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NUMERICAL STABILITY OF AN EXPLICIT FINITE DIFFERENCE  
SCHEME FOR THE SOLUTION OF TRANSIENT CONDUCTION  
IN COMPOSITE MEDIA

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November 1981



**NASA**

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## TECHNICAL MEMORANDUM

# NUMERICAL STABILITY OF AN EXPLICIT FINITE DIFFERENCE SCHEME FOR THE SOLUTION OF TRANSIENT CONDUCTION IN COMPOSITE MEDIA

### INTRODUCTION

Stability of numerical schemes plays a key role in the computer solution of transient heat conduction. A stability analysis relates permissible combinations of grid spacing and time step. For a given grid spacing, the time step must be smaller than a certain value for an explicit numerical scheme to be stable. A time step as close to the maximum as possible is desirable to reduce computer run times and cost.

In composite media, the transient heat conduction equation is not valid at interfaces between media with different heat conduction coefficients. In this case, a heat balance technique can be used to drive an explicit finite difference scheme. The derivation is presented herein, and a theoretical stability analysis is performed. Because of the complexity of the resulting relationships, a computer graphics code was developed to allow easy determination of an optimum time step. A sample problem is examined, and graphics output is presented.

### DERIVATION OF THE FINITE DIFFERENCE SCHEME

Figure 1 illustrates the general problem that is addressed herein. The following assumptions are made: (1) Media interfaces lie along two directions that are orthogonal, e.g., vertical or horizontal, as shown in Figure 1. (2) Interfaces lie along lines of grid points; and when more than two media intersect, they do so at a grid point. (3) Media interfacial contacts are perfect, i.e., infinite interfacial conductance. (4) Cartesian coordinates are used. (5) Heat conduction is in two dimensions.

The preceding assumptions were made for simplicity of presentation and are not absolute restrictions on the usefulness of the methods described. All results can be generalized so that none of the preceding assumptions is required.

Figure 1 shows a small element centered around grid point  $(i, j)$ . An explicit finite difference equation is desired involving the temperature at  $(i, j)$  and surrounding grid points at time  $t$  and at time  $t + \Delta t$ , where  $t$  is the time step. The desired result is accomplished using a heat balance. The heat balance is in the form

*Change of internal energy within the element during the time step =  
heat conduction into the element during the time step.*

A heat balance form is used because the heat conduction equation is not valid at the interface between media. In equation form, the preceding relation is

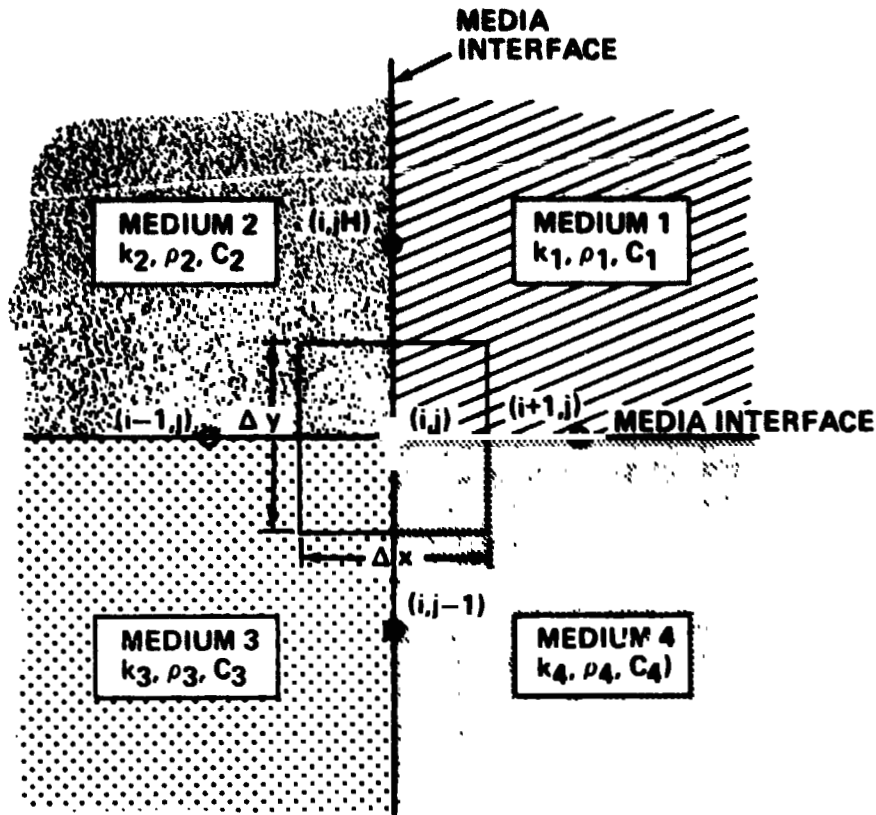


Figure 1. Finite difference grid arrangement for the general case of four intersecting media.

$$\begin{aligned}
 (T_{i,j}^{n+1} - T_{i,j}^n) \frac{\Delta x \Delta y}{4} (c_1 \rho_1 + c_2 \rho_2 + c_3 \rho_3 + c_4 \rho_4) &= \frac{(T_{ij}^n - T_{i-1,j}^n)}{\Delta x} \left[ \frac{-\Delta t \Delta y}{2} (k_2 + k_3) \right] \\
 + \frac{(T_{i,j}^n - T_{i,j-1}^n)}{\Delta y} \left[ \frac{-\Delta t \Delta x}{2} (k_3 + k_4) \right] &+ \frac{(T_{i+1,j}^n - T_{i,j}^n)}{\Delta x} \frac{\Delta t \Delta y}{2} (k_1 + k_4) \\
 + \frac{(T_{i,j+1}^n - T_{i,j}^n)}{\Delta y} \frac{\Delta t \Delta x}{2} (k_1 + k_2) & \quad (1)
 \end{aligned}$$

where

T = temperature

i = subscript corresponding to x direction

j = subscript corresponding to y direction

n = superscript corresponding to the time step

$\Delta x$  = x grid spacing

$\Delta y$  = y grid spacing

$\Delta t$  = time step

$k_1, k_2, k_3, k_4$  = thermal conductivity of the media in quadrants 1 through 4

$\rho_1, \rho_2, \rho_3, \rho_4$  = density of the media.

The terms in equation (1) are

*Change in the internal energy of the element during time  $\Delta t$  = heat flow into the element from the left-hand side during time  $\Delta t$  + heat flow into the element from the bottom + heat flow into the right-hand side + heat flow in from the top.*

Equation (1) is an explicit form; i.e., the temperature one time step ahead can be determined from temperatures at the grid point and adjacent points at the current time step. To see this more clearly, equation (1) is rewritten in the following form:

$$\begin{aligned} T_{i,j}^{n+1} \frac{(c_1\rho_1 + c_2\rho_2 + c_3\rho_3 + c_4\rho_4)}{4} &= T_{i+1,j}^n \frac{\Delta t}{(\Delta x)^2} \frac{(k_1 + k_4)}{2} + T_{i-1,j}^n \frac{\Delta t}{(\Delta x)^2} \frac{(k_2 + k_3)}{2} \\ &+ T_{i,j+1}^n \frac{\Delta t}{(\Delta y)^2} \frac{(k_1 + k_2)}{2} + T_{i,j-1}^n \frac{\Delta t}{(\Delta y)^2} \frac{(k_3 + k_4)}{2} \\ &+ T_{i,j}^n \left[ \frac{(c_1\rho_1 + c_2\rho_2 + c_3\rho_3 + c_4\rho_4)}{4} \right. \\ &\left. - \Delta t \left\{ \frac{k_2 + k_3}{2(\Delta x)^2} + \frac{k_3 + k_4}{2(\Delta y)^2} + \frac{k_1 + k_4}{2(\Delta x)^2} + \frac{k_1 + k_2}{2(\Delta y)^2} \right\} \right]. \end{aligned} \quad (2)$$

If the media in the four quadrants are the same, equation (2) reduces to

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \frac{k}{c\rho} \frac{T_{i+1,j}^n + T_{i-1,j}^n - 2T_{i,j}^n}{(\Delta x)^2} + \frac{k}{\rho c} \frac{T_{i,j+1}^n + T_{i,j-1}^n - 2T_{i,j}^n}{(\Delta y)^2}. \quad (3)$$

Observe that equation (3) is just the finite difference form of the heat conduction equation. The assumption of steady state heat flow with  $\Delta x = \Delta y$  reduces equation (3) to

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4} \quad (4)$$

The superscript is dropped from equation (4) because it is superfluous. Observe that the preceding equation is a well-known finite difference form of Laplace's equation, i.e., the steady state heat flow equation.

The finite difference form for the heat balance equation was derived and is given by equation (2). This equation is in explicit form and, hence, is only conditionally stable. An inequality relating  $\Delta t$ ,  $\Delta x$ ,  $\Delta y$ , and the material parameters must be found so that the marching technique suggested by equation (2) is stable. For equation (3), the relationship can be shown to be [1]

$$\frac{k \Delta t}{c \rho (\Delta x)^2} \leq \frac{1}{4} \quad (5)$$

The normal procedure for obtaining stable solutions is to select  $\Delta x$  for adequate resolution of the temperature field and use equation (5) to calculate the maximum  $\Delta t$ . The relationship analogous to equation (5) for the heat balance equation will be derived in the next section.

