# A GENERAL METHOD FOR COMPUTING THE TOTAL SOLAR RADIATION FORCE ON COMPLEX SPACECRAFT STRUCTURES

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## ABSTRACT

A general approach has been developed for computing the force due to solar radiation on an object of arbitrary shape. This method circumvents many of the existing difficulties in computational logic presently encountered in the direct analytical or numerical evaluation of the appropriate surface integral. It may be applied to complex spacecraft structures for computing the total force arising from either specular or diffuse reflection or even from non-Lambertian reflection and re-radiation.

## SECTION 1 - INTRODUCTION

The problem of computing the total force or total torque on a spacecraft due to solar radiation is, in general, very difficult. Mathematically, it requires the evaluation of a surface integral over only the illuminated region of the surface. Even if the illuminated region is known by some other means, the evaluation of the surface integral can still be very difficult analytically in the case of complex spacecraft structures. Moreover, if the illuminated region is not known a priori, the difficulties are compounded by having to determine self-shadowing. For non-convex objects, it is not trivially governed by a condition such as $\cos\theta \gtrless 0$ where $\theta$ is the angle between the sun vector and the outward vector normal to the surface. In fact, the logic in the present methods becomes extremely complicated and is also not fool-proof. Additional difficulties are introduced by choosing a set of points (vertices) on the surface to form a network in approximating it; this inadvertently leads to book-keeping problems associated with selecting appropriate sets of points for computing surface elements.

This paper presents a general method for performing the computations without encountering the difficulties described above. It does not attempt to evaluate the surface integral directly as it presents itself as done in the usual methods, but considers the same problem from a slightly different point of view which leads to the same results.

## SECTION 2 - ANALYSIS

Consider an arbitrarily shaped object as illustrated in Figure 1.1.

