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TIME-TEMPERATURE EFFECT IN ADHESIVELY BONDED JOINTS

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ABSTRACT

In this paper the viscoelastic analysis of an adhesively bonded lap joint is reconsidered. The adherends are approximated by essentially Reissner plates and the adhesive is assumed to be linearly viscoelastic. The hereditary integrals are used to model the adhesive. The problem is reduced to a system of linear integral-differential equations for the shear and the tensile stress in the adhesive. For a constant operating temperature, the equations are shown to have constant coefficients and are solved by using Laplace transforms. It is also shown that if the temperature variation in time can be approximated by a piecewise constant function, then the method of Laplace transforms could still be used to solve the problem. A numerical example is given for a single lap joint under various loading conditions and operating at temperatures 70, 100, 140 and 180°F.

1. Introduction

In most practical applications of adhesively bonded joints and epoxy-based composites the operating temperature is such that in the stress analysis of the structure any viscoelastic behavior which may be exhibited by the adhesive or the epoxy matrix may be neglected. On the other hand, depending on the time-temperature behavior of the particular epoxy, time-history of loading, and the level of accuracy required of the analysis, even at moderately low temperatures the viscoelastic effects may have to be taken into account in analyzing the structure. In particular, if the structure is subjected to a

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loading with a relatively high frequency cyclic component, temperature rise may occur due to internal heat generation and it may become necessary to investigate the time-temperature effects in the stress analysis. The objective of this paper is to study these effects by considering a relatively simple geometry. The corresponding elastic problem for the adhesively bonded joints have been studied quite extensively. Some typical models used in these studies may be found in [1-4].

2. Formulation of the Problem

For constant temperature the basic formulation of the adhesively bonded joints was considered in a previous paper [5] where it was assumed that the adhesive is a linear viscoelastic material which can be modeled by using differential operators. In this paper the hereditary integrals will be used to model the adhesive and the solution will be given for various temperatures in order to give some idea about the relative importance of the temperature changes or of the operating temperatures. It is assumed that the stress relaxation process in the viscoelastic adhesive takes place in a much slower rate than the heat conduction process in the bonded joint. Therefore, the spatial variation of temperature and its effect on the stress distribution may be neglected and it may be assumed that the adherends and the adhesive have the same temperature which is a function of the time only. Thus, the general formulation given in this section includes thermal stresses coming from differential thermal expansion only.

The problem under consideration is described in Figure 1. In this study, the adherends are treated as "plates" in which the transverse shear effects are taken into account. The adhesive is assumed to be a viscoelastic solid under in-plane deformations in which the thickness variation of stresses is neglected. The assumptions regarding the mechanical modeling of the adherends and the adhesive may be justified on the basis of the fact that generally the thickness of the adherends is one order of magnitude and that of the adhesive is approximately two orders of magnitude smaller than the characteristic "length" dimension of the joint. In the analysis it is also assumed that the dimension of the joint in z-direction is relatively large and the external loads are independent of z, meaning that the problem may be approximated by one of plane strain (Figure 1).

Referring to Figure 1, the equilibrium conditions for the plate elements representing the adherends 1 and 2 may be expressed as

$$\frac{\partial N_{1x}}{\partial x} = \tau, \quad \frac{\partial Q_{1x}}{\partial x} = \sigma, \quad \frac{\partial M_{1x}}{\partial x} = Q_{1x} - \frac{h_1+h_0}{2}\tau , \tag{1a-c}$$

$$\frac{\partial N_{2x}}{\partial x} = -\tau, \quad \frac{\partial Q_{2x}}{\partial x} = -\sigma, \quad \frac{\partial M_{2x}}{\partial x} = Q_{2x} - \frac{h_2+h_0}{2}\tau , \tag{2a-c}$$

where N_{ix}, Q_{ix}, M_{ix}, $(i=1,2)$ are respectively the membrane, the transverse shear, and the moment resultants for the adherends 1 and 2, and σ and τ are the normal and the shear stress in the adhesive.

Assume that at (the homogeneous) temperature T_0 the joint is free from temperature-induced stresses. If the temperature is raised to T at time t_1, the stress-displacement relations can be written as

$$\frac{\partial u_1}{\partial x} = C_1 N_{1x} + (1+\nu_1)\alpha_1(T-T_0)H(t-t_1),$$

$$\frac{\partial \beta_{1x}}{\partial x} = D_1 M_{1x} , \quad \frac{\partial v_1}{\partial x} + \beta_{1x} = Q_{1x}/B_1 , \tag{3a-c}$$

$$\frac{\partial u_2}{\partial x} = C_2 N_{2x} + (1+\nu_2)\alpha_2(T-T_0)H(t-t_1),$$

$$\frac{\partial \beta_{2x}}{\partial x} = D_2 M_{2x} , \quad \frac{\partial v_2}{\partial x} + \beta_{2x} = Q_{2x}/B_2 , \tag{4a-c}$$

where u_i, v_i, β_{ix}, $(i=1,2)$ are the x and y-components of the displacement vector, and the rotation of the normal at the midplane of the plates, α_i, (i=1,2) is the coefficient of thermal expansion, H(t) is the Heaviside function, and

$$C_i = \frac{1-\nu_i^2}{E_i h_i} , \quad D_i = \frac{12(1-\nu_i^2)}{E_i h_i^3} , \quad B_i = \frac{5}{6} \mu_i h_i , \quad (i=1,2) , \qquad (5a\text{-}c)$$

E_i, μ_i, and ν_i (i=1,2) being the elastic constants of the adherends.

To complete the formulation of the problem, the continuity conditions of the displacements in the region of adhesion have to be considered. Again, referring to Figure 1 the strains in the adhesive averaged over the thickness may be expressed as

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{1}{h_0} (u_1 - \frac{h_1}{2} \beta_{1x} - u_2 - \frac{h_2}{2} \beta_{2x}) ,$$

$$\varepsilon_y = \frac{1}{h_0} (v_1 - v_2) ,$$

$$\varepsilon_x = \frac{1}{2} (\frac{\partial u_1}{\partial x} - \frac{h_1}{2} \frac{\partial \beta_{1x}}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{h_2}{2} \frac{\partial \beta_{2x}}{\partial x}) , \qquad (6a\text{-}c)$$

where the remaining components of the adhesive strains are zero. The nonzero stress components in the adhesive, again averaged over the thickness, are σ_x, $\tau_{xy} = \tau$, $\sigma_y = \sigma$, and σ_z. Noting that $\varepsilon_z = 0$, in the standard fashion the hydrostatic and deviatoric components of the strain and the stress tensor for the adhesive may be defined as

$$e = (\varepsilon_x + \varepsilon_y)/3 , \quad e_{ij} = \varepsilon_{ij} - e\delta_{ij} , \quad (i,j = x,y,z) \qquad (7a,b)$$

$$s = (\sigma_x + \sigma + \sigma_z)/3 , \quad s_{ij} = \sigma_{ij} - s\delta_{ij} , \quad (i,j = x,y,z) . \qquad (8a,b)$$

Using now the hereditary integrals and observing that for the practical range of temperature and stress levels under hydrostatic stress, most viscoelastic materials behave elastically, the constitutive equations of the adhesive may be written as follows [6-8]:

$$s_{ij} = 2\int_{-\infty}^{t} G(T,\ t-\zeta)\ \frac{\partial e_{ij}}{\partial \xi}\ d\xi\ ,\ (i,j = x,y,z) \tag{9}$$

$$e = \frac{s}{3K(T)} + \alpha_3(T)\ (T-T_0)H(t-t_1)\ , \tag{10}$$

where the known functions $G(T,t)$, $K(T)$, and $\alpha_3(T)$ are respectively the relaxation modulus, the bulk modulus, and the coefficient of thermal expansion of the adhesive. Equations (9) and (10) may also be expressed in the following explicit form:

$$2\sigma_x-\sigma-\sigma_z = 2\int_{-\infty}^{t} G(T,\ t-\xi)(2\ \frac{\partial \varepsilon_x}{\partial \xi} - \frac{\partial \varepsilon_y}{\partial \xi})d\xi, \tag{11}$$

$$2\sigma-\sigma_x-\sigma_z = 2\int_{-\infty}^{t} G(T,\ t-\xi)(2\ \frac{\partial \varepsilon_y}{\partial \xi} - \frac{\partial \varepsilon_x}{\partial \xi})d\xi, \tag{12}$$

$$2\sigma_z-\sigma_x-\sigma = -2\int_{-\infty}^{t} G(T,\ t-\xi)(\frac{\partial \varepsilon_x}{\partial \xi} + \frac{\partial \varepsilon_y}{\partial \xi})d\xi, \tag{13}$$

$$\tau = \int_{-\infty}^{t} G(T,\ t-\xi)\ \frac{\partial \gamma_{xy}}{\partial \xi}\ d\xi\ , \tag{14}$$

$$\sigma_x + \sigma + \sigma_z = 3K(T)\ [\varepsilon_x + \varepsilon_y - 3\alpha_3(T)(T-T_0)H(t-t_1)]. \tag{15}$$