#### STABILITY OF THE EXPLICIT FINITE DIFFERENCE EQUATION

The stability analysis of the finite difference equation (2) will follow the procedure outlined by Richtmyer and Morton [1]. First consider the real solution of equation (2) given by  $T_{i,j}^n$ . The actual solution calculated on the computer will be  $\tilde{T}_{i,j}^n = T_{i,j}^n + \delta_{i,j}^n$ , where  $\delta_{i,j}^n$  is a small error that may be due to roundoff or truncation, function evaluation error, etc. If the solution with error is plugged into equation (2), it can be seen that  $\delta_{i,j}^n$  is a solution to the same equation as  $T_{i,j}^n$  since  $T_{i,j}^n$  is a solution of (2) and hence will cancel out. Reference 1 shows that the error function  $\delta_{i,j}^n$  can be represented in the form

$$\delta_{i,j}^n = \xi^n e^{i(i\beta\Delta x + j\gamma\Delta y)} \quad (6)$$

where  $i = \sqrt{-1}$ .

Here  $\xi$  is, in general, a complex amplitude factor; and  $\xi^n$  means  $\xi$  raised to the  $n$ th power (not  $\xi$  superscript  $n$ ).  $\beta$  and  $\gamma$  can be considered free parameters that can change independently of each other. Substituting equation (6) into equation (2) yields, after some rearrangement,

$$\xi = 1 + a \cdot \left[ \frac{(k_1 + k_2 + k_3 + k_4)}{2} (\cos B + \cos G - 2) \right] + i a \left[ \frac{(k_1 + k_4 - k_2 - k_3)}{2} \sin B + \frac{(k_1 + k_2 - k_3 - k_4)}{2} \sin G \right], \quad (7)$$

where

$$a = \frac{4\Delta t}{\Delta x \Delta y (c_1 \rho_1 + c_2 \rho_2 + c_3 \rho_3 + c_4 \rho_4)}$$

$$B = \beta \Delta x$$

$$G = \gamma \Delta y.$$

In the example of the discretized form of the heat conduction equation,  $\xi$  is real. For the current form,  $\xi$  is, in general, complex. For a given grid point, the amplification factor for any error is given by equation (7). If the modulus of  $\xi$  is greater than unity, any error present can amplify and disrupt the numerical solution of the temperature field. To assure stability of the solution,  $|\xi| \leq 1$  must be satisfied at every grid point in the interior of the composite region and at every boundary point (if heat flux or radiation boundary conditions are specified).

For a given grid point in a composite medium,  $\xi$  is a two-parameter function of  $B$  and  $G$  in the complex plane. If  $\Delta x$  and  $\Delta t$  are selected and  $|\xi| \leq 1$  for every value of  $B$  and  $G$ , then the stability condition is not violated.

To better understand equation (7), fix  $G$  at some arbitrary value. Then the following representations can be made:

$$\text{Re}[\xi] = c_1 \cos B + c_2$$

$$\text{Im}[\xi] = c_3 \sin B + c_4, \quad (8)$$

where

$$\text{Re}[\xi] = \text{real part of } \xi$$

$$\text{Im}[\xi] = \text{imaginary part of } \xi$$

$$c_1 = \frac{a(k_1 + k_2 + k_3 + k_4)}{2}$$

$$c_2 = 1 + c_1 (\cos G - 2)$$

$$c_3 = a(k_1 + k_4 - k_2 - k_3)/2$$

$$c_4 = a(k_1 + k_2 - k_3 - k_4) \sin G/2 .$$

Let

$$\xi = x + \hat{i} y = \text{Re}[\xi] + \hat{i} \text{Im}(\xi) . \quad (9)$$

Then

$$\frac{x - c_2}{c_1} = \cos B$$

$$\frac{y - c_4}{c_3} = \sin B .$$

Finally,

$$\frac{(x - c_2)^2}{c_1^2} + \frac{(y - c_4)^2}{c_3^2} = 1 . \quad (10)$$

This is the equation of an ellipse centered in the complex  $\xi$  plane at  $\xi = c_2 + \hat{i} c_4$  with semimajor axes  $c_1$  and  $c_3$ . Reference to the definitions of  $c_1$  and  $c_3$  indicates that the semimajor axes are constants for a given composite medium; i.e., they are not functions of  $G$ .  $c_2$  and  $c_4$  are functions of  $G$  and can be written in the form

$$c_2 = \alpha_1 \cos G + \alpha_0$$

$$c_4 = \beta_1 \sin G , \quad (11)$$



where

$$\alpha_1 = c_1$$

$$\alpha_0 = 1 - 2c_1$$

$$\beta_1 = \frac{a(k_1 + k_2 - k_3 - k_4)}{2}$$

Equation (11) is the parametric form for an equation of an ellipse. Equations (10) and (11) represent a series of ellipses centered on points on an ellipse. At this point, two approaches can be taken. The first to be considered involves trying to find an envelope for the series of ellipses defined by the preceding two equations. Determining an equation for the envelope involves combining equations (10) and (11) in the following form:

$$f(x,y,G) = \frac{(x - \alpha_1 \cos G - \alpha_0)^2}{c_1^2} + \frac{(y - \beta_1 \sin G)^2}{c_3^2} - 1 \quad (12)$$

If an envelope exists, it is necessary that it satisfy the following two equations (see Reference 2, for example):

$$f(x,y,G) = 0$$

$$f_G(x,y,G) = 0 \quad (13)$$

The subscript G refers to partial differentiation. Conditions (13) become

$$\frac{(x - \alpha_1 \cos G - \alpha_2)^2}{c_1^2} + \frac{(y - \beta_1 \sin G)^2}{c_3^2} - 1 = 0 \quad (14)$$

$$\tan G + k_1 \sin G - k_2 = 0$$

where

$$k_1 = \frac{-\left(\frac{\alpha_1}{c_1^2} + \beta_1\right)}{\left(\frac{x}{c_1^2} - \frac{\alpha_2}{c_1^2}\right)}$$

$$k_2 = \frac{2y}{c_3^2 \left(\frac{x}{c_1^2} - \frac{\alpha_2}{c_1^2}\right)}$$

The usual procedure for solving for the envelope involves elimination of G from the two equations in (14). Because of the transcendental nature of these equations, elimination of G is difficult. Rather than proceed in an attempt to obtain an analytical solution, a computer program was written to plot up the series of ellipses. This program is described in the following section.

#### COMPUTER GRAPHICS STABILITY DETERMINATION

The graphics program is based on equations (8) and (11), which are presented here in slightly different form for convenience,

$$x = c_1 \cos B + \alpha_1 \cos G + \alpha_0 \tag{15}$$

$$y = c_3 \sin B + \beta_1 \sin G$$

This represents a two-parameter family of curves. The finite difference form of equation (2) is stable, if the following inequality is true:

$$|\xi| = \sqrt{x^2 + y^2} \leq 1 \tag{16}$$

The preceding two-parameter family is plotted by holding G constant and varying B between 0 and  $2\pi$ . This plots one ellipse. After G is incremented between 0 and  $2\pi$ , a series of ellipses is drawn that allows visual determination of the  $\xi$  envelope. The program listing in the appendix draws the circle  $|\xi| = 1$  and the ellipses. If a part of the envelope falls outside the unit circle, instabilities can be expected. If this occurs,  $\Delta t$  must be reduced and a new graph drawn. When the envelope just stays in the unit circle, the corresponding value of  $\Delta t$  is nearly optimum.

To better illustrate the method, the following section outlines an example problem and indicates how optimum time increments can be obtained.

## EXAMPLE STABILITY ANALYSIS

The example geometry of Figure 2 is used to illustrate principles outlined in the preceding sections. The composite medium is made up of copper, plexiglass, cork for insulation, and Dow-Corning 200 Series oil (viscosity 1000 centistokes). Although the oil is a fluid and subject to convection, its high viscosity and the low temperature gradients of the problem (low Rayleigh number) mean that most heat transfer is by conduction.  $\Delta x$  and  $\Delta y$  are 0.1 cm for the example. Given these values for  $\Delta x$  and  $\Delta y$ , the object of the stability analysis is to determine the maximum  $\Delta t$  for a stable solution.

