A MATHEMATICAL MODEL OF LANDSAT-D ATTITUDE DYNAMICS WITH INTERNAL MOTION

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ABSTRACT

An algorithm to model the effects of internal motion by the solar array and the high-gain antenna on the attitude of the Landsat-D spacecraft is presented here. The relative torque and angular momenta arising from the internal motions are assumed to be attitude-independent but are considered to be a source of attitude perturbations. The equation of motion for the three-body problem is derived and then compared with the one-body case. The effect of the internal motion on the control of the spacecraft is shown in a computer study of the problem.

1. INTRODUCTION

The paper presents algorithms for modeling the effects of internally moving parts on the attitude of the Landsat-D (LSD) spacecraft. The internal motions considered here include the rotations of the solar array to follow the Sun and the gimballed high-gain antenna to communicate with the Tracking and Data Relay Satellite (TDRS) (Reference 1). The LSD system is treated as a rigid three-body system for describing the equation of motion. Modeling the disturbance torques produced by moving appendages is very important for missions such as Landsat-D, which require accurate knowledge of the attitude and precise control of the spacecraft.

The relative torques and angular momenta arising from the internal motions are considered as attitude-independent variables and as a source of attitude perturbations. The external disturbance torques and the angular momenta caused by the internal motions are generated in a profile program (called PROFILE) on an IBM S/360-95 computer, where null attitudes are assumed and are transmitted to a truth model on a DEC PDP-11/70 computer that simulates the effects on the attitude.

In this discussion, nonstandard rotations such as a 45-degree slew of the solar array to avoid interference with the antenna and the switching motion of the antenna from one TDRS to another are neglected. In addition to the rotational motions of the solar array and the antenna, the LSD spacecraft contains moving parts such as the thematic mapper and multispectral scanner (Reference 2). However, these motions are disregarded here because the motions are oscillatory with a high frequency (≈7 Hertz) and because they generate zero average angular momenta.

Section 2 discusses the mathematical derivations of the equation of motion and pertinent terms such as the moment of inertia (MOI) tensor and the center of mass (CM). When possible, these terms are compared with the form for the one-body system used by the Multimission Modular Spacecraft (MMS)/Solar Maximum Mission (SMM) spacecraft. Section 3 provides simulation results to compare the three-body and one-body cases. Conclusions resulting from the study are presented in Section 4.

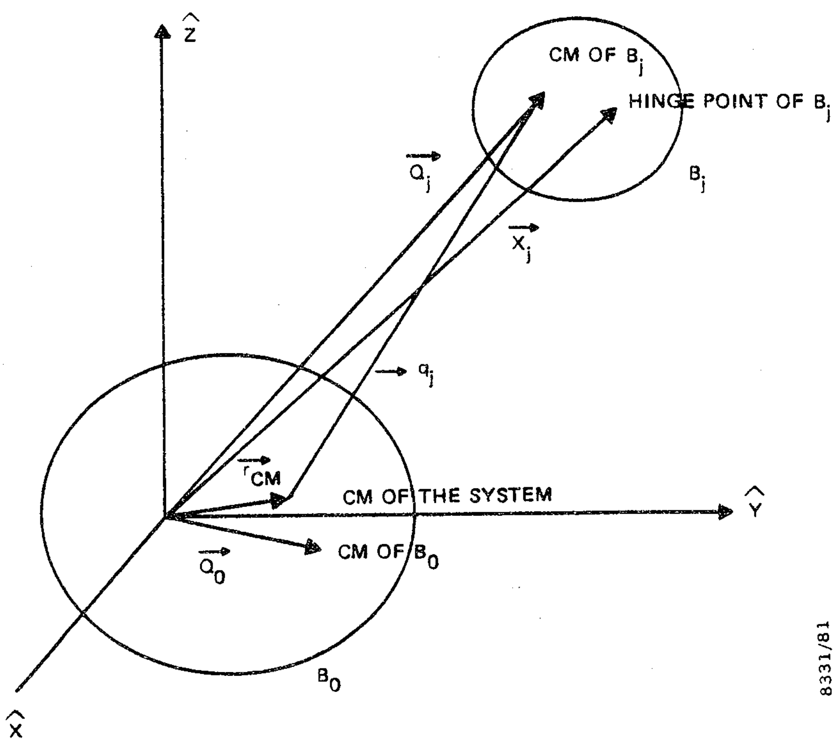
2. ANALYTICAL CONSIDERATIONS

This section presents the mathematical modeling to describe the dynamic effects of the moving parts on the motion of the spacecraft. The equation of motion for the LSD mission is referenced at the CM of the entire system but is represented in a coordinate system that is fixed in the main vehicle. The CM of the entire system is calculated as a function of time. The MOI tensors for the moving parts are reevaluated with respect to a set of time-independent axes parallel to a set in the main vehicle. Also calculated are the angular velocity of the appendages and the perturbation in the external torques due to the changing positions of the appendages. A comparison with the one-body problem is made.

