A LIQUID CRYSTAL ADAPTIVE LENS

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SUMMARY

Creation of an electronically controlled liquid crystal lens for use as a focusing mechanism in a multi-element lens system or as an adaptive optical element is analyzed. Varying the index of refraction is shown to be equivalent to the shaping of a solid refracting material. Basic characteristics of liquid crystals, essential for the creation of a lens, are reviewed. The required variation of index of refraction is provided by choosing appropriate electrode voltages. The configuration required for any incoming polarization is given and its theoretical performance in terms of modulation transfer function derived.

INTRODUCTION

Lenses play a critical role in almost all optical processing systems. From the crude and inexpensive plastic lens to the diffraction-limited lens, their role in the optical world is unique. The focusing of a lens is always accomplished by mechanical motion of one portion of the lens relative to the desired focal plane or correspondingly to other elements of the lens. This mechanical movement, to be precise, requires sophisticated and expensive mechanical hardware.

This paper addresses the electronically controlled creation of a lens. Since the actual focusing action is provided by electrode voltages, the image formation can be controlled. Also, since the index of refraction is controlled point-to-point on the lens, adaptive performance can be provided as well.

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SYMBOLS

f	focal length (meters)
k	wave number (meters^{-1})
L	diameter of lens (meters)
$n(x,y)$	position variable index of refraction
n_e	extraordinary index of refraction
n_o	ordinary index of refraction
n_θ	angle variable index of refraction
R_1	radius of curvature (meters)
$U(x,y)$	field distribution
V_T	threshold voltage (volts)
Δ	cell thickness (meters)
$\varepsilon_\perp$	component of the dielectric tensor perpendicular to the molecular axis
$\varepsilon_\parallel$	component of the dielectric tensor parallel to the molecular axis

LENS CREATION BY INDEX OF REFRACTION VARIATION

Lenses are classically created by varying the optical path length over an aperture by radially shaping a refractive medium. The same phase transformation can be obtained if the material has parallel faces but a varying index of refraction.

Consider the geometry shown in figure 1, where a material of width Δ and variable index of refraction (ref. 1)

$$n(x,y) = n_e - \frac{r^2}{2\Delta f} \qquad r^2 = x^2 + y^2 \tag{1}$$

is centered on the z-axis and lies in the x-y plane. Making the same approximation as in a phase analysis of a thin lens (ref. 2), namely,

1) Light passing through the device suffers only a phase transformation

2) Light rays are paraxial, that is,

$$\sqrt{1 - \frac{x^2 + y^2}{R_1^2}} \simeq 1 - \frac{x^2 + y^2}{2R_1^2} \tag{2}$$

results in a transmission function

$$t(x,y) = \exp\left(\frac{-jk}{2f}(x^2 + y^2)\right) \tag{3}$$

where

$$f \triangleq \frac{n_e - 1}{R_1} \tag{4}$$

LIQUID CRYSTAL CHARACTERISTICS

All crystals under the influence of an electric field or applied stress exhibit birefringence (ref. 3). That is, the index of refraction can be varied. In solids, however, this effect is small since only distortions of the indicatrix are made. Liquid crystals are unique since the molecules reorient under the influence of an applied field, either electric or magnetic (ref. 4). In addition, nematic liquid crystals are uniaxial and the optical axis is coincident with the molecular axis. Thus, as the molecules reorient in response to an applied field, the entire indicatrix is rotated, making large changes in the index of refraction.

Liquid crystals can be divided into two groups. If the dielectric tensor is such that the component along the molecular axis ($\varepsilon_{\parallel}$) is greater than the component perpendicular to the axis ($\varepsilon_{\perp}$), the crystal is said to be positive. These molecules tend to align parallel to an applied field. If the reverse is true, $\varepsilon_{\parallel} < \varepsilon_{\perp}$, the crystal is said to be negative. In this case, the molecules will align perpendicularly to an applied field. Both types of liquid crystals are positive uniaxial crystals. The extraordinary index of refraction, n_e (along the optical axis which is coincident with the molecular axis), is greater than the ordinary index of refraction, n_o.

