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VECTOR WIND PROFILE GUST MODEL

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SECTION I. INTRODUCTION

This report summarizes results from a study which had the objective of developing a vector wind gust model that is suitable for orbital flight test operations and trade studies. Detailed background information concerning earlier work can be found in References 1 and 2. In the work reported here, emphasis was given to verification of the hypothesis that gust component variables are gamma distributed, gust modulus is approximately Weibull distributed, and zonal and meridional gust components are bivariate gamma distributed. Section II describes a method of testing for bivariate gamma distributed variables; in Section III, two distributions for gust modulus are described, and the results of extensive hypothesis testing of one of the distributions are presented; Section IV establishes the validity of the gamma distribution for representation of gust component variables. Conclusions are presented in Section V.

SECTION II. TESTING FOR BIVARIATE GAMMA DISTRIBUTED VARIABLES

The hypothesis that absolute component gust and associated gust length are bivariate gamma distributed can be tested according to the procedure described below.

The probability density function of bivariate gamma distributed variables is

$$f(x,y;\gamma_x=\gamma_y=\gamma,\rho) = \frac{(\beta_x\beta_y)^{\gamma}}{\Gamma(\gamma)(1-\rho)}\left(\frac{xy}{\rho\beta_x\beta_y}\right)^{\frac{\gamma-1}{2}} \mathrm{EXP}\left(-\frac{\beta_x X+\beta_y Y}{1-\rho}\right)$$

$$\bullet \quad I_{\gamma-1}\left\{\frac{2\sqrt{\rho\beta_x\beta_y XY}}{\rho}\right\} \qquad (1)$$

where $I_n\{\}$ is the modified Bessel function of the first kind of order n; β_x and β_y are scale parameters; and γ_x and γ_y are shape parameters of the gamma distributions of x and y, respectively. The parameter γ is the geometric mean of γ_x and γ_y. Estimation of γ_x, γ_y, β_x, and β_y from sample statistics is discussed by Adelfang (Ref. 1).

Dimensionless variables T_1 and T_2 are defined by

$$T_1 = \beta_x X$$
$$T_2 = \beta_y Y \, . \qquad (2)$$

The variables T_1 and T_2 can be expressed in a coordinate system that is rotated by 45°; the transformed variables z_1 and z_2 are given by

$$z_1 = \frac{\sqrt{2}}{2}(T_1+T_2) = \frac{\sqrt{2}}{2}(\beta_x X + \beta_y Y)$$
$$z_2 = \frac{\sqrt{2}}{2}(T_2-T_1) = \frac{\sqrt{2}}{2}(\beta_y Y - \beta_x X) \quad . \tag{3}$$

The probability density function of the transformed variables is given by

$$f(z_1,z_2) = \left(z_1^2 - z_2^2\right)^{\frac{\gamma-1}{2}} \mathrm{EXP}\left(-\frac{\sqrt{2}}{1-\rho} z_1\right)$$
$$\bullet \quad I_{\gamma-1}\left\{\frac{\sqrt{2\rho}}{1-\rho} \sqrt{z_1^2 - z_2^2}\right\} \quad . \tag{4}$$

The probability that bivariate distributed variables z_1 and z_2 will occur within the area bounded by the lines $z_1=z_1^*$, $z_1=z_2$, and $z_1=-z_2$ (illustrated in Figure 1) can be calculated by numerical integration of the equation

$$P_\Delta = \frac{\sqrt{2\Pi} \int_0^{z_1^*} z_1^{\gamma-\frac{1}{2}} e^{\frac{-\sqrt{2}z_1}{1-\rho}} I_{\gamma-\frac{1}{2}} \left\{\frac{\sqrt{2\rho}}{1-\rho} z_1\right\} dz_1}{(1-\rho)^{\frac{1}{2}} (\sqrt{2\rho})^{\gamma-\frac{1}{2}} \Gamma(\gamma)} \quad , \tag{5}$$

where

$$I_n(W) = \sum_{k=0}^{\infty} \frac{W^{n+2k}}{2^{n+2k}\, k!\, \Gamma(n+k+1)} \tag{6}$$

$$n = \gamma - \tfrac{1}{2}$$

$$W = \frac{\sqrt{2\rho}}{1-\rho} z_1 \quad .$$

Alternatively, P_Δ can be estimated from the series:

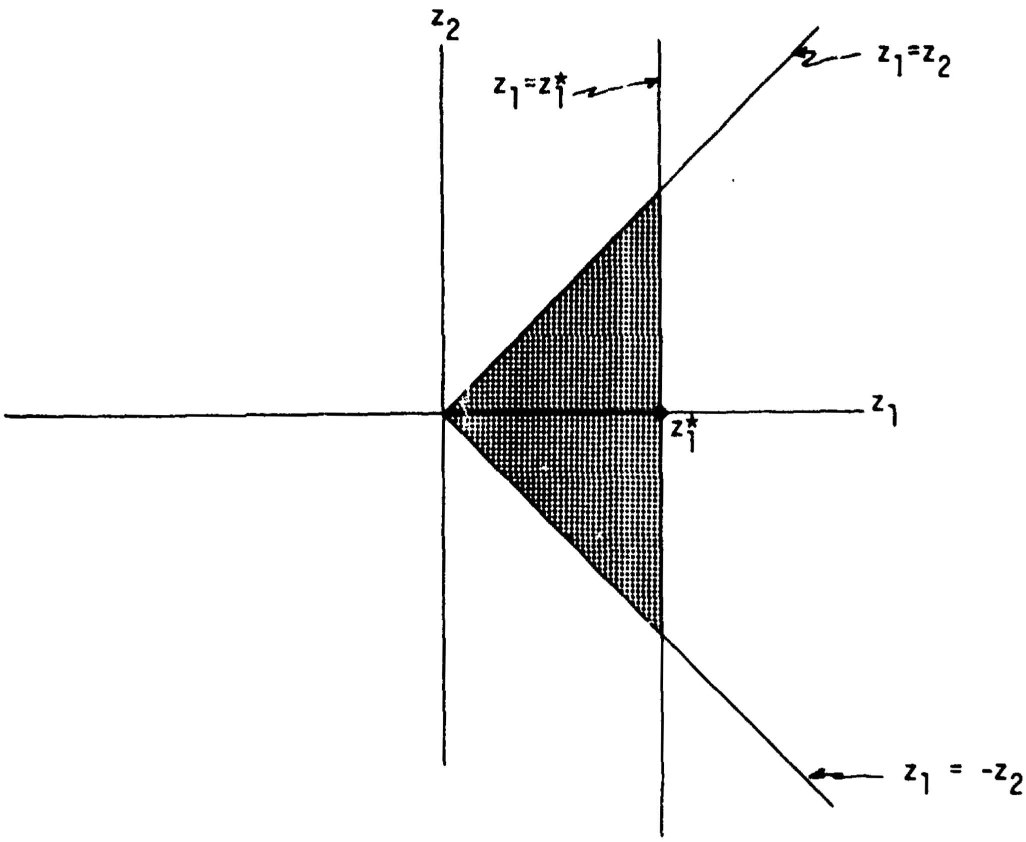


Figure 1. Area, Δ (Shaded), Which Bounds Bivariate Gamma Distributed Variables z_1 and z_2 for Which a Probability of Occurrence Can Be Calculated from Equation 7

$$P_\Delta = \frac{(1-\rho)^\gamma}{\Gamma(\gamma)} \sum_{m=0}^{\infty} \frac{\rho^m}{m!} \Gamma(\gamma+m) \; H\left(2(\gamma+m), \frac{\sqrt{2}}{1-\rho} z_1^*\right) . \qquad (7)$$

$H(a,X)$ is the incomplete gamma function which is given by the series

$$H(a,X) = X^a e^{-X} \sum_{n=0}^{\infty} \frac{X^n}{\Gamma(a+n+1)} \qquad (8)$$

where $a = 2(\gamma+m)$

$X = \frac{\sqrt{2}}{1-\rho} z_1^*$

ρ = correlation between variables x and y .

A computer program for calculation of P_Δ, using Equation (7), has been developed. A sample of the calculations of P_Δ as a function of ρ for $\gamma = 3$ is listed in Table 1.

