A DISTURBANCE ISOLATION CONTROLLER FOR THE SOLAR ELECTRIC PROPULSION SYSTEM FLIGHT EXPERIMENT

H. B. Waites
Marshall Space Flight Center
Huntsville, AL

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ABSTRACT

A disturbance isolation controller (DIC) is developed for a simplified model of the Solar Electric Propulsion System (SEPS) flight experiment which consists of a rigid Sperry gimbal torquer (AGS) mounted to a rigid Orbiter and the SEPS solar array (rigid) end mounted to the AGS. The main purpose of the DIC is to reduce the effects of Orbiter disturbances which are transmitted to the flight experiment. The DIC uses an observer, which does not require the direct measurement of the plant inputs, to obtain estimates of the plant states and the rate of the plant states.[1] The state and rate of state information is used to design a controller which isolates disturbances from specified segments of the plant, and for the flight experiment, the isolated segment is the SEPS solar array.

INTRODUCTION

The DIC design is an outgrowth of work form a dissertation[2] on model following controllers and from efforts to reduce the Orbiter acceleration disturbances for the European Instrument Pointing System (IPS) payloads. Model following controllers are required to have estimates of the plant states so that the plant can follow the model. If the inputs to the plant are not available for direct measurement, then the state estimator used in the model following controller can have biases; therefore, the model following controller performance is degraded. For the IPS controller, an accelerometer is mounted at the base of the IPS gimbals to measure accelerations caused by Orbiter disturbances. These disturbances are not available for direct measurement, so a state estimator, used in the IPS controller, would have biases and so would the rate of state information obtained by the estimator. With the apparent inability to estimate the states of the plant, an observer or filter is not useful for control of Orbiter disturbances.

An observer is developed for the flight experiment which does not require direct measurement of all the plant inputs. The observer's inputs are the output measurements of the states and the output measurements of the rate of the states. Using the output measurements of the AGS rate gyros and the AGS accelerometers, the observer converges to the plant states and the plant accelerations. This observer is a key ingredient in the DIC design.

The DIC is a force/torque feedback controller which uses the observer acceleration estimates and knowledge of the plant parameters to determine the appropriate torques for reducing the disturbance effects on certain segments of the plant. The reduction of acceleration effects not only decreases the loads on the payloads, but also improves payload pointing capability. The DIC can best be delineated by applying the control design to the simplified model of the flight experiment.

FLIGHT EXPERIMENT MODEL

The linear equations of motion for a rigid SEPS solar array connected to a rigid AGS which is connected to a rigid Orbiter are

$$\left[\begin{array}{c|c|c} m_{11} & m_{12} & m_{13} \\ \hline m_{21} & m_{22} & m_{23} \\ \hline m_{31} & m_{32} & m_{33} \end{array}\right] \left[\begin{array}{c} \ddot{r}_o \\ \hline \ddot{\Phi}_o \\ \hline \ddot{\Phi}_i \end{array}\right] = \left[\begin{array}{c} I_1 \\ \hline \tilde{r}_d \\ \hline 0_1 \end{array}\right] F + \left[\begin{array}{c} 0_1 \\ \hline 0_1 \\ \hline I_1 \end{array}\right] T_c, \qquad (1)$$

$$\left[\begin{array}{c} \Phi_T \\ \hline \dot{\Phi}_T \end{array}\right] = \left[\begin{array}{c|c|c|c|c|c} 0_1 & 0_1 & T_G{}^t & 0_1 & I_1 & 0_1 \\ \hline 0_1 & 0_1 & 0_1 & T_G{}^t & 0_1 & I_1 \end{array}\right] \left[\begin{array}{c} r_o \\ \hline \dot{r}_o \\ \hline \Phi_o \\ \hline \dot{\Phi}_o \\ \hline \Phi_i \\ \hline \dot{\Phi}_i \end{array}\right], \qquad (2)$$

$$A_g = \left[\begin{array}{c|c|c|c|c|c} 0_1 & I_1 & 0_1 & \tilde{d}_{sm} & 0_1 & 0_1 \end{array}\right] \left[\begin{array}{c} \dot{r}_o \\ \hline \ddot{r}_o \\ \hline \dot{\Phi}_o \\ \hline \ddot{\Phi}_o \\ \hline \dot{\Phi}_i \\ \hline \ddot{\Phi}_i \end{array}\right] \qquad (3)$$

where

0_1 = 3x3 zero matrix,

I_1 = 3x3 identity matrix,

m_{ij} = 3x3 mass matrix, $(i,j) \varepsilon (1,2,3)$,

$\tilde{d}_{sm}$ = 3x3 tilde matrix representing a distance measurement from the Orbiter c.g. to the AGS gimbal torquers,