2.1 COORDINATE SYSTEMS AND TRANSFORMATIONS MATRICES

The system under consideration, shown in Figure 1, consists of the main carrier vehicle, designated as body B_o, and $n(=2)$ moving bodies B_j $(j=1, n)$. Several coordinate systems are convenient for discussing the relative motions. These are as follows:

- Geocentric Inertial Coordinate System (GCI) (Reference 3)
- Orbit-Defined Coordinate System (OCS) where X (roll) is nearly along the spacecraft velocity vector, Y (pitch) is along the orbit normal vector, and Z (yaw) is along the nadir vector
- Spacecraft-Fixed Coordinate System (BCS), which is fixed in the main vehicle B_o
- Coordinate systems fixed in moving parts such as in the solar array (SACS) or in the high-gain antenna (ANTCS)



NOTE: $\vec{Q}_j$ = the CM of B_j
$\vec{r}_{CM}$ = the CM of the entire system
$\vec{q}_j$ = the CM of B_j from $\vec{r}_{CM}$
$\vec{X}_j$ = the hinge point of B_j
$\vec{\omega}_j$ = the angular velocity of B_j in inertial space
$\vec{\omega}_j$ = the angular velocity of B_j relative to the main body B_0 ($\vec{\omega}_j = \vec{\omega}_0 + \vec{\omega}'_j$)

Figure 1. Partitioning of the Satellite Into Main Body and Moving Parts

The transformation matrices (TRMA) to be used in this paper are defined as follows:

1. <u>TRMA from GCI to OCS : [O]</u>

$$[O] = \begin{bmatrix} \dfrac{(\hat{R}_I \times \hat{V}_I) \times \hat{R}_I}{|\hat{R}_I \times \hat{V}_I|} \\ \hline \dfrac{\hat{R}_I \times \hat{V}_I}{|\hat{R}_I \times \hat{V}_I|} \\ \hline \hat{R}_I \end{bmatrix} \tag{2-1}$$

where $\hat{R}_I$ and $\hat{V}_I$ denote the spacecraft position relative to the Earth and velocity unit vectors in the GCI frame, respectively.

2. <u>Attitude direction cosine matrix from the OCS to the BCS : [A]</u>. In the PROFILE Program [A] is given by the identity matrix because null attitudes are assumed. In the truth model, it is represented as

$$[A] = \begin{bmatrix} 1 & y & -p \\ -y & 1 & r \\ p & -r & 1 \end{bmatrix} \tag{2-2}$$

using the small angle approximation, which is sufficient and valid, since only small perturbations are assumed; r, p, and y denote roll, pitch, and yaw angles in radian units, respectively.

3. <u>TRMA from BCS to SACS : $[C^{(SA)}]$</u>. The solar array rotates around the $\hat{y}$-axis and is driven to follow the Sun. Thus, its orientation is determined from the Sunline angle α

$$[C^{(SA)}] = [\alpha]_y = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad (2\text{-}3)$$

Given the Sun unit vector, $\hat{S} = (S_x, S_y, S_z)^T$, in the BCS, the rotation angle α is given by

$$\alpha = \tan^{-1}\left(\frac{S_x}{S_z}\right) \quad (2\text{-}4)$$

because the Sun vector is perpendicular to the $\hat{x}$-axis of the SACS.

4. <u>TRMA from BCS to ANTCS : $[C^{(ANT)}]$</u>. The antenna has two gimbals with the inner gimbal angle, g_2, representing the elevation angle and the outer gimbal angle, g_1, representing the azimuth angle. The orientation of the antenna is determined from the gimbal angles

$$[C^{(ANT)}] = \left[g_2\right]_y \left[g_1\right]_z$$

$$= \begin{bmatrix} \cos g_1 \cos g_2 & \sin g_1 \cos g_2 & -\sin g_2 \\ -\sin g_1 & \cos g_1 & 0 \\ \cos g_1 \sin g_2 & \sin g_1 \sin g_2 & \cos g_2 \end{bmatrix} \quad (2\text{-}5)$$

The unit vector pointing from the spacecraft to TDRS is represented by $\hat{P}$ where $\hat{P} = (P_x, P_y, P_z)^T$ in the BCS.

