

ON THE SEAT OF THE SOLAR CYCLE

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This paper is a discussion of some of the issues that have been raised in connection with the seat of the solar cycle. Is the cycle controlled by a strictly periodic oscillator that operates in the core, or is it a turbulent dynamo confined to the convection zone and possibly a thin boundary layer beneath it? Sunspot statistics are discussed, with a view to ascertaining the length of the memory of the cycle, without drawing a definitive conclusion. Also discussed are some of the processes that might bring about variations δL and δR in the luminosity and the radius of the photosphere. It appears that the ratio $W = \delta \ln R / \delta \ln L$ increases with the depth of the disturbance that produces the variations, so that imminent observations might determine whether or not the principal dynamical processes are confined to only the outer layers of the sun.

INTRODUCTION

Early theories of the solar cycle were based on the idea that there is some periodic oscillation in which the entire sun participates. However, most solar physicists today probably believe that the cycle is the product of a turbulent dynamo in the convection zone. This belief appears to have been based originally on the premise that the solar interior could not possibly turn over in a time as short as 11 or 22 years, which would be necessary if the magnetic field external to the sun matched smoothly to the field beneath the convection zone. And the belief has been strengthened by the comparatively complicated theoretical edifice that has been erected to explain some of the observations in terms of a turbulent dynamo. That edifice has been of great use in helping us to understand the kinds of processes that are no doubt operative in the sun's convection zone, but one must be wary of taking too seriously the results of what are physically quite naive models. These models all neglect, often without serious discussion, what might be quite important phenomena, and one of those that may be of considerable interest is the coupling to the solar interior. It is to this issue that I intend to devote my discussion.

Aside from wanting to understand the solar cycle per se, a knowledge of the most important dynamical aspects is essential for any discussion of how, or whether, the cycle has any relevance to other issues that concern the sun. Does the mere existence of the cycle tell us anything about the conditions in the solar core? This question has been raised several times in connection with the solar neutrino problem, for example, and Dicke (1) has discussed it in connection with the 12^d.2 modulation of the Princeton oblateness data.

I might say in passing that an obvious point of interest in the solar cycle is its relation to the earth, and its influence on the climate and the ^{14}C production. These are discussed in other contributions to these proceedings. The only point I wish to add is that if any firm relation between the solar behaviour and measurable terrestrial records can be established, then the records might give us a measure of that behaviour that extends further back in the past than direct solar observations. This would be of obvious importance for improving our knowledge of the statistics of the cycle, to which I now turn my attention.

STATISTICS OF THE SOLAR CYCLE

Though theories of the solar cycle that depend on oscillations of the entire sun have not reached the level of sophistication attained by dynamo theories, and therefore may seem at first sight less plausible because they immediately raise unanswered questions in the minds of anyone who considers them, it does not necessarily follow that the ideas behind them are incorrect. To some extent dynamo theory may have suffered* the Gold effect (2), and to rescue it from this plight one should stand back and ask just what the predictions of the competing hypotheses are, and whether one really can discriminate between them by comparison with observation. There are many discussions of this issue, including the excellent critique by Cowling (3). Here I simply take up a point that Dicke (4) has raised, and ask whether the observed departure of the cycle from a regular oscillation can be used as a test.

If the dynamics of the cycle is controlled by a perfectly regular oscillation of the solar interior, then the manifestation of that oscillation by the sunspots ought to be closely linked to the state of the interior. I am not concerned here with whether the interior oscillation is itself directly responsible for the generation of the magnetic field, or whether it merely controls the dynamo in the convection zone. All I ask is whether the epochs of sunspot maxima and minima are closely linked in phase with a perfect clock.

A modern example of a model with an almost perfect clock is that proposed by Dicke (1): magnetic field of alternating polarity is released periodically from the core, and then rises slowly to the surface to produce the sunspots. The rise time is variable (5, 6): on the whole it is shorter the greater the total flux, which is what one might expect from magnetic buoyancy arguments, and provides a natural explanation for the correlation between the early onset of a new cycle and the sunspot number at the next sunspot maximum; in addition there are random fluctuations in the rise time induced by the turbulence in the convection zone. Associated with the release of the field is a contemporary variation in luminosity, which is presumed to be strictly periodic, and which is proposed to be responsible for climatic variation. Thus it is the interior oscillation itself that should be observable in climatic records, and not the sunspots. What must be somewhat disturbing to any proponent of

* or, perhaps, enjoyed

this theory therefore, is Murray Mitchell's report at this conference that the mean US drought record correlates better with the solar magnetic cycle than it does with a strictly periodic oscillator.

By comparing the sunspot data with mean $[D]/[H]$ ratios obtained from two bristlecone pines, averaged over samples representing non-overlapping ten-year growth intervals throughout a period of 1000 years, Dicke concluded that the mean rise time of the magnetic field is about 13 y. His argument is that the climatic record at the location of the pines, when viewed as ten-year averages, shows strong signs of a 22 y oscillation which might maintain phase, and whose maximum amplitude occurs about $2 + iP$ years prior to sunspot maximum, where $P \approx 11$ y is the mean duration of the sunspot cycle and i is an undetermined integer. Since the phase wandering of the sunspot cycle exceeds 2 years, i cannot be zero, so Dicke takes $i = 1$ to be the most plausible solution.

By contrast, a turbulent dynamo confined to the solar convection zone cannot be expected to maintain phase over long periods of time. Even though many theoretical idealizations of the dynamo, such as those based on mean-field electrodynamics, are described by equations that have periodic solutions, in reality one would expect turbulent fluctuations to destroy memory. Thus one might attempt to distinguish observationally between such dynamo hypotheses and the possibility of a regular oscillator by measuring the degree of phase maintenance of the sunspot cycle.

The first difficulty one encounters in such an endeavour is the problem of deciding how to define the phase of the cycle. Only the two most naive measures have been considered so far: the instant of field reversal which is estimated by the time of sunspot minimum, and the instant of greatest surface field which is estimated by the time of sunspot maximum.

Two independent analyses (5, 7) of the sunspot record have been carried out in an attempt to decide between the alternatives. Both used the same two statistical models to compare with the data, one with random fluctuations about a perfect clock and the other, which I shall misname the dynamo model, assumed random fluctuations in phase. The principle of both analyses was to choose a measure of the phase wandering of the cycle, and to compare the result with the expectations of the two models. The first discussion (7) was quite elementary, and used an obviously imperfect statistic that was chosen primarily for computational simplicity. The years of sunspot maxima and minima were used separately as tests, and it was found that the phase wandering of sunspot maxima lies closer to the expectation of the clock model, and that of sunspot minima is closer to the expectation of the turbulent dynamo model.* It was concluded, therefore, that the data is inadequate to support either model.

