On the Shape and Orientation Control of an Orbiting Shallow Spherical Shell Structure

by

Peter M. Bainum
Professor of Aerospace Engineering
Department of Mechanical Engineering
Howard University
Washington, D.C. 20059, USA

and

A.S.S.R. Reddy
Graduate Research Assistant
Department of Mechanical Engineering
Howard University
Washington, D.C. 20059, USA

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ON THE SHAPE AND ORIENTATION CONTROL OF AN ORBITING SHALLOW SPHERICAL SHELL STRUCTURE*

P.M. Bainum and A.S.S.R. Reddy

Department of Mechanical Engineering, Howard University
Washington, D.C., 20059, USA.

Abstract. The dynamics of orbiting shallow flexible spherical shell structures under the influence of control actuators is studied. Control laws are developed to provide both attitude and shape control of the structure. It is seen that the elastic modal frequencies for the fundamental and lower modes are closely grouped due to the effect of the shell curvature. The shell is also assumed to be gravity stabilized by a spring-loaded dumbbell type damper attached at its apex. Control laws are developed based on the pole clustering technique and it is assumed that the dumbbell state information may not be directly observable. Numerical results verify that a significant savings in fuel consumption can be realized by using the hybrid shell-dumbbell system together with point actuators. Other results indicate that for the less robust systems instability may result by not including the orbital and first order gravity-gradient effects in the plant prior to control law design.

Keywords. Modelling of orbiting flexible structures; pole placment; ~~modelling errors; hybrid control systems.~~

INTRODUCTION

Future proposed space missions would involve large inherently flexible systems for use in communications, radiometry, and in electronic orbital based mail systems. The use of very large shallow dish type structures to be employed as receivers/reflectors for these missions has been suggested. In order to satisfy mission requirements control of the shape as well as the over-all orientation will be often required. The proposed paper is devoted to a study of the shape and orientation control of such an orbiting shallow spherical shell structure and, to the authors' knowledge, represents the first such treatment of this subject.

A related recent paper (Kumar and Bainum, 1981) treated the dynamics and stability of a flexible spherical shell in orbit in the absence of active shape and orientation control. For small amplitude elastic displacements and rigid rotational modal amplitudes, it was seen that the roll-yaw (out-of-plane) motions completely separate from the pitch (in-plane) and elastic motions. Furthermore, the pitch and only the axi-symmetric elastic modes are coupled within the linear range.

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With the symmetry axis nominally following the local vertical, the structure is gravitationally unstable due to an unfavorable moment of inertia distribution. A rigid dumbbell connected to the shell at its apex by a spring loaded double gimballed joint with damping was proposed to gravitationally stabilize the structure (Fig. 1). It was noted that the dumbbell motion could excite only those elastic modes having a single nodal diameter (Reissner, 1955) and that to completely damp the system transient motion in all of the important lower frequency modes, the use of an active control system would be required.

The present paper represents an extension of the paper by Kumar and Bainum (1981) to include in the mathematical model of the dynamics the effects of point actuators located at pre-selected positions on the shell surface (Fig. 1).

DEVELOPMENT OF MATHEMATICAL MODEL OF THE PLANT

The mathematical model of an isotropic shallow flexible spherical shell in orbit was developed by Kumar and Bainum (1981) under the assumption that the shell's elastic displacements were principally in the transverse direction (parallel to the symmetry axis)

