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POSSIBILITY OF A NEW APPROACH TO PROCESSING SPACE PULSE RADIO SIGNALS

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16. Abstract
Proposed herein is a new approach to the processing of space pulse radio signals, which makes it possible to achieve a high level of time resolution. The approach is based on the utilization of the "Fourier transform of a radio pulse envelope in the interstellar medium" effect. The conditions of applicability of the proposed method and the requirements for recording equipment are substantiated. Also indicated is the possibility of obtaining "superresolution" with the utilization of the given method of signal processing.

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A. A. Bocharov

Proposed herein is a new approach to the processing of space pulse radio signals, which makes it possible to achieve a high level of time resolution. The approach is based on the utilization of the "Fourier transform of a radio pulse envelope in the interstellar medium" effect. The conditions of applicability of the proposed method and the requirements for recording equipment are substantiated. Also indicated is the possibility of obtaining "superresolution" with the utilization of the given method of signal processing. /2*

Introduction /3

During studies of the internal structure of pulse signals of cosmic origin, for example, pulsar radiation, substantial limitations are placed on time resolution by effects associated with the propagation of radio waves in the interstellar plasma,—with the dispersion and scattering on discontinuities. The dispersed state of the interstellar medium leads specifically to distortion of form and broadening of the pulses, in connection with which, in order to ensure a high level of time resolution, the development of special methods of "elimination of dispersion" [3] was required, which lead to an appreciable complication of both the recording equipment and the subsequent digital processing of the received signals.

However, as will be shown below, during the implementation of certain relationships between the parameters of the radiated pulses and the interstellar medium, another approach is possible to processing of the space pulse radio signals, with which the dispersed state of the medium may prove to be a "useful" effect, which makes it possible to study the internal structure of the pulses with a high level of time resolution, with a simultaneous reduction in the requirements for the recording equipment. The proposed method,

*Numbers in the margin indicate pagination in the foreign text.

which does not require complex specialized equipment and lengthy digital processing of the received signals, may, in our opinion, find application in studies of pulsar radiation, and possibly for problems of space communications as well. /4

1. Connection Between Emitted and Received Signals

We will mention at once that the entire subsequent examination will be carried out on the assumption of the narrow-bandedness of the examined signals, which is fully justified, insofar as in radioastronomy, as a rule, the band width of the receiving apparatus is small, as compared with the central frequency. The emitted signal, like a narrow-band signal, may be written in the form

$$S_1(t) \approx A_1(t)\cos(\omega_0 t + \varphi_1(t)),$$

and, accordingly, the received signal may be written as

$$S_2(t) = A_2(t)\cos(\omega_0 t + \varphi_2(t)),$$

where $A_{1,2}(t)$ and $\varphi_{1,2}(t)$ are the envelope and phase of the signals, ω_0 is the carrier frequency (for broad-band pulsar radiation, ω_0 is understood to mean the central frequency of the receiver).

In order to determine the connection between $S_1(t)$ and $S_2(t)$, it is advisable, in view of their narrow-bandedness, to utilize the method of complex envelopes [1]. Let $\tilde{g}_1(t)$ and $\tilde{g}_2(t)$ be the complex envelopes of the signals $S_1(t)$ and $S_2(t)$, determined by the expression

$$\tilde{g}_{1,2}(t) = A_{1,2}(t)e^{i\varphi_{1,2}(t)}.$$

The emitted and received signals are associated by the relationship

$$\tilde{G}_2(\omega) = \tilde{G}_1(\omega)\cdot\tilde{H}(\omega), \tag{1}$$

where $\tilde{G}_{1,2}(\omega)$ are the spectra of the complex envelopes $\tilde{g}_{1,2}(t)$; $\tilde{H}(\omega)$ is the equivalent low-frequency coefficient of transmission of the interstellar medium.

For narrow-band signals, in the approximation of geometric optics (i.e., with the condition that the parameters of the medium change little on the wave length, which is indisputably correct for the frequencies utilized in radioastronomy), $\tilde{H}(\omega)$ has the form [3], [5] /5

$$\tilde{H}(\omega) = e^{\frac{i\omega^2}{2\alpha_0}} \tag{2}$$

(In expression (2) for $\tilde{H}(\omega)$, two terms are omitted, which are insignificant for subsequent examination — the constant phase shift, which is lost during detection of the envelope, and also the term which determines the time lag between the moments of radiation and receiving of the pulse). The parameter α_0 in (2), which has the meaning of the derivative instantaneous frequency of the received signal according to time, is determined, in turn, as

$$\alpha_0 = \frac{Cm\varepsilon_0}{\bar{e}^2\,\mathcal{D}M}\,\omega_0^3 ,$$

where C, ε_0, m, $\bar{e}$ are the speed of light, the dielectric permeability of the vacuum, the mass and charge of the electron, respectively; $\mathcal{D}M = \int_0^{E_0} N_e(z)\,dz$ is the degree of dispersion of the radio source, which is the integral of the electron concentration along the trajectory of propagation of the radio wave.