$T_G{}^t$ = 3x3 transpose of the AGS gimbal angles,

r_o, $\dot{r}_o$, $\ddot{r}_o$ = 3x1 vector position, velocity, and acceleration of the Orbiter c.g.,

Φ_o, $\dot{\Phi}_o$, $\ddot{\Phi}_o$ = 3x1 vector angular displacement, velocity, and acceleration of the Orbiter,

Φ_i, $\dot{\Phi}_i$, $\ddot{\Phi}_i$ = 3x1 vector angular displacement, velocity, and acceleration of the AGS payload,

Φ_T, $\dot{\Phi}_T$ = 3x1 vector measurement of the plant states provided by the AGS rate gyros,

A_g = 3x1 vector measurements of the rate of the plant states provided by the AGS accelerometers,

F = 3x1 vector Orbiter disturbance which cannot be measured directly,

r_d = vector from Orbiter c.g. to application of F, and

T_c = 3x1 vector control torque provided by the AGS gimbal torquers.

The Orbiter linear displacements are eliminated from (1) by the relationship

$$\ddot{r}_o = m_{11}^{-1} (F - m_{12}\ddot{\Phi}_o - m_{13}\ddot{\Phi}_i). \tag{4}$$

Substituting (4) into (1) and into (3) gives

$$\left[\begin{array}{c|c} n_{11} & n_{12} \\ \hline n_{21} & n_{22} \end{array}\right]\left[\begin{array}{c} \ddot{\Phi}_o \\ \hline \ddot{\Phi}_i \end{array}\right] = \left[\begin{array}{c} p_{11} \\ \hline p_{21} \end{array}\right] F + \left[\begin{array}{c} 0_1 \\ \hline I_1 \end{array}\right] T_c \text{ and} \tag{5}$$

$$A_g = [0_1 \,\vdots\, V_{12} \,\vdots\, 0_1 \,\vdots\, V_{14}]\left[\begin{array}{c} \dot{\Phi}_o \\ \hline \ddot{\Phi}_o \\ \hline \dot{\Phi}_i \\ \hline \ddot{\Phi}_i \end{array}\right] + m_{11}^{-1} F \tag{6}$$

where

$n_{11} = m_{22} \quad -m_{21} \quad m_{11}^{-1} \quad m_{12}$,

$n_{12} = m_{23} \quad -m_{21} \quad m_{11}^{-1} \quad m_{13}$,

$n_{21} = m_{32} \quad -m_{31} \quad m_{11}^{-1} \quad m_{12}$,

$n_{22} = m_{33} \quad -m_{31} \quad m_{11}^{-1} \quad m_{13}$,

$$p_{11} = \tilde{r}_d \quad -m_{21} \quad m_{11}^{-1},$$

$$p_{21} = -m_{31} \quad m_{11}^{-1},$$

$$V_{12} = \tilde{d}_{sm} \quad -m_{11}^{-1} \quad m_{12}, \text{ and}$$

$$V_{14} = -m_{11}^{-1} \quad m_{13}$$

while the attitude measurement equation becomes

$$\left[\begin{array}{c} \Phi_T \\ \hline \dot{\Phi}_T \end{array}\right] = \left[\begin{array}{c|c|c|c} T_G^{\,t} & 0_1 & I_1 & 0_1 \\ \hline 0_1 & T_G^{\,t} & 0_1 & I_1 \end{array}\right] \left[\begin{array}{c} \Phi_o \\ \hline \dot{\Phi}_o \\ \hline \Phi_i \\ \hline \dot{\Phi}_i \end{array}\right] \tag{7}$$