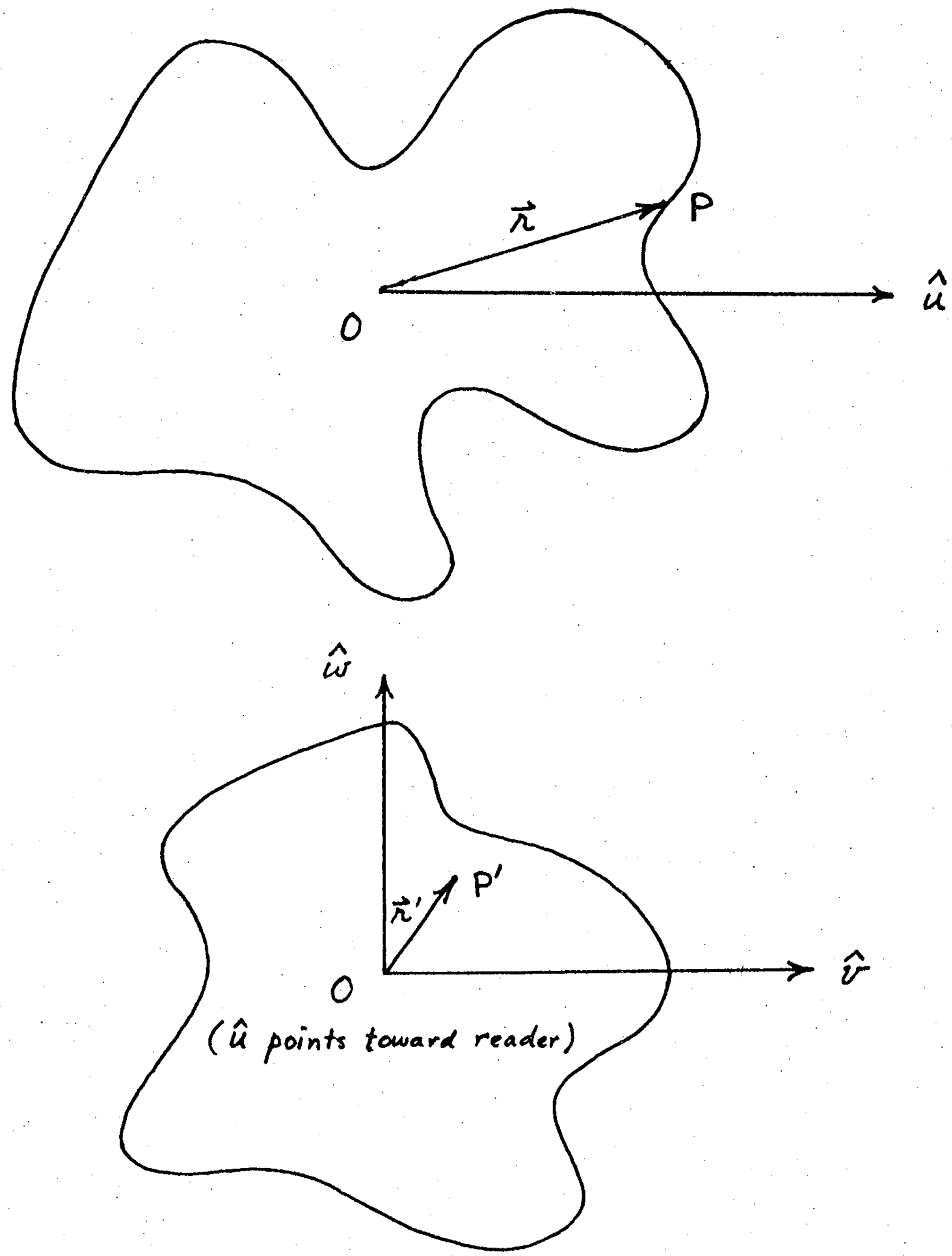


Figure 1.1 - Illustration of an Arbitrarily Shaped Object

For convenience, let us use the following notation:

$\hat{u}$ = unit vector along a specified direction

$\hat{v}$ = any unit vector orthogonal to $\hat{u}$, i.e., $\hat{u} \cdot \hat{v} = 0$

$\hat{w}$ = third unit vector forming orthogonal triad, i.e., $\hat{w} = \hat{u} \times \hat{v}$

O = origin of coordinate system

P = any point on object's surface

$\vec{r}$ = vector from O to P

P' = projection of P onto (v,w)-plane

$\vec{r}'$ = projection of $\vec{r}$ onto (v,w)-plane

(x,y,z) = reference orthogonal system for describing object's surface.

In the present analysis, it is advantageous to choose $\hat{u}$ to be opposite in direction to the incident solar radiation. (Alternatively, it can also be chosen to be in the same direction.)

The vectors $\hat{u}$ and $\vec{r}$ are known in the (x,y,z) system. In general, if $\vec{V}$ is any vector, then it may be more explicitly written in the (x,y,z)-space as $\vec{V}_{(x,y,z)}$ and has components $V_x$, $V_y$, $V_z$. That is, we implicitly mean

$$\vec{V}_{(x,y,z)} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z} \qquad (1.1)$$

In view of the definition of the vector $\hat{v}$, we may choose $v_z = 0$. Then, it may be shown that the other two components are given by

$$v_x = \pm \frac{u_y}{\sqrt{(u_x^2 + u_y^2)}} \qquad (1.2)$$

$$v_y = - \frac{u_x v_x}{u_y} \qquad (1.3)$$

From the definition of $\hat{w}$, we obtain

$$\hat{w} = - u_z v_x \hat{x} + u_z v_x \hat{y} + (u_x v_y - u_y v_x) \hat{z} \tag{1.4}$$

Therefore, any vector $\vec{V}_{(x,y,z)}$ can be transformed to $\vec{V}_{(u,v,w)}$ by the equation

$$\vec{V}_{(u,v,w)} = T \ \vec{V}_{(x,y,z)} \tag{1.5}$$

where the transformation matrix T is given by

$$T = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & 0 \\ w_x & w_y & w_z \end{bmatrix} \tag{1.6}$$

Then, using equation (1.6), the vector $\vec{r}_{(x,y,z)}$ is transformed and we obtain

