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# PREDICTION OF EARTH ROTATION AND POLAR MOTION

by

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## Abstract

Based on the analysis of the polar motion behavior, we found the possibility of predicting polar motion up to one year in advance. Comparing these predicted polar coordinates with the observed ones (smoothed), the rms of the differences is about $0\overset{''}{.}02$. The differences of the relative polar motion are much smaller. For any time interval of 20 - 30 days throughout the whole year, the rms of the relative polar motion differences is about $0\overset{''}{.}01$. Compared with the best available VLBI results (from 1977 to 1980), the rms of (pred. - obs.) is $0\overset{''}{.}013$, and the relative rms (for time intervals less than two months) is $0\overset{''}{.}008$ (here the observed data is unsmoothed).

It appears that 80 - 90% of the polar motion is composed of the stable, predictable Chandler and annual terms.

UT1-UTC has more complicated changes than polar motion making it difficult to find a satisfactory method of long-term prediction. So far the rms prediction error is $0\overset{s}{.}0023$ for up to 30 days.

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PART I

PREDICTION OF POLAR MOTION

# 1. THE BEHAVIOR OF POLAR MOTION AND THE MODEL FOR ITS PREDICTION

## 1.1 Introduction

It is a well known fact that the most important components of polar motion are the Chandler and annual terms. Polar motion may have other components of a periodic (e.g., semiannual), secular or random nature. The Chandler and annual terms themselves may vary with time, particularly their amplitudes and phases and possibly their periods too. Nevertheless, if the Chandler and annual terms contribute to a large percentage of the total polar motion, and if they do not change rapidly with time, polar motion prediction is feasible since these terms can be modeled quite well. Acquiring a better procedure to make polar motion predictions, assessing their accuracy, and at the same time finding the quantitative contribution of the Chandler and annual terms are the purposes of this investigation.

Markowitz [1976, 1979] pointed out the resemblance between the 1969-1976 IPMS polar motion and a theoretical curve (TC-2). We try to extend his idea in order to predict polar motion for practical applications.

## 1.2 Components of Polar Motion

### 1.2.1 Chandler motion

We extend the idea of Oesterwinter [1979] to determine the period of the Chandler motion, $P_C$, using longer time intervals of data. Our estimate is $P_C = 1.187 \pm 0.001$ yr (433.5 days). Taking $P_C$ as constant, we estimated the amplitudes and phases of the Chandler, annual and semiannual terms using ILS, IPMS, BIH and DMA 0.05 year data. The ILS, IPMS and BIH data sets start from 1962, and the DMA set from 1972.

The parameters of the Chandler motion are listed in Table 1 in which A is the amplitude, $\phi$ the phase, $\varepsilon$ the ellipticity of the Chandler ellipse (unitless), $\lambda_0$ is the (eastward) direction of the major axis of the Chandler ellipse, and $\sigma$ is the standard deviation of the estimated parameter. In computing the

Table 1 Parameters of Chandler Motion

| | DMA | | BIH | | IPMS | | ILS | | Mean * | |
|---|---|---|---|---|---|---|---|---|---|---|
| | x | y | x | y | x | y | x | y | x | y |
| A (0".001) | 136 | 134 | 135 | 133 | 134 | 133 | 127 | 138 | 135 | 133 |
| $\sigma_A$ | 2 | 3 | 2 | 2 | 2 | 2 | 3 | 3 | 1 | 1 |
| $\phi$ (°) | -89 | -88 | -93 | -89 | -93 | -90 | -92 | -90 | -92 | -89 |
| $\sigma_\phi$ | 0.9 | 1.3 | 0.9 | 0.7 | 0.8 | 0.8 | 1.4 | 1.0 | 0.6 | 0.5 |
| $\varepsilon$ | 0.015 | | 0.011 | | 0.007 | | 0.08 | | 0.01 | |
| $\sigma_\varepsilon$ | 0.028 | | 0.019 | | 0.019 | | 0.03 | | 0.01 | |
| $\lambda_0$ (°) | 1 | | 4 | | 3 | | 2 | | 3 | |
| $\sigma_{\lambda_0}$ | | | | | | | | | 1 | |

* ILS is not included in the mean.

phase, the adapted epoch is $t_0$ = 1962.0. To compute $\lambda_0$, we used the following equation:

$$\sin 2\lambda_0 = \frac{2 \sin(\phi_x - \phi_y)(A_x \cdot A_y)}{A_x^2 + A_y^2} \tag{1}$$

where $A_x$, $A_y$, $\phi_x$, $\phi_y$ are directly estimated from the data by a least squares fit.

Larmar [1896] has pointed out that the pole tide imposes a slight ellipticity on the path described by the pole of rotation. From theoretical consideration he calculated a value of 0.017 for the ellipticity $\varepsilon$, with the major axis pointing toward east longitude, $\lambda_0$ = 6°.

From the analysis of the unsmoothed latitude data 1899-1954, the ellipticity was estimated as $\varepsilon = 0.01 \pm 0.05$ [Munk and MacDonald, 1960]. From Table 1 our estimates are $\varepsilon = 0.01 \pm 0.01$, $\lambda_0 = 3° \pm 1°$. Although the precision of the results is much improved, we still cannot identify the ellipticity of the Chandlerian motion with great certainty. But, since all systems (except for the ILS) have shown that the Chandler amplitude in x is a little larger than that in y and all the $\lambda_0$ have the same sign, we suppose that the ellipticity hypothesis is probably correct.

### 1.2.2 Annual motion

The parameters of annual motion are listed in Table 2, with the same notations and units as in Table 1. From the table we can see that different

Table 2 Parameters of Annual Polar Motion*

| | DMA x | DMA y | BIH x | BIH y | IPMS x | IPMS y | ILS x | ILS y | Weighted Mean° x | Weighted Mean° y |
|---|---|---|---|---|---|---|---|---|---|---|
| A(0".001) | 117 | 101 | 111 | 95 | 105 | 91 | 92 | 76 | 110 | 95 |
| $\sigma$ | 2 | 3 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| $\phi$ (°) | 101 | 102 | 102 | 106 | 109 | 112 | 115 | 115 | 104 | 107 |
| $\sigma_\phi$ | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 3 |
| $\varepsilon$ | 0.131 | | 0.148 | | 0.135 | | 0.18 | | 0.14 | |
| $\sigma_\varepsilon$ | 0.033 | | 0.022 | | 0.023 | | 0.04 | | 0.014 | |
| $\lambda$ (°) | 1 | | 4 | | 3 | | | | 3 | |
| $\sigma_\lambda$ | | | | | | | | | 1 | |

* Here for the convenience of comparing it with the other results, the annual motion is described as $x = A \cos (2\pi t + \phi)$. Later we use $x = A \sin (2\pi t + \phi)$, so the phase may differ by 90°.

° ILS is not included in the mean.

polar motion services have significantly different estimates for amplitude and phase. The systematic difference between them is due to different observation procedures and/or data processing. It is apparent that the ILS has much larger systematic errors, its amplitude being about 20% smaller than the others. The BIH and IPMS systems may have amplitude fluctuations of about 0".01 which we shall discuss later.

Nevertheless, annual polar motion is elliptical without doubt. Our ellipticity estimate is 0.14 ± 0.014. Therefore, it is not quite suitable to take it as a circle in order to draw the theoretical curve.

The major axis of the annual ellipse does not exactly lie on the x axis, but points to $\lambda_0 = 3° \pm 1°$. That is, the major axes of the Chandler and annual ellipses are nearly coincident. In any case, the major axis of the annual ellipse is very near the x axis; thus we can approximate the major axis with the x axis.

### 1.2.3 Semiannual term

The parameters of the semiannual term may be found in Table 3. Such a term might exist in polar motion. If it exists, the amplitude must be smaller than 0".01. With the observation accuracies now available, it can hardly be determined with great certainty.

Table 3 Parameters of Semiannual Term

| | DMA x | DMA y | BIH x | BIH y | IPMS x | IPMS y | ILS x | ILS y |
|---|---|---|---|---|---|---|---|---|
| A (0".001) | 6.0 | 2.5 | 6.7 | 5.0 | 3.4 | 4.4 | 3.0 | 8.7 |
| $\sigma_A$ | 2.0 | 3.1 | 2.0 | 1.7 | 1.9 | 1.8 | 3.0 | 2.3 |
| $\phi$ (°) | -153 | -29 | -107 | 17 | -117 | 10 | -122 | -11 |
| $\sigma_\phi$ | 28 | 39 | 17 | 19 | 32 | 23 | 57 | 15 |

### 1.3 Model for Polar Motion Prediction

From the above it is clear that we can neglect the semiannual term in the model for polar motion prediction but must take the annual motion as an ellipse. The data also shows a linear trend, therefore it is better to include a linear term in the model. The final model we use for prediction is as follows:

$$\begin{aligned} x &= x_o + K_x(t-t_o) + b_x \sin 2\pi(t-t_o) + c_x \cos 2\pi(t-t_o) + \\ &+ B_C \sin \frac{2\pi(t-t_o)}{P_C} + C_C \cos \frac{2\pi(t-t_o)}{P_C} \\ y &= y_o + K_y(t-t_o) + b_y \cos 2\pi(t-t_o) - c_y \sin 2\pi(t-t_o) + \\ &+ B_C \cos \frac{2\pi(t-t_o)}{P_C} - C_C \sin \frac{2\pi(t-t_o)}{P_C} \end{aligned} \tag{2}$$

in which $P_C$ is constant (1.187). The parameters to be estimated are $K_x$, $b_x$, $c_x$, $K_y$, $b_y$, $c_y$, $B_C$ and $C_C$. Practical computation has shown that the mean prediction error is nearly the same whether we take the Chandler motion as elliptic or circular.

We use six or seven years of polar coordinates to estimate the above parameters, then use the parameters to calculate the next year's polar coordinates (or even the next two years).