* Formulae (8.7) and (8.8) in ref. 7 were quoted incorrectly. The ratio of the expectation of the square of the phase deviations to that of the period fluctuations should be $N(N+1)/[6(N-1)]$ and $N(5N-1)/[6(N^2-1)]$ for Models A and B respectively. Correcting these results does not alter the conclusion.

The second analysis (5) was more sophisticated. It employed a statistic that would have been less biased than that in (7) had the raw sunspot data been used. However, the presumed correlation in Dicke's model between the rise time of the field from the core and the sunspot number at subsequent sunspot maximum was used to adjust the dates of sunspot maxima to move them as close as possible to the clock model. These dates were used also to test the dynamo model; and the dates of sunspot minima were not considered. It is perhaps not surprising, therefore, in view of the results of (7), that the sunspot data appeared to be in closer agreement with the expectation of the clock model. It was concluded that the data tends to support the clock model. The data shows no statistical indication of random fluctuations in phase.

How confident can we be in this conclusion? I shall illustrate the use of the statistics in terms of the more elementary analysis of ref. 7. Consider a sequence of N successive sunspot cycles. For the clock model suppose that the time of occurrence of the n th maximum (or minimum) after the first (to which I assign $n = 0$) is $t_n = nT + \tau_n$, where T is constant and the τ_n are independent random variables with zero mean and standard deviation τ . The period of the n th cycle is $P_n = T + \tau_n - \tau_{n-1}$, and the mean period of the N cycles is $\bar{P}_N = T + N^{-1}(\tau_N - \tau_0)$. For the dynamo model, assume that the interval between two successive cycles is $P_n = \Psi + \psi_n$, where Ψ is constant and the ψ_n are also random variables with zero mean, but this time with standard deviation ψ . This model is really an extreme representation of a dynamo because it assumes that the sun has no memory of previous cycles at all. In this case

$$t_n = n\Psi + \sum_{m=1}^n \psi_m, \quad \bar{P}_N = \Psi + N^{-1} \sum_{m=1}^N \psi_m.$$

The object of the investigation is to compute a measure of the phase deviation from a perfect clock. In the context of the clock model, this is an obvious measure of τ . Of course we do not know which clock to choose, and for maximum simplicity I shall choose the clock that ticks at the average rate of the cycle, at times $T_n = n\bar{P}_N + \epsilon$, where ϵ is a constant. A measure of the phase deviation is the variance σ_ϕ^2 of $\phi_n = t_n - T_n$, defined by

$$\sigma_\phi^2 = (N+1)^{-1} \sum_{n=0}^N \phi_n^2 - \left[(N+1)^{-1} \sum_{n=0}^N \phi_n \right]^2, \quad (2.1)$$

which is independent of ϵ . This can be computed from the sunspot data, and its expectation can easily be evaluated for each model. The result is

$$E(\sigma_\phi^2) = \frac{(N-1)(5N-1)}{3N(N+1)} \tau^2, \quad \frac{N-1}{6} \psi^2 \quad (2.2)$$

for the clock and dynamo models respectively, irrespective of the forms of the probability distributions of the τ_n and ψ_n . As $N \rightarrow \infty$ the prediction of the clock model remains bounded, whereas the dynamo model predicts an increase without limit.

Of course we have only a limited amount of data at our disposal, but we could compare the predictions of the two models with the sunspot data by dividing the data into segments of N cycles and comparing the dependence of $\overline{\sigma_\phi^2}$ on N , where the overbar denotes an average over the segments. I shall not present the results of doing simply that, but instead I introduce a second statistic that relates more to the dynamo model. This is the variance of P_n , which is an obvious measure of ψ . It is given by

$$\sigma_p^2 = N^{-1} \sum_{n=1}^N (P_n - \overline{P_N})^2, \quad (2.3)$$

and its expectations are

$$E(\sigma_p^2) = 2(1-N^{-2})\tau^2, \quad (1-N^{-1})\psi^2 \quad (2.4)$$

for the two models. One can now consider the ratio $R \equiv E(\sigma_\phi^2)/E(\sigma_p^2)$, which is independent of τ or ψ , and compare it with $S \equiv \overline{\sigma_\phi^2}/\overline{\sigma_p^2}$ computed from the sunspot data.

The analysis is confined to the interval from the sunspot maximum of 1705.5 until the last sunspot minimum. The values of t_n have been taken from Allen (8), except that the dates of the first maxima in the nineteenth and twentieth centuries were replaced by 1803.5 and 1906.0*, and the date of the last sunspot minimum was taken to be 1976.5. In Figure 1 is shown the result of dividing the 24 cycles into q contiguous groups of $N = 24 q^{-1}$ cycles for $1 \leq q \leq 6$. The rhombuses represent the ratio of the mean variance $\overline{\sigma_\phi^2}$, (averaged over the q groups) to $\overline{\sigma_p^2}$ for sunspot maxima; and the squares are the ratios for sunspot minima. To give some idea of the scatter, the vertical lines show the standard deviation of σ_ϕ^2/σ_p^2 . Shown also are the values of R for the two models.

In this analysis the raw dates of sunspot maxima and minima have been used. If, as is implied by Dicke's theory for example, there is a physical relation between the phase delay of sunspot maxima and sunspot number, this should be taken into account. Thus one can consider the modified time sequence:

$$\tilde{t}_n = t_n - r(R_n - \overline{R_N}) \quad (2.5)$$

derived from the times t_n of sunspot maxima, where R_n is the sunspot number at the n th sunspot maximum, and $\overline{R_N}$ is the mean of R_n over the $N+1$ maxima. The

* The dates 1805.2 and 1907.0 are quoted by Allen (8). However, both these maxima are double (9, 10); the values for t_n used in the analysis here are better representations of the average dates, and are close to those used in ref. 7.

coefficient r was chosen by Dicke (5) such that σ_ϕ^2 was minimized for $q = 1$. Thus it is likely that this would reduce σ_ϕ^2/σ_p^2 , at least when $q = 1$, and so move the data closer to the prediction of the clock model (lower curve). The results of analyzing the sequence t_n with Dicke's value of r is shown also in Figure 1. Even though the results are typically closer to the clock model, they can hardly be said to confirm it.