According to (1) and (2), the expression for the complex envelope of the received signal takes on the form

$$\tilde{g}_2(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{G}_1(\omega)\, e^{\frac{i\omega^2}{2\alpha_0}}\, e^{i\omega t}\, d\omega . \tag{3}$$

2. Stationary Phase Method and Conditions of its Applicability

The stationary phase method (MSP) is widely utilized for approximate calculation of integrals of the type (3) [4]. Assume that we have the integral

$$I = \int_{-\infty}^{+\infty} R(\omega)\, e^{-i\Phi(\omega)} d\omega ,$$

where $\Phi(\omega)$ is the actual function, and $R(\omega)$ may also be complex. The stationary phase method is applicable in that case when $\Phi(\omega)$ contains even more rapidly changing components of the phase, and $R(\omega)$ is a slowly changing function. This shows that the basic contribution to the integral, in this case, is provided by the points of "stationariness" of the function $\Phi(\omega)$, in connection with which /6

$$I \cong R(\omega_s) \cdot \left[\frac{2\pi}{i\Phi''(\omega_s)} \right]^{1/2} \cdot \exp\left[-i\Phi(\omega_s)\right] ,$$

where ω_S is the point at which $\Phi(\omega)$ is stationary, i.e., $\frac{\partial\Phi(\omega)}{\partial\omega}\Big|_{\omega=\omega_S}=0$. With the presence in the function $\Phi(\omega)$ of several stationary points, it is necessary to sum up the contribution of each of them.

For simplicity, we will take a radio pulse without phase modulation as the emitted signal, i.e., we assume that $\varphi_1(t)=0$. (The examination with regard for the presence of phase modulation in the signal will be carried out below). We will rewrite (3) in the form

$$\tilde{g}_{2}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} W_1(\omega)\, e^{-iF(\omega)} d\omega , \qquad (4)$$

where $W_1(\omega)$ is the spectrum of the envelope $A_I(t)$ of the emitted radio pulse, and $F(\omega)=-(\frac{\omega^2}{2\alpha_0}+\omega t)$.

The problem consists of finding the condition of applicability of the stationary phase method for calculating the integral in (4). Comparison of the functions $W_1(\omega)$ and $e^{-iF(\omega)}$ will be carried out with t=0, which corresponds to the case of the "slowest change" in the function $F(\omega,t)$. Let the characteristic width of the emitted pulse be equal to t_{pulse}; then, accordingly, the characteristic width of its spectrum (which contains the basic portion of the pulse energy) $\Delta\omega_{en} \approx 2\pi/t_{pulse}$. In order for the function $W_1(\omega)$ to be assumed to be slowly changing, as compared to $e^{-iF(\omega)}$, it is natural to adopt the following condition (see Fig. 1):

/7

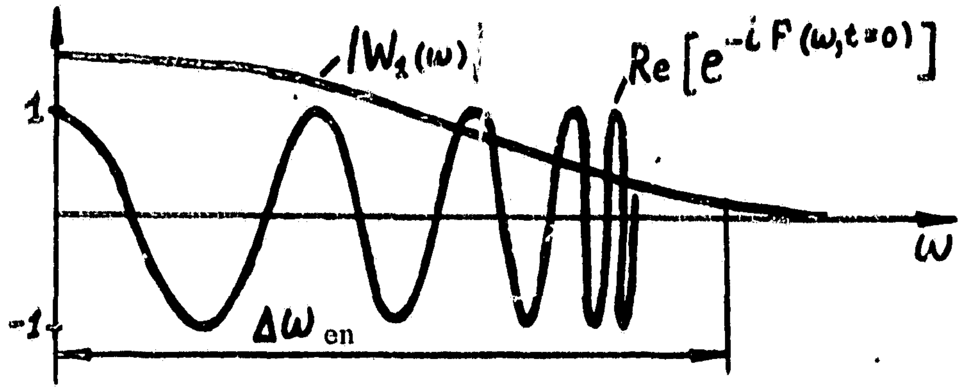

Fig. 1.

