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Radiative Transfer in Cometary Dust Atmospheres: Critique of Recent Developments

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CONTENTS

<u>Figures</u>

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Abstract.

The gas and dust production rates of comets near the sun are intimately linked to the fate of the solar radiation as it is scattered and partially absorbed by the cometary dust envelope or halo surrounding the comet's nucleus. This is so, because the radiative energy impinging on the surace of the nucleus governs the rate of evaporation of the nuclear matter (mostly water ice) and the release of dust particles embedded in the ice which are entrained by the escaping gas and subsequently form a dust atmosphere or halo. On the other hand, the interaction of the dust cloud with the solar radiation will in turn be responsible for the amount of radiative energy received by the nucleus and thus determine the evaporation rate. Here it will be shown, that recent attempts to quantitatively formulate this problem are in error. The correct formulation will be given and it will be pointed out that negative extinction (enhancement of the primary radiation received by the nucleus due to multiple scattering) may not be as large as predicted.

1.) Introduction.

The interaction of the solar radiation with a comet's nucleus as it approaches and recedes from perihelion, the build-up of a gaseous and dusty halo dwarfing the size of the nucleus, the interplay between trapped radiation within the dust atmosphere surrounding the nucleus and the amount of radiative energy delivered to the nucleus determining its rate of evaporation and production of dust by aerodynamic forces constitutes a topic of considerable research[1)]. But it is only recently that a comprehensive theory of radiative transfer within a cometary dust atmosphere has been promulgated[2)]. Absorption and scattering of light by the dust particles of the coma are important events, since they determine ultimately the amount of energy deposited on the surface of the cometary nucleus and thus, by this very fact, determine the density of the dust cloud and the gas production rate. In the chain of events, the radiative transfer throughout the dusty atmosphere of the comet constitutes one important link.

Unfortunately, the theory given in ref. 2 contains a number of errors and it is the aim of this paper to correct these and to provide a general framework for the evaluation of the light-dust interaction taking multiple scattering to arbitrary order into account. There are essentially two ways at our disposal to deal with radiative transfer, i.e., the analysis of a radiation field in a medium which absorbs and scatters radiation. One way, the customary way[3)], consists of writing down the energy balance for the intensity of the radiation (intensity ≡ energy flux per frequency interval and solid angle) thus:

$$\frac{dI_\nu}{ds} = \alpha_\nu I_\nu + \varepsilon_\nu \qquad (1)$$

where I_ν is the intensity of the radiation field at any given point in space, s the path length over which the change of the intensity is followed, α_ν the absorption coefficient and ϵ_ν the emission coefficient and finally ν is the frequency contemplated. In as much as ϵ_ν depends itself on the scattered intensity among other things, equation (1) constitutes a linear integral equation. The solution of this integral equation given appropriate boundary conditions can only be found approximately[3].

There exists however another method for dealing with radiative transfer. We call it the multiple scattering method. It consists of following each ray as it enters the medium, is scattered once then twice and so on, summing up all contributions of all configurations in an infinite series and constructing the radiation field at each point in space in this manner. This method is of an advantage when either the absorption cross-sections for the radiation are large or the scattering cross-sections are small. It is also advantageous if the assumption of local thermal equilibrium cannot be made. It has been used recently for evaluating the interaction of solar radition with the cometary dust clouds[2]. While the first method (eq. (1)) leads to an integral equation which is difficult to solve, the second method leads to multiple integrals which cannot be evaluated in closed form.

An analogy exists between the two methods of dealing with radiative transfer and a problem in optics. The diffraction of light by a plane parallel plate and the evaluation of transmission and reflection coefficients may be undertaken by either solving Maxwell's equations with the appropriate boundary conditions at the two surfaces of the plate or one follows a ray as it enters the plate via the first surface, being partially reflected and transmitted, the transmitted ray being again partially reflected and transmitted by the second surface and so on. Adding up all infinite many contributions leads to the same result as that obtained by the "boundary condition" method[4]. Obviously, the first method is analogous to the integral equation method (eq. (1)) and the second method to the multiple scattering method.