One is tempted to conjecture that the sun lies somewhere between the two models, having a memory of finite duration. If that is the case, how long is that memory? A step towards answering that question has been made by Barnes et al. (11, 12). They studied numerical simulations of a rectified oscillator that is randomly perturbed. They adjusted the bandwidth of the response to the perturbation such as to bring the variance of the fluctuations in period into agreement with the sunspot data, and found that with the same adjustment the variance in the simulated sunspot numbers at sunspot maximum also agreed with the real data. Moreover, the model produced intervals of about 50 years of continuous low sunspot activity, which occurred roughly once in 500 years. According to Barnes et al. the model has an inverse

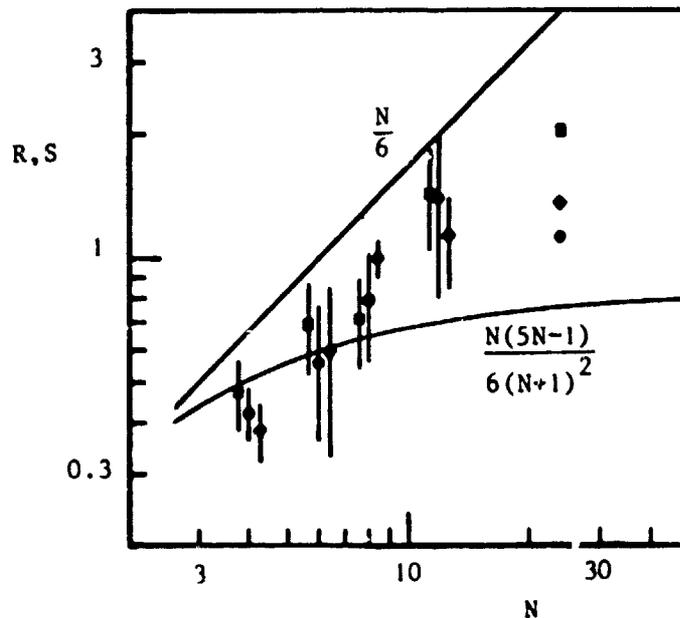


Figure 1. The ratios $S = \overline{\sigma_\phi^2}/\overline{\sigma_p^2}$. The rhombuses represent sunspot maxima and the squares sunspot minima. Except when $N = 24$ they have been displaced horizontally to the right or left of the value of N to which they pertain, to prevent cluttering the diagram. The circles represent S for the time sequence t_n defined by equation (2.5) with r chosen to minimize σ_ϕ^2 for $N = 24$. The vertical lines extend to plus and minus one standard deviation of σ_ϕ^2/σ_p^2 from S . The continuous curves represent the ratios R of the expectations of σ_ϕ^2 and σ_p^2 derived from the two extreme statistical models.

bandwidth of 500 years, which is a measure of the memory, and an engineering rule-of-thumb is that many inverse bandwidths are required to establish whether or not phase is maintained. This suggests that with the direct observations that are available, it may not be possible to measure the sun's memory accurately.

POTENTIAL LUMINOSITY AND RADIUS VARIATIONS

It is quite reasonable to expect there to be small variations in the luminosity L and the radius R associated with the solar cycle. If the seat of the cycle is in the core, then any change in the size of the core would force the envelope to expand or contract, thereby modifying the hydrostatic stratification and hence R and L . The earliest photospheric response to any such change to the core occurs after a delay equal to the sound travel time from the core to the surface, which is about half an hour. Similarly, any change to the convection zone brought about by a turbulent dynamo would also produce modifications to the state of the photosphere. The question to which I now address myself is whether from the variations in L and R one might infer anything about the nature of the perturbation.

I shall first discuss in broad terms the sequence of events after an imaginary instantaneous perturbation to the solar structure, and then I shall discuss some specific examples in greater detail. I should point out straightaway that I do not have a definitive answer to the question, but the results of the discussion below are perhaps suggestive.

RESPONSE OF THE SUN TO AN INTERNAL DISTURBANCE

I have already pointed out that the fastest response to a perturbation is dynamical. The response to any large-scale perturbation that varies on a timescale of more than a few hours can therefore be regarded as being instantaneous and hydrostatic. I am not going to discuss dynamical oscillations here, and from now on I shall disregard the manner in which the relaxation to the new hydrostatic state takes place.

After hydrostatic adjustment follows thermal relaxation. There are three obvious thermal timescales outside the energy generating core that can be relevant to the evolution; these are quite disparate and therefore their manifestations can be discussed separately. The first is the time τ_a required for the convection itself to attain a balance with the mean stratification. This is of the order of the turnover time of the largest convective eddies. Taking the convection zone as a whole, this time is about a month, and measures the duration of the transient response to any deep-seated event. The equilibration time for the eddies near the surface, such as the granulation, is very much shorter. I shall be considering only changes that occur after times much greater than τ_a .

The second adjustment is the coming into balance of the radiation from the photosphere with the changed internal heat flux. This is what is

sometimes called the Kelvin-Helmholtz time for the convection zone. I shall call it τ_c . To within a dimensionless factor it is the ratio of the thermal energy in the convection zone to the solar luminosity. This ratio is about 10^5 years for so-called standard solar models. The dimensionless factor is probably of order unity, though recently it has been suggested that for the sun it is of order 10^{-4} . I shall return to this point later.

The third and longest time is the Kelvin-Helmholtz time for the entire sun. It is the ratio of the magnitude of the total energy of the sun, which by the virial theorem is approximately equal to the thermal energy, to the luminosity. It is also the thermal diffusion time τ_d characteristic of the entire sun, and is approximately 3×10^7 years.

Outside the core, the sequence of events following a perturbation is likely to be thus: after the convection zone has readjusted itself internally, on the timescale of a month, and the radiative interior has responded adiabatically, the entire convection zone either cools or heats up on a timescale of 10^5 years until a stratification is achieved with an essentially divergence-free heat flux. Finally, the radiative interior relaxes to its new state of thermal balance, on its thermal diffusion timescale τ_d . Notice that this is the sequence of events wherever in the sun the instigating perturbation may be located, though of course if that perturbation were confined to the superficial layers of the sun the magnitudes of the longer thermal responses may be imperceptibly small.