Let there be a large number of "periods" of the function $e^{-iF(\omega)}$ in $\Delta\omega_{en}$ (here, "period" is understood to mean the distance between two adjacent points, in which the function $e^{-iF(\omega)}$ takes on equal values, for example, equal to I, which takes place with $F(\omega)=2\pi n$, where n is an integer). The number of these "periods" in $\Delta\omega_{en}$ will be equal to $F(\omega=\Delta\omega_{en})/2\pi$, and, consequently, the condition of the slow change in the function $W_1(\omega)$, as compared with $e^{-iF(\omega)}$, is written in the form

$$\frac{\Delta\omega^2_{en}}{2\alpha_0} \cdot \frac{1}{2\pi} \gg 1 ,$$

or, with regard for the connection between t_{pulse} and $\Delta\omega_{en}$

$$\alpha_0 t^2_{pulse} \ll \pi .$$

In practice, in the majority of cases, one may utilize the following condition of applicability of the stationary pulse method, with which we will subsequently operate, namely

$$\alpha_0 t^2_{pulse} \leq 1 \quad . \tag{5}$$

3. Fourier Transform of the Envelope of Pulse Radio Signals in the Interstellar Plasma

Let the parameters of the pulse and the medium satisfy condition (5). By calculating the interval in (4), using the stationary /8 phase method, and taking into account the fact that, in the given case, the function $F(\omega)$ has only one stationary point $\omega_S = -\alpha_0 t$, we obtain the following for the complex envelope of the received signal

$$\tilde{g}_2(t) = \sqrt{\frac{\alpha_0}{2\pi}} \cdot W_1(-\alpha_0 t) \cdot e^{-i\left(\frac{\alpha_0 t^2}{2} - \frac{\pi}{4}\right)} .$$

The "physical" envelope of the received signal (i.e., the voltage at the output of the linear detector) will have the form

$$A_2(t) = \sqrt{\frac{\alpha_0}{2\pi}} \left| W_1(\alpha_0 t) \right| \tag{6}$$

(It is taken into account here that $W_1(\omega)$ is the spectrum of the actual function $A_1(t)$, and, consequently, $|W_1(-\omega)| = |W_1(\omega)|$).

Ignoring the change in scales, one may establish that, with the implementation of condition (5), the envelope of the received signal is the modulus of the spectrum of the envelope of the emitted signal, i.e., the interstellar medium has accomplished its own type of Fourier transform of the envelope of the initial radio pulse.

By squaring both parts of (6), we will obtain the following for the square of the envelope of the received signal (i.e., the voltage at the output of the quadratic detector)

$$x(t) = A_2^2(t) = \frac{\alpha_0}{2\pi} \left| W_1(\alpha_0 t) \right|^2 . \tag{7}$$

The magnitude of $|W_1(\omega)|^2$ is, in essence, the energy spectrum of the signal $A_1(t)$. We will find the inverse Fourier transform from both parts of (7), taking into account the fact that, according to Fourier, the autocorrelation function (ACF) of the signal corresponds to its energy spectrum. We have

$$X(\tau) = \frac{1}{4\pi^2} \mathcal{K}\left(\frac{\tau}{\alpha_0}\right) \tag{8}$$

where $\mathcal{K}(\tau) = \int_{-\infty}^{+\infty} A_1(t)A_1(t+\tau)dt$ is the autocorrelation function of the envelope $A_1(t)$, and $X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathcal{X}(t)e^{i\tau t}dt$.

Thus, with the observance of condition (5), the inverse /9
Fourier transform from the square of the envelope of the received signal will give us the autocorrelation function of the envelope of the emitted signal. What is more, as will be shown below, the presence in (8) of the multiplier $1/\alpha_0$ in the argument of the autocorrelation function makes it possible to obtain the autocorrelation function of the envelope of the initial pulse with "superresolution", i.e., with a time resolution higher than at the recording equipment.

4. Basic Time Relationships

As was shown above, with the observance of condition (5), for obtaining the autocorrelation function of the envelope of the emitted radio pulse, it is sufficient to find the inverse Fourier transform from the square of the envelope of the received signal. We will now determine the time resolution obtained utilizing the given method of processing of the signals, and the requirements for the recording equipment. (Everywhere below, where it is not especially specified, we will be implying requirements for the low-frequency — i.e., after the detector of the envelope or heterodyning to low frequencies — portion of the equipment. In practice, with the utilization specifically of the method of "elimination of dispersion", it is namely the low-frequency portion of the receiving-recording equipment, which includes a rapid-action analog-digital converter, external information storage units with a device for controlling them, and so on, which is the basic complexity and determines, in the final analysis, the time resolution of the entire system).