In the following computations, we use 0.05 yr smoothed data to determine the parameters for prediction. (In the BIH they are called smoothed "normal values at 1/20 year interval".) To assess the accuracy, we compare the prediction both with next year's five-day data (Circular D) and with the 0.05 yr normal values and use the notations $\sigma_{5\text{-day}}$ and $\sigma_{0.05\text{-yr}}$ respectively to express the two prediction errors.

We also used the BIH Circular D 5-day polar motion data to compute the parameters and make predictions. The results were a little worse than those obtained by the above procedure. Therefore, we prefer to use normal values. However, normal values are only published once a year, and with several months of publishing delay. So practically, we use six years of normal values, plus several months of Circular D data to make predictions.

## 2. THE RESULTS OF PREDICTION

In this chapter we compare the predicted and observed polar motion. It must be kept in mind that the difference (pred.-obs.) is not only due to the model deficiencies, but also to the observing error. In the next chapter we will discuss this problem in detail. We will use the expression "prediction error" to describe the rms of (pred.-obs.), not the "absolute" error of prediction.

### 2.1 Prediction of the BIH Polar Coordinates

The prediction errors are given in Table 4. Data used for the prediction are from the BIH global solution given at 0.05-year intervals. The estimated parameters of the model (in equation (2)) are also given in Table 4. Because of the change of epoch the parameters of Chandler motion, B and C, change from year to year. For comparison, we give the amplitudes of the estimated Chandler motion in the last column.

Fig. 1 compares the predicted polar motion track with the observed one for the time interval 1979.00 - 1979.75. The predicted track closely resembles the observed one. The overall averages of the prediction errors are $\sigma_{0.05\text{-yr}} = 0\overset{''}{.}020$, $\sigma_{5\text{-day}} = 0\overset{''}{.}022$.

### 2.2 Prediction of the IPMS and DMA Polar Coordinates

The results when using IPMS 0.05-year smoothed data (provided by the IPMS Annual Reports) and using DMA 0.05-year smoothed data (provided by the BIH Annual Reports) are given in Table 5. Only the annual mean values of $\sigma_x$ and $\sigma_y$ are given. See also Fig. 2 for the IPMS predicted and observed pole track comparison for the interval 1979.00 - 1979.75. For comparison, we also list the BIH's $\sigma$ in the last column.

Table 4 Prediction of the BIH Polar Coordinates (unit 0".001)

| | | $\sigma_{0.05\text{-yr}}$ | $\sigma_{5\text{-day}}$ | $x_o(y_o)$ | K | b | c | B | C | $\sqrt{B^2+C^2}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1980 | x | -- | 15 | 23 | 1.4 | -115 | - 15 | - 96 | -105 | 142 |
| | y | -- | 31 | 259 | 4.0 | 11 | -103 | 110 | - 85 | 139 |
| | mean | -- | 24 | | | | | | | |
| 1979 | x | 15 | 14 | 23 | 0.3 | -115 | - 18 | 35 | -135 | 139 |
| | y | 19 | 22 | 257 | 2.3 | 15 | -102 | 128 | 43 | 135 |
| | mean | 17 | 18 | | | | | | | |
| 1978 | x | 18 | 16 | 23 | -0.3 | -124 | - 2 | 126 | - 45 | 134 |
| | y | 15 | 15 | 254 | 3.4 | 14 | - 97 | 37 | 127 | 132 |
| | mean | 17 | 16 | | | | | | | |
| 1977 | x | 22 | 24 | 22 | 1.6 | -118 | - 6 | 100 | 80 | 128 |
| | y | 14 | 14 | 251 | 2.6 | 12 | - 96 | - 83 | 101 | 131 |
| | mean | 18 | 19 | | | | | | | |
| 1976 | x | 31 | 37 | 18 | 3.0 | -109 | - 8 | - 21 | 125 | 127 |
| | y | 22 | 27 | 248 | 0.4 | 16 | - 94 | -132 | - 18 | 133 |
| | mean | 26 | 33 | | | | | | | |
| 1975 | x | 17 | 20 | 13 | 5.9 | -109 | - 8 | -116 | 51 | 127 |
| | y | 11 | 11 | 251 | -1.4 | 12 | - 96 | - 55 | -117 | 129 |
| | mean | 15 | 16 | | | | | | | |
| 1974 | x | 26 | 26 | 9 | 6.4 | -110 | - 17 | -110 | - 60 | 125 |
| | y | 17 | 20 | 252 | -0.5 | 18 | - 96 | 62 | -112 | 128 |
| | mean | 22 | 23 | | | | | | | |
| Overall Average | | 20 | 22 | | | | | | | |

Table 5 Prediction Errors for IPMS and DMA (unit 0".001)

| | IPMS | DMA | BIH |
|---|---|---|---|
| 1980 | -- | 20* | 24** |
| 1979 | 18 | 17 | 17 |
| 1978 | 15 | 15 | 17 |
| 1977 | 19 | 27 | 18 |
| 1976 | 24 | | 26 |
| 1975 | 16 | | 15 |
| 1974 | 27 | | 22 |
| Mean | 20 | 20 | 20 |

*Used DMA bi-daily solutions for making prediction and compared with BIH Circular D.