Table 1. $P_\Delta(\rho, \gamma=3)$ Calculated According to Equation 7

P DELTA - SERIES APPROXIMATION

GAMMA=3.0

\RHO Z\	.1	.2	.3	.4	.5	.6	.7	.8	.9
.25	.000002663	.000003641	.000005161	.000007655	.000012041	.000020487	.000038892	.000086626	.000242874
.50	.000122439	.000161260	.000218122	.000304586	.000442000	.000671951	.001079473	.001840323	.003268531
.75	.001006259	.001280495	.001660313	.002198620	.002979904	.004138859	.005880894	.008472993	.012122910
1.00	.004096778	.005051769	.006306665	.007973772	.010205460	.013194281	.017143814	.022178395	.028220989
1.25	.011373598	.013628834	.016449673	.019981516	.024383150	.029793530	.036265251	.043685778	.051778887
1.50	.024626401	.028987518	.033957309	.039852230	.046751108	.054646478	.063393999	.072710879	.082266417
1.75	.045972081	.052438397	.059834862	.068186850	.077437581	.087416003	.097837589	.108368992	.118727644
2.00	.075570901	.084417143	.094135205	.104628579	.115705275	.127081869	.138431603	.149467481	.160002507
2.25	.113559874	.124515460	.136103686	.148117352	.160276631	.172266847	.183605356	.194702851	.204875743
2.50	.159150511	.171659026	.184426945	.197180470	.209620247	.221475001	.232557615	.242788345	.252172884
2.75	.211018635	.224343151	.237492034	.250183273	.262161769	.273247857	.283365652	.292531900	.300818328
3.00	.267523069	.280862765	.293605171	.305513788	.316421274	.326257363	.335047193	.342879243	.349866703
3.25	.326909445	.339503709	.351151654	.361701947	.371091984	.379354101	.386590198	.392933127	.398515809
3.50	.387474723	.398678157	.408698678	.417485151	.425076645	.431585334	.437161103	.441956583	.446107242
3.75	.447684199	.457012288	.465046030	.471834525	.477495614	.482193664	.486099523	.489366144	.492119286
4.00	.506241910	.513387233	.519249007	.523954317	.527677067	.530603848	.532904647	.534718946	.536155000
4.25	.562122136	.566948146	.570601977	.573265314	.575137205	.576403409	.577217408	.577696733	.577928096
4.50	.614570901	.617093620	.618623890	.619379185	.619555540	.619320288	.618800826	.618088685	.617247865
4.75	.663087070	.663433147	.663023956	.662069164	.660748571	.659200028	.657520298	.655774407	.654004201
5.00	.707391016	.705783106	.703669488	.701240629	.698644474	.695984162	.693325408	.690707736	.688153610
5.25	.747387417	.744100653	.740553483	.736903206	.733259536	.729690202	.726231970	.722902142	.719706416
5.50	.783127092	.778464168	.773764975	.769145951	.764677390	.760394178	.756307907	.752417475	.748715110
5.75	.814770535	.809039071	.803462945	.798115805	.793031104	.788215362	.783659786	.779348925	.775264643
6.00	.842555657	.836050779	.829854354	.823999606	.818488121	.813303657	.808421969	.803817041	.799463555
6.25	.866770245	.859762490	.853176154	.847009242	.841237889	.835828938	.830747269	.825959936	.821437053
6.50	.887729567	.880457260	.873680890	.867369987	.861482039	.855972514	.850799583	.845926382	.841320775
6.75	[illegible]	[illegible]	.891625628	.885311343	.879426647	.873920493	.868747912	.863870710	.859256230
7.00	.921178930	.913948506	.907263815	.901060164	.895276234	.889858551	.884761751	.879948318	.875386573
7.25	.934299380	.927393307	.920839272	.914835684	.909229420	.903967835	.899007387	.894312605	.889853932
7.50	.945409432	.938745625	.932582162	.926845968	.921475857	.916422360	.911645308	.907112412	.902796686
7.75	.954775535	.948513106	.942706525	.937285751	.932194084	.927386947	.922828443	.918490104	.914348096
8.00	.962639138	.956822373	.951409065	.946335077	.941550314	.937015802	.932700731	.928580612	.924634971
8.25	.969216332	.963668730	.958868163	.954158798	.949697658	.945452109	.941396438	.937510319	.933776654
8.50	.974698171	.969826505	.965244934	.960906506	.956776207	.952827568	.949039914	.945396945	.941884898
8.75	.979252085	.974650170	.970683001	.966713086	.962913059	.959262468	.955745362	.952349223	.949063294
9.00	.983023658	.979075328	.975309961	.971699215	.968222871	.964866027	.961617045	.958466902	.955407530
9.25	.986136366	.982620426	.979238316	.975972302	.972808465	.969736934	.966749832	.963841176	.961005397
9.50	.988703705	.985588238	.982566930	.979627475	.976761632	.973963760	.971229337	.968554966	.965937190
9.75	.990811341	.988067620	.985381998	.982748590	.980163880	.977625825	.975132637	.972683184	.970275737
10.00	.992538847	.990174843	.987758510	.985409252	.983087391	.980793796	.978528969	.976293467	.974087149
10.25	.993951626	.991855174	.989761420	.987673827	.985595785	.983530454	.981479988	.979446605	.977431148
10.50	.995104663	.993284404	.991446741	.989598416	.987744942	.985891230	.984040812	.982196860	.980361275
10.75	.996043794	.994469784	.992862768	.991231762	.989583857	.987925164	.986260198	.984592862	.982925788
11.00	.996807367	.995451376	.994050808	.992616080	.991155304	.989675343	.988181368	.986677803	.985167749
11.25	.997427076	.996262968	.995046213	.993787810	.992496558	.991179645	.989842556	.988490127	.987125665
11.50	.997929141	.996933140	.995879106	.994778469	.993640013	.992471114	.991277330	.990063719	.988833651
11.75	.998335339	.997485757	.996575259	.995615050	.994613767	.993578677	.992515333	.991428688	.990322217
12.00	.998663440	.997940809	.997156389	.996320724	.995442122	.994527563	.993582487	.992611535	.991618313
12.25	.998928048	.998315148	.997640967	.996915370	.996146105	.995339796	.994501419	.993635707	.992745779
12.50	.999141239	.998622708	.998044510	.997415937	.996743791	.996034294	.995292127	.994521596	.993725799
12.75	.999312691	.998875067	.998360445	.997836925	.997250766	.996627651	.995971777	.995287254	.994576842
13.00	.999450497	.999081999	.998659678	.998190634	.997680418	.997134134	.996555611	.995948464	.995315298
13.25	.999561094	.999251477	.998891592	.998487532	.998044257	.997566126	.997056611	.996519051	.995955631
13.50	.999649711	.999390103	.999084033	.998736590	.998352073	.997934334	.997486323	.997011036	.996510431
13.75	.999720715	.999503464	.999243557	.998945333	.998612314	.998247577	.997854538	.997434936	.996990815
14.00	.999777518	.999596044	.999375738	.999120139	.998832151	.998514675	.998169839	.997799933	.997406438
14.25	.999822870	.999671549	.999485157	.999266431	.999017751	.998741545	.998439677	.998113975	.997765809
14.50	.999859124	.999733165	.999575660	.999388754	.999174304	.998934358	.998670399	.998384088	.998076320
14.75	.999887995	.999783367	.999650419	.999490969	.999306284	.999098157	.998867624	.998616166	.998344488
15.00	.999911048	.999824181	.999712214	.999576278	.999417476	.999237113	.999036036	.998815484	.998575911

Standard results for checking the computer programs are obtained from the closed-form solution for the case $\gamma=1$ which can be expressed in the form

$$P_\Delta(\gamma=1,\rho) = 1 - \frac{e^{-\frac{z_1^*}{\sqrt{\rho}}}}{2} \left[e^{z_1^*}\left(\frac{1}{\sqrt{\rho}} + 1\right) - e^{-z_1^*}\left(\frac{1}{\sqrt{\rho}} - 1\right)\right] . \quad (9)$$

Values for P_Δ are listed in Table 2 for selected values of z_1^* and ρ.

Table 2. $P_\Delta(\gamma=1, \rho)$ Calculated from Equation (9)

z_1^*	ρ				
	.1	.25	.50	.75	.875
1	.425980	.445255	.474470	.495090	.501805
2	.774586	.774144	.769772	.763367	.760084
3	.919308	.911445	.899446	.889099	.884464
4	.971975	.965471	.956084	.948025	.944361
5	.990370	.986548	.980820	.975641	.973206
6	.996704	.994760	.991624	.988584	.987097
7	.998874	.997959	.996342	.994650	.993786
8	.999615	.999205	.998402	.997493	.997008
9	.999869	.999690	.999302	.998825	.998559
10	.999955	.999879	.999695	.999449	.999306

Another useful special case is for $\rho=0$ and 2γ equal to an integer.

$$P_\Delta(\rho=0,\ 2\gamma = \text{an integer})$$

$$= 1 - e^{\sqrt{2}\, z_1^*}\left[\sum_{k=0}^{2\gamma-1} \frac{2^{k/2}(z_1^*)^k}{k!}\right] . \quad (10)$$

The variation of P_Δ as a function of correlation coefficient, ρ, (for $\gamma=2$) and as a function of shape parameter, γ, (for $\rho=0.5$) is illustrated in Figures 2 and 3, respectively.

A comparison of observed and expected P_Δ is illustrated in Figure 4; the line drawn at an angle of 45° to the abscissa represents perfect agreement between observed and expected values. Deviations of the plotted points from the line represent differences between the observed and expected values. The data plotted in Figure 4 show a consistent pattern at 10 and 12 km; for $P_\Delta < 0.3$, the observed is larger than the expected; for intermediate values ($0.3 < P_\Delta < 0.8$), the expected is larger than the observed. These results are the basis for initiating a more detailed analysis of the validity of the gamma distribution hypothesis for the marginal distributions (component gust and associated gust length). The results of this analysis are described in the next section.