Writing equation (5) in state form gives

$$\left[\begin{array}{c} \dot{\Phi}_o \\ \hline \ddot{\Phi}_o \\ \hline \dot{\Phi}_i \\ \hline \ddot{\Phi}_i \end{array}\right] = \left[\begin{array}{c|c|c|c} 0_1 & I_1 & 0_1 & 0_1 \\ \hline 0_1 & 0_1 & 0_1 & 0_1 \\ \hline 0_1 & 0_1 & 0_1 & I_1 \\ \hline 0_1 & 0_1 & 0_1 & 0_1 \end{array}\right] \left[\begin{array}{c} \Phi_o \\ \hline \dot{\Phi}_o \\ \hline \Phi_i \\ \hline \dot{\Phi}_i \end{array}\right] + \left[\begin{array}{c} 0_1 \\ \hline q_{21} \\ \hline 0_1 \\ \hline q_{41} \end{array}\right] F + \left[\begin{array}{c} 0_1 \\ \hline r_{21} \\ \hline 0_1 \\ \hline r_{42} \end{array}\right] T_c \tag{8}$$

where

$$\left[\begin{array}{c} q_{21} \\ \hline q_{41} \end{array}\right] = \left[\begin{array}{c|c} n_{11} & n_{12} \\ \hline n_{21} & n_{22} \end{array}\right]^{-1} \left[\begin{array}{c} p_{11} \\ \hline p_{21} \end{array}\right] \quad \text{and}$$

$$\left[\begin{array}{c} r_{21} \\ \hline r_{42} \end{array}\right] = \left[\begin{array}{c|c} n_{11} & n_{12} \\ \hline n_{21} & n_{22} \end{array}\right]^{-1} \left[\begin{array}{c} 0_1 \\ \hline I_1 \end{array}\right] .$$

Alas! Upon examiniation of (7) and (8), it is found that the system is not observable. To remove this thorn from one's side, consider adding feedback to (1) by defining

$$T_c = T_o + T_{cl} \tag{9}$$

where

$T_o = -K_1\dot{\Phi}_i - K_o\Phi_i$ and

T_{cl} is unspecified.

Substituting (9) into (5) and rearranging yields

$$\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}\begin{bmatrix} \ddot{\Phi}_o \\ \ddot{\Phi}_i \end{bmatrix} + \begin{bmatrix} 0_1 & 0_1 \\ 0_1 & K_1 \end{bmatrix}\begin{bmatrix} \dot{\Phi}_o \\ \dot{\Phi}_i \end{bmatrix} + \begin{bmatrix} 0_1 & 0_1 \\ 0_1 & K_o \end{bmatrix}\begin{bmatrix} \Phi_o \\ \Phi_i \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} F + \begin{bmatrix} 0_1 \\ I_1 \end{bmatrix} T_{cl} \tag{10}$$

and writing (1) in state form gives

$$\begin{bmatrix} \dot{\Phi}_o \\ \ddot{\Phi}_o \\ \dot{\Phi}_i \\ \ddot{\Phi}_i \end{bmatrix} = \begin{bmatrix} 0_1 & I_1 & 0_1 & 0_1 \\ k_{11} & d_{11} & k_{12} & d_{12} \\ 0_1 & 0_1 & 0_1 & I_1 \\ k_{21} & d_{21} & k_{22} & d_{22} \end{bmatrix}\begin{bmatrix} \Phi_o \\ \dot{\Phi}_o \\ \Phi_i \\ \dot{\Phi}_i \end{bmatrix} + \begin{bmatrix} 0_1 \\ q_{21} \\ 0_1 \\ q_{41} \end{bmatrix} F + \begin{bmatrix} 0_1 \\ r_{21} \\ 0_1 \\ r_{41} \end{bmatrix} T_{cl} \tag{11}$$

where

$$\begin{bmatrix} d_{11} & d_{21} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}^{-1} \begin{bmatrix} 0_1 & 0_1 \\ 0_1 & K_1 \end{bmatrix}, \text{ and}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}^{-1} \begin{bmatrix} 0_1 & 0_1 \\ 0_1 & K_o \end{bmatrix}.$$

With a suitable selection of K_o and K_1, the system equations (11) and (7) are observable. Once the observability criterion is met, the observer and the DIC can be formulated.

