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PROGRESS REPORT ON THE

FORMULATION OF A DYNAMIC ANALYSIS METHOD FOR A GENERIC FAMILY OF HOOP-MAST ANTENNA SYSTEMS

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INTRODUCTION

We are conducting analytical studies of "mast-cable-hoop-membrane" type antennae, using a transfer-matrix (Ref. 1) numerical analysis approach. This methodology has been chosen as particularly well-suited for handling a large number of antenna configurations of a generic type. While not capable of providing, in principle, more information than a proper NASTRAN formulation, a dedicated transfer matrix analysis, both by virtue of its specialization and the inherently easy compartmentalization of the formulation and numerical procedures, can be significantly more efficient not only in computer time required but, more importantly, in the time needed to review and interpret the results.

TRANSFER OF STATE VARIABLES AROUND THE HOOP

The analysis begins with the formulation of the typical element of the hoop assembly. Assumptions in this formulation are as follows:

1. The hoop is polygonal,

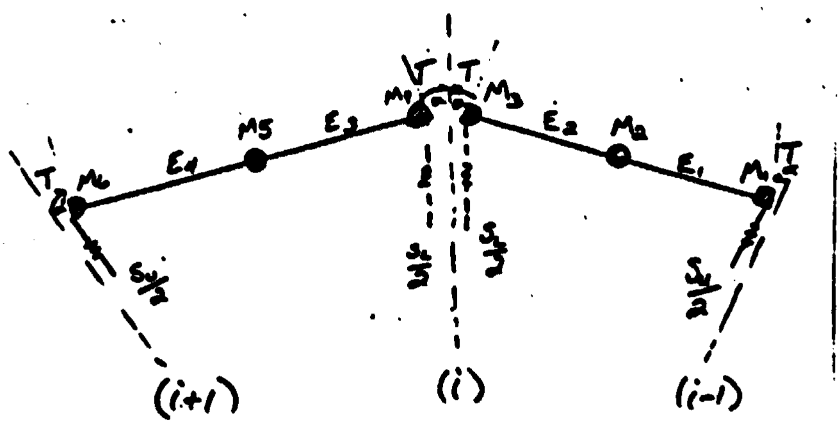
2. the hoop support cables alternate up and down with each segment of the hoop,

3. mass effects are approximated by a series of concentrated masses and mass moments of inertia,

4. stiffness transfer matrices account for the flexibility of hoop segments, which are modelled as equivalent beam-column-torsion members, including shear deflections, and

5. initial tensions in the cables and hoop are negligible compared to spring rates, (these tensions will be represented in subsequent analysis).

The typical hoop element model is conceived as beginning and ending in the middle of an upper cable position on the hoop, as shown below



Note that for hoop element #1 (ie $i = 1$) the right-hand most station is "0" and the left-hand most station is "2".

With a transformation of the state variables around the hoop, as follows:

$$
\left|\begin{array}{c} V \\ M \\ \phi \\ v \\ V \\ M \\ \phi \\ w \\ T \\ \theta \\ F \\ u \end{array}\right|_{i+1}
\begin{array}{l} \\ \\ \\ -y \\ \\ \\ \\ -z \\ \\ \\ \\ \\ \end{array}
= \left[\frac{S_u}{2}\right][T][M_6][E_4][M_5][E_3][M_4][T]\left[\frac{S_L}{2}\right]\left[\frac{S_L}{2}\right][T]
$$

$$
[M_3][E_2][M_2][E_1][M_1][T]\left[\frac{S_u}{2}\right]
\left|\begin{array}{c} V \\ M \\ \phi \\ v \\ V \\ M \\ \phi \\ w \\ T \\ \theta \\ F \\ u \end{array}\right|_{i-1}
\begin{array}{l} \\ \\ \\ -y \\ \\ \\ \\ -z \\ \\ \\ \\ \\ \end{array}
$$

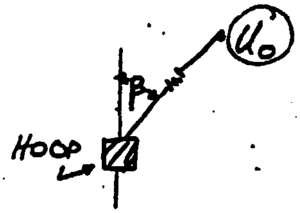
WHERE WE DEFINE,

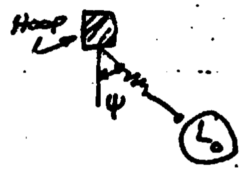
$$
[\Phi] = [T][M][E][M][E][M][T]
$$

DEFINITION OF TRANSFER MATRICES

THE $\left[\frac{S_U}{2}\right]$ & $\left[\frac{S_L}{2}\right]$ TRANSFER MATRICES IMPOS[E] UPPER AND LOWER CABLE FORCES ON THE HOOP, FOR THE MOMENT ASSUMING A FIXED MAST.

