NASA CR- 166,220

JOINT INSTITUTE FOR AERONAUTICS AND ACOUSTICS

NASA
National Aeronautics and Space Administration
Ames Research Center

NASA-CR-166220
19810021385

Stanford University

JIAA TR-25

A NOTE ON SOUND RADIATION FROM DISTRIBUTED SOURCES

H. Levine

STANFORD UNIVERSITY
Department of Aeronautics and Astronautics
Stanford, California 94305

May 1979

JIAA TR-25

A NOTE ON SOUND RADIATION FROM DISTRIBUTED SOURCES

HAROLD LEVINE

MAY 1979

The work here presented has been supported partially by
The National Aeronautics and Space Administration under
Grant NASA 2007.

N81-29923 #
SU-JIAA-TR-25

A NOTE ON SOUND RADIATION FROM DISTRIBUTED SOURCES

Harold Levine
Joint Institute for Aeronautics and Acoustics
Department of Aeronautics and Astronautics
Stanford University, Stanford, California 94305

Summary

The power output from a normally vibrating strip radiator is expressed in alternative general forms, one of these being chosen to refine and correct some particular estimates given by Heckl for different numberical ratios of strip width to wave length. An exact and explicit calculation is effected for sinusoidal velocity profiles when the strip width equals an integer number of half wave lengths.

The aspects of radiation from different acoustical sources and moving structural elements in particular are a matter of importance to practical ends such as control and also to the sharpening of conceptual views on the subject. Analytical representations that are effectively suited for wide ranges of the characteristic (or scale) parameters may be found only in connection with simple and idealized source models; and fully integrated or self-consistent calculations which account for the interaction between a source and its surroundings present a considerably more difficult undertaking. The purpose of this note is to correct and improve some estimates offered by Heckl, in the treatise "Structure-borne Sound" (1973), for the radiation associated with bending waves in finite plates. Specifically, the problem envisaged by Heckl postulates a normal velocity distribution everywhere on a full plane, or a line when the dependence on one of the coordinates is suppressed; and admits an explicitly realizable outgoing wave solution throughout the half-space, or half-plane, facing the source. The total radiated power can thus be exhibited in a quadrature or integral fashion and its measure sought in conjunction with assigned velocity profiles.

Let a time-periodic velocity potential involving two coordinate variables,

$$\Psi(x,y,t) = \mathrm{Re}\{\psi(x,y)e^{-i\omega t}\} ,$$

satisfy the homogeneous linear wave equation in a medium with constant acoustic speed c and equilibrium density ρ ; then a boundary value problem in the half-plane $-\infty < x < \infty$, $y > 0$ is uniquely posed by the differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\psi(x,y) = 0\ , \quad k = \omega/c \tag{1}$$

along with the specifications

$$-\left.\frac{\partial\psi}{\partial y}\right|_{y=0} = \begin{cases} v(x)\ , & |x| < \ell/2 \\ 0\ , & |x| > \ell/2 \end{cases} \tag{2}$$

and

$$\psi \simeq f(\theta)\,\frac{e^{ikr}}{\sqrt{kr}}\ , \ r = \sqrt{x^2+y^2} \to \infty\ , \ x = r\cos\theta\ , \ y = r\sin\theta\ . \tag{3}$$

Definite integral forms of solution to this ummixed Neumann type boundary value problem are given by

$$\begin{aligned}\psi(x,y) &= \frac{i}{2}\int_{-\ell/2}^{\ell/2} H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)\, v(x')\, dx' \\ &= \frac{i}{2}\int_{-\infty}^{\infty} \frac{\exp\{i\zeta x + i\sqrt{k^2-\zeta^2}\,y\}}{\sqrt{k^2-\zeta^2}}\,\bar{v}(\zeta)\, d\zeta\ , \ y \geq 0\ , \ \arg\sqrt{k^2-\zeta^2} \geq 0\end{aligned} \tag{4}$$

wherein $H_0^{(1)}$ denotes a Hankel function of the first kind and

$$\bar{v}(\zeta) = \int_{-\ell/2}^{\ell/2} e^{-i\zeta x}\, v(x)\, dx \tag{5}$$

represents a complex Fourier transform of the velocity or source function.