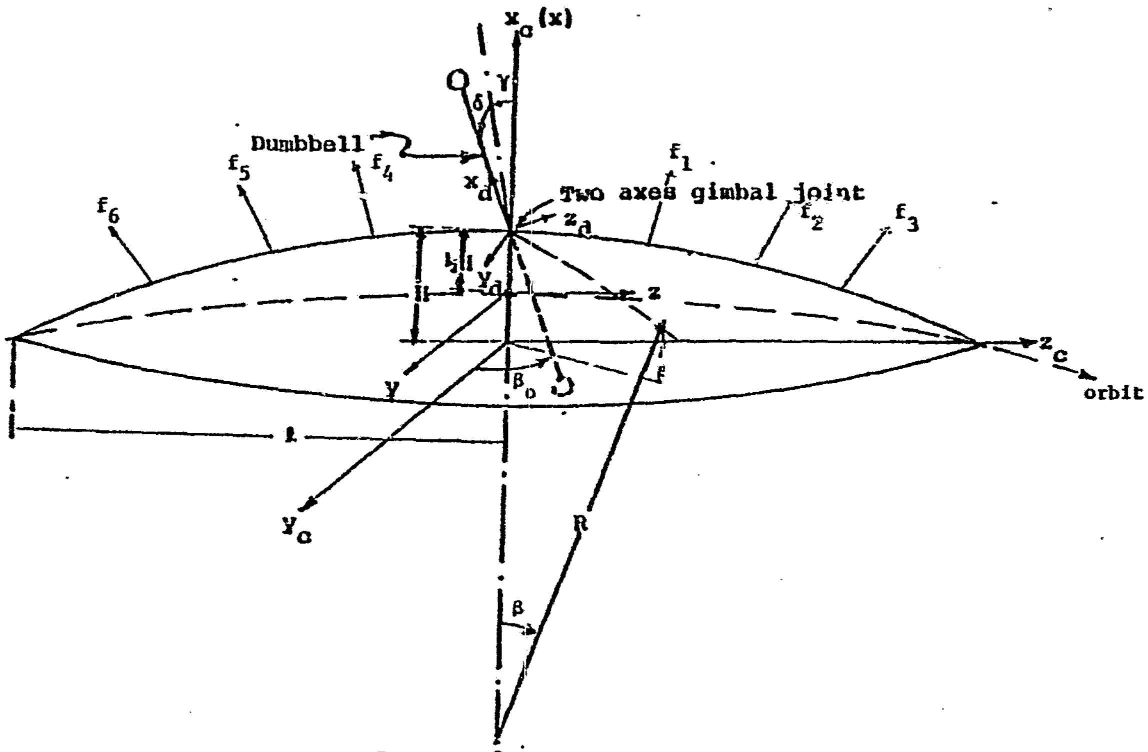

Fig. 1. Shallow spherical shell with dumbbell and actuators

and were small as compared with the other characteristic dimensions of the shell. The assumption of shallowness further insures that the ratio of the displacement of the shell's apex point above its base plane (H) is small as compared with the radius in the base plane, a, (Fig. 1).

The resulting linearized equations of motion for the rigid rotational and generic elastic modes were developed as:

$$\psi''-\Omega_x\psi-(1+\Omega_x)\phi' = C_x/J_x\omega_c^2$$
$$\phi''+4\Omega_z\phi+(1-\Omega_z)\psi' = C_z/J_z\omega_c^2$$
$$\theta''-3\Omega_y\theta-2\sum_i\epsilon_i' I_1^{(i)}\ell/J_y = C_y/J_y\omega_c^2$$
$$\epsilon_n''+(\Omega_n^2-3)\epsilon_n+2\theta' I_1^{(n)}/M_n\ell = 3I_1^{(n)}/M_n\ell + E_n/M_n\omega_c^2\ell \quad (n = 1,2,\ldots\infty) \tag{1}$$

where the variables and constant coefficients are defined in the Appendix . (An order of magnitude analysis also indicated that the coupling between the rigid (orbital) translational modes and those described in Eq.(1) was extremely small for structures with characteristic lengths of 100m. so that these modes are essentially governed by the orbital mechanics of the system mass center.)

It was further assumed that a dumbbell could be attached by a spring loaded gimbal damper to the shell at its apex and could provide both gravitational stability of the uncontrolled system as well as passive restoring and dissipative forces.

The linearized equations of motion for the shallow spherical shell-dumbbell system were developed as:

$$\psi''-\Omega_x\psi-(1+\Omega_x)\phi' = C_x/J_x\omega_c^2$$

$$\phi''+4\Omega_z\phi+(1-\Omega_z)\psi'-\bar{c}_z\delta' -\bar{k}_z\delta'-\sum_{i=1}^{N}(\bar{c}_z\epsilon_i'+\bar{k}_z\epsilon_i)C_y^{(i)} = C_z/J_z\omega_c^2$$

$$\theta''-3\Omega_y\theta-2\sum_i\epsilon_i' I_1^{(i)}\frac{\ell}{J_y}-\bar{c}_y\gamma'-\bar{k}_y\gamma + \sum_{i=1}^{N}(\bar{c}_y\epsilon_i'+\bar{k}_y\epsilon_i)C_z^{(i)} = C_y/J_y\omega_c^2$$

