MATRIX VECTOR MULTIPLIER

WITH TIME VARYING SINGLE DIMENSIONAL SPATIAL LIGHT MODULATORS

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ABSTRACT

Optical Matrix Vector Multipliers (OMVM) are of interest to real time adaptive array beam forming. Feasibility of high speed compact signal processing for applications such as sidelobe cancelling has yet to be demonstrated. The importance of the OMVM stems from the following:

(1)   Because of its optical parallelism it can operate at extremely high speeds, surpassing by several orders of magnitude competing electronic techniques.

(2)   The matrix-vector multiplication operation is basic to a variety of important signal processing applications, including adaptive array processing and other general control operations.

The very high speed of the OMVM strongly suggests its application to iterative processing, where in a very short time the system can cycle through perhaps 100 iterations, and assuming necessary convergence, yield a solution. This method has been suggested as a means of inverting the covariance matrix associated with noise samples from adaptive array antenna elements. A review of the jammer problem for radars indicates that solution is required in the 1 µsec to 10 µsec range. The spatial light modulator must be updated within this time frame.

We will describe here the optical architecture of an incoherent matrix vector multiplier where the matrix is complex and separable into outer product vectors.

In the late 1970's, researchers at Stanford University began investigation
of a new class of incoherent electro-optical processors for high speed matrix-
vector multipliers (ref. 1, 2). Work has continued, both at Stanford (ref. 3)
and Carnegie Mellon Institute, to the point were significant processing
capabilities have been demonstrated. It is not unreasonable to expect that
such systems could perform 10^8 matrix-vector multiplies per second with 100x100
element matrices.

One limitation of these processors as developed thus far is the relatively
fixed nature of the mask representing the matrix. In general photographic
masks have been employed, and processing where the matrix must change with some
rapidity has been impossible.

In our search for a matrix mask that can be updated rapidly, we have considered
several candidate spatial light modulators (ref. 4) (SLMs), including the
Itek PROM, and the Hughes LCLV. Neither the PROM nor the LCLV can be cycled
at rates approaching 1 MHz, and these candidates are thus unacceptable. A mosaic
of pockels cells could perhaps be made to operate in the desired fashion.
However, there are the usual problems of switching high voltages in microseconds,
and severe electronic crosstalk appears inevitable. We
have settled on a scheme whereby the mask is implemented acousto-optically.

Acousto-optic modulators possess several desirable features when used in signal
processing systems. These include reliability, availability, high response
speeds, and wide bandwidths. However, they are not easily adaptable to use as 2-D
spatial light modulators. The basic technique and system architecture for a
pair of 1-D SLMs is discussed in the following section.

ACOUSTO-OPTIC IMPLEMENTATION OF VECTOR MULTIPLICATION BY OUTER PRODUCT MATRICES

The operation to be implemented is:

$$\overline{Y} = \underline{H}\,\overline{X} \tag{1}$$

where $\overline{X}$ is a complex input column vector.

$\underline{H}$ is a complex matrix separable into outer products:

$$\underline{H} = \overline{A}\,\overline{B}^t \tag{2}$$

The multiplication of separable matrices would be possible if the OMVM were a
coherent processor. When the necessary complex to non-negative real encoding
of $\underline{H}$ is implemented for an incoherent processor, the separable form of the system
matrix is destroyed. However by using time multiplexing we can still avoid the
requirement for 2-D masks. Consider one possible method. We can define
expressions in which vectors and matrices are defined in the following form (ref 5,6).

$$\overline{X} = \overline{X}_0 + \overline{X}_1 e^{i\theta} + \overline{X}_2 e^{i2\theta} \qquad (3a)$$

$$\overline{Y} = \overline{Y}_0 + \overline{Y}_1 e^{i\theta} = \overline{Y}_2 e^{i2\theta} \qquad (3b)$$

$$\underline{H} = \underline{H}_0 + \underline{H}_1 e^{i\theta} + \underline{H}_2 e^{i2\theta} \qquad (3c)$$

where $\overline{X}_0$, $\overline{X}_1$, $\overline{X}_2$; $\overline{Y}_0$, $\overline{Y}_1$, $\overline{Y}_2$; $\underline{H}_0$, $\underline{H}_1$, $\underline{H}_2$ are all non-negative real and where $\theta = 2\pi/3$. The components $\overline{X}_i$, $\overline{Y}_i$, $\overline{H}_i$, $i = 0,1,2$ have the dimensions of $\overline{X}$, $\overline{Y}$, and $\underline{H}$, respectively.

