequencies, Hz (b)
1	14.18	13.83
2	21.50	22.78
3	23.26	23.83
4	24.36	24.37
5	24.63	26.10
6	26.29	28.76
7	27.66	28.90
8	28.67	29.77
9	29.22	29.87
10	33.42	29.96
11	34.49	30.79
12	35.37	31.85
13	41.04	35.58
14	42.11	38.46
15	42.99	
16	43.48	

[a] Frequencies are in numerical order, full correspondence between mode shapes is not implied.

[b] Computed from 1097 degree-of-freedom NASTRAN model. This data is not used in the AMI analysis.

TABLE X. - TEST DEGREES OF FREEDOM RELATED TO GRID POINTS AND MODEL DEGREES OF FREEDOM.

Test point	Test d.o.f. (a)	Grid point	Original model d.o.f.	Reordered model d.o.f.
1	Y,Z	2	50,51	1,2
2	Y	3	83	3
4	Y,Z	5	171,172	4,5
5	Y,Z	6	207,208	6,7
7	Y,Z	8	210,211	8,9
10	Y,Z	11	89,90	10,11
12	X,Y,Z	1	38,39,40	12,13,14
14	Y,Z	14	86,87	15,16
15	Y,Z	15	118,119	17,18
16	Y,Z	17	230,231	19,20
17	Y,Z	18	267,268	21,22
19	Y,Z	20	273,274	23,24
21	Y	22	188	25
22	Y,Z	23	129,130	26,27
24	Y,Z	13	62,63	28,29
25	Y,Z	26	126,127	30,31
26	Z	27	169	32
28	Y,Z	29	290,291	33,34
29	Y	30	327	35
31	Y,Z	32	336,337	36,37
33	Z	34	247	38
34	Y	35	185	39
36	Y	25	97	40
37	Y	38	182	41
38	Z	39	222	42
40	Y,Z	41	353,354	43,44
45	Z	46	304	45
46	Y,Z	47	243,244	46,47
48	Y	37	144	48
49	Y	50	238	49

TABLE X. - CONCLUDED.

Test point	Test d.o.f. (a)	Grid point	Original model d.o.f.	Reordered model d.o.f.
50	Z	51	285	50
52	Y,Z	53	400,401	51,52
53	Y	54	416	53
55	Y,Z	56	425,426	54,55
57	Z	58	331	56
58	Y,Z	59	281,282	57,58
60	Y	49	179	59
61	Y,Z	62	298,299	60,61
62	Y,Z	63	347,348	62,63
64	Z	65	438	64
65	Z	66	451	65
67	Y,Z	68	459,466	66,67
69	Y,Z	70	379,380	68,69
70	Y,Z	71	344,345	70,71
72	Y,Z	61	235,236	72,73
73	Y	74	359	74
74	Z,Y,Z	75	393,394,395	75,76,77
76	Y,Z	77	471,472	78,79
77	X,Y,Z	78	481,482,483	80,81,82
79	Y,Z	80	488,489	83,84
81	X,Y,Z	82	418,419,420	85,86,87
82	Y,Z	83	391,392	88,89
84	X,Y	74	294,295	90,91
86	Z	181	43	92
87	X,Y	184	7,8	93,94
88	Z	115	166	95
92	Y	10002	74	96
97	Z	7802	362	97
98	Y,Z	7902	479,480	98,99
99	Y,Z	8402	215,216	100,101

[a] X,Y,Z refer to axial, lateral, vertical deflections.

Summary of Improved Matrices

The mass and stiffness matrices were improved using several combinations of measured modes and with and without the frequency dependent terms in the full modes computation. Several combinations of modes were used prior to the inclusion of the 16 modes. The reason for this was to detect any sensitivities to particular combinations of modes. If, for example, one mode was nearly a linear combination of other modes (therefore not a true mode) one would expect large changes in the matrices.

Table XI summarizes the results, without frequency dependent terms, in terms of three measurements of the changes in the matrices. The first is simply the root mean square (rms) of the changes in the elements divided by the rms of the original matrix. This is not considered a meaningful measure of the changes in the mass matrix which is strongly diagonal.* The other two measures are the absolute mean ratio of the diagonal changes and the rms of the changes divided by the square root of product of the two corresponding diagonal elements. The last two are considered to be more meaningful descriptions of the changes in the matrices.

