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Kenneth J. Baumeister
Lewis Research Center
Cleveland, Ohio

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Numerical Techniques in Linear Duct

Acoustics - 1980-81 Update

Kenneth J. Baumeister

National Aeronautics and Space Administration

Lewis Research Center

Cleveland, Ohio 44135

A review is presented covering finite element and finite difference analysis of small amplitude (linear) sound propagation in straight and variable area ducts. This review stresses the new work performed during the 1980-1981 time frame, although a brief discussion of earlier work is also included. Emphasis is placed on the latest state of the art in numerical techniques.

E-1034

NOMENCLATURE

c_0^*	ambient speed of sound, m/s
c_g^*	group velocity, m/s
d_0^*	duct diameter, m
F	initial condition vector, equation (22)
f*	frequency, Hz
f	function of y, see equation (18)
H*	duct height, m
I	number of axial grid points
i	$\sqrt{-1}$
J	number of transverse grid points
K	coefficient matrix, equation (22)
L*	length of duct, m
N_{sub}	number of storage locations in sub matrix

n	normal
p^*	pressure, N/m^2
P	time dependent acoustic pressure, $P^*/\rho_0^* c_0^{*2}$
p	spatially dependent acoustic pressure, equation (3)
p_0	wave envelope pressure, equation (25)
r_0^*	radius of duct, m
t	dimensionless time, t^*/t_p^*
t_p	period $1/f^*$, sec
Δt	time step
U	time dependent axial dimensionless acoustic velocity, $U(x,y,t)$, U^*/c_0^*
U_0	Mach number, U_0^*/c_0^*
u	spatial dimensionless axial acoustic velocity $u(x,y)$, u^*/c_0^*
V	time dependent dimensionless transverse or radial dimensionless acoustic velocity, $V(x,y,t)$, V^*/c_0^*
v	spatial dimensional transverse or radial dimensionles acoustic velocity, v^*/c_0^*
x	axial coordinate, x^*/H^* or x^*/r_0^*
Δx	axial grid spacing
y	dimensionless transverse coordinate, y^*/H^*
Δy	transverse grid spacing
Z^*	impedance, kg/m^2 sec
ζ	dimensionless specific acoustic impedance
η	dimensionless frequency, $H^*/f^*\ c_0^*$ (cartesian) or $r_0^*/f^*\ c_0^*$ (cylindrical)
θ_r	dimensionless specific resistance
λ	dimensionless wave length

ρ_0^* ambient air density, kg/m^3

ϕ dimensionless acoustic potential function, $\phi^*/c_0^*/d_0^*$

χ dimensionless reactance

ω^* angular frequency ($2\pi f^*$)

Subscripts:

e exit condition

i axial index (fig. 1)

j transverse index (fig. 1)

o ambient condition or mean flow variables

* dimensional quantity

k time step index

(1) real part

(2) imaginary part

INTRODUCTION

"Steady" state finite element theories and transient finite difference theories are currently being applied to the design of loud speakers, ventillation systems, mufflers, and the acoustically treated nacelles of turbojet engines. In this paper, a review is presented covering both the finite difference and finite element analysis of small amplitude (linear) sound propagation in straight and variable area ducts with and without flow. Reference 1 contains a very recent literature review of the techniques, advantages, limitations and applications associated with the various numerical solutions of the sound propagation equations in ducts. In an extension of reference 1, this review emphasizes the new work performed during the 1980-1981 time period and the state of the art of numerical simulation of sound propagation in ducts.

Application of numerical theories to room acoustics, structural acoustic radiation systems and the determination of duct eigenvalues will not be included in the detailed discussions. However, since these research areas are often of interest to acousticians studying duct acoustics, these works have been included in the list of references (refs. 2-22) for completeness.

The numerical theories to be considered in detail in this paper employ full two or three dimensional finite element or difference theories which have provisions for inlet and outlet flow such as in a muffler or jet engine duct. To delineate the starting point of the present update, the literature cited in reference 1 is again tabulated herein (refs. 23 to 63). These references will also be cited in conjunction with brief summaries of earlier work. References 64 to 85 represent the new literature introduced during the 1980 to 1981 update period.

First some background information is briefly discussed relating to the properties of sound propagation in ducts and how they impact on numerical solutions. Next, the various numerical works developed in the 1980-1981 period are presented in the following order:

(1) Steady State Finite Difference Theory

(2) Steady State Finite Element Theory

(3) Special Numerical Transformations

(4) Transient Numerical Theory

BACKGROUND INFORMATION

In preparation for the new (1980-1981) literature on finite difference and finite element techniques as applied to duct acoustics, the simplest versions of the sound propagation equations and boundary conditions are first briefly reviewed and some inherent problems associated with numerical solutions of the sound propagation equations are high-lighted.

In the absence of flow, the wave equation in dimensionless form can be written as

$$n^2 \frac{\partial^2 P}{\partial t^2} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \tag{1}$$

where n is the dimensionless frequency defined as

$$n = \frac{H^* f^*}{c_0^*} \tag{2}$$

(1/n may also be thought of as a dimensionless speed of sound, in this case, equation (1) corresponds more closely to the dimensional form of the wave equation). The starred quantities represent dimensional variables. All symbols are defined in the nomenclature. The sound source is generally assumed to be harmonic in nature and to vary as $e^{i\omega^* t^*}$ (in dimensionless form as $e^{i2\pi t}$). In this case, the pressure and acoustic velocities will all be functions of $e^{i\omega^* t^*}$. If the acoustic pressure is assumed to be of the form

$$P(z,y,t) = p(x,y)e^{i2\pi t} \tag{3}$$

the wave equation (1) becomes

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + (2\pi n)^2 \, p = 0 \tag{4}$$

which is the classical Helmholtz equation.

Solutions to equation (4) are called "steady" state solutions because the equation is independent of time. In this case, p(x,y) actually represents the Fourier transform of P. On the other hand, solutions to equation (1) are time dependent or transient. Once the initial start up condition has died, the transient and "steady" state solutions are simply related by equation (3).