$$r_u = r_x u_x + r_y u_y + r_z u_z \tag{1.7}$$

$$r_v = r_x v_x + r_y v_y \tag{1.8}$$

$$r_w = r_x w_x + r_y w_y + r_z w_z \tag{1.9}$$

Consequently, the projection vector $\vec{r}'$ is simply given by

$$\vec{r}' = r_v \hat{v} + r_w \hat{w} \tag{1.10}$$

The component $r_u$ of the vector $\vec{r}$ is particularly important because, for a complex spacecraft structure, it can be used to yield the surface element which is directly exposed to solar radiation. This can be seen as follows: For any given point on the (v,w)-plane (i.e., for any given vector $\vec{r}'$), the point on the spacecraft which is not shadowed is the one which has the maximum value of $r_u$, independent of where the origin of the (u,v,w) coordinate system is chosen. (It would be the minimum value of $r_u$ if the vector $\hat{u}$ had been chosen to be in the same direction as the incident solar radiation.) To find the illuminated surface of the spacecraft, we proceed by dividing the (v,w)-plane into cells of area $\Delta v \Delta w$ with cell centers $(v_i, w_j)$. At these cell centers, the illuminated surface element is the one which has the maximum value of $r_u$. In this way, the logic of determining self-shadowing is extremely simple as compared to other methods which encounter considerable difficulty conceptually and computationally. Thus, given a vector $\vec{r}' = (0, v_i, w_j)$, the vector $\vec{r} = ([r_u]_{max}, v_i, w_j)$ corresponding to the illuminated point is determined. It is then transformed to the (x,y,z)-space by the equation

$$\vec{r}_{(x,y,z)} = T^T \vec{r}_{(u,v,w)} \tag{1.11}$$

At this point $\vec{r}_{(x,y,z)}$, the unit vector $\hat{n}_{(x,y,z)}$ normal to the surface is then obtained by

$$\hat{n}_{(x,y,z)} = \frac{\nabla \Phi}{|\nabla \Phi|} \tag{1.12}$$

where $\Phi(x,y,z) = 0$ denotes the equation of the surface in a region containing $\vec{r}$. For convenience, the direction of $\hat{n}$ is chosen such that

$$\hat{n} \cdot \hat{u} \geqslant 0 \tag{1.13}$$

This choice of direction automatically makes $\hat{n}$ the outward unit normal if the surface element belongs to a closed surface. Moreover, it establishes a direction for $\hat{n}$ in the case of a surface for which an outward unit normal is meaningless (such as a finite planar surface). The vector $\hat{n}_{(x,y,z)}$ is then transformed to the (u,v,w)-space using the equation

$$\hat{n}_{(u,v,w)} = T\,\hat{n}_{(x,y,z)} \tag{1.14}$$

The cell $(v_i, w_j)$ whose area is $\Delta v\,\Delta w$ corresponds to a surface element whose area is denoted by $\Delta A$. It is evident that we have

$$\Delta A = \frac{\Delta v\,\Delta w}{(\hat{n}\cdot\hat{u})} = \frac{\Delta v\,\Delta w}{n_u} \tag{1.15}$$

Therefore, the force $\Delta\vec{F}$ exerted on this surface element is given by

$$\Delta\vec{F} = \vec{p}\,\Delta A \tag{1.16}$$

where $\vec{p}$ is the solar radiation pressure vector acting on the surface element. Under very general conditions of surface reflection and re-radiation, it can be shown that this pressure vector has the form

$$\vec{p} = -\frac{S\cos\theta}{c}\left[C_1\hat{u} + (C_2\cos\theta + C_3)\hat{n}\right] \tag{1.17}$$

where S is the solar radiation flux per unit area normal to the flux, c is the velocity of light, and $\theta$ is the angle between the sun vector and the normal to the surface element, i.e.,

$$\cos\theta = \hat{n}\cdot\hat{u} \tag{1.18}$$

The coefficients $C_1$, $C_2$ and $C_3$ may change with time due to aging of of the surface material by some complex process.

For the case of specular reflection and diffuse (Lambertian) reflection, the $C_k$'s are given by [1]

$$C_1 = (1 - k_1) \tag{1.19}$$

$$C_2 = 2 k_1 \tag{1.20}$$

$$C_3 = \tfrac{2}{3} k_2 \tag{1.21}$$

where $k_1$ = the fraction of incident radiation reflected specularly

$k_2$ = the fraction of incident radiation reflected diffusely by a Lambertian surface.

It is to be noted that in equations (1.19) - (1.21), it is not implicitly assumed that the surface is radiating the entire energy incident on it, i.e., it is not necessary that we require the condition $k_1 + k_2 = 1$ in order to obtain these equations.

For the case of specular reflection and non-Lambertian reflection and re-radiation [2], a little consideration will reveal that the $C_k$'s are given by

$$C_1 = (1 - \beta\gamma) \tag{1.22}$$

$$C_2 = 2\beta\gamma \tag{1.23}$$

$$C_3 = B_f \left[ \gamma(1-\beta) + \frac{(B_f e_f - B_b e_b)}{B_f (e_f + e_b)} (1-\gamma) \right] \tag{1.24}$$

where $\gamma$ = the fraction of incident radiation reflected (specularly and otherwise)

$\beta$ = the fraction of reflected radiation that is specular

$B_f$, $B_b$ = non-Lambertian coefficients for front and back surfaces respectively

$e_f$, $e_b$ = emission values for front and back surfaces respectively.