Figure 2 illustrates the changing of index of refraction with orientation of the liquid crystal molecules. In this example, a liquid crystal is shown at the top of the figure in the homogeneous state. Correspondingly, the indicatrix is aligned so that a wave polarized in the x-direction is influenced by an index of refraction n_e. At the bottom of the figure, the orientation has been altered to the homeotropic state, in which case the same polarized light encounters an index of refraction n_o. At an intermediate point, the index of refraction is dependent on θ, the direction of the nematic director, and is given by (ref. 1)

$$\frac{1}{n_\theta^2} = \frac{\cos^2\theta}{n_e^2} + \frac{\sin^2\theta}{n_o^2} \tag{5}$$

Two important points should be noted. First, only the light polarized in the x-direction was influenced by a changing index of refraction. Light polarized in the y-direction suffered a constant phase delay. Because independent of the molecular orientation in the x-z plane, it was influenced by a constant index of refraction, n_o. The second important point is that the liquid crystal molecules do not align uniformly across a cell. As the voltage is increased above threshold (ref. 5), the molecules near the center react to the field first. At saturation, all of the molecules have responded except those tied by boundary conditions at the cell wall.

Figure 3 illustrates the index of refraction variation with voltage in one particular material.

LIQUID CRYSTAL LENS

Figure 4 shows a single stage of a liquid crystal lens. With voltages appropriately set on the electrode array, light polarized in the x-direction would be given the correct phase transformation. The y-polarized component would suffer a constant phase delay. To obtain total polarization capability, there must be a second stage with the nematic director having a preferential turn in the y-direction. This stage would vary the index of refraction for the y-component and give a constant phase delay to the component polarized in the x-direction.

It has been assumed to this point that electrodes could be configured to give the required radial variation in index of refraction. Since the voltage variation desired is radial, it would appear that radially symmetric electrodes would be ideal. However, radially symmetric electrodes require difficult mask fabrication and connections made to them create obscurations. Row and column addressable, rectangular grid electrodes like those used in commercially available liquid crystal displays could be considered. In this case, addressing individual electrodes is easy and mask fabrication is a simple process, but the device inherently does not match the symmetry of a lens. Thus, the structure would cause aberrations.

A simple structure can be implemented by using crossed linear arrays in tandem. Consider a square aperture of width L centered at the origin. Let the transmission of the aperture be given by

$$t(x,y) = \text{rect}\left(\frac{x}{L}\right) \text{rect}\left(\frac{y}{L}\right) \exp\left(jk\Delta n(x)\right) \tag{6}$$

where

k = wave number

Δ = thickness of the material

and

$n(x) = n_e - \frac{x^2}{2\Delta f}$ = index of refraction

Just past the aperture, if it is illuminated by a unit-amplitude, normally incident plane wave, the field distribution U' is

$$U'(x,y) = \text{rect}\left(\frac{y}{L}\right) \text{rect}\left(\frac{x}{L}\right) \exp\left(jk\Delta\left(n_e - \frac{x^2}{2\Delta f}\right)\right) \tag{7}$$

Ignoring the constant phase term,

$$U'(x,y) = \text{rect}\left(\frac{y}{L}\right) \text{rect}\left(\frac{x}{L}\right) \exp\left(\frac{-jkx^2}{2\Delta f}\right) \tag{8}$$

Now consider placing another transmission factor directly in front of the aperture with

$$n(y) = n_e - \frac{y^2}{2\Delta f} \tag{9}$$

then

$$U' = t_1(x,y)\ t_2(x,y) \tag{10}$$

$$U' = \mathrm{rect}\left(\frac{y}{L}\right) \mathrm{rect}\left(\frac{x}{L}\right) \exp\left(\frac{-jk(x^2 + y^2)}{2f}\right) \tag{11}$$

the ideal form of a lens is obtained with linear electrodes. By cascading two stages not only is the mask fabrication process simplified, the required symmetry is also maintained.

Figure 5 shows a single stage cross section and a representative electrode structure. Note that the ground electrode is uniform. Also, the variation in voltage profile can be used to generate either a change in index of refraction for x or y polarized light. If the preferential direction of the liquid crystal director is in the x-direction, only the component of the light polarized in that direction will be affected by a varying index of refraction.

To obtain a complete lens, four stages are required. "Complete" means that any incoming polarization is given the appropriate phase transform of a thin lens. Figure 6 illustrates the functions of the four stages. The first stage gives the appropriate variation in index of refraction in the x-direction for light polarized in the x-direction. Similarly, the second stage varies the index of refraction in the y-direction but still only for light polarized in the x-direction. Thus, if a lens was to be created for x-polarized light only, the first two stages would be sufficient. The last two stages repeat the process for the component polarized in the y-direction. Figure 7 shows the cross section of the four-stage liquid crystal lens.