The basic matrix vector multiplication operation can be written:

$$\begin{bmatrix} \overline{Y}_0 \\ \\ \overline{Y}_1 \\ \\ \overline{Y2} \end{bmatrix} = \begin{bmatrix} \underline{H}_0 & \underline{H}_2 & \underline{H}_1 \\ \\ \underline{H}_1 & \underline{H}_0 & \underline{H}_2 \\ \\ \underline{H}_2 & \underline{H}_1 & \underline{H}_0 \end{bmatrix} \begin{bmatrix} \overline{X}_0 \\ \\ \overline{X}_1 \\ \\ \overline{X}_2 \end{bmatrix} \qquad (4)$$

Consider the case where $\underline{H}$ is an outer product matrix (2). If we express (2) in terms of a three component decomposition, we have $\underline{H} = (\overline{A}_0 + \overline{A}_1 e^{i\theta} + \overline{A}_2 e^{i2\theta})\ (\overline{B}_0^t + \overline{B}_1^t e^{i\theta} + \overline{B}_2^t e^{i2\theta})^t$ (5)

Multiplying through and collecting terms in powers of exp $(i\theta)$ we find that

$$\underline{H}_0 = \overline{A}_0\,\overline{B}_0 + \overline{A}_1\,\overline{B}_2 + \overline{A}_2\,\overline{B}_1$$

$$\underline{H}_1 = \overline{A}_0\,\overline{B}_1 + \overline{A}_1\,\overline{B}_0 + \overline{A}_2\,\overline{B}_2 \qquad (6)$$

$$\underline{H}_2 = \overline{A}_0\,\overline{B}_2 + \overline{A}_1\,\overline{B}_1 + \overline{A}_2\,\overline{B}_0$$

To represent $\underline{H}$ by a three-component decomposition, we need nine outer products.

These nine outer products can be produced using a pair of crossed acousto-optic cells with multiple transducers. Figure 1 illustrates the case where $\overline{A}\ \overline{B}^t$ is a two-dimensional complex matrix. The desired outer products are formed at the intersection of the acoustic waves, in the following format:
Let $\underline{M}$ =

$$
\underline{M} = \begin{bmatrix} \overline{A}_0\ \overline{B}_0 & \overline{A}_0\ \overline{B}_1 & \overline{A}_0\ \overline{B}_2 \\[2em] \overline{A}_1\ \overline{B}_0 & \overline{A}_1\ \overline{B}_1 & \overline{A}_1\ \overline{B}_2 \\[2em] \overline{A}_2\ \overline{B}_0 & \overline{A}_2\ \overline{B}_1 & \overline{A}_2\ \overline{B}_2 \end{bmatrix} \tag{7}
$$

At this stage we encounter a difficulty in system architecture: although the various outer products of (3), together with input vector components $\overline{X}_0$, $\overline{X}_1$, $\overline{X}_2$, are sufficient to allow calculation of $\overline{Y}$, these outer products are not arranged so as to allow direct implementation of equation (4). The three-component form of matrix H

$$
\underline{H} = \begin{bmatrix} \underline{H}_0 & \underline{H}_2 & \underline{H}_1 \\[2em] \underline{H}_1 & \underline{H}_0 & \underline{H}_2 \\[2em] \underline{H}_2 & \underline{H}_1 & \underline{H}_0 \end{bmatrix} \tag{8}
$$

is itself not representable as an outer product. As a consequence, it appears as though system architecture will of necessity be more complicated than would be the case were $\underline{H}$ directly realized by, for example, a more general 2-D SLM.

Let us consider the problem further to gain a better understanding of its nature. We simplify notation by denoting the elements of equation (7) by $\underline{M}_{ij}$:

$$
\underline{M} = \begin{bmatrix} \underline{M}_{00} & \underline{M}_{01} & \underline{M}_{02} \\[2em] \underline{M}_{10} & \underline{M}_{11} & \underline{M}_{12} \\[2em] \underline{M}_{20} & \underline{M}_{21} & \underline{M}_{22} \end{bmatrix} \tag{9}
$$

