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Parameter Estimation of Large Flexible Aerospace Structures with Application to the Control of the Maypole Deployable Reflector

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1.0 Problem Description:

The Maypole Deployable Reflector is a large space structure consisting of a column and hoop structure which supports a circular antenna of 30-100 meters dia. It is a flexible structure and, hence, is described by partial, rather than ordinary, differential equations; such systems have a distributed parameter nature.

Flexible structures like the above can be described by partial differential equations of the form:

$$u_{tt} + D_0 u_t + A_0 u = F \tag{1.1}$$

where u(x,t) is the displacement of the structure subject to external forces F(x,t). The structural stiffness is determined by the differential operator A_0 and associated boundary conditions. An example of such an operator for a membrane antenna is the following:

$$A_0 u = \nabla \cdot E(x) \nabla u \tag{1.2}$$

where E(x) is the distributed "stiffness" of the membrane and

$$\nabla u = \text{gradient of } u = [\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \frac{\partial u}{\partial x_3}]^T.$$

The damping in the structure occurs due to material properties and

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construction techniques. It is represented by the differential operator $D_0 u_t$, but the actual form of D_0 in a given application is much more difficult to determine than that of the stiffness A_0. Some forms that D_0 might take are the following:

$$\begin{cases} D_0 u_t = \alpha_0 u_t & \text{(viscous damping)} \\ D_0 u_t = A_0^{1/2} u_t & \text{(visco-elastic damping)} \\ D_0 u_t = A_0 u_t & (?) \end{cases}$$

Of course, there are other possible forms for D_0 and, in general, the damping operator might turn out not to be a differential operator at all; this is the case for some types of damping which are related to frequency in a very nonlinear way.

The distributed parameter system (DPS) represented by (1.1) may be put into state variable form:

$$\begin{cases} \dfrac{\partial v(t)}{\partial t} = Av(t) + Bf(t) \\ y(t) = Cv(t) \end{cases} \tag{1.3}$$

where $A = \begin{bmatrix} 0 & I \\ -A_0 & -D_0 \end{bmatrix}$ and B, C represent the control actuator and sensor influence operators, respectively, with f(t) being the vector of M actuator commands and y(t) the vector of P sensor outputs. It is assumed that the operator A in (1.3) generates a C_0-semigroup U(t) which is similar to the matrix exponential e^{At} for lumped parameter systems.

2.0 Reduced-Order Modeling and Controller Design

Many reduced-order modeling techniques exist for large-scale systems; of particular interest are the ones based on asymptotic methods, such as multiple time scales and singular perturbations. Let H_N and H_R be the subspaces of the total state space H with $\dim H_N = N < \infty$ and $H = H_N \oplus H_R$. Define the projection operators P_N and P_R (not necessarily orthogonal) and let $v_N = P_N v$ and $v_R = P_R v$. This decomposes v into $v = v_N + v_R$ and the system (1.3) into

$$
\left\{
\begin{array}{ll}
\dot{v}_N = A_N v_N + A_{NR} v_R + B_N f, & v_N(0) = P_N v_0 \qquad (2.1) \\
\dot{v}_R = A_{RN} v_N + A_R v_R + B_R f, & v_R(0) = P_R v_0 \qquad (2.2) \\
y = C_N v_N + C_R v_R + Df & \qquad (2.3)
\end{array}
\right.
$$

The reduced-order model (ROM) for the system is (2.1) and (2.2) with C_R and A_{NR} assumed to be zero:

$$
\left\{
\begin{array}{ll}
\dot{\tilde{v}}_N = A_N \tilde{v}_N + B_N f, & \tilde{v}_N(0) = P_N v_0 \\
\tilde{y} = C_N \tilde{v}_N + Df &
\end{array}
\right. \qquad (2.4)
$$

Thus, the ROM depends on what choice of subspace H_N is made and what type of projection P_N is used (or, alternatively, what H_R is). The subspace H_N is called the ROM subspace and the subspace H_R, the residuals subspace. The terms $A_{NR} v_R$ and $A_{RN} v_N$ are called modeling error and $B_R f$ and $C_R v_R$ are called control and observation spillover, respectively. This gives a general format for reduced-order modeling.

The Controller Design is obtained by assuming the ROM (2.4) is the total system and using a linear controller of the form:

$$\begin{cases} f = H_{11}\, y + H_{12}\, z \\ \dot{z} = F\, z + Hy + Ef \quad , \quad z(0) = 0 \end{cases} \tag{2.5}$$

where dim $z = S \leq N$.