OBSERVER AND DIC DESIGN

The key factor in the DIC is an observer which does not require direct measurement of the plant inputs. The observer for the flight experiment is derived by first rewriting (11), (7), and (6) into generic system equations. The system equations are

$$\dot{x} = Ax + B_c\, T_{cl} + B_D\, F, \tag{12}$$

$$y = Cx, \text{ and} \tag{13}$$

$$z = P\dot{x} + P_D\, F. \tag{14}$$

The full state observer [1] for (12), (13), and (14) is of the form

$$\dot{\alpha} = E\alpha + Gy + Hz + JT_{cl} \tag{15}$$

where

E = 12x12 observer dynamic matrix,

G = 12x6 matrix (unknown for present),

H = 12x3 matrix (unknown for present),

J = 12x3 matrix (unknown for present), and

α = 12x1 vector of the observer states.

Let α be a linear combination of x such that

$$\alpha = Tx. \tag{16}$$

Substituting (12)-(14) and (16) into (15) gives

$$T\dot{x} = ETx + GCx + H[P(Ax + B_cT_{cl} + B_DF) + P_DF] + JT_{cl} \tag{17}$$

Premultiplying (12) by T gives

$$T\dot{x} = TAx + TB_cT_{cl} + TB_DF. \tag{18}$$

Subtracting (17) from (18), collecting terms, and equating variables yields the matrix relations

$$TA - ET = GC + HPA, \tag{19}$$

$$TB_c = HPA + J, \text{ and} \tag{20}$$

$$TB_D = H(PB_D + P_D). \tag{21}$$

If $(PB_D + P_D)^{-1}$ exists, then

$$H = TB_D(PB_D + P_D)^{-1}. \tag{22}$$

Substituting (22) into (19) and collecting terms gives

$$T[I - B_D(PB_D + P_D)^{-1}P]A - ET = GC. \tag{23}$$

Letting

$$A_1 = [I - B_D(PB_D + P_D)^{-1}P]A \tag{24}$$

and using direct products[3] on (23) yields

$$(I \otimes A_1{}^t - E \otimes I)\hat{T} = \hat{GC} \tag{25}$$

where $\hat{T}$ and $\hat{GC}$ are 144x1 vectors of the form

$$\hat{T} = \begin{bmatrix} t_{1*}{}^t \\ \hline \cdot \\ \cdot \\ \cdot \\ \hline t_{12,*}{}^t \end{bmatrix} \quad \text{and } \hat{GC} = \begin{bmatrix} (GC)_{1*}{}^t \\ \hline \cdot \\ \cdot \\ \cdot \\ \hline (GC)_{12,*}{}^t \end{bmatrix} .$$

If A_1 and E have no common eigenvalues, then

$$\hat{T} = (I \otimes A_1{}^t - E \otimes I)^{-1} \hat{GC}, \tag{26}$$

$$H = TB_D(PB_D + P_D)^{-1}, \text{ and} \tag{27}$$

$$J = TB_c - HPA. \tag{28}$$

If G is selected such that rank of C is equal to the rank of GC,[4] then T^{-1} exists.

For this paper, a reduced order observer is derived. The derivation is the same as the full state observer except that the dimension of T, E, and G are different. For the reduced order observer, the transformation is

$$\hat{T} = [I_6 \otimes A_1{}^t - E \otimes I_{12}]^{-1} \hat{GC} \tag{29}$$

where

I_6 = 6x6 identity matrix,

I_{12} = 12x12 identity matrix,

E = 6x6 stable matrix, and

G = 6x6 matrix.

For the simplified flight experiment model, the system parameters are shown in Table 1 and the observer parameters are shown in Table 2. Catenating the measurement equation (13) with the transformation obtained in (29) gives

$$\begin{bmatrix} y \\ \hline \alpha \end{bmatrix} = \begin{bmatrix} C \\ \hline T \end{bmatrix} x. \tag{30}$$

Since the rank of GC equals the rank of C, a solution for the plant states exists and it is

$$x = \left[\frac{C}{T}\right]^{-1} \left[\frac{y}{\alpha}\right]. \tag{31}$$

Table 3 shows the transformation and its inverse. This concludes the observer derivation for the flight experiment model.

To demonstrate the DIC design, consider the third vector equation in (1) which is

$$m_{31}\ddot{r}_o + m_{32}\ddot{\Phi}_o + m_{33}\ddot{\Phi}_i = T_o + T_{c1} = T_o + T_A + T_{DIC} \tag{32}$$

where

T_A = 3x1 attitude control torque and

T_{DIC} = 3x1 DIC torque.