It may be noted that since $\Sigma s_{ii} = 0$ and $\Sigma e_{ii} = 0$, equations (11-13) are not linearly independent; (12), for example, can be obtained by adding (11) and (13) and can, therefore, be ignored. Eliminating σ_x and σ_z, from (11), (13) and (15) it follows that

$$\sigma = K(T)(\varepsilon_x + \varepsilon_y) - 3K(T)\alpha_3(T)(T-T_0)H(t-t_1)$$
$$- \frac{2}{3}\int_{-\infty}^{t} G(T,\ t-\xi)(\frac{\partial \varepsilon_x}{\partial \xi} - 2\ \frac{\partial \varepsilon_y}{\partial \xi})d\xi\ . \tag{16}$$

Equations (14) and (16) with (6) provide the needed constitutive equations for the adhesive. Thus, (1-4), (14) and (16) (with (6)) give a system of fourteen equations to determine the fourteen unknown functions σ, τ, u_i, v_i, β_{ix}, N_{ix}, M_{ix} and Q_{ix}, $(i=1,2)$. Solution of the system of differential equations (1-4) contains twelve integration "constants" which are functions of time and are determined by using the boundary conditions for plates (1) and (2) at $x = \mp\ell$.

3. Solution for the Single Lap Joint

In the general formulation given in the previous section by eliminating all unknown functions other than $\sigma(x,t)$ and $\tau(x,t)$, it is possible to reduce the problem to a system of equations for σ and τ only. Even though relatively straightforward, this process is quite lengthy and the resulting equations are coupled. On the other hand, if the lap joint consists of two identical adherends, then the elimination process is somewhat simpler and the resulting equations for σ and τ are uncoupled. For this case, the problem is considerably simplified and at the same time still yields the main features of the solution. For the identical adherends, the material constants become

$$C_1 = C_2 = C = \frac{1-\nu^2}{Eh} \; , \; D_1 = D_2 = D = \frac{12(1-\nu^2)}{Eh^3} \; ,$$

$$B_1 = B_2 = B = \frac{5}{6}\mu h \; , \; \alpha_1 = \alpha_2 = \alpha \; , \qquad (17)$$

where $h = h_1 = h_2$, $E = E_1 = E_2$, $\nu = \nu_1 = \nu_2$, and $\mu = \mu_1 = \mu_2$.

For $t < t_1$ let the joint be at a homogeneous temperature $T = T_0$ and be stress-free. If the temperature is raised to T at $t = t_1 < 0$ and is held constant for $t > t_1$, then the particular adhesive model used in the analysis described in the previous section would give no temperature induced stresses. Therefore, if the external loads are applied at $t = 0$, in the constitutive equations (14) and (16) the lower limit of the integrals may be taken as zero. Needless to say, if the adherends are not identical, for $t_1 < t < 0$, the stress state in the joint is not zero. In this case, one can still use, for example, one-sided Laplace transform technique to solve the problem by shifting the time such that $t_1 = 0$.

In the problem under consideration, it is assumed that the external loads are applied at the ends of the composite plate. In particular, no external transverse shear load is applied to the plate in $-\ell < x < \ell$ (Figure 1). Thus, from the equilibrium of transverse shear resultants it follows that

$$Q_1(x,t) + Q_2(x,t) \equiv Q_0(t) \tag{18}$$

where $Q_0(t)$ is the transverse shear resultant applied to the ends of the plate (Figure 1). Using the relations (6), (17) and (18), the equations (1-4), (14) and (16) may now be reduced to

$$\frac{\partial^2\tau}{\partial x^2} - \frac{4C+h(h+h_0)D}{2h_0}\int_{0-}^{t} G(T,\ t-\xi)\,\frac{\partial\tau}{\partial\xi}\,d\xi = -\frac{hD}{2h_0}\int_{0-}^{t} G(T,\ t-\xi)\frac{dQ_0(\xi)}{d\xi}\,d\xi\ , \tag{19}$$

$$\frac{\partial^4\sigma}{\partial x^4} - K(T)[(\frac{2}{Bh_0} - \frac{hD}{2})\frac{\partial^2\sigma}{\partial x^2} - \frac{2D}{h_0}\sigma] - \frac{2}{3}\int_{0-}^{t} G(T,\ t-\xi)[(\frac{hD}{2} + \frac{4}{Bh_0})\frac{\partial^3\sigma}{\partial x^2\partial\xi} - \frac{4D}{h_0}\frac{\partial\sigma}{\partial\xi}]d\xi = 0. \tag{20}$$

First, we observe that if T is constant then the bulk modulus K(T) is constant and the relaxation modulus G is a function of time. Hence, the equations (19) and (20) have "constant coefficients" and a convolution type kernel. Therefore, the equations may be solved by using the standard Laplace transform technique. Let $F_1(x,s)$ and $F_2(x,s)$ be the Laplace transform of $\tau(x,t)$ and $\sigma(x,t)$, respectively. Noting that for $t < 0$ the composite plate is stress-free, from (19) and (20) we obtain

$$\frac{d^2F_1}{dx^2} - \gamma_1^2 F_1 = \beta , \tag{21}$$

$$\frac{d^4F_2}{dx^4} - 2\gamma_2^2 \frac{d^2F_2}{dx^2} + \gamma_3^4 F_2 = 0 , \tag{22}$$

where

$$\gamma_1^2 = \frac{1}{2h_o} [4C + hD(h + h_o)]s\ G_1(s) \tag{23}$$

$$\beta = - \frac{hD}{2h_o} s\ G_1(s)\ q_o(s) \tag{24}$$

$$\gamma_2^2 = (\frac{1}{h_oB} - \frac{hD}{4})K(T) + (\frac{hD}{6} + \frac{4}{3h_oB})s\ G_1(s) , \tag{25}$$

$$\gamma_3^4 = \frac{8D}{3h_o} s\ G_1(s) + \frac{2D}{h_o} K(T) , \tag{26}$$

and $G_1(s)$ and $q_o(s)$ are the Laplace transforms of G and Q_o, respectively. The solution of (21) and (22) is

$$F_1(x,s) = A_1 \sinh(\gamma_1 x) + A_2 \cosh(\gamma_1 x) - \frac{\beta}{\gamma_1^2} , \tag{27}$$

$$F_2(x,s) = A_3 \sinh(\phi_1 x) + A_4 \cosh(\phi_1 x) + A_5 \sinh(\phi_2 x) + A_6 \cosh(\phi_2 x), \tag{28}$$

where

$$\phi_1 = [\gamma_2^2 + (\gamma_2^4 - \gamma_3^4)^{1/2}]^{1/2} , \quad \phi_2 = [\gamma_2^2 - (\gamma_2^4 - \gamma_3^4)^{1/2}]^{1/2}. \tag{29}$$

The functions $A_1(s),\ldots,A_6(s)$ are unknown and are determined from the boundary conditions.

4. Examples

The solution of the lap joint problem shown in Figure 1 is obtained for three separate loading conditions. The solution given by (27) and (28) is derived under the assumption that the adherends are identical, the operating temperature T is constant, and the external loads are applied at $t = 0$ ($T \neq T_0$ where T_0 is a base temperature corresponding to zero stress and deformation state).

(a) Membrane Loading.