Let T be the observation interval, Δt is the quantization spacing (time resolution) of the recording equipment, and $N=T/\Delta t$ is the number of readings of the envelope $A_2(t)$ of the received signal, fixed during the interval T. After the inverse Fourier transform of the square of the envelope, we obtain the signal $X(\tau)$, determined in the interval Θ' with a quantization spacing $\Delta\tau'$, with /10

$$\Delta\tau' = 2\pi/T \quad ; \quad \Theta' = N\Delta\tau' = 2\pi N/T \ .$$

According to (8), the quantization spacing $\Delta\tau'$ of the function $X(\tau)$ corresponds to the fact that we obtain the values of the autocorrelation function of the envelope $A_1(t)$ with a quantization spacing $\Delta\tau=\Delta\tau'/\alpha_0$. We also take into account the fact that the discrete inverse Fourier transform of the function $\mathfrak{X}(t)$ will give us the function $X(\tau)$, which is symmetrical relative to the middle of the interval Θ', i.e., in actuality, we have only N/2+I independent readings of the autocorrelation function [2]. Thus, we obtain the autocorrelation function of the envelope of the initial radio pulse with a quantization spacing

$$\Delta\tau = \frac{2\pi}{\alpha_0 T} \tag{9}$$

and determined in the interval

$$\Theta = \left(\frac{N}{2}+1\right)\frac{2\pi}{\alpha_0 T} \approx \frac{\pi}{\alpha_0 \Delta t} \ .$$

In order to completely obtain the autocorrelation function (and, what is more, avoid undesirable effects associated with cyclic convolution), it is necessary to fulfill the condition $\Theta \geq t_{pulse}$, from which we obtain the requirement for the time resolution of the recording equipment

$$\Delta t \leq \frac{\pi}{\alpha_0 t_{pulse}} \ . \tag{10}$$

The number of readings of the envelope of the received signal, which must be fixed during the interval T, is accordingly equal to

$$N = \frac{T}{\Delta t} \geqslant \frac{\alpha_0 t_{pulse} T}{\pi} .$$

We will now explain the meaning of the term "superresolution", which has already been utilized. Let the equipment resolution be selected as the "worst" permissible according to (10), i.e.,

$$\Delta t_{opt} = \frac{\pi}{\alpha_0 t_{pulse}} \tag{11}$$

Then, from (9) and (11), we obtain the equation of "superresolution"

$$\Delta \tau = 2 \frac{t_{pulse}}{T} \Delta t_{opt}$$

/11

Consequently, taking the observation interval as sufficiently large, as compared with the duration of the emitted pulses, we may obtain the autocorrelation function of the envelope $A_1(t)$ with a time resolution considerably better than in the recording equipment. (In practice, the selection of a value of T which is as large as one pleases, and, consequently, obtaining a resolution as high as one pleases, is impossible because of the presence of noises). We would also note that the band of the receiver (prior to the envelope detector) Δf_{rec} should be matched with the magnitude $\Delta\tau$, i.e., $\Delta f_{rec} \approx 1/\Delta\tau$, and, thus, the term "superresolution" has meaning only as applied to the low-frequency portion of the recording equipment.

We will now evaluate the best resolution, which may be obtained utilizing the given method, for pulsars as periodic radio sources. It is evident that, for this purpose, one should (with the absence of noise) select an observation interval equal to the period of the pulsar P, i.e., $T_{opt}=P$. The best resolution, in that case, will be

$$\Delta \tau_{opt} = \frac{2\pi}{\alpha_0 P} \tag{12}$$

Taking Δt_{opt} according to (11), we obtained a number of readings of the envelope $A_2(t)$, which must be recorded during the interval T

$$N_{opt} = \frac{T_{opt}}{\Delta t_{opt}} = \frac{\alpha_0 t_{pulse} P}{\pi} \tag{13}$$

The equation of "superresolution" will now have the form

$$\Delta\tau_{opt} = 2\frac{t_{pulse}}{P}\Delta t_{opt}$$

Insofar as, for all pulsars, $t_{pulse}/P \ll 1$, consequently, $\Delta\tau_{opt} \ll \Delta t_{opt}$.

