OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

FEASIBILITY STUDY OF DEFORMATION MODES OF DEFORMING COMPOSITE MATERIALS USING VISIOPLASTICITY METHOD

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By

Surendra N. Dwivedi, Principal Investigator

Final Report
For the period May 16 - August 15, 1981

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
Research Grant NAG1-191
Walter Illg, Technical Monitor
Materials Division

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FEASIBILITY STUDY OF DEFORMATION MODES OF DEFORMING COMPOSITE MATERIALS USING VISIOPLASTICITY METHOD

By

Surendra N. Dwivedi*

INTRODUCTION

Due to the high static strength, high stiffness, and high strength-to-weight ratio of fiber-reinforced composite materials (FRCM), these materials are widely used in the aerospace industry and other commerical industries. Although sufficient work has been done in the past in evaluating different mechanical properties of composite materials, there is a dearth of information in respect to inpact resistance and an exact evaluation of the effect of impact on composite materials. Since even a small impact on composite materials tremendously reduces the life of an object made of composite materials, this study of impact on composites is very essential.

PREVIOUS WORK ON FOREIGN OBJECT DAMAGE TO COMPOSITES

Many research workers (ref. 1) have studied in detail the dynamic response of materials, including composites, to intensive loading. Other researchers have studied the mechanism of penetration of projectiles into targets (ref. 2), but this author has come across only a few research papers where the small foreign object damage on composite materials has been studied. The following paragraphs describe some of these earlier studies.

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Takeda et al. carried out studies on the impact of foreign objects on composites (ref. 3). Their studies revealed that the local damage produced is dependent on the geometry of the impactor. The delamination in laminates has been identified as sequential for all types of impactors. Takeda et al. studied delamination of composites for different ply orientations and found that it is similar in both cross-ply and angle-ply laminates with some difference in the shapes of delamination.

Avery and Porter (ref. 4) investigated the impact response of metals and composites and concluded that the composites are more resistant to crack-type impact damage than panels of aluminum, titanium, or steel. Due to limited data, quantitative comparison of the impact fracture behavior of composites and metals was not attempted.

Oplinger and Slepetz (ref. 5) conducted work on impact damage tolerance of graphite/epoxy sandwich panels. They observed that graphite sandwich panels exhibit marked susceptibility to foreign object impact damage. The surface damage included fiber fractures at nominal impact energy levels as low as 1.4 to 2.7 J (1-2 lbf) when indented by a steel ball of 5.1-cm (2-in.) diameter. Surface penetration occurred at energy levels above 9.5 J (7 lbf). The impact and static indentation tests also demonstrated the low energy absorption capability of the graphite sandwich panels compared to S-glass panels. Oplinger and Slepetz felt that the falling weight impact test serves reasonably well as a qualitative indicator of foreign object damage tolerance. However, the degree of damage depends on the radius of curvature of the impactor object, as well as on the impact energy level. Preston and

Cook (ref. 6) analyzed the impact response of graphite/epoxy laminates. According to them, an analysis of operating parameters of a typical turbine fan blade showed that small steel projectiles are most likely to cause delamination and penetration damage to unprotected graphite/ epoxy composite fan blades.

Numerous other works on foreign object damage to composites are given in reference 7.

RESEARCH PLAN

Most of the theoretical studies on foreign object damage of composites has been based on many assumptions in order to simplify the problem solution. In this study the project director has used the visioplasticity experimental method to study the deformation modes and transient impact distribution of deforming composite materials. This method was first introduced by Thomsen and Lapslley (ref. 8) in 1952. The author modified this method so that it can be used for dynamic problems. In this method the material flow field is determined experimentally either by placing a grid pattern on the meridian plane of a cylinder, as in the case of axisymmetric extrusion, or on a plane at right angles to the material movement, as in the case of plane strain deformation. The grid line patterns are photographed at each increment of deformation. The movement of grid points can be determined from consecutive photographs of the grid patterns, and hence the velocity field throughout the deformation zone can be found. The strain rates, effective strain rate, and total effective strain can thus be determined throughout the body. From strain field, the stress field may then be calculated. Details of the basic equations used in the visioplasticity method are given in the next section.

BASIC EQUATIONS

Using the usual notations for Young's modulus in the direction of fiber and perpendicular to the direction of fiber as E_L and E_T, shear modulus in LT and TT direction as G_{LT} and G_{TT}, and Poisson's ratios in LT and TT directions as ν_{LT} and μ_{LT}, the three-dimensional equations for composite materials can be modified into two-dimensional plane strain equations as follows:

$$
\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \frac{\gamma_{LT}}{2} \\ \varepsilon_z \\ \frac{\gamma_{Lz}}{2} \\ \frac{\gamma_{Tz}}{2} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{1}{\varepsilon_L} & \frac{\nu_{LT}}{E_L} & 0 & 0 & 0 & 0 \\
\frac{-\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2G_{LT}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{E_T} & 0 & 0 \\
0 & 0 & 0 & & \frac{1}{2G_{LT}} & 0 \\
0 & 0 & 0 & & 0 & \frac{1}{2G_{TT}}
\end{bmatrix}}_{S}
\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \\ \sigma_z \\ \sigma_{Lz} \\ \tau_{TT} \end{bmatrix}
\qquad (1)
$$

and

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \\ \varepsilon_z \\ \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yz}}{2} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
\cos^2\alpha & \sin^2\alpha & 2\sin\alpha\cdot\cos\alpha & 0 & 0 & 0 \\
\sin^2\alpha & \cos^2\alpha & -2\sin\alpha\cdot\cos\alpha & 0 & 0 & 0 \\
-\sin\alpha\cdot\cos\alpha & \sin\alpha\cdot\cos\alpha & \cos 2\alpha & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha \\
0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha
\end{bmatrix}}_{T}
\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \frac{\gamma_{LT}}{2} \\ \varepsilon^2{}_z \\ \frac{\gamma_{Lz}}{2} \\ \frac{\gamma_{Tz}}{2} \end{bmatrix}
\tag{2}
$$

Further,

$$
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix}
=
\begin{bmatrix} T \end{bmatrix}
\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \\ \tau_{Lz} \\ \tau_{Tz} \end{bmatrix}
\tag{3}
$$

and

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \\ \varepsilon_z \\ \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yz}}{2} \end{bmatrix}
=
\begin{bmatrix} S_{xyz} \end{bmatrix}
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_z \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix}
\tag{4}
$$

using equations (1) to (4),

$$S_{xyz} = TST^{-1} \tag{5}$$

Let us define a new matrix

$$C_{xyz} = S^{-1}_{xyz}$$

then stress can be represented in terms of strain as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xz} \\ \sigma_z \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} C_{xyz} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \\ \varepsilon_z \\ \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yz}}{2} \end{bmatrix} \tag{6}$$

But for plane strain conditions

$$\varepsilon_y = \gamma_{xy} = \gamma_{yz} = 0$$

so the above equation will reduce to

$$\begin{bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \end{bmatrix} \tag{7}$$

A computer program developed for the above computation is presented in the appendix.

PROCEDURE AND RESULTS OF DEFORMATION MODES AND STRESS DISTRIBUTIONS

Before making specimens from a composite material, the material was tested using an ultrasonic method for any possible defects (fig. 1). An impact probe was designed and fabricated (fig. 2) to suit the particular setup.

The specimens were made from composite materials. Details of the specimen are shown in figures 3 and 4. A square grid pattern was used on each specimen with the lines parallel to the x and y axes. Let IX be the number of lines parallel to the y axis and IY be the number of lines parallel to the x axis. The specimen was located on the lower platen of the apparatus with the grid pattern facing the camera.

The experiment was first performed at different loading conditions to establish a good deforming pattern. It was then decided that the same procedure would be applied for high-speed impact deformation. It was planned to record the motion of the impactor using a high-speed camera by special electronic circuitry in the case of the dynamic deformation. The impactor may be impacted at a predetermined speed and the deforming grid patterns recorded.

Suitable photographs (figs. 5,6) at constant time intervals were selected and the instantaneous coordinates of the grid points were digitized. The information recorded was used as input data for the computer program. The input data was then plotted as a check (fig. 7). The material movement was shown by plotting the instantaneous coordinates of particular grid points at different instances of time and different loading conditions.

A computer program has been developed to calculate the strains ϵ_x, ϵ_y, γ_{xy} using the digitized input data. The strain distributions for a particular load of 340 kg (750 lb) for different IX grid lines are shown in figures 7 to 13. Using these strains the stress can be determined. The computer program provided in the appendix still needs certain modifications and can be completed in the future.