In the next section we shall describe the multiple scattering method in detail and apply it to the interaction of light with the dust cloud of a comet.

2.) Analysis.

Consider fig. 1. The cometary nucleus, a sphere of radius r_k, defines a polar coordinate system as shown. The z-axis or polar axis is directed toward the sun. Therefore, the incident flux of the solar radiation E_0 in erg per cm^2 and sec is pointed into the negative z-direction. A dust cloud of number density N(r) per cm^3 surrounds the nucleus and we assume that N is only a function of r, the radial distance from the nucleus. For the formulation of the multiple scattering method this assumption is not necessary but it simplifies the formalism. We assume with ref. 2 that Mie scattering by the dust particles (considered spherical and of uniform density) constitutes the predominant mechanism by which the solar radiation interacts with the dust cloud. Consequently, we introduce an absorption cross-section $\sigma_A [cm^2]$ per dust particle and a differential cross-section for scattering $d\sigma_S [cm^2]$ defined by:

$$d\sigma_S = \sigma_S \, \eta \, (\cos \delta) \, \frac{d\omega}{4\pi} \tag{2}$$

where $d\omega$ signifies the solid angle element into which the incident radiation is scattered, δ the angle between incident and scattered radiation, the function η being a measure of anisotropy (for isotropic scattering $\eta = 1$ independent of δ). Since

$$\int_{4\pi} \eta \, d\omega = 4\pi, \tag{3}$$

where the integration takes place over all solid angles, σ_S of eq. (2) signifies the total scattering cross-section and therefore the extinction cross-section is given by:

$$\sigma_{ex} = \sigma_A + \sigma_S. \tag{4}$$

Of course, the solar flux and the cross-sections for Mie scattering (2) and and (4) are all very much frequency dependent. But since no inelastic scattering events take place we have refrained from explicitly showing this dependence; it is implicitly understood. Also, we assume, again for simplicity, that the dust particles consist of one species only with constant radius a. The generalization to a distribution of different grain sizes is trivial.

Suppose the intensity of radiation at position 1 and going into direction 12 is I_2 (1). Then, the amount of intensity arriving at point 2 having not been scattered is given by:

$$I(2) = I_2(1) \exp\left\{- \sigma_{ex} \int_1^2 N dz\right\}, \tag{5}$$

where the integration extends along the straight line between point 1 and point 2 and the integral is to be taken positive.

In order to calculate the total amount of energy per second $\dot{E}$ received by the nucleus from the sun taking multiple scattering into account, we proceed as follows. Without any dust cloud $\dot{E}$ would simply be:

$$\dot{E} = E_\theta\, r_K^2 \int \cos\theta\, d\omega = 2\pi E_\theta\, r_K^2 \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta = \pi r_K^2\, E_\theta, \tag{6}$$

since the angle between the normal to a surface element of the nucleus and the direction of the incoming radiation is θ. However, with the cloud of dust reinstated we may split the radiation impinging on the surface of the nucleus into parts of ever larger complexity. For instance that part of the radiation which has undergone no scattering or absorption is given by (using eq. (5)):

$$\dot{E}_o = E_\theta \, r_K^2 \int_{\text{surface}} \cos\theta \, \exp\left\{ - \sigma_{ex} \int^\infty N \, dz \right\} \sin\theta \, d\theta \, d\phi \quad . \tag{7}$$

We have put the upper limit of the integral over N to infinity for convenience because N decreases rapidly with distance. From geometrical considerations it can easily be gathered that: (8)

$$\dot{E}_o = 2\pi \, r_K^2 \, E_\theta \int_o^{\pi/2} d\theta \, \cos\theta \, \sin\theta \quad \exp\left\{ - \sigma_{ex} \int_o^\infty N((r_K^2 + z^2 + 2 \, r_K \, z \, \cos\theta)^{1/2}) dz \right\},$$

and if the number density N is known as a function of its argument, the integral (8) is also known in principle. For a quadratic dependence of N on r:

$$N = n_o/r^2, \tag{9}$$

eq. (8) goes over into:

$$\dot{E}_o = 2\pi \, r_K^2 \, E_\theta \int_o^{\pi/2} \cos\theta \, \sin\theta \, d\theta \, e^{-\varepsilon\theta/\sin\theta} \quad , \tag{10}$$

where

$$\varepsilon = \sigma_{ex} \, n_o/r_K. \tag{11}$$

Expression (8) is equivalent to eq. (5) of ref. 2.