The Appendix illustrates typical lists of the 50 largest changes in the elements of the mass and stiffness matrices.

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*The rms of the elements of a diagonal matrix, A, is $\left\{\sum_{i=1}^{N} a_{ii}^2/N^2\right\}^{1/2}$ but the rms of the diagonal elements is $\left\{\Sigma\, a_{ii}^2/N\right\}^{1/2}$ which is more representative of the "average" element. A diagonal matrix having 10 elements: 1, 2, ... 10, would have an rms of 1.96 and an rms of the diagonals elements of 6.20. This effect becomes greater as the matrix order increases.

TABLE XI. - MATRIX CHANGES WITHOUT FREQUENCY TERMS IN FULL MODE COMPUTATION

Modes (a)	Description	Mass matrix changes, % (b)	(c)	(d)	Stiffness matrix changes, % (b)	(c)	(d)
7,8,10,15	All vertical	.94	.04		.11	.07	
1,3,4,9	All torsion	1.73	.13		.08	.05	
2,5,11,12	All lateral	.45	.02		.09	.05	
All above		6.83	.46		.16	.15	
6,13,14,16	Misc. coupled	2.10	.11	.13	.10	.07	.08
1 - 4	By frequency	1.64	.11	.11	.08	.04	.08
5 - 8		2.02	.08	.10	.11	.06	.11
9 - 12		.40	.02	.03	.08	.05	.05
13 - 16		.99	.06	.06	.11	.08	.08
1 - 16	All modes	14.74	1.24	.95	.20	.24	.18

[a] See Table IX

[b] rms (changes)/rms (original elements)

[c] mean absolute ratio of diagonal changes/original diagonal

[d] rms change $(i,j)/(\text{original } (i,i) \times (j,j))^{1/2}$

Note that the data presented in Table XI is in percent. In the worst case, the most meaningful measure of the changes,(d), indicates a change of less than 1% in the mass matrix and less than .2% in the stiffness matrix. The changed matrices will have as eigensolutions the first 16 frequencies and mode shapes which were measured in test. This last statement is based on the analysis and has been verified for simple models but has not actually been verified for this data.

Some of the above conditions were analyzed using the frequency terms in the full mode computation. These cases are summarized in Table XII.

TABLE XII. - MATRIX CHANGES WITH FREQUENCY TERMS IN FULL MODE COMPUTATION

Modes (a)	Description	Mass matrix changes, % (b)	(c)	(d)	Stiffness matrix changes, % (b)	(c)	(d)
1 - 4	Low frequencies	1.20	.03	.06	.06	.03	.07
5 - 8	Higher frequencies	57.8	2.23	2.23	.07	.10	.12
9 - 12	Higher frequencies	.64	.04	.04	.07	.04	.06
13 - 16	Highest frequencies	1.63	.06	.10	.06	.02	.06
5,7,8,9	6 omitted	56.3	1.9	1.9	.09	.08	.10
5,6,7,9	8 omitted	4.2	.23	.24	.06	.03	.08

[a] See Table IX

[b,c,d] See Table XI

The data in Table XII requires some discussion. When the frequency dependent terms are included in the analysis, the unmeasured modal displacements will differ from those computed without these terms. However, those actually measured are not changed in either case. Thus, either improved model will predict the measured modes even though it is expected that the frequency dependent analysis will give a better model.

In general, the results in Table XII are of similar order of magnitude as Table XI, except for the case using modes 5 - 8 where the ratios of the rms values is 57.8% with frequency dependence and 2.02% without. Two further cases were run to try to determine if a particular mode is responsible for this large change. The last two cases in this table indicates that when mode 8 is omitted, the results are more compatible with the other data obtained. The frequency dependent algorithm should yield more accurate full mode shapes, thus, while this condition requires further investigation, it is possible that measured mode 8 is not a truly independent mode but is in reality a linear combination of other modes.* It is interesting that this effect did not show up in the simpler analysis without frequency dependent terms.