APPENDIX

DETAILED INFORMATION ABOUT THE COMPOSITE MATERIAL USED

AB
BD matrix

3.981E+08	1.265E+08	0.000E+00	0.000E+00	-1.000E-07	0.000E+00
1.265E+08	3.981E+08	0.000E+00	-1.000E-07	-1.000E-07	0.000E+00
0.000E+00	0.000E+00	1.358E+08	0.000E+00	0.000E+00	1.000E-07
0.000E+00	-1.000E-07	0.000E+00	1.498E+03	5.196E+02	3.942E+00
-1.000E-07	-1.000E-07	0.000E+00	5.196E+02	1.411E+03	3.942E+00
0.000E+00	0.000E+00	1.000E-07	3.942E+00	3.942E+00	5.547E+02

AB
BD matrix inverse

2.794E-09	-8.877E-10	3.047E-37	-1.217E-19	1.799E-19	-4.138E-22
-8.877E-10	2.794E-09	1.236E-36	1.601E-19	7.614E-20	-1.679E-21
3.047E-37	1.236E-36	7.364E-09	2.531E-21	2.777E-21	-1.328E-18
-1.217E-19	1.601E-19	2.531E-21	7.655E-04	-2.819E-04	-3.437E-06
1.799E-19	7.614E-20	2.777E-21	-2.819E-04	3.125E-04	-3.771E-06
-4.138E-22	-1.679E-21	-1.328E-18	-3.437E-06	-3.771E-06	1.803E-03

Laminate engineering constants:

Extensional		Flexural	
Ex =	5.326E+10	Efx =	5.166E+10
Ey =	5.326E+10	Efy =	4.867E+10
Gxy =	2.021E+10		
NUxy =	3.177E-01		
NUyx =	3.177E-01		
ETAx,xy =	4.138E-29		
ETAy,xy =	1.679E-28		
ETAxy,x =	1.091E-28		
ETAxy,y =	4.425E-28		

Lam.Coef.Therm.Exp.	ax =	7.819E-06	Curv. 1x =	-1.370E-15
	ay =	7.819E-06	1y =	1.618E-15
	axy=	-7.364E-19	1xy=	-1.805E-15

****

MATERIAL	E1	E2	G12	NU12	a1	a2
1	1.310E+11	1.300E+10	6.400E+09	3.800E-01	-1.060E-07	2.560E-05

K	Z(K)	T(K)	Th(K)	Material
0	-3.360E-03			
1	-3.220E-03	1.400E-04	45.0	1
2	-3.080E-03	1.400E-04	-45.0	1
3	-2.940E-03	1.400E-04	0.0	1
4	-2.800E-03	1.400E-04	90.0	1
5	-2.660E-03	1.400E-04	-45.0	1
6	-2.520E-03	1.400E-04	45.0	1
7	-2.380E-03	1.400E-04	0.0	1
8	-2.240E-03	1.400E-04	90.0	1
9	-2.100E-03	1.400E-04	45.0	1
10	-1.960E-03	1.400E-04	-45.0	1
11	-1.820E-03	1.400E-04	0.0	1
12	-1.680E-03	1.400E-04	90.0	1
13	-1.540E-03	1.400E-04	-45.0	1
14	-1.400E-03	1.400E-04	45.0	1
15	-1.260E-03	1.400E-04	0.0	1
16	-1.120E-03	1.400E-04	90.0	1
17	-9.800E-04	1.400E-04	45.0	1
18	-8.400E-04	1.400E-04	-45.0	1
19	-7.000E-04	1.400E-04	0.0	1
20	-5.600E-04	1.400E-04	90.0	1
21	-4.200E-04	1.400E-04	-45.0	1
22	-2.800E-04	1.400E-04	45.0	1
23	-1.400E-04	1.400E-04	0.0	1
24	0.000E+00	1.400E-04	90.0	1
25	1.400E-04	1.400E-04	90.0	1
26	2.800E-04	1.400E-04	0.0	1
27	4.200E-04	1.400E-04	45.0	1
28	5.600E-04	1.400E-04	-45.0	1
29	7.000E-04	1.400E-04	90.0	1
30	8.400E-04	1.400E-04	0.0	1
31	9.800E-04	1.400E-04	-45.0	1
32	1.120E-03	1.400E-04	45.0	1
33	1.260E-03	1.400E-04	90.0	1
34	1.400E-03	1.400E-04	0.0	1
35	1.540E-03	1.400E-04	45.0	1
36	1.680E-03	1.400E-04	-45.0	1
37	1.820E-03	1.400E-04	90.0	1
38	1.960E-03	1.400E-04	0.0	1
39	2.100E-03	1.400E-04	-45.0	1
40	2.240E-03	1.400E-04	45.0	1
41	2.380E-03	1.400E-04	90.0	1
42	2.520E-03	1.400E-04	0.0	1
43	2.660E-03	1.400E-04	45.0	1
44	2.800E-03	1.400E-04	-45.0	1
45	2.940E-03	1.400E-04	90.0	1
46	3.080E-03	1.400E-04	0.0	1
47	3.220E-03	1.400E-04	-45.0	1
48	3.360E-03	1.400E-04	45.0	1

ORIGINAL PAGE IS
OF POOR QUALITY

PROGRAM FOR DIGITIZING THE GRID POINTS OF ENLARGED PHOTOGRAPH

```
10 DIM X[200,2]
20 SCALE 0,12,0,12
30 XAXIS 0,1,0,12
40 YAXIS 0,1,0,12
50 PRINT "SET CURSOR TO ORIGIN; PRESS S"
60 ENTER (8,*)X,Y
70 PRINT "SET CURSOR TO MAXIMUM ORDINATE; PRESS S"
80 ENTER (8,*)X,Y1
90 PRINT X,Y1
100 IF (X>0.02) OR (X<-0.02) THEN 50
110 PRINT "MAX ORD IS",Y1,"IN."
120 PRINT "SET CURSOR TO MAX ABSCISSA; PRESS S"
130 ENTER (8,*)X1,Y
140 PRINT "MAX ABSCISSA IS",X1,"IN."
150 DISP "ENTER X AND Y ORIGIN";
160 INPUT X0,Y0
170 DISP "MAX VALUE ORD";
180 INPUT Y2
190 DISP "MAX ABSCISSA";
200 INPUT X2
210 FOR J=1 TO 200
220 DISP "MOVE CURSOR TO DATA POINT, PRESS S",J;
230 ENTER (8,*)X4,Y4
240 X4=X0+X4/X1*(X2-X0)
250 Y4=Y0+Y4/Y1*(Y2-Y0)
260 X5=X4
270 WRITE (15,280)X5,Y4
280 FORMAT 10X,2E20.6
290 PLOT X5,Y4
300 PEN
310 X[J,1]=X5
320 X[J,2]=Y4
330 NEXT J
340 DISP "COORDS TO BE STORED ? PRESS 1";
350 INPUT I9
360 IF I9=0 THEN 400
370 DISP "FILE NUMBER FOR STORAGE";
380 INPUT I1
390 STORE  DATA I1,X
400 END
```

CALCULATION OF STRAINS USING DIGITIZED GRID COORDINATES

```
10 DIM X[200,2],Y[200,2]
20 DIM E[10,3]
30 LOAD  DATA 1,X
40 DISP "NUMBER OF THE FILE";
50 INPUT I9
60 LOAD  DATA I9,Y
70 REM: AVERAGE FOR THE DX AND DY S
80 D1=0
90 FOR K=1 TO 19
100 K1=10*K
110 K2=10*(K-1)
120 FOR J=1 TO 10
130 I9=K1+J
140 I1=K2+J
150 Z9=X[I9,1]-X[I1,1]
160 D1=(X[I9,1]-X[I1,1])+D1
170 NEXT J
180 NEXT K
190 D1=D1/(19*10)
210 D2=0
220 FOR K=1 TO 20
230 K1=10*(K-1)
240 FOR J=1 TO 9
250 I1=K1+J
260 I9=K1+J+1
270 Z9=X[I9,2]-X[I1,2]
280 D2=(X[I9,2]-X[I1,2])+D2
290 NEXT J
300 NEXT K
310 D2=D2/(9*20)
320 PRINT D1,D2
330 PRINT
340 PRINT
350 PRINT
360 PRINT
370 FOR K=1 TO 20
380 K1=10*(K-1)
390 L=K-INT(K/5)*5
395 IF K=1 THEN 410
400 IF L#0 THEN 440
410 SCALE -4,0,1,10
420 XAXIS 1,0.5,-4,0
430 YAXIS 0,2,1,10
440 FOR J=1 TO 10
450 I=K1+J
460 X1=X[I,1]
470 Y1=X[I,2]
480 REM X9,Y9 ARE THE COORD OFTHE DEFORMED CONIGURATION
```

```
490 X9=Y[I,1]
500 Y9=Y[I,2]
510 REM STRAINS ARE AS FOLLOWS
520 E1=(X9-X1)/D1
530 E2=(Y9-Y1)/D2
540 E3=(X9-X1)/D2+(Y9-Y1)/D1
550 E[J,1]=E1
560 E[J,2]=E2
570 E[J,3]=E3
580 WRITE (15,590)I,E1,E2,E3
590 FORMAT 10X,F5.0,2X,3E15.5
600 NEXT J
610 PRINT
620 PRINT
630 IF K=1 THEN 650
640 IF L#0 THEN 730
650 FOR J=1 TO 3
660 FOR P=1 TO 10
670 PLOT E[P,J],P
680 PEN
690 NEXT P
700 DISP " CHANGE PEN ";
710 INPUT M9
720 NEXT J
730 NEXT K
740 END
```