Next we consider single scattering events. A volume element at position r_1, θ_1, ϕ_1 (point P_1) of magnitude d^3r_1 contains $N(r_1)d^3r_1$ dust particles. The incident solar radiation flux at this point has been attenuated and is given by:

$$E(1) = E_\theta \exp\left\{ - \sigma_{ex} \int_0^\infty N((r_1^2 + z^2 + 2r_1 z \cos\theta_1)^{1/2})dz \right\} \qquad (12)$$

The radiative energy per sec scattered into an arbitrary direction by the small amount of dust at this point is given by:

$$dE_1 = \sigma_s \eta (\cos\delta) \frac{d\omega}{4\pi} E(1) N(r_1) d^3r_1, \qquad (13)$$

where δ is the angle between the incident direction and the direction of scattering. The amount of radiation impinging on the nucleus due to the radiation having cuffered one scattering event is given by eq. (13) when integrated over all possible positions $\underset{\sim}{r}_1$ and all directions from point P_1 which intersect the nucleus (the cone emerging from P_3 in fig. 1) provided that properly taken into account is the attenuation of the scattered radiation as it emerges from point P_1 and strikes any part of the nuclear surface visible from P_1. It is now convenient to introduce another spherical coordinate system with origin at P_1 (P_3 in fig. 1) such that any line from P_1 intersecting the nucleus makes an angle θ with the line connecting the center of the nucleus with P_1 which evidently became the z-axis of this new coordinate system. It is clear from fig. 1 (where P_3 replaces P_1) that θ and ϕ can take on all values between 0 and θ_{max} and 0 and 2π respectively, where θ_{max} is determined by:

$$\sin \theta_{max} = r_K/r_1, \qquad (14)$$

(r_K/r_3 in fig. 1). Furthermore, the scattering angle δ obeys:

$$\cos\delta = \cos\theta \cos\theta_1 + \sin\theta \sin\theta_1 \cos\phi \ . \tag{15}$$

The attenuation of the scattered radiation emerging from point P_1 and heading into the direction θ, ϕ in the new coordinate system is given by:

$$e^{-F(\theta)} = \exp\left\{-\sigma_{ex}\int_0^{r_1\cos\theta - (r_K^2 - r_1^2 \sin^2\theta)^{1/2}} N((r_1^2 + z^2 - 2r_1 z \cos\theta)^{1/2})\, dz\right\}. \tag{16}$$

Collecting eqs. (12), (13) and (16) and integrating yields for that part of the energy impinging on the nuclear surface which underwent one scattering event:

$$\dot{E}_1 = \int' d^3r_1\, N(r_1)\, E(1)\, \frac{\sigma_s}{4\pi}\int_0^{\theta_{max}} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, e^{-F(\theta)}\, \eta(\cos\delta). \tag{17}$$

According to fig. 1 no scattering event can take place in the shadow region, at least not for the first scattering event. Therefore, in the integration over r_1 the shadow region must be excluded. The prime is a reminder of this fact. Analytically, this statement is expressed by the definition:

$$\int' d^3r_1 \ldots = \int_0^{2\pi} d\phi_1 \left[\int_0^{\pi/2} \sin\theta_1\, d\theta_1 \int_{r_K}^{\infty} r_1^2\, dr_1 \ldots + \int_{\pi/2}^{\pi} \sin\theta_1\, d\theta_1 \int_{r_K/\sin\theta}^{\infty} r_1^2 d\, r_1 \ldots\right] \ . \tag{18}$$