At this point I shall elaborate a little on what I have just said, in an attempt to dispel some common misconceptions about the meanings of these timescales. What I have to say is quite obvious to anyone who studies stellar evolution but does not appear to be common knowledge otherwise. The issue concerns whether the response of the photosphere to any deeply seated perturbation is evident in a timescale less than the Kelvin-Helmholtz time. I hope I have convinced you that in principle the answer must be yes, because the effect of any local change in the mass distribution of the sun will propagate with the sound speed. But suppose one considers a thermal perturbation somewhere in or at the base of the convection zone associated with which there is very little mass flow. In such a case there has been disagreement as to whether it is the thermal adjustment of the convection or the Kelvin-Helmholtz time for the zone that is important. This problem doesn't obviously arise when discussing the relaxation of the entire sun, because the analogous two times are the same. But surely the answer is this: both are important; the relaxation has two phases, and different processes control the evolution during the different phases.

In an efficient convection zone of any star, the convective adjustment time* τ_a is much less than the overall cooling time. Thus there is an initial internal redistribution of energy, on the timescale τ_a , followed by the slower evolution on the timescale τ_c which is controlled by the rate at which heat is radiated from the photosphere. This is the physics that describes evolution down the Hayashi track, for example, during which the structure of the entire fully-convective star is controlled by the radiation from the surface. It is analagous to the cooling of a hot block of copper: thermal conduction operates much faster than cooling from the surface, and the block is almost isothermal. It therefore cools at a rate that depends only on heat transfer processes at the surface and the thermal capacity of the block. The only essential differences in the case of a stellar convection zone are that the state of thermal balance is isentropic rather than isothermal and that the change in gravitational energy must be taken into account when assessing the thermal capacity of the convection zone.

In a radiative zone the evolution is quite different, for now it is the internal thermal readjustment that is the slowest. The analogy is now with the cooling of a block of wood. After an initial transient response during which the surface temperature adjusts to accommodate the heat flow from the interior, evolution proceeds on the thermal diffusion time, τ_d .

To summarize: a thermally relaxing convection zone adjusts its internal stratification in such a way as to supply the heat flow dictated by the surface conditions, whereas a radiative region adjusts its boundaries to transfer the heat that diffuses from within.

* In astronomy, this is often estimated as a thermal diffusion time obtained from a turbulent heat diffusivity computed from time-independent local mixing-length theory. If the action of the varying mean stratification on the dynamics of the turbulence is taken into account, still within the framework of local theory, the perturbation satisfies a wave equation instead, with a wave speed essentially equal to the rms convective velocity, w . The characteristic adjustment time of the entire convection zone is thus simply the advection timescale $\tau_w \equiv \int w^{-1} dr$, where r is a radial distance co-ordinate and the integral is over the vertical extent of the convection zone. It is likely that the dominant heat-carrying eddies in the main body of the convection zone actually extend from top to bottom. Thus aside from geometrical factors, τ_w is still a good estimate of τ_a , even though the local assumption is incorrect. Notice that the estimate of w via the relation $4\pi r^2 \rho w^3 = L$ (which follows from the usual considerations of the eddy dynamics that form part of mixing-length theory, but which does not depend on the detailed mixing-length assumptions about eddy breakup) is independent of the value of the assumed scale of the dominant eddies (or the mixing length) and is thus a fairly robust estimate. This estimate of w breaks down in the boundary layer at the top (and in the boundary layer at the bottom) of the convection zone, but that does not influence the value of the integral substantially.

COMPUTATION OF THE 'IMMEDIATE' RESPONSE TO A SPHERICALLY SYMMETRICAL PERTURBATION

Most of the numerical computations modelling the response of the sun on a timescale short compared with τ_c , but long compared with τ_a , have been integrations in time using a stellar evolution programme. The computations are quite expensive, and it is therefore worth contemplating other methods, even though they are more limited in scope.

If the disturbance is small everywhere, the obvious procedure is to perform a linearized perturbation analysis. I shall not discuss that here, simply because I do not yet have any results from this method. The reason is that my involvement in this problem stems from a search I once made for a non-linear relaxation oscillation involving the bottom boundary layer of the convection zone and its interaction with both the rest of the convection zone and the radiative region beneath. I was hoping to find a self-sustained oscillation in the solar luminosity with a period of about 10^5 years, with a view to explaining certain variations in the earth's climate. I therefore developed the following method, which treats the convection zone nonlinearly. My original investigation was never completed because I was unable to produce luminosity fluctuations with amplitudes greater than about 0.1 per cent, and at that time such changes were thought to be climatically insignificant.

The method involves only the computation of a few models of the convection zone, plus the linear adiabatic relaxation of the radiative interior. One is then able to estimate the change in the structure of the entire sun resulting from a given perturbation. Because one is seeking only the 'immediate' response, the total energy E of the star does not have time to change. Consequently, the idea is simply to compute the response of the model at constant E .

Let p_m and r_m be the pressure and radius of the model envelope at a fixed value of the mass co-ordinate m , whose value corresponds to the base of the convection zone in the unperturbed model. In addition consider a second envelope model with the same L and R into which a disturbance has been incorporated. Then if the pressure and radius at the same value of m in the second model are $p_m + \Delta p_m$, $r_m + \Delta r_m$, the actual change in p_m that would be produced by the same disturbance in a model of the entire sun is given by*

$$\delta \ln p_m \approx \Delta \ln p_m + \left(\frac{\partial \ln p_m}{\partial \ln L} \right)_{R,A} \delta \ln L + \left(\frac{\partial \ln p_m}{\partial \ln R} \right)_{A,L} \delta \ln R, \quad (3.1)$$

where $\delta \ln L$ and $\delta \ln R$ are the actual changes in $\ln L$ and $\ln R$. Similar relations give the variations in $\ln r_m$ and $\ln E$. Notice that for the linearized estimate (3.1) to be a good approximation it is no doubt necessary for the variations in E, L and R to be small, but it may be the case that in some limited regions there are properties of the envelope that vary by quite large amounts. Any localized nonlinearity that so arises is correctly taken into account.

The object now is simply to calculate $\delta \ln L$ and $\delta \ln R$ that result from the disturbance subject to the constraint $\delta \ln E = 0$. This can be done once the relation between $\delta \ln r$ and $\delta \ln p_m$ is known. To find that relation it is necessary to consider the response of the interior.

* The quantity A measures the amplitude of the disturbance. In practice the partial derivatives in equations (3.1) and (3.4) were evaluated at $A = 0$.