Let the composite medium be subjected to a constant membrane loading N_0 for $t \geq 0$. Reduced to the end points $x = \mp\ell$ the boundary conditions for the plates 1 and 2 may be expressed as

$$N_{1x}(\ell,t) = 0,\ M_{1x}(\ell,t) = 0,\ Q_{1x}(\ell,t) = 0,$$

$$N_{1x}(-\ell,t) = N_0H(t),\ M_{1x}(-\ell,t) = -N_0\,\frac{h_0+h}{2}\,H(t),\ Q_{1x}(-\ell,t) = 0, \tag{30}$$

$$N_{2x}(\ell,t) = N_0H(t),\ M_{2x}(\ell,t) = N_0\,\frac{h+h_0}{2}\,H(t),\ Q_{2x}(\ell,t) = 0,$$

$$N_{2x}(-\ell,t) = 0,\ M_{2x}(-\ell,t) = 0,\ Q_{2x}(-\ell,t) = 0\ , \tag{31}$$

where $H(t)$ is the Heaviside function. From the symmetry of the problem it can be shown that the conditions (30) and (31) are equivalent to

$$\tau(x,t) = \tau(-x,t)\ ,\ \int_{-\ell}^{\ell}\tau(x,t)dx = -N_0H(t), \tag{32a,b}$$

$$\sigma(x,t) = \sigma(-x,t)\ ,\ \int_{-\ell}^{\ell}\sigma(x,t)dx = 0\ , \tag{33a,b}$$

$$\frac{\partial^2}{\partial x^2}\,\sigma(\ell,t) = K(T)[(\frac{2}{h_0B} - \frac{hD}{2})\sigma(\ell,t) + \frac{h+h_0}{2h_0}\,DN_0H(t)]$$

$$+ \frac{2}{3}\int_{0-}^{t} G(T,\ t-\xi)[(\frac{hD}{2} + \frac{4}{Bh_0})\,\frac{\partial}{\partial\xi}\,\sigma(\ell,\xi) + \frac{h+h_0}{h_0}\,DN_0\delta(\xi)]d\xi. \tag{34}$$

Note that for the loading under consideration $\beta=0$. Thus, taking the Laplace transforms and substituting from (27) and (28), from (32-34) we obtain

$$A_1(s) = 0,\ A_3(s) = 0,\ A_5(s) = 0,\ A_2(s) = -\frac{\gamma_1 N_0}{2s\sinh(\gamma_1 \ell)},$$

$$A_4(s) = -\frac{(h+h_0)N_0\gamma_3{}^4\sinh(\phi_2\ell)}{4s\phi_2\Delta_a(s)},\ A_6(s) = \frac{(h+h_0)N_0\gamma_3{}^4\sinh(\phi_1\ell)}{4s\phi_1\Delta_a(s)}, \quad (35)$$

where

$$\Delta_a(s) = \phi_2\cosh(\phi_1\ell)\sinh(\phi_2\ell) - \phi_1\sinh(\phi_1\ell)\cosh(\phi_2\ell). \quad (36)$$

The adhesive stresses $\tau(x,t)$ and $\sigma(x,t)$ are obtained by substituting from (27), (28) and (35) into the corresponding inversion integrals.

(b) Bending.

In this case of the twelve stress and moment resultants which are prescribed on the boundaries (see (30) and (31)), the following are the only nonzero components:

$$M_{1x}(-\ell,t) = M_0H(t)\ ,\ M_{2x}(\ell,t) = M_0H(t). \quad (37)$$

Again, it can be shown that the boundary conditions are equivalent to

$$\tau(x,t) = -\tau(-x,t)\ , \quad (38)$$

$$\frac{\partial}{\partial x}\tau(\ell,t) = -\frac{hD}{2h_0}M_0\int_{0-}^{t} G(T,\ t-\xi)\delta\ (\xi)d\xi \quad (39)$$

$$\sigma(x,t) = -\sigma(-x,t),\ \int_{-\ell}^{\ell}\sigma(x,t)x dx = M_0H(t)\ , \quad (40a,b)$$

$$\frac{\partial^2}{\partial x^2}\sigma(\ell,t) = K(T)[(\frac{2}{Bh_o} - \frac{hD}{2})\sigma(\ell,t) + \frac{D}{h_o}M_oH(t)]$$

$$+ \frac{2}{3}\int_{0-}^{t} G(T,\ t-\xi)[(\frac{hD}{2} + \frac{4}{h_oB})\frac{\partial}{\partial\xi}\sigma(\ell,\xi) + \frac{2D}{h_o}M_o\delta(\xi)]d\xi. \qquad (41)$$

Substituting now from (27) and (28) into the Laplace transforms of (38-41) we find

$$A_2(s) = 0,\ A_4(s) = 0,\ A_6(s) = 0,\ A_1(s) = -\frac{hDM_oG_1(s)}{2h_o\gamma_1\cosh(\gamma_1\ell)},$$

$$A_3(s) = -\frac{\gamma_3{}^4M_o\cosh(\phi_2\ell)}{2s\phi_2\Delta_b(s)},\ A_5(s) = \frac{\gamma_3{}^4M_o\cosh(\phi_1\ell)}{2s\phi_1\Delta_b(s)}, \qquad (42)$$

where

$$\Delta_b(s) = \phi_2\sinh(\phi_1\ell)\cosh(\phi_2\ell) - \phi_1\cosh(\phi_1\ell)\sinh(\phi_2\ell). \qquad (43)$$

(c) Transverse shear loading.

For this loading condition the following are the only nonhomogeneous boundary conditions:

$$Q_{1x}(-\ell,t) = Q_oH(t),\ M_{1x}(-\ell,t) = -Q_o\ell H(t),$$

$$Q_{2x}(\ell,t) = Q_oH(t),\ M_{2x}(\ell,t) = Q_o\ell H(t). \qquad (44)$$

The equivalent conditions in terms of τ and σ may be shown to be

$$\tau(x,t) = \tau(-x,t),\ \int_{-\ell}^{\ell}\tau(x,t)dx = 0, \qquad (45a,b)$$

$$\sigma(x,t) = \sigma(-x,t),\ \int_{-\ell}^{\ell}\sigma(x,t)dx = -Q_oH(t), \qquad (46a,b)$$

$$\frac{\partial^2}{\partial x^2}\sigma(\ell,t) = K(T)[(\frac{2}{Bh_o} - \frac{hD}{2})\sigma(\ell,t) + \frac{D\ell}{h_o}Q_oH(t)]$$

$$+ \frac{2}{3}\int_{0-}^{t} G(T,\ t-\xi)[(\frac{hD}{2} + \frac{4}{Bh_o})\frac{\partial}{\partial\xi}\sigma(\ell,\xi) + \frac{2D\ell}{h_o}Q_o\delta(\xi)]d\xi. \qquad (47)$$

In this case

$$\beta = -\frac{hD}{2h_o}Q_oG_1(s) \qquad (48)$$

and the functions $A_1(s),\ldots,A_6(s)$ are obtained as

$$A_1(s) = A_3(s) = A_5(s) = 0\ ,\ A_2(s) = \frac{\beta\ell}{\gamma_1\sinh(\gamma_1\ell)}\ ,$$

$$A_4(s) = \frac{Q_o[\gamma_3{}^2\phi_1\cosh(\phi_2\ell) - \gamma_3{}^4\ell\ \sinh(\phi_e\ell)]}{2s\phi_2\Delta_c(s)}\ ,$$

$$A_6(s) = -\frac{Q_o[\gamma_3{}^2\phi_2\cosh(\phi_1\ell) - \gamma_3{}^4\ell\ \sinh(\phi_1\ell)}{2s\phi_1\Delta_c(s)}\ , \qquad (49)$$

where

$$\Delta_c(s) = \phi_2\cosh(\phi_1\ell)\sinh(\phi_2\ell) - \phi_1\sinh(\phi_1\ell)\cosh(\phi_2\ell)\ . \qquad (50)$$

5. The General Problem

As pointed out earlier, the general problem for dissimilar adherends under arbitrary temperature and loading conditions for $t > 0$ may be reduced to a system of coupled equations of the form (19) and (20). If the temperature T is constant for $t > 0$, then the equations have constant coefficients and may easily be solved by using Laplace transforms. On the other hand, if T is not constant, the equations would have time-dependent coefficients, and the Laplace transforms would not be applicable. In this case, to solve the problem, one may have to use a purely numerical technique.

There is one special case for which the solution may be obtained by following the procedure outlined in the previous section. Let the external loads be zero for $t < 0$, and be given arbitrary functions of t for $t > 0$. Also, let the temperature be a piecewise constant function of time, i.e., let

$$T(t) = T_i,\ t_{i-1} < t < t_i,\ t_0 = 0,\ (i=1,\dots,n)\ ,$$

$$T(t) = T_{n+1}\ ,\ t > t_n\ . \qquad (51)$$

Then, for $T=T_1$, the solution given in the previous section is valid in $0 < t < t_1$. After obtaining this solution the functions $\tau(x,t_1)$ and $\sigma(x,t_1)$ can be calculated. Using now this information as the "initial conditions" in time shifted to t_1, assuming $T = T_2$, and repeating the procedure of the previous section, the solution may be obtained which is valid for $t_1 < t < t_2$. The complete solution is obtained by repeating this process for the intervals $t_2 < t < t_3,\dots,t_n < t$.

6. Numerical Results.

Once the relaxation modulus G and the bulk modulus K are specified, the solution may be expressed in terms of Laplace inversion integrals. These integrals are much too complicated for closed form evaluation. However, they can easily be expressed in terms of real integrals and can be evaluated numerically [5]. The relaxation modulus of the adhesive is obtained from a torsion relaxation test. In practice $G(T,t)$ is generally measured in an interval $t_1 < t < t_2$ for different temperature levels. If the material is thermorheologically simple, the function G for $0 < t < \infty$ may then be obtained by using the time-temperature shift factor $a(T)$. For a reference temperature T_0, G is expressed as

$$G(T,t) = G(T_0,\eta),\ \eta = \frac{t}{a(T)}\ , \qquad (52)$$

η being the reduced time.