From equations (3), (6), (7) and (9) we have

$$\overline{Y}_0 = \underline{H}_0\overline{X}_0 + \underline{H}_2 X_1 + \underline{H}_1 \overline{X}_2 \tag{10a}$$

$$= \underline{M}_{00}\overline{X}_0 + \underline{M}_{12}\overline{X}_0 + \underline{M}_{21}\overline{X}_0 +$$
$$\underline{M}_{02}\overline{X}_1 + \underline{M}_{11}\overline{X}_1 + \underline{M}_{20}\overline{X}_1 +$$
$$\underline{M}_{01}\overline{X}_2 + \underline{M}_{10}\overline{X}_2 + \underline{M}_{22}\overline{X}_2$$

$$\overline{Y}_1 = \underline{H}_1\overline{X}_0 + \underline{H}_0\overline{X}_1 + \underline{H}_2\overline{X}_2 \tag{10b}$$

$$\underline{M}_{01}\overline{X}_0 + \underline{M}_{10}\overline{X}_0 + \underline{M}_{22}\overline{X}_0 +$$
$$\underline{M}_{00}\overline{X}_1 + \underline{M}_{12}\overline{X}_1 + \underline{M}_{21}\overline{X}_1 +$$
$$\underline{M}_{02}\overline{X}_2 + \underline{M}_{11}\overline{X}_2 + \underline{M}_{20}\overline{X}_2$$

$$\overline{Y}_2 = \underline{H}_2\overline{X}_0 + \underline{H}_1\overline{X}_1 + \underline{H}_0\overline{X}_2 \tag{10c}$$

$$\underline{M}_{02}\overline{X}_0 + \underline{M}_{11}\overline{X}_0 + \underline{M}_{20}\overline{X}_0 +$$
$$\underline{M}_{01}\overline{X}_1 + \underline{M}_{10}\overline{X}_1 + \underline{M}_{22}\overline{X}_1 +$$
$$\underline{M}_{00}\overline{X}_2 + \underline{M}_{12}\overline{X}_2 + \underline{M}_{21}\overline{X}_2$$

Inspection shows that component $\overline{Y}_n$ can be expressed by

$$\overline{Y}_n = \sum_{i,j,k} \underline{M}_{ij}\overline{X}_k \tag{11}$$

where i,j,k satisfy the relationship

$$(i+j+k) \bmod 3 = n$$

Given the matrix $\underline{M}$ and input $\overline{X} = (\overline{X}_0\ \overline{X}_1\ \overline{X}_2)^t$, it is not possible to generate all the terms of (10) by a single vector-matrix multiplication operation. It is, however, possible to generate $\overline{Y}$ with <u>three</u> vector-matrix multiplication operations, as follows.

First, input the vector $(\overline{X}_0\ \overline{X}_2\ \overline{X}_1)^t$ and calculate its product with $\underline{M}$. We write the resulting product as

$$
\begin{matrix}
\overline{Y}_{0a} & & \overline{X}_0 \\
\\
\overline{Y}_{1a} & = & \underline{M} & \overline{X}_2 \\
\\
\overline{Y}_{2a} & & \overline{X}_1
\end{matrix}
\tag{12a}
$$

This is followed by the operations

$$
\begin{matrix}
\overline{Y}_{1b} & & \overline{X}_1 \\
\\
\overline{Y}_{2b} & = & \underline{M} & \overline{X}_0 \\
\\
\overline{Y}_{0b} & & \overline{X}_2
\end{matrix}
\tag{12b}
$$

$$
\begin{matrix}
\overline{Y}_{2c} & & \overline{X}_2 \\
\\
\overline{Y}_{0c} & = & \underline{M} & \overline{X}_1 \\
\\
\overline{Y}_{1c} & & \overline{X}_0
\end{matrix}
\tag{12c}
$$

If the outputs of these three operations are properly combined, the result is the desired three-component representation for Y. Specifically,

$$
\begin{aligned}
\overline{Y}_0 &= \overline{Y}_{0a} + \overline{Y}_{0b} + \overline{Y}_{0c} \\
\\
\overline{Y}_1 &= \overline{Y}_{1a} + \overline{Y}_{1b} + \overline{Y}_{1c} \\
\\
\overline{Y}_2 &= \overline{Y}_{2a} + \overline{Y}_{2b} + \overline{Y}_{2c}
\end{aligned}
\tag{13}
$$

Study of these equations shows that as the components $\bar{X}_0$, $\bar{X}_2$, and $\bar{X}_1$ are stepped cyclically in the input, the components of $\bar{Y}_0$, $\bar{Y}_1$, and $\bar{Y}_2$ are themselves stepped cyclically. One method for performing the summation of (12) and (13) is to use charge-coupled integrating detectors in the output plane of the optical system. The integrated charges can then be stepped synchronously with the input components. Another method is to use high speed analog switches and short delay lines for storage.