The gimbal angles are thus given by

$$g_1 = \tan^{-1} (P_y/P_x) \tag{2-6a}$$

and

$$g_2 = -\sin^{-1} P_z \tag{2-6b}$$

since g_1, g_2 should align the antenna boresight (the $\hat{x}$-axis in ANTCS) with the normalized pointing vector $\hat{P}$. ($\hat{P}$ can be obtained from the spacecraft and TDRS ephemerides.)

2.2 ANGULAR VELOCITY OF MOVING PARTS

The angular velocity of the moving parts is used to calculate the internal angular momentum of the spacecraft for use in the equation of motion. It is easily seen from Equation (2-3) that the angular velocity of the solar array is as follows:

$$\vec{\omega}'_{SA} = \frac{d\alpha}{dt} \hat{y} \tag{2-7a}$$

The time derivative of the rotation angle α can be computed numerically

$$\frac{d\alpha}{dt} = \frac{\alpha(t) - \alpha(t - \Delta t)}{\Delta t} \tag{2-7b}$$

Using Equation (2-5) the angular velocity of the high-gain antenna is

$$\vec{\omega}'_{ANT} = \frac{dg_1}{dt}\,\hat{z} + \frac{dg_2}{dt}\left[g_1\right]_z\,\hat{y} = \begin{pmatrix} \sin g_1 & \frac{dg_2}{dt} \\ \cos g_1 & \frac{dg_2}{dt} \\ \frac{dg_1}{dt} & \end{pmatrix} \tag{2-8a}$$

where

$$\frac{dg_i}{dt} = \frac{g_i(t) - g_i(t - \Delta t)}{\Delta t} \tag{2-8b}$$

For SMM, the angular velocity of the moving parts was not calculated.

2.3 CENTER OF MASS

For LSD, the CM of appendage B_j in the BCS is given by

$$\vec{Q}_j(t) = \left[C^{(j)}(t)\right]^T (\vec{Q}_{j0} - \vec{X}_j) + \vec{X}_j \tag{2-9}$$

where $\vec{Q}_{j0}$ represents the CM of B_j at the initial time (see Figure 1). The rotation (or hinge) point is denoted by $\vec{X}_j$ and $\vec{Q}_{j0} - \vec{X}_j$ represents the CM of B_j from the hinge point at the initial time. Then, at any later time, the CM will be represented by the first term of the right-hand side of Equation (2-9). The CM of each appendage changes as a function of time because the high-gain antenna rotates to track the TDRS, and the solar array rotates to track the

Sun. Consequently, the CM of the system, $\vec{r}_{CM}$, changes in time and is represented by

$$\vec{r}_{CM}(t) = \sum_{r=0}^{n} M_r \, \vec{Q}_r(t) \Big/ \sum_{r=0}^{n} M_r \qquad (2\text{-}10a)$$

and the position of the CM of each appendage with respect to the CM of the system is

$$\vec{q}_j(t) = \vec{Q}_j(t) - \vec{r}_{CM}(t) \qquad (2\text{-}10b)$$

For SMM, the CM of the system was fixed in time in the BCS.

2.4 MOMENT OF INERTIA TENSOR OF THE SYSTEM

The MOI of the system, $[I_T]$, relative to axes parallel to the BCS axes passing through r_{CM} is expressed by

$$[I_T(t)]_{lm} = \sum_{r=0}^{n} \int dm_r \left\{ [\vec{q}_r(t) + \vec{\rho}_r]^2 \quad \delta_{lm} - [\vec{q}_r(t) + \rho_r]_l \; [\vec{q}_r(t) + \rho_r]_m \right\} \qquad (2\text{-}11)$$

where $\vec{\rho}_r$ is the position vector of the mass dm_r of body B_r relative to the CM of B_r and the subscripts l and m represent the l and m components of the vector or tensor. Note that because $\vec{q}_r$ is time-dependent, $[I_T]_{lm}$ is also dependent on time; in the remainder of this paper, the explicit time-dependence will be dropped.

The above equation can be written as

$$[I_T]_{lm} = \sum_{r=0}^{n} \left\{ M_r \left[q_r^2 \delta_{lm} - (q_r)_l \ (q_r)_m \right] + \left[\tilde{I}^{(r)} \right]_{lm} \right\} \quad (2\text{-}12)$$

since

$$\int dm_r \ \vec{\rho}_r = 0$$

$[\tilde{I}^{(r)}]$ is the MOI tensor of B_r represented in the BCS frame but relative to the CM of B_r:

$$[\tilde{I}^{(r)}] = [C^{(r)}]^T \ [I^{(r)}] \ [C^{(r)}] \quad (2\text{-}13)$$

where $[I^{(r)}]$ is the MOI of B_r represented in the coordinate system fixed in B_r. Equation (2-12) can be simply reexpressed by

$$[I_T] = \sum_{r=0}^{n} [J^{(r)}] \quad (2\text{-}14)$$

with

$$\left[J^{(r)} \right]_{lm} = \left[\tilde{I}^{(r)} \right]_{lm} + M_r \left\{ q_r^2 \delta_{lm} - (q_r)_l \ (q_r)_m \right\} \quad (2\text{-}15)$$

For the one-body problem, as represented by SMM, I is defined to be a constant in time.