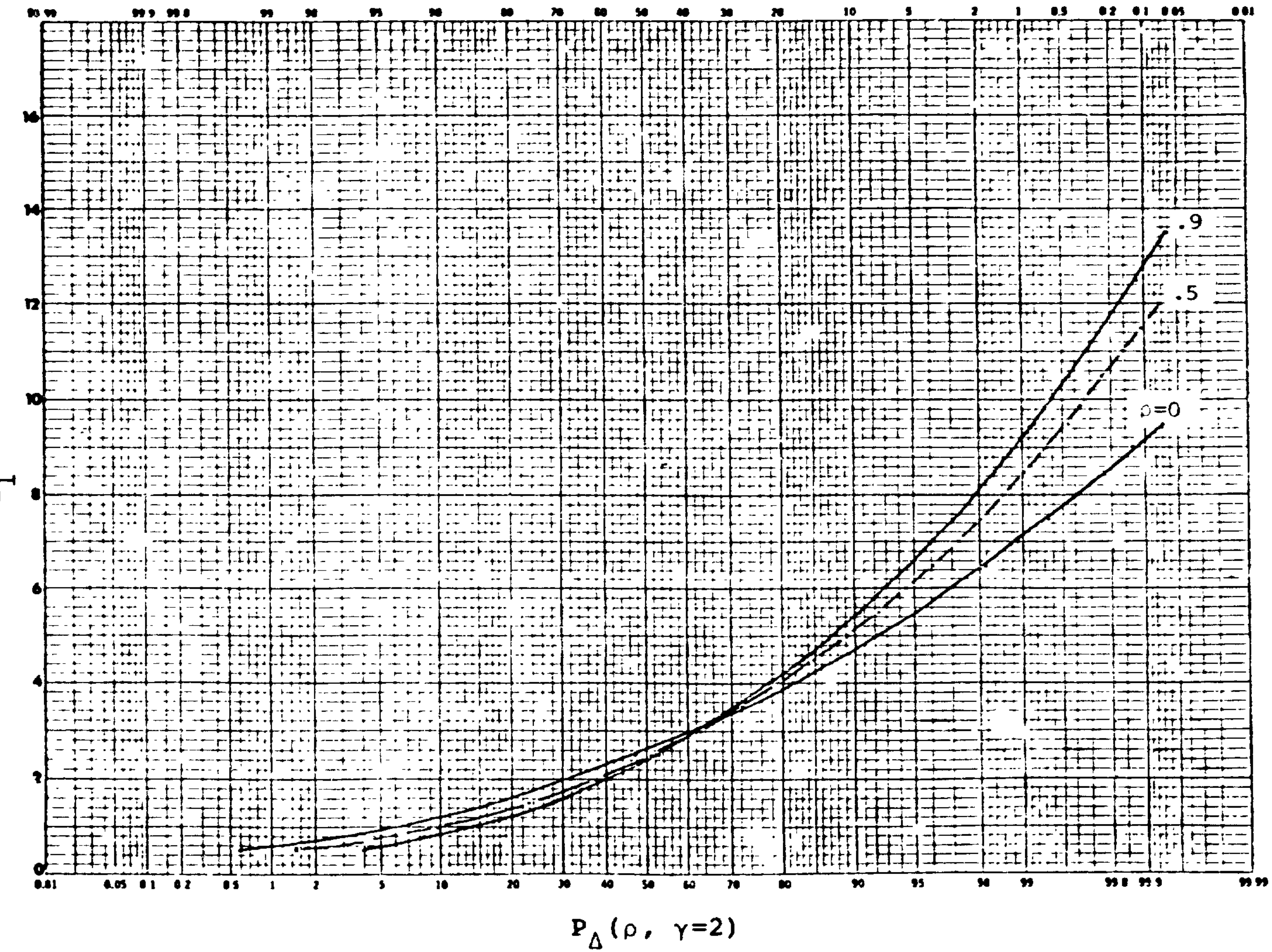


Figure 2. Series Approximation of P_Δ as a Function of z_1^* and ρ for $\gamma=2$

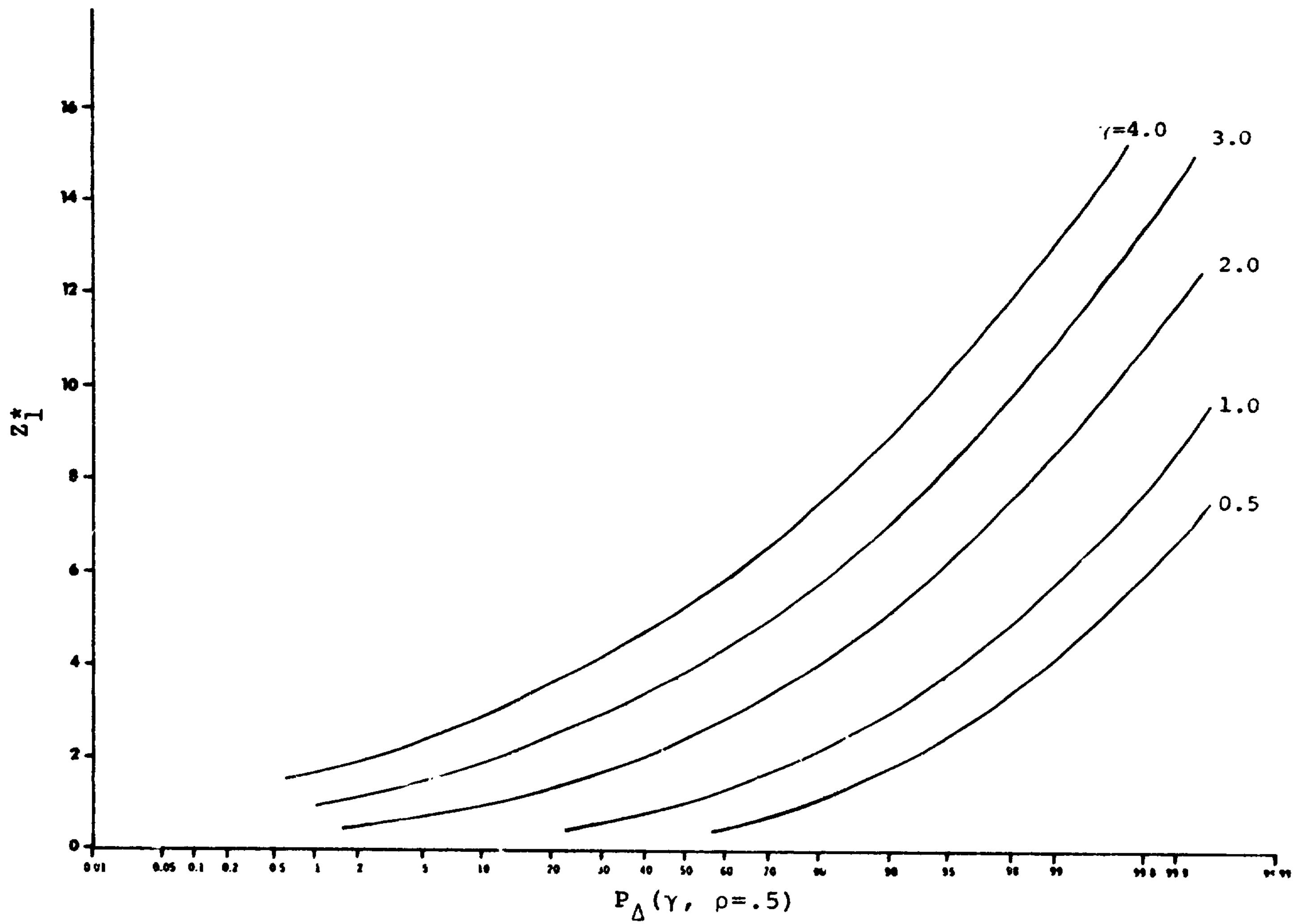


Figure 3. Series Approximation of P_Δ as a Function of z_1^* and γ for $\rho = 0.5$