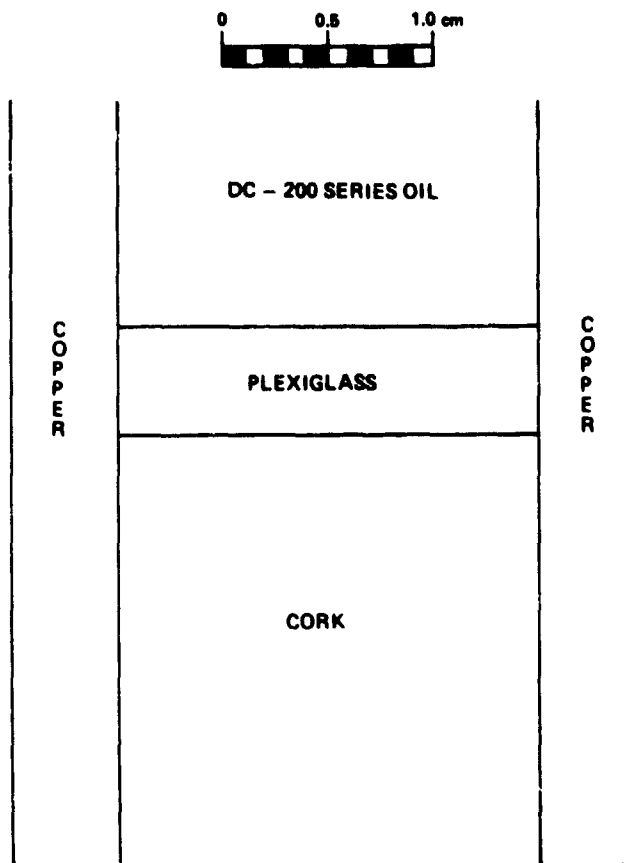


Figure 2. Composite medium geometry.

Each grid point can be classified into one of several categories, as indicated in Figure 3. To assure stability, each of these categories must be tested. For the example problem, stability considerations are dominated by copper because of its high conductivity relative to the other three materials of the composite medium. Experience shows that the only categories of interest are those containing copper in at least one of the four quadrants.

A trial and error procedure indicates that  $\Delta t = 0.002$  sec is nearly the maximum or optimum value. Figure 4 shows the stability plots for the interiors of each of the four materials.  $\xi$  is real in the interiors and becomes significant only for the case of copper. The plots of Figure 5 show the remaining categories listed in Figure 3.

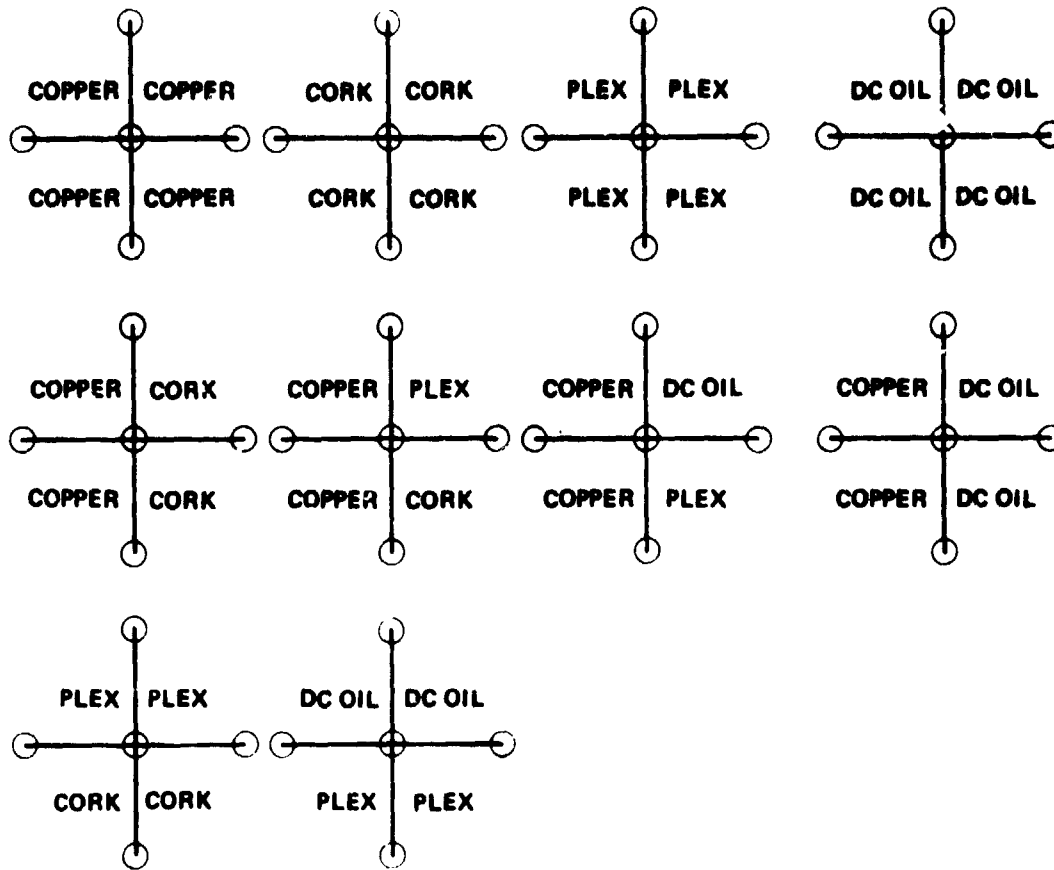
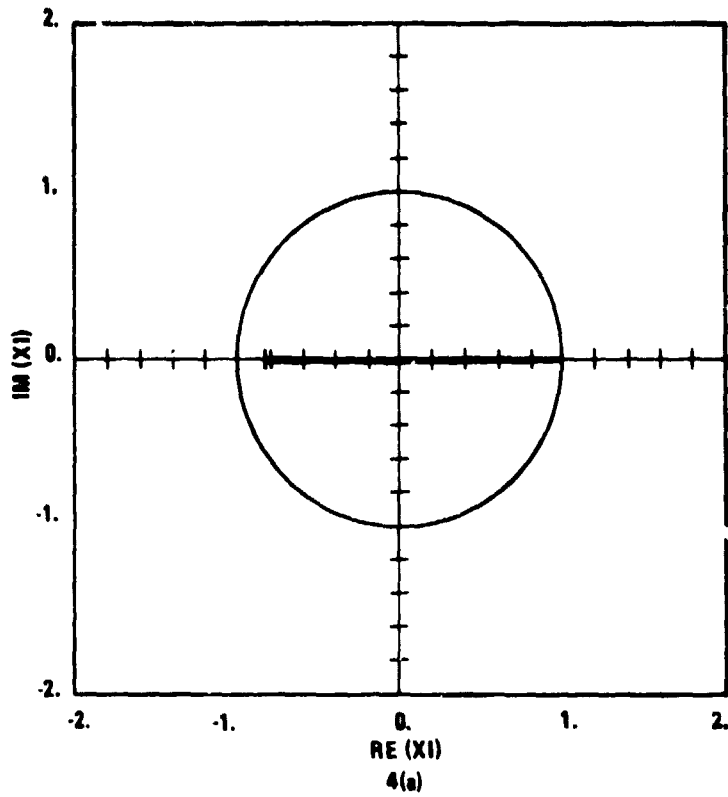


Figure 3. Classes of grid point arrangements that must be considered for stability.

Because of the dominance of copper, all curves for the interfaces appear to circles with centers on the real axis. Although not belonging to one of the categories, the plot of Figure 6 is included to show more fully the general character of the stability equations. Even this plot appears to be composed of circles. Again, the dominance of copper is evident. If two dominant materials existed with comparable but unequal conductivity, a series of ellipses could be expected.

To check the results of this stability analysis, the thermal code based on equation (2) was programmed. For a value of  $\Delta t = 0.002$  sec, the code was stable. For  $\Delta t = 0.00225$  sec, instabilities soon developed in the solution.

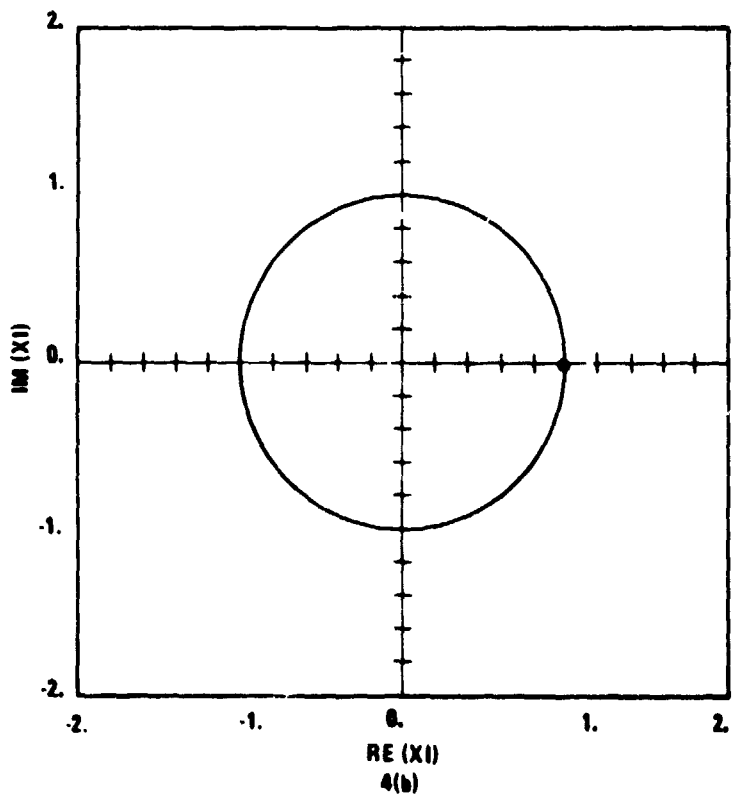
If inequality (5) for copper is used to find the maximum value of  $\Delta t$ , the result is  $\Delta t = 0.0021665$  sec. Figure 7 shows the copper category stability plots. The amplification factor closely approaches the unit circle. Figure 7a serves as a check on the code. These findings suggest that stability is determined by inequality (5) applied to the dominant material of the composite medium. The result is by no means proved, however.