PROGRAM FOR CALCULATION OF STRESSES OF DEFORMING PROJECTILES

```
      PROGRAM STRESS(INPUT,OUTPUT)
      REAL NULT,NUTT
      DIMENSION TH(50),ALPHA(50),S(6,6),T(6,6),TINV(6,6),SXYZ(6,6),
     1TEMP(6,6),KARRAY(7),C(3,3),KT(7),CXYZ(6,6)
C     FEED THE INDIVIDUAL LAMINA ELASTIC PROPERTIES
C     N=NO. OF LAYERS. OTHER SYMBOLS ARE SELF-EXPLANATORY.
      READ 1,N,EL,ET,GLT,GTT,NULT,NUTT
    1 FORMAT(I5,4E10.2,2F5.2)
      PRINT 2,N,EL,ET,GLT,GTT,NULT,NUTT
    2 FORMAT(//,* NUMBER OF LAYERS=*,I2,//,*EL=*,E10.4,5X,*ET=*,E10.4,
     1//,*GLT=*,E10.4,5X,*GTT=*,E10.4,//,* NULT=*,F4.2,
     25X,*NUTT=*,F4.2)
C     FEED THE LAMINATE STACKING SEQUENCE
C     ALPHA(I) IS FIBRE ANGLE OF THE I-TH LAYER
C     TH(I) IS THICKNESS OF THE I-TH LAYER
      READ 3,(ALPHA(I),TH(I),I=1,N)
    3 FORMAT(8F10.2)
      PRINT 4,(TH(I),I=1,N)
    4 FORMAT(* LAYER THICKNESSES*,/,(8F10.2))
      PRINT 5,(ALPHA(I),I=1,N)
    5 FORMAT(* LAYER ANGLES*,/,(8F10.2))
C     CONVERT FIBER ANGLES TO RADIANS
      DO 8 I=1,N
    8 ALPHA(I)=ALPHA(I)/57.29578
C     S IS THE LAMINA FLEXIBILITY MATRIX IN PRINCIPAL DIRECTIONS. INITIALIZE
C     MATRIX S TO ZERO
      DO 14 I=1,6
      DO 14 J=1,6
   14 S(I,J)=0
C     SET NONZERO ELEMENTS OF S TO APPROPRIATE VALUES
      S(1,1)=1/EL
      S(1,2)=-NULT/EL
      S(1,4)=S(1,2)
      S(2,1)=S(1,2)
      S(2,2)=1/ET
      S(2,4)=-NUTT/ET
      S(3,3)=1/GLT/2.0
      S(4,1)=S(1,4)
      S(4,2)=S(2,4)
      S(4,4)=S(2,2)
      S(5,5)=1/GLT/2.0
      S(6,6)=1/GTT/2.0
      PRINT 16
   16 FORMAT(//,*   INDIVIDUAL LAMINAE PLANE STRAIN STIFFNESS MATRICES*)
      DO 15 I=1,N
      ALP=ALPHA(I)
      CA=COS(ALP)
      SA=SIN(ALP)
      CS=CA*CA
      SS=SA*SA
      C2=CS-SS
      S2=SIN(2*ALP)
```

```
C   SET UP TRANSFORMATION MATRIX T FOR TRANSFORMING LAMINA STIFFNESS
C   MATRIX FROM PRINCIPAL DIRECTIONS INTO LOADING COORDINATES. T IS FIRST
C   INITIALIZED TO ZERO AND THE NONZERO ELEMENTS ARE THEN SET TO
C   APPROPRIATE VALUES
      DO 11 J=1,6
      DO 11 K=1,6
   11 T(J,K)=0
      T(1,1)=CS
      T(1,2)=SS
      T(1,3)=S2
      T(2,1)=SS
      T(2,2)=CS
      T(2,3)=-S2
      T(3,1)=-S2/2
      T(3,2)=S2/2
      T(3,3)=C2
      T(4,4)=1
      T(5,5)=CA
      T(5,6)=SA
      T(6,5)=-SA
      T(6,6)=CA
      DO 7 J=1,6
      DO 7 K=1,6
    7 TINV(J,K)=T(J,K)
      KARRAY(1)=10
      KARRAY(2)=6
      KARRAY(3)=6
      KARRAY(4)=0
      KARRAY(5)=6
      KARRAY(6)=0
      KARRAY(7)=0
      DO 9 J=1,7
    9 KT(J)=KARRAY(J)
      CALL MATOPS(KARRAY,TINV,P,Q)
      CALL MATMLT(T,S,TEMP,6,6,6)
C   SXYZ IS THE LAMINA FLEXIBILITY MATRIX FOR THE LOADING COORDINATES.
C   SXYZ IS NOW COMPUTED.
      CALL MATMLT(TEMP,TINV,SXYZ,6,6,6)
      DO 10 J=1,7
   10 KARRAY(J)=KT(J)
      DO 6 J=1,6
      DO 6 K=1,6
    6 CXYZ(J,K)=SXYZ(J,K)
      CALL MATOPS(KARRAY,CXYZ,P,Q)
C   CXYZ IS THE STIFFNESS MATRIX IN LOADING COORDINATES X,Y,Z
C   WE NOW FORMULATE THE PLANE STRAIN STIFFNESS MATRIX C IN THE X-Z COORDINATES
C   BY EXTRACTING THE RELEVANT ELEMENTS OF THE BIGGER MATRIX CXYZ
      C(1,1)=CXYZ(1,1)
      C(1,2)=CXYZ(1,4)
      C(1,3)=CXYZ(1,5)/2
      C(2,1)=C(1,2)
      C(2,2)=CXYZ(4,4)
      C(2,3)=C(4,5)/2
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
      C(3,3)=C(5,5)/2
      PRINT 12,((C(J,K),K=1,3),J=1,3)
   12 FORMAT(///,(3E20.5))
   15 CONTINUE
      STOP
      END
```