Sensitivity Studies

Measured mode shapes and natural frequencies cannot be precise. To qualitatively examine the effects of these errors in this process, two sets of data were synthesized. First, since frequencies are usually more accurately measured than mode shapes, all the mode shapes were rounded to 1 significant figure.

This data is shown in Table XIII. Note that the changes in the matrices are quite consistent with these obtained using the precise measured modes (Tables XI, XII).

TABLE XIII. - MATRIX CHANGES WITH 1 SIGNIFICANT FIGURE IN MEASURED MODE SHAPES.

Modes	Description	Mass matrix change %			Stiffness matrix change, %		
(a)		(b)	(c)	(d)	(b)	(c)	(d)
1 - 4	Without freq. terms	2.15	.14	.14	.09	.04	.09
5 - 8	"	1.93	.08	.10	.11	.06	.09
9 - 12	"	.38	.02	.03	.11	.08	.08
13 - 16	"	1.02	.05	.06	.13	.10	.11
5 - 8	With freq. terms	53.5	2.01	2.00	.08	.11	.13

[a]See Table IX
[b,c,d]See Table XI

*Information supplied related to the testing indicates that mode 8 has a predominately free-free bending shape and was quite difficult to isolate as such in the test.

The second set of data used the 1 significant figure mode shapes with a set of arbitrarily modified frequencies. These changes are of the order of ± 5% but were arranged to make sets of frequencies close together and in one case (mode 10) reversed the order of the modes. The modified frequencies are shown in Table XIV and the results of the AMI analysis are given in Table XV.

TABLE XIV. - ARBITRARY FREQUENCY CHANGES

Mode	Test frequency, Hz	Arbitrary changed frequency, Hz	Change, %
1	14.18	13.53	-4.5
2	21.50	22.28	+3.6
3	23.26	22.60	-2.8
4	24.36	23.08	-5.2
5	24.63	26.10	+6.0
6	26.29	26.42	0
7	27.66	26.73	-3.4
8	28.67	27.06	-5.6
9	29.22	30.24	+3.4
10	33.42	35.01	+4.8
11	34.49	32.63	-5.4
12	35.37	33.42	-5.5

TABLE XV. - MATRIX CHANGES WITH APPROXIMATE MODE SHAPES AND FREQUENCIES.

Modes (a)	Description	Mass matrix changes, % (b)	(c)	(d)	Stiffness matrix changes, % (b)	(c)	(d)
1 - 4	Without freq. terms	2.15	.14	.14	.09	.04	.09
5 - 8	" " "	1.93	.08	.10	.12	.06	.09
9 - 12	" " "	.38	.02	.03	.10	.08	.08

[a] Frequencies given in Table XII, mode shapes to 1 significant figure.

[b,c,d] See Table XI

Relationship to Structure

There is no physical reason for assuming that the changes in the matrices have any specific physical significance since the changes are only one set (a minimum) out of an infinity which will make the analytical model predict the test results. However, if the analytical model had errors at certain grid points, one might expect these grid points to be significantly changed.

An examination of the 50 largest errors in the first four cases of Table XI shows that several grid points appear more often than one would expect by pure chance. This information is shown in Table XVI.

TABLE XVI. - GRID POINTS APPEARING MOST OFTEN IN 50 LARGEST CHANGES(a)

Grid Point (b)	DOF (b)	4 Vertical (c)	4 Torsion (d)	4 Lateral (e)	12 Modes (f)
101	X,Y,Z	26	40	19	25
102	X,Y,Z	41	3	18	46
37	Y	19	22		17
8	Y			22	5
10002	Y		20		

(a) Values in table are the number of occurrences in the 50 largest changes.

(b) See Table X

(c),(d),(e),(f) Lines 1,2,3,4 of Table XI.

It should be emphasized that one should be extremely cautious in drawing conclusions from such data. However, it certainly would be appropriate to examine the modeling and test structure at such grid points to seek evidence of modeling or test error. Potential sources of such errors could be: errors in material properties; incorrect coordinates; key punch error; inproper location of transducer; incorrect calibration; electronic failure; structure loaded to nonlinear range.

Application to Reduced Model

There is another application of the AMI method which does not use test data but, instead, uses the computed frequencies and mode shapes of the full model to improve a reduced model. It is well known that reduced models do not have the same dynamic characteristics as the full models. This procedure modifies the reduced model so that it will duplicate specified frequencies and mode shapes of the complete analytical model.