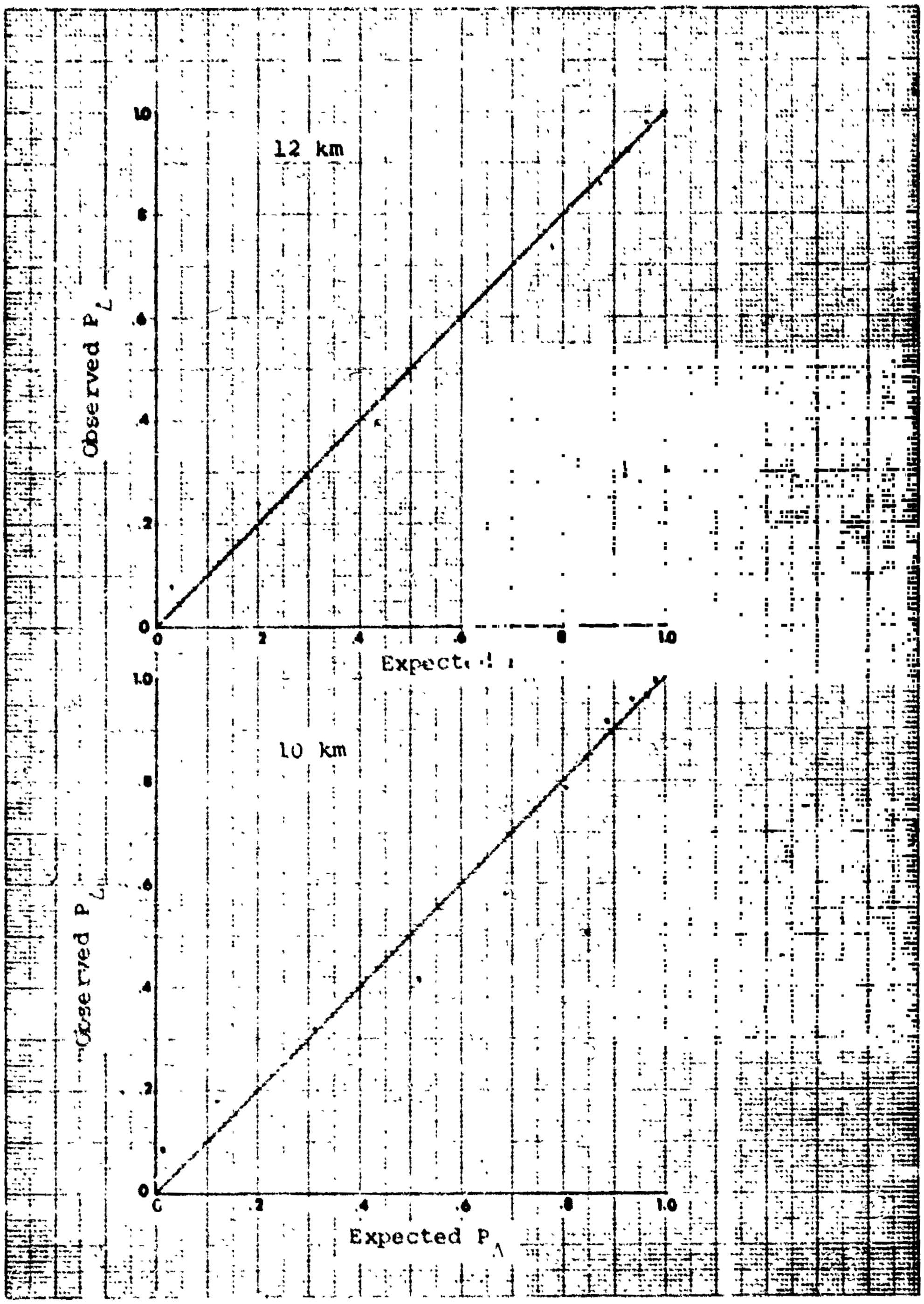


Figure 4. Observed and Expected P_Δ at 10 and 12 km Calculated from u Component Gust and Gust Length Data (λ_c = 2470 m) During February at Cape Kennedy

SECTION III. THE DISTRIBUTION OF GUST MODULUS

Given that the absolute gust components are uncorrelated bivariate gamma distributed, then the probability distribution of gust modulus is obtained by numerical integration of the joint distribution expressed in polar coordinates.

$$\Pr\{R \leq R^*\} = G(R^*) = \frac{\beta_1^{\gamma_1}\beta_2^{\gamma_2}}{\Gamma(\gamma_1)\Gamma(\gamma_2)} \int_0^{R^*} R^{\gamma_1+\gamma_2-1} \; [I] \, dR \qquad (11)$$

$$I = \int_0^{\pi/2} (\cos\theta)^{\gamma_1-1} (\sin\theta)^{\gamma_2-1} e^{-R(\beta_1\cos\theta+\beta_2\sin\theta)} d\theta$$

where β_1 and β_2 are the scale parameters and γ_1 and γ_2 are the shape parameters of the u and v component gamma distributions, respectively.

An expression that is approximately equivalent to Equation 11 is

$$G(R^*) = \frac{H(\gamma_1 + \gamma_2, \Lambda R^*)}{\Gamma(\gamma_1 + \gamma_2)} , \qquad (12)$$

where $H(\gamma_1 + \gamma_2, \Lambda R^*)$ is the incomplete gamma function which can be calculated accurately with the series approximation given in Section II.

$$A = \left[\frac{\Gamma\left(\frac{\gamma_1}{2}\right) \Gamma\left(\frac{\gamma_2}{2}\right) \Gamma(\gamma_1+\gamma_2) \beta_1^{\gamma_1} \beta_2^{\gamma_2}}{2\Gamma(\gamma_1) \Gamma(\gamma_2) \Gamma\left(\frac{\gamma_1+\gamma_2}{2}\right)} \right]^{\frac{1}{\gamma_1+\gamma_2}} \qquad (13)$$

Preliminary tests have indicated that reasonably accurate estimates of the probability distribution can be obtained from equation (12). However, it would be advantageous to determine if an alternative expression can be found which would not require as much computation. The Weibull distribution, widely used in wind energy studies (Reference 3) was chosen to represent gust modulus because of its relative mathematical simplicity and the availability of data for parameter estimation. The cumulative probability function for the Weibull distribution of gust modulus is

$$G(R^*) = 1 - \mathrm{EXP}\left[-\left| \frac{R^*}{c} \right|^k \right] . \qquad (14)$$

The parameters k and c are calculated according to the approximation given by Justus (Ref. 3)

$$k = \left(\frac{\sigma_R}{\bar{R}} \right)^{-1.086} \qquad (15)$$

$$c = \frac{\bar{R}}{\Gamma(1 + 1/k)} . \qquad (16)$$

It is noted that equation (15) implies the relation,

$$\left(\frac{\sigma_R}{\bar{R}} \right)^2 = k^{-1.84162} , \qquad (17)$$

whereas the exact relation for a Weibull distribution is given by

$$\left(\frac{\sigma_R}{\bar{R}}\right)^2 = \left[\frac{\Gamma(1 + 2/k)}{\Gamma^2(1 + 1/k)}\right] - 1 \quad . \tag{18}$$

The accuracy of the approximation has been evaluated for values of k from 0.5 to 10 by calculating the ratio, P, of the right side of equation (18) to the right side of equation (17). Perfect agreement is indicated when P=1. As illustrated in Figure 5, for $k > 1$, P is within a few percent of unity; for $k < 1$, P approaches ∞ as k approaches 0. Therefore, it is concluded that the approximation given by equation (15) is accurate for $k > 1$. The calculated values of k for gust modulus are between 2 and 3, which is within the range of acceptable accuracy of equation (15).

Parameters K and C, calculated from Equations (15) and (16), respectively, utilizing Cape Kennedy sample data are listed in Table 3.

A comparison of the Weibull, the probability distribution associated with the modulus of a bivariate normal, and the observed probability distribution is illustrated in Figure 6. It is indicated that there is little difference between the theoretical distributions for percentiles between 20 and 98; for percentiles outside that range, the distributions diverge; for this case, the observed distribution fits the Weibull slightly better than the bivariate gamma modulus distribution.

The hypothesis that gust modulus at a reference altitude is drawn from a Weibull distributed population was tested for 69 cases. The results summarized in Table 4 indicate that the hypothesis is accepted at the 0.05 level of significance in a large majority (65/69) of the cases.