**Used BIH global solution for prediction and compared with Circular D.

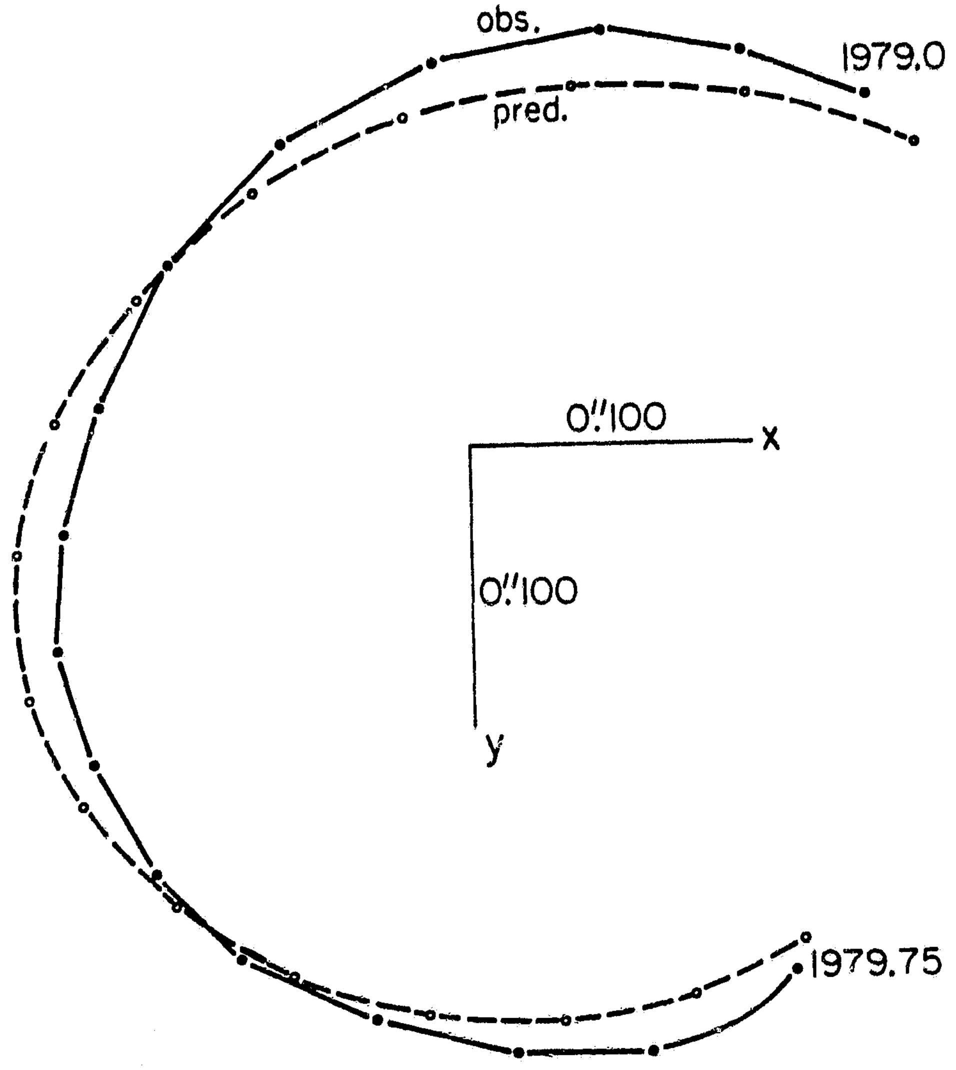


Fig. 1 Predicted polar motion versus observed, BIH, 1979.00 - 1979.75

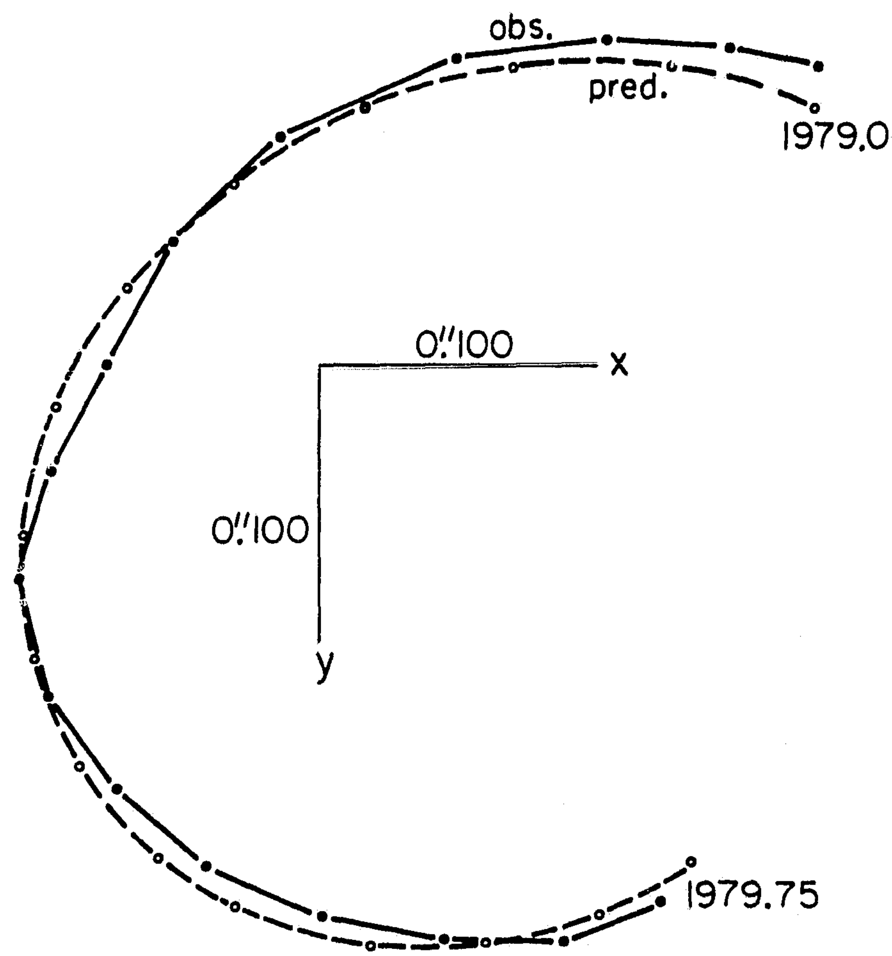


Fig. 2 Predicted polar motion versus observed,
IPMS 1979.00 - 1979.75

The average prediction errors for IPMS and DMA are also about 0".02. It should be mentioned that if we used, say, BIH data to make a prediction and compared it with the corresponding IPMS observation, the prediction error would increase slightly because the two polar motions services have different systematic (mostly annual) errors.

## 2.3 Relative Polar Motion

In some cases we are interested not in the polar coordinate itself but in the (relative) polar motion over a certain time interval. We predict the relative polar motion over one year and compare it with the observed values. The results are presented in Table 6. Data used are from the BIH.

Table 6 Prediction Errors for Relative Polar Motion (0".001)

| | | Time Interval | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 5-day | 10-day | 15-day | 0.05-yr | 20-day | 30-day | 0.1-yr | 40-day | 60-day |
| 1980 | x | 2.8 | 5.6 | 8.1 | -- | 10.5 | 14.6 | -- | 17.5 | 20.1 |
| | y | 2.2 | 4.2 | 6.3 | -- | 9.3 | 12.1 | -- | 15.9 | 22.7 |
| | mean | 2.6 | 5.0 | 7.3 | -- | 10.0 | 13.4 | -- | 16.8 | 21.5 |
| 1979 | x | 1.8 | 3.5 | 5.1 | 6.1 | 6.5 | 8.9 | 9.3 | 10.9 | 13.4 |
| | y | 1.8 | 3.4 | 5.1 | 6.3 | 6.8 | 10.0 | 11.7 | 12.9 | 17.9 |
| | mean | 1.8 | 3.5 | 5.1 | 6.2 | 6.7 | 9.5 | 10.6 | 12.0 | 15.8 |
| 1978 | x | 1.6 | 3.2 | 4.4 | 5.4 | 5.6 | 7.5 | 9.4 | 9.0 | 10.4 |
| | y | 2.0 | 3.9 | 5.8 | 6.4 | 7.7 | 11.2 | 12.3 | 14.2 | 20.2 |
| | mean | 1.8 | 3.6 | 5.2 | 6.0 | 6.8 | 9.5 | 10.9 | 11.9 | 16.1 |
| 1977 | x | 3.0 | 5.9 | 8.8 | 9.7 | 11.6 | 16.8 | 18.8 | 21.8 | 30.0 |
| | y | 1.4 | 2.7 | 3.9 | 4.5 | 5.1 | 7.4 | 8.3 | 9.6 | 13.9 |
| | mean | 2.3 | 4.5 | 6.6 | 7.6 | 9.0 | 13.0 | 14.5 | 16.8 | 23.4 |
| 1976 | x | 3.2 | 6.2 | 9.0 | 9.3 | 11.8 | 17.0 | 15.8 | 21.9 | 29.6 |
| | y | 2.1 | 4.1 | 6.0 | 6.0 | 7.8 | 11.0 | 11.8 | 13.8 | 17.6 |
| | mean | 2.7 | 5.2 | 7.6 | 7.8 | 10.0 | 14.3 | 14.1 | 18.3 | 24.4 |
| 1975 | x | 2.3 | 4.6 | 6.6 | 7.0 | 8.7 | 12.4 | 13.3 | 15.5 | 20.7 |
| | y | 1.8 | 3.6 | 5.3 | 4.2 | 6.9 | 10.0 | 8.0 | 12.6 | 15.5 |
| | mean | 2.1 | 4.2 | 6.0 | 5.8 | 7.9 | 11.3 | 11.0 | 14.1 | 18.3 |
| 1974 | x | 2.9 | 5.6 | 8.3 | 10.2 | 10.8 | 15.7 | 20.0 | 20.3 | 30.0 |
| | y | 2.4 | 4.7 | 6.7 | 7.2 | 8.5 | 11.5 | 13.5 | 13.9 | 18.0 |
| | mean | 2.7 | 5.2 | 7.6 | 8.9 | 9.8 | 13.9 | 17.1 | 17.4 | 24.7 |
| Overall Average | | 2.3 | 4.5 | 6.6 | 7.1 | 8.7 | 12.3 | 13.3 | 15.5 | 20.9 |

For the time interval of 20 - 30 days, the relative polar motion prediction error is only about 0."01 (30 cm). The polar motion prediction error is plotted in Fig. 3. With effort, the BIH might provide Circular D polar motion data with a delay of one month (or less). So the real time polar motion prediction error may reach about 0."01 which for any present application is more than sufficient.

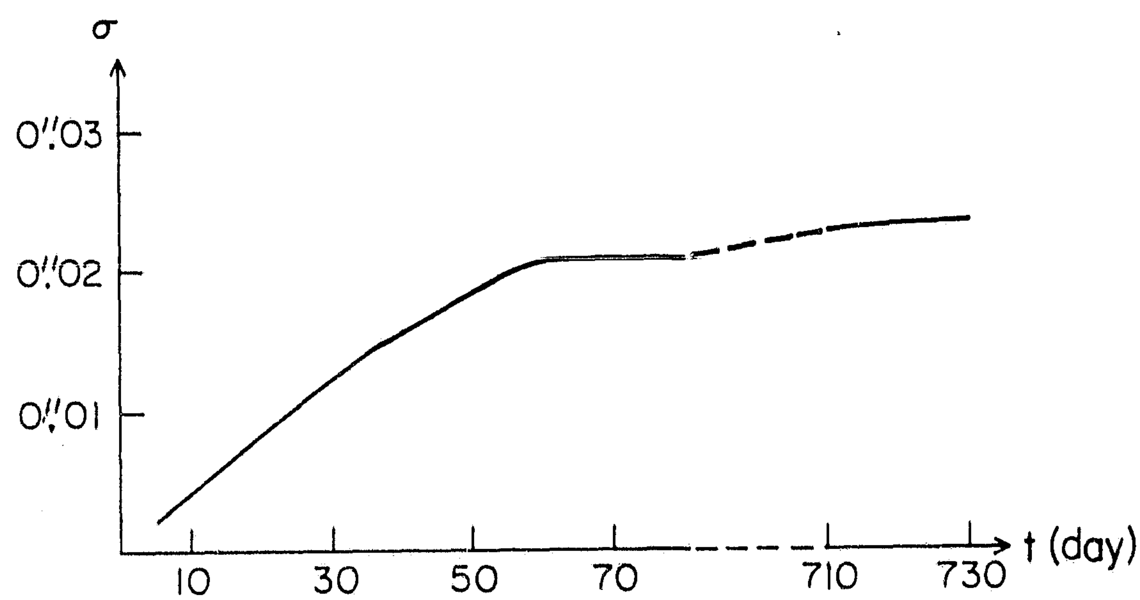


Fig. 3 Polar motion prediction error.
(time = 0 denotes the time of last data available)

## 2.4 Comparison with VLBI Observations

In Table 7 we list the DSN VLBI polar motion data SOLV 71809 F (provided by [Fanselow, 1981]) as well as the predicted polar coordinates of the same date. Predictions are based on BIH 0.05-year data. For comparison, we also give the BIH Circular D polar coordinates (interpolated to the same day). Although the SOLV 71809 F data is very preliminary, as pointed out by Fanselow, there is a significant improvement in the results with the introduction of the Wahr model (instead of the Woolard) for the nutation series.

Table 7 Pole Coordinates of DSN VLBI, Prediction and BIH (0".001)

| | VLBI | | Prediction | | BIH Circular D | |
|---|---|---|---|---|---|---|
| | x | y | x | y | x * | y |
| 1977/ 1/12 | | 97.4 | | 100 | | 94 |
| 1/21 | | 115.9 | | 121 | | 114 |
| 1/31 | -176.3 | 146.2 | -162 | 148 | -162 | 143 |
| 2/13 | | 186.5 | | 187 | | 187 |
| 2/28 | | 233.8 | | 237 | | 241 |
| 4/13 | -183.5 | | -180 | | -206 | |
| 1978/ 1/14 | | 17.4 | | 41 | | 21 |
| 1/24 | | 31.9 | | 52 | | 35 |
| 2/12 | | 66.6 | | 87 | | 78 |
| 5/15 | | 385.3 | | 377 | | 402 |
| 7/31 | 75.3 | | 53 | | 74 | |
| 9/04 | 172.5 | 414.5 | 170 | 422 | 178 | 419 |
| 10/28 | 236.0 | 251.8 | 247 | 269 | 249 | 265 |
| 11/05 | 242.5 | 218.1 | 245 | 244 | 245 | 236 |
| 12/31 | | 81.8 | | 97 | | 77 |
| 1979/11/25 | 125.1 | 318.2 | 141 | 290 | 141 | 325 |
| 12/28 | 131.4 | 255.6 | 136 | 234 | 131 | 258 |
| 1980/ 1/12 | | 232.2 | | 222 | | 233 |
| 1/26 | 110.7 | 209.2 | 101 | 205 | 100 | 210 |
| 2/14 | 67.4 | 187.6 | 73 | 189 | 78 | 187 |
| 2/24 | 58.8 | 180.0 | 58 | 184 | 70 | 179 |

* All the x coordinates of the BIH are changed into the BIH 1979 system.