In the case tested, modes 1, 2, 3, 4, 5, 6, and 8 which were computed by the 1097 degree of freedom model (Table IX) were used to correct the 508 degree of freedom reduced model. The changes, in terms of the ratios of the rms values were: mass matrix, 1.4%; stiffness matrix, 1.5%.

CONCLUDING COMMENTS

An analytical method has been developed which computes minimum changes in an analytical model of a structure to make it agree with modal test data. The method is direct and does not involve successive iteration.

The method was implemented on a computer and applied to a realistic structure. The changes in the model which were computed were generally well within the expected uncertainties in the analytical model.

Careful examination of the effects of using different combinations of modes may uncover poor interpretation of test modes. Careful examination of the elements of the model which change most may uncover inadequacies in the analytical model or in the test data.

A brief study of arbitrary changes in mode shapes and frequencies disclosed no numerical sensitivities.

The application of this method to improve reduced models may lead to better small dynamic analytical models of structures.

The results of this study are considered to have been very successful and a continuation of this area of research and applications to other structures is highly recommended.

APPENDIX

Sample Largest Change Output

Mass Matrix

Modes 1, 2, 3, 4 without frequency terms

50 Largest Changes

	Row	Col	Orig	Delta	Ratio
1	285	285	1.152E-00	1.134E-02	9.85E-03
2	285	96	7.654E-03	1.020E-02	1.33E 00
3	96	96	6.337E-02	7.265E-03	1.15E-01
4	285	139	3.208E-03	5.640E-03	1.76E 00
5	287	285	4.417E-07	-5.551E-03	-1.26E 04
6	285	193	9.774E-03	5.016E-03	5.13E-01
7	285	100	5.980E-06	4.819E-03	8.06E 02
8	393	285	-1.214E-07	4.646E-03	-3.83E 04
9	384	285	-1.598E-04	4.646E-03	-2.91E 01
10	285	123	3.313E-02	4.605E-03	1.39E-01
11	139	96	7.776E-05	4.053E-03	5.21E 01
12	333	285	1.609E-03	3.894E-03	2.42E 00
13	384	96	2.425E-05	3.823E-03	1.58E 02
14	97	96	2.940E-09	3.790E-03	1.29E 06
15	193	96	8.172E-05	3.569E-03	4.37E 01
16	123	96	1.131E-02	3.463E-03	3.06E-01
17	285	98	6.839E-06	3.436E-03	5.02E 02
18	285	99	1.045E-05	-3.427E-03	-3.28E 02
19	333	96	-1.232E-04	3.370E-03	-2.74E 01
20	391	96	5.973E-06	-3.347E-03	-5.60E 02
21	285	97	2.059E-02	3.321E-03	1.61E-01
22	391	100	7.653E-03	-3.242E-03	-4.24E-01
23	100	100	6.337E-02	-3.206E-03	-5.06E-02
24	287	96	-1.079E-03	-3.175E-03	2.94E 00
25	483	285	6.323E-02	3.124E-03	4.94E-02
26	99	96	2.351E-11	-3.007E-03	-1.28E 08
27	483	96	2.244E-06	2.960E-03	1.32E 03
28	403	285	6.043E-05	2.953E-03	4.89E 01
29	285	258	3.026E-07	-2.893E-03	-9.56E 03
30	285	279	4.308E-07	-2.868E-03	-6.66E 03
31	285	35	-3.917E-06	-2.810E-03	7.17E 02
32	398	285	-3.662E-07	-2.743E-03	7.49E 03
33	335	285	-3.561E-08	-2.706E-03	7.60E 04
34	285	48	-1.281E-07	-2.701E-03	2.11E 04
35	285	264	-1.243E-08	-2.678E-03	2.15E 05
36	358	285	1.130E-06	-2.632E-03	-2.33E 03
37	100	97	2.487E-11	2.619E-03	1.05E 08
38	285	81	-1.316E-02	-2.595E-03	1.97E-01
39	285	21	1.544E-05	-2.572E-03	-1.67E 02
40	285	18	-6.726E-06	2.557E-03	-3.80E 02
41	285	179	-3.357E-08	2.524E-03	-7.52E 04
42	325	285	2.648E-06	2.498E-03	9.43E 02
43	285	248	-8.519E-06	2.492E-03	-2.93E 02
44	285	28	4.125E-06	2.483E-03	6.02E 02
45	372	285	-1.907E-08	2.416E-03	-1.27E 05
46	285	253	1.282E-03	2.407E-03	1.88E 00
47	285	33	7.018E-06	-2.384E-03	-3.40E 02
48	393	96	-2.510E-08	2.358E-03	-9.39E 04
49	403	96	-6.782E-06	2.353E-03	-3.47E 02
50	302	285	7.495E-06	2.352E-03	3.14E 02