OPTICAL PERFORMANCE ANALYSIS

Due to the electrode structure used to create the index of refraction variation in the liquid crystal, a diffraction-limited lens will not be formed. The wavefront leaving the lens will have phase distortions caused by the approximation of a smoothly varying index of refraction by a sampled function. Thus, the liquid crystal lens will inherently have aberrations. However, the smaller the electrodes and the spacing between them, the better the index of refraction is matched to a smooth curve and therefore the more nearly the lens approaches the diffraction limit. Figures 8 and 9 show the predicted MTF for a 2-cm wide lens with a 10-meter focal length constructed with 501 and 101 electrodes. Spaces between electrodes are assumed to be the same size as the electrodes. Thus, fabrication of the 501 electrode (20-micron width) is easily achievable with today's microelectronics capability.

CONCLUSION

An electronically controlled liquid crystal lens will provide direct control on focusing and point-to-point control on index of refraction for potential adaptive optical techniques. The lens has all the favorable characteristics of liquid crystal displays, such as operation at low voltages and low power dissipation. With microprocessor control, it could adapt in real time and be electronically calibrated. Using current microelectronic technology, near diffraction-limited performance is predicted.

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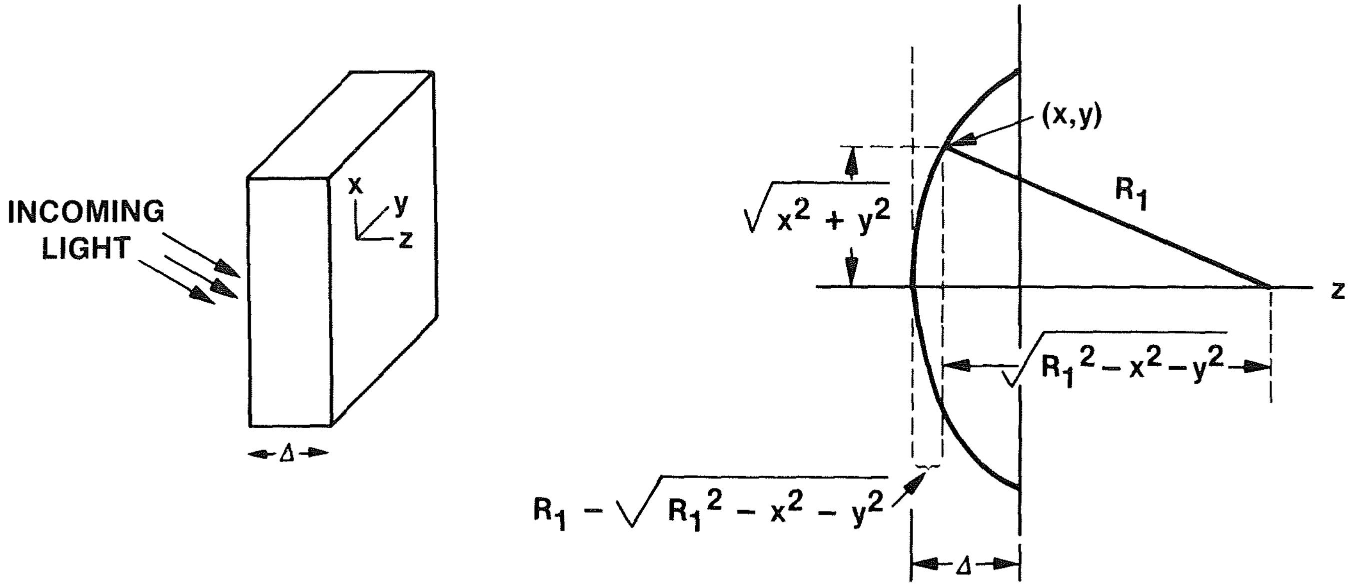

(a) (b)

Figure 1.- Basic configuration (a) and geometry of variation (b).