THE HOOP SUPPORT CABLES ARE APPROXIMATED AS SPRINGS.





$$\left[\frac{S_U}{2}\right] = S_{\text{UPPER CABLE}} = \begin{bmatrix} 1 & 0 & 0 & -K_u\cos^2\beta & 0 & 0 & 0 & K_u\sin\beta\cos\beta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_u\cos\beta\sin\beta & 1 & 0 & 0 & -K_u\sin^2\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\frac{S_L}{2}\right] = S_{\text{LOWER CABLE}} = \begin{bmatrix} 1 & 0 & 0 & -K_L\cos^2\psi & 0 & 0 & 0 & -K_L\sin\psi\cos\psi & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_L\cos\psi\sin\psi & 1 & 0 & 0 & -K_L\sin^2\psi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

THE $[T]$ TRANSFER MATRIX IS AN AXIS TRANSFORMATI[ON] MATRIX, ACCOUNTING FOR CHANGES IN ORIENTATION OF HOOP SEGMENT AXES AROUND THE HOOP AZIMUTH.

$$[T] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos\alpha & 0 & 0 & 0 & 0 & 0 & 0 & \sin\alpha & 0 & 0 & 0 \\
0 & 0 & \cos\alpha & 0 & 0 & 0 & 0 & 0 & 0 & -\sin\alpha & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos\alpha & 0 & 0 & 0 & 0 & 0 & -\sin\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha & 0 & 0 & 0 & -\sin\alpha \\
0 & -\sin\alpha & 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha & 0 & 0 & 0 \\
0 & 0 & \sin\alpha & 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & \sin\alpha & 0 & 0 & 0 & 0 & 0 & \cos\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\alpha & 0 & 0 & 0 & \cos\alpha
\end{bmatrix}$$

AND WE FURTHER DEFINE

$[T_1^{(i)}]$ AS THE SAME AS $[T]$ WHERE THE TRANSFORMATION ANGLE $\alpha = (4i-4)\alpha$

AND

$[T_2^{(i)}]$ WHERE $\alpha = (4i-2)\alpha$

AND

$[T_3^{(i)}]$ WHERE $\alpha = (4i)\alpha$

THE MASS TRANSFER MATRICES [M] INCORPORATE D'ALEMBERT FORCE AND MOMENT (REF. 1), ~~WHICH~~ AND REVEAL~~S~~ THE ASSUMPTION OF INFINITESIMAL DIMENSIONS OF THE CONCENTRATED MASSES.

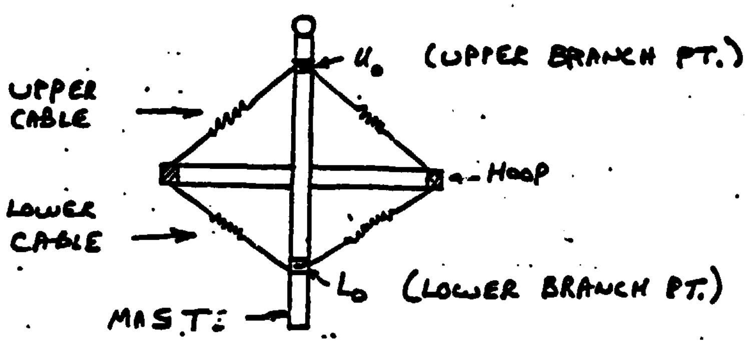
$$[M] = \begin{bmatrix} 1 & 0 & 0 & \omega^2 m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\omega^2 I_{y_m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \omega^2 m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\omega^2 I_{z_m} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \omega^2 I_{x_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \omega^2 m \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

THE ELASTIC TRANSFER MATRICES [E] TREAT THE HOOP SEGMENTS AS MASSLESS EQUIVALENT BEAMS AND INCLUDE CONSIDERATION OF AXIAL, TORSIONAL AND SHEAR DEFLECTIONS. (REF 2)

$$[E] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \ell & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\ell^2}{2EI} & \frac{\ell}{EI} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\ell^3}{6EI}-\frac{F\ell}{AG} & \frac{\ell^2}{2EI} & \ell & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ell & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\ell^2}{2EI} & \frac{\ell}{EI} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\ell^3}{6EI}-\frac{F\ell}{AG} & \frac{\ell^2}{2EI} & \ell & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\ell}{GJ_T} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\ell}{AE} & 1 \end{bmatrix}$$

INTRODUCING THE EFFECT OF CABLE FORCES

The analysis proceeds by accounting for the transverse motions of the mast as they affect cable forces. Note that axial motions of the mast are neglected.