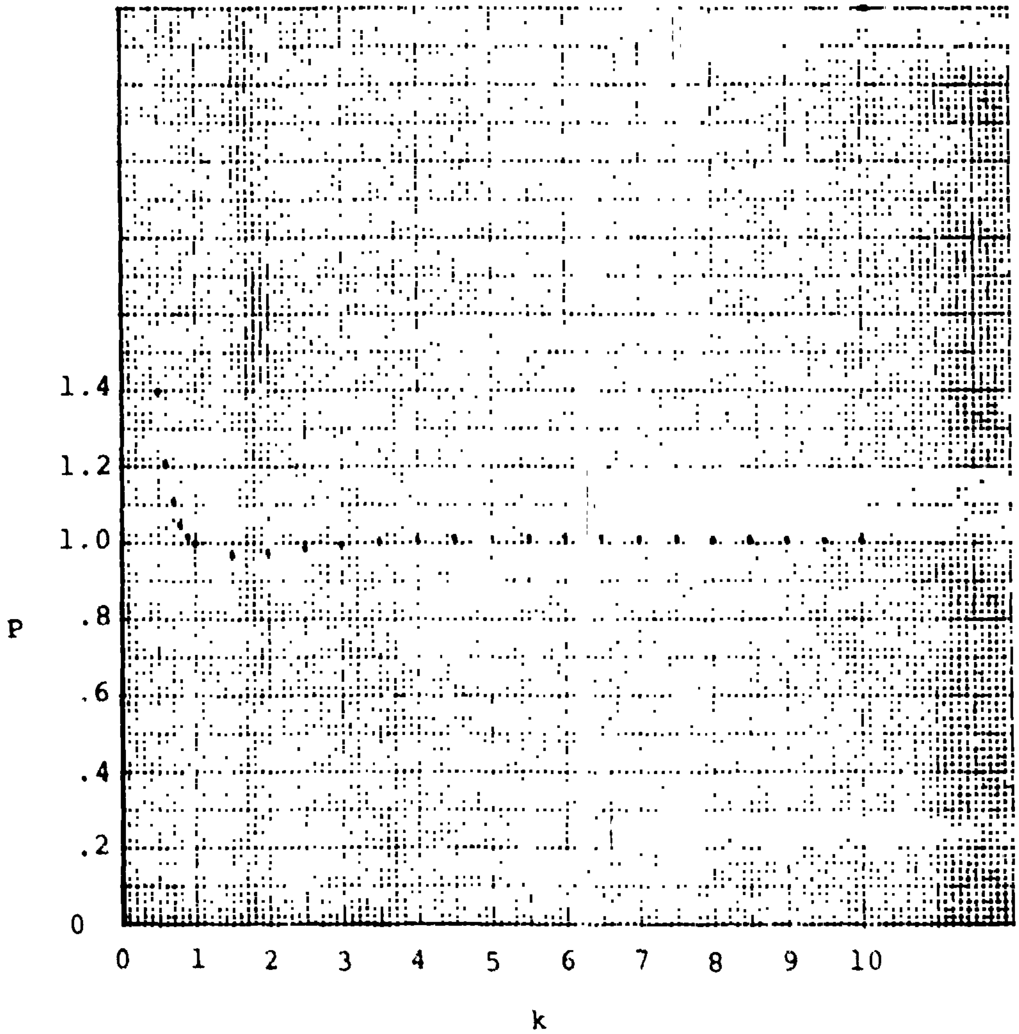


Figure 5. Ratio P as a Function of Shape Parameter, k

Table 3. Parameters K and C of the Weibull Distribution for Gust Modulus at Cape Kennedy

Filter Cutoff Wavelength λ_c (m)		Altitude (km)											
		4		6		8		10		12		14	
		K	C(m/s)	K	C	K	C	K	C	K	C	K	C
February	420	2.2812	.6737	2.4065	.6053	2.3033	.6577	2.0882	.7617	2.2963	1.1098	2.2743	1.2555
	997	2.4716	1.4711	2.6186	1.3742	2.3686	1.3610	2.1574	1.4895	2.3406	2.1723	2.4070	2.4489
	2470	2.7278	2.8597	2.6919	2.9165	2.6825	2.9958	2.3357	3.4963	2.2864	4.8499	2.4854	5.4242
	6000	2.1948	3.5185	2.3282	4.9945	2.3869	5.5529	1.9893	6.8036	2.2852	8.5558	2.8019	[illegible]
April	420	2.2575	.5965	2.4877	.6264	1.9536	.5551	2.5255	.5646	1.9330	.8594	2.3434	[illegible]
	997	2.5140	1.3690	2.9526	1.4104	2.3241	1.2216	2.6265	1.1899	2.0577	1.8545	2.3870	[illegible]
	2470	2.8052	2.7906	2.9528	2.9339	2.5155	2.7722	2.4631	2.7887	2.3803	4.2895	2.5189	[illegible]
	6000	2.1703	2.7986	2.9758	4.8125	2.6556	4.8202	2.6730	5.4216	2.6051	7.3551	2.8779	[illegible]
July	420	2.6524	.5256	2.7129	.5604	2.6012	.5372	2.3230	.4696	2.1920	.5082	1.9689	[illegible]
	997	2.6489	1.0502	2.5272	1.1329	3.2253	1.1497	2.4713	1.0338	2.6973	1.1182	2.2996	[illegible]
	2470	2.3573	2.0264	2.9588	2.0706	2.8330	2.1746	2.3203	2.2526	2.2491	2.4230	2.2845	[illegible]
	6000	2.3016	2.5464	2.9172	3.3690	2.7821	3.4551	2.3934	3.8710	2.5393	5.2454	2.6671	[illegible]

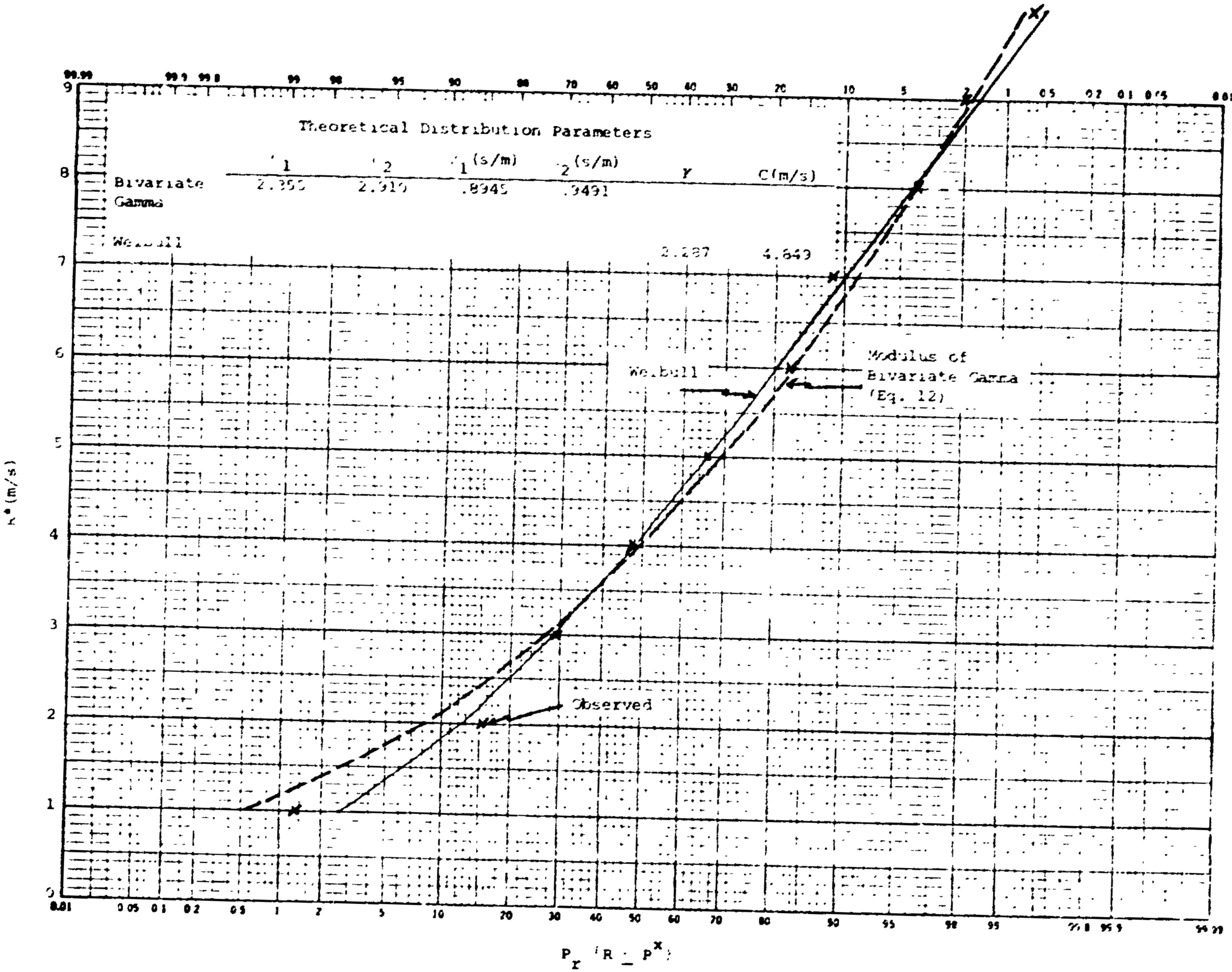


Figure 6. Observed and Theoretical Distribution of Gust Modulus at 12 km During February at Cape Kennedy for λ_c = 2,470 m

Table 4. Summary of Results of Testing the Hypothesis* That Gust Modulus at a Reference Altitude (4, 6 ... 14 km) Is Drawn From a Weibull Distributed Population

Month	λ_c (m)	Number of Cases: Hypothesis Accepted	Number of Cases: Hypothesis Rejected	Number of Cases: Insufficient Data
Feb	420	6	0	0
	997	5	1	0
	2470	5	1	0
	6000	5	0	1
	Total	21	2	1
Apr	420	5	1	0
	997	6	0	0
	2470	6	0	0
	6000	4	1	1
	Total	21	2	1
Jul	420	6	0	0
	997	6	0	0
	2470	6	0	0
	6000	5	0	1
	Total	23	0	1
	Grand Total	65	4	3

*For the 0.05 level of significance for a χ^2 variate with m degrees of freedom, m = r-1-b, where r = number of class intervals, b = number of parameters of the Weibull distribution = 2.