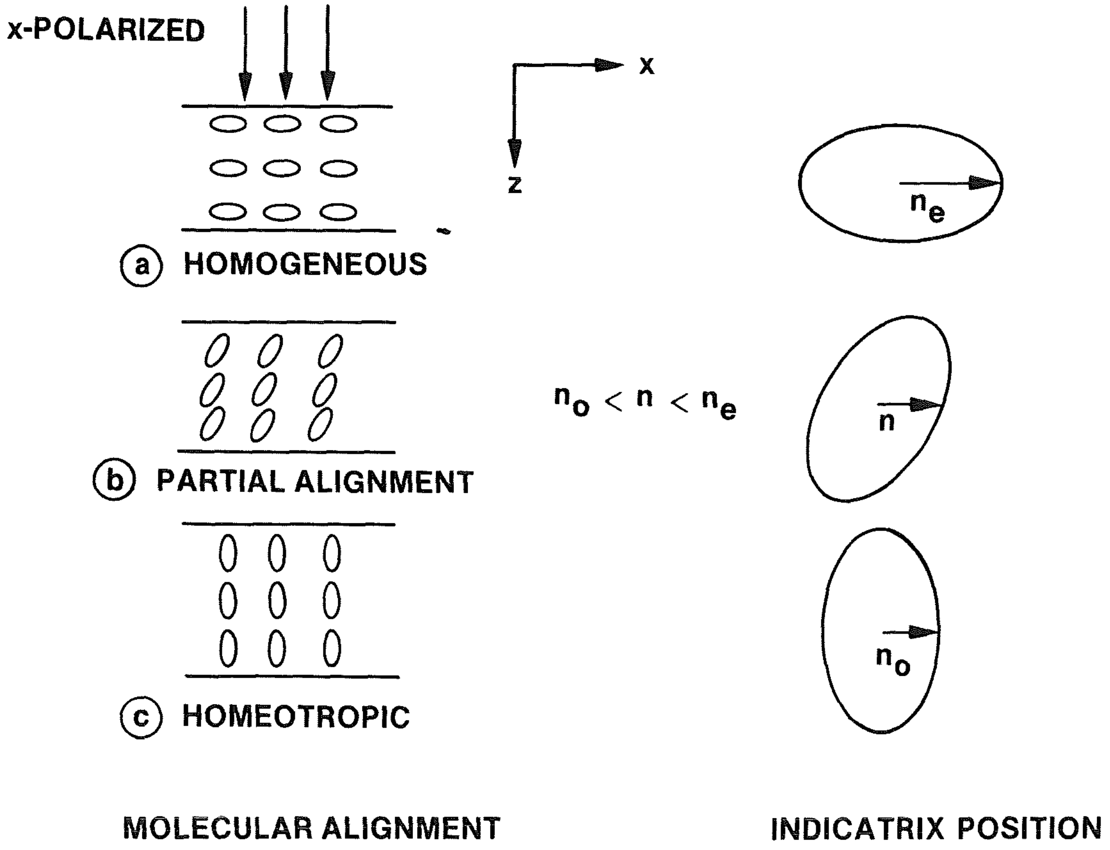

Figure 2.- Liquid crystal orientation for variation of the index of refraction.