$$\epsilon_n''+(\Omega_n^2-3)\epsilon_n+2\theta'\frac{I_1^{(n)}}{M_n\ell}-\frac{3I_1^{(n)}}{M_n\ell} -(\bar{c}_y\gamma'+\bar{k}_y\gamma)\frac{J_y}{M_n\ell^2}C_z^{(n)} -(\bar{c}_z\delta'+\bar{k}_z\delta)\frac{J_z}{M_n\ell^2}C_y^{(n)}+\sum_m(\bar{c}_y\epsilon_n'+\bar{k}_y\epsilon_n)C_z^{mn} + \sum_m(\bar{c}_z\epsilon_n'+\bar{k}_z\epsilon_n)C_y^{mn} = E_n/M_n\ell\omega_c^2 \tag{2}$$

$$\gamma''+\bar{c}_y(1+c_1)\gamma'+\{3+\bar{k}_y(1+c_1)\}\gamma+3(1+\Omega_y)\theta -(1+c_1)\sum_{i=1}^{N}(\bar{c}_y\epsilon_i'+\bar{k}_y\epsilon_i)C_z^{(i)}+2(\sum_i\epsilon_i' I_1^{(i)}\ell/I_y)=0$$

$$\delta''+\bar{c}_z(1+c_2)\delta'+\{4+\bar{k}_z(1+c_2)\}\delta+4(1-\Omega_z)\phi -(1-\Omega_z)\psi-(1+c_2)\sum_{i=1}^{N}(\bar{c}_z\epsilon_i'+\bar{k}_z\epsilon_i)C_y^{(i)}= 0$$

It is seen from Eq. (1) that for the uncontrolled system without the dumbbell that the out-of-plane roll-yaw motions are completely decoupled from the in-plane pitch (θ) and elastic motions (ϵ_n). Within the linear range only the axisymmetric elastic modes ($I_1^{(i)}\neq 0$) are coupled to the pitch motion. Futhermore from the analysis of Eq. (2) by Kumar and Bainum (1981) it was concluded that the dumbbell motion could excite only those elastic modes having a single nodal diameter and that to completely damp the system transient motion in all of the important lower frequency modes, the use of an active control system would also be required. However, it was hoped that a properly designed hybrid control system consisting of the passive dumbbell and active control actuators could provide satisfactory performance with a savings in fuel consumption as compared with the active thrusters operating alone.

The formulation of the uncontrolled dynamics assumes an a priori knowledge of the frequencies of all the elastic modes to be incorporated within the system model. The frequencies (p) of the spherical shell are evaluated using the following identities, as presented by Johnson and Reissner (1958):

$$\mu = \left[\frac{h\rho a^4}{D}(p^2-p_\infty^2)\right]^{1/4} \tag{3}$$

where the μ's are calculated from

$$\frac{\mu}{2}\left[\frac{J_n(\mu)}{J_{n+1}(\mu)}+\frac{I_n(\mu)}{I_{n+1}(\mu)}\right]= 1-\nu \quad \text{for } n = 0,1 \tag{4}$$

where n represents the number of nodal diameters (meridians), $D=Eh^3/12(1-\nu^2)$, and $p_\infty^2=E/\rho R^2$. For n>1, Eq. (4) must be replaced by a more complex form as follows:

$$\frac{\mu^4}{K^4} = \frac{S_n(\mu)}{R_n(\mu)} -1 \tag{5}$$

where
$$S_n(\mu)= 4n^2(n^2-1)(1-\nu)\{\mu[J_n(\mu)I_n'(\mu) -J_n'(\mu)I_n(\mu)]+(n+1)(1-\nu)[I_n'(\mu)-\frac{n}{\mu}I_n(\mu)]\times [J_n'(\mu)-\frac{n}{\mu}J_n(\mu)]\}$$
$$R_n(\mu) = \{(1-\nu)[\mu J_n'(\mu) -n^2J_n(\mu)]+\mu^2J_n(\mu)\}\{(1-\nu)n^2[\mu I_n'(\mu) -I_n(\mu)]-\mu^3I_n'(\mu)\}-\{(1-\nu)n^2[\mu J_n'(\mu) -J_n(\mu)]+\mu^3J_n'(\mu)\}\{(1-\nu)]\mu I_n'(\mu) -n^2I_n(\mu)]-\mu^2I_n(\mu)\}$$

and J_n, I_n are Bessel functions of the first kind and modified Bessel functions of the first kind, respectively. Eq. (4) or Eq. (5) is satisfied by an infinite number of the parameter, μ, for every value of n ($j=1,2,...\infty$). For the sample calculations in this paper, we will consider only three such values of μ ($j= 1,2,3$) for the cases where $n = 0,1$.