**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

1	1
1	1

**TIME INC =**  
**.00200 SEC**

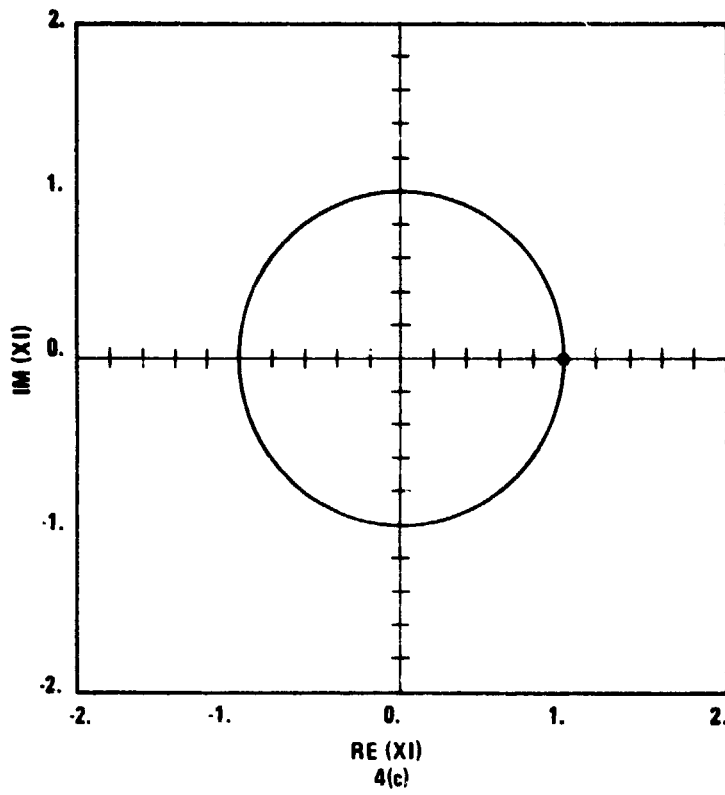


**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

2	2
2	2

**TIME INC =**  
**.00200 SEC**

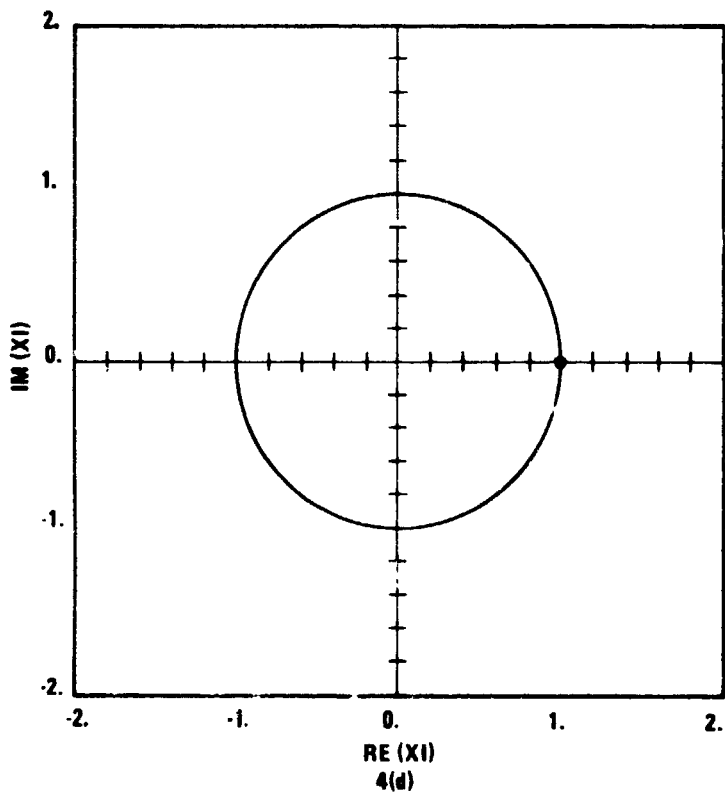
Figure 4. Stability of the finite difference scheme in the interiors of the four materials.



**MATERIALS**  
 1=CU  
 2=CORK  
 3=DC OIL  
 4=PLEX

3	3
3	3

**TIME INC =**  
**.00200 SEC**



**MATERIALS**  
 1=CU  
 2=CORK  
 3=DC OIL  
 4=PLEX

4	4
4	4

**TIME INC =**  
**.00200 SEC**

Figure 4. (Concluded).

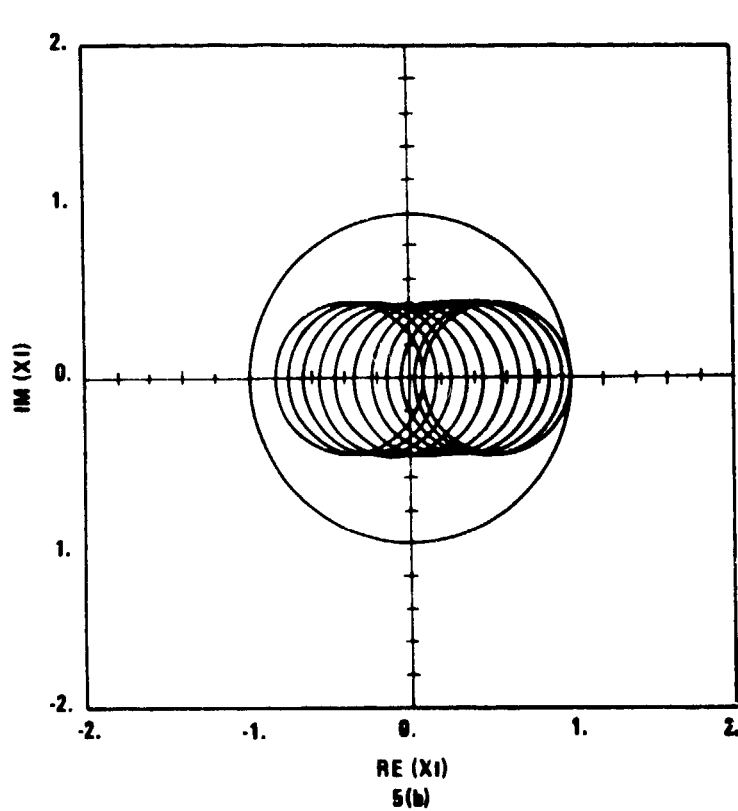
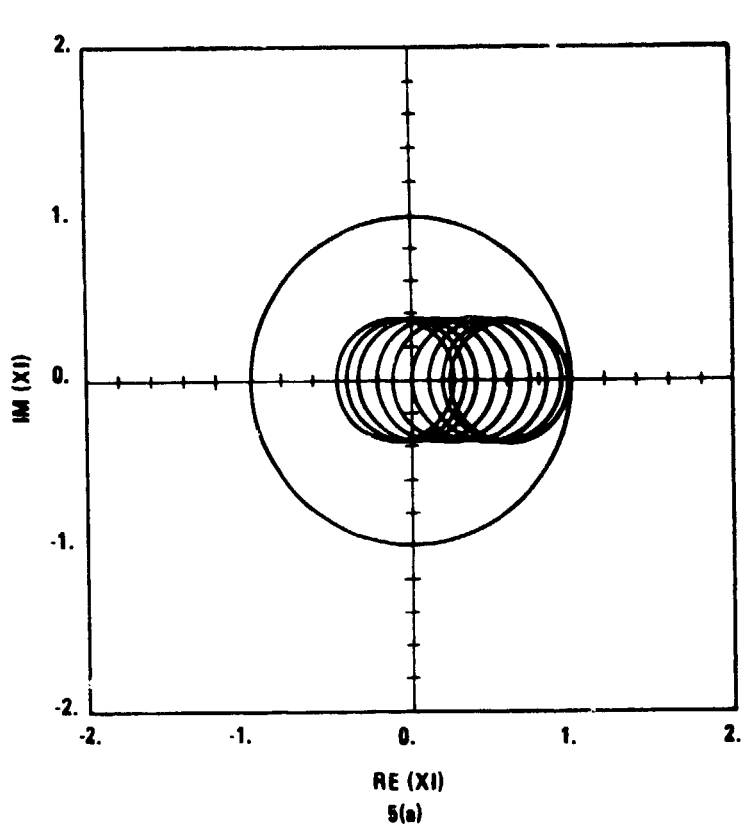
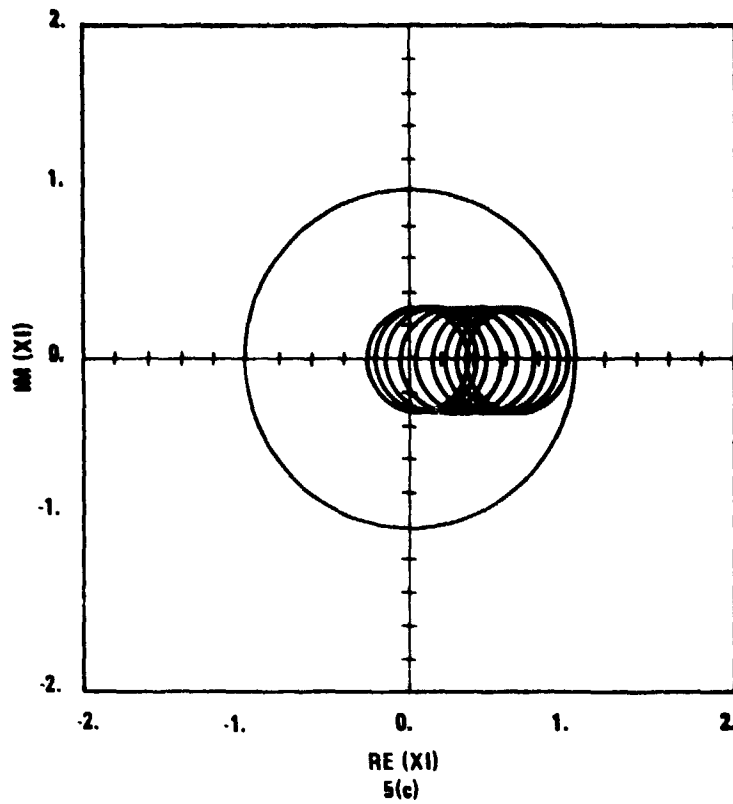