We computed the rms of (VLBI - BIH), of (VLBI - pred.), and of (BIH - pred.). The results are:

$$\sigma\ (\text{VLBI - pred.}) = 0''.013$$

$$\sigma\ (\text{VLBI - BIH}) = 0''.009$$

$$\sigma\ (\text{BIH - pred.}) = 0''.013$$

We also computed the relative polar motion. We computed the prediction errors of relative polar motion for all time intervals of less than two months. The results are as follows:

$$\sigma \text{ relative (VLBI - pred.)} = 0''.008$$
$$\sigma \text{ relative (VLBI - BIH)} = 0''.011$$
$$\sigma \text{ relative (BIH - pred.)} = 0''.009$$

These results are very encouraging. It seems that predicted polar motion fits the VLBI observation quite well, even better than VLBI versus BIH or BIH versus prediction in the relative polar motion case.

## 2.5 Comparison of Different Methods of Prediction

Besides our prediction method, there are two widely used prediction methods. One is the BIH rapid service (referred to as BIH); the other is predicted by the U.S. Naval Observatory (referred to as USNO, see [McCarthy, 1980]). Each prediction method uses nearly the same idea, considering some model, estimating the parameters by a least squares fit, then using the estimates to predict the polar motion. The main difference is in the length of the data used. Using long time interval data, the observing errors will be greatly reduced, but the parameters are not instantaneous --they are mean values over the long time intervals. On the other hand, using short time interval data, the estimates are nearly instantaneous, but they are much more influenced by observing errors.

We made some calculations using only 3-6 months data and found that sometimes the estimated Chandler (or annual) amplitudes were less than 0''.02 which is unreasonably small. This implies that the estimates must not be the real instantaneous parameters, but the error contaminated ones, which means that the observing error is more serious than the real change in the Chandler and annual parameters. Therefore, we propose to use longer intervals of data. Other methods using a short time span of data still can make fairly good predictions for short time periods although the estimated parameters seem to be meaningless. The reason is the correlation compensation. Once the prediction time is extended, the correlation is decreased, and the prediction error increases quickly. (This situation is more or less the same in our UT1 prediction (see Part II)).

## 2.6 Prediction Versus MERIT Short Campaign Results

Using the BIH Circular D series as a common reference, the rms difference of the MERIT results are calculated (see BIH "MERIT Short Campaign - Final Status Report"). We used predicted polar motion as a common reference to do the same calculation. The predictions are based on 1974-1979 BIH global results. The results are tabulated in Table 8. The numbers are rms differences in units of 0.″001.

Table 8 Comparison of Prediction with MERIT Short Campaign Results

| Observing Technique | x BIH (D) | x Prediction | y BIH (D) | y Prediction |
|---|---|---|---|---|
| BIH Classical | 15 | 18 | 27 | 25 |
| DMA 5-day mean | 8 | 9 | 7 | 12 |
| SAO SLR | 8 | 6 | 10 | 5 |
| UTX SLR | 11 | 13 | 14 | 13 |

Fig. 4 depicts the detailed comparison between the prediction and other observing techniques. From the figure and Table 8 we see that the polar motion prediction is quite meaningful. As with the VLBI results in Section 2.4, SLR results seem to fit the predicted polar motion better than the BIH data.

We also compared the observed relative polar motion with its predicted counterpart. The results are given in Table 9.

## 2.7 Prediction for Two Years in Advance

We predicted polar motion for two years in advance and computed the prediction errors. The results are presented in Table 10 and Table 11 for relative polar motion.

Comparing Table 5 with 10 and 6 with 11, we see that two-year prediction errors are practically the same as the one-year prediction. Our method is able to predict polar motion two years in advance with approximately 0.″02 accuracy.

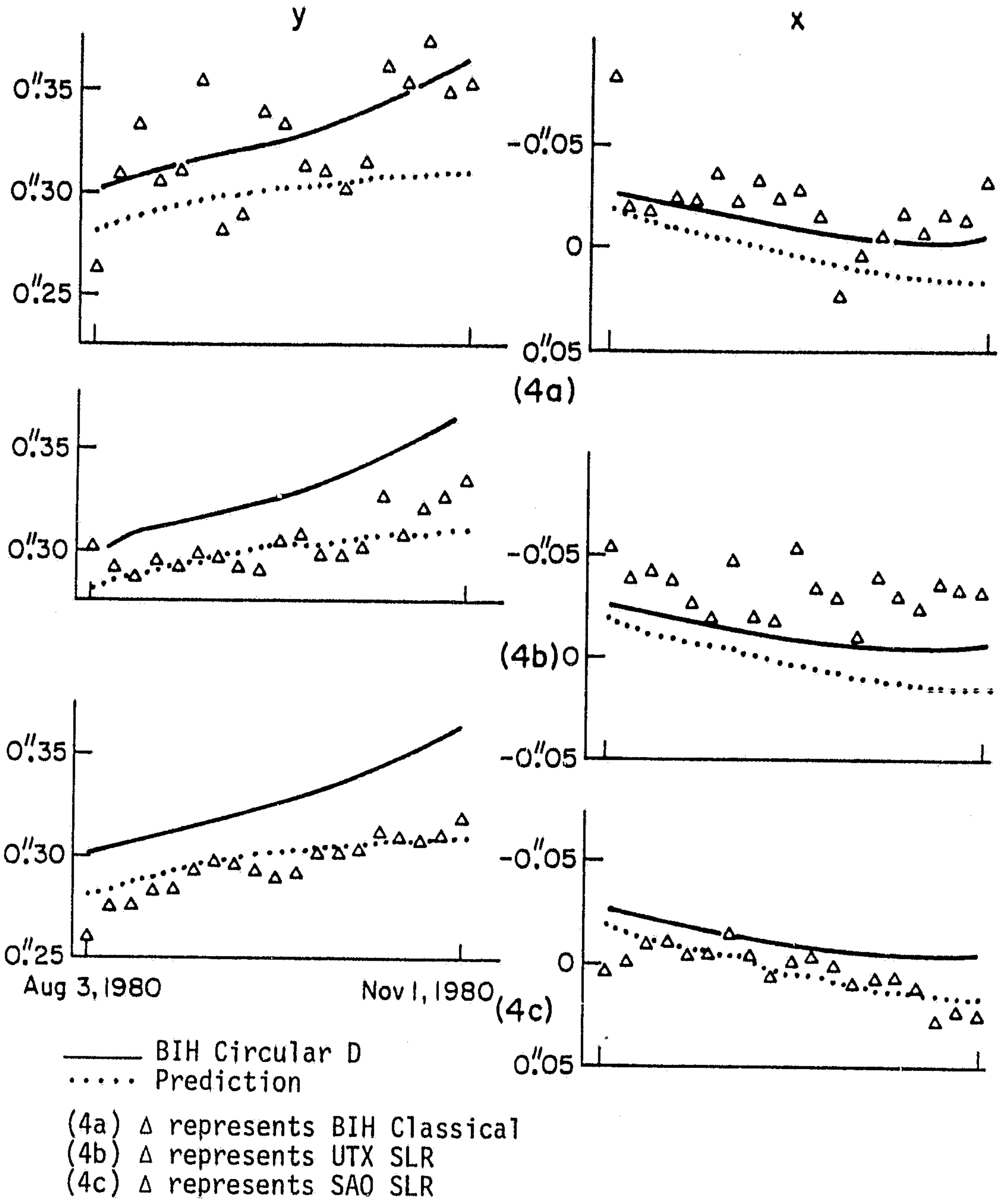


Fig. 4 Prediction versus MERIT Short Campaign.

Table 9   Rms of Relative Polar Motion from the MERIT Short Campaign (0".001)

| Time interval: | 5-day | | | | 15-day | | | | 30-day | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference: | BIH Circular D | | Prediction | | BIH Circular D | | Prediction | | BIH Circular D | | Prediction | |
| | x | y | x | y | x | y | x | y | x | y | x | y |
| BIH Circular D | -- | -- | 1.2 | 2.7 | -- | -- | 3 | 8 | -- | -- | 5 | 14 |
| Prediction | 1.2 | 2.7 | -- | -- | 3 | 8 | -- | -- | 5 | 14 | -- | -- |
| BIH (raw) | 13 | 16 | 12 | 16 | 14 | 18 | 16 | 19 | 20 | 14 | 17 | 22 |
| BIH Optical | 22 | 31 | 22 | 31 | 24 | 31 | 26 | 31 | 29 | 25 | 31 | 30 |
| DMA 5-day Mean (raw) | 12 | 11 | 12 | 12 | 11 | 9 | 11 | 11 | 12 | 8 | 14 | 16 |
| UTX | 16 | 14 | 16 | 14 | 15 | 14 | 15 | 15 | 19 | 16 | 22 | 18 |
| SAO | 7 | 5 | 7 | 5 | 11 | 9 | 10 | 8 | 14 | 12 | 14 | 10 |

Table 10 Two-year Prediction Errors for IPMS and BIH

| System: | BIH | IPMS |
|---|---|---|
| 1975 | 18 | 13 |
| 1976 | 21 | 22 |
| 1977 | 25 | 24 |
| 1978 | 22 | 18 |
| Mean | 22 | 20 |

Table 11 Relative Polar Motion Errors for Two-year Predictions ($0\overset{''}{.}001$)

| Time interval: | 5-day | 10-day | 20-day | 30-day | 40-day | 60-day |
|---|---|---|---|---|---|---|
| 1979 | 1.8 | 3.6 | 7 | 10 | 13 | 17 |
| 1980 | 2.8 | 5.4 | 10 | 14 | 17 | 22 |

Fig. 5 gives the long-term (four-year) prediction results. The data used are BIH polar motion from 1972.0 to 1977.95; the predictions are extended from 1978.0 to 1981.9. Compare the predicted pole path with the BIH results. The BIH pole path 78.0 - 81.0 is from BIH "Annual Report for 1980"; the path from 81.0 to 81.6 is drawn according to BIH Circular D data. From the curve we see that the predicted polar motion generally follows the trend of the observed polar motion quite well. It shows that although the real polar motion path might have some changeable complicated details, the main trend is regular and predictable.