For isotropic scattering ($\eta = 1$) eq. (18) goes over into:

$$\dot{E}_1 = \frac{\sigma_s}{2} E_\theta \int d^3r_1\, N(r_1) \exp\left\{-\sigma_{ex}\int_0^{\infty} N((r_1^2 + z^2 + 2r_1 z \cos\theta_1)^{1/2}) dz\right\} \int_0^{\theta_{max}} \sin\theta\, e^{-F(\theta)} d\theta . \tag{19}$$

It is eq. (19) which must be compared with its equivalent expression (16) of ref. 2.

If we set $F(\theta) = 0$ in the last (angular) integral of eq. (20) we obtain:

$$\int_0^{\theta_{max}} \sin\theta \, d\theta = 1 - \sqrt{1 - r_K^2/r_1^2} \quad , \tag{20}$$

and eq. (19) goes over into eq. (6) of ref. 2. But putting $F(\theta) = 0$ is equivalent to setting $\sigma_{ex} = 0$ according to eq. (16) which means that the scattered light is <u>not</u> attenuated as it traverses the dust cloud toward the surface of the nucleus! When the dust particle density N is given by eq. (9), the integral of eq. (16) can be performed and yields:

$$F(\theta) = \varepsilon(r_K/r_1\sin\theta)(\pi/2-\theta-\tan^{-1}\left[r_K^2/r_1^2\sin^2\theta-1\right]^{1/2}), \tag{21}$$

with ε given by eq. (11).

We now turn to higher order scattering contributions. Consider the light being scattered at P_1 into the direction of P_2 (fig. 1). The amount of radiation scattered into this direction is given by (see eq. (13)):

$$dE_1 = \sigma_S \, \eta(\cos\delta_1) \, E(1) \, N(r_1) \, d^3r_1 \frac{d\omega}{4\pi} \, , \tag{22}$$

where

$$\cos\delta_1 = r_{12}^{-1}(r_1 \cos\theta_1 - r_2 \cos\theta_2). \tag{23}$$

The latter equation follows from simple geometrical considerations. r_{12} is the length of the vector $\underset{\sim}{r}_2 - \underset{\sim}{r}_1$:

$$r_{12} = (r_1^2 + r_2^2 - 2r_1r_2 \cos\gamma_{12})^{1/2}, \tag{24}$$

with

$$\cos\gamma_{12} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1-\phi_2). \tag{25}$$

The amount of radiative <u>flux</u> arriving at P_2 will be attenuated in going from P_1 to P_2 and is given by:

$$\frac{dE_1}{dA} \exp\left\{-\sigma_{ex}\int_{P_1}^{P_2} N(r)dz\right\} = \sigma_s\ \eta(\cos\delta_1)\ E(1)N(r_1)\ d^3r_1\ (4\pi r_{12}^{\ 2})^{-1} \exp\left\{-\sigma_{ex}\int_{P_1}^{P_2} N\ dz\right\}, \tag{26}$$

where $dA = r_{12}^{\ 2}d\omega$. The presence of the factor $r_{12}^{\ -2}$ accounts for the divergence of the beam emerging from the point P_1. At point P_2 the second scattering event takes place which may either lead to a third scattering event elsewhere (at P_3 in fig. 1) or scatter the light directly to the surface of the nucleus. The analysis from this point on is identical to the one which lead to eq. (17), so we will not expatiate on it any further. The result is:

$$\dot{E}_2 = \frac{\sigma^2_s}{(4\pi)^2}\int' d^3r_1\int' d^3r_2 N(r_1)N(r_2)E(1)\,\eta\,(\cos\delta_1)\ \exp\left\{-\sigma_{ex}\int_{P_1}^{P_2} N(r)dz\right\} r_{12}^{-2}\int_0^{\theta_{max}}\sin\theta\, d\theta\int_0^{2\pi} d\phi\,\eta\,(\cos\delta_2)\, e^{-F(\theta)}. \tag{27}$$