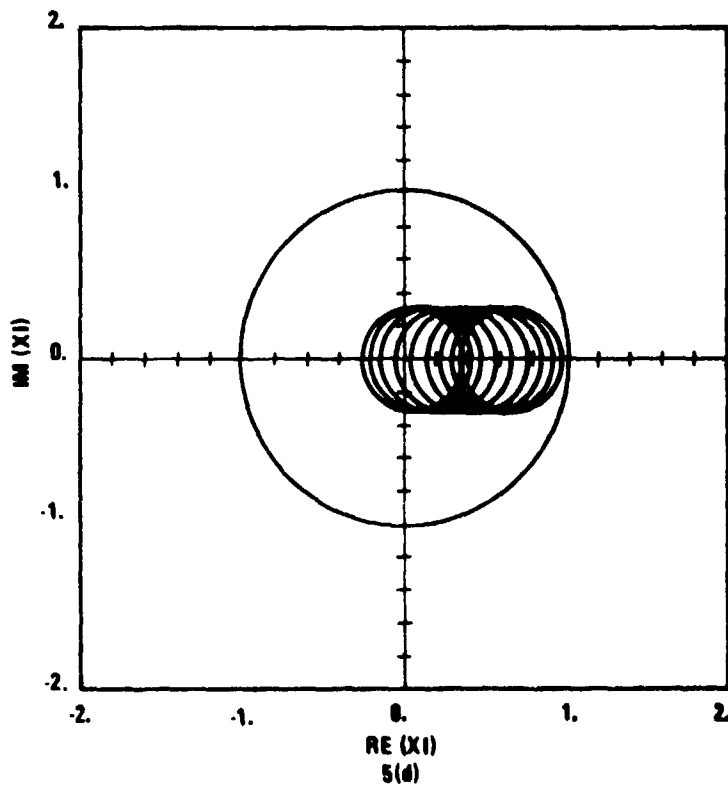
Figure 5. Stability at the interfaces.



**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

1	3
1	4

**TIME INC -**  
**.00200 SEC**



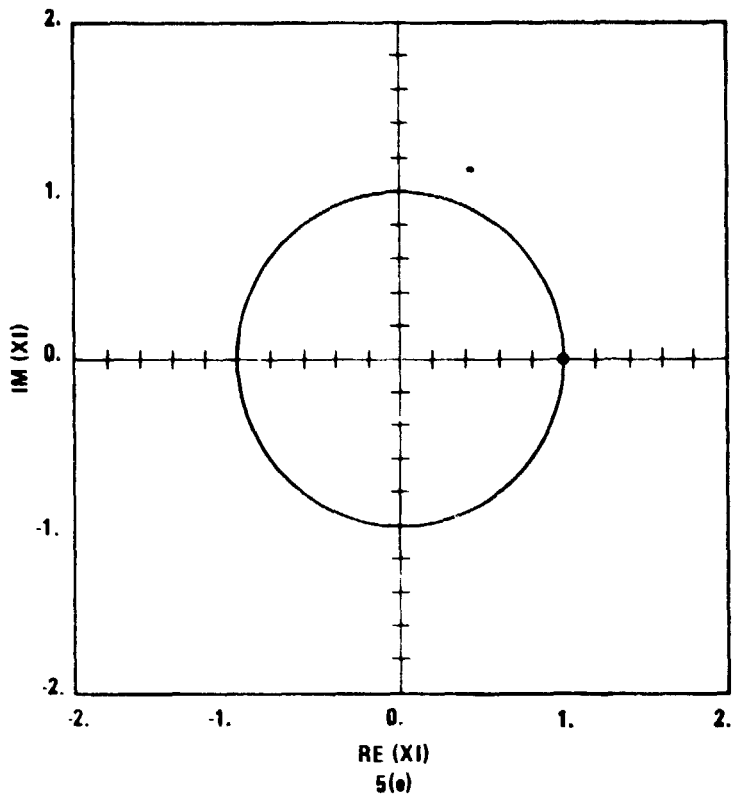
**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

1	3
1	3

**TIME INC -**  
**.00200 SEC**

Figure 5. (Continued).

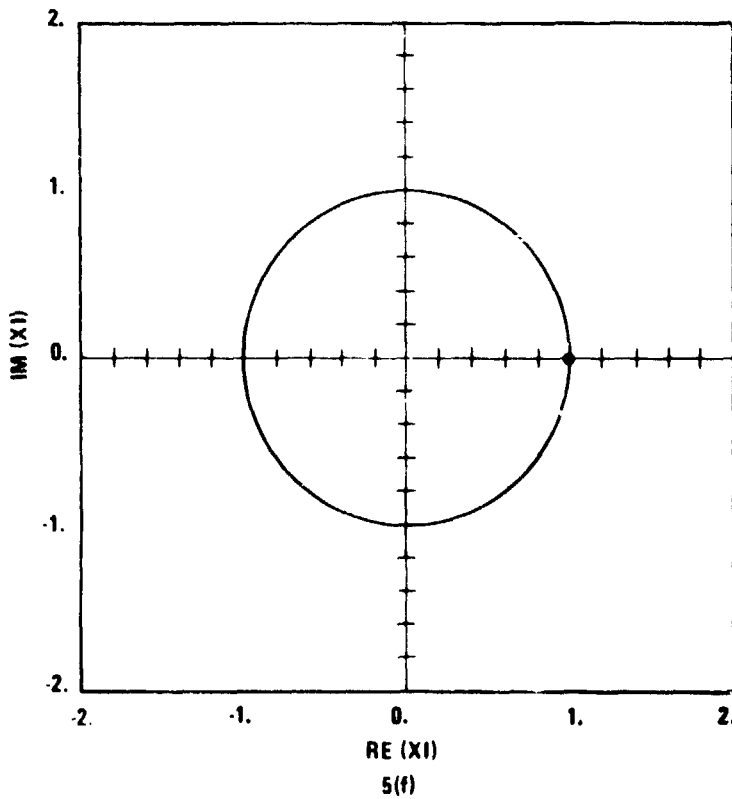




**MATERIALS**  
 1=CU  
 2=CORK  
 3=DC OIL  
 4=PLEX

4	4
2	2

TIME INC =  
 .00200 SEC



**MATERIALS**  
 1=CU  
 2=CORK  
 3=DC OIL  
 4=PLEX

3	3
4	4

TIME INC =  
 .00200 SEC

Figure 5. (Concluded).