SECTION IV. DISTRIBUTION OF GUST COMPONENT VARIABLES

Four variables associated with gusts at a reference height, H_0, have been studied to establish the validity of the hypothesis that they are samples from gamma distributed populations. The four variables are illustrated in Figure 7. The variable u_1 is the largest u component excursion with sign equal to the sign of u at H_0; u_2 is the largest u component excursion of sign opposite u_1 found by scanning upward after the second zero crossing associated with u_1. The vertical distance between u_1 and u_2 is defined as L Range; the sum of the absolute values of u_1 and u_2 is defined as u Range. The variables u Range and L Range represent wind shear and wind shear altitude interval associated with gusts in the vicinity of H_0. Each of the four variables defined above have been calculated at six reference altitudes from a sample (150/month) of February, April, and July Jimsphere wind profile data from Cape Kennedy. These data sets were tested to establish the validity of the hypothesis that each variable is drawn from a gamma distributed population. Acceptance or rejection of the hypothesis is at the 0.05 level of significance for a χ^2 variate defined by

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} \quad ; \qquad (19)$$

O_i = Observed frequency in the ith class interval

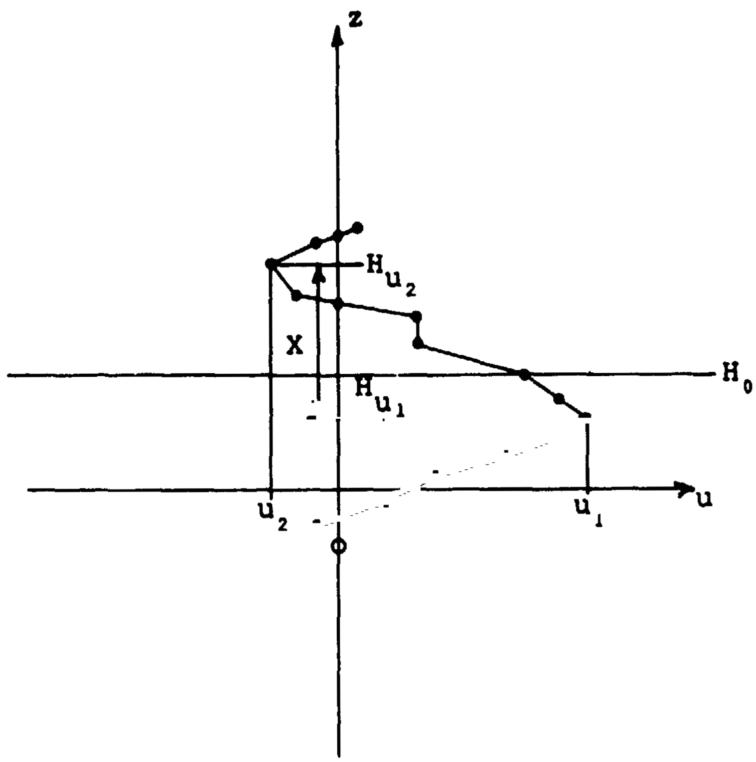
E_i = Expected frequency in the ith class interval (of the theoretical-gamma distribution).

The results of the hypothesis testing are described later.

A. Absolute Gust Component and Associated Gust Length

Gust, defined as the maximum excursion between successive zero crossings in the vicinity of a reference altitude, and associated gust length, defined as the distance between zero crossings, are each hypothesized to be drawn from a gamma[1] distributed population. The hypothesis is accepted at the

1. Sample estimates for the parameters of the gamma distribution are given in the Appendix.



$$u \text{ Range} = |u_1| + |u_2|$$

$$L \text{ Range} = X = H_{u_2} - H_{u_1}$$

Figure 7. Schematic Definition of u Range and L Range

0.05 level of significance, in a large majority of cases, for gust component magnitude ($|u'|$); specifically, the accept/reject ratio is 47/22 and 46/23 for u and v component magnitudes, respectively. As indicated in Table 5, the ratio is significantly smaller for gust length (Lu and Lv) with rejections exceeding acceptances (for method I). The large number of rejections is attributed to large differences between observed and expected frequency of occurrence in the first few class intervals; the observed frequencies are always much larger than the expected frequencies. Small gust magnitudes are associated with small gust lengths that are observed as a consequence of the definition of gust used in this study. These small gust lengths are not measurable with the Jimsphere system; therefore, they are not considered to be valid data for hypothesis testing. By neglecting these data, we obtain the results summarized under II in Table 5 which indicate acceptance in a much larger proportion of the cases.

B. U Range and L Range

A summary of results of testing the hypothesis that the variables, u Range and L Range, are drawn from gamma distributed populations is given in Table 6. It is indicated that the hypothesis for u Range is accepted at the 0.05 level of significance in 66 of the 72 cases. Acceptance is not a function of altitude except in July when the number of samples accepted at 14 km was less than at the other altitudes. Acceptance was not related to filter choice with only slight exceptions (for λ_c = 2470 during July and λ_c = 6000 m during February one-third of the samples were rejected). Based on these results, it is concluded that u Range is gamma distributed.

The results for L Range summarized in the lower half of Table 6 indicate acceptance of the hypothesis (46 of the 72 cases) with not as strong a tendency as that indicated previously for u Range. Acceptance is an irregular function of altitude which is a minimum at 12 km where 50 percent is accepted to a maximum at 8 km where 75 percent is accepted. Acceptance is greater in July (75 percent) than in either April or February (58 percent for both months). Acceptance is weak or non-existent for λ_c = 420 m and is strong for λ_c large (2470 and 6000 m).

Table 5. Summary of Results of Testing the Hypothesis[1] that u and v Component Absolute Gust and Gust Length are Drawn from Gamma Distributed Populations

A/R[2]

|u'|

Filter λ_c (m)	Method							
	I				II			
	Month				Month			
	2	4	7	All Months	2	4	7	All Months
420	4/2	6/0	5/1	15/3	5/1	6/0	6/0	17/[illegible]
997	6/0	5/1	4/2	15/3	6/0	6/0	5/1	17[illegible]
2470	5/1	1/5	2/4	8/10	6/0	4/2	5/1	15/[illegible]
6000	3/2	5/0	1/4	9/6	5/0	5/0	5/0	15/[illegible]
All Filters	18/5	17/6	12/11	47/22	22/1	21/2	21/2	64/5

Lu

420	5/1	4/2	4/2	13/5	6/0	5/1	5/1	16/2
997	1/5	1/5	4/2	6/12	2/4	1/5	6/0	9/9
2470	0/6	1/5	3/3	4/14	4/2	4/2	5/1	13/5
6000	3/2	4/1	2/3	9/6	5/0	5/0	5/0	15/0
All Filters	9/14	10/13	13/10	32/37	17/6	15/8	21/2	53/16

|v'|

Filter λ_c (m)	Method							
	I				II			
	Month				Month			
	2	4	7	All Months	2	4	7	All Months
420	6/0	4/2	6/0	16/2	6/0	4/2	6/0	16/2
997	4/2	5/1	2/4	11/7	5/1	6/0	5/1	16/2
2470	3/3	4/2	3/3	10/8	4/2	5/1	5/1	14/4
6000	3/2	4/1	2/3	9/6	5/0	4/1	3/2	12/3
All Filters	16/7	17/6	13/10	46/23	20/3	19/4	19/4	58/11

Lv

420	5/1	4/2	3/3	12/6	5/1	5/1	5/1	15/3
997	0/6	2/4	2/4	4/14	1/5	5/1	4/2	10/8
2470	1/5	3/3	3/3	7/11	3/3	4/2	5/1	12/6
6000	2/3	1/4	4/1	7/8	4/1	4/1	4/1	12/3
All Filters	8/15	10/13	12/11	30/39	13/10	18/5	18/5	49/20

(1) At the .05 level of significance for χ^2 variate with m degrees of freedom; m = n-1-b, where n = number of class intervals, b = number of parameters of the gamma distribution = 2.

(2) A/R is the ratio of the number of cases accepted to the number rejected.