2.5 EXTERNAL TORQUES

Two external torques are discussed: the gravity gradient torque and the aerodynamic torque. The solar radiation torque is similar to the aerodynamic torque, and the other external torques are not sensitive to the three-body problem.

The gravity gradient torque, N_{GG}, can be computed by

$$\vec{N}_{GG} = -\mu \sum_{r=0}^{n} \int (\vec{q}_r + \vec{\rho}_r) \times \frac{\vec{R} + \vec{\rho}_r + \vec{q}_r}{|\vec{R} + \vec{\rho}_r + \vec{q}_r|^3} \, dm_r \qquad (2\text{-}16)$$

where μ is the Earth gravitational constant ($\simeq 3.986005 \times 10^{14}$ m^3/sec^2). $\vec{R}$ is the spacecraft position vector from the Earth. Considering that $|\vec{R}| >> |\vec{q}_r + \vec{\rho}_r|$, $\vec{N}_{GG}$ is simply,

$$\begin{aligned} \vec{N}_{GG} &= \frac{3\mu}{R^3} \sum_{r=0}^{n} \int dm_r (\vec{q}_r + \vec{\rho}_r) \times \hat{R}\left[\hat{R} \cdot (\vec{q}_r + \vec{\rho}_r)\right] \\ &= \frac{3\mu}{R^3} \sum_{r=0}^{n} \left\{ M_r \, \hat{q}_r \times \hat{R}(\hat{q}_r \cdot \hat{R}) + \hat{R} \times [\tilde{I}^{(r)}] \, \hat{R} \right\} \qquad (2\text{-}17) \\ &= \frac{3\mu}{R^3} \sum_{r=0}^{n} \hat{R} \times [J^{(r)}] \, \hat{R} \\ &= \frac{3\mu}{R^3} \hat{R} \times [I_T] \, \hat{R} \end{aligned}$$

The expression for the one-body system has the same form except for the replacement of $[I_T]$ by the constant $[I]$.

To simplify the calculation of the solar radiation and aerodynamic torques, the LSD spacecraft is modeled as an assembly of a cylinder for the main vehicle, flat plates for the solar array panels, and a sphere for the antenna. Only the aerodynamic torque is discussed here because the modifications to the center of pressure (CP) are common in solar radiation and aerodynamic torques.

The aerodynamic torque, $\vec{N}_{aero}$, is

$$\vec{N}_{aero} = -\frac{1}{2} C_D \rho v^2 \sum_{i=1}^{8} \int \hat{n}_i \cdot \hat{v}(\vec{q}_{cp,i} \times \hat{v}) \, dA_i \qquad (2\text{-}18)$$

Here, $\hat{v}$ denotes the spacecraft velocity unit vector, $\hat{n}_i$ denotes the normal unit vector for the ith surface, $\vec{q}_{cp,i}$ denotes the CP of the ith surface from $\vec{r}_{CM}$, ρ denotes the atmospheric density, and C_D denotes the drag coefficient. The normal vectors, $\hat{n}_i$, for the solar array and antenna surfaces are dependent on time by

$$\hat{n}_i = \left[C^{(i)}\right]^T \hat{n}_{io} \qquad (2\text{-}19)$$

where $\hat{n}_{io}$ represents the initial normal vector for the ith surface. $\vec{q}_{cp,i}$ for the solar array and antenna are computed by

$$\vec{q}_{cp,i} = \vec{Q}_{cp,i} - \vec{r}_{CM} \qquad (2\text{-}20)$$

with

$$\vec{Q}_{cp,i} = \left[C^{(i)}\right]^T (\vec{Q}_{cp,io} - \vec{X}_i) + \vec{X}_i \qquad (2\text{-}21)$$

More consideration is required to specify $[C^{(i)}]$ for the solar array surfaces that are canted. The transformation matrix from BCS to these surfaces, $[C^{(i)}]$, is given by

$$[C^{(i)}] = [C^{(SA)}] \; [\theta_C]_X \qquad (2\text{-}22)$$

with the canted angle θ_C.

For the one-body case of SMM, n_i and $\vec{q}_{cp,i}$ are constants.