Because τ_d is large compared with the initial response time, the reaction of the radiative interior is essentially adiabatic. Moreover, since $\delta \ln p_m$ and $\delta \ln r_m$ are small, linearized theory may be used. The calculation is simply to solve the adiabatic radial pulsation problem at zero frequency in the radiative interior, without the mechanical boundary condition at the surface. This gives a relation between the pressure and radius perturbations on the radiative side of the base of the convection zone, which I represent by

$$\delta \ln r_m = \lambda \delta \ln p_m. \quad (3.2)$$

Since r_m and p_m must be continuous functions of m , condition (3.2) provides the information necessary for matching the perturbed convection zone onto its radiative interior, and together with $\delta \ln E = 0$ can be used to eliminate $\delta \ln r_m$ and $\delta \ln p_m$ from the relation (3.1) and its companions. The result is

$$\delta \ln L = \frac{D_R \Delta \ln E - (\Delta \ln r_m - \lambda \Delta \ln p_m) E_R}{D_L E_R - D_R E_L}, \quad (3.3)$$

with a similar equation for $\delta \ln R$, where

$$D_R \equiv \left(\frac{\partial \ln r_m}{\partial \ln R} \right)_{A,L} - \lambda \left(\frac{\partial \ln p_m}{\partial \ln R} \right)_{A,L}, \quad E_R \equiv \left(\frac{\partial \ln E}{\partial \ln R} \right)_{A,L}, \quad (3.4)$$

and D_L and E_L are obtained by interchanging R and L. Notice that in general the procedure leads formally to a discontinuity in temperature at the base of the convection zone. In reality radiative diffusion and convective overshooting must smooth that out. If radiative diffusion alone were operating, after 10 years the jump in temperature would be spread over a layer only about 1000 km thick. Since this distance is only about one-third the mesh spacing near the base of the convection zone in my programme, the approximation is good. The artificial diffusion introduced by numerical differencing in a typical stellar evolution programme with a similar mesh spacing is likely to exceed greatly the radiative diffusion that is implied by the original differential equations.

It is also possible to estimate the relation between $\delta \ln L$ and $\delta \ln R$ that results from a disturbance that is confined to the solar core. Once again the expansion or contraction of the radiative envelope, outside the region of the disturbance, is adiabatic, and must match onto the perturbed envelope. If one were to assume that the perturbation could be linearized in the convection zone too, the problem would reduce to the nonadiabatic radial pulsation problem, with any period much greater than τ_g but much less than τ_c and without the boundary condition at $r = 0$. However, to get a rough idea of the result it is not necessary to solve that problem precisely. It is probably adequate simply to assume a homologous expansion or contraction of the convection zone, which yields equation (3.2) again, but with $\lambda = -0.25$. One can

then repeat the calculation, but with $\Delta \ln p_m = 0$, etc., to obtain

$$\frac{\delta \ln R}{\delta \ln L} = - \frac{D_L}{D_R}, \quad (3.5)$$

where D_L and D_R are now computed with the new value of λ .

It is thus possible to estimate the response of the entire sun from numerical models of only the solar envelope and a knowledge of λ . The partial derivatives D_R , E_R etc. are computed by finite differences using undisturbed model envelopes with different values of L and R . Notice that although E , which is the energy of the entire star, can never be computed from envelope models alone, this does not matter because only differences of E appear in equation (3.3). These are simply the differences between the energies of the two appropriate envelopes above the matching point, plus the differences in the energies of the interiors. The latter can be computed as the work done by the envelope on the interior, which requires a knowledge of only p_m and r_m .

My computations reported below were performed with an early version of the computer programme used by Baker and myself (13) to model RR Lyrae pulsations. The programme had not been designed for this purpose, and several interpolations, which could have been avoided by rewriting the programme, were performed. Therefore I make no claims to high accuracy. The unperturbed model was chosen with abundances $X = 0.745$, $Z = 0.02$ of hydrogen and heavy elements, which are approximately the values that would have produced the correct luminosity in an evolved model of the entire sun. Cox-Stewart (14) opacities were used, and the equation of state was of the type discussed by Eggleton et al. (15). A mixing length of 2 pressure scale heights was chosen so as to yield a convection zone about 2×10^5 km deep, in accordance with the dictates of the high-degree five-minute oscillation data (16,17).

POTENTIAL INTERNAL CHANGES DURING THE SOLAR CYCLE

Convective inhibition by sunspots

The mechanism that has received most attention is the direct blocking of the heat flux by magnetic fields. This is particularly apparent in sunspots. Whether sunspots do actually reduce the energy flux has been questioned, the possibility being that the deficit in the radiative flux is made up by extra wave energy. The issue is probably not completely resolved, but I think the observational evidence is weighted towards a net reduction of energy flow within sunspots. I shall accept that here, and ignore wave transport entirely.

Notice that I have not yet said that the local reduction of the heat flow in sunspots necessarily implies a significant diminution in the solar luminosity. The local reduction is countered by a tendency for more heat to flow around the edges of the spot, producing a circumsistent bright ring.

However, the excess heat output by the identifiable part of the ring is quite inadequate to make up for the deficiency in the spot; inhibition of the flux in the spot extends deep into the convection zone,* so any excess flux can be distributed widely by the time it reaches the photosphere. Also any change in the mean (averaged over a spherical surface concentric with the photosphere) efficacy with which heat is transported in the convection zone leads to a change in the stratification of the convection zone that modifies the luminosity in such a sense as to oppose the original change. Nevertheless one expects that on average the opposing reaction is less than the perturbing influence, because the convection zone is apparently stable (though we are not absolutely sure of this). Thus it does appear that if the local inhibition of heat flow were the sole influence sunspots exerted, some reduction in luminosity would accompany an increase in sunspot numbers. But how substantial this reduction is cannot be judged without careful calculation.

It is very difficult to perform a realistic calculation to assess the effect of sunspot creation. What has been tried is to consider the effects of sunspots to be averaged over spherical surfaces $r = \text{constant}$, and to modify the standard techniques for studying spherically symmetrical stellar evolution to model the overall response of the sun. Thus, the magnetic inhibition of convective heat transport has been modelled by artificially reducing the mixing length l in the usual time-independent heat flux formula. If such a procedure is a reasonable approximation to reality, it would be valid for studying variations on any timescale greater than τ_a , and would therefore be adequate for solar cycle variations.