The azimuthal direction corresponding to station "0" on the hoop is chosen as the reference direction; and state variables on the mast at the upper branch point, U_o, and the lower branch point, L_o, are referred to that direction.

A cable force on the hoop is equal and opposite to that on the mast. Thus, we define

$$\left[\frac{\bar{S}_u}{2}\right] = -\left\{\left[\frac{S_u}{2}\right] - [I]\right\}$$

$$\left[\frac{\bar{S}_L}{2}\right] = -\left\{\left[\frac{S_L}{2}\right] - [I]\right\}$$

Here, subtracting the unit matrix from $\left[\frac{S_u}{2}\right]$ and $\left[\frac{S_L}{2}\right]$ insures that in calculating cable forces only transverse displacements come into play.

WE CAN NOW WRITE STATE VARIABLES FOR THE LEFT END OF A TYPICAL HOOP ELEMENT WITH THE ALGORITHM BELOW.

FOR TYPICAL HOOP ELEMENT NO. "N"

$$
\Big\|_{2N} = \left[\frac{S_u}{2}\right]\left[\Phi\right]\left[\frac{S_L}{2}\right]\left[\frac{S_L}{2}\right]\left[\Phi\right]\left[\frac{S_u}{2}\right]\Big\|_{2N-2}
$$

$$
+\left\{\left[\frac{S_u}{2}\right]\left[\Phi\right]\left[\frac{S_L}{2}\right]\left[\frac{S_L}{2}\right]\left[\Phi\right]\left[\frac{\bar{S}_u}{2}\right]\left[T_1^{(N)}\right] + \left[\frac{\bar{S}_u}{2}\right]\left[T_3^{(N)}\right]\right\}\Big\|_{u_0}
$$

$$
+\left\{\left[\frac{S_u}{2}\right]\left[\Phi\right]\left[\frac{S_L}{2}\right]\left[\frac{\bar{S}_L}{2}\right]\left[T_2^{(N)}\right] + \left[\frac{S_u}{2}\right]\left[\Phi\right]\left[\frac{\bar{S}_L}{2}\right]\left[T_2^{(N)}\right]\right\}\Big\|_{L_0}
$$

THE ALGORITHM IS FURTHER REFINED, SETTING

$$[\psi] = \left[\frac{S_u}{2}\right][\Phi]\left[\frac{S_L}{2}\right]\left[\frac{S_L}{2}\right][\Phi]\left[\frac{S_u}{2}\right]$$

$$[\bar{\psi}] = \left[\frac{S_u}{2}\right][\Phi]\left[\frac{\bar{S}_L}{2}\right]\left[\frac{S_L}{2}\right][\Phi]\left[\frac{\bar{S}_u}{2}\right]$$

$$[\bar{\bar{\psi}}] = \left[\frac{S_u}{2}\right][\Phi]\left[\frac{S_L}{2}\right]\left[\frac{\bar{S}_L}{2}\right] + \left[\frac{S_u}{2}\right][\Phi]\left[\frac{\bar{S}_L}{2}\right]$$

WE GET

$$\Big\|_{2N} = [\psi]^N \Big\|_0 + \overbrace{\sum_{i=1}^{N} [\psi]^{N-i}[\bar{\bar{\psi}}][T_2^{(i)}]}^{[\bar{P}_N]} \Big\|_{L_0}$$

$$+ \underbrace{\sum_{i=1}^{N} \left\{[\psi]^{N-i}[\bar{\psi}][T_1^{(i)}] + [\psi]^{N-i}\left[\frac{\bar{S}_u}{2}\right][T_3^{(i)}]\right\}}_{[P_N]} \Big\|_{u_0}$$