Stiffness Matrix

Mods 1, 2, 3, 4 without frequency terms

50 largest changes

	Row	Col	Orig	Delta	Ratio
1	392	8	-3.924E 01	1.777E 04	-4.53E 02
2	286	8	7.459E 00	1.699E 04	2.28E 03
3	43	8	-4.352E 01	1.254E 04	-2.88E 02
4	411	8	-9.045E 02	1.174E 04	-1.30E 01
5	404	8	-1.101E 02	1.120E 04	-1.02E 02
6	432	8	-2.635E 02	1.118E 04	-4.24E 01
7	334	8	4.157E 00	1.097E 04	2.64E 03
8	371	8	5.505E 01	1.093E 04	1.99E 02
9	8	8	1.233E 06	-1.079E 04	-8.75E-03
10	285	8	-1.625E 02	-1.069E 04	6.58E 01
11	46	8	-1.360E 01	1.024E 04	-7.53E 02
12	392	6	2.494E 02	-9.884E 03	-3.96E 01
13	286	6	3.414E 03	-9.772E 03	-2.86E 00
14	41	8	-4.839E-02	9.676E 03	-2.00E 05
15	401	8	-3.870E-02	9.281E 03	-2.40E 05
16	264	8	7.276E 00	9.032E 03	1.24E 03
17	392	47	1.363E 06	8.947E 03	6.57E-03
18	392	86	-3.100E 01	-8.911E 03	2.87E 02
19	286	47	3.633E 04	8.441E 03	2.32E-01
20	286	86	1.669E 00	-8.424E 03	-5.05E 03
21	392	45	-9.298E 05	-8.399E 03	9.03E-03
22	392	48	-1.910E 04	8.299E 03	-4.35E-01
23	392	4	7.479E 00	-8.168E 03	-1.09E 03
24	285	61	-3.065E 03	8.015E 03	-2.62E 00
25	286	48	2.445E 05	7.943E 03	-3.25E-02
26	286	45	-5.603E 03	-7.890E 03	1.41E 00
27	286	4	-3.934E 01	-7.570E 03	1.92E 02
28	180	6	-3.083E 01	-7.156E 03	2.32E 02
29	86	43	-6.144E-01	-7.016E 03	1.14E 04
30	344	8	-7.547E 01	6.900E 03	-9.14E 01
31	411	86	1.650E 01	-6.893E 03	-4.18E 02
32	404	86	-5.094E 00	-6.875E 03	1.35E 03
33	432	86	-1.286E 01	-6.857E 03	5.33E 02
34	48	48	9.561E 06	6.748E 03	7.06E-04
35	392	76	1.674E 00	6.694E 03	4.00E 03
36	410	6	-2.628E 02	-6.667E 03	2.54E 01
37	43	6	-4.076E 02	-6.609E 03	1.62E 01
38	411	6	-9.256E 01	-6.569E 03	7.10E 01
39	284	8	-8.510E 00	6.524E 03	-7.67E 02
40	96	69	8.886E-06	6.517E 03	7.33E 08
41	47	43	1.094E 02	6.473E 03	5.92E 01
42	100	18	-8.888E-06	-6.472E 03	7.28E 08
43	96	71	8.966E-06	-6.370E 03	-7.10E 08
44	286	76	-2.950E 01	6.322E 03	-2.14E 02
45	411	47	4.613E 03	6.285E 03	1.36E 00
46	48	8	3.493E 00	6.285E 03	1.80E 03
47	285	71	3.647E 01	-6.283E 03	-1.72E 02
48	45	43	-8.323E 00	-6.275E 03	7.54E 02
49	285	60	1.774E 03	6.269E 03	3.53E 00
50	371	6	1.577E 01	-6.192E 03	-3.93E 02