Governing Equations with Flow

The wave equation (1) takes on slightly different forms for the cases of uniform mean flow and for irrotational flow (see ref. 40). For uniform mean flow the wave equation becomes

$$n^2 \frac{\partial^2 p}{\partial t^2} = \left(1 - U_0^2\right) \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - 2nU_0 \frac{\partial^2 p}{\partial t \partial x} \tag{5}$$

As is equation (1), this form of the wave equation is ideally suited for numerical analysis in that only one dependent variable is involved in the solution of the propagation equations. The wave equations are obtained by combining the linearized continuity and momentum equations which describe the propagation of small acoustic disturbances. Unfortunately, for realistic shear flows, the continuity and momentum equations cannot be combined into a single second order wave equation. In these cases, the continuity and momentum equations must be solved simultaneously. For parallel shear flow in a two-dimensional rectangular coordinate system, these equations are

$$\frac{\partial P}{\partial t} = -\frac{1}{n}\frac{\partial U}{\partial n} - \frac{1}{n}\frac{\partial V}{\partial y} - \frac{U_0}{n}\frac{\partial P}{\partial x} \tag{6}$$

$$\frac{\partial U}{\partial t} = -\frac{1}{n}\frac{\partial P}{\partial x} - \frac{U_0}{n}\frac{\partial U}{\partial x} - \frac{1}{n}\frac{\partial U_0}{\partial y} V \tag{7}$$

$$\frac{dV}{dt} = -\frac{1}{n}\frac{\partial P}{\partial y} - \frac{U_0}{n}\frac{\partial V}{\partial x} \tag{8}$$

For "steady" state, these equations become

$$(i2\pi n)p = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - U_o \frac{\partial p}{\partial x} \tag{9}$$

$$(i2\pi n)u = -\frac{\partial p}{\partial x} - U_o \frac{\partial u}{\partial x} - \frac{\partial U_o}{\partial y} v \tag{10}$$

$$(i2\pi n)v = -\frac{\partial p}{\partial y} - U_o \frac{\partial v}{\partial x} \tag{11}$$

As will be shown later in detail, the "steady" state solution of the sound propagation equations requires storage of large matrices to obtain a solution. Consequently, the inclusion of three dependent variables in the shear flow problem requires an order of magnitude increase in the storage capability and solution times of a computer over a similar problem in the absence of shear. Therefore, to obtain optimum storage and run times, separate programs are required for no flow, irrotational flows, and shear flows. Fortunately, as will be shown shortly, the transient solution to the governing equations does not require the storage of large matrices. Therefore, the generation of a single optimum program to cover the complete range of no flow and shear flow seems a reasonable goal.

<u>Discretize the Continuum</u>

In either the finite difference or finite element numerical analysis, the continuous acoustic flow field is lumped into a series of grid points or elements as shown in figure 1. Instead of obtaining a continuous solution for the acoustic pressure, for example, its values are obtained only at isolated node points. In the finite difference analysis, a simple rectangular pattern of discrete points is almost always used. The finite difference approach is generally restricted to uniform ducts unless special bookkeeping or transformations are employed. On the other hand, a great variety of finite element patterns (see table I) can be easily employed to handle complex geometric variations in duct walls. Two such element patterns are displayed in figure 1. The more nodes per element the fewer will be the number of elements needed to resolve the acoustic field. However, since the size of the solution (global) matrix is proportional to the number of nodes, considerable more computational time and computer core memory are required for the higher order elements.

Grid Point Requirements

In sound propagation in ducts, the acoustic pressures and velocities oscillate down the duct. The question naturally arises as to how many grid points or elements are required to accurately resolve these oscillations. For example, consider a hard wall duct, semi-infinite in extent with a sinusodial plane wave pressure at $x = o$, and a uniform Mach number U_o in the duct. The one dimensional "steady" state plane wave solution of equation (5) in conjunction with equation (3) yields

$$p = p^{(1)} + i\, p^{(2)} = e^{-\frac{i2\pi n x}{1 + U_o}} \tag{12}$$

Thus, the real and imaginary components of the acoustic pressure oscillate in the axial direction with a wave length

$$\lambda = \frac{1 + U_o}{n} \tag{13}$$

The number of axial grid points to obtain accurate acoustic pressure and velocity profiles may be estimated from the following "rule of thumb" given in references 24 and 25

$$I = \frac{12\ (L^*/H^*)}{\lambda} \tag{14}$$

Thus, for the unit frequency ($\eta = 1$), unit length ($L^*/H^* = 1$) and no mean flow, 12 grid points are necessary to describe adequately the sinusoidal form of the spatial pressure dependence, as shown in figure 2. If the frequency or length is doubled, or the Mach number U_0 is -0.5 (flow towards the source), clearly twice as many grid points will be required to describe the wave, since two wave lengths of sound must now be resolved. When higher order transverse acoustic modes are present, the axial wave length of sound will be longer and the required number of axial grid points will be significantly reduced. Therefore, equation (13) represents a conservative estimate of the grid point requirements.

The number of transverse grid points will depend on both the source transverse modal distribution and the generation of higher order modes in the duct. To resolve all the higher order propagating modes that could occur in the most general case, the number of grid points in the transverse direction would be proportional to the frequency η (refs. 61 and 63). The total number of grid points would be the product of the points in the axial and transverse directions. In high frequency (large η) cases, the required number of grid points or elements can become very large.

In determining the required number of finite elements, however, the constant 12 in equation (6) will be smaller because the interpolation function in finite element theory is generally of higher order than the first order linear difference approximation used to establish equation

(7). For a particular finite element, some numerical experimentation will be required to determine if sufficient numbers of elements have been employed.

Wall Boundary Conditions

The boundary condition at a hard wall duct is simply that the normal acoustic pressure gradient is zero

$$\frac{\partial P}{\partial n} = \frac{\partial p}{\partial n} = 0 \tag{15}$$

or that the normal acoustic velocity is zero.

$$v_n = 0 \tag{16}$$

For a soft wall (absorbing) duct, an impedance is specified at the duct wall

$$\zeta = \frac{Z^*}{\rho_o^* c_o^*} = \theta_r + i\chi = \frac{p}{v_n} \tag{17}$$

The wall resistance is represented by θ_r while the reactance by χ. Both θ_y and χ are assumed known input quantities.

In the "steady" state analyses, application of equation (17) is straight-forward and offers no computational problem. In the transient analysis, however, direct application of equation (17) can lead to numerical instabilities. The integration technique presented in reference 61 seems to alleviate this problem.

Entrance Conditions

For a rectangular duct, the boundary condition at a liner entrance P(o,y,t) can be of the form

$$P(o,y,t) = f(y)^{i\omega^* t^*} \tag{18}$$

where $f(y)$ represents the transverse variation in acoustic pressure. Similar expressions for the velocity potential or acoustic velocity can be used. In cylindrical coordinates, the radial coordinate r would replace y in equation (18). Often the function form $f(y)$ is associated with the normal-mode analytical solution-cosines for the rectangular coordinate system and Bessel functions for the cylindrical coordinate system.

Physically, boundary condition (18) represents the sum of a forward and reflected acoustic wave at $x = 0$. In a uniquely different approach, Eversman, et al. (ref. 49), assumed a uniform infinite hard wall duct upstream of the duct section of interest and that equation (18) represents only the forward propagating wave. The amplitudes of a truncated series of normal mode reflection coefficients were determined by matching the pressure and pressure gradients in the infinite duct to the finite element values at the entrance. This form of termination is particularly useful in conjunction with many experiments which employ an anechoic entrance or exit condition.