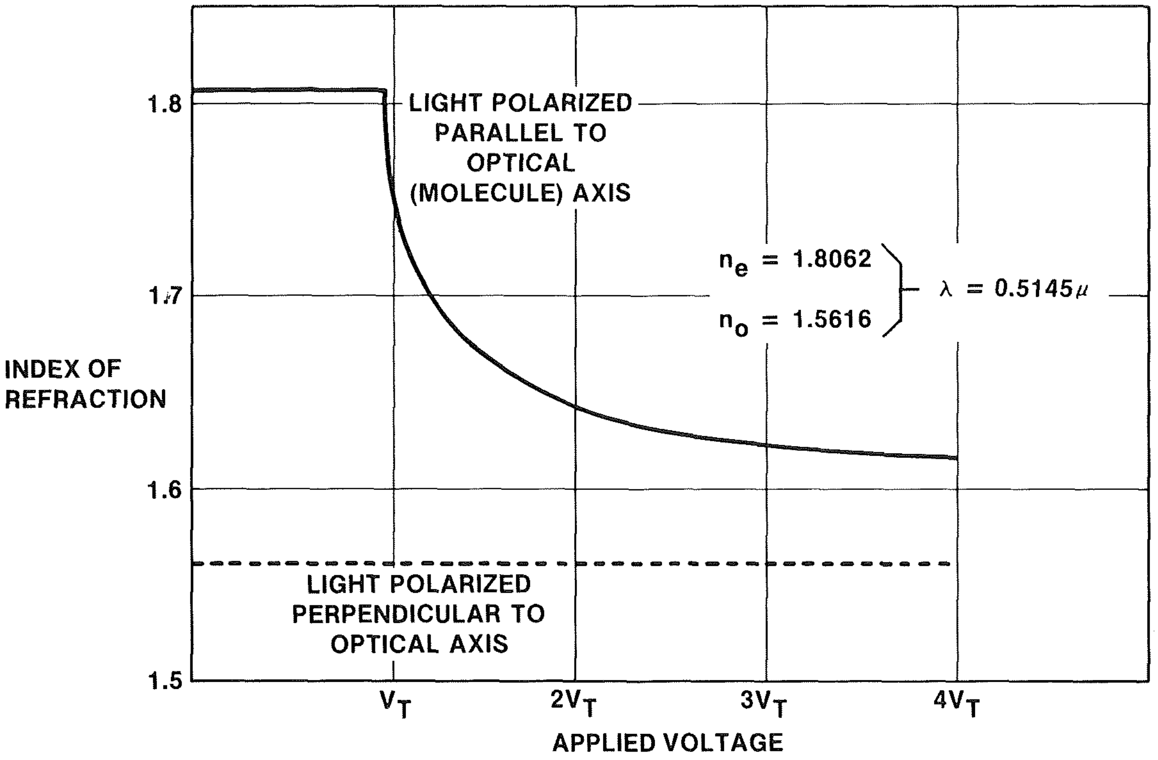

Figure 3.- Index of refraction versus cell voltage using MBBA.

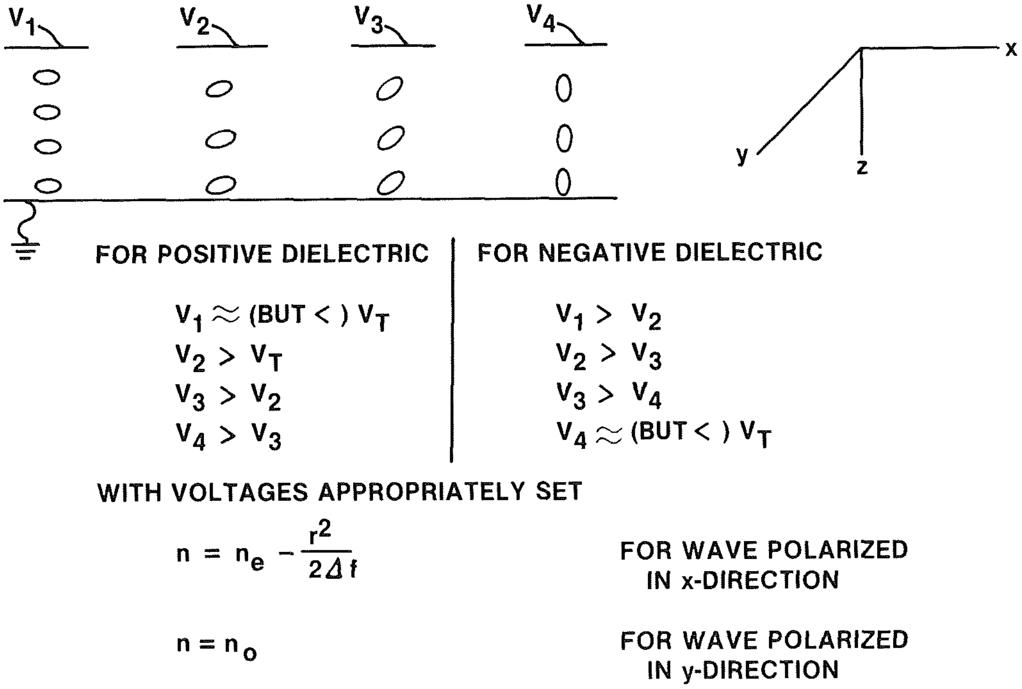

Figure 4.- Cross section of liquid crystal lens.