$E(1)$ in eq. (27) is given by eq. (12). θ_{max} is determined by $\sin\theta_{max} = r_K/r_2$. δ_2 is the scattering angle between the direction $r_2 - r_1$, and the θ, ϕ direction of the new polar coordinate system within the cone as discussed previously is given by:

$$\cos\delta_2 = (r_1\ \cos\gamma_{12} - r_2)r_{12}^{-1}\ \cos\theta + (r_1/r_{12})\ (\sin\theta\ \cos\phi\ [\sin\theta_2\ \cos\theta_1 - \cos\theta_2\ \sin\theta_1\cos(\phi_2-\phi_1)] + \sin\theta\ \sin\phi\ \sin\theta_1\ \sin(\phi_2-\phi_1)). \tag{28}$$

The primes at the integration signs indicate that the appropriate shadow zones must be excluded. For the integration over r_1 it is shown in fig. 1 and for the integration over r_2 it is shown in fig. 2. The analytic representation, simple in the case of fig. 1 and given by eq. (18), may be formulated in the following manner in case of fig. 2: Given the angles θ_2 and ϕ_2, determine γ_{12} from eq. (25). If $\gamma < \pi/2 - \theta_{max}$ where θ_{max} is given by eq. (14), with r_1 replaced by r_2, the integration over r_2 extends from r_K to infinity. If $\gamma > \pi/2 - \theta_{max}$, the range of integration over r_2 extends from

$$r_K/\sin(\gamma + \theta_{max}) < r_2 < \infty. \tag{29}$$

If $\theta_1 = \theta_{max} = 0$, the prescription (29) goes over into eq. (18) as it must.

The integral (27) constitutes the second term in an infinite series of multiple scattering contributions toward the insolation of the nucleus. It is by now clear how to generalize eq. (27) to triple scattering contributions for instance. We will refrain from doing so, since the expressions become quickly impractical for computer work and ref. 2 stopped at this point too. It is not very easy to compare our final expression (27) with the work of ref. 2, since the results there have not been put into closed form as we have done in eq. (27). However, eq. (26) and eq. (13) of ref. 2 should be equivalent. Comparing the two expressions, it becomes evident that indeed they are essentially equivalent, were it not for the fact that the divergence factor ${r_{12}}^{-2}$ is missing in ref. 2. But the radiation emerging from a point diverges; the flux becomes smaller the further away from the point source and this must be taken into account. As the points P_1 and P_2 coalesce and $r_{12} \to 0$ eq. (26) is not valid anymore, we are in the presence of the near field of the scattered radiation. Obviously, no contributions toward scattering have developed yet. We must exclude a small region about the singularity at $r_{12} = 0$, and the integration over r_2 is understood as a principal value as far as the singularity is concerned. For a quadratic dependence on distance of the dust density eq. (9), the attenuation along the path 1-2 is readily obtained and is given by:

$$\exp\left\{-\sigma_{ex}\int_{P_1}^{P_2} N(r)dz\right\} = \exp\left[-\varepsilon(r_{12}\, r_K/r_1\, r_2)(\gamma_{12}/\sin\gamma_{12})\right]. \tag{30}$$

Again ε is defined by eq. (11). In case of isotropic scattering we obtain from eq. (27) the somewhat simpler formula:

$$\dot{E}_2 = \frac{\sigma_s^2}{8\pi}\int d^3r_1\int d^3r_2\ N(r_1)N(r_2)\ E(1)\ \exp\left\{-\sigma_{ex}\int_{P_1}^{P_2} N(r)dz\right\} r_{12}^{-2}\int_0^{\theta_{max}} \sin\theta\ d\theta\ e^{-F(\theta)} \tag{31}$$

3.) Discussion.