Exit Condition

In applying the finite difference or finite element analyses to a turbojet engine inlet, for example, the grid system has generally been confined to the internal portion of the inlet. Thus, the engine has been modeled as a short duct. In this case, an impedance of the form

$$\zeta_e = \frac{P(L^*)}{v(L^*)} \tag{19}$$

is often used to close the boundary. The value of ζ_e is chosen to approximate an anechoic termination. General values for the exit impedance for single modes in an infinite hard wall duct (refs. 38 and 40) are often used for the anechoic approximation. For arbitrary multi-modal wave forms, however, a general impedance equation is not available for an exact simulation of an anechoic termination.

As with the entrance condition, Eversman, et al. (ref. 22) use the technique of modal decomposition to obtain an accurate simulation of an anechoic termination.

Reference 24 (Appendix E) suggests another possibility for simulating a non-reflecting interface at the duct exit. By adding on an additional length of duct with acoustical damping, the reflections from the duct exit will be effectively damped before they can re-enter the original portion of the duct. This technique has worked extremely well in soft wall duct problems (ref. 24) and is presently being employed in the transient analysis. Also, in the transient analysis, an anechoic termination can be modeled exactly by extending the duct such that the steady state solution is obtained before the arrival of a reflected wave from the duct exit. This approach, however, is costly from the standpoint of computer storage and run times.

All the previous cases attempted to eliminate or at least reduce reflections at the duct exit. However, in many practical applications, such as a turbofan inlet, reflections could be important for certain modes. Consequently, continuing the grid structure from inside the duct into the far field (ref. 31) would simulate the actual dynamic process occurring at the lip of the duct. In the far field, all duct modes propagate and have identical $\rho_0^* c_0^*$ exit impedance far from the exit. Therefore, the closure problem is simplified but at the expense of a greater number of elements and a larger solution matrix. To eliminate this large matrix requirement, Kwgawa, et al. (ref. 60) developed a combination of finite element and analytical methods to analyze sound propagation into the far field.

Methods of Solution

The methods of solution for the various "steady" and transient theories will be discussed in detail later. Generally, the "steady" state finite difference and finite element numerical algorithms have been limited to low frequencies (small η) because of the grid point requirements and more importantly because of the large matrices associates with the solution of the time independent equations. Special transformations have also been developed (refs. 54-60) to either reduce the size of the "steady" state matrix or to eliminate it. As an alternative to the "steady" state theories, time dependent numerical algorithms (refs. 61-63) were developed which eliminate the need to store a large solution matrix.

STEADY STATE FINITE DIFFERENCE THEORY

In the finite difference theory, the governing equation is written in difference form at each grid point. For the previous no flow example, equation (3) can be written in difference form by the usual 5-point difference approximations,

$$\left(\frac{p_{i-1,j} - 2p_{i,j} + p_{i+1,j}}{\Delta x^2}\right) + \left(\frac{p_{i,j-1} - 2p_{i,j} + p_{i,j+1}}{\Delta y^2}\right) + (2\pi\eta)^2\, p_{i,j} = 0 \qquad (20)$$

or for Δx equal to Δy

$$p_{i-1,j} + p_{i,j-1} - 4[1 - (\pi\eta\Delta x)^2]\, p_{i,j} + p_{i,j+1} + p_{i+1,j} = 0 \qquad (21)$$

Equation (21) applies away from the cell boundaries. For grid points on the boundary, the hard (eq. (15)) or soft wall (absorbing) impedance condition (eq. (17)) is imposed. Methods for generating the difference equations on the boundary are fully documented in references 23-28.

Matrix Solution

The collection of the various difference equations at each grid point forms a set of simultaneous equations that can be expressed as

$$[K] \{\bar{p}\} = \{\bar{F}\} \tag{22}$$

where [K] is the known coefficient matrix, $\{\bar{p}\}$ is the pressure vector containing the $P_{i,j}$ unknowns, and $\{\bar{F}\}$ is the known column vector containing the various boundary conditions. This matrix is complex because of the complex nature of the source and impedance boundary conditions (eq. 17). As seen in equation (21), the frequency term $(\pi n \Delta x)^2$ subtracts from the main diagonal element such that the coefficient matrix is not diagonally dominant. As a result, iteration solutions cannot generally be used. Equation (22) is usually solved by some elimination technique. Unfortunately, the elimination technique requires the storage of all the matrix elements.

The total storage will be proportional to the square of the total number of grid points. Because of the banded nature of the difference matrix, sparse matrix techniques have been employed to reduce computer storage and run times as much as possible. Quinn (ref. 28) partitioned the matrix K into tridiagonal form, which reduced the elements in the submatrix storage to

$$N_{sub} = J^2 = 144n^2 \tag{23}$$

which led to a total storage requirement of

$$N_{\substack{total\\complex}} = \frac{3456\ n^3 (L^*/H^*)}{1 + U_0} \tag{24}$$

For $n = 10$, $L^*/H^* = 1$ and $U_0 = 0$, the total storage will be 3.456×10^6, which is quite large. Higher frequencies and flows will greatly enlarge the storage. These submatrices are read in one at a time to obtain a solution of the matrix. Thus, the input-output time of the computer solution can also become very large.

Update 1980-1981

Finite difference solutions (ref. 64) were used to study plane wave sound propagation in a curved bend joined to two straight sections of duct. A rectangular mesh and a cylindrical mesh were used in the straight and curved sections of the duct respectively. At the discontinuities in the coordinate systems, a linear interpolation was used in the radial direction in conjunction with unequal meshes in the axial direction to derive the appropriate finite difference approximation. Equation (18) was employed at the entrance condition while modal decomposition and matching were used for the anechoic exit condition. Unfortunately, the algorithms for the difference methods are not presented in reference 64. For a variety of curved ducts, the numerical results agreed closely with the experimental data for the reflection coefficient and the pressure amplitude across the duct. For frequencies just below the first cut-on mode frequency, the magnitude of the cut-off modes generated at the interface were found to be greater in magnitude than the incident plane wave.

With the exception of reference 64, basic research on the "steady" state finite difference method appears on the wane. The bulk of recent "steady" state acoustic analyses employ the finite element theory. The finite element theory can readily handle duct area variation or curvature changes without resorting to special bookkeeping procedures. However, finite difference theory is again receiving renewed interest in the transient solution of the wave equation. This topic will be covered later in the paper.

Status

A finite difference program is available in the literature for handling rectangular and circular ducts with uniform flow. Reference 28 contains a complete list of fortran statements plus example problems to illustrate the use of the program. The program also has provisions for analyzing variable area ducts if a mapping function is available. The mapping function for a cone and hyperbolic horn are listed. As written, the program is limited to 100 axial grid points and 20 grid points in the transverse directions. This limited number of transverse grid points restricts the number of higher order modes which can be resolved. Therefore, some modification of the program would be necessary in the analysis of high frequency sound.