Table 6. Summary of Results of Testing the Hypothesis That the Variables, u Range and L Range, at a Reference Altitude (4, 6, ... 14 km) are Drawn from Gamma Distributed Populations

Variable	Month	Filter λ_c (m)	Reference Altitude (km) 4	6	8	10	12	14	Summary A	R
U range	2	420	A*	A	A	A	A	A	6	0
		997	A	A	A	A	A	A	6	0
		2470	A	A	A	A	A	A	6	0
		6000	R*	A	A	A	R	A	4	2
Accept/Reject (all filters)			3/1	4/0	4/0	4/0	3/1	4/0	22 /	2
	4	420	A	A	A	A	A	A	6	0
		997	A	A	A	A	A	A	6	0
		2470	A	A	A	A	A	A	6	0
		6000	A	A	A	A	R	A	5	1
Accept/Reject			4/0	4/0	4/0	4/0	3/1	4/0	23 /	1
	7	420	A	A	A	A	A	A	6	0
		997	A	A	A	A	A	R	5	1
		2470	A	A	A	R	A	R	4	2
		6000	A	A	A	A	A	A	6	0
Accept/Reject			4/0	4/0	4/0	3/1	4/0	2/2	21 /	3
Accept/Reject (all months)			11/1	12/0	12/0	11/1	10/2	10/2	66 /	6
L range	2	420	R	R	R	R	R	R	0	6
		997	A	A	R	A	R	R	3	3
		2470	A	A	A	A	A	A	6	0
		6000	R	A	A	A	A	A	5	1
Accept/Reject			2/2	3/1	2/2	3/1	2/2	2/2	14 /	10
	4	420	R	R	A	R	R	R	1	5
		997	A	A	R	A	R	R	3	3
		2470	A	A	R	R	A	A	4	2
		6000	A	A	A	A	A	A	6	0
Accept/Reject			3/1	3/1	2/2	2/2	2/2	2/2	14 /	10
	7	420	R	R	A	R	R	R	1	5
		997	A	A	A	A	R	A	5	1
		2470	A	A	A	A	A	A	6	0
		6000	A	A	A	A	A	A	6	0
Accept/Reject			3/1	3/1	4/0	3/1	2/2	3/1	18 /	6
Accept/Reject (all months)			8/4	9/3	8/4	8/4	6/6	7/5	46/	26

*Accept (A) or Reject (R) hypothesis at the .05 level of significance for χ^2 variate with m degrees of freedom, m = n-1-b, where n = number of class intervals, b = number of parameters of the gamma distribution = 2.

SECTION V. CONCLUSIONS

This report has emphasized methods for establishing the validity of the hypothesis that observed gust variables, including gust component magnitude, gust length, u Range, and L Range, have been drawn from gamma distributed populations and that observed gust modulus has been drawn from a bivariate gamma distributed population that can be approximated with a Weibull distribution. An analytical procedure has been proposed for testing for the bivariate gamma distribution. The procedure has the advantage of not requiring frequency counts within narrow cells defined by the intersection of intervals of the marginal distribution; these frequency counts would be impractical and unreliable because of the limited sample size (150) of the available data. Instead, the new method requires theoretical and observed frequency counting over larger areas associated with non-dimensionalized and transformed variables. Preliminary results utilizing this method have indicated larger observed than expected frequencies for small gust lengths and associated small gust magnitudes; this is attributable to the definition of gust used in this study. These small gust lengths are not measurable with the Jimsphere system and, as indicated in Section IV, the results of hypothesis testing for the marginal distributions are improved greatly by eliminating them from the data sample. The hypothesis that gust component (u and v) magnitudes are drawn from a gamma distributed population is accepted at the 0.05 level of significance in 122 of the 136 cases tested; for gust length (Lu and Lv), 102 of the 136 cases were accepted.

The variables u Range and L Range have been used to represent component wind shear and shear interval associated with gusts. The hypothesis that u Range observations were drawn from a gamma distributed population was accepted at the 0.05 level in 66 of the 72 cases tested; the acceptance ratio was somewhat smaller for L Range with acceptance in 46 of the 72 cases tested.

Testing of the hypothesis that gust modulus is drawn from a Weibull distributed population has yielded highly favorable results with acceptance of the hypothesis at the 0.05 level in 65 of the 69 cases tested.

SECTION VI. REFERENCES

1. Adelfang, S. I., and Evans, B.: Vector Wind Profile Gust Model, Final Report (for Period April 10, 1979 – April 9, 1980), prepared under Contract NAS8-33433 for NASA/MSFC, April 9, 1980.

2. Smith, O. E., and Adelfang, S. I.: A Model for Gust Amplitude and Gust Length Based on the Bivariate Gamma Probability Distribution Function. Presented at the AIAA 19th Aerospace Sciences Meeting, January 1981, St. Louis, Missouri. AIAA Paper 81-0299.

3. Justus, C. G., Hargraves, W. R., Mikhail, A., and Graber, D.: Methods for Estimating Wind Speed Frequency Distributions. JAM, Vol. 17, pp. 350-353, March 1978.

APPENDIX

Parameters γ and β (calculated from sample moments) for hypothetical gamma distributions of gust component variables $|u'|$, $|v'|$, Lu, Lv, u Range, and L Range defined in Section IV are listed in Tables A-1 through A-6.

The parameters in the tables can be used to derive the gamma probability density function of the form

$$g(x) = \frac{\beta^{\gamma}}{\Gamma(\gamma)} x^{\gamma-1} \text{EXP}(-\beta x) \quad . \tag{A-1}$$

Equation (A-1) can be expressed in terms of a nondimensional variable y, i.e., $y = \frac{x}{\beta}$, such that

$$g(y) = \frac{1}{\Gamma(\gamma)} y^{\gamma-1} \text{EXP}(-y) \quad . \tag{A-2}$$

The probability that y does not exceed a specified value, Y, is given by

$$P_r \{y \leq Y\} = \int_0^Y g(y)\, dy = \frac{1}{\Gamma(\gamma)} \int_0^Y y^{\gamma-1} \text{EXP}(-y)\, dy \, . \tag{A-3}$$

The integral on the right side of Equation (A-3) is the incomplete gamma function, $H(\gamma, Y)$, which can be approximated with the series summation given by Equation 4 in Section II with the substitution

$$a = \gamma$$

$$X = Y \; .$$

Table A-1. Gamma Distribution Parameters γ and β of Absolute u Component Gust Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	2.7977	7.1252	3.0595	8.2808	2.6387	6.5191	2.7884	6.1971	2.2586	4.3472	2.7237	4.4131
	997	3.6720	4.3885	3.0471	3.7889	2.7925	3.3954	2.6372	3.1212	2.5924	2.2480	2.7435	2.2276
	2470	3.4160	2.0864	3.2470	2.0025	3.3461	1.9497	3.0639	1.6141	2.3545	.8945	3.3385	1.1593
	6000	1.3784	.7212	2.5834	.9603	2.9254	.9140	2.4424	.6500	2.6651	.5478	4.1791	.8345
April	420	2.2160	6.8163	2.6129	7.8253	2.4453	7.7683	2.8283	8.4910	2.7139	6.0048	3.3506	5.3996
	997	2.8800	3.7674	3.9797	5.0300	2.9474	4.2228	2.9914	4.5243	3.0542	3.0214	2.8570	2.0031
	2470	3.2557	2.1546	3.3992	2.1361	3.5606	2.2367	3.1450	2.1659	3.2043	1.3532	3.1588	.9373
	6000	1.4722	1.1660	3.4500	1.2714	2.9691	1.1169	3.1542	1.0743	3.4673	.7980	4.3829	.7665
July	420	3.0155	9.7748	3.1550	9.3360	3.3174	10.4939	3.1022	10.6578	2.4241	7.9454	2.1704	4.7490
	997	3.0537	4.7798	2.9116	4.3739	3.9496	5.7012	3.0069	4.9926	3.2366	4.9563	3.0064	2.7846
	2470	3.0713	2.6264	4.0635	3.3023	3.2331	2.4762	2.6744	2.0260	2.7080	1.9894	3.0995	1.3168
	6000	2.3696	1.5587	3.6039	1.8240	2.9507	1.4285	2.6261	1.1393	2.8570	.8762	3.0623	.7489

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-2. Gamma Distribution Parameters γ and β of Gust Length, Lu, Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km) 4		6		8		10		12		14	
		γ	β (1/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	4.3144	.0303	4.7477	.0317	3.5532	.0262	3.9387	.0338	2.7802	.0259	2.3970	.0227
	997	5.0961	.0191	4.5724	.0173	4.3199	.0168	3.2904	.0143	3.4236	.0143	2.8273	.0120
	2470	3.5379	.0059	3.2987	.0057	2.8203	.0047	3.0325	.0047	2.9236	.0051	2.9723	.0046
	6000	2.2881	.0037	2.1950	.0020	2.0270	.0017	2.8136	.0020	1.9954	.0015	2.2030	.0017
April	420	3.8465	.0287	4.3296	.0320	4.4428	.0332	4.2881	.0337	3.6501	.0299	3.6396	.0265
	997	4.9658	.0190	5.6832	.0196	4.7895	.0172	3.9201	.0158	3.6556	.0146	4.4410	.0161
	2470	3.5516	.0059	3.0867	.0051	3.7403	.0059	2.4735	.0040	3.77[illegible]	.0059	3.7691	.0059
	6000	1.0203	.0020	2.9714	.0026	2.6921	.0023	2.9223	.0022	2.6511	.0018	2.4178	.0019
July	420	5.2320	.0367	6.1092	.0415	5.1708	.0379	4.1033	.0302	4.0764	.0321	3.6856	.0259
	997	3.9741	.0160	3.7563	.0146	5.1326	.0191	3.7614	.0139	4.4005	.0154	4.2432	.0145
	2470	3.0248	.0056	2.6623	.0048	2.9303	.0051	2.9148	.0047	2.8370	.0044	3.2452	.0050
	6000	1.9617	.0029	2.3853	.0023	2.1322	.0021	2.4274	.0022	3.6950	.0023	2.5452	.0019

