

SHORT AND LONG TERM VARIATIONS IN THE "SOLAR CONSTANT"

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ABSTRACT

Short and long term variations in the solar constant are examined theoretically. The variations observed by the Solar Maximum Mission, lasting several days and associated with the passage of sunspot groups, strikingly demonstrates the well known lack of a "bright ring" effect around sunspots. This suggests that sunspot magnetic fields do not simply block the heat flowing upward into the photosphere. Rather, it is suggested that gravitational draining occurs; this cools sunspots and transports downward the heat that would otherwise flow into the photosphere. A model of sunspot temperature with depth shows modest support when compared with the empirical model of Van't Veer. Secular trends in the solar constant may occur and be associated with the influence of the convection zone magnetic field upon convective heat transport. As a start to understanding this problem, the Schwarzschild criterion has been modified to include the effects of magnetic field.

INTRODUCTION

Recently investigations by Livingston (ref. 1), Kosters and Murcray (ref. 2), Livingston et al. (ref. 3), Dicke (ref. 4), and Willson et al. (ref. 5) suggest that short term and secular changes in solar luminosity may be occurring. Theoretically, the influence of the solar cycle magnetic field on solar luminosity is a multifaceted question, since there are numerous ways in which magnetic field can affect the sun's luminosity.

Thomas (ref. 6) has theoretically investigated luminosity changes associated with effective solar radius changes produced by magnetic buoyancy in spots. Hoyt (ref. 7) looked at differences in luminosity associated with observable photospheric features—spots, faculae, etc. — to obtain a measure of possible global luminosity differences. He found a relationship between umbra-penumbra area ratios and terrestrial temperature variations. Willson et al. (ref. 5) have examined dips in the solar luminosity associated with the passage of spot groups past central meridian. These dips are a "short term" influence we shall discuss shortly. "Long term" influences may arise from changes in the convection zone. Schatten (ref. 8) suggested that magnetic buoyancy may influence convection; Spiegel and Weiss (ref. 9) argue that the heat transport by convection can be affected by the magnetic field, and that the influence is most significant at the base of the convection zone. We shall discuss, later, a mechanism that allows the long term influence of the magnetic field upon the convection zone to be calculated from stellar models. First, let us examine the short term variations associated with spot groups.

SHORT TERM VARIATIONS ASSOCIATED WITH SPOT GROUPS

As the photospheric material cannot serve as a reservoir for the vast flow of energy on time scales longer than about a second, a "bright ring" around sunspots might be expected if the Biermann (ref. 10) mechanism involving field inhibition of convective heat transport occurred.

A new school of thought developed, beginning with Danielson (ref. 11), in which the convective generation of Alfvén waves below the sunspot consumed so much energy that cooling of the spot occurred with an attendant field concentration. It also eliminated the "bright ring" effect wherein the blocked heat would flow around the spot into the surrounding photosphere. The lack of a bright ring was strikingly demonstrated by the recent "solar constant" observations of Willson et al. where dips in the solar constant occurred when sunspots approached central meridian. If any bright ring were present, it would cancel the spot energy deficit and no dip should occur. It is these dips that are the subject of the present section. Other investigators have also found changes in the solar constant with sunspot visibility. Foukal and Vernazza (ref. 12) found a level of 3×10^{-4} change in the solar constant (similar to the SMM findings) and Chapman (ref. 13) found a dip in the sun's brightness equal to 62% of a sunspot's area.

Here we have followed much of the theoretical guidance suggested by Meyer et al. (ref. 14) and by Parker (ref. 15, 16). We further this third view that pores, knots and sunspots are a kind of dynamical solar sink in which material drains gravitationally downward. This utilizes a dynamical gravitational draining as the mechanism to explain 1) the cooling of features (ref. 15) 2) the elimination of the bright ring around sunspots, and 3) the field concentration mechanism (ref. 14). It enables a calculation of the temperature of these downdrafts to be roughly 1000°K cooler than the photosphere from basic principles and that the downflow in fluxtubes should be roughly several km/s.