For each space pulse radio source at a fixed frequency $f_0=\omega_0/2\pi$, one may introduce the parameter $t_{pulse\ max}$, which is the maximum duration of the emitted pulse, with which condition (5) is still fulfilled, i.e., $t^2_{pulse\ max}=1/\alpha_0$. The values of $t_{pulse\ max}$ for some values of $\mathcal{D}M$ and f_0 are given in the Table. /12

MAXIMUM PULSE DURATION

$\mathcal{D}M$ [ps/cm^3]	5	10	20	50	100	200	500
$t_{pulse\ max}$ [msec] (f_0 = 100 MHz)	0,08	0,11	0,16	0,25	0,35	0,5	0,8
$t_{pulse\ max}$ [msec] (f_0 = 40 MHz)	0,32	0,45	0,64	1,0	1,4	2,0	3,2
$t_{pulse\ max}$ [msec] (f_0 = 20 MHz)	0,9	1,3	1,8	2,8	4,0	5,6	9,0

Equation (12) may then be rewritten in the form

$$\Delta\tau_{opt} = \frac{2\pi t^2_{pulse\ max}}{P}$$

and relationships (11) and (13) will be written as

$$\Delta t_{opt} \geqslant \pi t_{\text{pulse max}}$$
$$N_{opt} \leqslant \frac{P}{\pi t_{\text{pulse max}}} \qquad (14)$$

with the sign "=" in (14) occurring with $t_{pulse}=t_{\text{pulse max}}$ and corresponding to the "worst" case, i.e., the highest requirements for the recording equipment.

5. Comparison with the Method of "Elimination of Dispersion"

We will now examine some advantages and shortcomings of the proposed method for processing pulse radio signals of cosmic origin. We will carry out the comparison with the aforementioned method of "elimination of dispersion" of Hankins (which is, /13 in essence, inverse filtration of the received signals, i.e., multiplication of their spectra by the function which is inverse to the coefficient of transmission of the interstellar medium), which makes it possible to achieve considerably higher resolution than other processing methods.

The two basic shortcomings of the proposed method are evident — first, the impossibility of recreating the very form of the envelope of the initial radio pulse (we obtain only its autocorrelation function); second, an entire series of restrictions on the applicability of the given method, which may be utilized only with fully determined relationships between the parameters of the pulse and the medium. The Hankins method is completely devoid of both these shortcomings.

The basic advantage of the processing method examined above consists of the considerable reduction in the requirements for the recording equipment (more precisely, as was noted above, for its low-frequency portion). Let $\Delta\tau$ be the obtained resolution of the autocorrelation function of the envelope of the initial signal (or the very envelope for the Hankins method), and Δt is the time resolution of the equipment. With utilization of the procedure of "elimination of dispersion", the obtained resolution of the envelope, in the best case, is equal to that of the equipment, i.e., $\Delta\tau \geq \Delta t$, whereas, for the proposed processing method according to the equation of "superresolution", the requirements for the equipment resolution are reduced by $k = T/2t_{pulse}$ times, which, in practice, may comprise two or more orders.

The advantages of the given method may include the fact that, in this case, it is not necessary to know the degree of dispercion of the radio source with very high accuracy, insofar as it determines only the time scale of the attained autocorrelation function of the envelope; at the same time, utilization of the procedure of "elimination of dispersion" requires the knowledge of the degree of dispersion with very low absolute error. /14

It makes sense to dwell here on the matter of the effect of scattering of radio waves in the interstellar medium. Scattering, like dispersion, leads to broadening of the pulses, which will be characterized by the magnitude of τ_{scatt}, which is the scattering time constant. We would note that, insofar as there is no functional dependence between the degree of dispersion and the scattering time constant, the magnitude of τ_{scatt} for a specific radio source at a given frequency may only be determined experimentally.

Scattering places a natural restriction on the time resolution of the received signals, which, irrespective of the parameters of the receiving equipment, may not be better than the scattering time constant. Consequently, with utilization of the method of "elimination of dispersion", it is impossible to achieve a time resolution better than the magnitude of τ_{scatt}; at the same time, the proposed method, which places requirements $T/2t_{pulse}$ times lower on the time resolution of the received signal, makes it possible, accordingly, to achieve resolution several times greater than the magnitude of τ_{scatt}. In this case, however, the scattering places an evident restriction on the applicability of the given method, and namely, $\tau_{scatt} \leq \Delta t_{opt}$, where the magnitude of Δt_{opt} is determined from expression (11), i.e.,

$$\tau_{scatt} \leq \frac{\pi}{\alpha_0 t_{pulse}}$$

6. General Case of Radio Pulses with Phase Modulation

Examined above was the case when phase modulation is absent in the emitted pulses; we will now make an attempt to evaluate to what consequences its presence leads. In this case, for the complex envelope of the received signal, we have /15

$$\tilde{g}_2(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{G}_1(\omega)\, e^{-iF(\omega)}\, d\omega, \tag{15}$$

where $\tilde{G}_1(\omega) = \int_{-\infty}^{+\infty} \tilde{g}_1(t)\, e^{-i\omega t}\, dt;\quad F(\omega) = -\left(\frac{\omega^2}{2\alpha_0} + \omega t\right).$