Premultiplying A_g in (3) by m_{31} yields

$$m_{31}A_g = m_{31}\ddot{r}_o + m_{31}\tilde{d}_{sm}\ddot{\Phi}_o. \tag{33}$$

Using this result and the estimates of the plant states in (31), let

$$T_A = -L_1x_4 - L_ox_3 + m_{31}A_g \tag{34}$$

where

$$\begin{bmatrix} x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline x_4 \end{bmatrix} \rightarrow \begin{bmatrix} \Phi_o \\ \hline \dot{\Phi}_o \\ \hline \Phi_i \\ \hline \dot{\Phi}_i \end{bmatrix},$$

L_o = 3x3 attitude position gain, and

L_1 = 3x3 attitude rate gain.

Substituting (34) into (32) and collecting terms in the resulting equation gives

$$(m_{32} - m_{31}\tilde{d}_{sm})\ddot{\Phi}_o + m_{33}\ddot{\Phi}_i = -L_1x_4 - L_ox_3 + T_o + T_{DIC} \tag{35}$$

Differentiating (31) yields

$$\dot{x} = \left[\frac{C}{T}\right]^{-1} \left[\frac{\dot{y}}{\dot{\alpha}}\right] \tag{36}$$

where

$$\begin{bmatrix} x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline x_4 \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\Phi}_o \\ \hline \Phi_o \\ \hline \dot{\Phi}_i \\ \hline \Phi_i \end{bmatrix}$$

Since $\dot{x}_2$ converges to $\ddot{\Phi}_o$, select

$$T_{DIC} = -T_o + (m_{32} - m_{31}d_{sm})\dot{x}_2. \tag{37}$$

Substituting (37) into (35) gives

$$m_{33}\ddot{\Phi}_i + L_1x_4 + L_ox_3 + (m_{32} - m_{31}\tilde{d}_{sm})(\ddot{\Phi}_o - \dot{x}_2)$$

$$= m_{33}\ddot{\Phi}_i + L_1\dot{\Phi}_i + L_o\Phi_i = 0 \tag{38}$$

which shows that the Φ_i variable is isolated from the dynamic variables r_o and Φ_o and the Orbiter disturbance F which is the goal of the DIC.

SUMMARY

This paper contains the development of a disturbance isolation controller for a simplified model of the SEPS flight experiment. The DIC design primarily consists of a method to obtain observer estimates which will converge to the plant states and the plant accelerations. Using the acceleration estimates, the DIC isolates the AGS payload from the Orbiter disturbances. Future considerations will include the effects of system noises, nonlinear plants and measurements, and a determination of the observer robustness.

REFERENCES

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A

0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	1	0	0	3	0	0
0	0	0	0	0	0	0	2	0	0	6	0
0	0	0	0	0	0	0	0	3	0	0	9

BC

0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
¯6.19E¯7	0.00E0	¯1.86E¯11
0.00E0	¯1.38E¯7	0.00E0
¯2.70E¯8	0.00E0	¯1.33E¯7
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
1.43E¯5	0.00E0	2.70E¯8
0.00E0	1.39E¯5	0.00E0
2.70E¯8	0.00E0	4.95E¯3

BD

0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	¯5.99E¯7	0.00E0
1.34E¯7	0.00E0	¯5.80E¯7
0.00E0	5.26E¯7	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	9.35E¯7	0.00E0
¯6.15E¯7	0.00E0	6.53E¯7
0.00E0	¯6.26E¯7	0.00E0

C

1	0	0	0	0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	1

P

0	0	0	0	¯0.963	¯0	0	0	0	0	¯0.0356	0
0	0	0	0.963	0	¯4.71	0	0	0	0.0356	0	0
0	0	0	0	4.71	0	0	0	0	0	0	0

PQ

1.13E¯5	0.00E0	0.00E0
0.00E0	1.13E¯5	0.00E0
0.00E0	0.00E0	1.13E¯5

TABLE 1. SYSTEM PARAMETERS

E

0	1	0	0	0	0
¯2	¯2	0	0	0	0
0	0	0	1	0	0
0	0	¯2	¯2	0	0
0	0	0	0	0	1
0	0	0	0	¯2	¯2

G

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

H

0	0	0
0	0	0
0	0	0
0	0.0431	0
¯0.0431	0	0
0	0	0
¯0.00241	0.00952	0.0342
0.00541	¯0.0357	¯0.0385
0	¯0.0578	0
0	0.066	0
0.00481	0.0378	¯0.0685
¯0.0108	¯0.00621	0.077