Parker (ref. 16) has suggested that the " $6-8 \text{ km s}^{-1}$... downdrafts in the fluxtubes (leading to) concentration to 1500 G may be a direct dynamical consequence". This downdraft inferred by Deubner (ref. 17) from observations may also be important in stabilizing fluxtubes insofar as the magnetic pressure of the fields, in a solar atmosphere with no inward and downward flow, would tend to disperse the fields in a short amount of time. Meyer et al. (ref. 14) discuss the stabilizing effect of this inflow of material as a hydrodynamic "collar" which the spot wears.

We discuss here, that this inward and downward flow as shown in Figure 1, may also play a role in the cooling mechanism for spots, as well as pores where Frazier (ref. 18) has observed downdrafts up to 3 km s^{-1} . For pores and knots, large velocities in the photosphere are found, or at least inferred however, for spots Beckers (ref. 19) notes that they "show no vertical motions ... exceeding 25m/sec." Some confusion has arisen "because of the limb effect of the surrounding photosphere, sunspots appear to have a downflow of $\sim 400 \text{ m/sec}$ " Thus downflows are seen or inferred for small flux tubes, and not for spots where we hypothesize a deeper gravitational draining occurs owing to the larger size of the magnetic object. In this view, the field

originates from dynamo processes, the final stage of which is magnetic buoyancy, whereby a large concentrated field erupts at the sun's surface. In the absence of a suitable cooling and field stabilization mechanism, the strong field would quickly dissipate. However, as the material near the sun's surface is continually radiating luminous energy into space, the gases cool and with their increased density return in conduits to the heat source from which they came, thereby completing the convection cycle. Thus by analogy with any thermal cycle, the material moving towards the heat source would be the coolest. In the absence of sunspot fields, the cooled photospheric gases return at the boundaries of the supergranulation, and so form pores. In the presence of sunspot fields, they are aided in their return by this field conduit, which tends to reduce the turbulent viscosity and so provides an easier pathway. Owing to the larger size of a spot, not enough material appears capable of congregating into an observable downflow in the photosphere and we hypothesize it does so at depth.

To calculate the size of the effect, we take a volume element within the magnetic region, as shown in Figure 1. It is bounded by 4 sides (1-4) with no interaction on the 5th and 6th sides due to azimuthal symmetry. We shall simplify the picture by assuming the following. The flow is roughly inward to the box at 1, and outward at 4. If the photosphere were not yet cool, region 2 would radiate the same energy rate, F , as the remainder of the photosphere. Due to the inward and downward flow, little or no convective energy is being transported into the region through 3 (where in the normal photosphere, a rate, F , balances the outflow through the top). For the purpose of the calculations, the volume extends from the photosphere $\tau = 1$ to roughly $\tau = 3$ so that most of the radiant flux, $F = 6.4 \times 10^{10}$ erg $\text{cm}^{-2}\text{s}^{-1}$ (ref. 20), is indeed emitted from the volume through 2. Further, the material flows inward through 1 at several km sec^{-1} and out through 2 at the same rate and pressure, so that we can ignore compressive heating or expansive cooling as a consideration. Then the temperature difference can be shown to be:

$$\Delta T = \frac{2 FL}{5 \bar{\rho} h v R} = 1,100^\circ \text{K} \quad (1)$$

where R is the gas constant, L and h are the length and height of the volume, taking $L/h = 1$ and $v = 7 \text{ km s}^{-1}$. Thus for a spot, an umbral temperature of $4,600^\circ \text{K}$ is calculated showing good agreement with observed umbral temperatures. Put simply, the mechanism can provide adequate cooling for pores, magnetic knots and sunspot umbra. We now examine the question of the downflow. The cooled photospheric gases, being denser than the hotter photospheric gases, would tend to descend. Will this gas reach a terminal velocity of several km sec^{-1} as it descends? We consider a calculation for the terminal velocity of a blob of gas in a viscous stratified atmosphere.