The values of the natural frequencies and mode shape functions of the axisymmetric modes will be slightly modified by the presence of the dumbbell. However, for this application, an order of magnitude analysis for the system parameters involved, indicates that the coupling between the axisymmetric modes and the rigid pitch mode is extremely weak and that, to a good first approximation, the small number of axisymmetric modes included can be considered independently of all the rigid rotational modes. In view of this the axisymmetric frequencies and mode shapes as given Johnson and Reissner (1958) are used here as a first approximation to the actual values in the presence of the dumbbell. The natural frequencies and mode shapes of the other elastic modes characterized by nodal diameters (meridians) remain unaffected by the presence of the dumbbell (Kumar and Bainum, 1981).

The point actuators are modelled as follows. An actuator located at (x,y,z) with components (f_x,f_y,f_z) provides the following torques,

$$T_x = yf_z-zf_y; \; T_y = -xf_z+zf_x;$$
$$T_z = xf_y-yf_x \tag{6}$$

and the corresponding generic force in the nth mode,

$$E_n = \int \phi_n \cdot \bar{f} \, dm \tag{7}$$

For the shallow spherical shell it is assumed that the major elastic displacement occurs in a direction normal to the base (y,z) plane. - i.e. $\phi_n \doteq \phi_x^{(n)} \bar{i}$. Thus for a point actuator located at (x,y,z)

$$E_n \doteq \phi_x^{(n)}(x,y,z) f_x M_n \tag{8}$$

where $\phi_x^{(n)}(x,y,z)$ is the nth modal shape function evaluated at (x,y,z), f_x represents the component of $\bar{f}$ in a direction normal to the base plane and M_n is the modal mass, here considered to be the total mass of the shell, since the shell mode shape functions have each been normalized with respect to the mass of the shell.

NUMERICAL EXAMPLE

As an example a large flexible shallow shell is selected with the following dimensions (Fig. 1) and material properties:

$H = 1m$; $a = 100m$; $\rho = 27.68 kg/m^2$; $h = 1cm$;
$\nu = 1/3$; $E = 0.744 \times 10^{10} kg/m^2$; $R = 5000.0m$

Six elastic modes are included in the truncated model and are selected to have either no or a single nodal diameter. The number of nodal circles is varied from 1 to 3 such that the elastic modal frequencies are calculated using Eqs. (3) and (4). The results are summarized in Table 1.

TABLE 1 - Elastic Modal Frequencies of the Shell (Six Modes)

# of nodal diameters (n)	# of nodal circles (j)	$\rho^2(H_z^2)$
0	1	0.107593022
1	1	0.107678417
0	2	0.107946351
1	2	0.108475664
0	3	0.109516711
1	3	0.111142560

It is observed that all the frequencies are grouped with only a difference of 3% between maximum and minimum values of the flexible frequencies considered in this model. This close grouping is due to the dominance of the curvature of the shell on the assumed model, and further emphasizes the importance of a careful consideration of potential modelling errors on the design of the control system..

The resulting state vectors for the system now take the form:

$$X^T = [\psi,\phi,\theta,\varepsilon_1 \ldots \varepsilon_6,\psi',\phi',\theta',\varepsilon_1',\ldots\varepsilon_6'] \quad 18x1 \tag{9}$$

without the dumbbell; and

$$X^T = [\gamma,\delta,\psi,\phi,\theta, \varepsilon_1 \ldots \varepsilon_6,\gamma',\delta',\ldots\varepsilon_6'] \quad 22x1 \tag{10}$$

with the dumbbell.

The equations of motion can be written in the state vector format:

$$\dot{X} = AX+BU \tag{11}$$

where the B matrix is evaluated using the location of the actuators, the values of the elastic modal functions at these locations (Johnson and Reissner (1958) and Itao and Crandall (1979)) and the direction cosines of the actuator thrust vectors.

It is assumed that six actuators are positioned along the surface of the shell (in the x,z plane as shown in Fig. 1). Force directions are selected so that when the actuators are operating torques will be provided directly about each of the shell's principal axes. The assumed actuator locations and force directions are summarized in Table 2.