For the numerical calculations the relaxation modulus of the epoxy is assumed to have the following form:

$$G(T,t) = \{[(\mu_0(T)-\mu_\infty(T)]e^{-t/\varepsilon(T)} + \mu_\infty(T)\}H(t), \tag{53}$$

where

$$\varepsilon(T) = \frac{\mu_\infty(T)}{\mu_0(T)} t_0(T) , \tag{54}$$

$\mu_0(T)$ represents the shear modulus at $t=0$, $\mu_\infty(T)$ at $t=\infty$, and $t_0(T)$ corresponds to the retardation time. For such a material the bulk modulus may be expressed as

$$K(T) = \frac{E_0(T)\mu_0(T)}{3[3\mu_0(T) - E_0(T)]} \ . \tag{55}$$

The constants E_0, μ_0, μ_∞, and t_0 for the temperature levels used in the calculations are given in Table 1. The adherends are assumed to be aluminum plates for which $E = 10^7$ psi, $\nu = 0.3$. The lap joint is assumed to have the following dimensions:

Table 1. Viscoelastic constants of the adhesive

T(°F)	E_0(psi)	μ_0(psi)	μ_∞(psi)	t_0(hours)
70	4.65×10^5	1.80×10^5	0.8×10^5	0.5
100	4.40×10^5	1.70×10^5	0.7×10^5	0.5
140	4.10×10^5	1.58×10^5	0.58×10^5	0.5
180	3.85×10^5	1.50×10^5	0.50×10^5	0.5

$h = 0.09$ in., $\ell = 0.5$ in., $h_0 = 0.004$ in.

For the adhesive model used the Laplace transform of the relaxation modulus is

$$G_1(s) = \frac{\mu_o(T) - \mu_\infty(T)}{s+1/\varepsilon(T)} + \frac{\mu_\infty(T)}{s} . \qquad (56)$$

The calculated results are shown in Tables 2-7. To show the trends some limited results are also shown in Figures 2-4. For a lap joint under membrane loading Figure 2 shows the distribution of the shear stress in the adhesive at various times and for various operating temperatures. Figures 3 and 4 show the "relaxation" of the adhesive stresses $\tau(x,t)$ and $\sigma(x,t)$ for various operating temperatures. These figures indicate that after approximately one hour the stress state in the joint reaches a steady-state. From the figures, it may be observed that the peak values of σ and τ (which are at the end points $x = \mp\ell$) at t=0 may be considerably greater than the corresponding peak stresses at steady-state. Except for the variation with time and temperature, as expected the distribution of stresses in the adhesive has the same trends as those obtained from the related elastic problem [4,5].

Table 2. Variation of $\tau(x,t)/(N_0/\ell)$ for the case of membrane loading (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	-6.98×10^{-4}	-9.10×10^{-4}	-1.27×10^{-3}	-4.27×10^{-3}	-7.17×10^{-3}	-9.54×10^{-3}
0.1	-1.04×10^{-3}	-1.33×10^{-3}	-1.82×10^{-3}	-5.62×10^{-3}	-9.06×10^{-3}	-0.012
0.2	-2.40×10^{-3}	-2.97×10^{-3}	-3.91×10^{-3}	-0.011	-0.016	-0.019
0.3	-6.10×10^{-3}	-7.32×10^{-3}	-9.30×10^{-3}	-0.022	-0.030	-0.035
0.4	-0.016	-0.018	-0.023	-0.046	-0.060	-0.066
0.5	-0.041	-0.046	-0.055	-0.098	-0.119	-0.127
0.6	-0.106	-0.116	-0.132	-0.205	-0.234	-0.243
0.7	-0.273	-0.292	-0.318	-0.425	-0.458	-0.465
0.8	-0.708	-0.732	-0.764	-0.871	-0.890	-0.888
0.9	-1.831	-1.837	-1.829	-1.761	-1.714	-1.695
1.0	-4.740	-4.603	-4.358	-3.501	-3.277	-3.233
			T=100°F			
0.	-9.06×10^{-4}	-1.21×10^{-3}	-1.73×10^{-3}	-6.11×10^{-3}	-0.010	-0.014
0.1	-1.32×10^{-3}	-1.72×10^{-3}	-2.41×10^{-3}	-7.85×10^{-3}	-0.013	-0.016
0.2	-2.94×10^{-3}	-3.71×10^{-3}	-4.98×10^{-3}	-0.014	-0.021	-0.025
0.3	-7.26×10^{-3}	-8.85×10^{-3}	-0.011	-0.028	-0.039	-0.044
0.4	-0.018	-0.022	-0.027	-0.056	-0.073	-0.080
0.5	-0.046	-0.052	-0.063	-0.114	-0.138	-0.147
0.6	-0.115	-0.128	-0.146	-0.230	-0.261	-0.269
0.7	-0.289	-0.310	-0.340	-0.456	-0.488	-0.493
0.8	-0.727	-0.754	-0.788	-0.895	-0.908	-0.903
0.9	-1.828	-1.831	-1.818	-1.728	-1.673	-1.653
1.0	-4.594	-4.440	-4.173	-3.279	-3.063	-3.024

T=140°F						
0.	-1.26×10^{-3}	-1.74×10^{-3}	-2.58×10^{-3}	-9.81×10^{-3}	-0.017	-0.022
0.1	-1.79×10^{-3}	-2.41×10^{-3}	-3.49×10^{-3}	-0.012	-0.020	-0.025
0.2	-3.81×10^{-3}	-4.95×10^{-3}	-6.85×10^{-3}	-0.020	-0.031	-0.037
0.3	-9.04×10^{-3}	-0.011	-0.015	-0.038	-0.053	-0.060
0.4	-0.022	-0.026	-0.033	-0.073	-0.094	-0.102
0.5	-0.053	-0.062	-0.075	-0.140	-0.168	-0.176
0.6	-0.128	-0.144	-0.167	-0.266	-0.298	-0.305
0.7	-0.310	-0.335	-0.371	-0.498	-0.527	-0.529
0.8	-0.752	-0.781	-0.819	-0.921	-0.924	-0.917
0.9	-1.821	-1.819	-1.800	-1.674	-1.609	-1.589
1.0	-4.408	-4.229	-3.928	-2.985	-2.784	-2.752
T=180°F						
0.	-1.59×10^{-3}	-2.26×10^{-3}	-3.45×10^{-3}	-0.014	-0.024	-0.030
0.1	-2.21×10^{-3}	-3.07×10^{-3}	-4.57×10^{-3}	-0.017	-0.027	-0.034
0.2	-4.58×10^{-3}	-6.09×10^{-3}	-8.65×10^{-3}	-0.027	-0.040	-0.048
0.3	-0.011	-0.013	-0.018	-0.048	-0.066	-0.074
0.4	-0.025	-0.030	-0.039	-0.088	-0.112	-0.120
0.5	-0.058	-0.069	-0.085	-0.162	-0.192	-0.199
0.6	-0.138	-0.156	-0.184	-0.295	-0.327	-0.332
0.7	-0.326	-0.354	-0.395	-0.529	-0.553	-0.553
0.8	-0.769	-0.801	-0.841	-0.936	-0.931	-0.921
0.9	-1.814	-1.808	-1.782	-1.626	-1.555	-1.534
1.0	-4.277	-4.075	-3.744	-2.770	-2.582	-2.555

Table 3. Variation of $\sigma(x,t)/(N_0/\ell)$ for membrane loading (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	1.93×10^{-4}	2.18×10^{-4}	2.55×10^{-4}	4.07×10^{-4}	4.57×10^{-4}	4.70×10^{-4}
0.1	2.30×10^{-4}	2.67×10^{-4}	3.24×10^{-4}	5.51×10^{-4}	6.24×10^{-4}	6.41×10^{-4}
0.2	9.78×10^{-5}	1.70×10^{-4}	2.81×10^{-4}	7.15×10^{-4}	8.45×10^{-4}	8.71×10^{-4}
0.3	-1.69×10^{-3}	-1.60×10^{-3}	-1.44×10^{-3}	-8.77×10^{-4}	-7.31×10^{-4}	-7.10×10^{-4}
0.4	-0.011	-0.011	-0.011	-0.012	-0.012	-0.012
0.5	-0.050	-0.051	-0.052	-0.056	-0.057	-0.057
0.6	-0.174	-0.177	-0.181	-0.194	-0.197	-0.198
0.7	-0.495	-0.502	-0.507	-0.526	-0.530	-0.531
0.8	-1.032	-1.034	-1.030	-1.013	-1.006	-1.005
0.9	-0.640	-0.621	-0.587	-0.473	-0.448	-0.445
1.0	7.870	7.806	7.649	7.151	7.044	7.033
			T=100°F			
0.	2.41×10^{-4}	2.72×10^{-4}	3.18×10^{-4}	5.01×10^{-4}	5.57×10^{-4}	5.70×10^{-4}
0.1	3.01×10^{-4}	3.47×10^{-4}	4.17×10^{-4}	6.87×10^{-4}	7.67×10^{-4}	7.83×10^{-4}
0.2	2.33×10^{-4}	3.21×10^{-4}	4.53×10^{-4}	9.54×10^{-4}	1.09×10^{-3}	1.11×10^{-3}
0.3	-1.52×10^{-3}	-1.41×10^{-3}	-1.23×10^{-3}	-6.19×10^{-4}	-4.83×10^{-4}	-4.68×10^{-4}
0.4	-0.011	-0.012	-0.012	-0.012	-0.012	-0.012
0.5	-0.051	-0.052	-0.054	-0.058	-0.060	-0.060
0.6	-0.179	-0.183	-0.187	-0.201	-0.205	-0.205
0.7	-0.503	-0.509	-0.515	-0.535	-0.539	-0.539
0.8	-1.026	-1.028	-1.022	-1.002	-0.995	-0.994
0.9	-0.596	-0.574	-0.537	-0.419	-0.396	-0.394
1.0	7.673	7.599	7.429	6.920	6.823	6.816