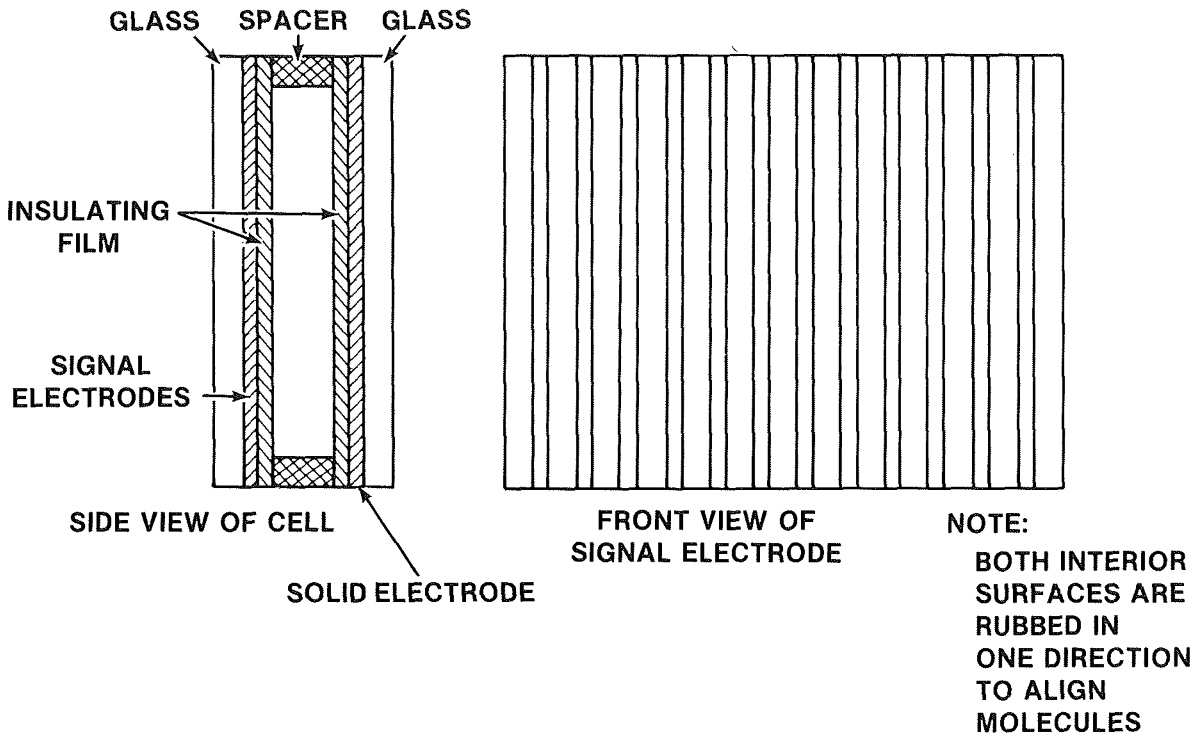

Figure 5.- Single stage of liquid crystal lens.