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```
      SUBROUTINE MATMLT(A,B,C,I,J,K)
C  THIS SUBROUTINE MULTIPLIES MATRICES A AND B OF SIZES (I,J) AND (J,K)
C  RESPECTIVELY AND STORES THE PRODUCT IN A MATRIX C OF SIZE (I,K)
      DIMENSION A(I,J),B(J,K),C(I,K)
      DO 1`L=1,I
      DO 1 M=1,K
    1 C(L,M)=0
      DO 2 L=1,I
      DO 2 M=1,K
      DO 2 N=1,J
    2 C(L,M)=C(L,M)+A(L,N)*B(N,M)
      RETURN
      END
```

REFERENCES

1. Chou, P.C.; and Hopkins, A.K.: Dynamic Response of Materials to Intense Impulsive Loading. Air Force Materials Laboratory, Aug. 1972.

2. Bachman, M.E.; and Goldsmith: The Mechanics of Penetration of Projectiles on Targets. Int. J. Eng. Sci., Vol. 16, 1978, pp. 1-99.

3. Takeda, Nobuo; Sierakowski, R.L.; Malvern, L.E.: Studies of Impacted Glass Fiber Reinforced Composite Laminates. SAMPLE Quarterly, Jan. 1981, pp. 9-17.

4. Avery, J.G.; and Porter, T.R.: Comparison of the Ballistic Response of Metals and Composites for Military Aircraft Applications. Foreign Object Damage to Composites, ASTM STP 568, Amer. Soc. for Testing and Materials, 1975, pp. 3-29.

5. Oplinger, D.W.; and Slepetz, J.M.: Impact Damage Tolerance of Graphite/Epoxy Sandwich Panels. Foreign Object Impact Damage to Composites, ASTM STP 568, Amer. Soc. for Testing and Materials, 1975, pp. 30-48.

6. Preston, J.L.,Jr.; and Cook, T.S.: Impact Response of Graphite/Epoxy Flat Laminates Using Projectiles That Simulate Aircraft Engine Encounters. Foreign Object Impact Damange to Composites, ASTM STP 568, Amer. Soc. for Testing and Materials, 1975, pp. 49-71.