In general for ducts with variable area, the finite element theory is the most convenient to use. However, as pointed out by Quinn (ref. 53), the advantage of finite elements is not as great when a mapping function is used to compute the mean flow field. In such a case, where the conformal transformation is available, the use of finite difference theory would have an advantage over finite element theory. In this case, the program cited in reference 28 could be used very effectively.

"STEADY" STATE FINITE ELEMENT THEORY

Finite element theory is reviewed in depth in many references 29-50 as well as the previous review paper (ref. 1). Consequently, only a very brief introduction to the finite element theory is now presented.

In the finite element theory, an appropriate element with its associated nodes is distributed in the duct, such as shown in figure 1. At each node labeled i, an unknown value of pressure p_i is assigned. The acoustic velocities would also be assigned to the nodes in the more general shear flow problem. Table I shows some additional elements currently being used in acoustic studies. In contrast to the finite difference theory, interpolation functions are used to determine the value of pressure between the nodes at any position inside the element. Again, table I lists some typical interpolation functions.

Generally, Lagrange polynomials are used for interpolation when only the magnitudes of the dependent variables, such as pressure, are desired at each node. This is commonly called the C° continuity problem. In inlet ducts, Astley and Eversman (ref. 51) showed that C° continuity leads to smooth solutions for the acoustic pressure provided a sufficiently small mesh is employed. If the mesh size is fixed, however, and the Mach number is increased, the solutions for pressure become cusped (discontinuous) at each node. Improvements in resolution for the high Mach number cases require either greater mesh refinement with a corresponding increase in dimensionality or the introduction of more sophisticated Hermitian elements.

When using Hermite polynomials, items 1 and 6 in table I, the values of slope are also calculated at each node, commonly called the C' problem. In this case, fewer elements are required to give smooth solutions at the nodes. However, C' continuity is more difficult to construct, so with the exception of items 1 and 6 in Table I, Lagrange polynominals are used. Also, C' increases the unknowns by a factor of 4 (item 6, table I), and the resulting solution matrix will increase by about 16.

Next, either the Galerkin, least squares or variational finite element methods is used to constrain the pressures at the node points such that they satisfy the Helmholtz equation (3) or a more general form of the wave propagation equation. Generally, the variational techniques have been used for problems without mean flow, while the Galerkin formulation is used when mean flow is present, see table I. Then the elemental equations at each node are combined into a general matrix (global) similar to equation (22). Again, this matrix must be solved to determine the unknown pressure p_i.

Matrix Solution

With the exception of the least squares approach, the global matrix must be solved by some elimination technique just like the finite difference solution. For the larger problems, an out-of-core banded solver is generally required with a moderate amount of in-core storage but much more input/output time. To reduce computer storage and run times, the nodes should have been labeled in a manner to obtain the minimum possible banded matrix.

Update 1980-1981

No Flow

In reference 65, Shepherd and Cabelli presented numerical and experimental techniques to determine the acoustic characteristics of discontinuities in rigid curved wall duct systems when several modes propagate. The finite element technique was used to solve the two-dimensional Helmholtz equation and determine the acoustic behavior of arbitrarily shaped duct elements in terms of modal amplitudes. Eight-node isoparametric rectangular elements were used to describe the solution with hard wall boundary conditions. Shepherd and Cabelli applied the method to a 90° mitred bend without mean flow and obtained good agreement between numerical and experimental results. In references 66 and 67, Cabelli and Shepherd extended the experiments and theory of reference 65 to a range of mitre bend geometries. The influence of change in geometry on the acoustic characteristics of 90° bends was established over a range of inner and outer radii and the effects of including a turning vane were examined.

Figure 3 displays a layout of the experimental apparatus used by Shepherd and Cabelli (ref. 65) to measure the propagation of higher order duct modes through mitred bands. In the finite element simulation of this experiment, the modal match boundary conditions developed by Astley and Eversman (ref. 50), infinite duct modal solutions coupled to finite element solution, were used to model the anechoic entrance and exit condition. Good agreement between experiment and theory was obtained. In contrast, it would be difficult to employ an exit impedance boundary condition, equation (19), in this problem because of the difficulty of specifying an impedance when multiple modes are present.

In addition to the theoretical studies, some duct experiments were conducted which were compared to previously-developed finite element programs. In reference 68 and its improved version reference 69, the absorbing properties of a linear locally reacting honeycomb liner were measured for an incident plane wave (no mean flow). The 5.08 cm square anechoic test section supported plane wave propagation up to about 3.5 KHz; therefore, the standard plane wave $\rho_0^* c_0^*$ exit impedance was used to terminate the solution. The impedance of the liner was determined in a separate normal incidence measurement. Very good agreement between finite element theory and experiment was observed at 0.9 KHz while in the high frequency tests of 2.5 KHz, significant discrepancy existed in both the pressure level and phase at the trailing edge of the liner.

Because of the rotational nature of the blades in a turbofan jet engine, much of the emitted turbomachinery noise is concentrated in spinning acoustic modes. The NASA Langley Spinning Mode Synthesizer shown in figure 4 has the capability to generate individually dominant circumferential modal patterns. In reference 70, comparisons of the transmission and reflection coefficients with calculations based on finite element models (refs. 37 and 46) were made. In this case, the experiment was conducted without mean flow with hard walls and a dominant spinning mode of order (1,0). In general, reasonable agreement between experiment and theory was obtained.

Uniform Flow

Ling and Hamilton (refs. 71 and 72) have developed a finite element model for steady uniform flow based on the "steady" state form of equation (5). The finite element formulation is based on the Galerkin method using cubic isoparametric elements. The model was also used to approximate sound propagation in a converging-diverging duct with flow. Since the $\partial U_0/\partial x$ term has been neglected in their analysis, a comparison with the more exact theories of references 45 to 50 will be required to establish the range of validity of their finite element program as applied to flow with area variations.

Potential Flow

Sigman and Zinn (ref. 73) have applied their potential flow finite element program (refs. 40 and 43) to the problem of describing wave propagation in axisymmetric nozzles. Agreement between values of nozzle admittances computed from finite element solutions experimental admittances are extremely good.

Shear Flow

Astley and Eversman (ref. 74) have extended and improved their earlier analysis (ref. 51) on the implementation of the finite element method for acoustic transmission through a non-uniform duct carrying a high speed subsonic compressible sheared flow. A finite element scheme based on both the Galerkin method and the residual least squares method using an eight model isoparametric element was described. Multimodal propagation was investigated by coupling of the solution in the duct non-uniform section to modal expansions in uniform sections. As in references 48 and 53, they found that the residual least squares formulation performs poorly in comparison with the Galerken approach. At high Mach numbers, evidence was provided that multi-modal interactions become important necessitating a proper handling of the radiation condition.