The results of several independent computations have been published (18-22), and more are reported in this conference. The published results are summarized in Table 1, together with my own unpublished values. The diversity in the results arises partly from differences in the unperturbed models and partly from numerical error. For example, Dearborn and Newman (18) and Dearborn and Blake (21) used a small mixing length and consequently had a thin convection zone. In such a model the heat transport is more sensitive to the mixing length and it is therefore to be expected that $\delta \ln L / \delta \ln \alpha$ would be overestimated. To test my procedure I recently repeated the calculation with the ratio α of mixing length to pressure scale height equal to unity, yielding a convection zone with mass $9 \times 10^{-3} M_{\odot}$. Increased sensitivity was found, but my value of $\delta \ln L / \delta \ln \alpha$ was only 0.35, nearly a factor 2 less than that found by Dearborn and Newman (18) and Dearborn and Blake (21). I suspect that most of the scatter in the values of $\delta \ln R / \delta \ln \alpha$ and hence in

* By the argument in the anteprecedent footnote one expects the time taken for convection to adjust to the creation of a sunspot to be of order $\int w^{-1} ds$, where s is a distance co-ordinate and the integral is over a path that represents a typical heat-flow line that starts at the base of the spot and ends in the photosphere near the spot. The two independent observations reported at this conference that substantial transient reductions in the solar constant associated with large sunspot groups can last at least ten days therefore indicates that the spots responsible are not superficial phenomena.

TABLE 1 - Summary of published results from perturbing the mixing length

Authors	$\delta \ln L / \delta \ln \alpha$	$\delta \ln R / \delta \ln \alpha$	W	M_c / M_\odot
Dearborn and Newman (18)	0.46α †			0.008
Dearborn and Blake (21)	0.64	3×10^{-3}	5×10^{-3}	0.0065
Sofia <i>et al.</i> (19) Sofia and Endal (20)	1.5	0.11	7.5×10^{-2}	
Gilliland (22)			8.5×10^{-4}	
Gough	0.22	$\leq 1 \times 10^{-4}$	$\leq 5 \times 10^{-4}$	0.03

The last column is the mass of the convection zone in units of the solar mass. †The formula for $\delta \ln L / \delta \ln \alpha$ quoted by Dearborn and Newman was found to hold for $1 \leq \alpha \leq 1.5$; the value of α corresponding to the quoted value of M_c / M_\odot was not given. Dearborn and Blake used $\alpha = 1.4$.

$W \equiv \delta \ln R / \delta \ln L$ is a product of numerical error. In my calculations the numerator in the equation similar to (3.3) for $\delta \ln R$ is a small difference between two quantities of order unity (each of which is a numerical derivative computed from interpolated quantities). The result I obtain is sensitive to the interpolation formula I use, and I would therefore not be surprised if I have underestimated the degree of cancellation considerably. Thus at present I would summarize the results for a model with an adequately deep convection zone thus:

$$\frac{\delta \ln L}{\delta \ln \alpha} \approx 0.3, \quad W \equiv \frac{\delta \ln R}{\delta \ln L} \approx 0. \quad (3.6)$$

In none of the publications is a relation between α and sunspot number derived. It is very difficult to do this theoretically, but one can be guided by observation. According to Allen (8) the mean intensity in a sunspot (umbra and penumbra combined) is about 70 per cent of that in the unperturbed photosphere. I shall take this as a measure of the inhibition of the convection at fixed entropy gradient. A medium-sized sunspot occupies about 6×10^{-6} of the surface of the sun, and at sunspot maximum there are present the equivalent of about 200 such sunspots.* Hence at sunspot maximum

* That is to say, about 100 on the side we can see and about 100 on the other.

the luminosity of the sun, ignoring faculae and the bright rings, is

$$L = (1 - \beta) \tilde{L}, \quad (3.7)$$

where \tilde{L} is the luminosity the sun would have had had the sunspots been absent and the radiative flux been everywhere equal to the flux in the apparently undisturbed photosphere; β , which measures the mean reduction of the heat flux by the sunspots, is $200 \times (1 - 0.7) \times 6 \times 10^{-6} \approx 4 \times 10^{-4}$. Throughout almost all the convection zone the convective heat flux is proportional to α^2 , and consequently the observed mean flux inhibition is obtained by setting

$$2 \frac{\delta\alpha}{\alpha} = -\beta. \quad (3.8)$$

We can now combine equation (3.8) with (3.6) to yield an estimate of the difference δL between the luminosities at sunspot maximum and sunspot minimum:

$$\frac{\delta L}{L} \approx -0.15\beta \approx -6 \times 10^{-5}. \quad (3.9)$$

Notice that the actual decrease in the luminosity from sunspot minimum to sunspot maximum is only 15 percent of the apparent blocking of the luminosity βL caused by the sunspots.

The exclusion of material from sunspots

It was stated by Jensen (23) that because the matter density in sunspots is lower than outside, the sun must have a larger volume at sunspot maximum. This idea has been elaborated on by Thomas (24) and Dearborn and Blake (25), who estimate the expansion to be not insubstantial. I do not understand their arguments, and I think it might be instructive if I say why.

Let us first consider the balance of energy in a star in hydrostatic equilibrium. In common with almost everyone else I shall ignore the turbulent stresses in the convection zone. I shall also assume that the pressure and the magnetic stress on the surface (i.e. the photosphere) can be ignored. Then the virial theorem takes the form

$$\frac{1}{2} \ddot{I} = 2T + \Omega + M, \quad (3.10)$$

where I is the so-called spherical moment of inertia, T , Ω and M are the kinetic, gravitational and magnetic energies, and the dots on I denote time derivatives. The kinetic energy T is comprised of the energy of thermal motion plus the energy of macroscopic motion, the latter residing mainly in

rotation. In equilibrium $\ddot{I} = 0$, whence

$$2T + \Omega + M = 0. \quad (3.11)$$

Suppose now that sunspots are created, and that at the same time the magnetic energy of the sun is increased by $\delta M > 0$. According to most theories of the sunspot cycle, the field is produced by the stretching that results from differential rotation. Thus the reaction of the field is to oppose gradients in angular velocity without changing the angular momentum, and hence to reduce the kinetic energy of the rotation. Just as the reduction of the luminosity by sunspots discussed above is not as great as the degree of direct inhibition of the heat flux, so the depletion of the rotational kinetic energy in this case is not as great as the energy imparted to the magnetic field. The continual driving of the large scale flow in the convection zone tends to restore the internal differential rotation to its original state, at the expense of other forms of energy. Let us assume, therefore, that only a fraction η (which I presume is positive) of the magnetic energy is extracted from T , and that the rest comes from Ω .^{*} The virial balance is now

