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EVOLUTIONARY VARIATIONS OF SOLAR LUMINOSITY*

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ABSTRACT

Theoretical arguments for a 30% increase in the solar luminosity over the past 4.7 billion years are reviewed. A scaling argument shows that this increase can be predicted without detailed numerical calculations. The magnitude of the increase is independent of nuclear reaction rates, as long as conversion of hydrogen to helium provides the basic energy source of the Sun.

The effect of the solar luminosity increase on the terrestrial climate is briefly considered. It appears unlikely that an enhanced greenhouse effect, due to reduced gases (NH_3, CH_4), can account for the long-term paleoclimatic trends.

INTRODUCTION

Climatically significant changes of the solar luminosity (L) have been postulated to occur on time scales ranging from a few years to billions of years. The shorter time scales have been discussed extensively at this conference. In the present review, I will restrict myself to the longest time scales ($\lesssim 10^9$ yr.) and discuss the basis for the astrophysical conclusion that the Sun was $\sim$ 30% fainter 4.7×10^9 yr. ago and that the evolution since the Sun's formation requires a slow, but steady, increase in L.

I should note that this is the only change in L predicted by stellar evolution theory, in its standard form. This prediction is common to all modern calculations and is supported by a large body of data from observational stellar astronomy (see reference 1 for a review of the observational evidence). Nevertheless, the validity of this result has been questioned because of the apparent conflict with proxy indicators of the Earth's past climate (ref. 2-4). For this reason, a review of the theoretical arguments for the long-term increase of L is in order.

Stellar evolution is governed by nonlinear differential equations derived from conservation laws and considerations of energy transport processes. Analytic solutions do not exist for any cases relevant to the Sun so numerical solutions must be used. Modern calculations require complex computer codes incorporating a variety of physical data on nuclear parameters, transport coefficients, and thermodynamic properties. In this respect, the situation is

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similar to that encountered in current theoretical investigations of the terrestrial climate. This may, in fact, explain the reluctance of the climatologists to accept the astrophysical result: climatologists understand the pitfalls of accepting solutions obtained from complex computer codes at face value. For this reason, I will largely avoid discussion of numerical models and base the astrophysical case on simple scaling laws.

A SCALING MODEL OF THE SUN

We begin by requiring that the Sun be in hydrostatic equilibrium, with gravitational forces balanced by the pressure gradient. The free-fall time of the Sun is on the order of an hour and any departures from hydrostatic equilibrium would show up as luminosity and radius changes on this time scale. For the spherically symmetric case, hydrostatic equilibrium is expressed as

$$\frac{dP}{dr} = -\frac{Gm_r}{r^2}\,\rho\,, \tag{1}$$

where G is the gravitational constant, P and ρ are the pressure and density at a distance r from the center and m_r is the mass interior to r. Measurements of the visible solar disk show that the Sun is spherical to within 1 part in 10^5 (ref. 5).

We can construct a one-zone model by replacing (1) by a finite-difference equation evaluated between the center c and surface s, with mean values enclosed in brackets $\langle\ \rangle$:

$$\frac{P_c - P_s}{r_c - r_s} = -G\left\langle \frac{m_r\rho}{r^2}\right\rangle .$$

Applying the boundary conditions $P_s = 0$, $r_s = R$ (radius), and $r_c = 0$ gives

$$P_c = G\left\langle \frac{m_r\rho}{r^2}\right\rangle R. \tag{2}$$

The scaling laws for the mean values are:

$$\langle m_r\rangle \propto m, \tag{3}$$

$$\langle r\rangle \propto R, \tag{4}$$

and

$$\langle \rho \rangle = \frac{m}{\frac{4}{3}\pi R^3} \propto \frac{m}{R^3} , \tag{5}$$