TABLE 2 - Actuator Locations and Force Directions

Actuator No.	β_o	β	x	y	z
1)	90^o	0.28	f_1	0	0
2)	90^o	0.57	f_2	0	0
3)	90^o	0.84	f_3	0	f_3
4)	270^o	0.28	f_4	0	f_4
5)	270^o	0.57	f_5	f_5	0
6)	270^o	0.84	f_6	f_6	f_6

The location of the non-zero elements of the system "A" matrix is illustrated in Figs. 2 and 3 for the system without the dumbbell and containing the dumbbell, respectively. A comparison of these figures illustrates the greater coupling with the rigid and flexible modes that is provided by the dumbbell. Numerical values for the complete A and B matrices are listed in Bainum and co-workers (1981). It is seen that the B matrix of the shell with the dumbbell has the form:

$$B_d = \begin{bmatrix} 0_{13x6} \\ \hline B' \end{bmatrix} \qquad (12)$$

where B' is the (9x6) lower portion of the B matrix without the dumbbell.

Control laws are selected using the pole clustering algorithm developed by Armstrong (1978) for the spherical shell without the dumbbell. This same control law is then applied to the case of the shell with the dumbbell, i.e.- it is assumed that the dumbbell position and rate information may not be directly observable and is not included in the control law. (As long as the dumbbell-shell system is controllable and stabilizable, the dumbbell will return to its desired local vertical equilibrium orientation after the transients have been removed. It is possible to design a control law for the dumbbell-shell system that results in a controllable-stabilizable system without including the dumbbell information within the control law.)

A typical time history of the required control forces for the shell without the dumbbell is shown in Fig. 4 for non-dimensional initial position displacements in all state components of 0.01, with the control law based on placement of all the poles so as to have a dimensionless negative real part of (-1.72). This will provide a system time constant of 461 seconds in all modes. A similar response was generated using the same control law for the case of the shell with the dumbbell where the dumbbell inertia ratios (c_1=c_2) were assumed to be 0.9, the dumbbell spring constants, $\bar{k}_y = \bar{k}_z = 100$, and damping coefficients selected so as to provide 0.1 critical damping.

A comparison of the maximum force amplitude in each of the actuators and also of the total force impulse required is given in Table 3 for the two cases. It can be seen that although there is little difference in the maximum force amplitudes, there is approximately a 25 percent savings in fuel consumption by using the shell-dumbbell system.

TABLE 3 - Maximum Force Amplitudes of the Shell with 6 Actuators

	Without dumbbell	With dumbbell
f_1	103.411Nt.	103.410Nt.
f_2	105.398	105.398
f_3	317.83	317.83
f_4	107.55	107.55
f_5	77.09	79.09
f_6	127.32	127.318
Total Force impulse:	432.8 Nt-sec	320.49 Nt-sec (25% savings with dumbbell.)

Many current investigations of the shape and orientation control of flexible orbiting structures do not incorporate the effects of the gravity-gradient and orbital dynamic coupling into the linear model of the plant. -i.e. the poles of the rigid rotational modes of the open loop system are at the origin. A study was made to determine the effects of omitting these terms in the development of control laws. Control laws based on pole placement were first developed based on the shell model which did not include the gravity-gradient and orbital coupling terms in the plant model. The control laws thus developed were then inserted into the previously developed models which contain both first order gravity-gradient and orbital dynamic coupling terms. Typical results are illustrated in Figs. 5 and 6. In all cases studied, there is a general tendency for some of the poles of the rigid modes to shift towards the imaginary axis when the gravity-gradient and orbital effects are superimposed into the plant. For the less robust systems (Fig. 5) instability may result. In general, there is no noticeable shift in the poles corresponding to the flexible modes. As expected, for the more robust systems, the relative effect of this shift is less apparent (Fig. 6), but at the expense of greater control force effort (Table 4).

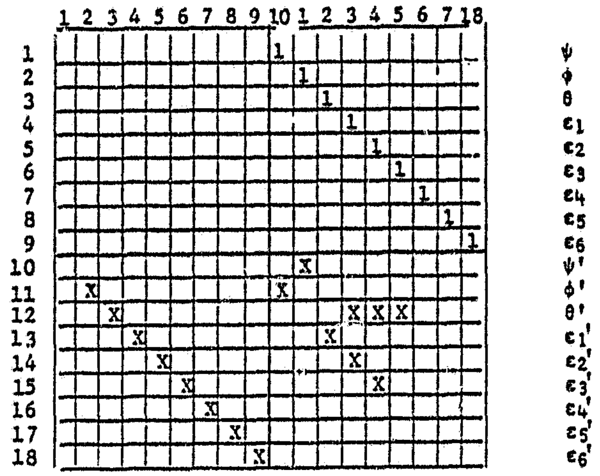

Fig. 2. Location of the non-zero elements of the "A" matrix of shell without the dumbbell.