J

0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
0.00E0	0.00E0	0.00E0
1.36E 5	0.00E0	0.00E0
0.00E0	1.36E 5	0.00E0
0.00E0	0.00E0	4.95E¯3
¯3.13E¯6	¯1.29E¯6	¯1.25E¯8
4.29E¯6	5.95E¯7	3.16E¯8
1.16E¯6	0.00E0	¯5.36E¯4
1.97E¯6	0.00E0	6.94E¯4
6.73E¯10	2.58E¯6	1.59E¯4
¯4.62E¯9	¯1.19E¯6	3.78E¯4

TABLE 2. OBSERVER PARAMETERS

T

1	0	0	0	0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	1
1	0.5	0	-0.5	-0.5	0	1.07	0.65	0	-0.241	-0.0999	0
-1	0	0	1	0.5	0	-1.24	-0.2	0	0.344	0.0499	0
0	0	1	0.5	0	-0.5	-0.173	0	1.11	0.102	0	-0.108
0	0	-1	0	0	1	0.104	0	-1.32	0.14	0	0.14
0	0	0	0	1	0.5	0.000442	-0.3	-0.21	0.00104	0.2	0.0318
0	0	0	0	-1	0	-0.000399	0.4	0.0955	-0.000486	-0.0997	0.0754

TI

-14.3	-12.6	-4.99	4.01	0	0	25.3	9.98	-2.67	-7.66	12.6	4.99
1.34	5	10.7	0	2	0	12	10.7	-9.33	1.34	-12	-10.7
-9.14	0.864	9.9	0	0	1.33	1.73	-10.9	20	9.1	0.864	5.43
6.66	4.66	1.33	-2	0	0	-9.32	2.66	2	3.33	-4.66	-1.33
1.33	2.67	-4	0	-1	0	-5.33	-4	2.67	-1.33	5.33	4
2.68	-1.1	4.89	-0.00287	0	-0.667	2.2	4.88	-7.55	-2.66	1.1	2.44
15.3	12.6	4.99	-4.01	0	0	-25.3	-9.98	2.67	7.66	-12.6	-4.99
1.34	6	-10.7	0	-2	0	-12	-10.7	9.33	-1.34	12	10.7
9.14	-0.864	10.9	0	0	-1.33	1.73	10.9	-20	-9.1	0.864	5.43
-6.66	-4.66	1.33	3	0	0	9.32	2.66	2	-3.33	4.66	1.33
-1.33	-2.67	4	0	2	0	5.33	4	-2.67	1.33	-5.33	-4
-2.68	1.1	-4.89	0.00287	0	1.67	-2.2	-4.88	7.55	2.66	-1.1	-2.44

TABLE 3. OBSERVER/STATE TRANSFORMATION AND ITS INVERSE

NOMENCLATURE

$M_{11}\ddot{r}_o + M_{12}\ddot{\Phi}_o + M_{13}\ \Phi_i = F$: ORBITER CG TRANSLATIONAL EQUATION

$M_{21}\ddot{r}_o + M_{22}\ \ddot{\Phi}_o + M\,23\ \ddot{\Phi}_i = \tilde{r}_d\,F$: ORBITER ROTATIONAL EQUATION

$M_{31}\ \ddot{r}_o + M_{32}\ \ddot{\Phi}_o + M_{33}\ \ddot{\Phi}_i = T_C + T_o$: PAYLOAD ATTITUDE EQUATION

F: FORCE WHICH IS NOT DIRECTLY MEASUREABLE i.e., CREW MOTION OR RCS

T_C: CONTROL TORQUE

T_o: OBSERVABILITY TORQUE (ADDED)