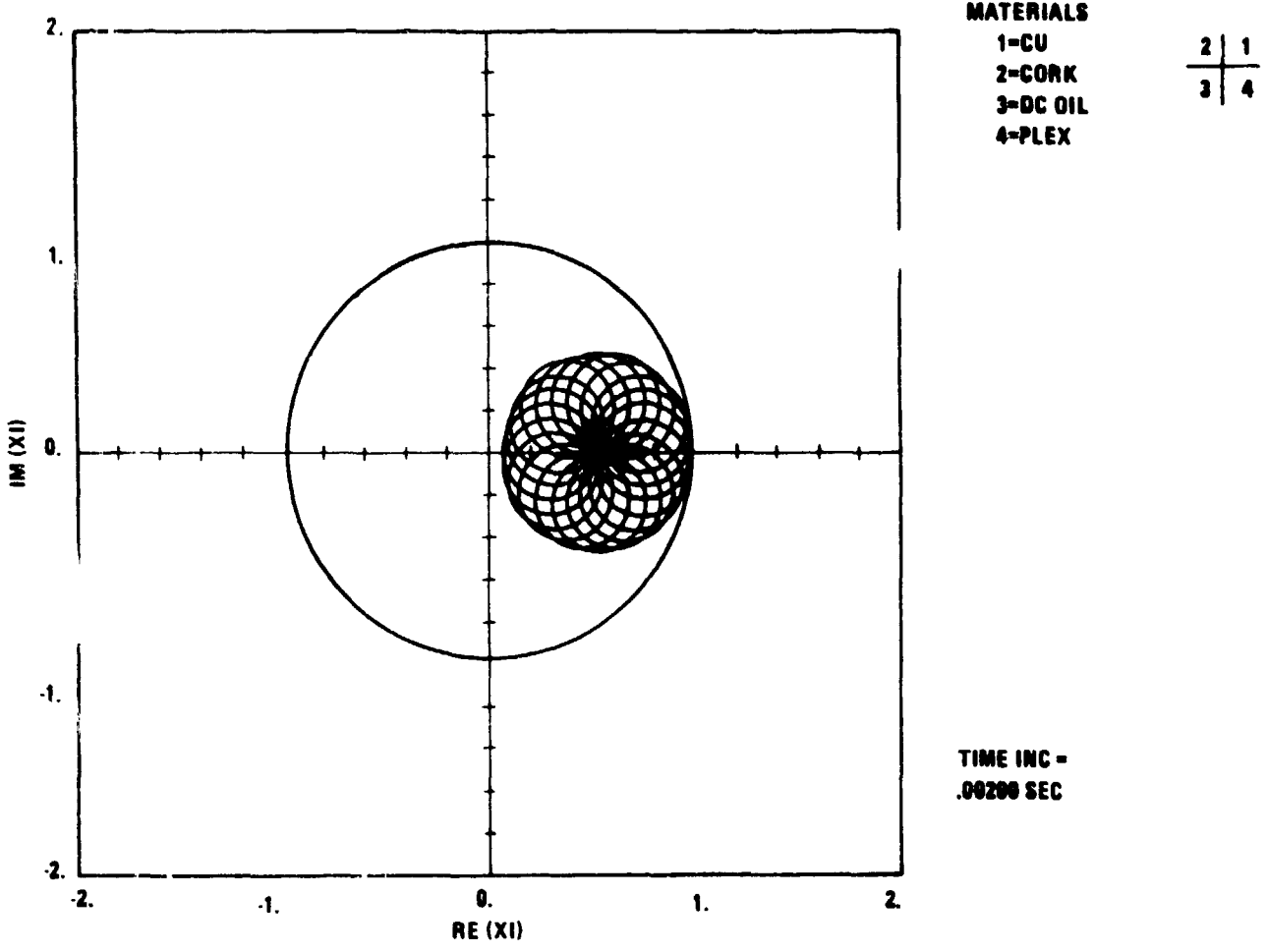
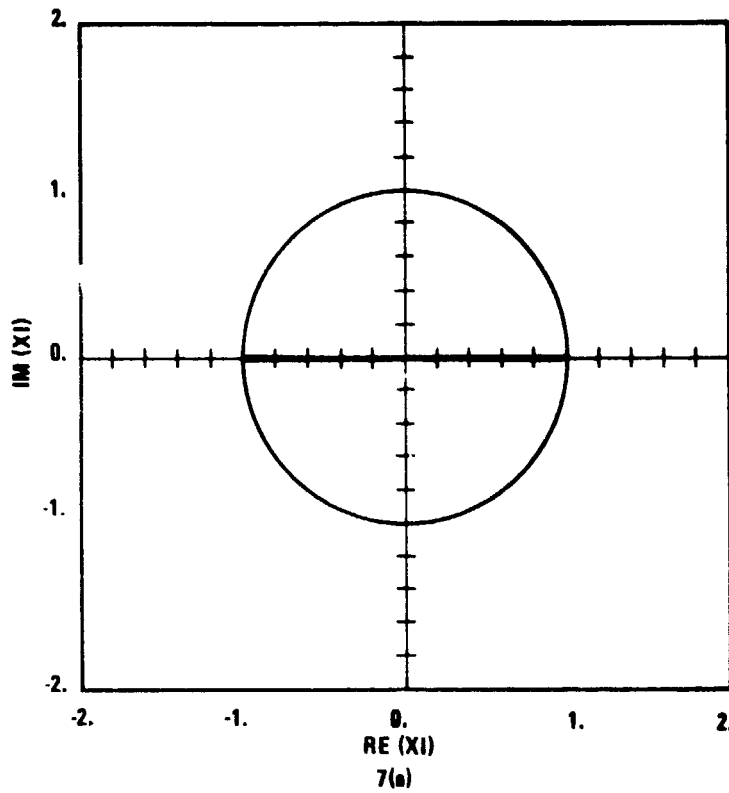


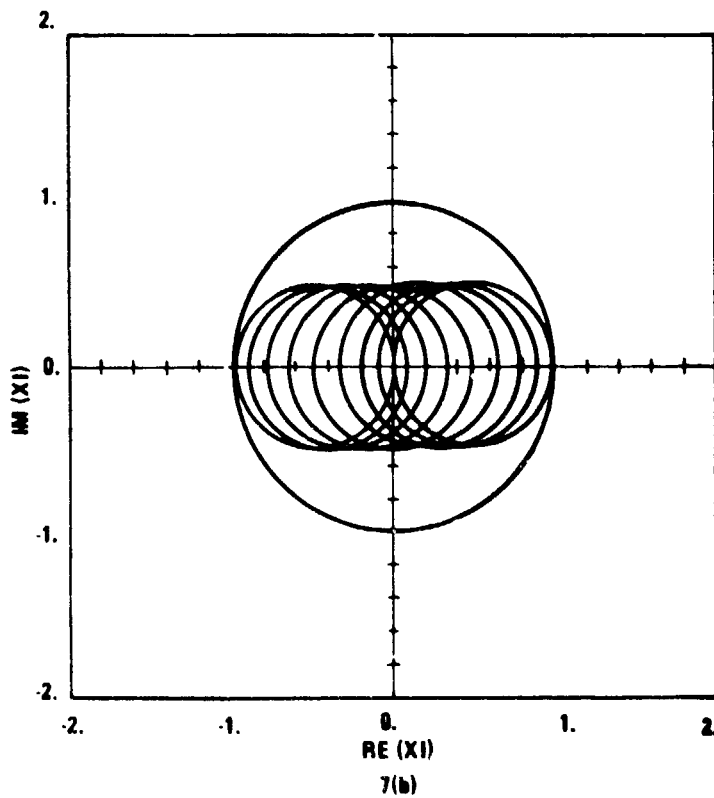
Figure 6. Hypothetical grid point with a different material in each of the four quadrants.



**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

1	1
1	1

**TIME INC =**  
**.00217 SEC**

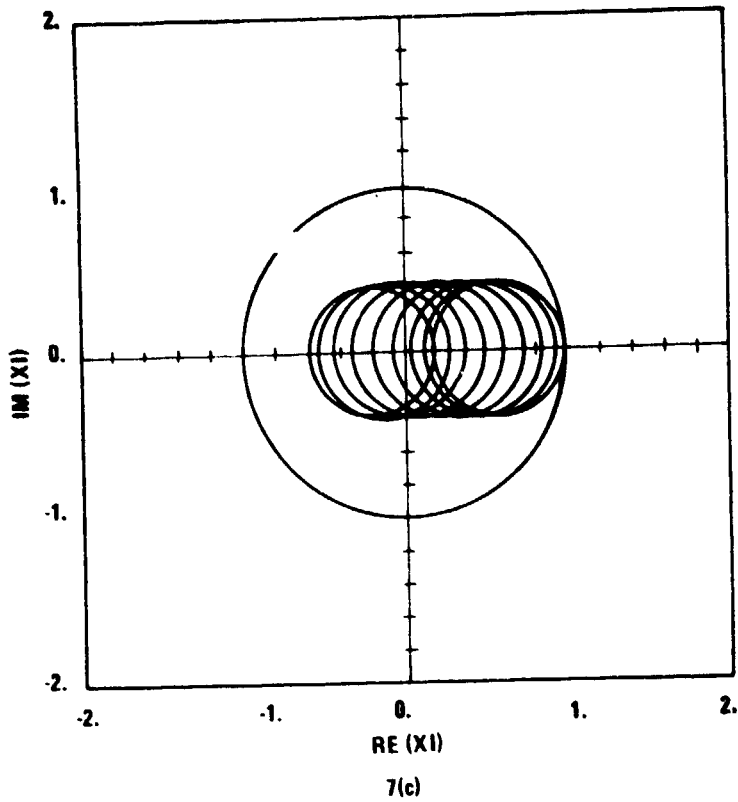


**MATERIALS**  
 1-CU  
 2-CORK  
 3-DC OIL  
 4-PLEX

1	2
1	2

**TIME INC =**  
**.00217 SEC**

Figure 7. Stability plots for the optimum  $\Delta t$ .