$$\begin{aligned} \frac{1}{2}\ddot{I} &= 2(T - \eta\delta M) + [\Omega - (1 - \eta)\delta M] + (M + \delta M) \\ &= -\eta\delta M < 0, \end{aligned} \quad (3.12)$$

so I must decrease. Hence, on average, the star shrinks, which might seem contrary to Jensen's assertion. It does not necessarily follow that there is a contradiction, however, for the adjustment might deviate substantially from being homologous. Nevertheless, it does illustrate a possible pitfall that might be encountered if one does not take into account that the magnetic energy in the sunspots must be provided from within the sun. Had I forgotten the changes in the other forms of energy in the star, as did Jensen and Thomas, I would have deduced that $\frac{1}{2}\ddot{I} = +\delta M > 0$, and then perhaps I might have been happy that this was apparently consistent with Jensen's claim.

Let us now look a little more carefully at the recent arguments. Thomas (24) evaluated the mass defect in toroidal flux ropes about 1000 km beneath the photosphere, and assumed that the total volume of the sun is simply increased by the volume that the missing mass would occupy at the ambient density. The estimated expansion was $\delta R/R = 5 \times 10^{-4}$. Thomas's neglect of the change in the balance of forces in the interior consequent to the creation of flux ropes is tantamount to ignoring the fact that the region beneath the flux rope contracts as a reaction to the attempt to raise the height of the photosphere. Thus his estimates of the expansion must be exaggerated, and I shall now argue that the error may be quite large.

* Strictly speaking, the argument should be complicated further by considering also the interchange with the energies of ionization and the electrostatic interactions amongst electrons, ions and neutral atoms. These forms of energy do not appear in the virial theorem.

Suppose one simply implants a flux rope in pressure balance in the convection zone, and removes the mass defect ΔM from the star entirely. The perturbation to the gravitational field is negligible. Hence the stratification far from the flux tube is unaffected, and in particular the position of the photosphere is unchanged: one cannot detect a stationary submarine by observing the position of the surface of the ocean. Thus to compute the true effect of the flux tube on the photospheric radius one must merely replace the mass ΔM in the star. To within a factor of order unity, the resulting relative radius change will be $\Delta M/M_{\odot}$, and therefore the volume change is roughly equal to the volume occupied by ΔM at the mean solar density $\bar{\rho}$, rather than at the density ρ in the vicinity of the flux rope. If this argument is correct, Thomas has overestimated the expansion by a factor $\bar{\rho}/\rho$, which at 1000 km beneath the photosphere is about 5×10^5 .

I am certainly not concluding from this exercise that the magnetic stresses of sunspots do actually have so miniscule an influence on the solar radius. All I am saying is that the exclusion mechanism discussed by Jensen and Thomas, taken in isolation, appears to be unimportant.

Dearborn and Blake (25) modelled the effect by including 'a global magnetic pressure term in a stellar structure code'. At first sight this appears to be the product of considering the sunspot magnetic field as providing an additional pressure contribution to influence directly the mean hydrostatic balance. But at this conference Dearborn has argued that his procedure can also be regarded as the excluded volume effect that Jensen and Thomas have discussed. He showed that if one integrates the stellar structure equations outside sunspots, where the magnetic field is negligible, then in order to relate the density ρ to dm/dr correctly one must add to ρ a term that takes account of the mass that has been pushed aside by the sunspots. This term is proportional to the magnetic pressure in the sunspots, and so appears as an additional contribution to the pressure in the equation of state. Precisely what this implies is difficult to judge, because we have not been told exactly how the additional term has been incorporated into the other equations. Dearborn and Blake find $\delta R/R \leq 10^{-4}$ associated with a 0.1 per cent reduction in the luminosity. Thus $|W| \leq 0.1$, and possibly $W = 0$.

Other magnetic processes

Spiegel and Weiss (26) have considered recently the importance of the interaction between the convection zone and the radiative interior. They discussed the implications of the idea that magnetic field is compressed into a thin layer at the base of the convection zone by the combined action of topological pumping and field expulsion. After a sufficient amount of field has accumulated, hydromagnetic instabilities driven by magnetic buoyancy cause some of the field to rise to the surface to produce active regions and sunspots. Spiegel and Weiss suggest that the instability occurs when the magnetic layer is about a pressure scale height thick.

The cycle is complicated, and Spiegel and Weiss emphasize one aspect of it that may be important in causing luminosity and radius variations. It is that the magnetic layer will inhibit motion, in part by modifying the potential temperature gradient indirectly via the change in the hydrostatic

balance, and so cause the convection zone to recede. The flux expulsion process involves magnetic diffusion of stretched field that varies on a relatively short length scale. Since the magnetic Prandtl number is small, one might also expect thermal diffusion to be significant, so that the relatively quiescent magnetic layer would not be adiabatically stratified. But how it is stratified is hard to assess. A certain degree of mixing with the material in the radiative zone may well have taken place, so the resulting temperature gradient is presumably somewhere between the radiative and the adiabatic values.

An estimate of the manifestations of part of this process can be obtained from a somewhat different model I have constructed by suppressing the motion in a layer of thickness d at the bottom of the convection zone and assuming the reclaimed quiescent region to have achieved the radiative temperature stratification. Only one temperature discontinuity, at the base of the thermally mixed layer, was created. Magnetic stresses were not included in the hydrostatic balance in that region, so the calculation is not internally consistent. However, since the process being investigated is primarily thermal, it is not unreasonable to consider it in isolation from the balance of forces. Moreover, it is no less consistent than imagining α to have been changed without taking into account the stresses responsible. Indeed, the perturbation is equivalent to a drastic modification to α in a limited region of the convection zone. The result of a calculation with $d = 7000$ km is listed in Table 2. The increase in luminosity is about $5 \times 10^{-4} L_{\odot}$ somewhere near sunspot maximum, and is probably proportional to d^2 .