* $\hat{\gamma} = (\overline{x}/\sigma)^2$

$\hat{\beta} = \overline{\gamma}/\overline{x}$

Table A-3. Gamma Distribution Parameters γ and β of Absolute v Component Gust Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β (s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	2.5620	6.367	3.0842	8.7604	2.8059	7.5590	2.3220	5.0906	2.4888	3.3212	2.7977	3.2771
	997	3.1964	3.4383	3.3494	4.0559	3.3089	4.1330	2.3731	2.6057	3.1270	2.2114	3.1354	1.9026
	2470	3.4847	1.9691	3.6922	1.9896	2.8574	1.5778	2.8406	1.2632	2.9095	.9491	3.1772	.8909
	6000	2.2621	1.1301	3.0577	.9520	2.5705	.7725	2.0864	.4907	2.6933	.5152	3.1304	.5772
April	420	2.6285	7.1176	3.3563	8.2955	2.2458	6.4842	3.1248	9.3733	2.1858	3.8315	2.3588	3.0760
	997	3.3134	3.8589	4.8051	5.2364	2.8348	3.7631	3.4730	4.6320	2.3209	1.9349	2.8295	1.7494
	2470	3.5359	1.9731	3.4691	1.8528	3.0296	1.7655	2.7126	1.4948	2.9190	1.0677	3.1823	.8619
	6000	3.3614	1.7161	3.5757	1.1955	3.7780	1.2287	3.3991	.9762	2.5541	.5941	3.3257	.5816
July	420	3.4726	11.0219	3.6591	11.1260	3.2813	10.1280	2.5372	9.5756	2.9943	10.0276	2.2201	4.7901
	997	3.2128	5.3395	3.4127	5.0117	4.6553	6.7185	2.8180	4.5618	3.6180	5.3203	2.7254	2.3321
	2470	2.6506	2.1572	3.6841	2.9425	3.7912	2.9586	2.7195	2.0371	2.3367	1.5839	2.7649	1.0942
	6000	1.7155	1.2577	3.9569	1.9228	3.5513	1.7642	3.2635	1.4297	3.3935	1.1520	3.5878	.8447

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-4. Gamma Distribution Parameters γ and β of Gust Length, Lv, Estimated from Sample Moment Statistics*

Month	Filter λ_c (m)	Altitude (km) 4		6		8		10		12		14	
		γ	β (1/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	4.2668	.0276	4.3845	.0329	4.6915	.0365	3.3980	.0300	2.8752	.0257	3.1804	.0327
	997	5.4600	.0189	5.2302	.0181	4.0927	.0153	2.7340	.0120	3.1941	.0134	2.5454	.0114
	2470	3.2958	.0057	4.2853	.0064	3.2358	.0052	3.1330	.0046	2.3618	.0039	2.8339	.0048
	6000	.8325	.0014	3.4616	.0029	2.8211	.0023	2.8833	.0023	2.5287	.0022	2.1470	.0021
April	420	4.4899	.0306	4.5662	.0283	4.0750	.0292	3.2733	.0282	2.7620	.0221	2.3890	.0212
	997	4.8218	.0166	6.8564	.0237	4.1517	.0146	3.7680	.0141	3.6093	.0150	2.8605	.0122
	2470	3.5608	.0056	3.7188	.0061	3.4439	.0050	3.4641	.0055	3.0020	.0049	3.0412	.0055
	6000	1.7891	.0027	3.4475	.0034	4.0519	.0034	3.0236	.0022	1.8364	.0017	3.4754	.0039
July	420	5.4864	.0401	5.5545	.0383	4.2290	.0288	4.2895	.0307	4.3395	.0306	3.7989	.0266
	997	4.6205	.0181	4.8734	.0183	5.7390	.0209	4.5953	.0164	4.9991	.0178	4.6217	.0158
	2470	3.0105	.0055	3.1497	.0057	3.4228	.0063	3.5367	.0057	2.7405	.0042	3.0952	.0050
	6000	1.2473	.0020	2.8210	.0028	2.4524	.0026	3.6418	.0031	3.6045	.0026	2.6019	.0022

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

Table A-5. Gamma Distribution Parameters γ and β of u Range Estimated from Sample Moment Statistics*

Month	Filter λ_c(m)	Altitude (km)											
		4		6		8		10		12		14	
		γ	β(s/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	3.4988	4.7377	3.3888	4.6909	3.4400	4.3525	3.1709	3.6078	3.4380	3.3728	3.9035	3.4181
	997	4.0685	2.5484	3.1495	2.0366	3.2842	2.1506	2.8046	1.7967	2.6382	1.2044	2.9463	1.3421
	2470	3.8216	1.3669	3.1250	1.0598	3.2096	1.0171	2.6697	.83208	2.2354	.48073	3.0640	.5888
	6000	1.6391	.48268	2.5218	.52372	2.0067	.37436	2.0257	.30665	2.5443	.33335	3.5214	.4435
April	420	2.7635	4.4602	3.2748	5.3127	2.7241	4.6830	3.3368	5.4695	3.1181	3.5458	3.6009	3.0546
	997	3.3845	2.4067	4.0804	2.7498	3.7056	2.9160	3.3044	2.7948	2.7672	1.4059	3.5335	1.3483
	2470	3.3141	1.1613	3.3726	1.1900	3.7203	1.2763	2.6586	1.0445	2.5676	.57766	3.3004	.5385
	6000	2.7461	.73656	3.3468	.74479	3.2969	.69900	2.4936	.46891	3.4774	.45412	5.2357	.5934
July	420	3.5465	5.8122	3.7867	5.7765	3.8386	6.3667	3.1673	5.8122	2.6331	4.6180	2.4736	2.7753
	997	3.3767	2.8885	3.7713	2.9707	3.7837	2.9990	3.3852	3.0288	3.0171	2.4227	3.3480	1.5222
	2470	2.7029	1.2414	3.7104	1.7019	3.0316	1.3083	2.9147	1.2418	2.8121	1.1409	3.3170	.7530
	6000	2.4536	1.0152	3.2221	.97725	3.0657	.84117	2.3522	.57890	2.7642	.48606	2.4447	.3796

* $\hat{\gamma} = (\overline{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\overline{x}$

Table A-6. Gamma Distribution Parameters γ and β of L Range (Associated with U Range) Estimated from Sample Moment Statistics*

Month	Filter λ_c(m)	Altitude (km) 4		6		8		10		12		14	
		γ	β(1/m)	γ	β	γ	β	γ	β	γ	β	γ	β
February	420	3.5129	.0269	3.8620	.0318	3.4447	.0301	3.7603	.0376	2.1597	.0215	2.2610	.0240
	997	3.5618	.0147	3.2469	.0147	2.8123	.0124	2.2186	.0111	2.4612	.0117	1.9530	.0099
	2470	2.5643	.0053	2.2883	.0046	2.2325	.0042	1.7932	.0037	1.9100	.0042	2.3743	.0045
	6000	1.6415	.0030	1.5704	.0017	1.5307	.0016	1.5881	.0015	1.8765	.0022	2.0555	.0022
April	420	3.5102	.0293	3.9302	.0319	3.3691	.0288	3.2901	.0323	3.1661	.0293	2.2937	.0198
	997	3.2268	.0130	4.1280	.0168	2.6893	.0111	2.3915	.0116	2.5769	.0110	2.5890	.0108
	2470	2.7068	.0051	2.6437	.0051	3.1583	.0058	2.3049	.0048	2.9662	.0063	2.7228	.0051
	6000	1.6792	.0025	2.0037	.0021	1.9754	.0019	2.1335	.0021	2.1085	.0022	3.1420	.0039
July	420	5.1746	.0398	3.9245	.0287	3.8306	.0304	3.7633	.0320	3.6531	.0325	3.0540	.0240
	997	2.7890	.0121	2.8991	.0123	3.6725	.0154	2.4720	.0101	3.1053	.0128	3.1303	.0114
	2470	2.3830	.0053	2.0355	.0046	2.5076	.0054	2.2080	.0042	2.1111	.0040	2.6624	.0049
	6000	.9255	.0016	1.6771	.0022	2.0761	.0026	2.0056	.0020	2.8401	.0025	2.1455	.0026

* $\hat{\gamma} = (\bar{x}/\sigma)^2$

$\hat{\beta} = \hat{\gamma}/\bar{x}$

APPROVAL

VECTOR WIND PROFILE GUST MODEL

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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