Astley, Walkington and Eversman (ref. 75) investigated the computational efficiency and possible improvements to existing finite element models. Numerical results were presented in an attempt to evaluate four computational schemes comprising all combinations of Galerkin and residual least squares with both Lagrangian (C°) and Hermitian (C^{1}) elements. A computationally inexpensive one dimensinal model was used for most of the results. It was demonstrated that an inevitable consequence of the Galerkin schemes appears to be the presence of spurious oscillations excited by insufficient number of elements or inaccurate matching at the boundaries. Also, the wave envelope technique, to be discussed in the next section, showed a significant increase in accuracy over the conventional solutions when a fixed number of elements was employed.

Abrahamson (ref. 76) attempted the development of a 3 dimensional finite element program to analyze acoustic propagation in an axisymmetric duct with circumferential variations in wall impedance. Galerkin or least squares element formulations combined with Gaussian elimination, successive over-relaxation, or conjugate gradient solution algorithms were investigated. Unfortunately, all the techniques proved impractical for handling realistic three dimensional problems, because of both very large storage and run times.

Status

Nearly all the finite element investigations have been concerned with the theory and accuracy of various forms of the finite element method for the calculation of the acoustic transmission in ducts. In most cases, only in-core implementation of the finite element method has been employed. Abrahamson (ref. 46), however, has developed a large scale computational scheme utilizing an out of core banded solver for handling sound propagation in ducts containing a compressible mean sheared flow. The program has the capacity to handle relatively large frequencies (η = 10), duct lengths, and Mach numbers. Both a computer program manual (ref. 77) and detailed flow charts (ref. 78) are available. The detailed Fortran listing can be obtained directly from the author.

The program solves the separate linearized continuity and momentum equations using the Galerkin finite element method. As shown in figure 5, a simple linear rectangular Lagrangian element is used for which the acoustic pressures and velocities are to be determined at each node. The numbering system shown in figure 5 leads to a global matrix which consists of 3 diagonal bands. For a typical two dimensional problem, the width of each band is 9. The structure of the global matrix is the same for the uniform, non-uniform and variable area case shown in figure 5. These elements are stored in a packed matrix which ignores the large number of zeros in the general global matrix. The packed blocked tri-diagonal matrix is then solved in the usual manner by forward and backward substitution.

At the present time, the program has been successfully applied to a variety of no-flow and uniform flow problems for straight and variable area ducts. Additional checking and debugging may still be necessary whe the program is applied in a sheared flow situation. It would be useful if the modal matching boundary condition could be incorporated into this program so that anechoic termination could be conveniently simulated when higher order modes can propagate.

"STEADY" STATE SPECIAL NUMERICAL TRANSFORMATIONS

In a turbofan inlet, high frequencies (n = 30 to 50) and flow Mach numbers lead to pressure and velocity oscillations where many axial wave lengths variations are present in the duct. The number of grid points or elements required to track these oscillations requires extensive computer storage and run times which prohibit the analysis of many duct problems. Also, in a computer optimization process, hundreds of calculations are often required to determine a desired liner configuration. Therefore, any reduction in the number of grid points or elements in a numerical analysis will significantly reduce the cost of obtaining the desired solution. As cited in reference 1, three special numerical techniques have been developed in order to reduce the storage requirement and increase the accuracy of the numerical solution.

Wave Envelope Technique

The wave envelope technique attempts to reduce the axial oscillatory part of the wave pressure profile by transforming the wave equation into a new form whose solution is non-oscillatory. For example, if one dominant mode is present in a duct, the pressure can be assumed to be of the form

$$p(x,y) = p_0(x,y)\, e^{\frac{-i2\pi x}{\lambda}} = p_0(x,y)\, e^{i2\pi nx} \tag{25}$$

where $p(x,y)$ is the oscillatory pressure shown by the solid line in figure 6 (real part only) and p_0 represents the pressure amplitude or envelope shown by the dashed line in figure 6. The amplitude in figure 6 falls off because of absorption by the soft wall. Substituting equation (25) into the Helmholtz equation (4) yields the wave envelope equation

$$\frac{\partial^2 p_o}{\partial x^2} + \frac{\partial^2 p_o}{\partial y^2} - i4\pi n \frac{\partial p_o}{\partial x} = 0 \tag{26}$$

Solutions of equation (26) have lead to order of an magnitude reduction in grid points and computer storage compared to conventional solutions of the Helmholtz equation, equation (4), as discussed in references 54-47. The technique is limited to those cases where some reasonable estimate of the wavelength can be made a priori or by some simpler analysis. Re-running a problem with increased grid points will generally be necessary to check the validity of the solution.

Marching Technique

Although initial value numerical solutions of elliptic equations are generally unstable, the two-dimensional Helmholtz wave equation can be solved as an initial value problem using explicit marching techniques (refs. 58 and 59). Compared to standard finite-difference or finite element boundary value approaches, this numerical technique is orders of magnitude shorter in computational time and required computer storage. This technique is limited, however, to high frequencies and to cases where reflections are small.

Far Field Coupling

To obtain far field acoustic radiation patterns and to eliminate the necessity of specifying the impedance of the duct at its exit, finite elements can be extended from the internal portion of a duct into the far field (ref. 31, fig. 16). Generally, the increased computer storage requirements make this approach very costly. Kagawa, et al. (ref. 31) recently developed a combination of finite element (in duct) and analytical methods (Green-theorem-far field) to analyze sound propagation from an acoustic horn. Baumeister and Majjigi (ref. 44) presented a matrix partitioning approximation that separates the duct and far field into two or more regions and thereby reduces total matrix storage.

Update 1980-1981

Wave Envelope Technique

Astley and Eversman (ref. 79) have adapted the wave envelope approach to their finite element program which was documented in reference 74. In contrast to the earlier works which considered mainly plane wave applications in uniform ducts, they considered multimodal propagation in soft walled straight and variable area ducts with flow. Astley and Eversman reported that significant increases in accuracy result from a relatively crude wave envelope modification to conventional finite element schemes. In practical terms, as suggested in previous studies, Astley and Eversman agree that this technique could significantly reduce computer storage and run times in realistic aeroacoustic configurations where the propagation of high frequency modes places great demands on the axial resolution of conventional numerical schemes.

Far Field Coupling

In determining far field radiation patterns from inlets, generally the "interior" region of the duct and unbounded "exterior" region of the inlet are treated separately. In the absence of flow, Horowitz, Sigman and Zinn (ref. 80) developed a hybrid iterative solution approach using finite elements for the interior of the duct and the integral technique for the exterior region in order to obtain a continuous numerical solution for the acoustic field. In the first iteration, an impedance boundary condition was assumed at the interface between the interior and exterior regions. The boundary condition was then used to obtain interior and exterior (radiation) acoustic solutions. Since the prescribed boundary condition at the interface was an approximation, a discontinuity arises in the computed acoustic fields at the interface. In this case, both the interior and exterior acoustic fields were in error. This error was eliminated by an iterative matching of the computed acoustic fields at the interface which also provides the correct boundary condition at that location.