Since the time average energy flux vector (* symbolizing complex conjugation

$$\vec{N} = -\frac{1}{2}\rho\omega\ \mathrm{Im}\{\psi\nabla\psi^*\}$$

is divergenceless throughout the half-plane $y > 0$, the total radiated power P can be expressed as an integral of the radial component of $\vec{N}$ over all directions in the far field or, equivalently, as an integral of the y component of $\vec{N}$ along the source line itself; from the latter follow, on invoking (2) and (4), the alternative versions

$$P = -\frac{1}{2}\rho\omega \int_{-\ell/2}^{\ell/2} \mathrm{Im}\{\psi(x,0)\frac{\partial}{\partial y}\psi^*(x,0)\}\,dx$$

$$= \frac{\rho\omega}{4}\,\mathrm{Re}\int_{-\ell/2}^{\ell/2} v(x)H_0^{(1)}(k|x-x'|)\,v^*(x')\,dx\,dx' \tag{6}$$

$$= \frac{\rho\omega}{4}\int_{-k}^{k}\frac{|\bar{v}(\zeta)|^2}{\sqrt{k^2-\zeta^2}}\,d\zeta\ .$$

Heckl chooses the coordinate function

$$v(x) = v_n \sin\frac{n\pi}{\ell}\left(x+\frac{\ell}{2}\right)\ ,\ n = 1,2,\ldots\ ,\ v_n = v_n^*\ ,\ -\frac{\ell}{2} < x < \frac{\ell}{2} \tag{7}$$

to simulate the modal velocity profiles of a simply supported plate and substitutes the appertaining transform function

$$|\bar{v}(\zeta)|^2 = v_n^2\left(\frac{2\pi n\ell}{\zeta^2\ell^2-n^2\pi^2}\right)^2\sin^2\left(\frac{\zeta\ell-n\pi}{2}\right) \tag{8}$$

into the final version of (6); he thus arrives at a single integral expression

$$P = \rho\omega(\pi v_n)^2\int_{-k}^{k}\frac{n^2\ell^2}{(\zeta^2\ell^2-n^2\pi^2)^2}\sin^2\left(\frac{\zeta\ell-n\pi}{2}\right)\frac{d\zeta}{\sqrt{k^2-\zeta^2}} \tag{9}$$

for the power output per unit length of the plate parallel to its edges, and remarks that "this integral unfortunately cannot be evaluated in closed form; therefore, several approximations will be employed here." The latter, differentiated by magnitudes of the characteristic parameter $k\ell$, include a first estimate

$$P \doteq \frac{1}{2}\rho\omega\, v_n^2\,(\ell/n\pi)^2\quad ,\quad k\ell \ll n\pi \tag{10}$$

which is achieved through a preliminary averaging of the function (8) prior to substitution in (6). Another estimate

$$P \doteq \rho c \ell v_n^2 \quad , \quad k\ell >> n\pi \tag{11}$$

is given, in keeping with appropriate views about the relative importance of contributions to the integral (9) near $\zeta\ell = \pm n\pi$; and it is merely asserted, as regards the last estimate, that

$$P \doteq \frac{1}{6}\,\rho c \ell v_n^2 (k\ell/\pi)^{1/2} \quad , \text{ for } \quad k\ell = n\pi \tag{12}$$

with a reference to Lyon and Maidanik (1962).

If the second of the power representations (6) be adopted, on the other hand, and made explicit with the source velocity (7), the consequent double integral

$$P = \frac{1}{8}\,\rho\omega v_n^2 \int_{-\ell/2}^{\ell/2} [\cos\frac{n\pi}{\ell}(x-x') - \cos\frac{n\pi}{\ell}(x+x'+\ell)]J_0(k|x-x'|)\,dx\,dx' \quad ,$$