All controller synthesis assumptions are made in terms of the ROM parameters (A_N, B_N, C_N, D) alone. This allows the synthesis to be carried out when these parameters are known. When the ROM parameters are not known or are poorly known, they must be estimated on-line or the controller must adapt itself in the presence of unknown parameters.

3.0 Digital Parameter Estimation and Control

The implementation of any control or parameter estimation scheme will be achieved with one or more on-line digital computers; consequently, all such schemes must be discrete-time. A discrete-time version of (1) may be obtained by assuming a constant input f(k) over the uniform time interval $(k-1)\,\Delta t \leq t < k\,\Delta t$:

$$\begin{cases} v(k+1) = \Phi\, v(k) + Hf(k) \\ y(k) \quad = C\, v(k) \end{cases} \tag{3.1}$$

where $\Phi = U(\Delta t)$ and $H = \int_0^{\Delta t} U(\tau)\, Bd\tau$. Other discrete-time versions can be obtained using <u>non-uniform</u> time steps.

Now the model reduction described in (2.0) becomes:

$$\begin{cases} v_N(k+1) = \Phi_N\, v_N(k) + \Phi_{NR}\, v_R(k) + H_N f(k) \\ v_R(k+1) = \Phi_{RN}\, v_N(k) + \Phi_R\, v_R(k) + H_R f(k) \\ y(k) = C_N\, v_N(k) + C_R\, v_R(k) \end{cases} \tag{3.2}$$

where $\Phi_N = P_N\, \Phi\, P_N$, etc. The reduced-order model is

$$\begin{cases} v_N(k+1) = \Phi_N\, v_N(k) + H_N f(k) \\ y(k) = C_N\, v_N(k) \end{cases} \tag{3.3}$$

It is this approximation of the flexible structure whose parameters (Φ_N, H_N, C_N) we will want to estimate on-line and use in any control schemes. The dimension N of this reduced-order model is related to the on-line computational capacity.

Most parameter estimation schemes for lumped parameter systems depend on an <u>Autoregressive Moving Average</u> (ARMA) model of the system. For distributed parameter systems, such as flexible structures, we have developed the <u>quasi-ARMA</u>:

$$y(k+N) = \sum_{r=1}^{N} \alpha_r\, y(k+r-1) + \sum_{r=1}^{N} \beta_r\, f(k+r-1) + R(k) \tag{3.4}$$

where the coefficients α_r and β_r are obtained from (3.3) and the residual interaction term R(k) is given by

$$R(k) = C_R v_R(k+N) + \sum_{r=1}^{N} \Delta_r \; v_R(k+r-1) \tag{3.5}$$

with $\Delta_r = \Gamma_r \; \Phi_{NR} - \alpha_r \; C_r$ and $\beta_r = \Gamma_r \; H_N$.

By ignoring the term R(k), we have an approximate ARMA model of the flexible structure whose parameters α_r and β_r we can estimate via least-squares or model reference techniques. However, the residual interaction terms will cause errors in these parameter estimates; hence, the effects of these terms must be analyzed to determine acceptable levels of residual interaction and to aid in the selection of the model dimension N. Compensation in the form of (digital) prefiltering may be added to reduce the residual interactions with the parameter estimation scheme.

Once the parameters are adequately estimated, they may be used in a discrete-time feedback controller of the form:

$$\begin{cases} f(k) = H_{11} \; y(k) + H_{12} \; z(k) \\ z(k+1) = H_{21} \; y(k) + H_{22} \; z(k) \end{cases} \tag{3.6}$$

Such a controller can substantially improve the performance of the flexible mechanical structure, as long as control or observation spillover is not excessive. Spillover is related to the model dimension N and the actuator-sensor locations which are part of the control design.

The use of adaptive structure controllers, i.e., controllers which tune themselves to the structure, raises even more complicated residual interaction issues. In particular, it has been shown via numerical simulation that adaptive control based on a reduced-order model can cause unstable operation of the flexible structure if the spillover issue is not carefully considered.

4.0 Recent Progress

In the preliminary phase of this study, we have decided to consider two phases of operation of the Reflector: deployment and on-orbit operation. Deployment would be characterized by (possibly) high transient disturbances and rapidly-changing configuration. Control applied during this phase would need to be done on a fast time-scale; hence, a low-order model of the reflector dynamics would be incorporated in the identification algorithm to enable the calculations to proceed rapidly. On the other hand, once deployed the reflector would operate on-orbit in a relative steady-state. Hence, the identification (and control) could be done in a quasi-static mode.

We feel this time-scale separation is a useful way to break down the identification problem. In the dynamic (deployment) mode, we have developed rapid, reduced-order identification schemes based on reduced-order lumped parameter models of the distributed parameter description of the reflector. I discussed some of the theoretical developments on this at the Yale Adaptive Control Workshop.