Here, it is difficult to clearly formulate the condition of applicability of the stationary phase method for the general case without any information on the law of phase change. In the extreme case, one may assume that, insofar as the spectra of a radio pulse of one and the same form, with the presence in it of phase modulation, will be broader than in the case of its absence, and, consequently, the characteristic width of the spectrum of the complex envelope $\tilde{G}_1(\omega)$ should be greater than the spectrum of the "physical" envelope $W_1(\omega)$, the function $\tilde{G}_1(\omega)$ is "slower changing" than $W_1(\omega)$, and one may utilize condition (5). But, strictly speaking, it is impossible to recognize similar discussions as correct, and condition (5) here is inapplicable in the general case.

Let the condition of applicability of the stationary phase method be observed. By calculating the integral in (15) using the stationary phase method, we obtain the following for the complex envelope of the received signal

$$\tilde{g}_2(t) = \sqrt{\frac{\alpha_0}{2\pi}} \cdot \tilde{G}_1(-\alpha_0 t) \cdot e^{-i\left(\frac{\alpha_0 t^2}{2} - \frac{\pi}{4}\right)}$$

and, accordingly, for the square of the "physical" envelope

$$\mathcal{X}(t) = A_2^2(t) = \frac{\alpha_0}{2\pi} \cdot \left| \tilde{G}_1(-\alpha_0 t) \right|^2 \tag{16}$$

As is evident from (16), the square of the envelope of the received signal is proportional to the energy spectrum of the complex envelope of the emitted signal, but from a negative argument. The energy spectrum and the autocorrelation function of the complex signals are associated between themselves by a Fourier transform, i.e.,

$$\tilde{K}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |G_1(\omega)|^2 e^{i\omega t} d\omega \tag{17}$$

where $|\tilde{G}_1(\omega)|^2$ is the energy spectrum of the complex signal $\tilde{g}_1(t)$; /16
and $\tilde{K}(t) = \int_{-\infty}^{+\infty} \tilde{g}_1^*(t)\, \tilde{g}_1(t+\tau)dt$ is its autocorrelation function. In addition, the function $\tilde{K}(-\tau)$ will correspond to the function $|\tilde{G}_1(-\omega)|^2$, according to Fourier, with $\tilde{K}(-\tau) = \tilde{K}^*(\tau)$.

From (16) and (17), we obtain the following expression for the function $X(\tau)$, which is the inverse Fourier transform of the function $\mathcal{X}(t)$ — the square of the envelope of the received signal:

$$X(\tau) = \frac{1}{4\pi^2} \tilde{K}^*\left(\frac{\tau}{\alpha_0}\right)$$

or

$$X^*(\tau) = \frac{1}{4\pi^2} \tilde{K}\left(\frac{\tau}{\alpha_0}\right).$$

In the given case, consequently, we obtain the autocorrelation function of the complex envelope of the received signal, and not its "physical" envelope, like in the case of pulses without phase modulation. One may establish that such a complex autocorrelation function for pulses with phase modulation is more informative than simply the autocorrelation function of the envelope, insofar as it carries information on the law of phase change. The time resolution of the obtained autocorrelation function, and all other time relationships, are completely identical to those obtained above, and, accordingly, the phenomenon of "superresolution" may also occur here.

Conclusion

Proposed in the present study is a new method for processing space pulse radio signals for the study of their internal structure with a high level of time resolution. The proposed method is based on the utilization of the effect of the Fourier transform of the envelope of the radio pulses in the interstellar plasma, which takes place with the observance of definite relationships between the parameters of the pulse and the medium. The conditions of applicability of the method are formulated, and requirements are also substantiated for the receiving-recording /17 equipment, which proved considerably lower than for the processing methods being utilized at the present time. It is shown that the proposed method makes it possible to obtain the autocorrelation function of the envelope of the emitted radio pulses with "super-resolution", i.e., with better time resolution than in the recording equipment.

The obtained results may also find use for problems of the synthesis and processing of signals in space communications systems.

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