A GENERAL FORM FOR THE MIDDLE DISPLACEMENT OF A TYPICAL HOOP ELEMENT WOULD THEN BE:

$$\Big\|_{(2N-1)} = \left[\frac{S_L}{2}\right][\Phi]\left[\frac{S_u}{2}\right][\psi]^{N-1}\Big\|_0 + \overbrace{\left\{\left[\frac{\bar{S}_L}{2}\right][T_2^{(N)}] + \sum_{i=1}^{N-1}[\psi]^{N-i}[\bar{\bar{\psi}}][T_2^{(i)}]\right\}}^{[\bar{\bar{P}}_N]}$$

$$+ \left\{\left[\frac{S_L}{2}\right][\Phi]\left[\frac{\bar{S}_u}{2}\right][T_1^{(N)}] + \sum_{i=1}^{N-1}\left\{[\psi]^{N-i}[\bar{\psi}][T_1^{(i)}] + [\psi]^{N-i}\left[\frac{\bar{S}_u}{2}\right][T_3^{(i)}]\right\}\right\}\Big\|_{u_0}$$

LET M = TOTAL NUMBER OF HOOP ELEMENTS (WHERE WE RECALL THAT AN ELEMENT IS DEFINED AS THE SECTION OF THE HOOP BETWEEN TWO ADJACENT UPPER CABLES). SINCE THE HOOP CLOSES, WE KNOW THAT THE STATE VARIABLES AT HOOP STATION ZERO ARE THE SAME AS THOSE AT HOOP STATION $2M$, SO WE CAN WRITE THE STATE VARIABLES AT STATION ZERO AS;

$$\Big\|_{0} = \left\{[I] - [\varphi]^{M}\right\}^{-1}\left\{[P_M]\Big\|_{u_0} + [\bar{P}_M]\Big\|_{L_0}\right\}$$

SUBSTITUTING $\|_{0}$ INTO PREVIOUS EXPRESSIONS FOR $\|_{2N}$ & $\|_{2N-1}$; WE CAN WRITE THESE AS FUNCTIONS OF $\|_{u_0}$ & $\|_{L_0}$

$$\Big\|_{2N} = [\Lambda_N]\Big\|_{u_0} + [\bar{\Lambda}_N]\Big\|_{L_0}$$

AND

$$\Big\|_{2N-1} = [\bar{\bar{\Lambda}}_N]\Big\|_{u_0} + [\bar{\bar{\bar{\Lambda}}}_N]\Big\|_{L_0}$$

TRANSFER OF STATE VARIABLES ALONG THE MAST

The analysis now focuses on the transfer of state variables along the mast. The following assumptions are made for the mast configuration:

1. The feed assembly has its unique mass,
2. the feed mast has a uniform mass distribution,
3. the upper mast has a mass distribution that increases in sections as we proceed down the mast,
4. the hub has a unique mass,
5. the lower mast has a mass distribution that decreases in sections as we proceed down the mast,
6. the bottom mast has a uniform mass distribution, and
7. the mast may be assumed infinitely stiff in the axial direction.

The transfer of state variables from the feed assembly to just before the upper branch point takes the form:

$$\left\| \quad \right\|_{u_0} = \left[X \right] \left\| \quad \right\|_{F.A.}$$

To obtain the state variables at the branch point we add the cable forces from all the upper cables.

SO,

$$\Big\|_{u_0} = [X]\Big\|_{F.P.} + \Big\|_{\substack{\text{UPPER}\\ \text{CABLE FORCES}}}$$

INTRODUCING $[T_4^{(i)}]$ AS THE SAME AS PREVIOUSLY DEFINED $[T_3^{(i)}]$, EXCEPT THE SIGNS ARE REVERSED FOR ALL ±

WE DERIVE

$$\Big\|_{u_{C.F.}} = \overbrace{\sum_{N=1}^{M} [T_4^{(N)}]\Big\{(2)\Big\{[I]-\Big[\frac{S_u}{2}\Big]\Big\}[\Lambda_N] + (2)\Big\{\Big[\frac{S_u}{2}\Big]-[I]\Big\}[T_3^{(N)}]\Big\}}^{[\beta]}\Big\|_{u_0}$$

$$+ \underbrace{\sum_{N=1}^{M} [T_4^{(N)}]\Big\{[I]-\Big[\frac{S_u}{2}\Big]\Big\}[\bar{\Lambda}_N]}_{[\bar{\beta}]}\Big\|_{L_0}$$

SUBSTITUTING INTO EQN. FOR $\Big\|_{U_o}$ WE GET

$$\Big\|_{U_o} = \left\{[I]-[\beta]\right\}^{-1}[X]\Big\|_{F.A.} + \left\{[I]-[\beta]\right\}^{-1}[\bar{\beta}]\Big\|_{L_o}$$