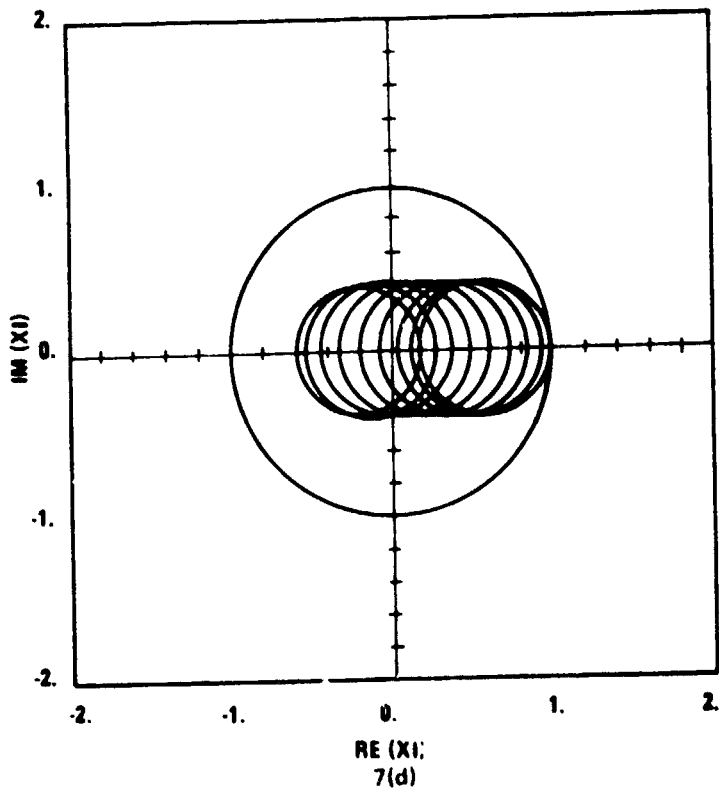


**MATERIALS**

- 1-CU
- 2-CORK
- 3-DC OIL
- 4-PLEX

1	4
1	2

TIME INC =  
.00217 SEC



**MATERIALS**

- 1-CU
- 2-CORK
- 3-DC OIL
- 4-PLEX

1	4
1	2

TIME INC =  
.00217 SEC

Figure 7. (Continued).

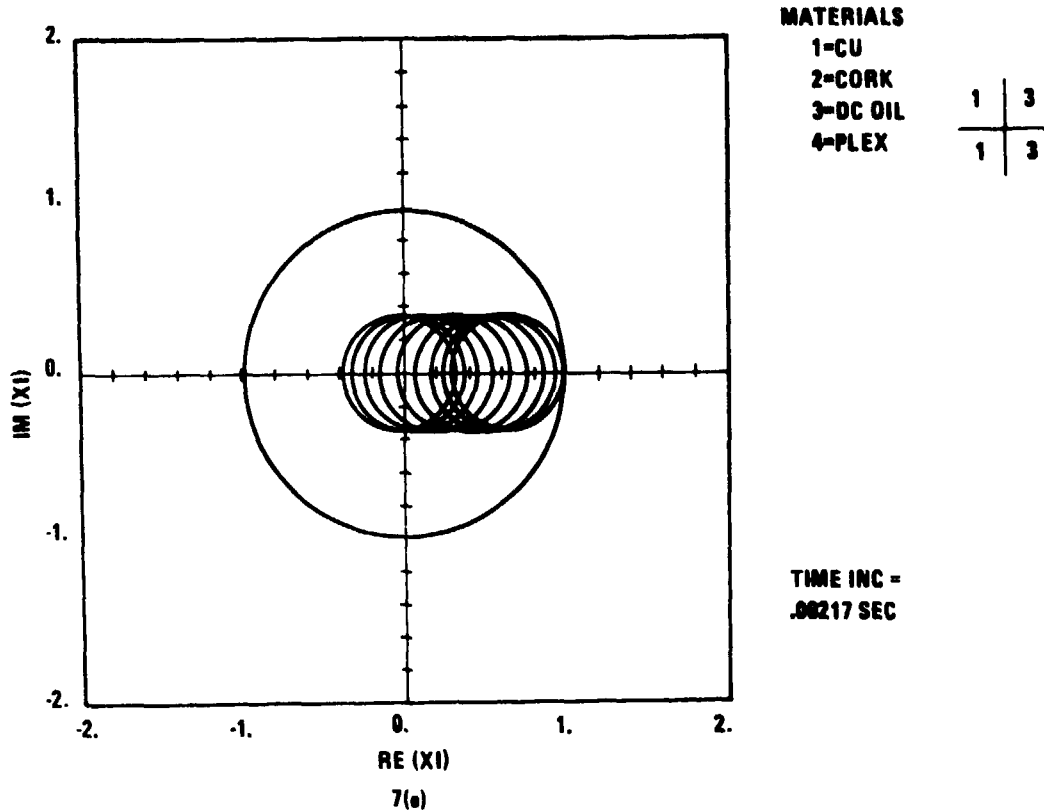


Figure 7. (Concluded).

### SUMMARY

A computer graphics technique was derived that was useful for determining the maximum time step in an explicit finite difference equation describing transient conduction in a composite medium. An example was used to illustrate the method. For the example, two codes were programmed. The first was the explicit, conditionally stable thermal code based on equation (2). The second, or stability code, was a computer graphics program for determining the optimum time increment which was an input for the thermal code. Values of  $\Delta t$  for which the stability code indicated stability ( $\Delta t = 0.002$  sec) and instability ( $\Delta t = 0.00225$  sec) were input to the thermal code. The results from the thermal code were consistent with the predictions from the stability code.

For the example problem, an accurate stability prediction could be achieved by applying inequality (5) to the dominant material, which in this case was copper. This result may or may not be a universal one.

The stability of the conditionally stable explicit finite difference equation (2) was controlled by the dominant material in the example problem. For the other three materials, a much larger maximum time step would be calculated from (5). Two possibilities exist for overcoming this problem. One is to use a larger  $\Delta x$  and  $\Delta y$  in the dominant material. For the example, this procedure was not feasible because sufficient resolution could not be achieved. The second possibility is to use a different time step in the different materials. This possibility means that, while the solution is updated every 0.002 sec in the copper of the example, it is only updated every 0.2 sec in the other materials. This variable time method should be explored.