T=140°F						
0.	3.10×10^{-4}	3.52×10^{-4}	4.13×10^{-4}	6.45×10^{-4}	7.09×10^{-4}	7.20×10^{-4}
0.1	4.03×10^{-4}	4.64×10^{-4}	5.55×10^{-4}	8.91×10^{-4}	9.79×10^{-4}	9.93×10^{-4}
0.2	4.21×10^{-4}	5.35×10^{-4}	7.04×10^{-4}	1.30×10^{-3}	1.44×10^{-3}	1.45×10^{-3}
0.3	-1.29×10^{-3}	-1.16×10^{-3}	-9.46×10^{-4}	-2.86×10^{-4}	-1.71×10^{-4}	-1.66×10^{-4}
0.4	-0.012	-0.012	-0.012	-0.013	-0.013	-0.013
0.5	-0.053	-0.055	-0.056	-0.062	-0.064	-0.064
0.6	-0.186	-0.190	-0.195	-0.211	-0.214	-0.214
0.7	-0.512	-0.519	-0.526	-0.546	-0.549	-0.549
0.8	-1.018	-1.018	-1.011	-0.985	-0.978	-0.977
0.9	-0.541	-0.515	-0.472	-0.350	-0.331	-0.330
1.0	7.428	7.338	7.150	6.631	6.549	6.546
T=180°F						
0.	4.42×10^{-4}	5.04×10^{-4}	5.95×10^{-4}	9.35×10^{-4}	1.02×10^{-3}	1.04×10^{-3}
0.1	5.91×10^{-4}	6.80×10^{-4}	8.12×10^{-4}	1.28×10^{-3}	1.39×10^{-3}	1.41×10^{-3}
0.2	7.48×10^{-4}	9.05×10^{-4}	1.13×10^{-3}	1.90×10^{-3}	2.05×10^{-3}	2.07×10^{-3}
0.3	-9.54×10^{-4}	-7.89×10^{-4}	-5.38×10^{-4}	1.56×10^{-4}	2.36×10^{-4}	2.31×10^{-4}
0.4	-0.012	-0.012	-0.013	-0.014	-0.014	-0.014
0.5	-0.057	-0.059	-0.061	-0.069	-0.071	-0.071
0.6	-0.196	-0.202	-0.208	-0.227	-0.230	-0.231
0.7	-0.526	-0.534	-0.542	-0.562	-0.564	-0.564
0.8	-1.002	-1.000	-0.989	-0.954	-0.946	-0.946
0.9	-0.454	-0.423	-0.374	-0.246	-0.229	-0.229
1.0	7.053	6.944	6.733	6.195	6.124	6.123

Table 4. Variation of $\tau(x,t)/(M_0/\ell^2)$ for bending (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	0.	0.	0.	0.	0.	0.
0.1	-6.21×10^{-3}	-7.80×10^{-3}	-0.010	-0.030	-0.045	-0.055
0.2	-0.018	-0.023	-0.030	-0.078	-0.113	-0.134
0.3	-0.049	-0.059	-0.074	-0.173	-0.238	-0.272
0.4	-0.127	-0.148	-0.181	-0.372	-0.481	-0.531
0.5	-0.329	-0.372	-0.440	-0.788	-0.957	-1.022
0.6	-0.851	-0.936	-1.064	-1.653	-1.889	-1.960
0.7	-2.204	-2.353	-2.565	-3.429	-3.696	-3.749
0.8	-5.706	-5.906	-6.162	-7.028	-7.177	-7.161
0.9	-14.77	-14.81	-14.75	-14.20	-13.82	-13.67
1.0	-38.22	-37.12	-35.15	-28.23	-26.42	-26.07
			T=100°F			
0.	0.	0.	0.	0.	0.	0.
0.1	-7.75×10^{-3}	-9.92×10^{-3}	-0.014	-0.040	-0.060	-0.073
0.2	-0.023	-0.028	-0.038	-0.102	-0.148	-0.173
0.3	-0.058	-0.071	-0.091	-0.219	-0.300	-0.340
0.4	-0.147	-0.173	-0.215	-0.453	-0.583	-0.638
0.5	-0.369	-0.422	-0.505	-0.921	-1.113	-1.179
0.6	-0.928	-1.028	-1.179	-1.853	-2.103	-2.168
0.7	-2.333	-2.502	-2.743	-3.681	-3.939	-3.976
0.8	-5.864	-6.080	-6.356	-7.217	-7.319	-7.283
0.9	-14.74	-14.76	-14.66	-13.94	-13.49	-13.33
1.0	-37.04	-35.81	-33.66	-26.44	-24.70	-24.39

T=140°F

0.	0.	0.	0.	0.	0.	0.
0.1	-0.010	-0.013	-0.019	-0.059	-0.088	-0.103
0.2	-0.029	-0.037	-0.051	-0.145	-0.208	-0.239
0.3	-0.072	-0.090	-0.119	-0.298	-0.405	-0.450
0.4	-0.176	-0.212	-0.269	-0.586	-0.746	-0.803
0.5	-0.426	-0.496	-0.604	-1.128	-1.346	-1.408
0.6	-1.033	-1.159	-1.347	-2.143	-2.402	-2.452
0.7	-2.503	-2.704	-2.990	-4.019	-4.247	-4.260
0.8	-6.062	-6.302	-6.606	-7.428	-7.452	-7.390
0.9	-14.68	-14.67	-14.51	-13.50	-12.98	-12.81
1.0	-35.55	-34.11	-31.68	-24.08	-22.45	-22.19

T=180°F

0.	0.	0.	0.	0.	0.	0.
0.1	-0.012	-0.017	-0.024	-0.078	-0.115	-0.132
0.2	-0.035	-0.046	-0.064	-0.188	-0.266	-0.299
0.3	-0.084	-0.107	-0.145	-0.373	-0.500	-0.547
0.4	-0.199	-0.245	-0.317	-0.706	-0.887	-0.942
0.5	-0.471	-0.556	-0.688	-1.305	-1.536	-1.591
0.6	-1.113	-1.262	-1.484	-2.376	-2.629	-2.665
0.7	-2.628	-2.857	-3.182	-4.268	-4.459	-4.451
0.8	-6.200	-6.459	-6.786	-7.547	-7.503	-7.423
0.9	-14.63	-14.58	-14.37	-13.11	-12.54	-12.37
1.0	-34.49	-32.86	-30.19	-22.34	-20.82	-20.60