r_d: VECTOR FROM ORBITER CG TO FORCE (F) APPLICATION

M_{ij}: MASS MATRIX COMPONENTS

$\Phi_T = T_G^t\ \Phi_o + \Phi_i$: DIGITAL RATE GYRO MEASUREMENTS

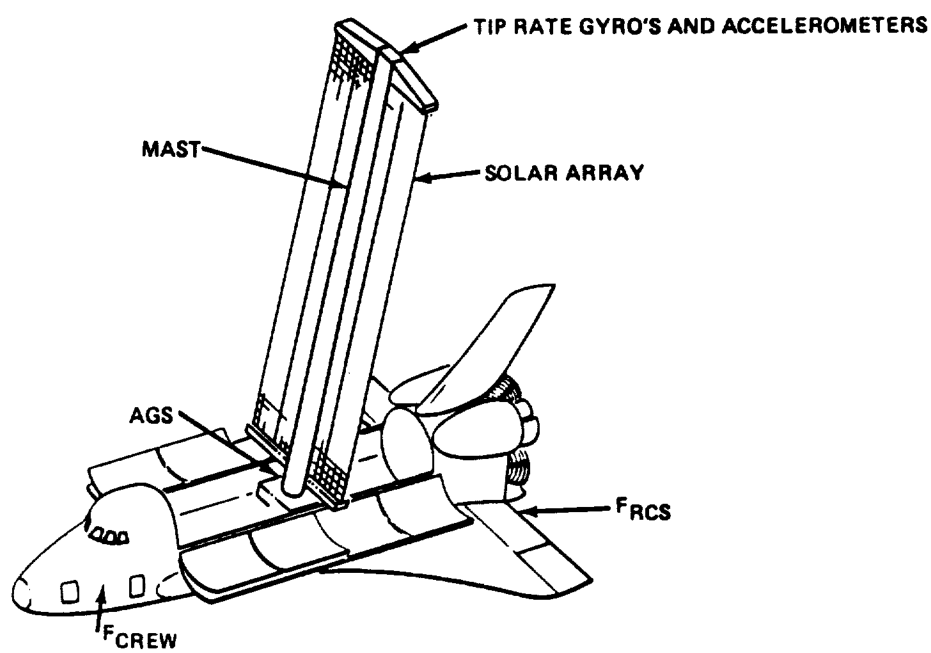
$\dot{\Phi}_T = T_G{}^t\,\dot{\Phi}_o + \dot{\Phi}_i$: ANALOG RATE GYRO MEASUREMENTS

$T_G{}^t$: TRANSFORMATION FROM INERTIAL TO BODY FRAME

$A_g = \ddot{r}_o + \tilde{d}_{SM}\,\ddot{\Phi}_o$: AGS BASE ACCELEROMETER MEASUREMENTS

d_{SM} = VECTOR FROM ORBITER CG TO AGS ACCELEROMETERS

- FLIGHT EXPERIMENT CONFIGURATION



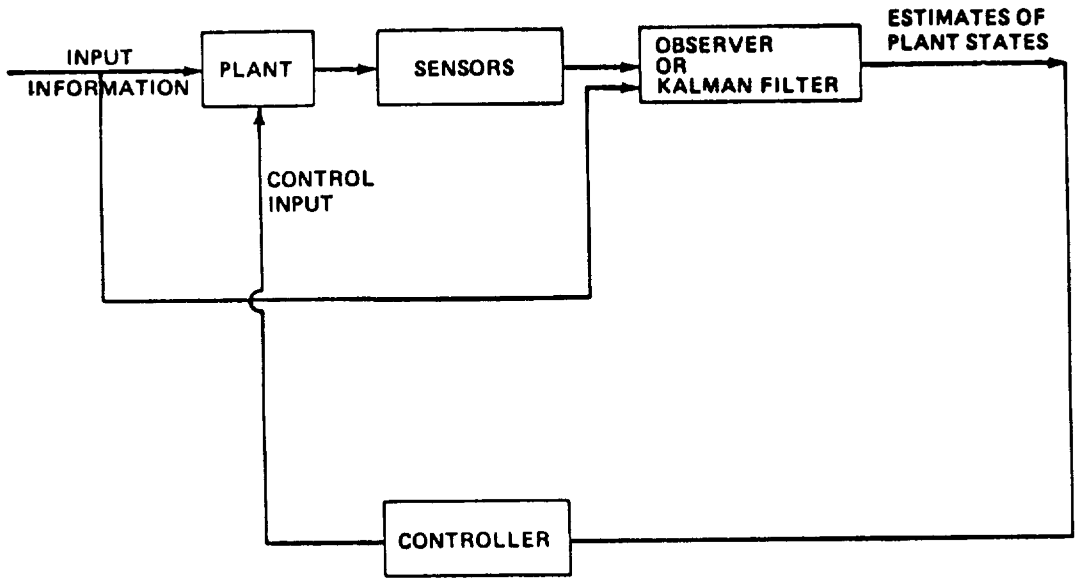
- EXPERIMENT OBJECTIVES
  - SUITABLE STRUCTURE
  - ASPECTS OF LSS CONTROL
- CONTROL OBJECTIVES
  - DISTRIBUTED SENSOR CONTROL
  - MODAL DAMPING
  - NESTED CONTROLLER
  - DISTURBANCE ISOLATION CONTROL
    - ORBITER DISTURBANCES
    - ACCELERATION REDUCTION
- SYSTEM EQUATION FOR SIMPLIFIED MODEL