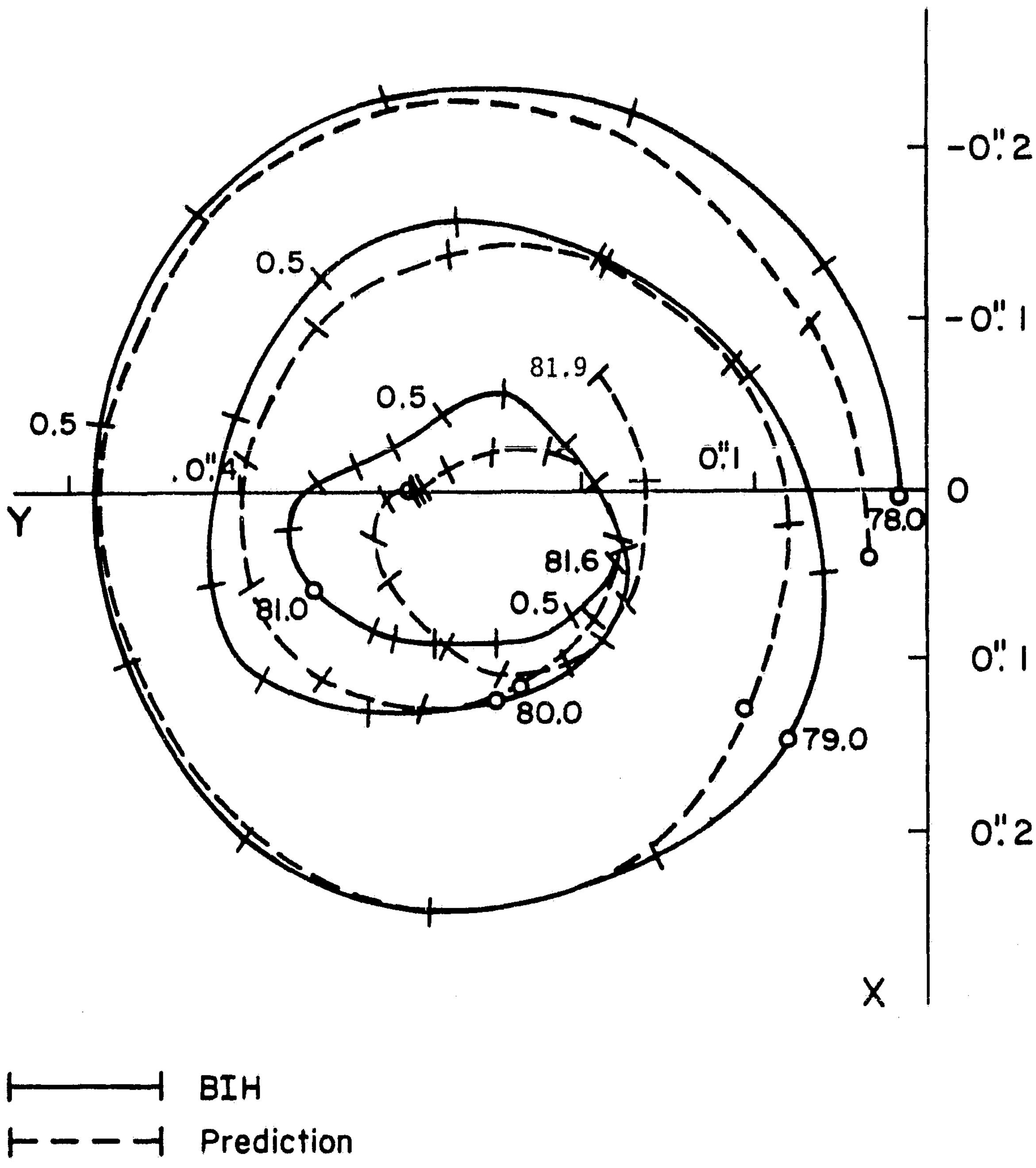


Fig. 5 Long-term (4-yr) predicted pole path versus BIH pole path (1978.0 - 1981.9)

## 3. ERROR ANALYSIS

As mentioned before, besides the Chandler motion and annual motion, real polar motion has other components. Furthermore, the Chandler and annual components may vary with time so that in the prediction there are modeling errors. At the same time, the prediction error also includes observing errors.

From (2) we get:

$$
\begin{aligned}
dx &= dx_o + dK_x(t-t_o) + db_x \sin 2\pi(t-t_o) + dc_x \cos 2\pi(t-t_o) + \\
&\quad + dB_C \sin \frac{2\pi(t-t_o)}{P_C} + dC_C \cos \frac{2\pi(t-t_o)}{P_C} + \delta x \\
dy &= dy_o + dK_y(t-t_o) + db_y \cos 2\pi(t-t_o) - dc_y \sin 2\pi(t-t_o) + \\
&\quad + dB_C \cos \frac{2\pi(t-t_o)}{P_C} - dC_C \sin \frac{2\pi(t-t_o)}{P_C} + \delta y
\end{aligned}
\tag{3}
$$

The error in $P_C$ is included in $\delta x$ and $\delta y$; these two terms may also include other modeling errors, random errors, etc. Averaged over one year (or two) we obtain:

$$
\begin{aligned}
\sigma_x^2 &\approx \sigma_{x_o}^2 + \sigma_{x_K}^2 \overline{(t-t_o)^2} + \tfrac{1}{2}(\sigma_{b_x}^2 + \sigma_{c_x}^2 + \sigma_{B_C}^2 + \sigma_{C_C}^2) + \sigma_{\delta_x}^2 \\
\sigma_y^2 &\approx \sigma_{y_o}^2 + \sigma_{y_K}^2 \overline{(t-t_o)^2} + \tfrac{1}{2}(\sigma_{b_y}^2 + \sigma_{c_y}^2 + \sigma_{B_C}^2 + \sigma_{C_C}^2) + \sigma_{\delta_y}^2
\end{aligned}
\tag{4}
$$

Each of the above coefficients may contain modeling errors as well as observing error ingredients. We will try to estimate how much might be ascribed to modeling errors and how much to observing errors. In the case of observing errors, we are mainly concerned with systematic errors.

### 3.1 Systematic Errors in the BIH and IPMS Results

It is known that the BIH 1968 system has systematic errors, mostly in x (and UT1 - UTC) (see BIH Annual Report for 1979, p. D-77, and [Feissel, 1980]). We refer to these as the constant parts of systematic errors.

Besides these, there remains a changing part. Writing the difference of IPMS - BIH results in the form

$$\text{difference} = A + B \sin 2\pi T + C \cos 2\pi T + D \sin 4\pi T + E \cos 4\pi T \quad (5)$$

Table 12 is extracted from BIH Annual Report for 1979, page D-98. From this table we can see that the coefficients of A, B, C, D, E not only have a non-zero mean value but also change from year to year. For the purpose of prediction, the mean values are not important--they are cancelled in the (prediction - observation)[1] but the changing part of the

Table 12 Comparison to the BIH·Global Solution (IPMS - BIH)

**UNITS : 0.001 ARCSECOND**

| | X | | | | | Y | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **YEAR** | **A** | **B** | **C** | **D** | **E** | **A** | **B** | **C** | **D** | **E** |
| 1962 | 36 | -4 | -2 | 8 | -2 | -19 | -8 | 43 | 2 | -2 |
| 1963 | 46 | 0 | 15 | 5 | -4 | -23 | -20 | 20 | -2 | -3 |
| 1964 | 44 | -1 | 15 | 18 | 4 | -20 | 17 | 17 | 1 | -1 |
| 1965 | 26 | -14 | 16 | 10 | -10 | -12 | 3 | 20 | -1 | -7 |
| 1966 | 24 | -5 | 15 | 6 | -11 | -3 | 8 | 12 | 0 | -4 |
| 1967 | 33 | -2 | 5 | 12 | 2 | 1 | -7 | 13 | -2 | -6 |
| 1968 | 33 | 4 | 11 | 3 | -1 | -6 | 19 | 12 | 12 | -1 |
| 1969 | 30 | -9 | 10 | 3 | -7 | -8 | 12 | -5 | 6 | -7 |
| 1970 | 23 | -17 | 3 | 0 | -4 | 6 | 10 | -19 | -2 | -12 |
| 1971 | 20 | -13 | 6 | -4 | -6 | 11 | 23 | 3 | 1 | 0 |
| 1972 | 24 | -4 | 0 | -3 | -8 | 5 | 11 | -4 | 1 | -1 |
| 1973 | 28 | -8 | -2 | -5 | -2 | 10 | 9 | -2 | -3 | -9 |
| 1974 | 36 | -3 | 7 | -6 | -8 | 11 | 2 | -7 | 1 | -1 |
| 1975 | 37 | -5 | 14 | 1 | 3 | 9 | 11 | -2 | -7 | 0 |
| 1976 | 25 | -4 | 11 | -6 | 7 | 6 | 12 | 0 | -1 | 3 |
| 1977 | 39 | 6 | 20 | -3 | 1 | -4 | 15 | 10 | -1 | 3 |
| 1978 | 43 | 13 | 8 | 6 | 0 | -7 | 11 | 10 | 0 | 7 |
| 1979 | 37 | 13 | -4 | 6 | -3 | -3 | 19 | 3 | 8 | 4 |
| **MEAN** | 32 | -3 | 8 | 3 | -3 | -3 | 8 | 7 | 1 | -2 |
| $\sigma$ | 8 | 7 | 7 | 7 | 7 | 13 | 11 | 14 | 4 | 5 |

[1]If we use BIH data to make predictions and compare with other techniques, say VLBI, then the constant part will remain in the prediction error. However, for the BIH 1979 system such systematic differences between the BIH and new techniques (see [Robertson et al., 1980]) have already been removed by an ad hoc correction [Feissel, 1980].