Resulting solutions for several simple cases were compared with known exact solutions and good agreement was demonstrated. In all cases, the convergence was very rapid. Under a NASA grant, work is continuing to adapt this matching procedure for the case when flow enters the duct.

Status

At the present time, these special techniques have not been incorporated into a documented computer program. Therefore, if desired, individual programs must be revised to accommodate these special techniques just discussed.

TRANSIENT FINITE DIFFERENCE THEORY

The transient analysis begins with a harmonic ($e^{i2\pi t}$) sound source, equation (18), at $x = 0$ radiating into an initially quiescent duct. Next, an explicit iteration solution of the wave equation, equation (1) is used to calculate both the transient as well as the "steady" state solution. The "steady" state is obtained when the value of

$$p(x,y) = P(x,y,t)/e^{i2\pi t} \tag{27}$$

at the duct exit reaches a constant value. For comparison and reporting purposes, all the transient pressures are converted into "steady" pressures according to equation (27) thereby eliminating the temporal variations.

To illustrate how the transient equations are programmed, the second derivatives in the wave equation, equation (1), can be represented by the usual central differences in time (superscript k) and space (subscripts i, j)

$$n^2\left(\frac{P^{k+1}_{i,j} - 2P^{k}_{i,j} + P^{k-1}_{i,j}}{\Delta t^2}\right) = \left(\frac{P^{k}_{i+1,j} - 2P^{k}_{i,j} + P^{k}_{i-1,j}}{\Delta x^2}\right) + \left(\frac{P^{k}_{i,j+1} - 2P^{k}_{i,j} + P^{k}_{i,j-1}}{\Delta y^2}\right) \tag{28}$$

where Δt, Δx, Δy are the time and space mesh spacings, respectively. Solving equation (28) for $P^{k+1}_{i,j}$ yields

$$P^{k+1}_{i,j} = 2P^{k}_{i,j} - P^{k-1}_{i,j} + \left(\frac{\Delta t}{n\Delta x}\right)^2\left[P^{k}_{i-1,j} + P^{k}_{i,j-1} - 4P^{k}_{i,j} + P^{k}_{i,j+1} + P^{k}_{i+1,j}\right] \tag{29}$$

where Δx equals Δy. The iteration procedure is explicit since all the past values of P^k are known as the new values of P^{k+1} are computed. Since equation (29) is a simple algebraic equation, storage is only required for the solution vectors $P^{k}_{i,j}$ and $P^{k-1}_{i,j}$; that is, matrix storage and manipulation are not required.

As with any explicit iteration solution, the time step Δt must be chosen sufficiently small to insure numerical stability. Nevertheless, the transient solutions have been found to be an order of magnitude faster than the similar "steady" solution, ref. 61, for plane wave propagation. Using the von Neumann stability theory as a guide (refs. 61-63), the time dependent solutions have been found to be stable without flow and with uniform flow. When axial and transverse variations in the mean flow are considered, some numerical experimentation will no doubt be required to determine the maximum time increment associated with numerical stability.

1981 Update

As previously formulated in the literature (ref. 63), the transient method did not converge to the "steady state" solution for cut-off acoustic modes. This has implications as to its use in a variable area duct where modes may become cut off in the small area portion of the duct, or for situations where cut-off modes may be generated such as duct discontinuities or large velocity gradients. For single cut off mode propagation, Baumeister (ref. 81) found that the "steady state" impedance boundary condition produced acoustic reflections during the initial transient which caused finite instabilities in the numerical calculations. In reference 81, extending the duct length to prevent transient reflections resolved this stability problem.

Reference 81 also addresses the problem of how long the transient calculation must continue for the initial transient to die out in order to obtain a "steady state" solution. In agreement with analytical predictions (ref. 86), numerical calculation showed that the time to reach a steady state solution will be

$$t^* > L^*/c_g^* \tag{30}$$

where c_g^* is the axial component of the group velocity. Since this group velocity becomes very small near the cut off frequency, the transient solution will required long calculation times to resolve modes near cut off.

In problems with duct area variations, the finite element method is very convenient for "steady state" solutions of the wave equation. Unfortunately, the transient finite element method is inappropriate for two-dimensional problems because the solutions are implicit requiring storage of the usual large matrix for each time iteration. Therefore, the transient finite difference theory must be adapted to duct problems with area variations.

With area variations, the boundary can be located in an approximate manner with a uniform grid or can be located exactly by use of a variable spaced grid. Both approaches are inaccurate and are cumbersome to use. In reference 82, White has developed a numerical mapping procedure which transforms a complex duct geometry into a simple rectangular form. Using the results of the numerical mapping, a transformed wave equation is solved in the rectangular system with a standard uniform rectangular grid.

Prof. White of the University of Tennessee (Knoxville) under NASA Grant NAG3-18, is extending the procedure of reference 82 to the problem of sound propagation from ducts into the far field. Figure 7 shows mapping for a straight two-dimensional duct into the far field while figure 8 illustrates the mapping for a variable area duct coupled to the far field. In both cases, the wave equation is solved in a rectangular grid system and transformed back to the appropriate grid locations shown in figures 7 and 8.

In reference 83, as a check on both the steady state finite element and transient finite difference theories, plane wave sound propagation was studied experimentally in a rectangular duct with a converging-diverging area variation for no mean flow. The 0.5 area contraction was of sufficient magnitude to produce large reflections and induce some modal scattering. Figure 9 from reference 83 shows a comparison of White's (ref. 82) transient finite difference program and Astley-Eversman finite element program (ref. 74) with experimental data. Both theories accurately predict the standing wave pattern upstream of the test section ($x < 0$) and the pressure distribution through the area variation ($0 \leq x \leq 1$).

The transient theory has recently been applied to study the behavior of a sound pulse propagating through the shear layer of an axisymmetric jet (ref. 84). Both experimentation and numerical analysis show that in the low and intermediate frequencies the far field acoustic power exhibits a marked amplification as the flow velocity increases. The amplification is traced to shear noise terms which trigger the instability waves that are inherent within the flow. The experimental results were qualitatively in agreement with the numerical simulation.

As discussed earlier in conjunction with the exit impedance, continuing the grid structure from inside the duct into the far field would simulate the actual dynamic process of reflection and transmission occurring at a duct lip. This is one of the strong motivations for Prof. White's study (ref. 82) and the far field mapping shown in figures 7 and 8. In the far field, care must also be exercised to prevent false reflection generated at the termination of the far field. Maestrello, Bayliss and Turkel (ref. 84) have included in their paper an excellent discussion of the problem involved in the far field termination as well as a correction to the exit impedance termination to improve the accuracy of the numerical results. In addition, reference 85 contains a comprehensive literature summary of recent work on the external radiation boundary condition.