7. Foreign Object Damage to Composites. ASTM, Special Technical Publication 568. Amer. Soc. for Testing and Materials.

8. Thomson, E.G.; and Lapslley, J.T.: Experimental Stress Determination Within a Metal during Plastic Flow. Proc. Soc. Exptl. Stress Analysis, Vol. 11, No. 2, 1954.

9. Dwivedi, S.N.: Plane Strain Visioplasticity and Quasi-static Deformation Process, M.S. Thesis, Univ. of British Columbia (Canada).

10. Dwivedi, S.N.: Determination of Impact Deformation Modes and Transient Stress Distribution. Proc. of Seventh Canadian Congress of Applied Mechanics (Sherbrooke, Canada), 1979.

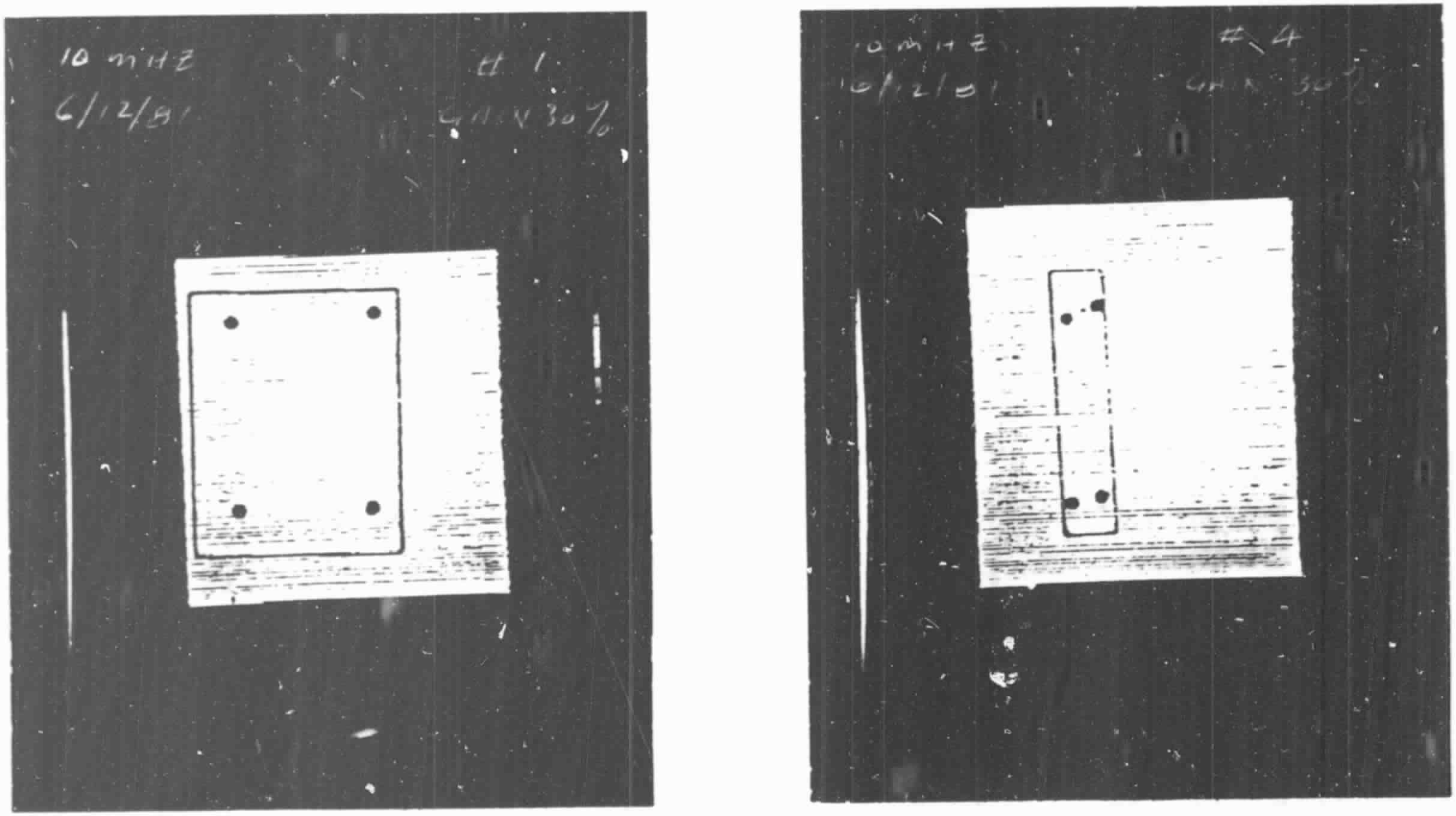

Figure 1. Ultrasonic testing of the composite materials before making specimens.

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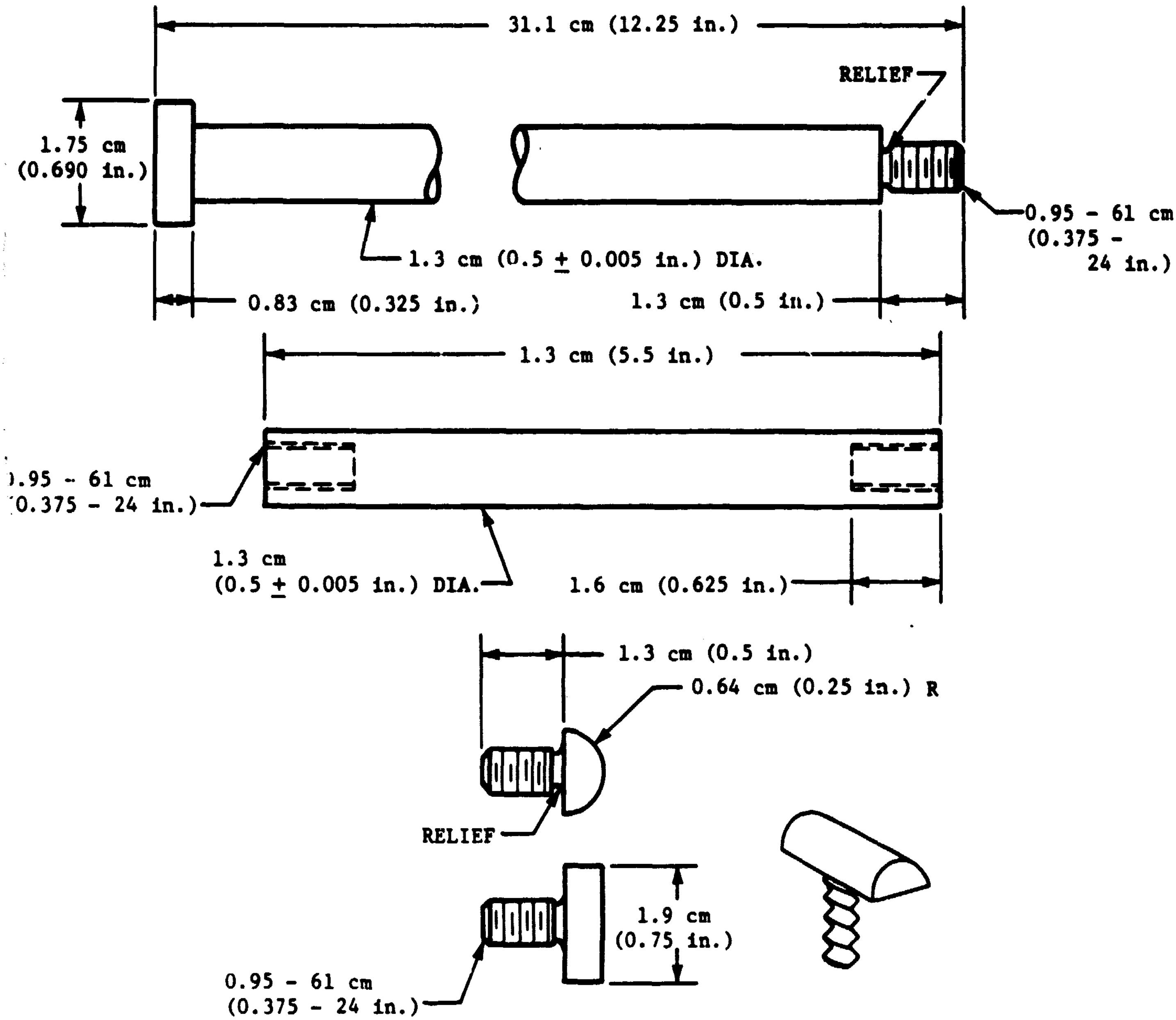


MATERIAL: STAINLESS STEEL