Mass Matrix

Modes 1, 2, 3, 4 (1 significant figure) without frequency terms

	Row	Col	Orig	Delta	Ratio
1	285	285	-1.152E 00	1.408E-02	1.22E-02
2	285	96	7.654E-03	1.227E-02	1.60E 00
3	96	96	6.337E-02	8.931E-03	1.41E-01
4	287	285	4.417E-07	-8.768E-03	-1.99E 04
5	393	285	-1.214E-07	7.844E-03	-6.46E 04
6	285	139	3.208E-03	6.817E-03	2.13E 00
7	285	100	5.980E-06	6.092E-03	1.02E 03
8	285	193	9.774E-03	6.024E-03	6.16E-01
9	285	123	3.313E-02	5.693E-03	1.72E-01
10	384	285	-1.598E-04	5.516E-03	-3.45E 01
11	287	96	-1.079E-03	-5.316E-03	4.93E 00
12	285	99	1.045E-05	-5.242E-03	-5.01E 02
13	139	96	7.776E-05	4.983E-03	6.41E 01
14	391	96	5.973E-06	-4.821E-03	-8.07E 02
15	333	285	1.609E-03	4.720E-03	2.93E 00
16	97	96	2.940E-09	4.586E-03	1.56E 06
17	384	96	2.425E-05	4.521E-03	1.86E 02
18	285	97	2.059E-02	4.463E-03	2.17E-01
19	393	96	-2.510E-08	4.447E-03	-1.77E 05
20	193	96	8.172E-05	4.388E-03	5.37E 01
21	99	96	2.351E-11	-4.358E-03	-1.85E 08
22	123	96	1.131E-02	4.289E-03	3.79E-01
23	391	100	7.653E-03	-4.164E-03	-5.44E-01
24	391	287	1.095E-09	4.151E-03	3.79E 06
25	285	98	6.839E-06	4.129E-03	6.04E 02
26	391	285	3.574E-04	-4.064E-03	-1.14E 01
27	493	285	6.323E-02	4.013E-03	6.35E-02
28	333	96	-1.232E-04	4.000E-03	-3.25E 01
29	372	285	-1.907E-08	3.992E-03	-2.09E 05
30	335	285	-3.561E-08	-3.952E-03	1.11E 05
31	393	391	-1.989E-04	-3.930E-03	1.98E 01
32	100	100	6.337E-02	-3.701E-03	-5.84E-02
33	287	97	-1.731E-03	-3.629E-03	2.10E 00
34	483	96	2.244E-06	3.619E-03	1.61E 03
35	285	258	3.026E-07	-3.598E-03	-1.19E 04
36	285	279	4.308E-07	-3.566E-03	-8.28E 03
37	285	35	-3.917E-06	-3.501E-03	8.94E 02
38	285	254	2.004E-08	-3.488E-03	-1.74E 05
39	285	81	-1.316E-02	-3.473E-03	2.64E-01
40	398	285	-3.662E-07	-3.372E-03	9.21E 03
41	393	97	-1.045E-07	3.355E-03	-3.21E 04
42	412	285	-2.042E-09	3.352E-03	-1.64E 06
43	285	42	4.738E-08	-3.295E-03	-6.96E 04
44	285	18	-6.726E-06	3.267E-03	-4.86E 02
45	285	33	7.018E-06	-3.271E-03	-4.60E 02
46	285	264	-1.243E-08	-3.226E-03	2.60E 05
47	384	287	3.185E-10	-3.185E-03	-1.00E.07
48	285	248	-8.519E-06	3.173E-03	-3.72E 02
49	403	285	6.043E-05	3.169E-03	5.24E 01
50	100	97	2.487E-11	3.169E-03	1.27E 08