Table 5. Variation of $\sigma(x,t)/(M_0/\ell^2)$ for bending (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	0.	0.	0.	0.	0.	0.
0.1	1.71×10^{-3}	2.03×10^{-3}	2.51×10^{-3}	4.47×10^{-3}	5.09×10^{-3}	5.23×10^{-3}
0.2	8.28×10^{-4}	1.58×10^{-3}	2.73×10^{-3}	7.25×10^{-3}	8.61×10^{-3}	8.89×10^{-3}
0.3	-0.018	-0.017	-0.015	-9.38×10^{-3}	-7.83×10^{-3}	-7.60×10^{-3}
0.4	-0.120	-0.121	-0.121	-0.124	-0.126	-0.126
0.5	-0.529	-0.539	-0.550	-0.594	-0.607	-0.610
0.6	-1.854	-1.888	-1.925	-2.064	-2.100	-2.106
0.7	-5.269	-5.335	-5.393	-5.599	-5.640	-5.646
0.8	-10.98	-11.00	-10.95	-10.77	-10.71	-10.70
0.9	-6.813	-6.606	-6.241	-5.032	-4.768	-4.737
1.0	83.72	83.04	81.37	76.07	74.93	74.82
			T=100°F			
0.	0.	0.	0.	0.	0.	0.
0.1	2.31×10^{-3}	2.71×10^{-3}	3.30×10^{-3}	5.62×10^{-3}	6.30×10^{-3}	6.43×10^{-3}
0.2	2.23×10^{-3}	3.14×10^{-3}	4.52×10^{-3}	9.74×10^{-3}	0.011	0.011
0.3	-0.016	-0.015	-0.013	-6.64×10^{-3}	-5.18×10^{-3}	-5.02×10^{-3}
0.4	-0.122	-0.123	-0.123	-0.128	-0.130	-0.130
0.5	-0.545	-0.557	-0.570	-0.621	-0.635	-0.638
0.6	-1.905	-1.944	-1.987	-2.141	-2.176	-2.181
0.7	-5.349	-5.420	-5.484	-5.695	-5.730	-5.735
0.8	-10.92	-10.94	-10.88	-10.65	-10.58	-10.57
0.9	-6.343	-6.108	-5.707	-4.455	-4.213	-4.191
1.0	81.63	80.84	79.03	73.61	72.58	72.51

T=140°F						
0.	0.	0.	0.	0.	0.	0.
0.1	3.18×10^{-3}	3.70×10^{-3}	4.48×10^{-3}	7.34×10^{-3}	8.06×10^{-3}	8.18×10^{-3}
0.2	4.18×10^{-3}	5.36×10^{-3}	7.12×10^{-3}	0.013	0.015	0.015
0.3	-0.014	-0.012	-0.010	-3.09×10^{-3}	-1.86×10^{-3}	-1.80×10^{-3}
0.4	-0.124	-0.125	-0.126	-0.134	-0.136	-0.137
0.5	-0.567	-0.582	-0.599	-0.661	-0.676	-0.678
0.6	-1.973	-2.019	-2.071	-2.244	-2.277	-2.281
0.7	-5.448	-5.526	-5.599	-5.813	-5.841	-5.843
0.8	-10.83	-10.83	-10.75	-10.48	-10.41	-10.40
0.9	-5.751	-5.474	-5.021	-3.726	-3.522	-3.511
1.0	79.02	78.07	76.06	70.54	69.67	69.64
T=180°F						
0.	0.	0.	0.	0.	0.	0.
0.1	4.76×10^{-3}	5.51×10^{-3}	6.62×10^{-3}	0.011	0.011	0.012
0.2	7.58×10^{-3}	9.20×10^{-3}	0.012	0.020	0.021	0.021
0.3	-0.010	-8.45×10^{-3}	-5.78×10^{-3}	1.64×10^{-3}	2.51×10^{-3}	2.46×10^{-3}
0.4	-0.129	-0.132	-0.135	-0.148	-0.153	-0.153
0.5	-0.609	-0.628	-0.653	-0.736	-0.754	-0.756
0.6	-2.089	-2.146	-2.212	-2.417	-2.450	-2.453
0.7	-5.600	-5.685	-5.766	-5.981	-6.000	-6.001
0.8	-10.65	-10.63	-10.52	-10.15	-10.06	-10.06
0.9	-4.828	-4.497	-3.980	-2.617	-2.439	-2.434
1.0	75.03	73.87	71.63	65.91	65.14	65.13

Table 6. Variation of $\tau(x,t)/(Q_0/\ell)$ for transverse shear loading (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	4.027	4.025	4.022	3.998	3.974	3.955
0.1	4.024	4.022	4.018	3.987	3.959	3.938
0.2	4.013	4.008	4.001	3.948	3.906	3.878
0.3	3.983	3.973	3.957	3.856	3.788	3.750
0.4	3.905	3.884	3.851	3.659	3.548	3.496
0.5	3.703	3.660	3.592	3.244	3.073	3.007
0.6	3.181	3.096	2.968	2.379	2.143	2.071
0.7	1.828	1.680	1.467	0.603	0.336	0.283
0.8	-1.674	-1.873	-2.129	-2.996	-3.145	-3.129
0.9	-10.74	-10.78	-10.71	-10.17	-9.792	-9.637
1.0	-34.19	-33.09	-31.11	-24.20	-22.39	-22.04
			T=100°F			
0.	4.025	4.023	4.018	3.983	3.949	3.922
0.1	4.022	4.018	4.013	3.969	3.929	3.901
0.2	4.009	4.002	3.992	3.919	3.863	3.828
0.3	3.974	3.961	3.940	3.808	3.721	3.675
0.4	3.885	3.859	3.816	3.577	3.444	3.385
0.5	3.663	3.610	3.527	3.110	2.917	2.848
0.6	3.104	3.004	2.852	2.179	1.928	1.862
0.7	1.699	1.531	1.289	0.351	0.093	0.055
0.8	-1.832	-2.048	-2.324	-3.185	-3.287	-3.252
0.9	-10.71	-10.73	-10.63	-9.905	-9.463	-9.299
1.0	-33.01	-31.78	-29.62	-22.41	-20.67	-20.35

T=140°F						
0.	4.022	4.018	4.011	3.953	3.898	3.858
0.1	4.018	4.013	4.004	3.934	3.872	3.830
0.2	4.002	3.992	3.977	3.867	3.785	3.737
0.3	3.959	3.941	3.912	3.724	3.607	3.550
0.4	3.856	3.820	3.762	3.442	3.276	3.211
0.5	3.606	3.536	3.428	2.902	2.681	2.614
0.6	2.999	2.874	2.686	1.888	1.628	1.574
0.7	1.530	1.329	1.042	0.013	-0.217	-0.231
0.8	-2.030	-2.269	-2.574	-3.396	-3.421	-3.360
0.9	-10.65	-10.64	-10.48	-9.470	-8.947	-8.779
1.0	-31.52	-30.07	-27.64	-20.04	-18.42	-18.16

T=180°F						
0.	4.019	4.014	4.004	3.920	3.843	3.791
0.1	4.014	4.007	3.995	3.896	3.811	3.758
0.2	3.995	3.983	3.963	3.814	3.707	3.649
0.3	3.947	3.924	3.885	3.643	3.499	3.436
0.4	3.832	3.787	3.714	3.319	3.127	3.061
0.5	3.561	3.476	3.344	2.724	2.486	2.424
0.6	2.919	2.770	2.548	1.654	1.398	1.357
0.7	1.405	1.175	0.850	-0.236	-0.430	-0.425
0.8	-2.168	-2.427	-2.754	-3.515	-3.473	-3.394
0.9	-10.59	-10.55	-10.34	-9.082	-8.507	-8.340
1.0	-30.46	-28.83	-26.16	-18.31	-16.79	-16.57

Table 7. Variation of $\sigma(x,t)/(Q_0/\ell)$ for transverse shear loading (t in hours)

x/ℓ	t=0.01	t=0.05	t=0.1	t=0.5	t=1	t=2
			T=70°F			
0.	1.94×10^{-3}	2.19×10^{-3}	2.56×10^{-3}	4.05×10^{-3}	4.54×10^{-3}	4.65×10^{-3}
0.1	2.37×10^{-3}	2.74×10^{-3}	3.31×10^{-3}	5.58×10^{-3}	6.30×10^{-3}	6.47×10^{-3}
0.2	1.34×10^{-3}	2.09×10^{-3}	3.23×10^{-3}	7.69×10^{-3}	9.02×10^{-3}	9.30×10^{-3}
0.3	-0.015	-0.014	-0.013	-6.49×10^{-3}	-4.85×10^{-3}	-4.59×10^{-3}
0.4	-0.109	-0.109	-0.109	-0.109	-0.110	-0.111
0.5	-0.487	-0.495	-0.504	-0.540	-0.551	-0.554
0.6	-1.727	-1.757	-1.789	-1.912	-1.944	-1.950
0.7	-4.980	-5.041	-5.094	-5.283	-5.321	-5.327
0.8	-10.67	-10.70	-10.66	-10.51	-10.45	-10.44
0.9	-8.223	-8.048	-7.720	-6.638	-6.400	-6.373
1.0	71.59	70.97	69.47	64.74	63.73	63.63
			T=100°F			
0.	2.42×10^{-3}	2.72×10^{-3}	3.18×10^{-3}	4.97×10^{-3}	5.51×10^{-3}	5.63×10^{-3}
0.1	3.08×10^{-3}	3.54×10^{-3}	4.23×10^{-3}	6.93×10^{-3}	7.72×10^{-3}	7.88×10^{-3}
0.2	2.74×10^{-3}	3.64×10^{-3}	5.01×10^{-3}	0.010	0.012	0.012
0.3	-0.014	-0.012	-0.010	-3.55×10^{-3}	-1.98×10^{-3}	-1.79×10^{-3}
0.4	-0.109	-0.110	-0.110	-0.112	-0.113	-0.113
0.5	-0.499	-0.510	-0.520	-0.563	-0.575	-0.578
0.6	-1.772	-1.806	-1.844	-1.980	-2.011	-2.016
0.7	-5.053	-5.119	-5.177	-5.372	-5.405	-5.409
0.8	-10.62	-10.64	-10.59	-10.41	-10.35	-10.34
0.9	-7.802	-7.601	-7.242	-6.120	-5.903	-5.884
1.0	69.71	68.99	67.38	62.54	61.63	61.56