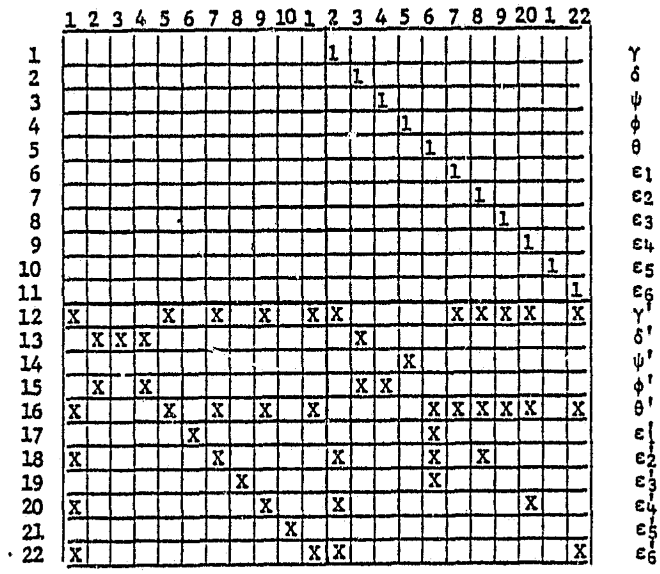

Fig. 3. Location of non-zero elements of the "A" matrix of the shell with the dumbbell.