is reducible to a single integral after effecting the change of variables

$$\xi = \frac{x-x'}{\sqrt{2}} \quad , \quad \eta = \frac{x+x'}{\sqrt{2}} \; ;$$

and this operation yields a replacement for Heckl's expression (9), namely

$$P = \frac{1}{4}\,\rho\omega(v_n\ell)^2 \int_0^1 J_0(k\ell\zeta)[(1-\zeta)\cos n\pi\zeta + \frac{\sin n\pi\zeta}{n\pi}]\,d\zeta \tag{13}$$

or

$$P = \frac{1}{4}\,\rho c \ell v_n^2 \int_0^{k\ell} J_0(\mu)[(1-\frac{\mu}{k\ell})\cos\frac{n\pi\mu}{k\ell} + \frac{1}{n\pi}\sin\frac{n\pi\mu}{k\ell}]\,d\mu \tag{14}$$

involving the zero order Bessel function J_0 . The above integrals are preferable to (9) insofar as they can be estimated with facility and precision when $k\ell << n\pi$ or $k\ell >> n\pi$; and permit an exact evaluation if $k\ell = n\pi$, i.e.,

$$\begin{aligned} P &= \frac{1}{4}\,\rho c v_n^2 (n\pi/k) \int_0^{n\pi} J_0(\mu)[(1-\frac{\mu}{n\pi})\cos\mu + \frac{1}{n\pi}\sin\mu]\,d\mu \\ &= \frac{\rho c}{6k}(-1)^n (n\pi v_n)^2 \{J_0(n\pi) - \frac{2}{n\pi}J_1(n\pi)\} \end{aligned} \tag{15}$$

where J_1 denotes the first order Bessel function. It is a simple matter to recover the estimate (12) by disregarding the second term of (15) and utilizing the asymptotic approximation

$$J_0(n\pi) \sim (2/n\pi)^{1/2} \cos(n\pi - \frac{\pi}{4}) = (-1)^n (n\pi)^{-1/2} \ , \ n\pi >> 1$$

in the first term.

Heckl's prediction (11), which is independent of the wave length $\lambda = 2\pi/k$ and representative of the leading term in a short wave expansion, receives support from the analysis of (14); however, the estimate (10) does not adequately distinguish between even/odd symmetry of the velocity (source) profile, or parity of the integer n, at long wave lengths. To elucidate the latter aspect in mathematical fashion, a power series development for the Bessel function in (13) is indicated, with rapid convergence of the resulting (integrated) terms when $k\ell << n\pi$ The measure of power obtained on setting $J_0(x) = 1$ for $x << 1$,

$$\begin{aligned} P &\doteq \frac{1}{4} \rho\omega(v_n \ell)^2 \int_0^1 [(1-\zeta) \cos n\pi\zeta + \frac{1}{n\pi} \sin n\pi\zeta] \, d\zeta \\ &= \frac{1}{2} \rho\omega(v_n \ell)^2 \cdot \frac{1}{n\pi} \int_0^1 \sin n\pi\zeta \, d\zeta \\ &= \frac{1}{2} \rho\omega(v_n \ell)^2 \frac{1 - \cos n\pi}{(n\pi)^2} \ , \quad k\ell << n\pi \end{aligned}$$

possesses an order of magnitude comparable with that in (11), so long as n is odd and the corresponding velocity is symmetric relative to the midpoint of its range. If n assumes an even value and the two term approximation $J_0(x) \doteq 1 - \frac{1}{4} x^2$ be employed it turns out that

$$P \doteq \frac{1}{8} \rho\omega(v_n \ell)^2 (k\ell/n\pi)^2 \ , \ k\ell << n\pi \ , \ n \text{ even}$$

which implies a significantly lesser output from asymmetrical sources at long wave lengths. Thus, the complementary estimates

$$P \doteq \begin{cases} \rho\omega(v_n \ell/n\pi)^2 & , \quad n = 1,3,\ldots \\ \frac{1}{8}\,\rho\omega(v_n \ell/n\pi)^2 (k\ell)^2 & , \quad n = 2,4,\ldots \end{cases} \qquad k\ell \ll n\pi \tag{16}$$

supersede the single one proposed by Heckl*, and their refinements are readily secured through the inclusion of additional terms in a Bessel function polynomial approximation.