where m is the total mass. Inserting these scaling laws into equation (2) gives

$$P_c \propto \frac{m^2}{R^4} . \tag{6}$$

To proceed further, we need an equation of state, relating the pressure to the density and temperature (T). For typical conditions characterizing the bulk of the solar interior ($\rho \simeq 1$ g/cm^3 and $T \simeq 10^6$ to 10^7 °K), Coulomb interaction energies are at least 2 orders-of-magnitude smaller than particle kinetic energies. Thus, the ideal gas law is an excellent approximation and this is what differentiates a star from a planet. Applying the ideal gas law to the center gives

$$P_c = \frac{N_A k}{\mu} \rho_c T_c , \tag{7}$$

where k is the Boltzmann constant and the particle density is expressed as the Avogadro number N_A divided by the mean mass μ (in atomic mass units) per free particle. Eliminating P_c between (6) and (7) and noting that the central density must scale as the mean density gives

$$T_c \propto \frac{\mu m^2}{R^4} \frac{1}{\langle \rho \rangle}$$

If the scaling law (5) is used to replace R, we get

$$T_c \propto \mu m^{2/3} \langle \rho \rangle^{1/3} . \tag{8}$$

We now turn to the question of how energy is transported from the core, where nuclear reactions produce energy, to the surface. Due to the high temperatures, radiative transport of energy is very efficient and dominates over the bulk of the interior. The mean free path ℓ of a photon is typically 1 cm so the photon diffusion approximation is valid to order $\ell/R \simeq 10^{-11}$. The radiative diffusion equation with spherical symmetry is

$$L_r = \frac{64\pi\sigma}{3} \frac{r^2 T^3}{\kappa\rho} \frac{dT}{dr} , \tag{9}$$

where L_r is the total flux across a spherical surface at distance r from the center, σ is the Stefan-Boltzmann constant and κ is the Rosseland-mean opacity coefficient. Again, we use the one-zone difference approximation to write this equation as

$$\langle L_r \rangle = \frac{16\pi\sigma}{3} \left\langle \frac{r^2 T^3}{\kappa\rho} \right\rangle \frac{T_c - T_s}{r_c - r_s} \quad .$$

Application of our previous boundary conditions plus $T_s = 0$ (i.e. $T_s << T_c$) gives

$$L \propto \frac{R^2 T^3}{\kappa\rho} \; \frac{T}{R} = \frac{RT^4}{\kappa\rho} \tag{10}$$

where, since we are now dealing with a scaling law (proportionality), L_r can be replaced by L, r by R, etc.

To evaluate the opacity coefficient κ, we note that, in the solar interior, hydrogen and helium will be completely ionized and the heavier ions will be stripped of most of their electrons. Hydrogen and helium affect κ through free-free transitions while the heavier elements contribute primarily through bound-free transitions. Both processes are reasonably represented by the hydrogenic approximation so the absorption coefficient for a given ion varies inversely with the cube of the frequency. Although individual ionization states may contribute "noise" to the detailed dependence of κ on ρ and T, the broad dependence is given by Kramers' opacity:

$$\kappa = \kappa_o \rho \; T^{-3.5} \quad . \tag{11}$$

Putting this result into equation (10), and using equation (8) to eliminate the temperature, gives

$$L \propto m^{5.33} \; \rho^{0.17} \; \mu^{7.5} \quad . \tag{12}$$

The present rate of mass loss, due to the solar wind, is roughly $10^{-14} m_o$/yr. (ref. 6) and there is no reason to believe that the mass loss rate in the past was great enough to significantly affect m. The density dependence in (12) is so weak that we may also neglect changes in this parameter. Thus, the luminosity is primarily dependent on the mean molecular weight μ and we rewrite (12) as

$$L(t) = L(o) \left[\frac{\mu(t)}{\mu(o)}\right]^{7.5} \tag{13}$$

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We let X, Y, and Z denote the fractional abundances, by mass, of hydrogen, helium, and metals (X+Y+Z = 1). For a fully ionized gas,

$$\mu \simeq \frac{1}{2X + \frac{3}{4}Y + \frac{1}{2}Z} = \frac{2}{1 + 3X + \frac{1}{2}Y} \qquad (14)$$