Status

Although no documented computer program is available at the present time, the transient technique is relatively easy to program using the published theoretical results. The theory has been applied and checked for cases involving soft walls (ref. 61), uniform flow (ref. 62), variable area propagation (ref. 79) and sheared mean flow (refs. 62 and 85). As yet, however, the theory has not been checked in the combined case of a sheared mean flow with soft walls in a variable area duct.

CONCLUDING REMARKS

The finite difference and finite element theories are ideally suited for predicting sound propagation in ducts particularly in low frequency applications such as mufflers, expansion chambers, and exhaust ducts of turbofan engines. Using available computer programs, sound attenuations can now be easily and precisely calculated for ducts with a variety of complexities, such as variation in the wall liner impedances, axial area changes or large variations in the mean flow field. Of course, the sound source distribution and acoustic liner parameters must be accurately known.

On the theoretical side, at the present time, research priorities include extending the numerical theories to higher frequencies, coupling the internal portion of the duct to the far field, and including non-linear effects. Experimentally, testing with large area and mean flow variations is continuing.

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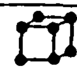
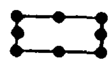
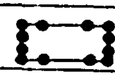
TABLE I. - FINITE ELEMENT SUMMARY

Item number	Element type	Interpolation function	Dependent variables	Finite element method	Types of acoustic propagation	References
1		Hermitian fourth order	$p, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}$	Variational	No flow, hard wall	(30)
2		Linear ring	p	Variational	No flow, hard wall	(31)
3		Quadratic ring	p	Variational	No flow, soft wall	(32), (60)
4		Hexahedral isoparametric	p	Variational	No flow, soft walls, non-local	(33), (34), (35) (36)
5		Linear isoparametric	p	Variational	No flow (plug)	(38), (39)
6		Hermitian fourth order	$p, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial^2 p}{\partial x \partial y}$	Galerkin	No flow, soft walls	(37)
7		Quadratic isoparametric	p	Galerkin	No flow, soft walls	(50), (65)
8		Cubic isoparametric	p	Galerkin	Uniform	(72)
9		Linear	ϕ	Galerkin	Irrotational, hard walls	(40)
10		Quadratic	ϕ	Galerkin	Irrotational, soft walls	(42), (43), (44)
11		Linear isoparametric	ϕ	Galerkin	Irrotational, hard walls	(41)
12	 	Linear quadratic isoparametric	p, u, v	Least squares Galerkin	General flow	(53)
13		Linear	p, u, v, w	Galerkin	General flow	(46), (47), (48)
14		Quadratic isoparametric	p, u, v	Least squares Galerkin	General flow	(49), (50), (51), (52), (74)

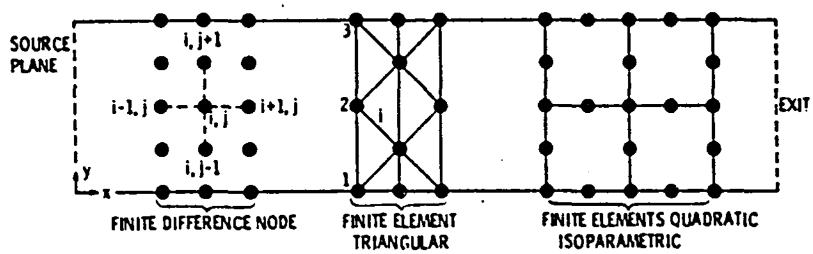


Figure 1. - Lumped-parameter representation of continuous acoustic flow field.

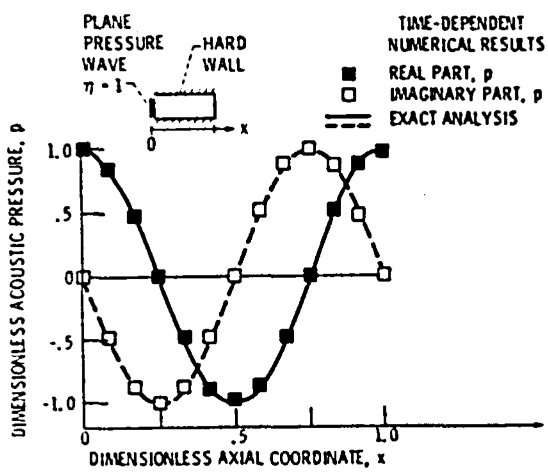


Figure 2. - Analytical and numerical pressure profiles for one-dimensional sound propagation in hard-wall duct, $\eta = 1$.

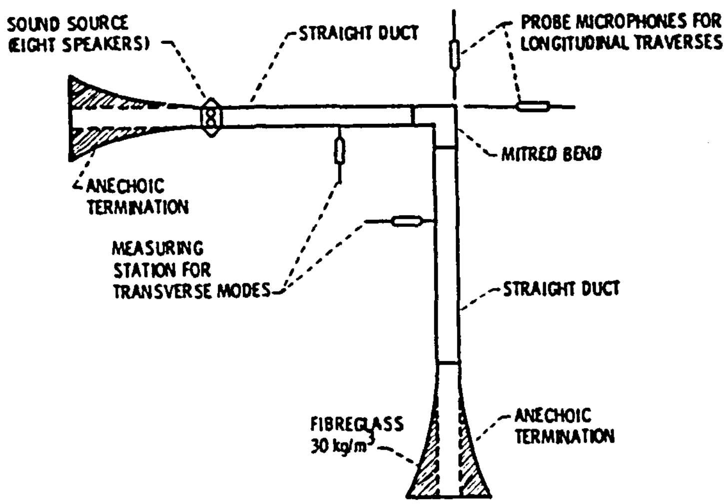


Figure 3. - Apparatus for measuring higher order modes through mitred bends.