As the terminal velocity is governed by the interaction with the stratified medium, we choose the scale height, H , to be a characteristic length dimension for consideration. The downward force upon the gas is:

$$F = g \Delta \rho H^3 \quad (2)$$

where g is the acceleration of gravity = $2.7 \times 10^4 \text{ cm s}^{-2}$, $\Delta \rho$ is the

additional density of the blob of gas $= \frac{\Delta T}{T} \rho = \frac{1100^\circ\text{K}}{5700^\circ\text{K}} 4 \times 10^{-7} \text{ gm cm}^{-3} = 0.7 \times 10^{-7} \text{ gm cm}^{-3}$, and H is the scale height $= 100 \text{ km}$. The viscous balancing upward force is:

$$F = \frac{\eta v A}{d} = \eta v H \quad (3)$$

where η is dynamic viscosity, ρv , and v is the kinematic viscosity $1/10 \text{ vl}$. Busse (ref. 21) obtained a value of $10^{12} \text{ cm}^2 \text{ s}^{-1}$ for v , yielding $3 \times 10^5 \text{ gm cm}^{-1} \text{ s}^{-1}$ for η . The quantity v is the velocity difference between the blob and the surrounding material, and H is the scale height of the photosphere again. Equations 2 and 3 can be solved for v , yielding:

$$v = g \Delta \rho H^2 / \eta. \quad (4)$$

This gives 7 km s^{-1} for the terminal downward velocity of gases, a sufficient downflow to provide cooling. It is comparable with the 2 km s^{-1} velocity that Parker (ref. 15) required for a downflow within spots to account for their cooling.

The lack of a bright ring around the gas central to the problem of solar constant dips is understood in this model by examining Figure 1. As the upward heat flux, F, transported principally by convection, encounters the sunspot magnetic field, the energy and gas are entrained amongst the descending gases where the energy is carried down toward the base of the convection zone and so is lost to the sunspot umbra.

Table 1 shows the temperature and density of the umbra and surrounding photospheric material. The photospheric parameters are shown from the solar model of Endal and Sofia (ref. 22). The umbral parameters were obtained from a computer model having been calculated utilizing the simple theoretical model outlined. That is, there is no convective heat transport into the spot from below and the photospheric material is cooled by radiation into space.

For comparison, the semi-empirical model of Van't Veer as outlined in Tandberg-Hanssen (ref. 23) is shown in table 2. It is based on a method developed by Van't Veer that determines the umbral parameters using the measured intensity in the wings of certain Fraunhofer lines. Near $\tau = 4$, the values agree remarkably well; however, near $\tau = 1$ the agreement is less good.

LONG TERM VARIATIONS IN THE SOLAR CONSTANT

We consider, in this section, the effects of the solar cycle magnetic field embedded within the convection zone upon the solar luminosity. We develop a version of the Schwarzschild criterion which includes the effects of magnetic field. We consider the approximation that the magnetic field provides a net isotropic pressure.

The Schwarzschild criterion (ref. 24) is obtained by considering density changes associated with a rising convective element of the star, leading to instability if the radiative temperature gradient exceeds the adiabatic gradient:

$$\left. \frac{d \ln T}{d \ln P} \right|_r > \left. \frac{d \ln T}{d \ln P} \right|_a = \frac{\gamma-1}{\gamma} \quad (5)$$