The exact formula (15) which manifests the relative predominance of power output when the constructive interference condition $k\ell = n\pi$ holds, is directly forthcoming once the definite integrals

$$\int_0^x J_0(\mu)\ \sin(x-\mu)\ d\mu = xJ_1(x)$$

and

$$\int_0^x \mu J_0(\mu)\ \sin(x-\mu)\ d\mu = \frac{1}{3}\,x^3 J_1(x)$$

are available. A power estimate on the basis of (14), appropriate to the short wave lengths specified by the inequality

$$\varepsilon = n\pi/k\ell \ll 1\ ,$$

rests on those of the functions

$$K(x) = \int_0^x J_0(\mu)\ (x-\mu)\ \cos\varepsilon\mu\ d\mu$$

and

$$L(x) = \int_0^x J_0(\mu)\ \sin\varepsilon\mu\ d\mu$$

*It is apparent from Heckl's integral (9) that the distinction between the symmetry cases originates in the trigonometric factor $\sin^2\left(\frac{\zeta\ell - n\pi}{2}\right)$, which acquires the forms $\cos^2(\zeta\ell/2)$ and $\sin^2(\zeta\ell/2)$ according as n is odd or even; and these are evidently of different magnitudes at small values of $\zeta\ell$.

for $x \to \infty$. Taking account of the representation

$$J_0(x) = \frac{2}{\pi} \int_1^\infty \frac{\sin \zeta x}{\sqrt{\zeta^2-1}} \, d\zeta$$

it can be demonstrated that

$$K(x) = \frac{x}{\sqrt{1-\varepsilon^2}} + \mathrm{Re}\, \frac{i}{\pi} \int_1^\infty \left(\frac{e^{ix(\zeta+\varepsilon)}}{(\zeta+\varepsilon)^2} + \frac{e^{ix(\zeta-\varepsilon)}}{(\zeta-\varepsilon)^2} \right) \frac{d\zeta}{\sqrt{\zeta^2-1}}$$

and

$$L(x) = \mathrm{Re}\, \frac{i}{\pi} \int_1^\infty \left(\frac{e^{ix(\zeta+\varepsilon)}}{\zeta+\varepsilon} - \frac{e^{ix(\zeta-\varepsilon)}}{\zeta-\varepsilon} \right) \frac{d\zeta}{\sqrt{\zeta^2-1}}$$

where the principal contributions to the latter integrals stem from the neighborhood of $\zeta = 1$ if $x >> 1$. The deductions made in accord with the latter circumstance, viz.

$$K(x) \simeq \frac{x}{\sqrt{1-\varepsilon^2}} - \frac{1}{\sqrt{2\pi x}} \left\{ \frac{\cos[x(1+\varepsilon)-\pi/4]}{(1+\varepsilon)^2} + \frac{\cos[x(1-\varepsilon)-\pi/4]}{(1-\varepsilon)^2} \right\}$$

and $\qquad x(1-\varepsilon) >> 1$

$$L(x) \simeq -\frac{1}{\sqrt{2\pi x}} \left\{ \frac{\cos[x(1+\varepsilon)-\pi/4]}{1+\varepsilon} - \frac{\cos[x(1-\varepsilon)-\pi/4]}{1-\varepsilon} \right\} ,$$

underlie the estimate

$$P \simeq \frac{1}{4} \rho c \ell v_n^2 \left\{ \frac{1}{\sqrt{1-(\frac{n\pi}{k\ell})^2}} + 4\, \frac{(-1)^{n+1}}{\sqrt{2\pi}(k\ell)^{3/2}} \frac{(n\pi/k\ell)^2}{(1-(\frac{n\pi}{k\ell})^2)^2} \cos(k\ell-\pi/4) \right\} , \qquad (17)$$

$$k\ell >> n\pi$$

which surpasses the companion one furnished by Heckl.

References

1. Cremer, L and Heckl, M. Structure-borne Sound, translated and revised by E. E. Ungar, Springer-Verlag (1973), Chapter VI.

2. Lyon, R. H. and Maidanik, G. J. Acoust. Soc. Amer. 34 (1962), 623.

End of Document