PLANS FOR IMPLEMENTATION

The basic optical layout of the iterative processor is shown schematically in figure 2. Light emitted from the sources is collimated into N parallel beams by means of a set lens L1. Each lens is a segment of a spherical lens. Thus, each row of the first SLM is illuminated by a collimated beam from one of the light sources. The output beams from the AO Bragg cell, A, are focused by lens L4, spatially filtered by S1, and recollimated by L5. The lens set L4-L5 images the aperture of Bragg cell A at the second Bragg cell, B. The modulated beam from the second Bragg cell is spatially filtered and brought to focus on the linear detector array. The spatial fitlers are necessary only to remove the zero and second order beams. The output beams are modulated in intensity at NxN points within the optical aperture. This representation is equivalent to having both masks located at the same plane. The outputs from the light emitters are modulated by vectors whose elements are $\bar{X}_j$. On multiplication, the output terms are $\bar{Y}_i = \sum_j \bar{A}_j \bar{B}_i \bar{X}_j$.

When a non-negative real approach is implemented, the above scheme must be modified. The speed of the processor is slowed down by approximately a factor of three, since for each iteration of three successive multiplications must be carried out. After each multiplication the analog data must be stored while the input vector is shifted. The three successive outputs are added as in equation (18). The output vector then becomes the input to the light emitters as in other iterative optical processors.

The time multiplexed method described can be implemented using CCD shift registers. Unfortunately, state of the art CCD shift registers operating at the necessary speed lack the dynamic range and noise properties necessary for our applications. We have thus chosen to implement the previously mentioned procedure utilizing fast four channel CMOS analog multiplexer switches arranged in a switching matrix configuration. Referring to figure 3, as the MUX is clocked, its output is shifted along the linear laser diode arrays. Similarly, at the linear photodiode array, the outputs $(\bar{Y}_{0a}\ \bar{Y}_{1a}\ \bar{Y}_{2a})^t$ from the first input cycle $(\bar{X}_0\ \bar{X}_2\ \bar{X}_1)^t$ are stored in coaxial cable delay lines. The input X vector is then switched to $(\bar{X}_1\ \bar{X}_0\ \bar{X}_2)^t$ and the output $(\bar{Y}_{1b}\ \bar{Y}_{2b}\ \bar{Y}_{0b})^t$ is switched to the proper location and stored in another delay line. This procedure is repeated a third time, with the appropriate combinations taken to yield the desired outputs.

Some of the problems particular to the implementation involve non-linearities in the Bragg diffraction process, electrical and acoustic crosstalk, and limited dynamic range due to light scattering.

CONCLUSIONS

An analysis has been performed of the applicability of iterative optical processors in environments requiring a short transient response time. An approach has been proposed utilizing realizable acousto-optic single dimension light modulators where the matrix is the outer product of two vectors. The problems associated with a non-negative real representation have been addressed. A solution has been proposed albeit at some cost of processor speed. A proof of concept experiment is underway which will better determine the limitations and advantages of this method.

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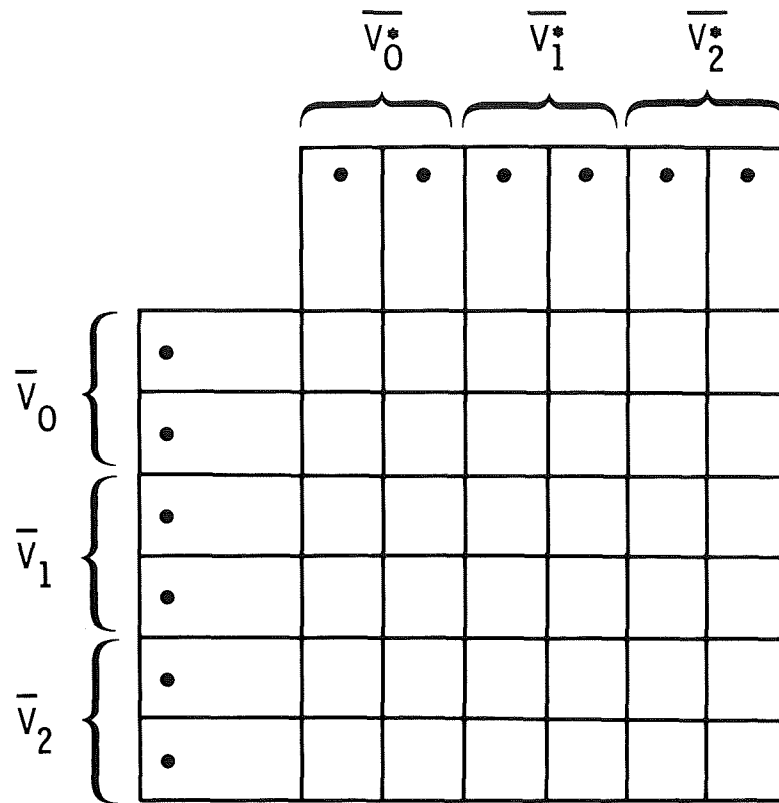
6.   Ref. 3, p. 13.