If we now include a magnetic field, there will be a magnetic pressure P_B , associated with a rising bubble. In any discussion of the magnetic stresses, we may neglect the tension term in the stress tensor because the field is continuous and divergence B equals zero. Thus across any small element, the field tension balances on either side and imparts no force to the gas. The total pressure P_T , equals the gas pressure plus magnetic pressure:

$$P_T = P_G + P_M = P_G (1 + \beta^{-1}) \quad (6)$$

where β is the gas to magnetic pressure ratio. The adiabatic density gradient now becomes:

$$\left. \frac{d \ln \rho}{dr} \right|_a = \frac{d \ln P}{\gamma dr} \Big|_a - \frac{d \ln(1+\beta^{-1})}{\gamma dr} \Big|_a \quad (7)$$

where we have used the adiabatic relation, $P_G V^\gamma = \text{constant}$. The radiative gradient is similarly modified:

$$\left. \frac{d \ln \rho}{dr} \right|_r = \frac{d \ln P_T}{dr} \Big|_r - \frac{d \ln T}{dr} \Big|_r - \frac{d \ln(1+\beta^{-1})}{dr} \Big|_r \quad (8)$$

Instability occurs if the adiabatic density gradient has a larger magnitude than the radiative gradient. From equations 7 and 8 this implies:

$$\left. \frac{d \ln T}{d \ln P_T} \right|_r > \frac{\gamma-1}{\gamma} + \frac{d \ln(1+\beta^{-1})}{\gamma d \ln P_T} \Big|_a - \frac{d \ln(1+\beta^{-1})}{d \ln P_T} \Big|_r \quad (9)$$

where adiabatic and radiative subscripts on β allow for differing values between convective bubbles and the surrounding environment. Equation 9, our modified Schwarzschild criterion, can be rewritten as:

$$\left. \frac{d \ln T}{d \ln P_T} \right|_r > \frac{\gamma-1}{\gamma} \left[1 - \frac{d \ln(1+\beta^{-1})}{\delta d \ln P_T} \Big|_a \right] \quad (10)$$

where we have incorporated the two expressions on the right of 9 into one through the utilization of a parameter, δ . Here δ equals one under the assumed approximation, for the onset of instability. For the more general case δ depends on the ratio of field inside the rising gas bubble to field outside, and geometry. Other formulae can be developed to include the effects of other geometries (see Thomas and Nye (ref. 25)). Further, if one excludes the interaction of the bubble with the material, an adiabatic equation of state can be written, but we have allowed this to be incorporated into δ . As we have developed our criterion for the onset of instability when the field is not perturbed, we have ignored the possible restoring force (or tension) between the base field and a rising bubble which may ensue with turbulence. The above criterion assumes that once the magnetic field and temperature gradient are suitable for instability, the Rayleigh-Taylor instability or another plasma interchange instability will develop to allow the bubble to rise.

The term on the right of equation 10 modifies the adiabatic condition in the following fashion. Let us consider β^{-1} to approach 1 in a region of the sun and examine how this affects the onset of instability. Figure 2 shows the magnetic field, the magnetic to gas pressure ratio (β^{-1}), and the term in equation 10 modifying the Schwarzschild criterion. As the gas pressure decreases radially outward, the whole term in the square bracket will first exceed one, and then fall below one, to possibly a negative value. Thus a single layer or region of magnetic field will form a relatively stable layer in its lower side and a relatively unstable layer in its upper side. The word

relative is used to indicate that these layers may be stable or unstable, depending upon the remainder of the Schwarzschild condition, and thus, they are merely modifying the condition. However, in case the modifying term reaches a negative value, as the left hand side of equation 10 is positive definite, instability is required. This is conventionally referred to as magnetic buoyancy, and in the usual treatment only the field considerations are provided. We see here, however, that ordinary convection may also be affected by the presence of a subsurface field.

If we consider the fields in the solar interior suggested by dynamo theory (ref. 26), the term on the right of equation 10 appears to be significant only at the base of the convection zone (except in centers of activity). If the gas pressure variation has a scale height H_G , and the magnetic pressure a scale height H_B , the term on the right can reach a magnitude of order H_B/H_G . One of the problems associated with solar dynamo theory is the formation of a region where magnetic fields may regenerate without being lost to magnetic buoyancy (ref. 26, 27). In this result the underside of a magnetic field region would be a stable location where fields could regenerate. Above the region, if the field approached a high enough value, instabilities could form with a balance developing between regeneration and deterioration. Unno and Ribes (ref. 28) have examined magnetic buoyancy and have found that 200-300 gauss dynamo field can be retained in the convection zone for time scales up to about 80 years due to turbulent viscosity. The large scale field, although retained, could affect the onset of convective instability through the above criterion, thus the process of magnetic stabilization and destabilization may influence heat transfer at the base of the convection zone with solar cycle phase much as Spiegel and Weiss (ref. 9) suggest.