TABLE 2 - Summary of the responses of the sun to various disturbances

Disturbance	$\delta \ln L$	$\delta \ln R$	W
Inhibition of convection by sunspots modelled by reducing α according to equation (3.8)	-6×10^{-5}	≈ 0	≈ 0
Introduction of equipartition tangled magnetic field into convection zone	2.6×10^{-2}	1.2×10^{-4}	4.4×10^{-3}
Replacement of temperature gradient in the bottom eighth of a pressure scale height of the convection zone by the radiative gradient	-4.5×10^{-4}	-9.1×10^{-5}	0.20
Any disturbance that is confined to the energy generating core			0.53

Although the magnitude of the luminosity perturbation is comparable with that deduced by Spiegel and Weiss, the mechanism by which it was obtained is rather different. Replacing what was essentially the adiabatic temperature gradient by the radiative gradient in the lower boundary layer of the convection zone results in a change in the stratification which is locally much greater than Spiegel and Weiss envisaged. Consequently, a greater redistribution of energy takes place. For example, subsequent to the magnetic instability, the temperature of the material near the base of the convection zone increases by about 10^4K and the internal energy in the convection zone is decreased by about 10^{42} erg. If this estimates the energy available to supply the increment δL in the luminosity, it would imply that $\delta L \approx 10^{-4}L_{\odot}$ if the increment were spread uniformly over 10^5 years. The interchange between the different forms of energy is brought about by a force provided from a comparatively small energy reservoir: to suppress the convection at the base of the zone and change the stratification to the radiative gradient requires only about 3×10^{33} erg of work. Thus magnetic energies as great as the total change in the energy radiated (if subsequently the star were to remain unperturbed until the relaxation time τ_c had elapsed) are not necessary to bring about that change. This result provides some a postericri justification for ignoring the magnetic stresses in the hydrostatic equation.

In contrast, Spiegel and Weiss imagine the perturbation to cause a redistribution of the energy in the convection zone of just 10^{39} erg. This they assume is radiated in only about 10 years. The crux of the disagreement between their ideas and the assumptions of the calculation described above, therefore, is that Spiegel and Weiss assert that under these conditions real convection, unlike the predictions of the mixing-length formalism, reacts very sensitively to perturbations from beneath. This difference of opinion has little to do with the efficacy with which convection transports heat down a gradient of potential temperature. It concerns the degree to which convection modifies the photospheric temperature and so changes the rate at which heat is radiated from the star. The very high sensitivity of the state of the photosphere must be related to the small nonzero divergence of the heat flux in the convection zone, for if in the steady state the sensitivity of real convection were 10^4 times greater than the prediction of mixing-length theory, it would be unlikely that the latter could have been used to reproduce successfully the slope of the lower half of the main sequence in the Hertzsprung-Russell diagram.

Another phenomenon of interest is the influence of the small-scale tangled magnetic field in the convection zone. One effect is that the magnetic pressure modifies the hydrostatic balance, and another is that magnetic buoyancy enhances the driving force on the turbulent eddies and so increases the efficacy of the convection. I have modelled these processes by adding to the free energy of the fluid, from which all thermodynamical state variables are calculated, the energy of a tangled magnetic field in equipartition with the kinetic energy of convection. This increases the fluid pressure by an amount equal to the magnetic pressure. It also reduces the adiabatic temperature gradient and thus enhances the buoyancy forces acting on the convective eddies. As in the case of changing the mixing length to scale height ratio by a constant amount, this perturbation has a significant influence on the stratification only in the upper boundary layer of the convection zone. The idea is

that there will be more tangled field in the convection zone at sunspot maximum. Once again, to compare such a model with one having no magnetic field at all overestimates the difference between sunspot maximum and sunspot minimum. And indeed, the luminosity enhancement by the magnetic field at sunspot maximum of more than 2 per cent (see Table 2) is greater than the limits set by observation.

Perturbations to the core

Table 2 also contains an entry corresponding to the response to a perturbation that is confined to the core. The perturbation is presumed to provide only a mechanical disturbance to the base of the envelope. Thus no rising magnetic field of the kind envisaged by Dicke (1), for example, is accounted for. Without specifying the amplitude of the core perturbation one cannot set absolute values to the perturbations in L and R, but provided linear theory is valid, their ratio is independent of the nature of the perturbation.

CONCLUSION

We do not yet know whether the solar cycle is controlled in the convection zone or the radiative interior. It cannot be claimed that the sunspot statistics support either view convincingly, though they do hint that the sun does not keep perfect time. If that is indeed the case, one might regard it as evidence that a turbulent dynamo is operative, and that the wandering of the phase of the cycle is produced by the dynamical effect of the turbulent fluctuations on the oscillation. A convincing demonstration that the phase of the cycle is not maintained would not close the case, however, because it is quite common for nonlinear systems to oscillate almost but not exactly periodically without any stochastic interactions. The potential diagnostic power of the sunspot statistics lies mainly in the possibility of demonstrating phase maintenance, for in that case stochastic interactions must necessarily be unimportant.

Studies of the luminosity and radius variations associated with the cycle will probably be more fruitful. Some work has already been done, but mainly with only superficial perturbations meant to represent the magnetic inhibition of convection in the upper boundary layer of the convection zone. It may be that plausible variations in luminosity can be engineered, though the associated radius variations are very small: $W \approx 0$. The response of the sun to a few other types of disturbance have been discussed in this paper, but no systematic investigation has yet been undertaken. In all cases it is hard to estimate the absolute magnitudes of the resulting luminosity and radius perturbations, but their ratio W is more clearly determined. The examples suggest that W increases as the depth of the disturbance increases, and if that tendency is ever demonstrated to hold universally, it seems likely that imminent observations will enable us to decide at least whether part of the dynamo process operates deep in the sun.

Other diagnostics that might be of use in this respect come to mind. The low-degree five-minute oscillations provide integral measures of the solar

interior that must vary over the solar cycle. It has not yet been demonstrated, however, whether they remain coherent for long enough to have sufficiently accurate frequencies to measure the solar change. Another indicator may be the apparent quasi-biennial variation in the solar neutrino flux. Sakurai (27,28) has found a 26 month variation in the measurements of Davis and his colleagues which appears to be correlated with the residuals in the sunspot numbers that remain after subtraction of a 5-month running mean. If there is a causal connection between the variations of sunspots and the neutrino flux, its discovery would clearly be important. One conclusion we can draw straight away, however, is that none of the disturbances seated outside the core that have been considered here is of a magnitude anywhere near to being adequate to cause any perceptible variation in the neutrino flux.

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