For the time being, we consider the reflector to be a circular membrane; later, as the modeling aspect of the study develops, we expect to use more detailed models. Nevertheless, the generic behavior of a structure like the reflector can be revealed by even simple distributed models. In the future, I think we will try to fit distributed parameter models to the finite-element data on the reflector.

4.1 Quasi-Static DPS

In the quasi-static (on-orbit) mode, we do not have to rush to identify and control. Consequently, a least squares approach where a distributed parameter (partial differential equation) model is fitted to the sensor data seems reasonable. In discussions with H. T. Banks in May, we decided that a singular perturbations approach might yield some useful model reduction since the stiffness of the membrane is quite high along the radii where stringers are attached but much lower elsewhere on the reflector. We are using this idea to reduce the computational load of the identification problem.

The quasi-static (or steady state) identification problem can be described by setting $u(x,t) = u(x)$, i.e., no time-variation, in (1.1):

$$A_0 u = F = Bf \tag{4.1}$$

where A_0 is described, for example, as the membrane operator (1.2) with $E(x)$ unknown. Assume

$$E(x) = \sum_{k=1}^{N_E} \alpha_k \Phi_k(x) \tag{4.2}$$

where α_k are unknown scalars but $\Phi_k(x)$ are known functions (e.g., cubic splines or trigonometric functions). This reduces the problem to the estimation of the parameters α_k.

From the <u>Galerkin</u> or <u>finite-element</u> method, we approximate $u(x)$ in (4.1) by

$$\hat{u}(x) = \sum_{k=1}^{N} u_k \, \theta_k(x) \tag{4.3}$$

where $\theta_k(x)$ are known functions (although θ_k is not necessarily equal to Φ_k in (4.2)). The Galerkin method yields: for $\ell = 1, 2, \ldots, N$

$$\sum_{k=1}^{N} u_k \, \langle A_0 \, \theta_k \, , \, \theta_\ell \rangle = \langle F, \, \theta_\ell \rangle \tag{4.4}$$

where $\langle u,v \rangle \equiv \int_\Omega u(x) \, v(x) \, dx$ is the inner product on $L^2(\Omega)$ with Ω being the structure described in R^3 (or R^2). Substitution of (1.2) and (4.2) into (4.4) yields:

$$\sum_{i=1}^{N_E} \alpha_i \sum_{k=1}^{N} u_k \, \langle q_{ik}, \, \theta_\ell \rangle = \langle F, \, \theta_\ell \rangle \tag{4.5}$$

where the known functions yield

$$q_{ik}^{(x)} \equiv \sum_{j=1}^{3} \frac{\partial \theta_k(x)}{\partial x_j} \frac{\partial \phi_i(x)}{\partial x_j} + \phi_i(x) \, \Delta\theta_k(x)$$

and

$$\Delta u = \sum_{j=1}^{3} \frac{\partial^2 u}{\partial x_j^2}.$$

From (4.3), it is possible to obtain the u_k by knowledge of u(x) at various points on the structure (at least N of them) if we assume $\hat{u}(x) \cong u(x)$; this is true for N sufficiently large. Therefore solve:

$$\sum_{k=1}^{N} u_k \ \theta_k(x_i) = \hat{u}(x_i) \cong u(x_i) \tag{4.6}$$

for u_k when $1 \leq i \leq N$. Furthermore, we need knowledge of the loads f(x) applied by the actuators; this can be approximated and, hence, we assume $\langle F, \theta_\ell \rangle$ is also known. Then (4.5) takes the form:

$$L \ \underline{\alpha} = \underline{F} \tag{4.7}$$

where L is an N_E x N matrix and $\underline{\alpha}$, $\underline{F}$ are N_E x 1 and N x 1 vectors respectively. Since in general $N_E \neq N$ there may be no unique solution for (4.7); however, a "least-squares" solution can be obtained via the pseudo-inverse matrix $L^{\#}$ of L, i.e.,

$$\underline{\hat{\alpha}} = L^{\#} \ \underline{F} \tag{4.8}$$

is the best mean-square solution of (4.7).

The above describes our basic approach to the quasi-static problem. Once the values of $\hat{\alpha}_i$ are determined from (4.8), they are used to approximate E(x) by $\sum_{i=1}^{N_E} \hat{\alpha}_i \ \phi_i(x)$. Once E(x) is identified, it is possible to develop a quasi-static DPS control approach for (4.1). Thus, the control problem becomes the determination of actuator control commands $f_1, \ldots, f_M$ in (4.1) such that

$$\min_{f_i} \| u(x) - u_D(x) \|^2$$

is achieved where u(x) is the actual antenna shape and $u_D(x)$ is the desired shape, e.g., parabolic or spherical. We are in the process of evaluating this identification and control approach for the circular membrane model of the antenna. The singular perturbations idea for the reduction of (1.1) into two separate uniform problems is also under consideration.