CONCLUSIONS

Downflow of material within sunspots is seen as a key to the understanding of their stability, temperature and heat flow. The inward velocity prevents the magnetic field from expanding into the surrounding surface by balancing the spot's magnetic pressure against the velocity change from an inflow into a downflow. The cool, dark characteristics of sunspots are to be explained by their being the cool end of a thermal cycle. In this cycle, the gases have radiated their energy into space, are allowed to descend along field conduits into a cooler, denser state and cut off the heat flow. The heat flux that would normally be convected into and radiated by the sunspot is carried downward by the flow to deep layers in the convection zone. Thus, the minima in the solar output associated with sunspot groups observed by Willson et al. (ref. 5) are explicable.

Further, for secular changes, we consider the magnetic field at the base of the convection zone to affect the Schwarzschild criterion and form stabilizing and destabilizing layers that may cause a significant variation in the solar luminosity similar to that in the model of Spiegel and Weiss.

REFERENCES

1. Livingston, W. C.: 1978, Nature, 272, 340.
2. Kosters, J. J., and Murcray, D. G. 1979: Geophys. Res. Lett., 6, 382.
3. Livingston, W., Milkey, R. and Slaughter, C.: 1977, Astrophys. J., 211, 281.
4. Dicke, R. H.: 1978, Nature, 276, 676.
5. Willson, R. C., Gulkis, S., Janssen, M., Hudson, H. S., and Chapman, G. A.: Science, 211, 700, 1981.
6. Thomas, J. H.: 1979, Nature, 280, 662.
7. Hoyt, D. V.: 1979, Climate Change, 2, 79.
8. Schatten, K. H.: 1973, Solar Physics, 33, 305.
9. Spiegel, E. A. and Weiss, N. O.: 1980, Nature, 287, 616.
10. Biermann, L.: 1941, Vierteljahrsschr. Astron. Ges., 76, 194.
11. Danielson, R. E.: 1962, Astron. J., 67, 574.
12. Foukal, P. V., and Vernazza, J. E.: 1979, Astrophys. J., 234, 707.
13. Chapman, G. A.: 1980, Astrophys. J. Lett., 242, 245.
14. Meyer, F., Schmidt, H. U., Weiss, N. O., and Wilson, R. P.: 1974, Mon. Not. R. Astr. Soc., 169, 35.
15. Parker, E. N.: 1979, Astrophys. J., 204, 259.
16. Parker, E. N.: 1979, Cosmical Magnetic Fields, Clarendon Press, Oxford, p. 213, p. 255.
17. Deubner, F.: 1976, Astron. and Astrophys., 47, 475.
18. Frazier, E. N.: 1970, Solar Physics, 14, 89.
19. Beckers, J. M.: 1981, The Sun as a Star, Ed. S. Jordan, NASA/CNRS, GPO, CH. 2.
20. Allen, C. W.: 1963, Astrophysical Quantities, Athlone Press, University of London, Ch. 9.
21. Busse, F. H.: 1969, Astrophys. J., 159, 629.

22. Endal, A. S. and Sofia, S.: 1981, Astrophys. J., in press.
23. Tandberg-Hanssen, E.: 1966, Solar Activity, Blaisdell Pub. Co., Waltham, p. 199.
24. Schwarzschild, K.: 1906, Gottinger Nachr., 41.
25. Thomas, J. H. and Nye, A. H.: 1975, Physics of Fluids, 18, 490.
26. Parker, E. N.: 1976, in Basic Mechanisms of Solar Activity, Ed. Bumba and Kleczek, 3.
27. Parker, E. N.: 1975, Astrophys. J., 198, 205.
28. Unno, W. and Ribes, E.: 1976, Astrophys. J., 208, 222.