Figure 1.- Outer product geometry for producing elements of $\underline{M}$ with a pair of six transducer acousto-optic cells.
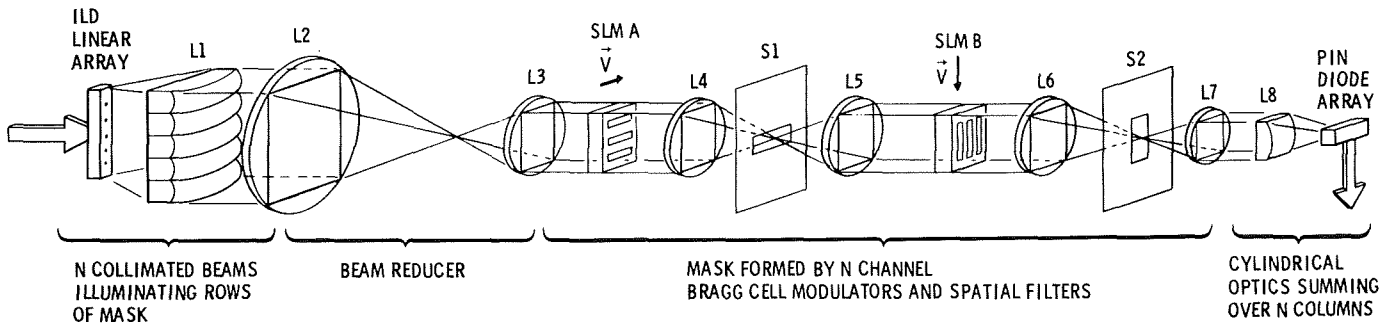

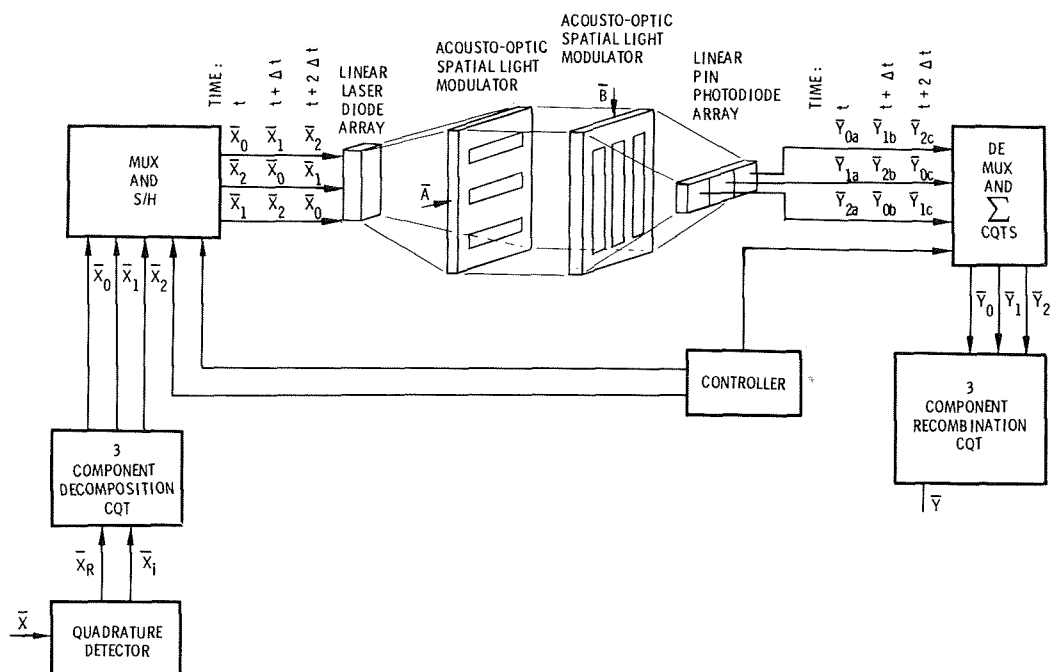Figure 2.- Optical system for matrix vector processor with separable masks.

Figure 3.- System architecture for OMVM, with time varying mask.