CONTINUING THE TRANSFER DOWN THE MAST,

$$\Big\|_{L_o} = [Z_1]\Big\|_{U_o} + \Big\|_{\substack{\text{LOWER} \\ \text{CABLE FORCES}}}$$

WHERE $\Big\|_{L_{CF}}$ IS DERIVED IN THE SAME MANNER AS $\Big\|_{U_{CF}}$;

$$\Big\|_{L_{CF}} = [\bar{\bar{\beta}}]\Big\|_{L_o} + [\bar{\bar{\bar{\beta}}}]\Big\|_{U_o}$$

SUBSTITUTING FOR $\Big\|_{U_o}$ & $\Big\|_{L_{CF}}$ INTO THE EQN FOR $\Big\|_{L_o}$; WE CA

REARRANGE TERMS AND GET A NEW EXPRESSION FOR $\Big\|$

IN TERMS OF $\Big\|_{F.A.}$ & $\Big\|_{LO}$. TAKING THE TWO

FOR $\Big\|$, WE SOLVE FOR $\Big\|_1$ IN TERMS OF $\Big\|$

AND GET

$$\Big\|_{L_0} = \left\{\left[\bar{\bar{\bar{\beta}}}\right]^{-1}\left\{\left([I]-\left[\bar{\bar{\beta}}\right]-[Z_1]\left\{[I]-[\beta]\right\}^{-1}\left[\bar{\beta}\right]\right)-\left\{[I]-[\beta]\right\}^{-1}\left[\bar{\beta}\right]\right\}\right\}^{-1} \times$$

$$\left\{\left\{[I]-[\beta]\right\}^{-1}[X]+\left[\bar{\bar{\bar{\beta}}}\right]^{-1}[Z_1]\left\{[I]-[\beta]\right\}^{-1}[X]\right\}\Big\|_{F.A.}$$

RENAMING THE ABOVE,

$$\Big\|_{L_0} = [H]\Big\|_{F.A.}$$

AND CONTINUING THE TRANSFER TO THE BOTTOM OF THE MAST, WE GET

$$\Big\|_{BM} = [Y]\,[H]\Big\|_{F.A.}$$

WHERE, $[Y][H] = \left[D(\omega_n)\right]$

SUMMARY

Note that in free vibration, the forces and moments at the extremes of the antenna assembly are zero. So those state variable vectors are

$$
\begin{array}{c|c|l}
 & 0 & \\
 & 0 & \\
 & \phi & \\
 & v & y \\
 & 0 & \\
 & 0 & \\
 & \phi & \\
 & w & z \\
 & 0 & \\
 & \theta & \\
 & 0 & \\
 & u & \\
\end{array}_{B.M}
=
\overset{12 \times 12}{\left[D(\omega_n) \right]}
\begin{array}{c|c|l}
 & 0 & \\
 & 0 & \\
 & \phi & \\
 & v & y \\
 & 0 & \\
 & 0 & \\
 & \phi & \\
 & w & z \\
 & 0 & \\
 & \theta & \\
 & 0 & \\
 & u & \\
\end{array}_{FA}
$$

Which can be reduced to

$$
\begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}
=
\overset{6 \times 6}{\left[\bar{D}(\omega_n) \right]}
\begin{array}{c|c|l}
 & \phi & \\
 & v & y \\
 & \phi & \\
 & w & z \\
 & \theta & \\
 & u & \\
\end{array}
$$

The matrix $\bar{D}(\omega_n)$ consists of elements of $D(\omega_n)$ at the intersections of its 3rd, 4th, 7th, 8th, 10th & 12th columns and its 1st, 2nd, 5th, 6th, 9th & 11th rows. For a non-trivial solution the determinant of $\bar{D}(\omega_n)$ must equal zero. Given values for the geometrical and physical characteristics of the system, we can now determine natural frequencies of the structure, by trial and error, seeking those values of ω_n which make the determinant zero.

FINALIZING THE FORMULATION OF THE ANALYSIS

The mast and hoop structure are, in one sense, the skeleton on which the antenna reflecting surface is hung. It now remains to represent the mass and stiffness characteristics of the antenna surface in the analysis. We anticipate doing this by dealing with the antenna reflecting surface as a series of pie-shaped segments supported at state-variable stations on the hoop and mast.

REFERENCES

1. E. C. Pestel and F. A. Leckie, Matrix Methods in Elastomechanics (McGraw-Hill Book Company, Inc., New York, 1963).

2. Roark, Formulas for Stress & Strain (McGraw-Hill Book Company, Inc, New York, New York)