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{r}_o \\ \ddot{\Phi}_o \\ \ddot{\Phi}_i \end{bmatrix} = \begin{bmatrix} I_1 \\ \tilde{r}_d \\ 0_1 \end{bmatrix} F + \begin{bmatrix} 0_1 \\ 0_1 \\ I_1 \end{bmatrix} T_c + \begin{bmatrix} 0_1 \\ 0_1 \\ I_1 \end{bmatrix} T_o$$

$$\begin{bmatrix} \Phi_T \\ \dot{\Phi}_T \end{bmatrix} = \begin{bmatrix} 0_1 & 0_1 & T_G^t & 0_1 & I_1 & 0_1 \\ 0_1 & 0_1 & 0_1 & T_G^t & 0_1 & I_1 \end{bmatrix} \begin{bmatrix} r_o \\ \dot{r}_o \\ \Phi_o \\ \dot{\Phi}_o \\ \Phi_i \\ \dot{\Phi}_i \end{bmatrix}$$

$$A_g = \begin{bmatrix} 0_1 & I_1 & 0_1 & \tilde{d}_{sm} & 0_1 & 0_1 \end{bmatrix} \begin{bmatrix} \dot{r}_o \\ \ddot{r}_o \\ \dot{\Phi}_o \\ \ddot{\Phi}_o \\ \dot{\Phi}_i \\ \ddot{\Phi}_i \end{bmatrix}$$

SYSTEM BLOCK DIAGRAM



- GENERIC SYSTEM EQUATIONS

$$\dot{x} = Ax + B_c T_{c1} + B_D F$$

$$y = Cx$$

$$z = P\dot{x} + P_D F$$

- OBSERVER FORM

$$\dot{\alpha} = E\alpha + Gy + Hz + JT_{c1}$$

- OBSERVER CONSTRAINT EQUATIONS

$$TA - ET = GC + HPA$$

$$TB_c = HPB_c + J$$

$$TB_D = H(PB_D + P_D)$$

- CONSTRAINT EQUATION SOLUTIONS

$$\hat{T} = (I \oplus A^T - E \oplus I)^{-1} \widehat{GC}$$

$$H = TB_D (PB_D + P_D)^{-1}$$

$$J = TB_C - HPB_C$$

- **ESTIMATOR EQUATIONS**

$$\alpha = T x + e^{Et}\left(\alpha(0) - T x(0)\right)$$

$$\dot{\alpha} = T \dot{x} + E e^{Et}\left(\alpha(0) - T x(0)\right)$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \rightarrow T \begin{bmatrix} \Phi_o \\ \Phi_o \\ \Phi_i \\ \Phi_i \end{bmatrix} \quad \text{AND} \quad \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{bmatrix} \rightarrow T \begin{bmatrix} \dot{\Phi}_o \\ \ddot{\Phi}_o \\ \dot{\Phi}_i \\ \ddot{\Phi}_i \end{bmatrix}$$

- **DISTURBANCE ISOLATION CONTROL TORQUE**

$$m_{31} \ddot{r}_o + m_{32} \ddot{\Phi}_o + m_{33} \ddot{\Phi}_i = T_o + T_A + T_{DIC}$$

$$A_g = \ddot{r}_o + \tilde{d}_{sm} \ddot{\Phi}_o$$

$$T_A = -L_1 \alpha_4 - L_o \alpha_3 + m_{31} A_g$$

$$T_{DIC} = -T_o + (m_{32} - m_{31} \tilde{d}_{sm}) \dot{\alpha}_2$$

- **FUTURE INVESTIGATION**
  - COMPLEX SAFE MODEL
  - OBSERVER SENSITIVITY (PARAMETERS)
  - OBSERVER TRUNCATION
    - MODES
    - WORD LENGTH
  - NOISE