Results on the convergence and "well-posed"-ness of the above scheme and other similar ones are presented in "A Survey for Parameter Estimation and Optimal Control in Delay and Distributed Parameter Systems" by H. T. Banks, ICASE-NASA Langley Research Center, Report No. 81-26, August 17, 1981.

4.2 Dynamic DPS

For the dynamic case (in which u(x,t) is a function of time, as well as x), it is necessary to identify the terms in the linear differential equations that define the structural time variations in (1.1). In discussing potential identification procedures, one must keep in mind that identification of the dynamics of a large space structure is usually performed for one or more of the following reasons:

(1) To build a simulation model of the system which can be used for predicting the response to various types of inputs.

(2) To build a controller for the structure.

(3) To design adaptive control to account for structural parameter changes. (This purpose clearly requires an on-line identification procedure.)

Relative to these purposes, various identification and adaptive control procedures have been considered. These include:

(1) Adaptive observer - this procedure can be used either off-line or on-line.

(2) Autoregressive moving-average (ARMA) identification - such an approach can be done off-line or recursively on-line. It has been applied by several investigators to adaptive control.

(3) Frequency domain identification - although this is strictly an off-line procedure, it has the potential for producing a high order model without the inherent computational problems of many on-line procedures.

(4) Indirect or implicit adaptive control - this procedure is useful for on-line direct computation of the control gains without the need for explicit system parameter identification.

These procedures will now be discussed in more detail.

(1) <u>Autoregressive Moving Average (ARMA) Identification (Balas-Kaufman)</u>

A typical ARMA model between some output quantity y and a forcing function f can be written as:

$$y(k+N) = \sum_{i=1}^{N} a_i \, y(k+i-1) + \sum_{i=1}^{N} b_i \, f(k+i-1) + R(k) \tag{4.9}$$

where R(k) incorporates the residual effects. If R(k) can be modeled as the correlated noise sequence

$$R(k) = \sum_{i=1}^{N} c_i \, e(k+i-1)$$

where e_i is a sequence of independent Gaussian variables, then maximum likelihood estimation can be used. Thus, a_i, b_i, c_i would be determined so as to minimize

$$L = 1/2 \sum_{k} (y(k) - \hat{y}(k/k-1))^2$$

where $\hat{y}(k/k-1)$ is the predicted Kalman estimate of y. The optimum model order N can then be determined by evaluating Akaike's criteria (- Log L + 2 * number of parameters) which weights both the error index and a measure of model complexity.

(2) <u>Frequency Domain Procedures (Kaufman)</u>

Alternately, if sinusoidal test signals can be used, it is suggested that frequency domain identification procedures also be considered. Previous results[1,2] have shown that for flexible aircraft this approach is advantageous in that

- the input design problem is eliminated;
- fewer parameters need be identified per computational cycle;
- Relatively simple algebraic least squares approaches can be used very effectively;

- results from several controllers excited separately can be combined to improve accuracy.

One disadvantage, however, is that the number of independent sensors must be equal to the assumed process order. Furthermore, this procedure is strictly an off-line type procedure and therefore is not suitable for use in an adaptive control scheme.

(3) Direct or Implicit Adaptive Control (Kaufman-Balas)

An adaptive control procedure which appears appropriate for large space structures is the direct or implicit adaptive controller proposed by Sobel, Kaufman, and Mabius.[3] This is a model reference procedure that adaptively tunes control gains so that the structural outputs follow the corresponding outputs of a reference model (which can be of lower order than the process). Stability is guaranteed provided that the structure has an input-output transfer function matrix that is positive real for some feedback gain matrix. Preliminary analysis indicates that this positive real property should hold for large space structural components provided that actuators and sensors are collocated and provided that velocity sensors are available.