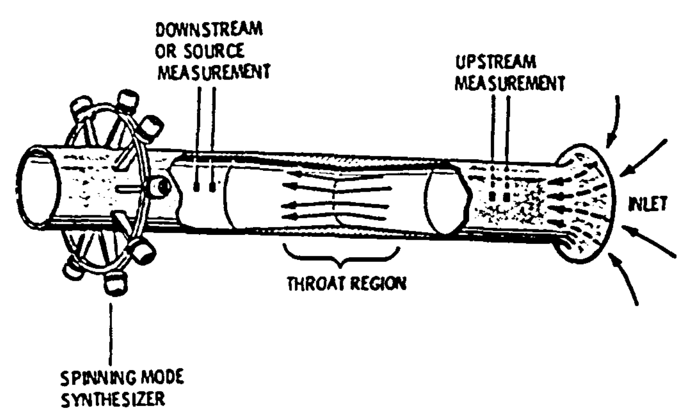
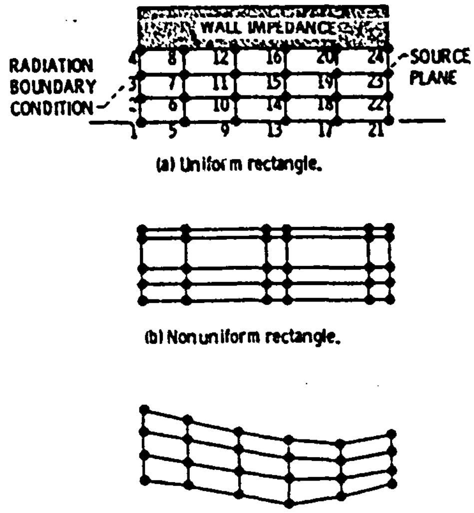


Figure 4. - A cutaway view of the experimental hardware to study spinning mode sound propagation through a variable area duct.



(a) Uniform rectangle.

(b) Nonuniform rectangle.

(c) Variable area duct.

Figure 5. - Abrahamson's rectangular discretization of variable area duct.

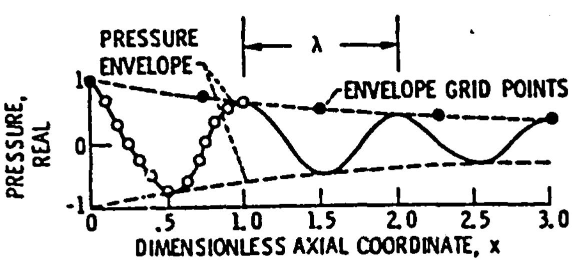


Figure 6. - Illustration of Wave Envelope method in determining the pressure profile for sound propagation in a soft-wall duct for dimensionless frequency $\eta = 1$ and duct length $L^*/H^* = 3$.

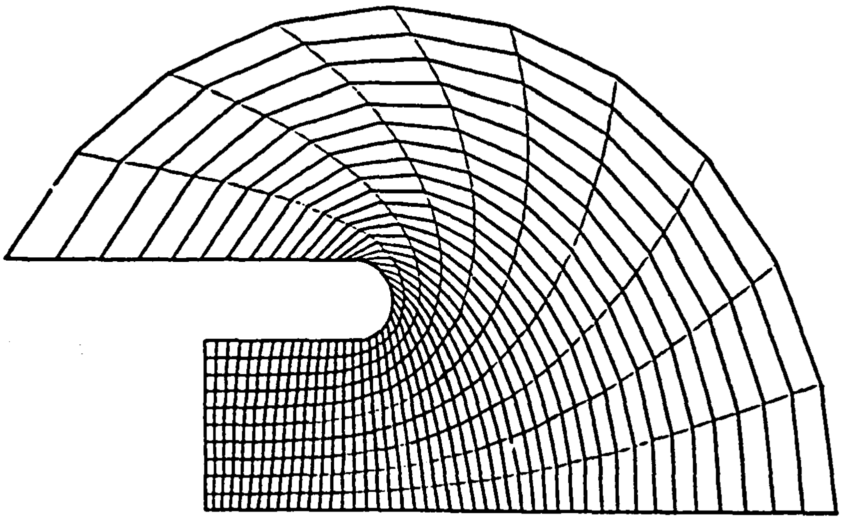


Figure 7. - Prof. White's numerical map of straight inlet coupled to far field.

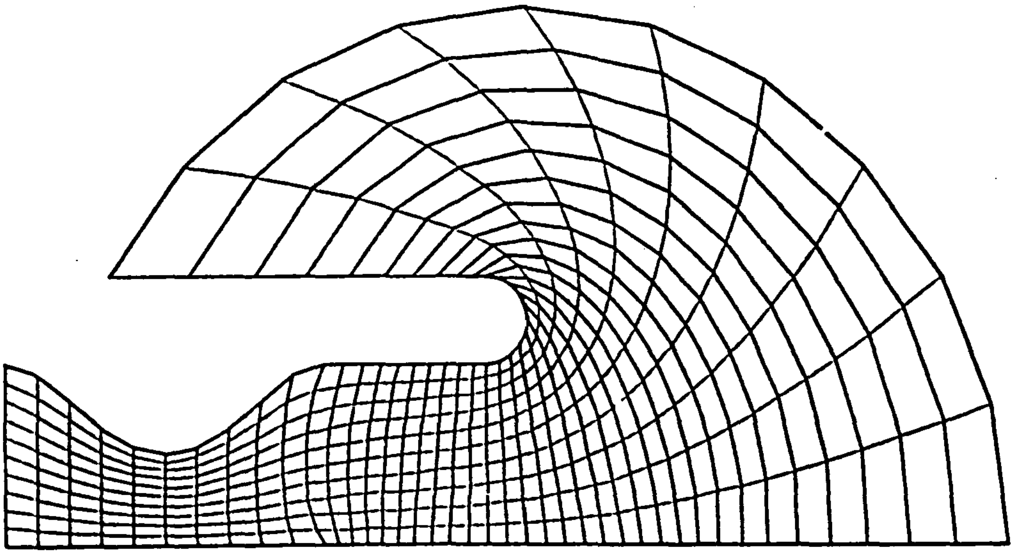


Figure 8. - Prof. White's numerical map of variable area inlet coupled to far field.

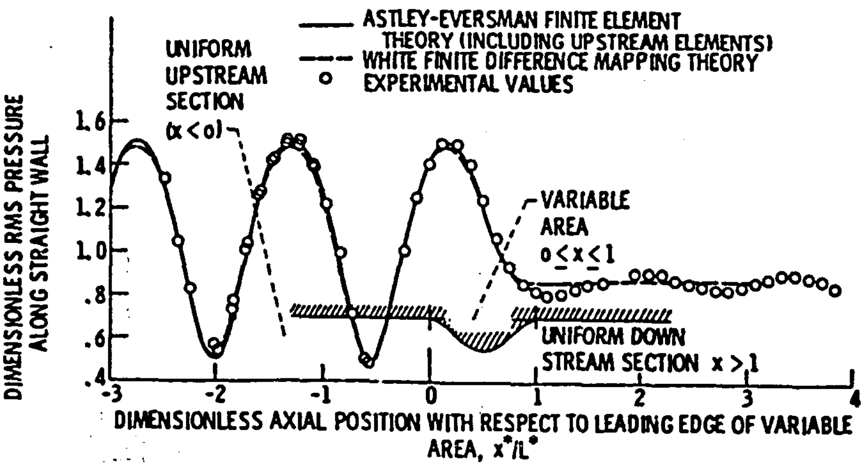


Figure 9. - Experimental and theoretical axial standing wave pressure profiles.

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