T=140°F						
0.	3.10×10^{-3}	3.51×10^{-3}	4.11×10^{-3}	6.37×10^{-3}	6.98×10^{-3}	7.09×10^{-3}
0.1	4.10×10^{-3}	4.71×10^{-3}	5.62×10^{-3}	8.95×10^{-3}	9.81×10^{-3}	9.95×10^{-3}
0.2	4.68×10^{-3}	5.85×10^{-3}	7.59×10^{-3}	0.014	0.015	0.015
0.3	-0.011	-9.50×10^{-3}	-7.15×10^{-3}	3.28×10^{-4}	1.69×10^{-3}	1.79×10^{-3}
0.4	-0.110	-0.111	-0.112	-0.116	-0.117	-0.118
0.5	-0.519	-0.531	-0.545	-0.597	-0.610	-0.612
0.6	-1.832	-1.873	-1.919	-2.072	-2.101	-2.105
0.7	-5.144	-5.217	-5.283	-5.482	-5.508	-5.510
0.8	-10.55	-10.56	-10.49	-10.26	-10.20	-10.19
0.9	-7.272	-7.033	-6.627	-5.467	-5.283	-5.273
1.0	67.38	66.51	64.72	59.80	59.03	59.00
T=180°F						
0.	4.39×10^{-3}	4.99×10^{-3}	5.88×10^{-3}	9.18×10^{-3}	0.010	0.010
0.1	5.98×10^{-3}	6.86×10^{-3}	8.17×10^{-3}	0.013	0.014	0.014
0.2	8.06×10^{-3}	9.68×10^{-3}	0.012	0.020	0.022	0.022
0.3	-7.11×10^{-3}	-5.22×10^{-3}	-2.37×10^{-3}	5.81×10^{-3}	6.90×10^{-3}	6.89×10^{-3}
0.4	-0.114	-0.116	-0.117	-0.127	-0.130	-0.130
0.5	-0.554	-0.570	-0.591	-0.662	-0.676	-0.678
0.6	-1.935	-1.985	-2.044	-2.226	-2.256	-2.258
0.7	-5.284	-5.364	-5.439	-5.639	-5.658	-5.659
0.8	-10.40	-10.39	-10.29	-9.980	-9.908	-9.904
0.9	-6.443	-6.157	-5.693	-4.472	-4.311	-4.306
1.0	63.80	62.75	60.75	55.66	54.98	54.97

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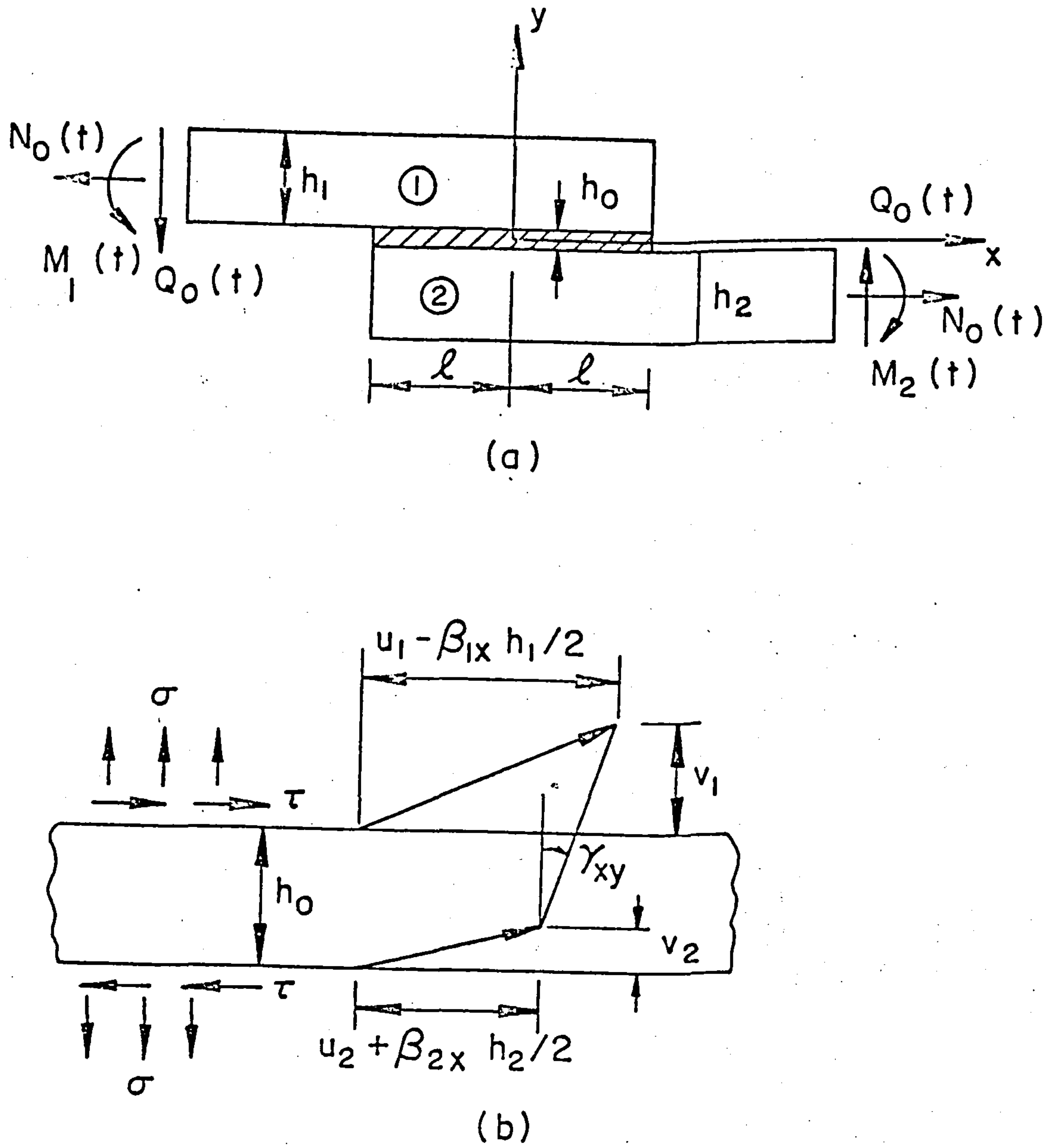

Figure 1. The geometry of the bonded joint.