A, B, C, D, E coefficients will influence the prediction error. Therefore this part is more dangerous. From the given coefficients, it is easy to calculate the $\sigma_A$, $\sigma_B$, etc. These sigma values are given at the bottom of Table 12.

Since all these values are calculated from (IPMS - BIH), any real polar motion ingredients are cancelled. Therefore, $\sigma_A$, $\sigma_B$, etc. only reflect the observing systematic error. $\sigma_A$ corresponds to $\sigma_{x_o}$ (or $\sigma_{y_o}$) in equation (4); similarly $\sigma_B$ corresponds to $\sigma_{b_x}$ (or $\sigma_{b_y}$), $\sigma_C$ to $\sigma_{c_x}$ (or $\sigma_{c_y}$). $\sigma_D$, $\sigma_E$ will go into $\sigma_{\delta_x}$ (or $\sigma_{\delta_y}$).

Suppose that $\sigma_A$, etc. are half due to IPMS and half due to BIH, then by simple calculation we can see that these changing systematic errors above will cause prediction errors of 0".009 - 0".013. The prediction error due to all systematic and random errors together will reach about 0".015 for the BIH and IPMS.

## 3.2 Systematic Errors in DMA Polar Motion

The polar motion determined by DMA is better than by optical instruments. Nevertheless, Doppler polar motion also suffers from systematic errors as can be seen below by comparing polar motion results determined from different satellites.

In Fig. 6 we plot the differences of Satellites 34-67, 92-67, and 81-67. We can see that from 1976.0 to 1976.40 every x coordinate of Satellite 34 is larger than that of Satellite 67, while the difference of y coordinates shows a linear trend. From 1976.45 to 1977.45 the difference in x between Satellite 34 and Satellite 67 has a periodic feature. In 1979, every x coordinate of Satellite 92 is larger than that of Satellite 67, and the difference in y still shows periodic properties. In 1980, the systematic differences between Satellites 92 and 62 have approximately the same feature as in 1979, especially the y components. For comparison we put the two-year differences together in Fig. 7 (the scale is different from that in Fig. 6). The periodic feature is prominent.

As in the BIH Annual Reports, we use the differences to form A, B, C, D, E of equation (5). The results are listed in Table 13. When these systematic errors are removed, the rms of residuals reduced to 0".002 - 0".007. It then matches well with the (random) standard error of DMA.

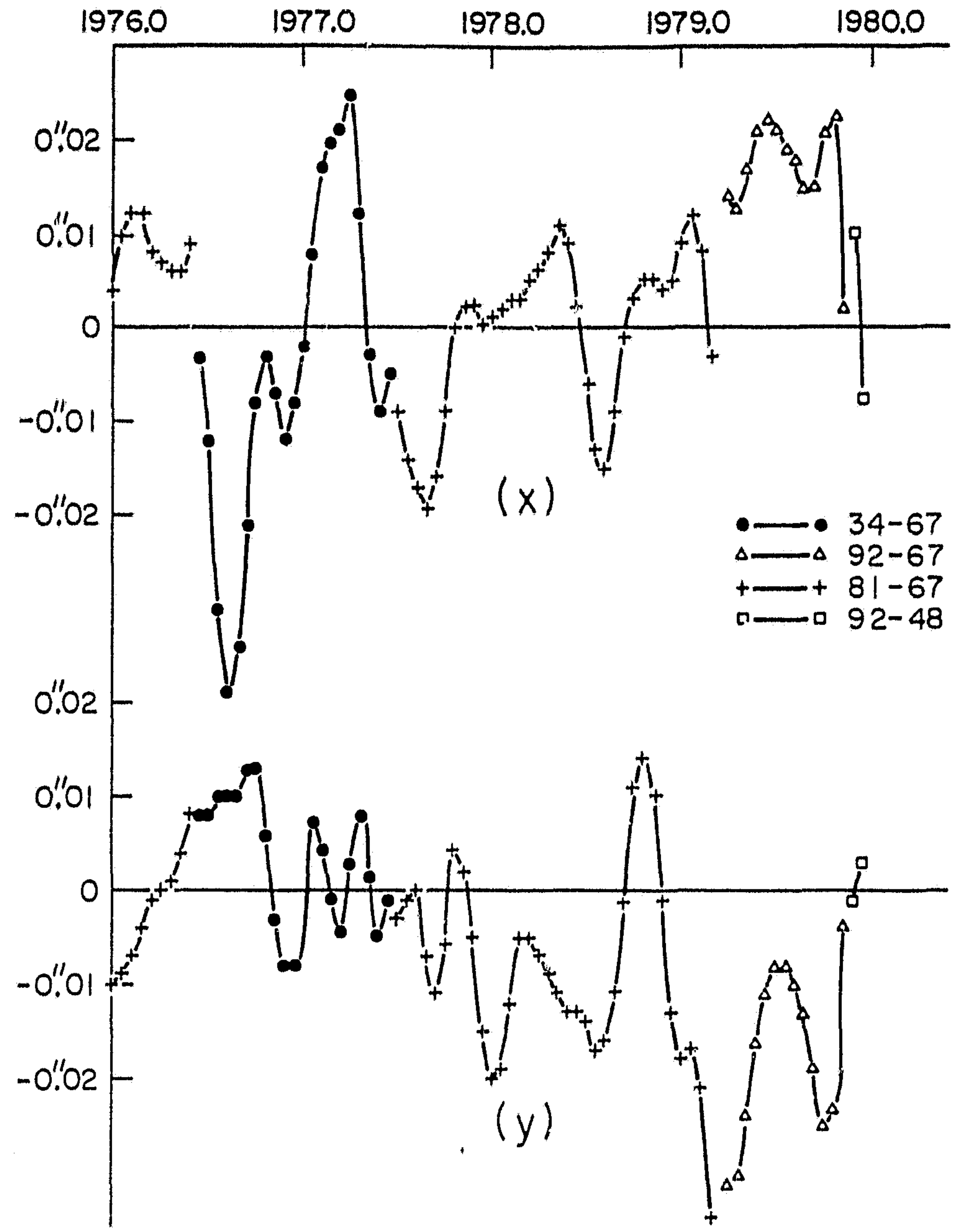


Fig. 6 Polar motion differences between different Transit satellites.

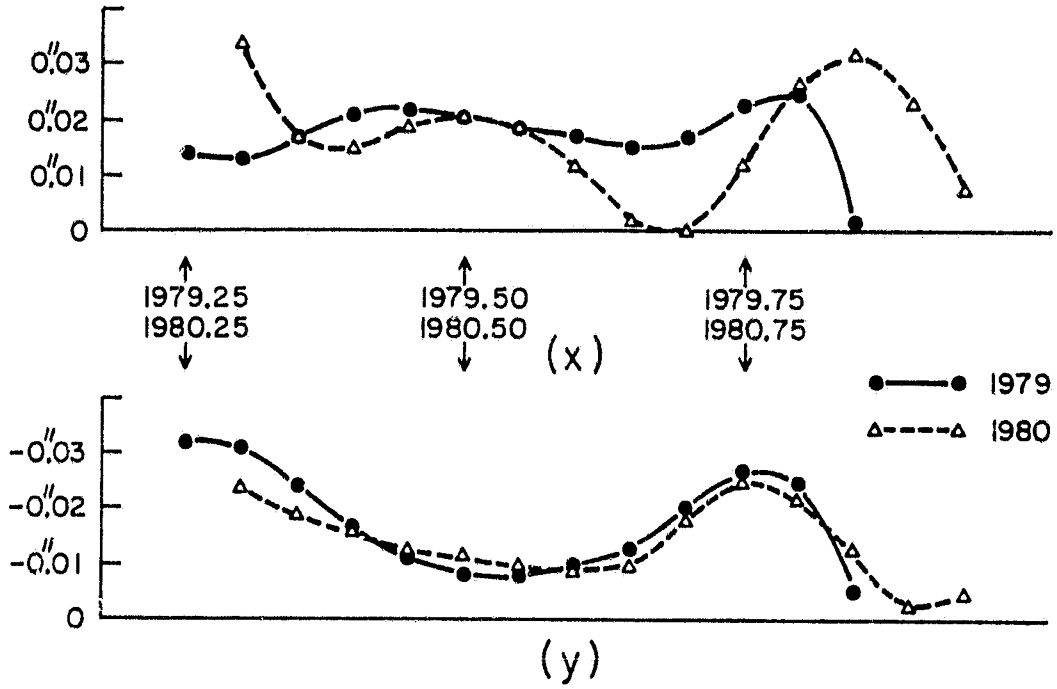


Fig. 7 Polar motion differences between Satellites 92 and 67.

Table 13 Systematic Differences Between Different Satellites (0".001)

| Date | Satellites | x A | x B | x C | x D | x E | y A | y B | y C | y D | y E |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1976.00-76.40 | 81-67 | 8 | (0) | (1) | (2) | (-1) | 2 | (1) | (6) | (6) | (-2) |
| 1976.45-77.45 | 34-67 | -4 | 21 | 1 | -7 | -1 | 3 | 5 | 2 | 0 | -5 |
| 1977.50-79.15 | 81-67 | -1 | -8 | -5 | -5 | 3 | -8 | 7 | -1 | -8 | 2 |
| 1979.25-79.85 | 92-67 | 17 | (3) | (-1) | (-2) | (1) | -17 | (1) | (-6) | (-7) | (-7) |

( ) indicates the time interval is not long enough to make appropriate estimates.