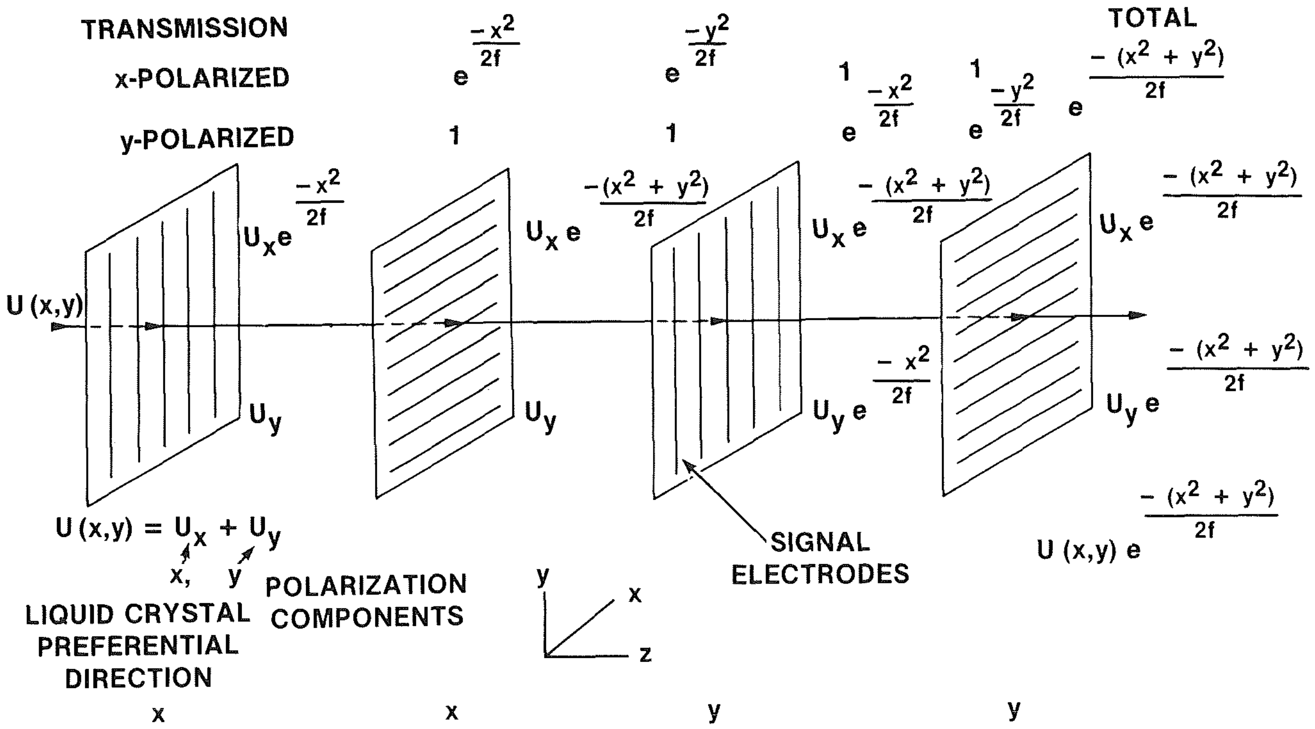

Figure 6.- Functions of the four stages in a liquid crystal lens.

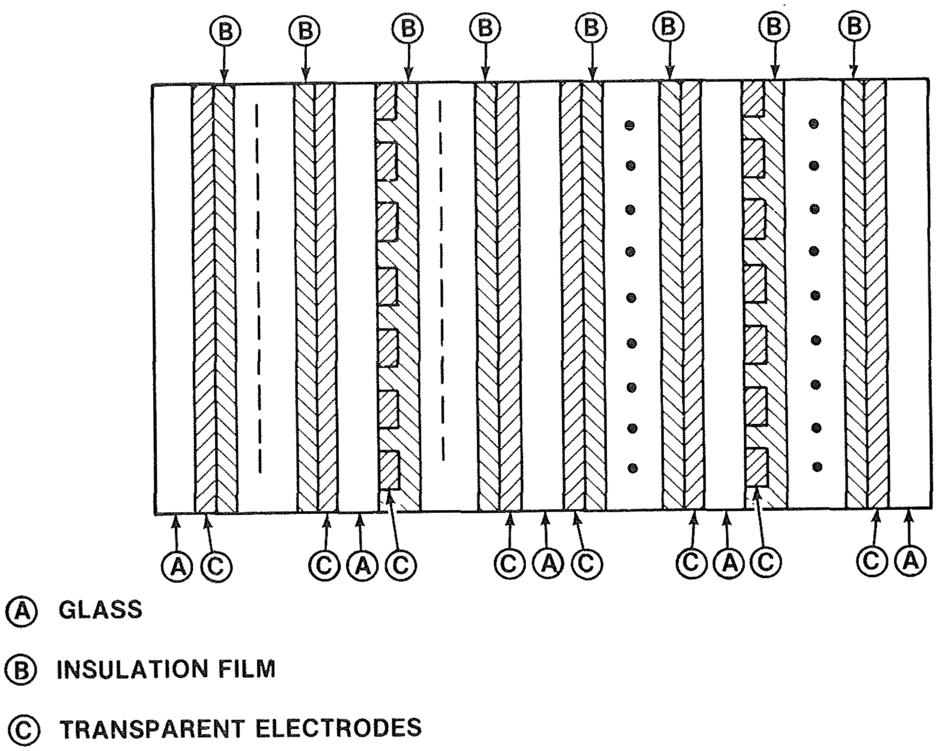

Figure 7.- Four-stage liquid crystal lens.

