FLAP-LAG-TORSIONAL DYNAMICS OR EXTENSIONAL AND INEXTENSIONAL ROTOR BLADES IN HOVER AND IN FORWARD FLIGHT

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PRINCIPAL INVESTIGATOR: M.R.M. CRESPO DA SILVA
PROFESSOR

NASA TECHNICAL OFFICER FOR THIS GRANT: DR. DEWEY H. HODGES
AEROMECHANICS LABORATORY
MAIL STOP 215-1
AMES RESEARCH CENTER
MOFFETT FIELD, CALIFORNIA
94035

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Principal Investigator: M.R.M. Crespo da Silva
Professor

NASA Technical Officer for This Grant: Dr. Dewey H. Hodges
Aeromechanics Laboratory
Mail Stop 215-1
Ames Research Center
Moffett Field, California
94035

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DEPARTMENT OF AEROSPACE ENGINEERING
AND APPLIED MECHANICS
UNIVERSITY OF CINCINNATI
CINCINNATI, OHIO 45221

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The differential equations describing the flap-lag-torsional motion of a flexible rotor blade including third-order non-linearities were derived in [1] for both cases of hover and of forward flight [equations (5.2a-d)]. Making use of the two boundary conditions $G_u(x = 1, \tau = \tau) = 0$ and $u(x = 0, \tau = \tau) = 0$, those equations were reduced to a set of three integro partial differential equations written in terms of the flexural deflections $v(x,\tau)$, $w(x,\tau)$ and the torsional variable $\theta(x,\tau)$. These latter equations are of the form (equations (5.10) to (5.12) in [1])

$$G'_\alpha = h_\alpha \quad (\alpha = v,w) \tag{1}$$

$$A_{\theta_x} = Q_{\theta_x} \tag{2}$$

For a blade with the end x = 1 free, $G_\alpha(x = 1, \tau = \tau) = 0$. The expressions for G_α, h_α $(\alpha = v,w)$, A_{θ_x} and the aerodynamic term Q_{θ_x} are given in [1].

Since the publication of [1] the case of hover is being addressed. Because of two of the boundary conditions for equations (1) are $G_\alpha(x = 1, \tau = \tau) = 0$ for $\alpha = v$ and $\alpha = w$, it is convenient to first integrate equations (1) from x = 1 to x = x to obtain

$$G_\alpha = \int_1^x h_\alpha dx \qquad (\alpha = v,w) \tag{3}$$

It should be noted that the expressions for h_α in equations (3) contain the generalized aerodynamic forces Q_v and Q_w. Explicit forms for the third order expressions for Q_v and Q_w in terms of the deflections v and w, and their derivatives, were developed by making use of the implicit expressions given in Chapter VI of [1].

An approximate solution to equations (2) and (3) was written in terms of a set of orthogonal functions $f_j(x)$ and $g_j(x)$ ($j = 1,2,\ldots$) as

$$v(x,\tau) = \sum_{j=1}^{n} f_j(x) v_{tj}(\tau) \; ; \quad w(x,\tau) = \sum_{j=1}^{n} f_j(x) w_{tj}(\tau) \tag{4}$$

$$\theta_x(x,\tau) = \sum_{j=1}^{n} g_j(x) \theta_{tj}(\tau) \tag{5}$$

and equations (2) and (3) were then reduced to a set of ordinary differential equations for v_{tj}, w_{tj} and θ_{tj} ($j = 1,2,\ldots,n$). For this, we found it convenient in terms of obtaining somewhat simpler "Galerkin integrals" and also in terms of comparing the present work with that of other authors such as [2], to premultiply equations (3) by $f_i'(x)$ ($i = 1,2,\ldots,n$) to obtain

$$\int_0^1 f_i'(x) G_v \, dx = \int_0^1 f_i'(x) \int_1^x h_v \, dx \, dx \tag{6a}$$

$$\int_0^1 f_i'(x) G_w \, dx = \int_0^1 f_i'(x) \int_1^x h_w \, dx \, dx \tag{6b}$$

As an illustration of the above process consider the right hand side of equation (6a) where, as obtained from [1],

$$h_v = \ddot{v} - v - 2\dot{w}\, s\beta - 2(c\beta) \int_0^x [v'\dot{v}' + w'\dot{w}' + \frac{2c\beta}{EA} \int_1^x \ddot{v} \, dx] dx - Q_v + O(\varepsilon^4) \tag{7}$$

Making use of equations (4) and (7), equation (6a) yields, for $i = 1,2,\ldots,n$

$$\int_0^1 f_i'(x) G_v \, dx + \sum_{j=1}^{n} [\delta_{ij}(\ddot{v}_{tj} - v_{tj} - 2\dot{w}_{tj}\, s\beta) - \frac{4c^2\beta}{EA} M_{3,ij} \ddot{v}_{tj}]$$

$$- 2(c\beta) \sum_{j=1}^{n} \sum_{k=1}^{n} F_{jki}(v_{tj}\dot{v}_{tk} + w_{tj}\dot{w}_{tk}) = - \int_0^1 F_i' \int_1^x Q_v \, dx \, dx + O(\varepsilon^4) \tag{8}$$