Figure 2. Details of impact probe used in the present investigation.

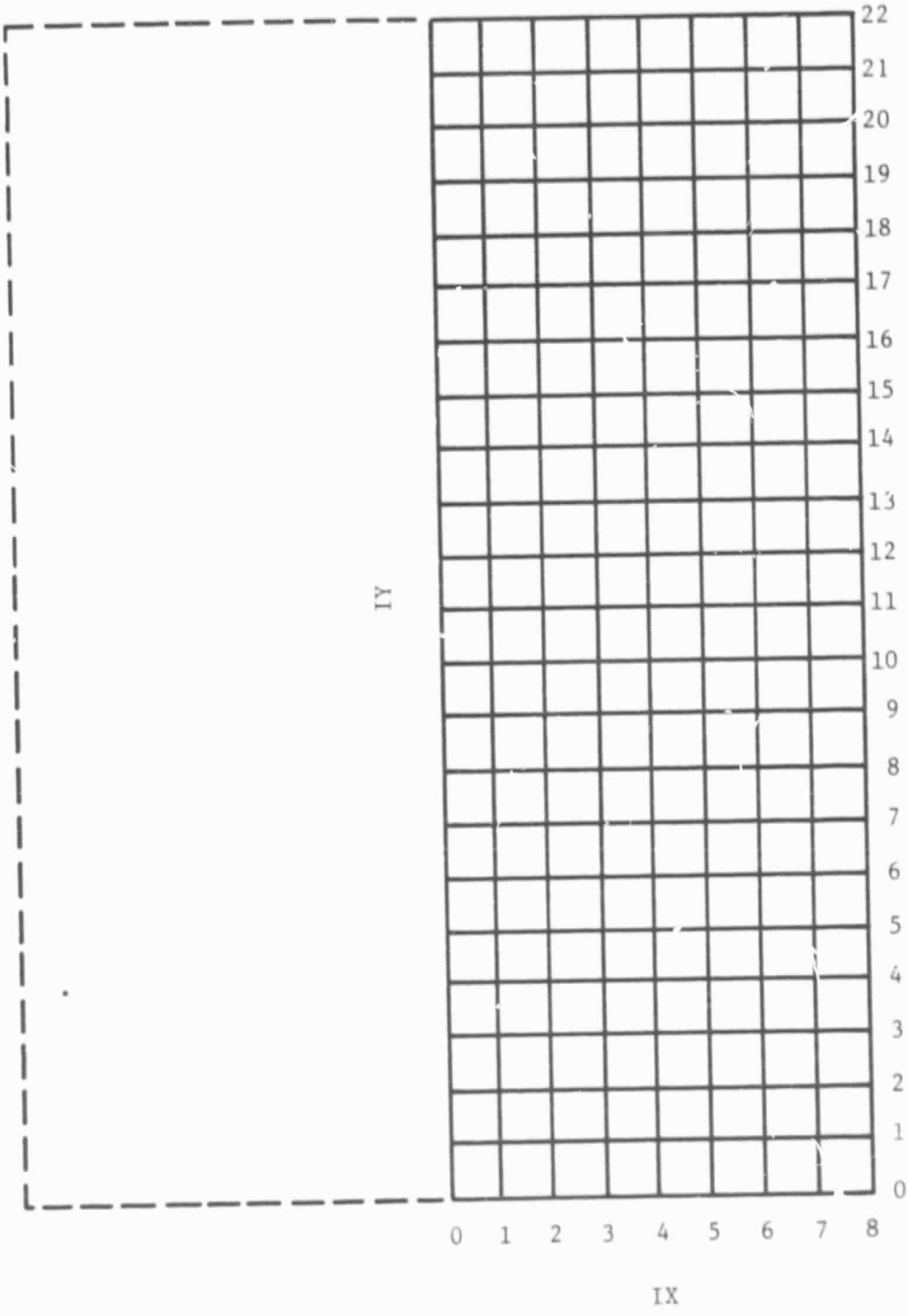
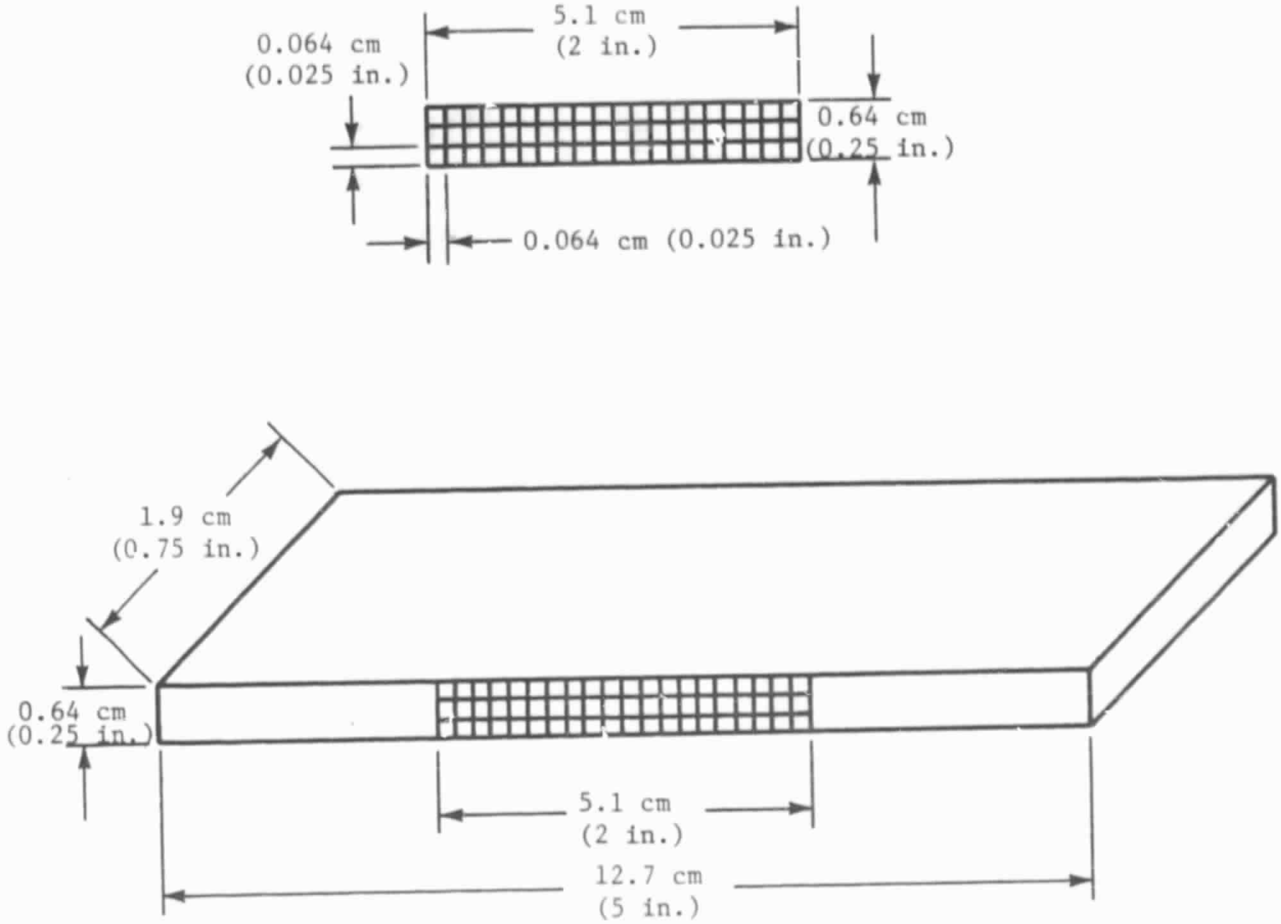


Figure 3. Grid lines on the specimen.



DRAWING NOT TO SCALE

NUMBER REQUIRED = FIVE

MATERIAL: COMPOSITE PLATE

Figure 4. Details of specimens used.

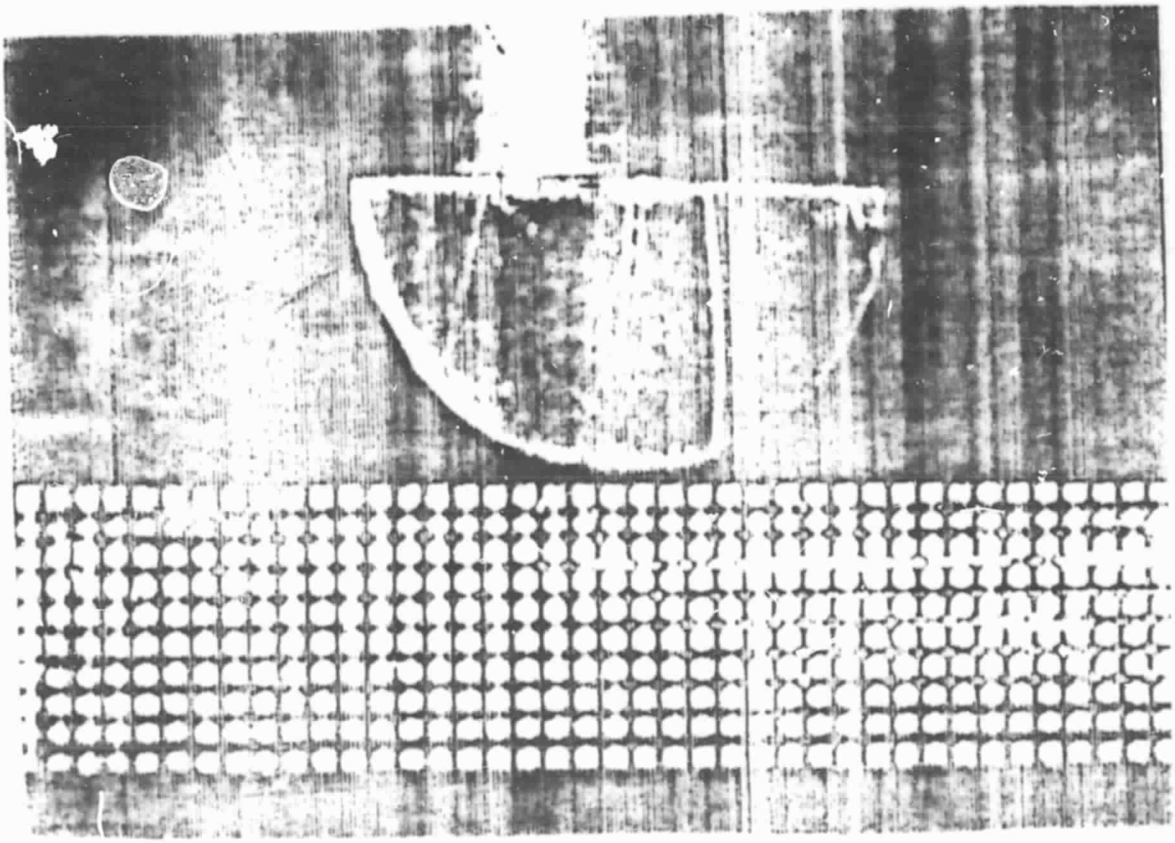


Figure 5. Enlarged photograph of undeformed specimen.

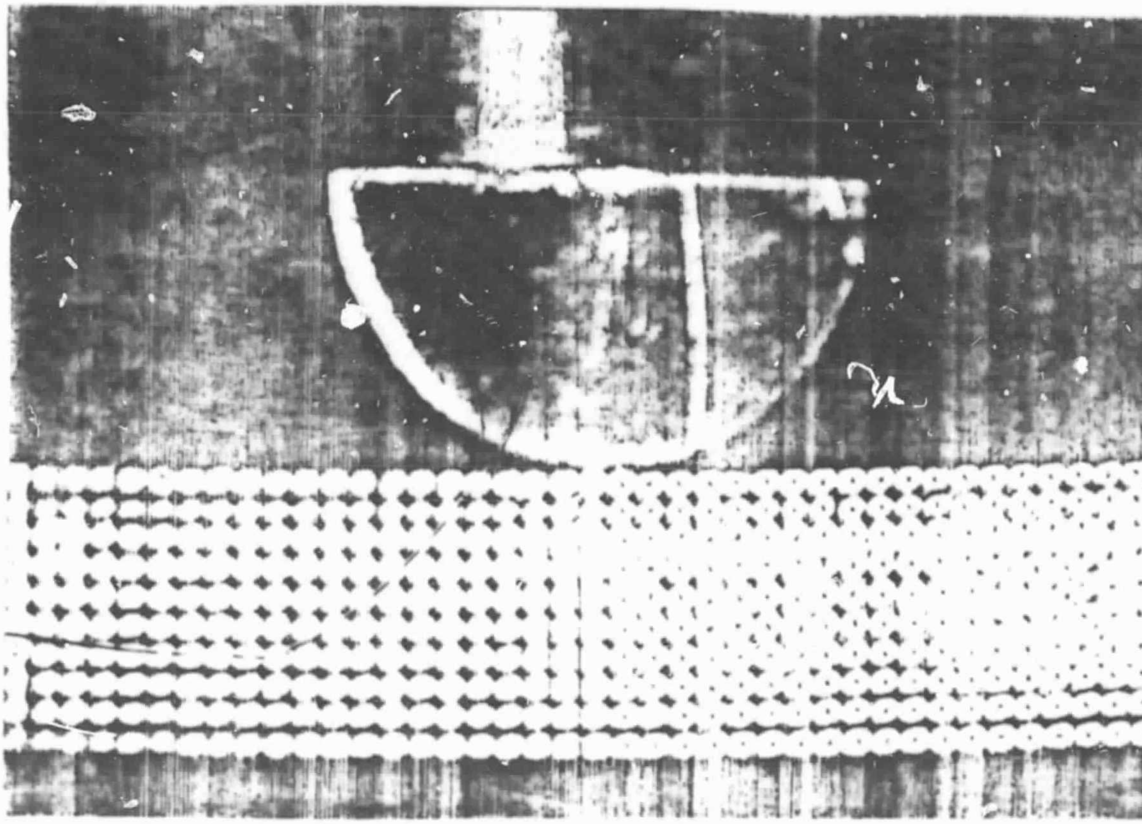


Figure 6. Enlarged photograph of deformed specimen for a deforming load of 340 kg (750 lb).

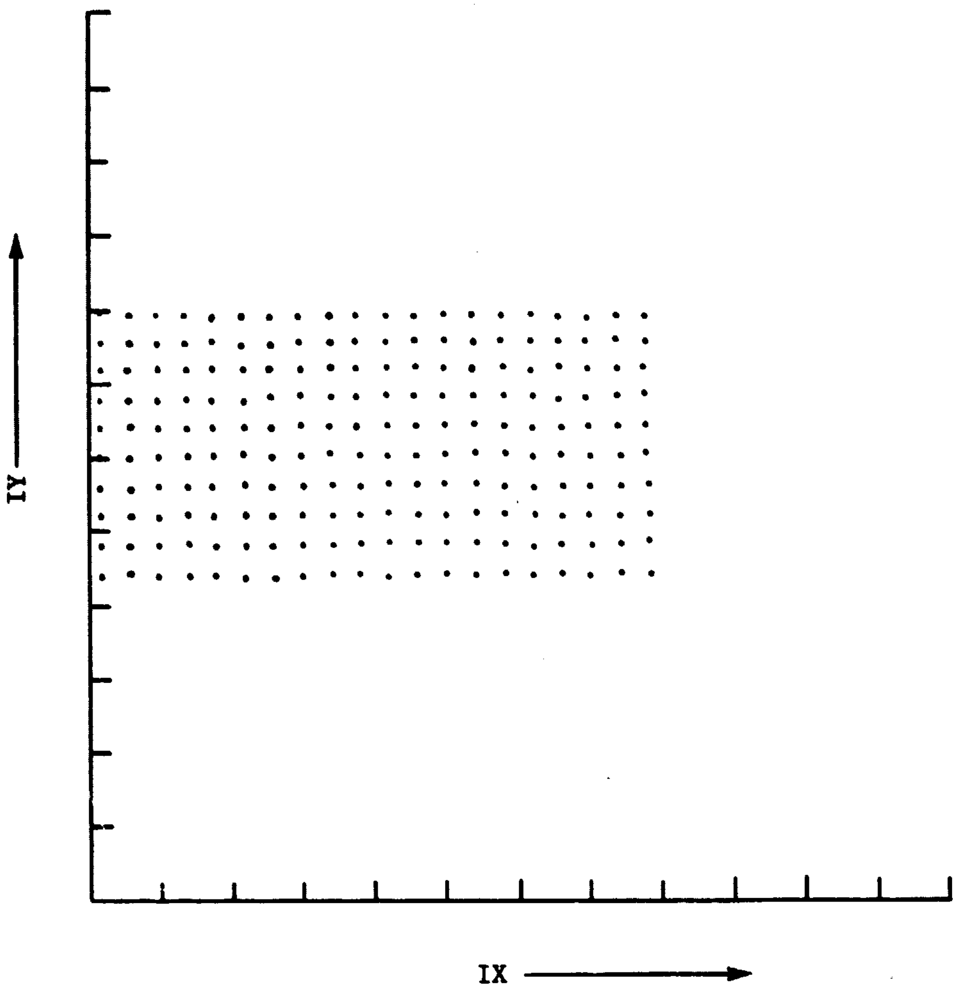


Figure 7. Digitized points of enlarged photograph of the undeformed test piece.

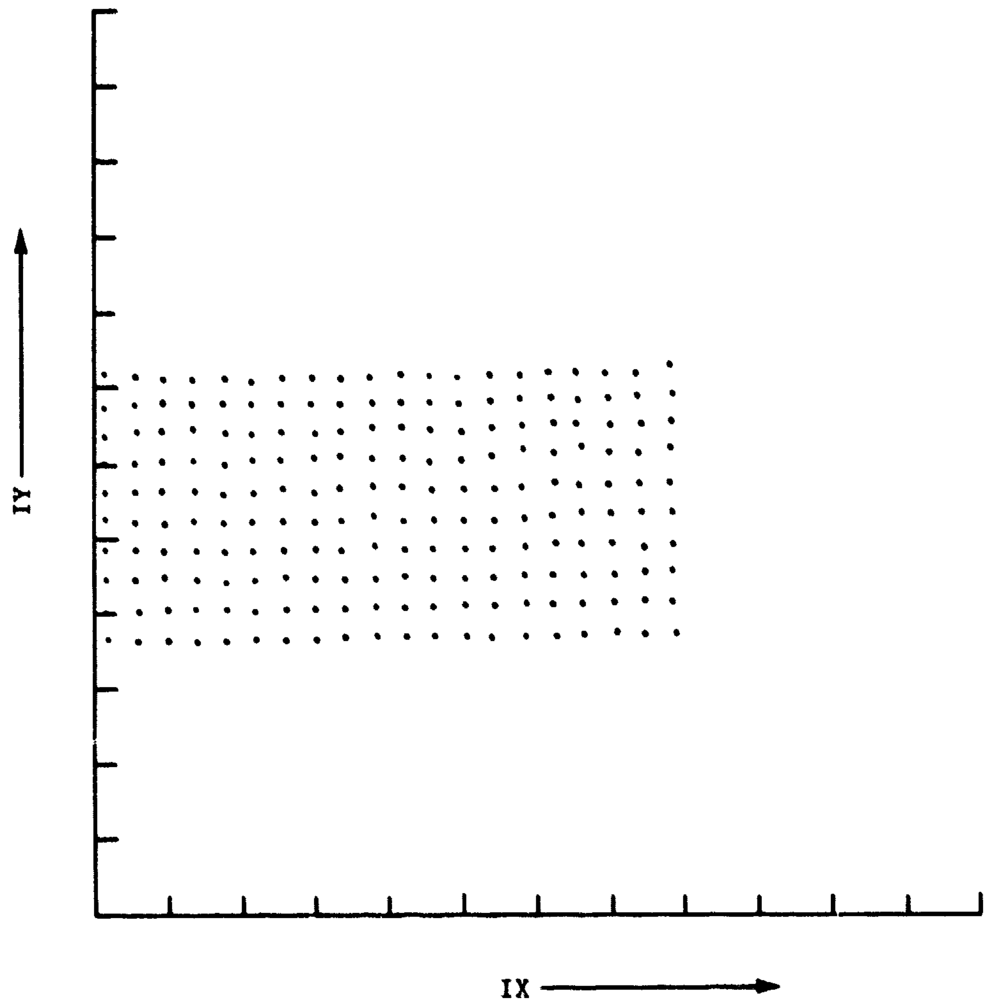


Figure 8. Digitized points of enlarged photograph of the deformed test piece under a load of 340 kg (750 lb).

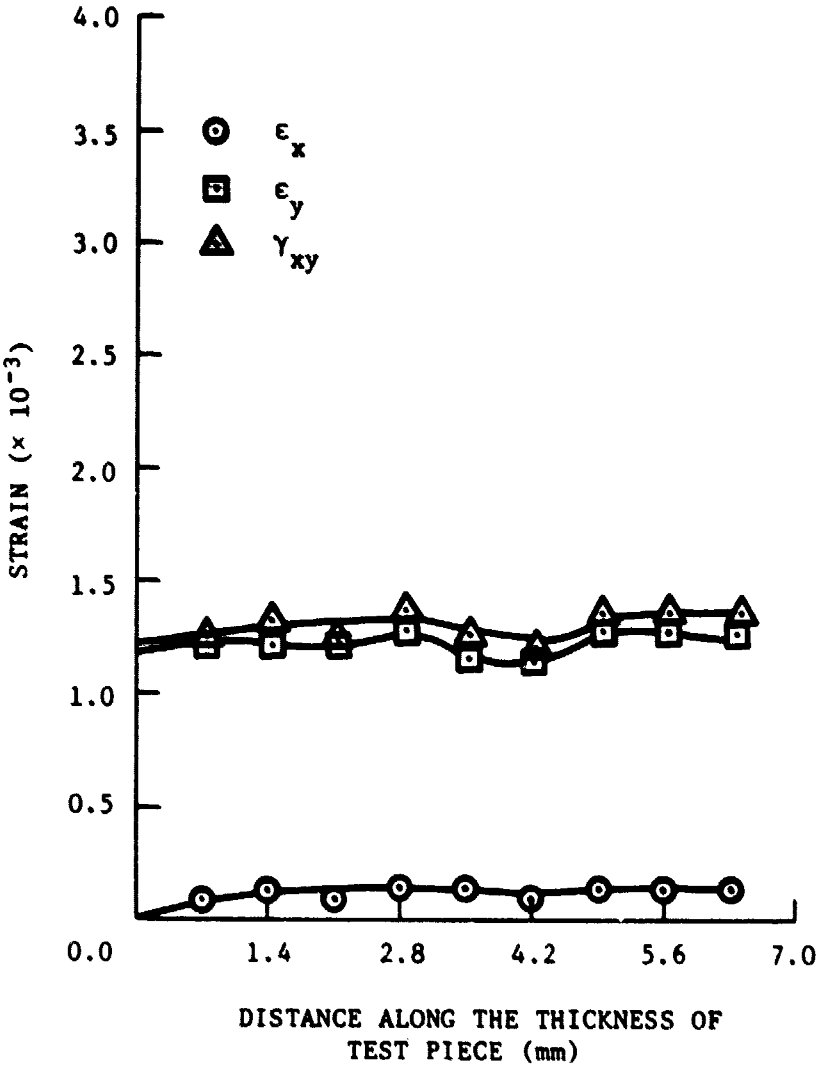