In passing, it may be noted that we have the relations

$$k_1 = \beta \gamma \tag{1.25}$$

$$k_2 = \gamma (1-\beta) \tag{1.26}$$

Moreover, it may be remarked that the form of equation (1.17) is valid for the more general non-Lambertian reflection and re-radiation which have a period of $\pi$ in the azimuthal variable. In other words, Lambertian reflection means that the intensity I of the reflection is given by

$$I = I_0 \cos\theta \tag{1.27}$$

Then, the case of non-Lambertian reflection and re-radiation expressed by equation (1.24) would correspond to an intensity which is independent of the azimuthal variable $\phi$ and is of the form

$$I = I_0 f(\theta) \tag{1.28}$$

where implicitly we exclude the case of Lambert's law, i.e.,

$$f(\theta) \neq \cos\theta \tag{1.29}$$

The even more general case means that we can have reflection and re-radiation for which the intensity is of the form

$$I = I_0 f(\theta, \phi) \tag{1.30}$$

where $$f(\theta, \phi) = f(\theta, \phi + \pi) \tag{1.31}$$

Finally, to compute the total force $\vec{F}$ due to solar radiation, we obtain from equations (1.15) - (1.19) the following expressions

$$F_u = -\Delta v \, \Delta w \, \frac{S}{c} \sum_{\{v_i, w_j\}} \left[ C_1 + (C_2 n_u + C_3) n_u \right] \tag{1.32}$$

$$F_v = -\Delta v \, \Delta w \, \frac{S}{c} \sum_{\{v_i, w_j\}} \left[ (C_2 n_u + C_3) n_v \right] \tag{1.33}$$

$$F_w = -\Delta v \, \Delta w \, \frac{S}{c} \sum_{\{v_i, w_j\}} \left[ (C_2 n_u + C_3) n_w \right] \tag{1.34}$$

It is also trivial to compute the total torque $\vec{M}$ on the spacecraft by using the equations

$$\Delta \vec{M} = \vec{r} \times \Delta \vec{F} \tag{1.35}$$

$$\vec{M} = \sum_{\{v_i, w_j\}} (\vec{r} \times \Delta \vec{F}) \tag{1.36}$$

but this will not be done here.

## SECTION 3 - DISCUSSION

It is obvious that the method just discussed does not encounter logic problems in determining self-shadowing. Moreover, because the points $\{v_i, w_j\}$ are first chosen on the projection plane, it circumvents the difficulties in book-keeping experienced in the other method of choosing vertices on the surface of the object. Furthermore, it does not require excessive core for storing the vertex data such as coordinates, area of surface element, normal vector, solar incidence angle, etc. This advantage becomes evident by evaluating the expressions in equations (1.32) - (1.34) using three accumulators (one for each force component), not having to store the set of points $\{v_i, w_j\}$ . Finally, if greater accuracy is desired, it suffices only to choose smaller values $\Delta v^* \Delta w^*$, multiply the previous result by the factor $\left(\frac{\Delta v^* \Delta w^*}{\Delta v \ \Delta w}\right)$ , and then perform computations only for the additional points newly introduced into the set $\{v_i, w_j\}$. This advantage cannot be realized in the other method of choosing vertices on the surface of the object. In that case, in going to a refined model with additional vertices, it is necessary to perform the entire computations starting from the beginning each time.

## SECTION 4 - CONCLUSION

From the foregoing discussion, it may be concluded that the present method has the following advantages:

1. It does not experience logic problems in determining self-shadowing.
2. It does not encounter the book-keeping problems arising in the case of choosing vertices on the surface of the object.
3. It does not require excessive core for storing vertex data.
4. It can utilize previously obtained results in going to progressively more refined models.

## REFERENCES

(1) Eliasberg, P. E., "Introduction to the Theory of Flight of Artificial Earth Satellites," Moscow (1965). Translation, Jerusalem (1967).

(2) Wright, J., "Solar Sail Force Model," Jet Propulsion Laboratory Interoffice Memorandum 312/77.3-201 (28 March 1977).