According to the explanation of Anderle [1981], the bias between Satellites 92 and 62 may be caused by the modeling error (mostly the gravity field model). that is, different satellites have different modeling errors.

If DMA had continued to observe the same satellite, its A, B, C, D, E coefficients would not have changed radically. In this case, as stated before, these A, B, C, D, E coefficients would not have caused the prediction error. But over the years, DMA has changed satellites several times, and the A, B, C, D, E coefficients have changed from satellite to satellite and from year to year. This situation would cause significant prediction error. This might be the reason why the prediction using DMA 0.05-yr data did not show a large improvement over that using BIH data.

The systematic error between different satellites implies that every new technique might in some way suffer from unexpected systematic errors. Before we use any new technique for constructing a polar motion service, we must have certain time intervals of simultaneous observations and must analyze them carefully for possible systematic errors.

### 3.3 Modeling Error

As mentioned before, in prediction the constant part of (annual and other) systematic errors is canceled; only the changing part remains. At the same time there is the real change of polar motion parameters (the modeling error). To detect these, we use BIH data to make predictions and compare it with BIH observations. We do the same for IPMS and DMA data.

Fig. 8 shows the prediction error of BIH, IPMS and DMA in 1979. We know that BIH and DMA are correlated. BIH and IPMS may also be correlated due to many common observing instruments, but DMA and IPMS must be independent. The similarity of the three curves reflects the influence of modeling errors.

In the common part, we find an annual term with a semi-amplitude of 0".013 for x and 0".017 for y. Because the time interval of the above calculations is only one year, we could not determine whether the modeling error is due to the Chandler motion change or annual change, or both.

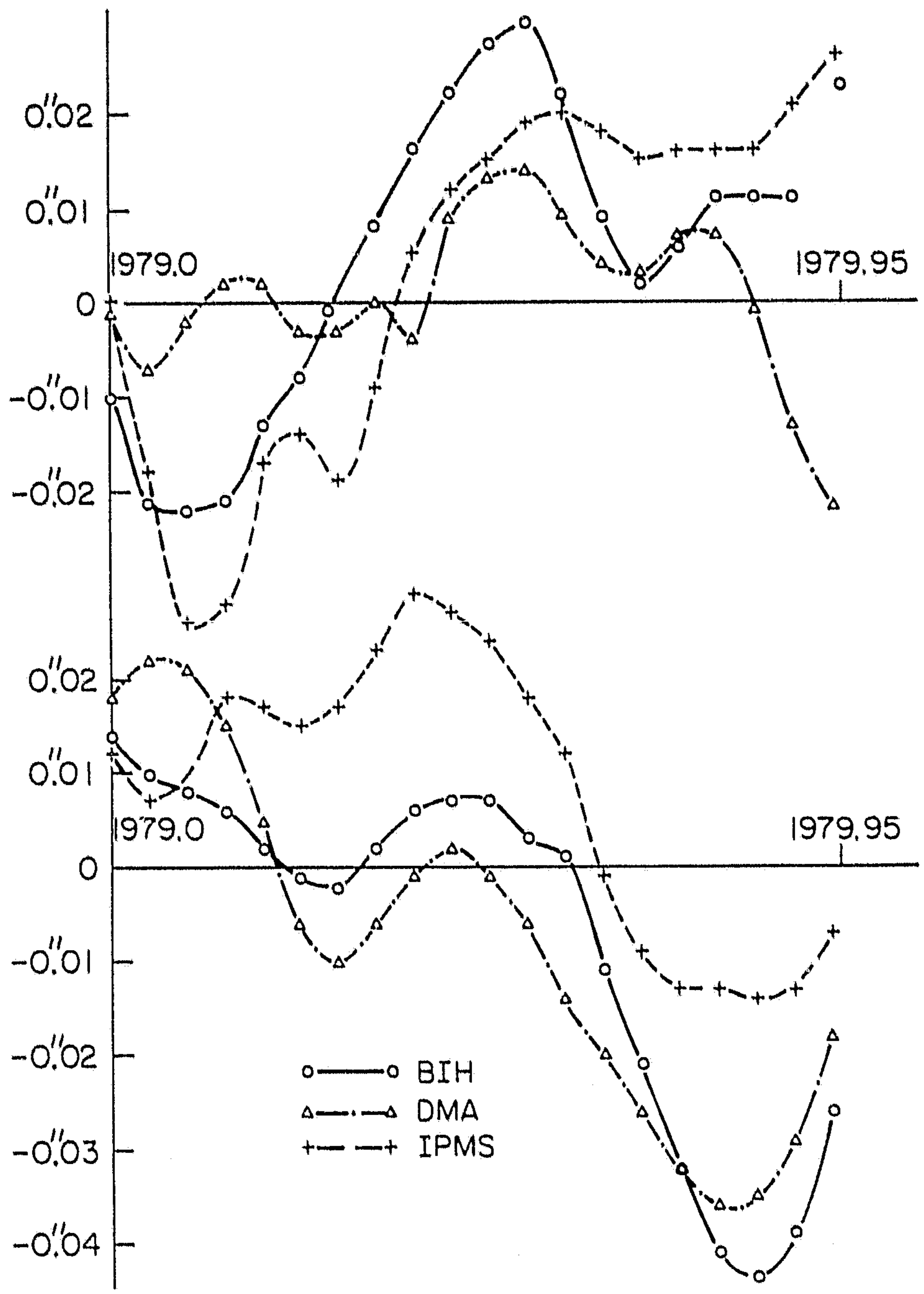


Fig. 8 (Predicted - observed) in 1979.

The modeling error depends on the excitation and dissipation of the polar motion so it varies with time. Since the overall prediction error is about 0".02 and the part produced by the observing error is about 0".015, the average modeling error in these years is also about 0".015.

Comparing the polar motion amplitude with the modeling error, we conclude that 80 - 90% of polar motion is composed of the stable (at least stable within several years) Chandler and annual motions, and so it is predictable.

## 3.4 The Problem of ILS Data

### 3.4.1 Systematic error in ILS

We know that the most important source of polar motion data before 1962 is provided by the ILS. We do not know directly the quality of the data before 1962. After 1962 we have simultaneous polar motion data provided by the ILS and by the BIH. Therefore, we can use the BIH as a reference to estimate the error in the ILS data.

As in Section 3.1 we used the BIH Annual Report for 1979 (page D-97) to get the changing part of the systematic error in (BIH - ILS). The results are listed at the bottom of Table 14.

In this case we think the systematic difference is mostly due to the ILS rather than to the BIH. In several instances there occur large changes in the coefficients. Take the x coordinates of 1976 and 1979 as an example:

$$B_{x_{1979}} = -0''.098 \qquad C_{x_{1979}} = -0''.037$$

$$B_{x_{1976}} = 0''.072 \qquad C_{x_{1976}} = 0''.063$$

$$\Delta B_{x_{1976-79}} = 0''.170 \qquad \Delta C_{x_{1976-79}} = 0''.100$$

$$\sqrt{\Delta B_x^2 + \Delta C_x^2} \approx 0''.2$$

We see that the changing part of the systematic error itself is an annual term with a semi-amplitude of 0".2. With such poor quality, one can hardly determine the real change of the polar motion parameter from the pseudo-polar motion change caused by nonpolar error.

The above systematic errors together with the random error (0".03) may cause a 0".04 - 0".05 prediction error.

Table 14 Comparison to the BIH Global Solution (ILS - BIH)

UNITS : 0.001 ARCSECOND

| | X | | | | | Y | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| YEAR | A | B | C | D | E | A | B | C | D | E |
| 1962 | 54 | 5 | 6 | 6 | 8 | -22 | 15 | 50 | 4 | 3 |
| 1963 | 64 | -7 | 9 | 11 | 5 | -28 | -8 | 16 | 2 | 5 |
| 1964 | 56 | 4 | 11 | 8 | 5 | -22 | 15 | 22 | -10 | -8 |
| 1965 | 25 | -25 | 20 | 10 | -4 | -3 | 3 | 16 | -5 | -5 |
| 1966 | 21 | 0 | 20 | 0 | -24 | 0 | 0 | 32 | 14 | 5 |
| 1967 | 11 | -4 | -12 | 25 | 7 | -4 | -13 | 41 | 3 | -4 |
| 1968 | 7 | 11 | -2 | 6 | -3 | 3 | 7 | 40 | 7 | 4 |
| 1969 | 25 | 10 | -14 | 9 | -8 | 2 | 14 | 14 | 3 | 2 |
| 1970 | 13 | -22 | 2 | -3 | -2 | 13 | 6 | 3 | 0 | -17 |
| 1971 | 23 | 5 | -27 | -16 | 5 | 9 | -30 | 13 | -15 | 8 |
| 1972 | 45 | 23 | 2 | -10 | 1 | 8 | -13 | 12 | -19 | -2 |
| 1973 | 58 | 31 | 3 | 7 | -6 | -10 | 2 | 5 | 11 | 1 |
| 1974 | 61 | -13 | 2 | -11 | -5 | -21 | -19 | 18 | -9 | -5 |
| 1975 | 49 | 2 | 23 | -4 | -1 | -5 | 4 | 9 | -13 | 3 |
| 1976 | 7 | 72 | 63 | -14 | -14 | -4 | 11 | 20 | -18 | -6 |
| 1977 | 40 | 56 | -14 | 3 | -8 | -11 | 42 | 42 | -11 | 7 |
| 1978 | 55 | 13 | -30 | 14 | -4 | 5 | 33 | 18 | -5 | 9 |
| 1979 | 78 | -98 | -37 | -14 | -28 | 6 | 11 | 10 | -2 | -20 |
| MEAN | 38 | 4 | 1 | 1 | -4 | -5 | 4 | 21 | -3 | -1 |
| $\sigma$ | 22 | 34 | 22 | 11 | 10 | 12 | 17 | 14 | 9 | 8 |

3.4.2 Using previous data to make predictions

We used ILS data to make predictions and compared them with ILS observations. The average rms of (pred.-obs.) for the time interval 1962-1979 is 0".05. Using the data for 1949-1962, we made the same comparison and obtained an average rms of 0".05.