(4) Adaptive Observer (Balas)

On-line adaptive parameter estimation of the DPS (1.3) cannot be done since this system is infinite dimensional. However, an adaptive observer based on a reduced-order finite-dimensional model (2.4) can be accomplished. Such an adaptive observer has the form:

$$
\begin{cases}
\dot{\hat{v}}_N(t) = K_N \hat{v}_N(t) + (k_N - \hat{a}_N(t))y(t) \\
\qquad + \hat{b}_N(t)\ f(t) + w(t) \\
\hat{y}(t) = C_N \hat{v}_N(t)
\end{cases}
\tag{4.10}
$$

where $K_N = [-k_N \mid Q_N]$ is a known stable matrix and $\hat{a}_N(t)$, $\hat{b}_N(t)$ are the parameter estimates obtained from the following nonlinear adaptation laws:

$$
\begin{cases}
\hat{a}_N(t) = \Gamma_1\ (y(t)-\hat{y}(t))\ v^1(t) \\
\hat{b}_N(t) = \Gamma_2\ (y(t)-\hat{y}(t))\ v^2(t)
\end{cases}
\tag{4.11}
$$

where $w(t)$ in (4.10) and $v^1(t)$, $v^2(t)$ in (4.11) are auxiliary signals obtained by filtering the inputs $f(t)$ and the outputs $y(t)$.

The parameter errors introduced by the reduced-order observer have been analyzed in Refs. 4 and 5.

(5) Parameter Estimation Using a Linear Reinforcement Learning Factor (Desrochers)

Recursive estimates which minimize

$$
\begin{cases}
J = \sum_{i=1}^{N} (x_i^T\ \hat{a}_i - y_i^*)^2\ (1-\alpha)^{N-1}\alpha \\
0 < \alpha < 1
\end{cases}
\tag{4.12}
$$

are considered where x_i and y_i^* are known and a_i are the parameters to be determined. This leads to the following recursive algorithm for parameter estimates:

$$\left\{\begin{aligned} \hat{a}_k &= \hat{a}_{k-1} - \frac{\alpha}{1-\alpha} P_{k-1}\, x_k \left(1 + \frac{\alpha}{1-\alpha} x_K^T P_{k-1}\, x_k\right)^{-1} \cdot \\ &\qquad (x_k^T \hat{a}_{k-1} - y_k^*) \\ P_k &= \frac{1}{1-\alpha} P_{k-1} - \frac{\alpha}{(1-\alpha)^2} P_{k-1}\, x_k \left(1 + \frac{\alpha}{1-\alpha} x_k^T P_{k-1}\, x_k\right)^{-1} \cdot \\ &\qquad x_k^T P_{k-1} \end{aligned}\right. \tag{4.13}$$

The effect of α (Forgetting Factor) is to weight newer data more heavily than past measurements in order to better adapt to changing parmaeters.

Of course, this approach has the disadvantage that the effect of noise will also be detected and used to modify the estimate $\hat{a}$. Therefore, the scheme functions best when the parameter variations are larger than the residual fluctuations due to noise. The factor α is a variable and it is updated through a linear reinforcement algorithm. Such an algorithm makes decisions about the reliability of the present data and adjusts the factor α accordingly.

(6) Feedback Control (Balas)

Analysis of the stability of feedback control based on finite element (Galerkin) reduced-order models of DPS such as flexible structures is considered in Ref. 6. This analysis assumes that the identification problem has been successfully completed by one of the above-mentioned procedures. Control procedures for DPS are evaluated in Ref. 7.

(7) Reduced-Order Modeling for Nonlinear Systems (Desrochers)

The first case is concerned with nonlinear systems that can be modeled by

$$x(k+1) = \sum_{j=1}^{n} A_j \; F_j(x_j(k)) + Bu(k) \tag{4.14}$$

where A_j is an n x m matrix of constants, $x(k)\varepsilon R^n$, F_j $(\cdot)$ are real valued nonlinear vector functions of dimension m, B is n x 1, and u(k) is the scalar input. Consider the equation error for a single non-linearity in (4.14)

$$e_j(k) = x(k+1) - [A_j \;\; B]\begin{bmatrix} F_j(k) \\ u(1) \end{bmatrix} j = 1, 2, \ldots , n \tag{4.15}$$

Minimizing

$$\sum_{k=0}^{N-1} e_j^T(k)e_j(k) \tag{4.16}$$

leads to an algorithm for assigning costs to the $F_j(\cdot)$ in order to retain the most dominant non-linearities. Since each $F_j(\cdot)$ is a function of only one x_j this leads to the retention of the most dominant states.

The more general case of

$$x(k+1) = \sum_{j=1}^{n} A_j \; F_j(x_1,x_2, \ldots , x_n) + Bu(k) \tag{4.17}$$

can also be handled this way. Note that now, model reduction may not necessarily lead to an elimination of state variables, but the number of terms, n, will be reduced.

These techniques are described in Refs. 8, 9 and 10. We feel that in the case of nonlinear deformation of the antenna it is wise to make a nonlinear model reduction before linearizing the system for control. Also, the nonlinear reduction can be used to obtain a nonlinear version of the ARMA described above, this could be useful in the identification of nonlinear models of the DPS.

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