Figure 9. Variations of ϵ_x, ϵ_y, and γ_{xy} along the thickness of the test piece for IX = 2 and a load of 340 kg (750 lb).

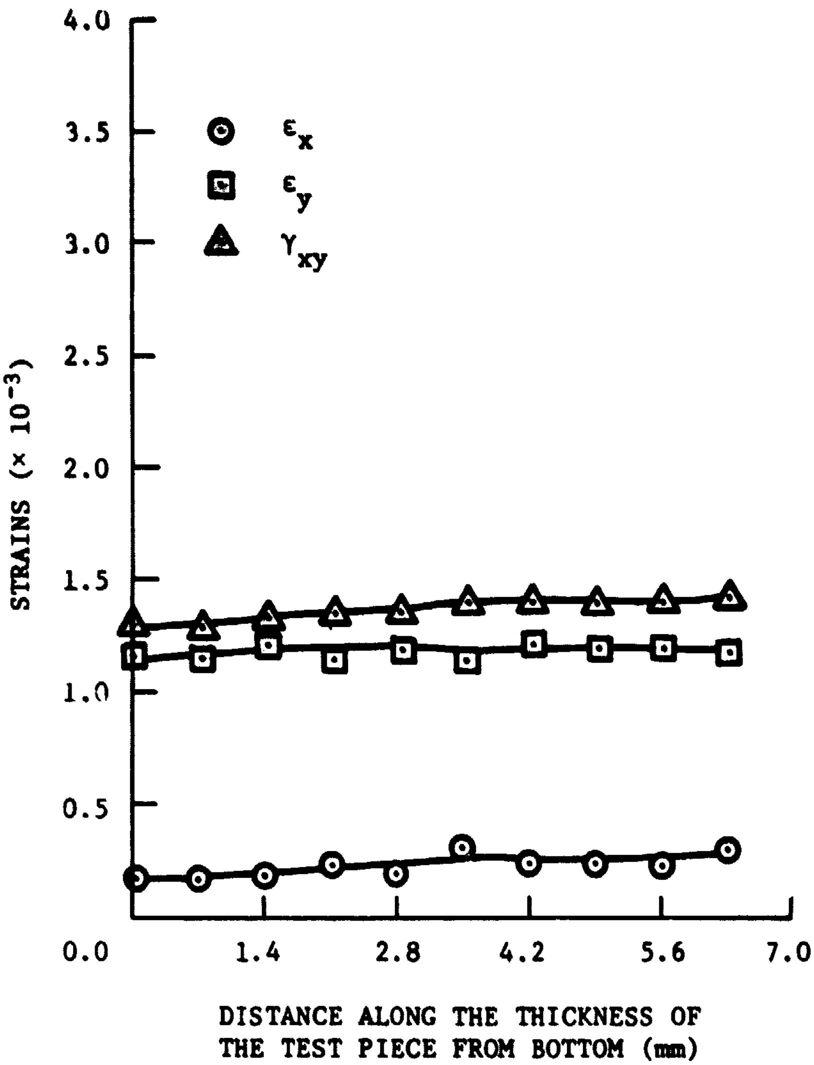


Figure 10. Variations of ϵ_x, ϵ_y, and γ_{xy} along the thickness of the test piece for IX = 4 and a load of 340 kg (750 lb).

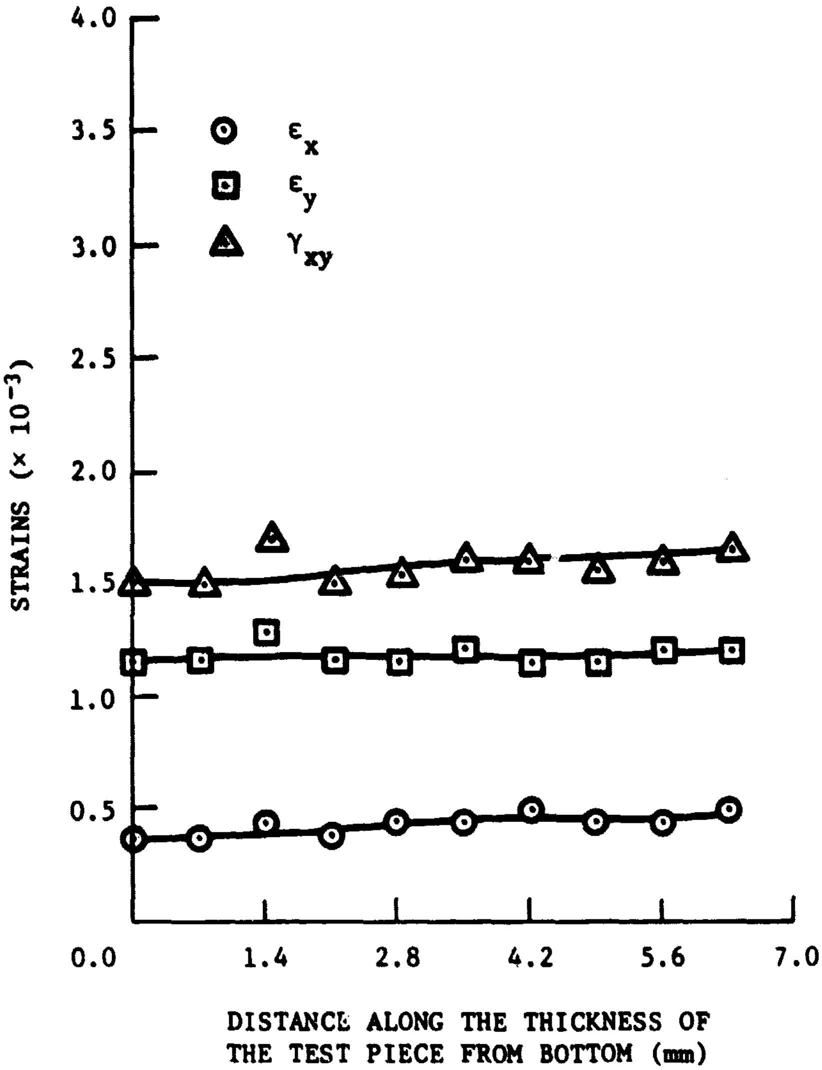


Figure 11. Variations of ϵ_x, ϵ_y, and γ_{xy} along the thickness of the test piece for IX = 6 and a load of 340 kg (750 lb).

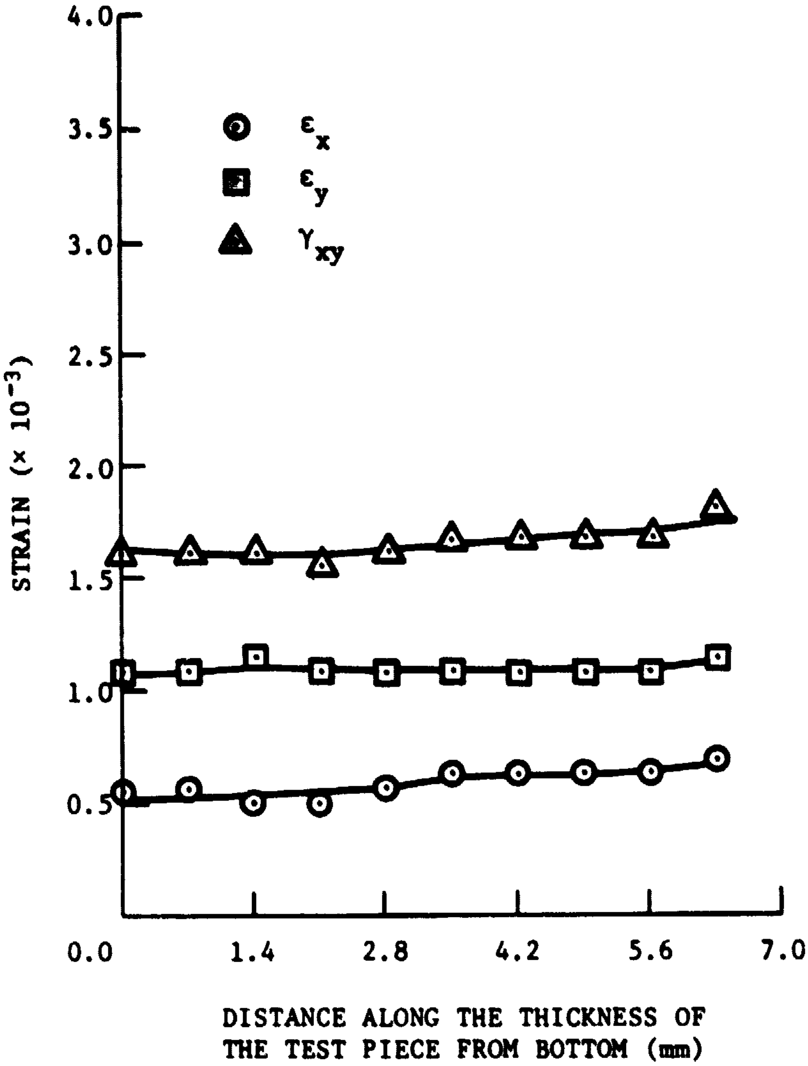


Figure 12. Variations of ε_x, ε_y, and γ_{xy} along the thickness of the test piece for IX = 8 and a load of 340 kg (750 lb).

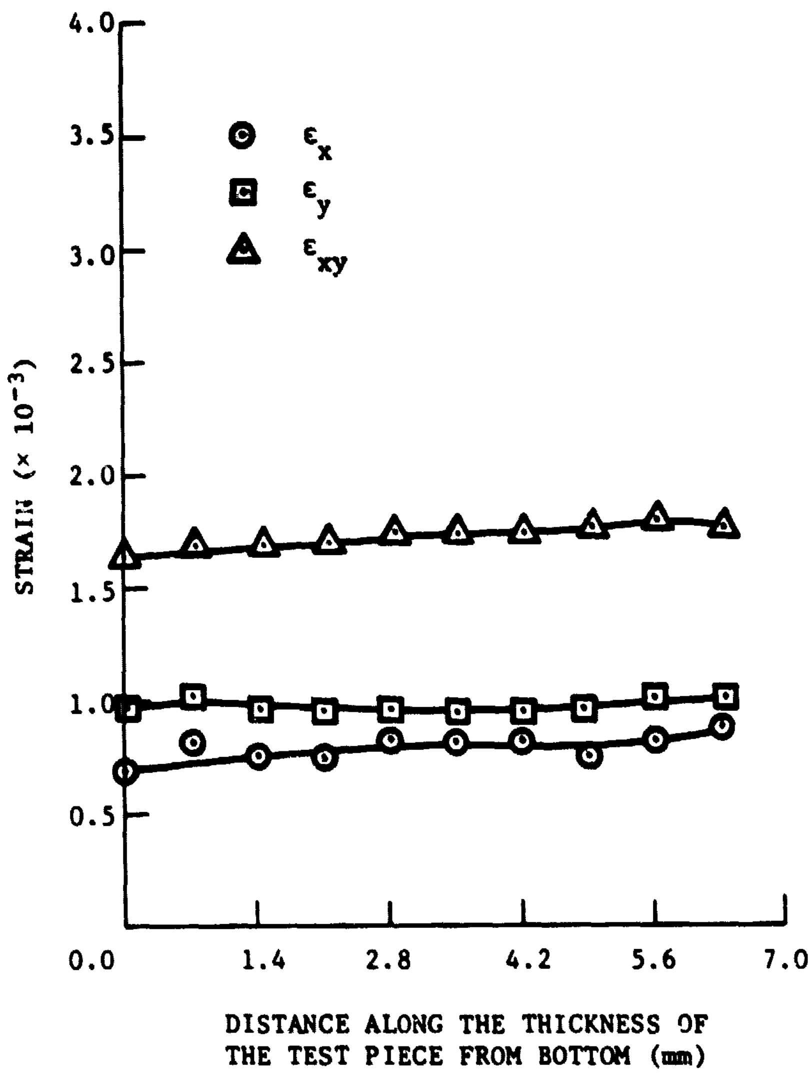


Figure 13. Variations of ε_x, ε_y, and γ_{xy} along the thickness of the test piece for IX = 10 and a load of 340 kg (750 lb).