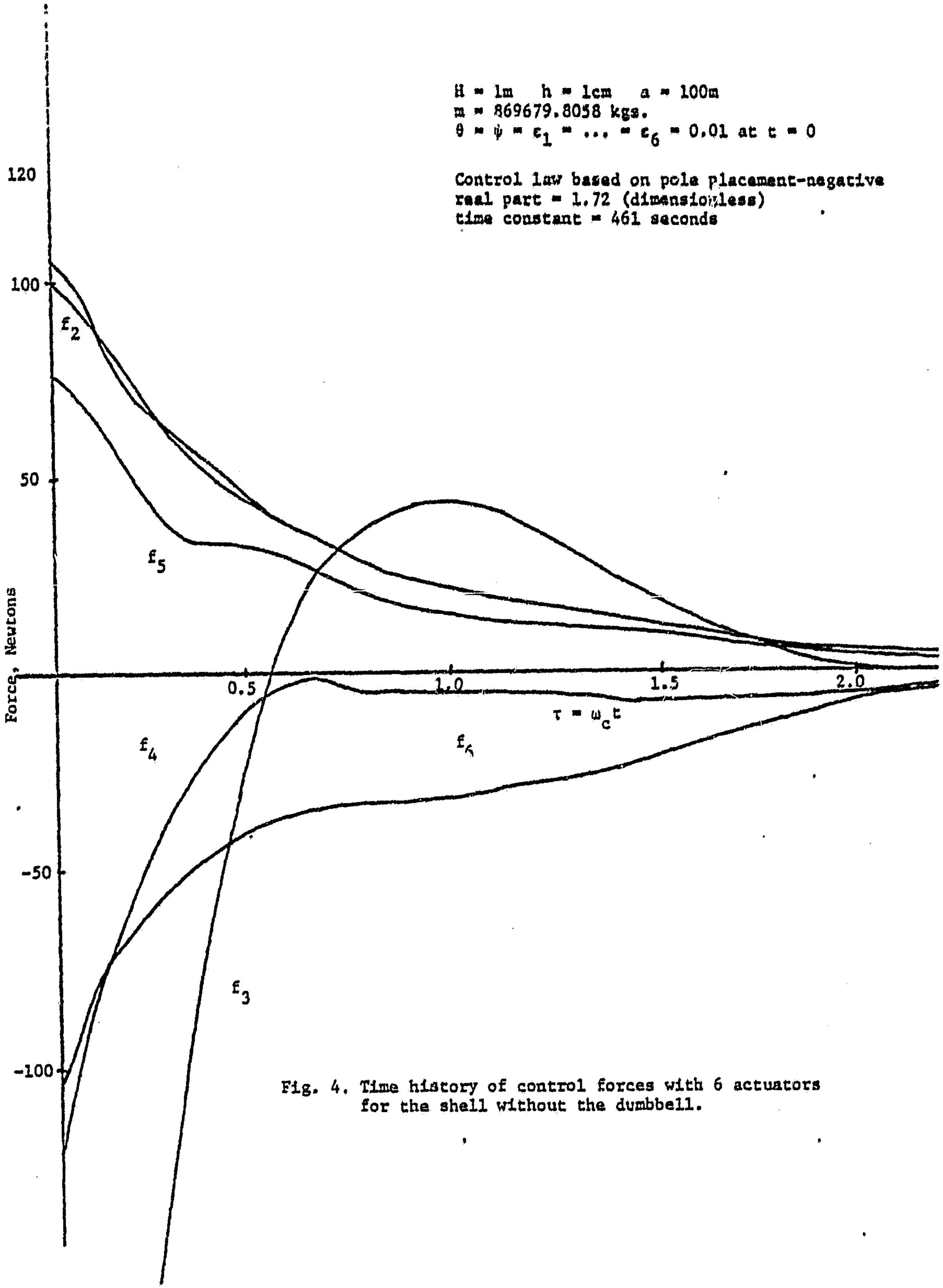

Fig. 4. Time history of control forces with 6 actuators for the shell without the dumbbell.

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3 rigid + 6 flexible modes
250n. mile altitude orbit

⊙ (a) designed closed loop poles
model does not include orbital
and gravity-gradient effects

⊡ (b) closed loop poles resulting from
the control law of (a) when applied
to a model which includes orbital
and gravity-gradient effects

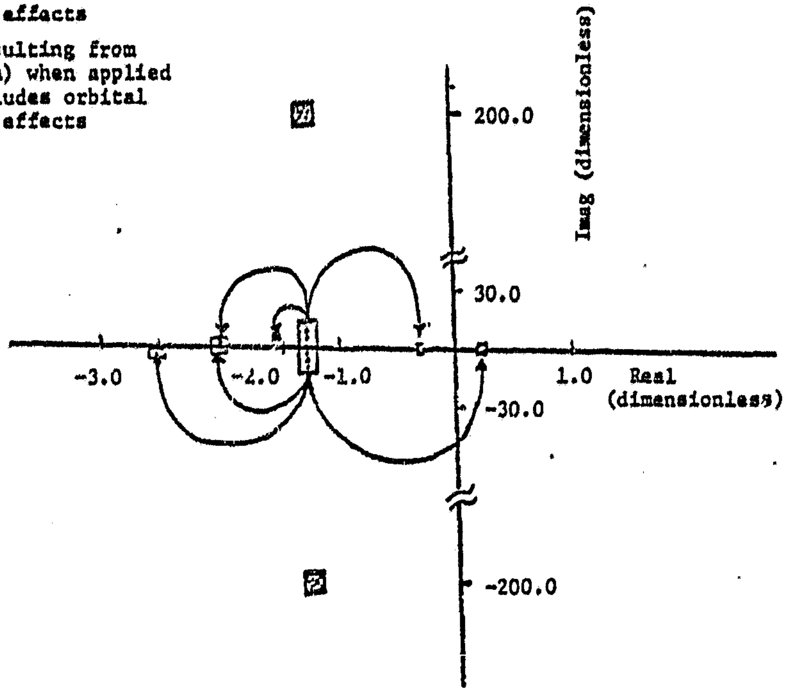

Fig. 5. Shift in closed loop poles due to orbital and gravity-gradient effects-designed response time = 615 secs.