## APPENDIX

This program listing is written in Hewlett-Packard (HP) Fortran 4X, which is described in Reference 3. Except for the multiple statement lines in which statements are separated by the dollar sign (\$), the program uses standard Fortran IV. The graphics subroutines are part of a special package called Graphics 1000, which is described in Reference 4.

```

0001 FTN4.L
0002 PROGRAM TSTAB
0003 C ****
0004 C ***
0005 C *** PROGRAMMER: WARREN CAMPBELL/3-1886
0006 C ***
0007 C *** PROGRAM PURPOSE: TO CALCULATE STABILITY FOR FINITE DIFFERENCE
0008 C *** SCHEME FOR SOLUTION OF TRANSIENT CONDUCTION EQUATION IN
0009 C *** COMPOSITE MEDIA WHOSE INTERFACES ARE VERTICAL OR HORIZONTAL.
0010 C ***
0011 C ****
0012 C ***
0013 C *** DEFINITION OF VARIABLES:
0014 C ***
0015 C *** NOTE: ALL UNITS ARE CGS
0016 C ***
0017 C *** RHO CU = DENSITY OF COPPER
0018 C *** RHO CK = DENSITY OF CORK
0019 C *** RHO DC = DENSITY OF DOW CORNING 200 SERIES OIL
0020 C *** RHO PL = DENSITY OF PLEXIGLASS
0021 C *** CCU = SPECIFIC HEAT OF COPPER
0022 C *** CCK = SPECIFIC HEAT OF CORK
0023 C *** CDC = SPECIFIC HEAT OF DOW CORNING 200 SERIES OIL
0024 C *** CPL = SPECIFIC HEAT OF PLEXIGLASS
0025 C *** KCU = THERMAL CONDUCTIVITY OF COPPER
0026 C *** KCK = THERMAL CONDUCTIVITY OF CORK
0027 C *** KDC = THERMAL CONDUCTIVITY OF DOW CORNING 200 SERIES OIL
0028 C *** KPL = THERMAL CONDUCTIVITY OF PLEXIGLASS
0029 C *** RHO1,RHO2,RHO3,RHO4 = DENSITY IN 1ST,2ND,3RD,4TH QUADRANT
0030 C *** C1,C2,C3,C4 = SPECIFIC HEAT IN " " " " "
0031 C *** K1,K2,K3,K4 = THERMAL CONDUCTIVITY IN 1ST...ETC QUADRANT
0032 C *** DELX = X INCREMENT IN FINITE DIFFERENCE EQUATIONS
0033 C *** DELY = Y " " " " "
0034 C *** DELT = TIME INCREMENT
0035 C *** A = DEFINED PARAMETER
0036 C *** XI = COMPLEX AMPLIFICATION FACTOR
0037 C *** X = REAL PART OF THE AMPLIFICATION FACTOR
0038 C *** Y = IMAGINARY PART OF AMPLIFICATION FACTOR
0039 C ****
0040 C ****
0041 DIMENSION IPLTR(192),C(4),RHO(4),IM(4)
0042 REAL K(4),KCU,KCK,KDC,KPL
0043 PI=3.14159
0044 DELX=0.1
0045 DELY=0.1
0046 WRITE(1,999)
0047 999 FORMAT(" ENTER TIME INCREMENT B_")
0048 READ(1,*) DELT
0049 C *** COPPER PARAMETERS ***
0050 KCU=0.9512
0051 RHO CU=8.96
0052 CCU=0.092
0053 C *** CORK PARAMETERS ***
0054 KCK=0.000103
0055 RHO CK=0.1602
0056 CCK=0.04
0057 C *** DL 200 OIL PARAMETERS ***
0058 KDC=0.00037

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0059      RHODC=0.971
0060      CDC=0.379
0061 C *** ENTER PLEXIGLASS PARAMETERS ***
0062      KPL=4.475E-4
0063      RHOPL=1.19
0064      CPL=0.35
0065 C *** ENTER MATERIAL FOR EACH OF THE FOUR QUADRANTS ***
0066      WRITE(1,1000)
0067 1000  FORMAT(" CU=1, CORK=2, DC OIL=3, PLEX=4, QUAD 1 MAT=#_")
0068      READ(1,*) IM(1)
0069      WRITE(1,1100)
0070 1100  FORMAT(" QUAD 2 MAT=#_")
0071      READ(1,*) IM(2)
0072      WRITE(1,1200)
0073 1200  FORMAT(" QUAD 3 MAT=#_")
0074      READ(1,*) IM(3)
0075      WRITE(1,1300)
0076 1300  FORMAT(" QUAD 4 MAT=#_")
0077      READ(1,*) IM(4)
0078      DO 130 I=1,4
0079      IF(IM(I).NE.1) GO TO 100
0080      K(I)=KCU
0081      RHO(I)=RHOCU
0082      C(I)=CCU
0083      GO TO 130
0084 100   IF(IM(I).NE.2) GO TO 110
0085      K(I)=KCK
0086      RHO(I)=RHOCK
0087      C(I)=CCK
0088      GO TO 130
0089 110   IF(IM(I).NE.3) GO TO 120
0090      K(I)=KDC
0091      RHO(I)=RHODC
0092      C(I)=CDC
0093      GO TO 130
0094 120   K(I)=KPL
0095      RHO(I)=RHOPL
0096      C(I)=CPL
0097 130   CONTINUE
0098      A=4.*DELTA/(DELTA*DELY*(C(1)*RHO(1)+C(2)*RHO(2)+C(3)*RHO(3)+C(4)*
0099      1RHO(4)))
0100 C *** SET UP PLOTTER ***
0101      WRITE(1,1111)
0102 1111  FORMAT(" ENTER ID NUMBER #_")
0103      READ(1,*) ID
0104      WRITE(1,1112)
0105 1112  FORMAT(" ENTER LU NUMBER #_")
0106      READ(1,*) LU
0107      CALL PLOTTR(IPLTR,ID,1,LU)
0108      CALL VIEWP(IPLTR,15.,85.,15.,85.)
0109      CALL WINDW(IPLTR,-2.,2.,-2.,2.)
0110      IPEN=1
0111      CALL PEN(IPLTR,IPEN)
0112      CALL LAXES(IPLTR,-2.,0.,2.,0.,0.,5.,5.,1.)
0113      CALL FRAME(IPLTR)
0114      CALL MOVE(IPLTR,1.,0.)
0115      DO 200 I=1,101
0116      ARG=(I-1)*2.*PI/100.
0117      X=COS(ARG)
0118      Y=SIN(ARG)

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0119      CALL DRAW(IPLTR,X,Y)
0120  200  CONTINUE
0121      CALL PENUP(IPLTR)
0122      DO 210 I=1,21
0123      CALL PEN(IPLTR,IPEN)
0124      G=(I-1.)*2.*PI/20.
0125      DO 220 J=1,51
0126      B=(J-1)*2.*PI/50.
0127      X=1.+A*(0.5*(K(1)+K(2)+K(3)+K(4))*(COS(B)+COS(G)-2.))
0128      Y=A*(K(1)+K(4)-K(2)-K(3))*0.5*SIN(B)+(K(1)+K(2)-K(3)-K(4))*0.5
0129      1*SIN(G)
0130      CALL DRAW(IPLTR,X,Y)
0131  220  CONTINUE
0132      CALL PENUP(IPLTR)
0133      IPEN=IPEN+1
0134      IF(IPEN.LE.4) GO TO 210
0135      IPEN=1
0136  210  CONTINUE
0137      CALL VIEWP(IPLTR,0.,152.,0.,100.)
0138      CALL WINDOW(IPLTR,0.,152.,0.,100.)
0139      IPEN=1
0140      CALL PEN(IPLTR,IPEN)
0141      CALL MOVE(IPLTR,50.,10.)
0142      CALL CPLOT(IPLTR,-3.,0.)
0143      CALL LABEL(IPLTR)
0144      WRITE(LU,4000)
0145  4000  FORMAT("RE(XI)")
0146      CALL MOVE(IPLTR,10.,50.)
0147      CALL CPLOT(IPLTR,0.,-3.)
0148      CALL LDIR(IPLTR,1.57)
0149      CALL LABEL(IPLTR)
0150      WRITE(LU,4100)
0151  4100  FORMAT("IM(XI)")
0152      CALL LDIR(IPLTR,0.)
0153      CALL MOVE(IPLTR,50.,90.)
0154      CALL CPLOT(IPLTR,-13.,0.)
0155      CALL LABEL(IPLTR)
0156      WRITE(LU,4200)
0157  4200  FORMAT("AMPLIFICATION FACTOR PLOT")
0158      CALL MOVE(IPLTR,90.,85.)
0159      CALL LABEL(IPLTR)
0160      WRITE(LU,4300)
0161  4300  FORMAT("MATERIALS")
0162      WRITE(LU,4400)
0163  4400  FORMAT("  1=CU")
0164      WRITE(LU,4500)
0165  4500  FORMAT("  2=QORR")
0166      WRITE(LU,4600)
0167  4600  FORMAT("  3=DC OIL")
0168      WRITE(LU,4700)
0169  4700  FORMAT("  4=PLEX")
0170      CALL MOVE(IPLTR,110.,75.)
0171      CALL DRAW(IPLTR,120.,75.)
0172      CALL MOVE(IPLTR,115.,70.)
0173      CALL DRAW(IPLTR,115.,80.)
0174      CALL MOVE(IPLTR,117.,77.)
0175      CALL LABEL(IPLTR)
0176      WRITE(LU,4800) IM(1)
0177  4800  FORMAT(I1)
0178      CALL MOVE(IPLTR,112.,77.)

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0179      CALL LABEL(IPLTR)
0180      WRITE(LU,4800) IM(2)
0181      CALL MOVE(IPLTR,112.,72.)
0182      CALL LABEL(IPLTR)
0183      WRITE(LU,4800) IM(3)
0184      CALL MOVE(IPLTR,117.,72.)
0185      CALL LABEL(IPLTR)
0186      WRITE(LU,4800) IM(4)
0187      CALL MOVE(IPLTR,90.,30.)
0188      CALL LABEL(IPLTR)
0189      WRITE(LU,4900) DELT
0190 4900  FORMAT("TIME INC = "F8.5" SEC")
0191      CALL PEN(IPLTR,0)
0192      STOP
0193      END
0194      END*
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## REFERENCES

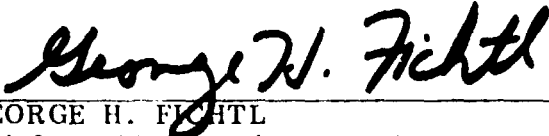
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## APPROVAL

# NUMERICAL STABILITY OF AN EXPLICIT FINITE DIFFERENCE SCHEME FOR THE SOLUTION OF TRANSIENT CONDUCTION IN COMPOSITE MEDIA

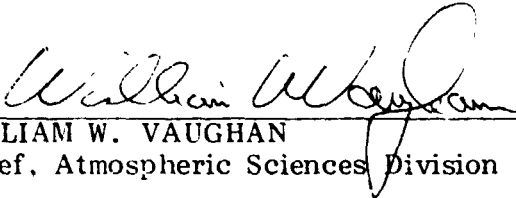
By Warren Campbell

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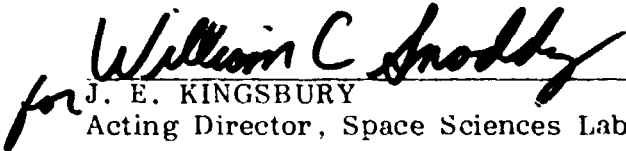
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