The mean molecular weight increases with time due to conversion of hydrogen ($\mu = 1/2$) into helium ($\mu = 4/3$), by nuclear reactions, producing $Q = 6\times10^{18}$ erg per gram of hydrogen consumed. This energy must supply the luminosity of the Sun. Since Xm is the total mass of hydrogen,

$$\frac{d(Xm)}{dt} = m\,\frac{dX}{dt} = -\frac{L}{Q} \quad ,$$

or

$$\frac{dX}{dt} = -\frac{L}{mQ} \qquad (15)$$

Differentiating (14) with respect to time, and noting that $dY/dt = -\,dX/dt$, we get

$$\frac{d\mu}{dt} = -\frac{5}{4}\,\mu^2\,\frac{dX}{dt} = \frac{5}{4}\,\frac{\mu^2 L}{mQ} \quad . \qquad (16)$$

Finally, we can eliminate μ between equations (13) and (16). The resulting differential equation can be directly integrated to give

$$L(t) = L(o)\left[1 - \frac{85}{8}\;\frac{\mu(o)\,L(o)}{mQ}\;t\right]^{-15/17} \qquad (17)$$

Since nuclear reactions are confined to the core, the present photospheric abundances should reflect the initial composition. Thus, we may evaluate μ_o using $X \simeq 0.71$ and $Z = 0.02$. Equation (17) becomes

$$L\left(\frac{t}{t_\odot}\right) = L(o)\left[1 - 0.35\;L(o)\left(\frac{t}{t_\odot}\right)\right]^{-15/17} \qquad (18)$$

where L is expressed in units of the present solar luminosity (taken as 3.9×10^{33} erg/s) and $t_\odot$ is the present solar age (4.7×10^{9} yr.). The initial luminosity required to match the present solar luminosity at $t_\odot$ is $L(o) = 0.76$.

The scaling arguments predict that the Sun was initially 24% fainter than the present luminosity. A comparison of the luminosity evolution according to equation (18) with results from detailed numerical models (ref. 7) is shown in figure 1. As noted by D. O. Gough, the evolution predicted by numerical models is accurately represented by

$$L = \left[1 + \frac{2}{5}\left(1 - \frac{t}{t_{\odot}}\right)\right]^{-1} L_{\odot} , \qquad (19)$$

where $L_{\odot}$ is the present solar luminosity. This formula, rather than equation (18), is recommended for studies of the evolution of planetary atmospheres.

SUMMARY OF THE ASTROPHYSICAL CASE

The above analysis shows that a quantitative prediction of the evolutionary increase of the Sun's luminosity may be made without detailed knowledge of the physical processes taking place in the interior. Therefore, this prediction is not affected by the uncertainties in this knowledge. In particular, we did not have to specify any nuclear reaction rates since the net reaction rate, integrated over the solar mass, is determined by the measured solar luminosity. This is quite different from the case of the solar neutrino prediction, which is very sensitive to detailed nuclear reaction rates (ref. 8). The discrepancy between the predicted and observed neutrino flux should not be used to argue that the luminosity prediction is also questionable.

APPLICATION TO THE EARTH'S CLIMATE

Sagan and Mullen (ref. 9) pointed out that an enhanced greenhouse effect, due to higher concentrations of NH_3 and CH_4 in the Earth's atmosphere, could have maintained a warm climate even with a lower solar luminosity. A similar conclusion was reached by Hart (ref. 10). This mechanism cannot, however, compensate for all of the solar luminosity evolution.

Paleological evidence (ref. 11) shows that the Earth's atmospheric chemistry changed from reducing to oxidizing some 1.5 to 2 billion years ago and this would have removed the enhanced greenhouse effect due to reduced compounds. Roughly one-half of the solar luminosity increase occurs during the last 2 billion years but there is no evidence for a parallel increase in the Earth's mean surface temperature. Indeed, isotopic studies of Precambrian samples by Knauth and Epstein (ref. 12) indicate that the mean surface temperature has been decreasing during this time. Clearly, there is a need for further studies of the effects of crustal movements and volcanism, biological activity, etc. on the long-term evolution of the Earth's climate. At present, it appears that the effects of solar evolution are still buried in the "noise" due to other uncertainties in paleoclimatic models.