Table 1

Depth km	Density $10^{-7} \text{ gm cm}^{-3}$	T_p 10^{30} K	T_u 10^{30} K
0	1.69	5.75	4.60
42	1.94	6.46	5.04
98	2.08	8.17	5.28
238	2.98	10.13	6.94
448	5.18	11.57	9.73

Table 2

$\tau(5000)$	Depth, km from $\tau=.5$	T_p 10^{30} K	T_u 10^{30} K
.01	-220	4.70	3.57
.05	-135	4.93	3.70
.1	-100	5.07	3.78
.3	-50	5.51	4.02
.5	0	5.83	4.18
1.0	40	6.41	4.48
2.0	60	7.18	4.84
4.0	100	8.10	5.24

CROSS SECTION OF A SUNSPOT

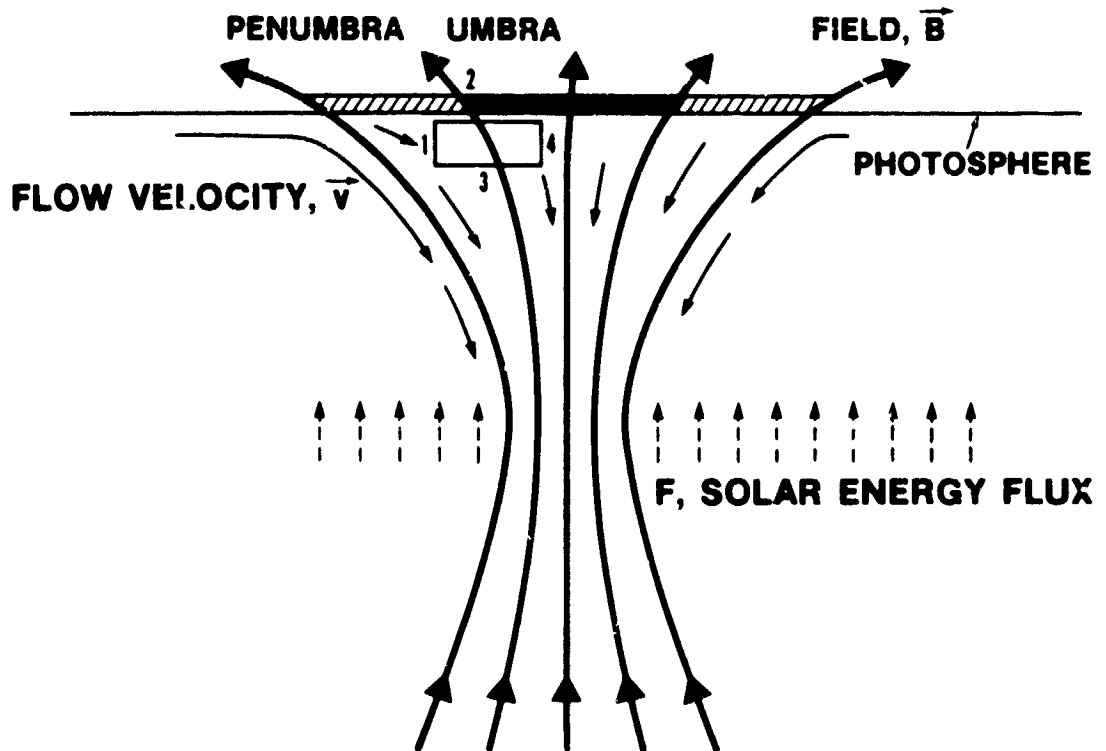


Figure 1. Shown is the field geometry and downflow velocities in the vicinity of a sunspot or pore. The inward and downward flow beneath the photosphere and below acts as a "collar," stabilizing the sunspot. This flow also returns the cooled gases to the base of the convection zone. It is by the liberation of radiant heat that the gases are cooled to umbral temperatures. The energy flux that would flow upward into the sunspot is entrained within the cooler gases and descends to the convection zone base to be reheated. The box at the top labelled with sides, 1-4, exhibits a simple heat calculation for the sunspot.