Stiffness Matrix

Modes 1, 2, 3, 4 (1 significant figure) without frequency terms

50 Largest Changes

	Row	Col	Orig	Delta	Ratio
1	392	8	-3.924E 01	1.778E 04	-4.53E 02
2	286	8	7.459E 00	1.747E 04	2.34E 03
3	43	8	-4.352E 01	1.258E 04	-2.89E 02
4	411	8	-9.045E 02	1.177E 04	-1.30E 01
5	8	8	1.233E 06	-1.175E 04	-9.53E-03
6	100	45	1.534E-01	-1.175E 04	-7.66E 04
7	392	45	-9.298E 05	-1.147E 04	1.23E-02
8	404	8	-1.101E 02	1.112E 04	-1.01E 02
9	432	8	-2.635E 02	1.110E 04	-4.21E 01
10	334	8	4.157E 00	1.106E 04	2.66E 03
11	371	8	5.505E 01	1.102E 04	2.00E 02
12	100	47	-2.245E-01	1.059E 04	-4.72E 04
13	392	47	1.363E 06	1.047E 04	7.68E-03
14	285	8	-1.625E 02	-1.003E 04	6.17E 01
15	393	45	-3.296E 07	-9.984E 03	3.03E-04
16	41	8	-4.839E-02	9.887E 03	-2.04E 05
17	264	8	7.276E 00	9.725E 03	1.34E 03
18	285	61	-3.065E 03	9.703E 03	-3.17E 00
19	100	42	-9.024E-04	9.652E 03	-1.07E 07
20	287	45	-1.879E 02	9.482E 03	-5.05E 01
21	46	8	-1.360E 01	9.440E 03	-6.94E 02
22	401	8	-3.870E-02	9.215E 03	-2.38E 05
23	392	4	7.479E 00	-9.140E 03	-1.22E 03
24	393	47	-4.423E 04	9.132E 03	-2.06E-01
25	286	6	3.414E 03	-9.111E 03	-2.67E 00
26	96	45	9.029E-04	-9.095E 03	-1.01E 07
27	432	45	4.552E 02	-8.958E 03	-1.97E 01
28	404	45	-5.441E 00	-8.931E 03	1.64E 03
29	392	86	-3.100E 01	-8.749E 03	2.82E 02
30	392	6	2.494E 02	-8.727E 03	-3.50E 01
31	287	47	1.149E 03	-8.668E 03	-7.54E 00
32	286	4	-3.934E 01	-8.667E 03	2.20E 02
33	48	8	3.493E 00	8.515E 03	2.44E 03
34	286	45	-5.603E 03	-8.390E 03	1.50E 00
35	180	6	-3.183E 01	-8.382E 03	2.72E 02
36	47	43	1.094E 02	8.340E 03	7.62E 01
37	432	47	-6.627E 02	8.203E 03	-1.24E 01
38	404	47	-8.637E 00	8.178E 03	-9.47E 02
39	96	47	-6.049E-03	8.156E 03	-1.35E 06
40	411	45	-3.151E 03	-8.097E 03	2.57E 00
41	285	60	1.774E 03	7.998E 03	4.51E 00
42	286	86	1.669E 00	-7.813E 03	-4.68E 03
43	45	45	3.716E 07	-7.790E 03	-2.10E-04
44	393	42	-1.875E 02	7.772E 03	-4.14E 01
45	287	42	-3.296E 07	-7.724E 03	2.34E-04
46	286	47	3.633E 04	7.675E 03	2.11E-01
47	45	43	-8.323E 00	-7.579E 03	9.11E 02
48	100	44	-9.491E-04	-7.531E 03	7.94E 06
49	96	42	-1.535E-01	7.459E 03	-4.86E 04
50	100	18	-8.888E-06	-7.423E 03	8.35E 08

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16. Abstract

A method is developed and illustrated which finds minimum changes in analytical mass and stiffness matrices to make them consistent with a set of measured normal modes and natural frequencies. The corrected model will be an improved base for studies of physical changes, changes in boundary conditions, and for prediction of forced responses. Features of the method are: efficient procedures not requiring solutions of the eigenproblem; the model may have more degrees of freedom than the test data; modal displacements at all the analytical degrees of freedom are obtained; the frequency dependence of the coordinate transformations are properly treated.

The method has been applied to the LDEF structure using several combinations of modes. The changes found were generally within the limits of the uncertainties in the analytical model.

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