2.6 EQUATION OF MOTION

The equation of motion for the LSD spacecraft is written in the form

$$\frac{d\vec{Y}}{dt} = \vec{f}(\vec{Y}(t), t) \tag{2-23}$$

where $\vec{Y} = (q_\mu, \vec{L}_T, \vec{L}_W)^T$; q_μ (μ = 1, 2, 3, 4) denotes the Euler symmetric parameters representing a rotation from the GCI to the spacecraft-fixed coordinate frame, $\vec{L}_T$ is the total angular momentum of the spacecraft, and $\vec{L}_W$ is the wheel momentum.

The body angular momentum of the main vehicle, $\vec{L}_B$, is given by the total spacecraft angular momentum minus the sum of the wheel momentum, payload momentum, $\vec{L}_R$, and the angular momentum, $\vec{L}_{INT}$, caused by the internal motions

$$\vec{L}_B = \vec{L}_T - \vec{L}_{INT} - \vec{L}_W - \vec{L}_R \tag{2-24}$$

$\vec{L}_B$ depends on the angular velocity of the main vehicle, $\vec{\omega}_0$, and $\vec{L}_{INT}$ depends on the angular velocity of moving parts, $\vec{\omega}_j'$. To formulate these mathematically, the angular momentum of the total system, $\vec{L}_T$, ignoring wheel and payload momenta, is considered

$$\begin{aligned} \vec{L}_T' &= \vec{L}_B + \vec{L}_{INT} \\ &= \sum_{r=0}^{n} \int (\vec{q}_r + \vec{\rho}_r) \times (\dot{\vec{q}}_r \times \dot{\vec{\rho}}_r)\, dm_r \\ &= \sum_{r=0}^{n} \left\{ M_r\, \vec{q}_r \times \dot{\vec{q}}_r + [\tilde{I}^{(r)}]\, (\vec{\omega}_o + \vec{\omega}_r') \right\} \end{aligned} \tag{2-25}$$

With some computation, L'_T can be shown as

$$\vec{L}'_T = [I_T]\ \vec{\omega}_o + \sum_{r=0}^{n} \left\{ [J^{(r)}] + [K^{(r)}]\ \vec{\omega}'_r \right\} \tag{2-26}$$

where

$$[K^{(r)}]_{lm} = M_r \left\{ \vec{q}_r \cdot (\vec{r}_{CM} - \vec{X}_r)\ \delta_{lm} - (\vec{r}_{CM} - \vec{X}_r)_l\ q_m \right\}$$

Thus, the body rate of the main carrier is simply

$$\vec{\omega}_o = [I_T]^{-1}\ \vec{L}_B \tag{2-27}$$

and $\vec{L}_{INT}$ caused by the internal motion, is

$$\vec{L}_{INT} = \sum_{r=1}^{n} \left\{ [J^{(r)}] + [K^{(r)}]\ \ \vec{\omega}'_r \right\} \tag{2-28}$$

The time derivatives of the Euler symmetric parameter, q_μ, can be obtained as

$$\frac{dq_\mu}{dt} = \frac{1}{2}\ [\Omega(\vec{\omega}_o)]_{\mu\nu}\ q_\nu \tag{2-29}$$

with

$$[\Omega(\vec{\omega})] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & \omega_4 \end{bmatrix} \tag{2-30}$$

The time derivative of the total angular momentum of the spacecraft is given by the Euler equation as

$$\frac{d\vec{L}_T}{dt} = \vec{N}_{ext} + \vec{L}_T \times \vec{\omega}_o \tag{2-31}$$

For SMM, the body angular momenta, $\vec{L}_B$, is given by

$$\vec{L}_B = \vec{L}_T - \vec{L}_W - \vec{L}_R \tag{2-32}$$

with the payload momentum $\vec{L}_R$. The spacecraft body rate, $\vec{\omega}$, is determined by

$$\vec{\omega} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = [I]^{-1} \vec{L}_B \tag{2-33}$$

where $[I]^{-1}$ is the inverse of the spacecraft MOI tensor. The time derivatives of the Euler symmetric parameters, q_μ, can be obtained as

$$\frac{dq_\mu}{dt} = \frac{1}{2} [\Omega(\vec{\omega})]_{\mu\nu} q_\nu \tag{2-34}$$

The time derivatives of the total angular momentum of the spacecraft are given by the Euler equation as

$$\frac{d\vec{L}_T}{dt} = \vec{N}_{ext} + \vec{L}_T \times \vec{\omega} \tag{2-35}$$

with the external torque, $\vec{N}_{ext}$.

3. SIMULATION RESULTS

A computer study of the effect of the three-body problem on the motion of the spacecraft has been made using the general equations derived here. Since the spacecraft is subject to noticeable external torques, a control law that provides compensatory torques was necessary to keep the spacecraft near null attitude. The one-body case, using the same control law, was also studied.