An approximate solution to equations (2) and (3) was written in terms of a set of orthogonal functions $f_j(x)$ and $g_j(x)$ $(j = 1,2,\ldots)$ as

$$v(x,\tau) = \sum_{j=1}^{n} f_j(x) v_{tj}(\tau) \; ; \quad w(x,\tau) = \sum_{j=1}^{n} f_j(x) w_{tj}(\tau) \tag{4}$$

$$\theta_x(x,\tau) = \sum_{j=1}^{n} g_j(x)\theta_{tj}(\tau) \tag{5}$$

and equations (2) and (3) were then reduced to a set of ordinary differential equations for v_{tj}, w_{tj} and θ_{tj} $(j = 1,2,\ldots,n)$. For this, we found it convenient in terms of obtaining somewhat simpler "Galerkin integrals" and also in terms of comparing the present work with that of other authors such as [2], to premultiply equations (3) by $f_i'(x)$ $(i = 1,2,\ldots,n)$ to obtain

$$\int_0^1 f_i'(x) G_v \, dx = \int_0^1 f_i'(x) \int_1^x h_v \, dx\, dx \tag{6a}$$

$$\int_0^1 f_i'(x) G_w \, dx = \int_0^1 f_i'(x) \int_1^x h_w \, dx\, dx \tag{6b}$$

As an illustration of the above process consider the right hand side of equation (6a) where, as obtained from [1],

$$h_v = \ddot{v} - v - 2\dot{w}\, s\beta - 2(c\beta)\int_0^x [v'\dot{v}' + w'\dot{w}' + \frac{2c\beta}{EA}\int_1^x \ddot{v}\, dx]\, dx - Q_v + O(\varepsilon^4) \tag{7}$$

Making use of equations (4) and (7), equation (6a) yields, for $i = 1,2,\ldots,n$

$$\int_0^1 f_i'(x) G_v \, dx + \sum_{j=1}^{n} [\delta_{ij}(\ddot{v}_{tj} - v_{tj} - 2\dot{w}_{tj}\, s\beta) - \frac{4c^2\beta}{EA} M_{3,ij}\ddot{v}_{tj}]$$

$$- 2(c\beta)\sum_{j=1}^{n}\sum_{k=1}^{n} F_{jki}(v_{tj}\dot{v}_{tk} + w_{tj}\dot{w}_{tk}) = -\int_0^1 F_i' \int_1^x Q_v \, dx\, dx + O(\varepsilon^4) \tag{8}$$

where

$$\zeta_{ij} = - \int_0^1 f_i' \int_1^x f_j dx = f_i(0) \int_0^1 f_j dx + \int_0^1 f_i f_j dx \tag{9a}$$

$$F_{jki} = - \int_0^1 f_i' \int_1^x \int_0^x f_j' f_k' \, dx\, dx\, dx \tag{9b}$$

$$M_{3,ij} = - \int_0^1 f_i' \int_1^x \int_0^x \int_1^x f_j \, dx\, dx\, dx\, dx \tag{9c}$$

To $O(\varepsilon^2)$ equation (8) is of the same form as equation (B3a) in [2].

A complete set of shifted Legendre polynomials, defined in the interval $0 \le x \le 1$ and normalized, for convenience, so that $\int_0^1 f_i f_j \, dx = 0$ for $i \neq j$ and 1 for $i = j$, was chosen as the set $\{f_i\}$. In addition, normalized non-rotating torsional beam modes was taken as the set $\{g_i\}$. The "Galerkin coefficients" that appear in the reduced set of ordinary differential equations for $v_{ti}(\tau)$, $w_{ti}(\tau)$ and $\theta_{ti}(\tau)$ were evaluated and stored on tape for use in the subsequent analysis of the motion. The number of different coefficients in the equations is considerably reduced when orthogonal polynomials is used instead of non-rotating flexural beam nodes. Analytical expressions for the coefficients, that were evaluated recursively, were developed. As a result, the numerical value of each coefficient obtained is essentially "exact." For those coefficients that were expressed as integrals of a product of a polynomial in x and $\sin a x$ ($a = (i - \frac{1}{2})\pi$), the analytical expressions developed contained the sum of two series, one being the product of a polynomial in x and $\sin a x$, and the other the product of another polynomial in x and $\cos a x$. Each integration process was then transformed into an algebraic manipulation of these series.

It is worth noting that a number of numerical difficulties affecting the accuracy of several of the coefficients was initially encountered when attempting to evaluate them numerically. At first the numerical evaluation was done by making use of the IBM SSP subroutine DQATR. A number of coefficients were evaluated analytically in order to ascertain the validity of the numerical results obtained. Use of DQATR for this purpose was eventually abandoned when several of the numerical computations presented accuracy problems that yielded unacceptable results. At that point we have decided to change the numerical scheme for evaluating any coefficient, say $\alpha = \int_0^1 g(x)\,dx$, as $\alpha = \gamma(1)$ where $\gamma(x) = \int_0^x g(x)\,dx$. The coefficient α exemplified here was then evaluated by numerically integrating the differential equation $\gamma'(x) = g(x)$, with $\gamma(0) = 0$, from $x = 0$ to $x = 1$. The numerical integrations were performed by using the IMSL routine DASCRU. This process yielded accurate results for a considerable number of the coefficients with which numerical problems were encountered with DQATR, and even required less computation time. However, it was eventually abandoned after we have found several coefficients determined numerically to be in error when a number of spot checks were made, and after several attempts to uniformly correct the problem by re-adjusting the integration step size failed.

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