From ILS data it has been concluded that in 1949-1962 the polar motion parameters, especially the Chandler amplitude, were changing rapidly. It has been thought that polar motion was somehow stable from 1962 to 1976. But the fact is that in both the 1949-1962 and

1962-1979 periods, the overall errors of prediction are nearly the same (0".05). The poor prediction accuracy of the ILS may be due mostly to observing errors in the ILS.

The same situation exists for BIH predictions. In Table 5 we have listed only the results using the data from 1968. Using data before 1968, for instance the BIH 1962-1968 data, to make predictions, the rms increases to 0".03 because of the relatively poor quality at that time. Therefore, we assume that in the past if high quality observations had been provided, high accuracy predictions could also have been made.

## PART II

## PREDICTION OF EARTH ROTATION

### 1. IMPROVING THE RAPID SERVICE OF EARTH ROTATION (UT1 - UTC)

One of the factors that influences the accuracy of the rapid service of earth rotation is the error of polar motion provided. To obtain UT1 from UT0, a correction caused by polar motion must be added. From [Mueller, 1969]

$$\Delta\Lambda_p = -(x_p \sin \Lambda + y_p \cos \Lambda) \tan \phi \tag{6}$$

From the above we get

$$\sigma^2_{\Delta\Lambda_p} = (\sigma^2_{x_p} \sin^2 \Lambda + \sigma^2_{y_p} \cos^2 \Lambda) \tan^2 \phi \tag{7}$$

Assuming that $\sigma_{x_p} = \sigma_{y_p} = \sigma$ and $\sigma_{x_p y_p} = 0$

then

$$\sigma_{\Delta\Lambda_p} = \sigma \tan \phi \tag{8}$$

Since the error of the now available extrapolated or predicted polar motion is much larger than that of Circular D, its influence on UT1 is also large. Using our method for polar motion prediction, this error source can be reduced.

### 2. EARTH ROTATION PREDICTION

We attempted to use BIH Circular D UT1-TAI data to make predictions. The variation of the earth rotation is more complicated than that of polar motion and the accuracy with which UT1 is determined is worse than that of polar motion. Therefore, the earth rotation prediction is more difficult and less satisfactory than for polar motion. So far we have not succeeded in finding a method for long-period interval prediction. Prediction errors increase rapidly with time.

The best results we have obtained up to now are shown in Table 15 and Fig. 8.

Table 15 Errors of Earth Rotation Prediction (unit 0.0001 s)

| Time | 5-day | 10-day | 15-day | 20-day | 25-day | 30-day | 35-day | 40-day |
|---|---|---|---|---|---|---|---|---|
| $\sigma$ | 1.8 | 4 | 7 | 12 | 17 | 23 | 30 | 37 |

Here $\sigma$ is the rms of (pred.-obs.).

The principle of the method used is first to estimate the annual, semiannual and linear parameters by a least squares fit. Then taking the residuals of the above step as input data, we predict the future residuals by linear filtering methods.

As pointed out by Guinot [1979], the seasonal variation of UT1 is variable from year to year. Therefore, instead of using the conventional value

$$UT2 - UT1 = 0.022 \sin 2\pi t - 0.012 \cos 2\pi t - 0.006 \sin 4\pi t + + 0.007 \cos 4\pi t$$

we estimate the annual and semiannual coefficients from the data provided. In addition, we also estimate the linear trend from data of the past three years. The equation used is as follows:

$$UT1 - TAI = a_0 + a_1 (t-t_o) + b \sin 2\pi (t-t_o) + c \cos 2\pi (t-t_o) + + d \sin 4\pi (t-t_o) + e \cos 4\pi (t-t_o) \tag{9}$$

Then, form the residuals

$$v_t = (UT1 - TAI)_t - a_0 - a_1 (t-t_o) - b \sin 2\pi (t-t_o) - c \cos 2\pi (t-t_o) - d \sin 4\pi (t-t_o) - e \cos 4\pi (t-t_o) \tag{10}$$

Taking the residuals as input data, we used a linear filtering method to predict the future residuals. The filtering model used is

$$R_{jt} = \sum_k P_{jk} R_{jt-k} \tag{11}$$

Experiments have shown that when the number of coefficients $P_{jk}$ equals about 20, the results are the best. To estimate $P_{jk}$, only about one and one-half year's residuals are needed.

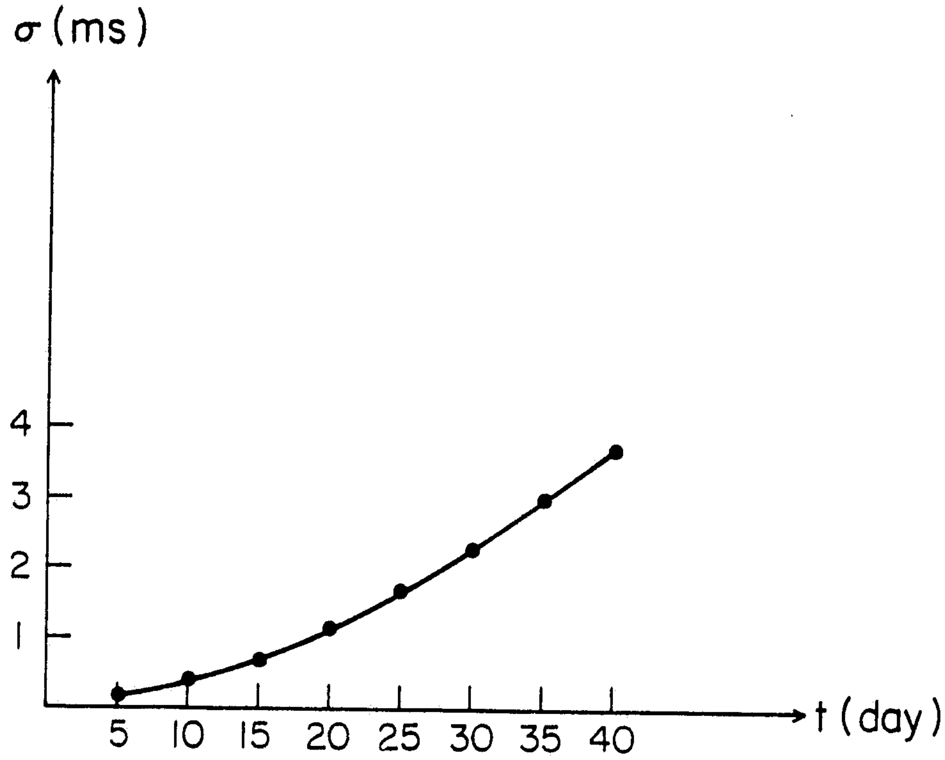


Fig. 8 Prediction error for UT1 - TAI.
(Time = 0 means the time of last data available.)

Only the first one or two 5-day residual predictions are made by this method; then the linear model is used for predicting the other days' residuals. Finally, adding

$$a_0 + a_1(t-t_0) + b \sin 2\pi(t-t_0) + c \cos 2\pi(t-t_0) + d \sin 4\pi(t-t_0) + e \cos 4\pi(t-t_0)$$

to the predicted residuals, we get the predicted values of UT1 - TAI.

We used the BIH Circular D data set beginning from 1974 to make about 200 sets of predictions. Each set includes predictions from five days up to 40 days. We then compared the predicted values with the later observed ones. From 200 such values we arrived at the $\sigma$ values listed in Table 15.

We take one of the worst and one of the best cases as examples. One of the best involves BIH Circular D No. 147 dated Feb. 2, 1979, giving UT1 - TAI up to Jan. 1. We used the data set from 1977 Jan. 6 to 1980 Jan. 1 to make predictions. The results are listed in Table 16.

Table 16 One of the Best Predictions of UT1 - TAI (0.001 s)

| Predicted Date | Predicted UT1-TAI | Observed UT1-TAI | Pred.-Obs. |
|---|---|---|---|
| 1980 Jan 6 | 17418.85 | 17419.0 | -0.15 |
| 11 | 433.21 | 433.4 | -0.19 |
| 16 | 477.55 | 477.9 | -0.35 |
| 21 | 461.92 | 462.2 | -0.28 |
| 26 | 476.37 | 476.6 | -0.23 |
| 31 | 490.92 | 491.0 | -0.08 |
| Feb 5 | 505.64 | 505.6 | 0.04 |
| 10 | 520.54 | 520.1 | 0.44 |

The observed UT1-TAI were taken from BIH Circular D Nos. 148 and 149 published later.

One of the worst cases involves Nov. 1, 1980, data published on Dec. 4, 1980, in Circular D No. 169. The results are shown in Table 17.

Table 17 Worst Prediction of UT1-TAI (0.001 s)

| Predicted Date | Predicted UT1-TAI | Observed UT1-TAI | Pred.-Obs. |
|---|---|---|---|
| 1980 Nov 6 | 19061.66 | 19061.2 | 0.46 |
| 11 | 075.17 | 074.1 | 1.07 |
| 16 | 088.72 | 086.9 | 1.82 |
| 21 | 102.25 | 099.5 | 2.75 |
| 26 | 115.75 | 111.9 | 3.85 |
| Dec 1 | 129.19 | 124.1 | 5.09 |
| 6 | 142.56 | 136.2 * | 6.36 |
| 11 | 105.84 | 148.2 * | 7.64 |

* The observed UT1-TAI were taken from Circular D Nos. 170 and 171. (No. 171 has printing errors. We used the corrected values.)

The time delay of BIH Circular D is about one month. From Table 15 and Fig. 8 we can see that the real time prediction accuracy ($\sigma$) is about 2.3 ms. From Fig. 8 it is seen that if we need real time prediction errors of less than 1.5 ms, then the BIH results must be available within 23 days.

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