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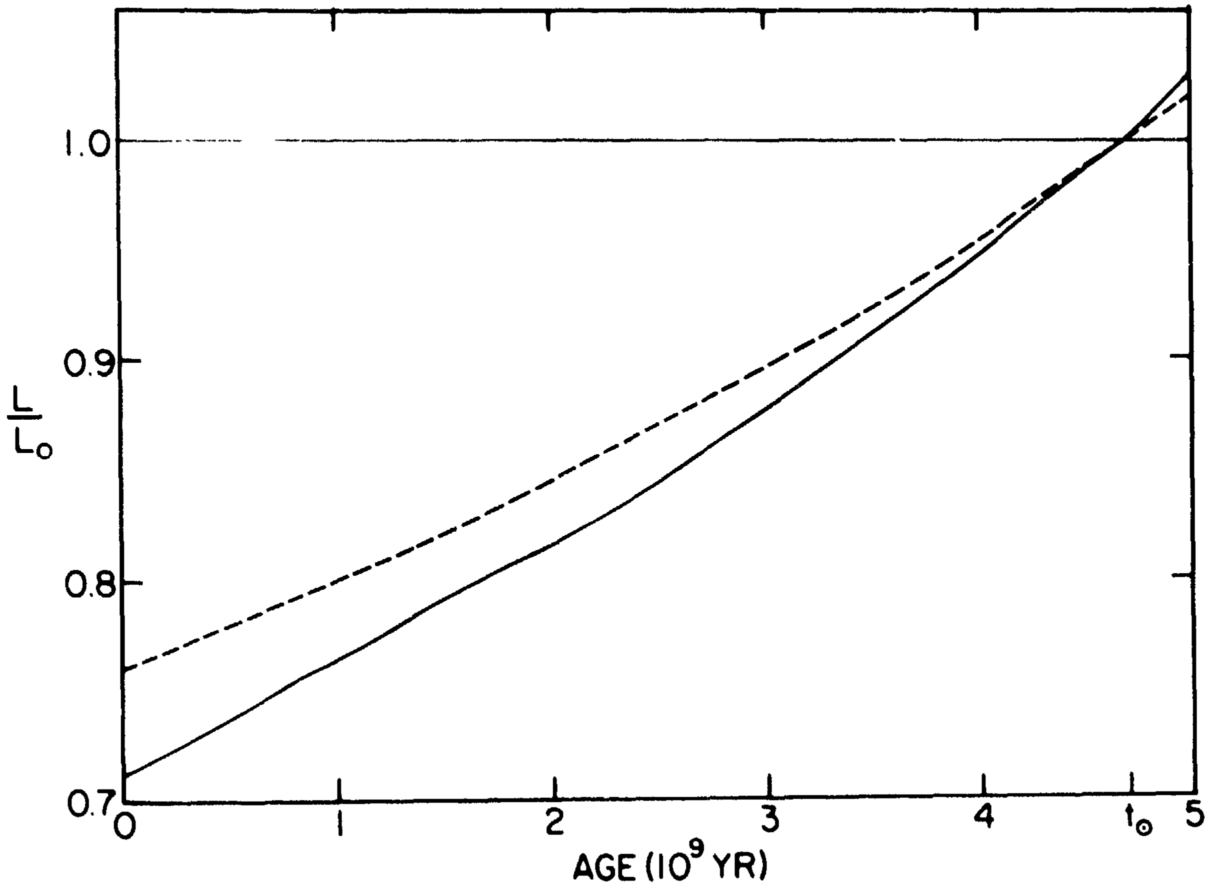

Figure 1. Long-term evolution of the solar evolution. The evolution predicted by the scaling model (Equation (18)) is shown by the dashed line and the prediction from a detailed computer model is shown by the solid line.