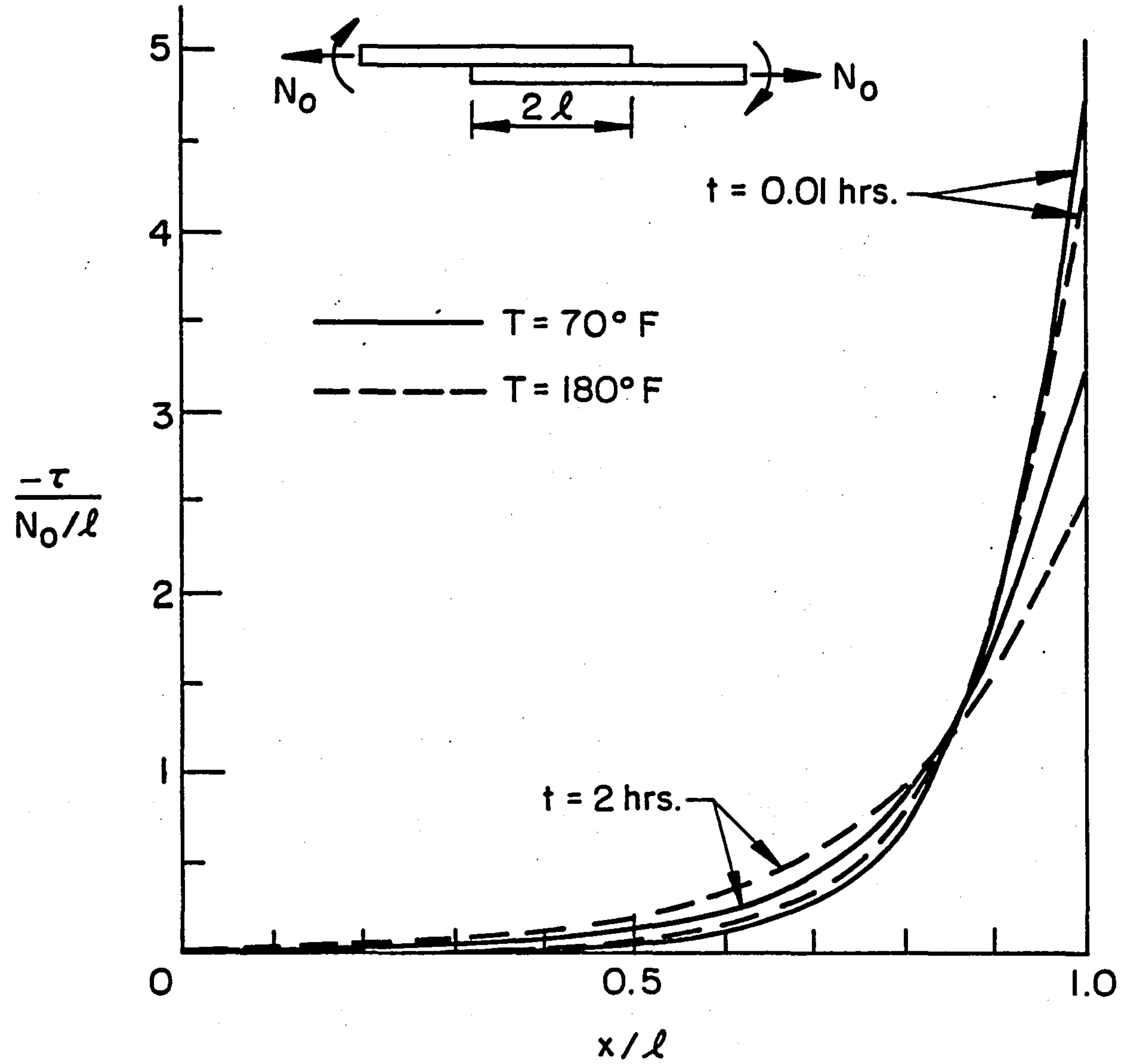

Figure 2. Distribution of the shear stress in the adhesive in a bonded joint under uniform membrane loading.

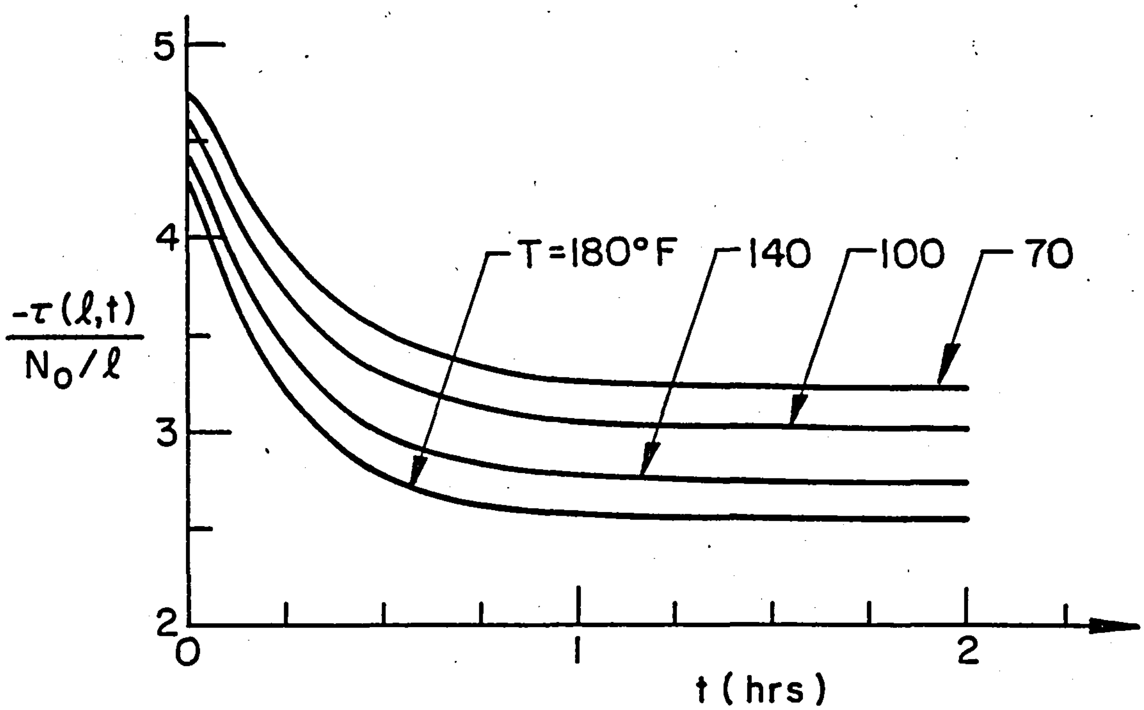

Figure 3. Relaxation of the peak value of the adhesive shear stress for various operating temperatures.

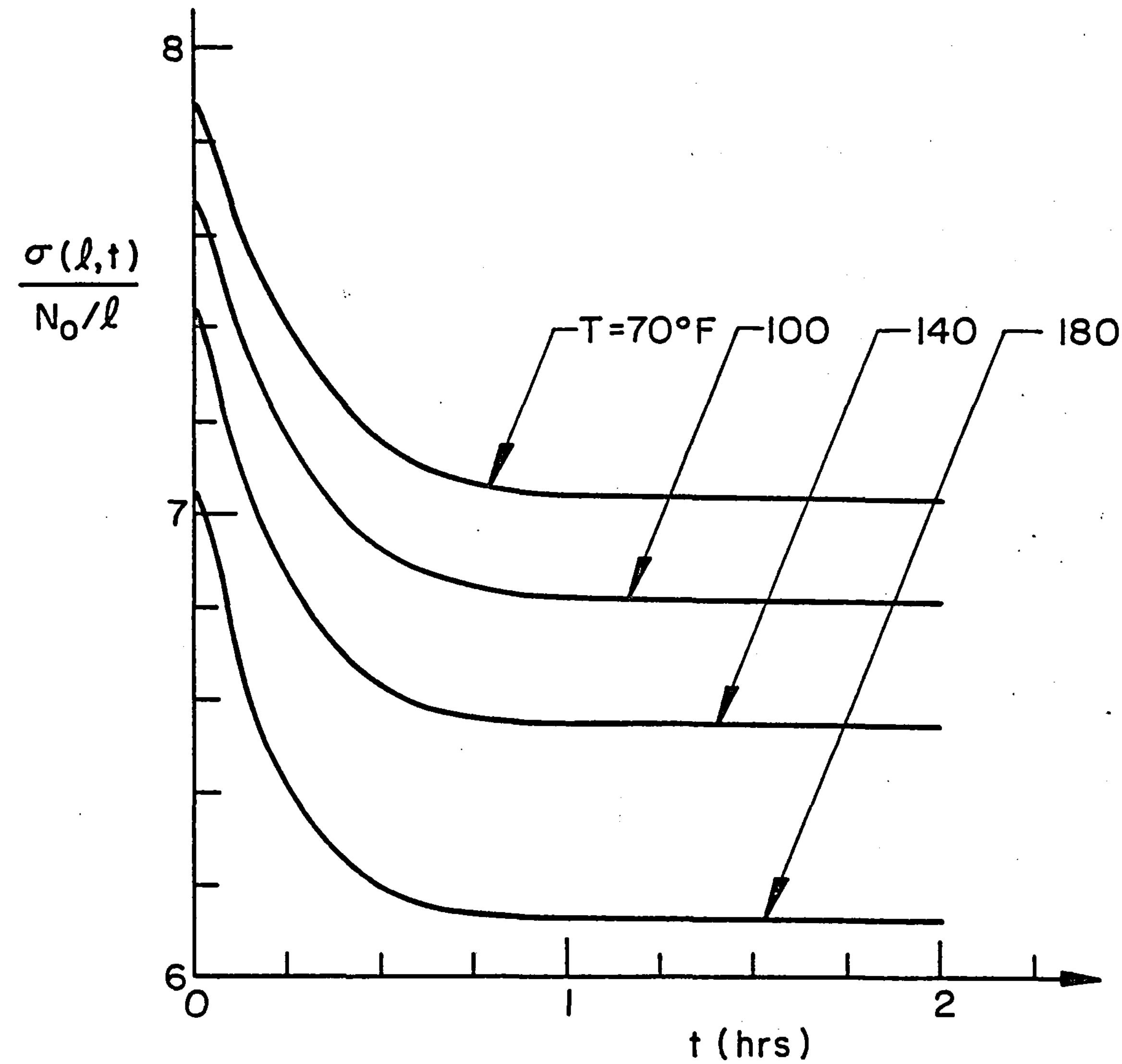

Figure 4. Relaxation of the peak value of the normal stress in the adhesive for various operating temperatures.