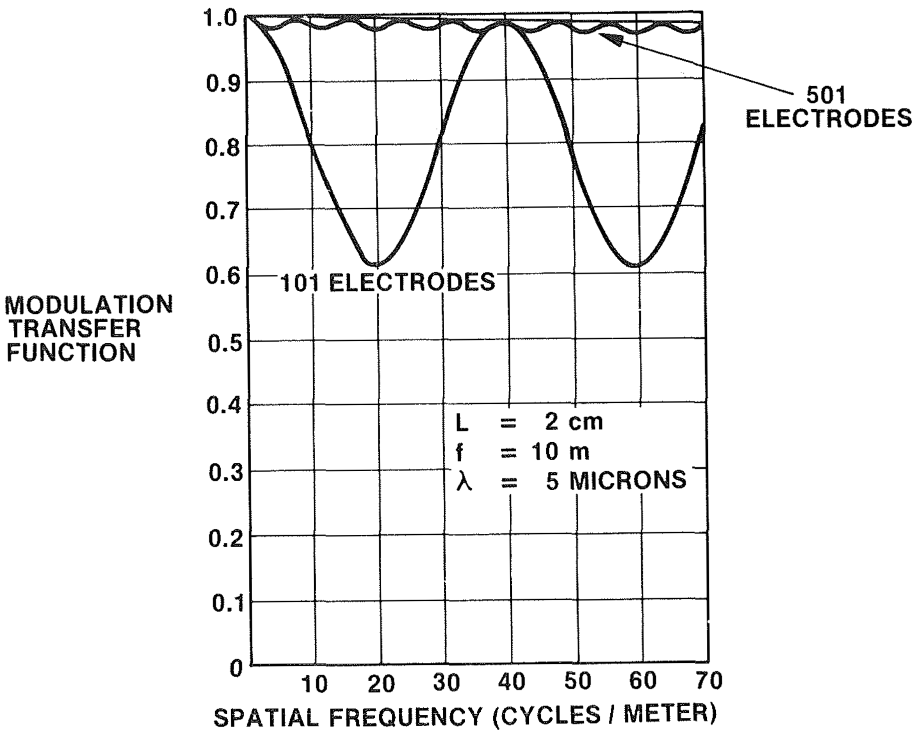

Figure 8.- Modulation transfer function of liquid crystal lens. λ=5.0 µm.

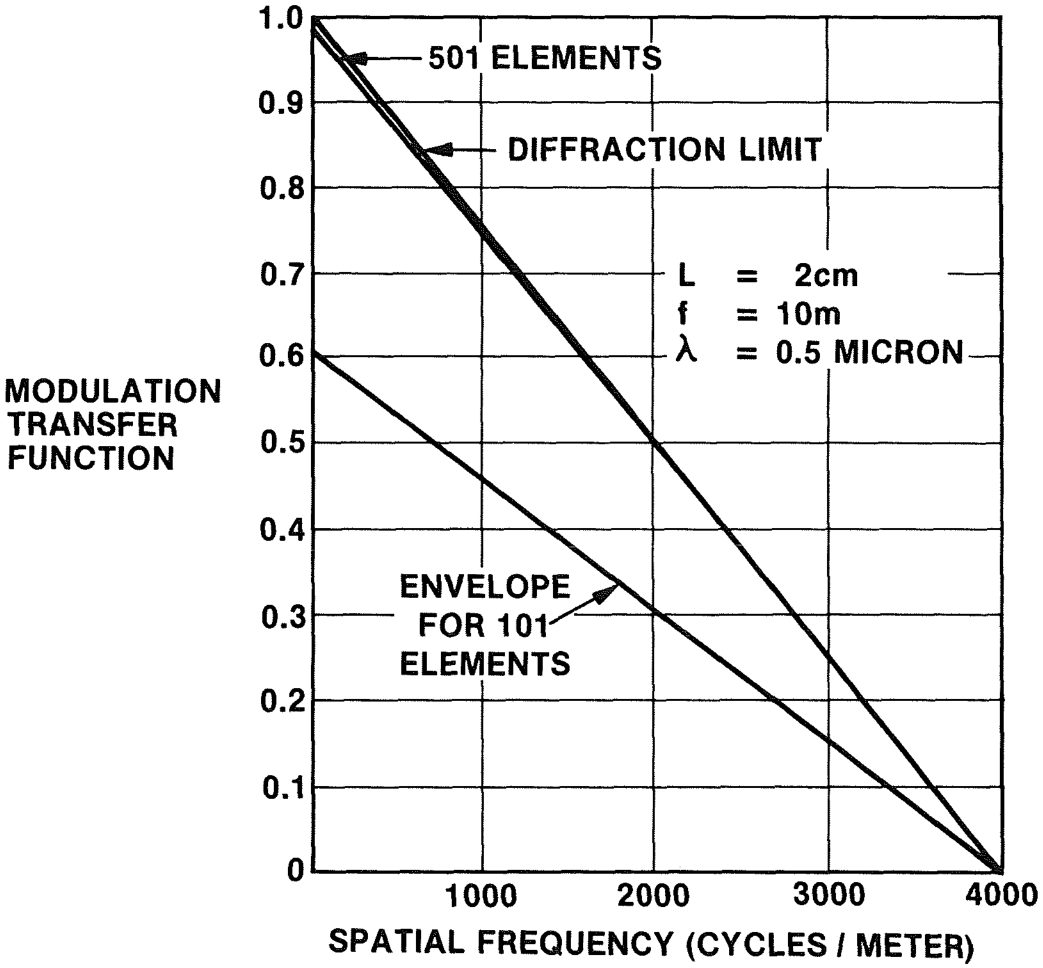

Figure 9.- Modulation transfer function of liquid crystal lens. $\lambda=0.5$ μm.