The first few terms of a series of contributions to the total insolation of a cometary nucleus surrounded by a dust cloud have been derived. Each subsequent contribution takes one more scattering event into account than the previous one and the total sum determines the total insolation. Of course, the terms of this series become very rapidly very complex and even high speed computers could not possibly cope with the higher order terms. The only hope for keeping computer times low and still obtain reasonable results, is a rapid convergence of the series and this is assured when the absorption is large or the scattering cross-section small. Just where we stand in this respect with the dust cloud may be seen from the following argument. Assume the dust cloud to obey the law (9). Assume also that there are approximately 20 dust particles per m^3 at a distance of 1000 km from the center of the nucleus[5]. This determines n_o to be $2x10^{11}$ cm^{-1}. Assuming furthermore the dues to consist of spheres of Olivin[2] with a refractive index of slightly less than two and a radius of 1 μm, we obtain for the extinction cross-section[6]:

$$\sigma_{ex} = \sigma_A + \sigma_S \approx \sigma_S = \pi\ a^2 Q_{ex} = 6.3x10^{-8}\ cm^2 \tag{32}$$

For a nuclear radius r_K of 3 km, we obtain for the quantity ε

$$\varepsilon = \sigma_{ex}\ n_o/r_K = 0.04 \tag{33}$$

and we see that the attenuation as for instance expressed in eq. (30) is not very large. But it is easy to see that the general term $\dot{E}_n$ is proportional to ε^n and because of eq. (33) rapid convergence is assured.

So far we have only considered the irradiation of the nucleus by the sun. But, the nucleus itself radiates too. Partially, through its albedo, the nuclear surface will reradiate energy which will be subjected to multiple scattering in the dust atmosphere just as the sun's radiation previously considered. Partially some of the solar energy will be used in melting the water ice and some of the energy will penetrate the interior of the nucleus. Also, the nucleus will emit radiation by virtue of its temperature. Because of the linearity of the radiative transfer equations, the nuclear radiation field due to reflection and temperature-radiation may be treated independently from the solar radiation in the same manner as discussed in section 2. One must only insure that the proper energy balance is maintained. But, this is not the place to discuss these items any further. Our aim was to derive and describe a correct formulation of the radiative transfer theory based on the concept of multiple scattering events. We will therefore discuss here briefly the two points in which the present theory differs from published results[2)]. It is clear from the exposition in section 2, that the two points in question are: (1.) the neglect of attenuation of the radiation as it is scattered toward the nucleus (the cone in fig. 1) and (2.) the disregard of the beam divergence, the flux of radiation in going from point P_1 to P_2 for instance (see fig. 1) being inversely proportional to the area $r_{12}^2 d\omega$ at the point P_2 where the second scattering event is taking place.

While the neglect of attenuation of the radiation during its last leg of travel after having undergone n scattering events (points P_1, P_2 and P_3 of fig. 1 for instance) always increases the amount of radiation the nucleus receives, the omission of the beam divergence tends also to overestimate the insolation of the nucleus by scattered radiation if the extinction is not too large since the volume sampled by the integration over r_2 is large. For large absorptivities however, the influence may be in the opposite direction because now smaller distance r_{12} are weighted more heavily on account of the decrease with distance of the attenuation factor

$$\exp\left(-\sigma_{ex}\int_{P_1}^{P_2} N\,dz\right)$$

(see eq. (31)). But in this case higher order contributions (double or more scatterings) tend to be small anyway.

In conclusion, it must be said, that the numerical calculations and the numerical results given in ref. 2 are rather doubtful. If the extinction of solar radiation in the cometary dust cloud (both light scattering and absorption) is of any significance to the gas and dust production and ultimately responsible for the net loss of matter during each perihelion passage of the comet, then, only new extensive numerical calculations with the aid of the correct theory, as propounded on the previous pages, will give an adequate picture of the situation.