The roll, pitch, and yaw of the spacecraft main carrier for both cases is shown in Figures 2 through 4. The results of the three-body case are represented by the "X" points and the results of the one-body case are shown as open circles. Note that both cases are subject to the same control law. This control law attempts to make the pitch, roll, and yaw zero and to bring the spacecraft rate to null. This control law is the same one (Reference 4) that Landsat-D will use during its acquisition phases. The torque applied to each reaction wheel is as follows:

for the roll axis,

$$T_r = K_r(k_r \Delta r + \omega_r) \tag{3-1a}$$

for the pitch axis,

$$T_p = K_p \; [k_p(\Delta p + B) + \omega_p] \tag{3-1b}$$

and for the yaw axis,

$$T_y = K_y \; [\omega_y - k \; k_y \; \omega_r] \tag{3-1c}$$

where Δr and Δp are the roll and pitch attitude errors as determined by an Earth sensor; K_r, K_p, K_y, k_r, k_p, k_y, and k

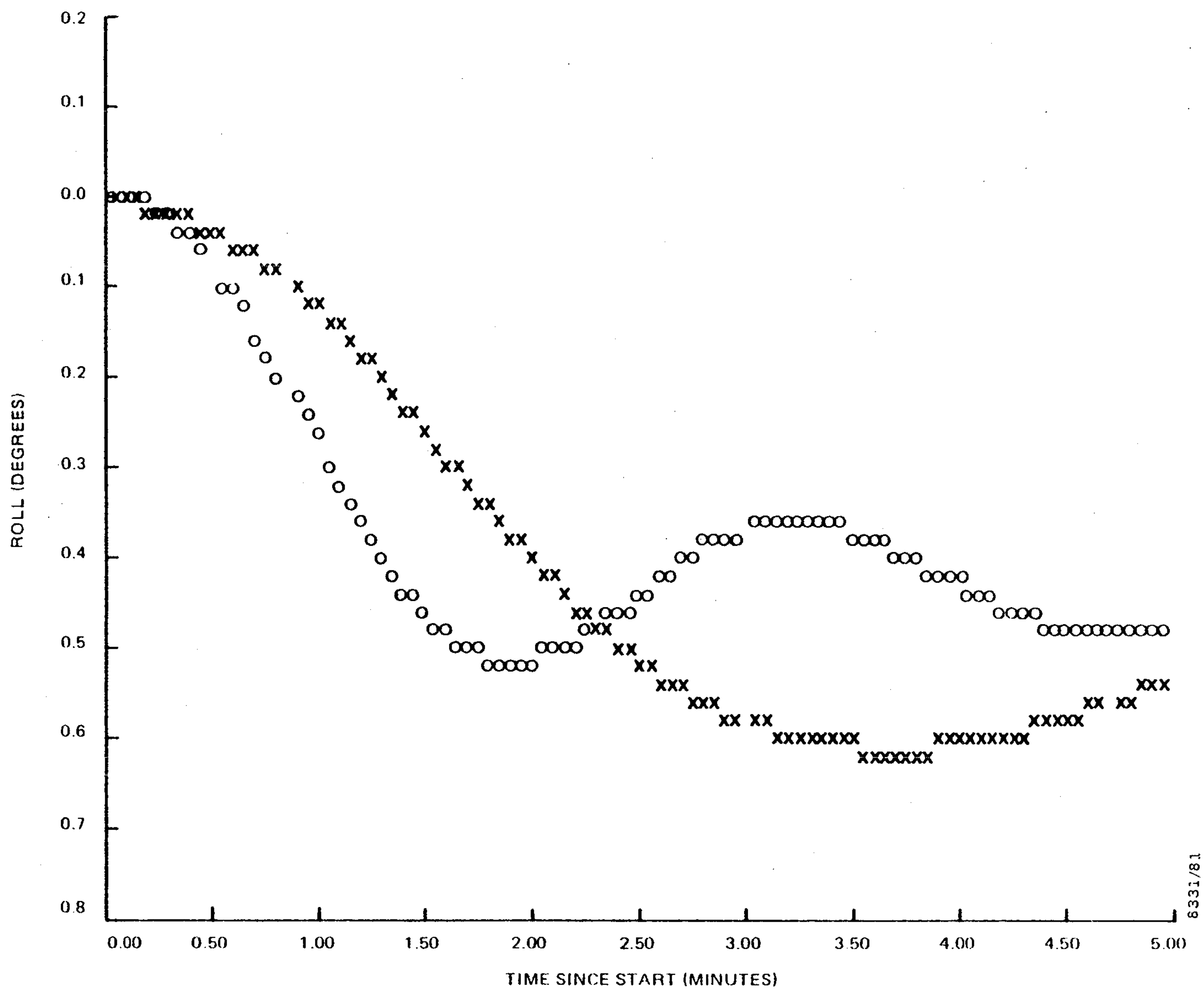


Figure 2. Spacecraft Roll Versus Time

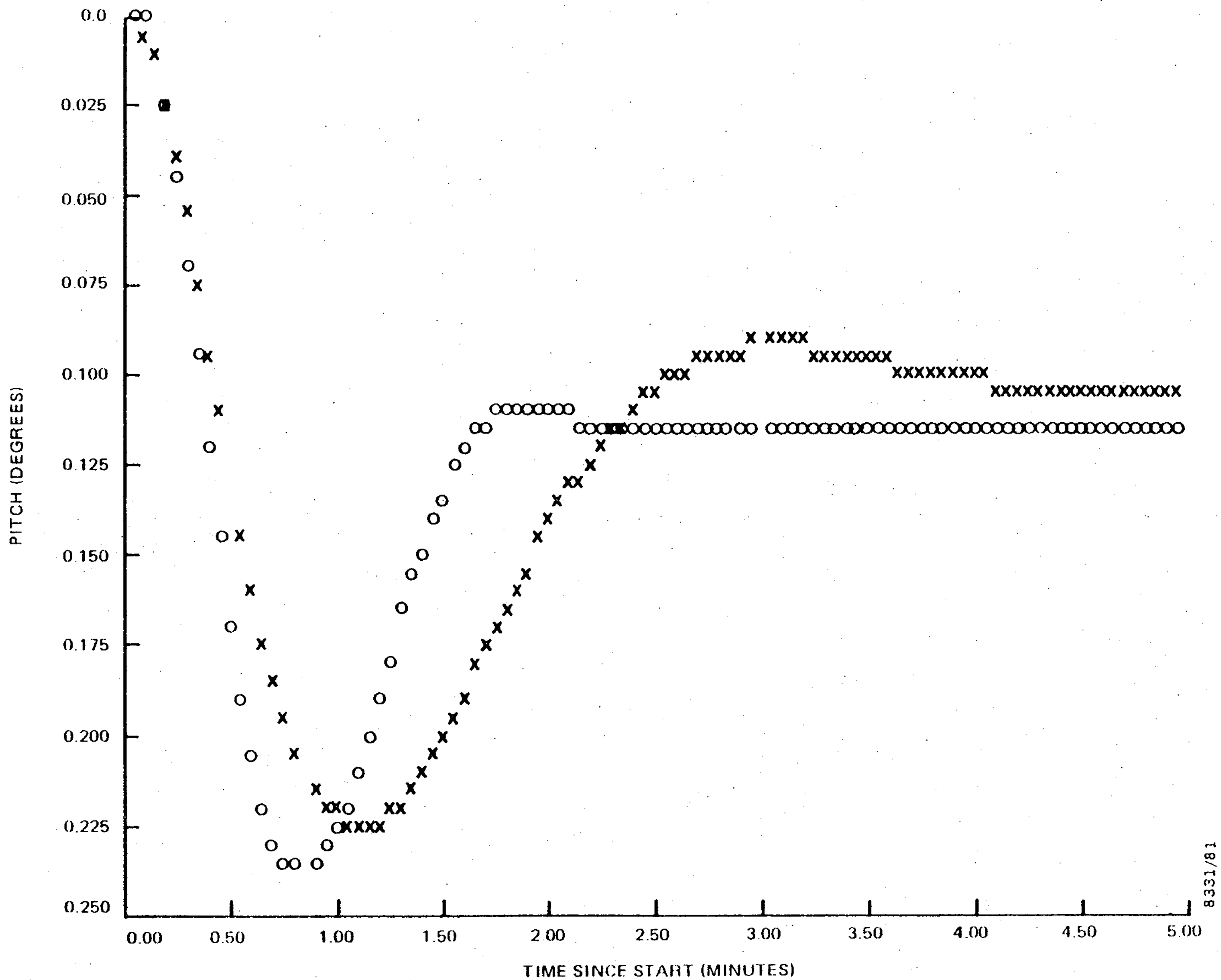


Figure 3. Spacecraft Pitch Versus Time

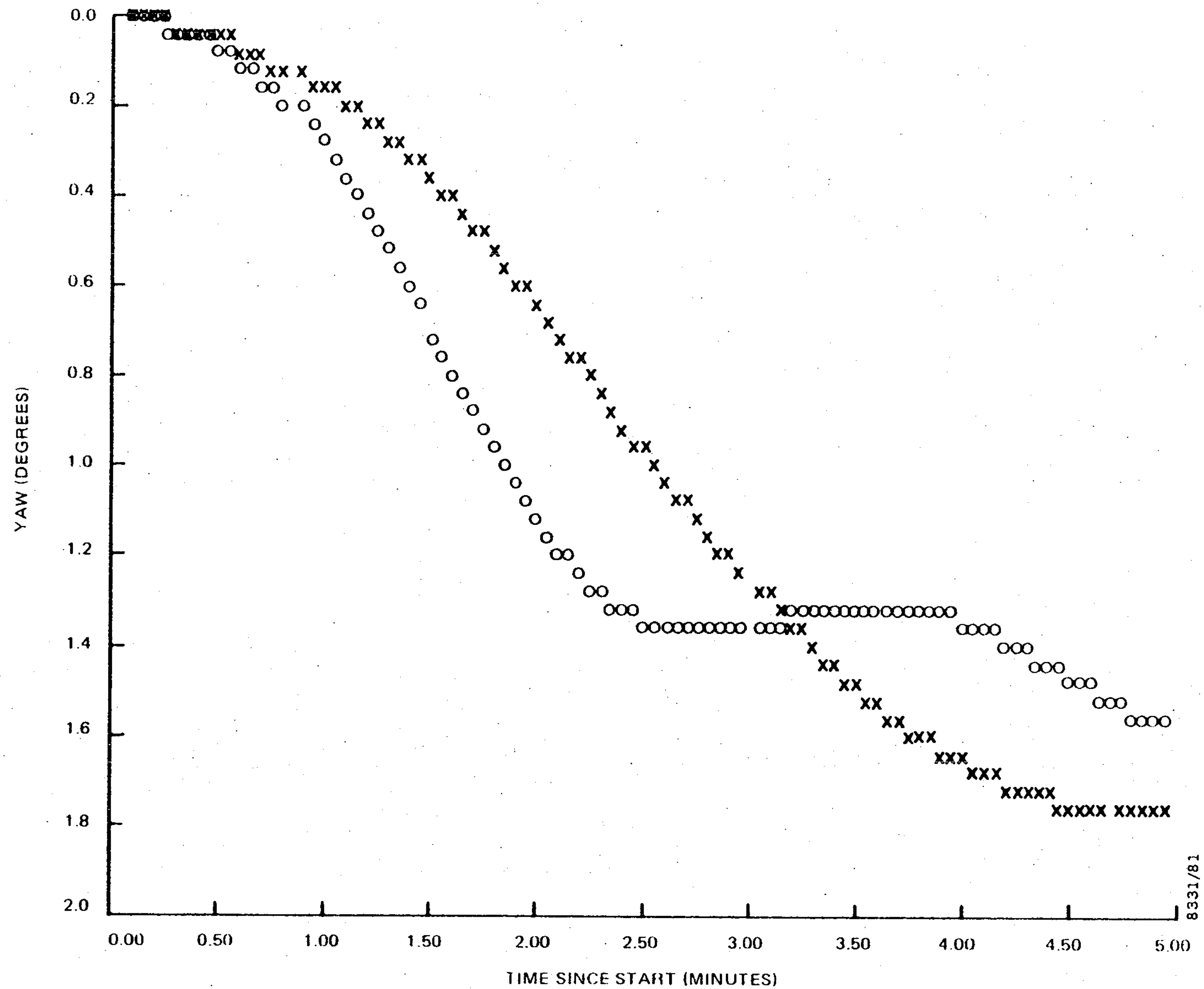


Figure 4. Spacecraft Yaw Versus Time

are constants; B is a bias to compensate for the orbital rotation; and ω_r, ω_p, and ω_y are the angular velocity along the roll, pitch, and yaw data. Because of the values used for k_r, k_p, and k_y, the control law is much more sensitive to the spacecraft rate than to the attitude error.

Most of the structure seen in the plots is a result of the control law. However, since the control law is the same, the differences in the plots are a result of the three-body problem. Note in Figure 2 that after 4.5 minutes the control law has the roll rate to zero for the one-body problem but not the three-body problem. Likewise, after 2.5 minutes, the pitch rate of the one-body problem is under control.

4. CONCLUSIONS

The conversion of the rigid one-body problem to the three-body problem has added another dimension to the study of dynamics. Although the exact perturbations in motion are obscured by the control law used, the effects are still important in control of the spacecraft.

The algorithms used in this paper can be applied to other spacecraft such as the Space Telescope to study important low-frequency effects, as in this paper, and also higher frequency effects that will cause jitter.

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