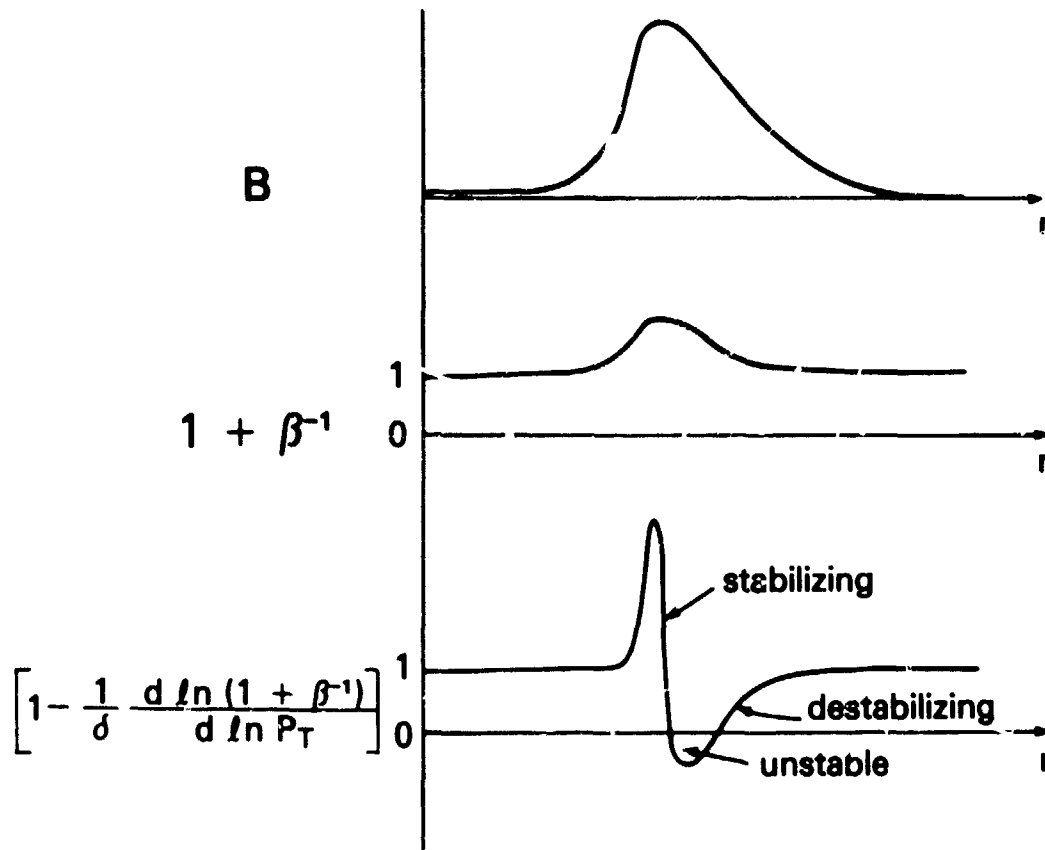


Figure 2. Schematic showing the radial variation of magnetic field B , magnetic to gas pressure ratio β^{-1} , and the term modifying the Schwarzschild criterion. As can be seen when the modifying term exceeds 1, the instability is more difficult to attain as convective elements have difficulty rising through a more tenuous field saturated layer. On the upper side of a field layer, however, destabilization occurs, and convective elements may more easily rise. When the modifying term becomes negative, then absolute buoyancy occurs as the field strength is sufficient to cause instability even against an unfavorable temperature gradient.