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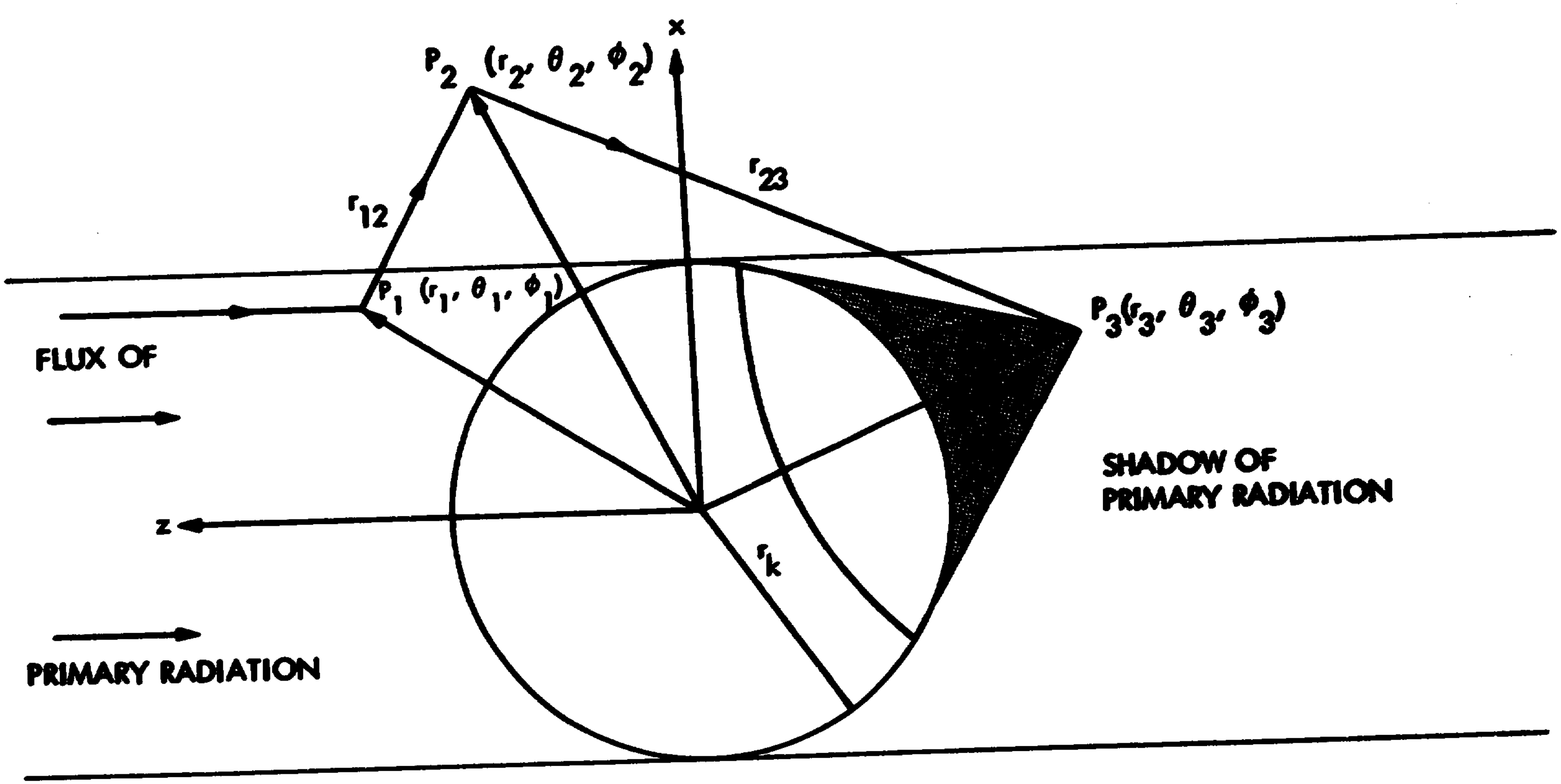

Fig. 1. Geometry for the multiple scattering approach used in this paper. The incident (solar) radiation of $E_\odot$ [erg/cm^2 sec] impinges from the negative z-direction. Depicted is a contribution toward the total irradiance of the nucleus from a triple scattering event.