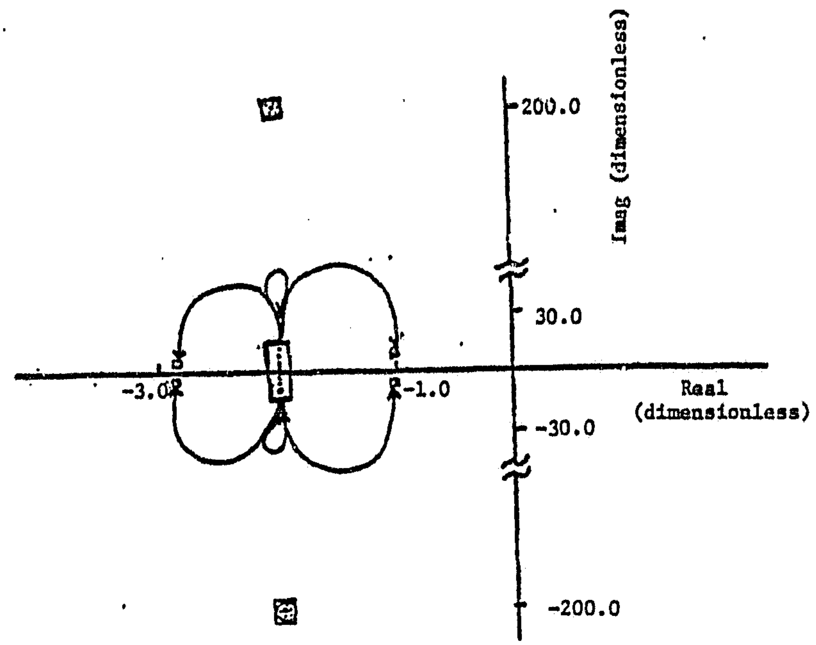

Fig. 6. Shift in closed loop poles due to orbital and gravity-gradient effects-designed response time = 400 secs.

TABLE 4 - Peak Force Amplitudes

Control law developed without orbital and gravity-gradient effects in the plant and then applied to a model including these.

	Without orbital & gravity-gradient	With orbital & gravity-gradient
f_1	516.40Nt.	565.63Nt.
f_2	73.30	164.76
f_3	239.50	321.11
f_4	117.54	117.07
f_5	132.45	153.84
f_6	146.82	431.61
Total force impulse:	7020.8Nt-sec	19373.0Nt-sec

CONCLUSIONS

Orientation and shape control of an orbiting shallow spherical shell system may be accomplished by using appropriately positioned actuators on the surface of the shell. A gimballed spring-loaded dumbbell damper connected at the shell's apex can provide gravitational stabilization together with a source of passive damping. A significant savings in fuel consumption can be realized by using the combined active and passive control systems. For less robust systems instability may result by not including the orbital and gravity-gradient effects in the plant prior to the design of the active control law.

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APPENDIX - NOMENCLATURE

A_n	:	modal amplitudes
a	:	base radius of shell
$\overline{C}$	:	external torques with components (C_x, C_y, C_z)
c, c_y, c_z	:	coefficients of viscous damping
$\overline{c}, \overline{c}_y, \overline{c}_z$	:	$c/J_y\omega_c, c_y/J_y\omega_c, c_z/J_z\omega_c$
$C_y^{(n)}, C_z^{(n)}$	:	$\frac{\ell \partial \phi_x^{(n)}}{\partial y}(0,0)$, $\frac{\ell \partial \phi_x^{(n)}}{\partial z}(0,0)$, respectively
$C_y^{(mn)}, C_z^{(mn)}$	:	$\frac{J_y}{M_m \ell^2} C_y^{(m)} C_y^{(n)}$, $\frac{J_z}{M_m \ell^2} C_z^{(m)} C_z^{(n)}$
c_1, c_2, c_3	:	J_y/I_d; J_z/I_d; $J_y/m\ell^2$, respectively
D	:	flexural rigidity
E	:	modulus of elasticity
E_n	:	modal component of external forces
$I_1^{(n)}$	:	$\int_{vol} x_c \phi_x^{(n)} \rho dv$
I_d	:	moment of inertia of the dumbbell
J_x, J_y, J_z	:	principal moments of inertia of the undeformed shell
k, k_y, k_z	:	torsional spring constants
$\overline{k}, \overline{k}_y, \overline{k}_z$	:	$k/J_y\omega_c^2, k_y/J_y\omega_c^2, k_z/J_z\omega_c^2$, respectively
ℓ	:	characteristic length (=a, the base radius)
M_n	:	modal mass of n^{th} mode
R	:	radius of curvature of the shell

x_c	coordinate of differential area on the surface above the base plane
t	time
β	polar angle of particular location on shell
β_o	angle measured in base plane of shell
γ, δ	dumbbell deflection angles
ϵ_n	A_n/ℓ
θ, ψ, ϕ	pitch, yaw and roll angles, respectively
ν	Poisson's ratio
ρ	mass density
τ	$\omega_c t$ (dimensionless time)
$\bar{\omega}$	body angular velocity vector, $(\omega_x \omega_y \omega_z)$ or $(\omega_r \omega_p \omega_x)$
ω_c	orbit angular velocity
ω_n'	natural frequency of n^{th} mode
$\Omega_x, \Omega_y, \Omega_z$	$(J_z - J_y)/J_x$, $(J_x - J_z)/J_y$, $(J_y - J_x)/J_z$, respectively
$(\dot{\ }), (\ )'$	$\frac{d}{dt}$, $\frac{d}{d\tau}$, respectively
$\phi_x^{(n)}$	transverse component of the n th modal shape function