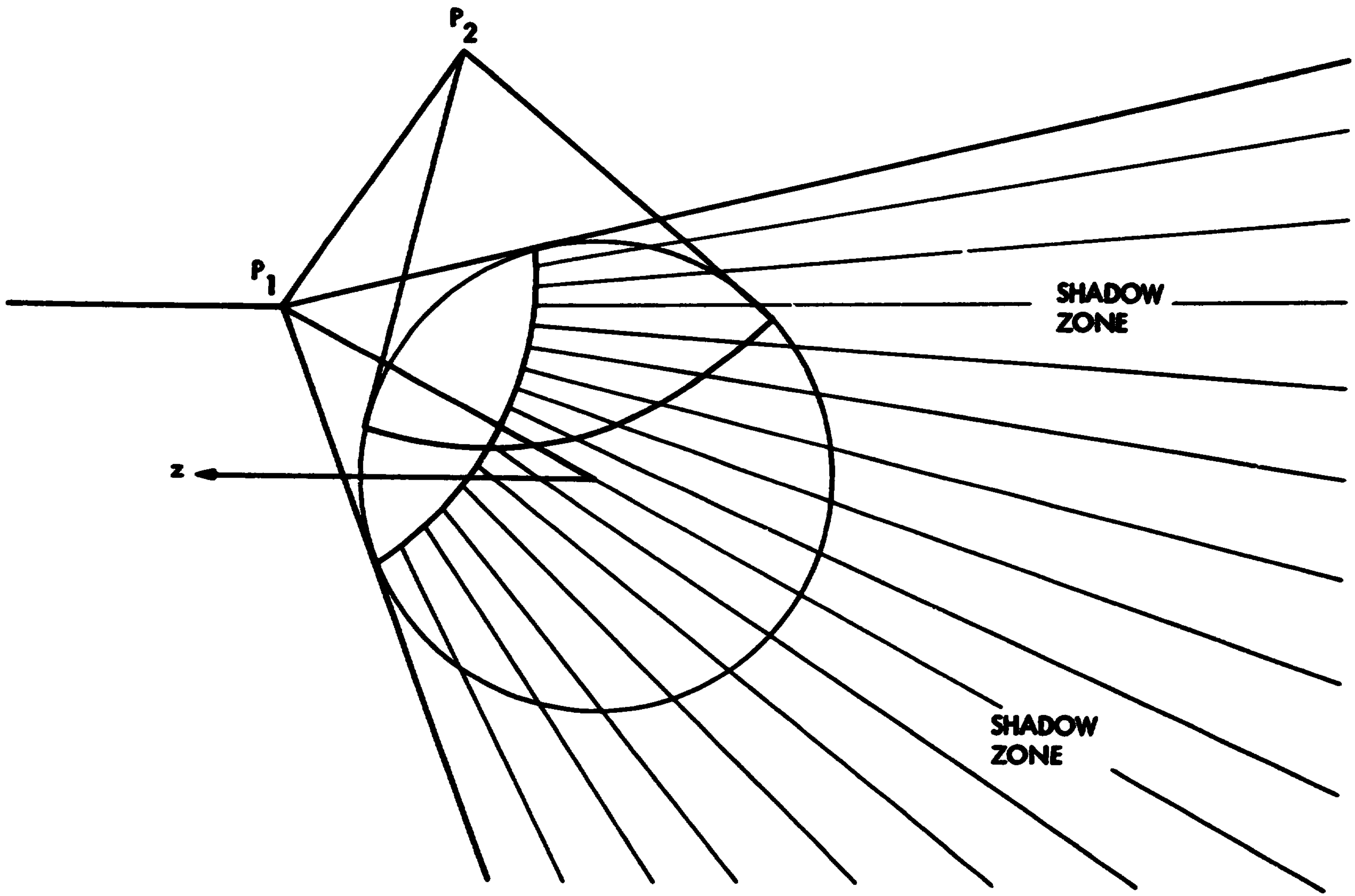

Fig. 2. The integration over r_2 excludes the shadow zone defined by the lower part of the cone with the tip at P_1.