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ANALYTIC OPTICAL POTENTIALS FOR NUCLEON-NUCLEUS AND NUCLEUS-NUCLEUS COLLISIONS INVOLVING LIGHT AND MEDIUM NUCLEI

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NOMENCLATURE

A	nuclear mass number, dimensionless
a	oscillator parameter, fm
$B(e)$	average slope parameter of nucleon-nucleon scattering amplitude, fm^2
B_1	defined in equation (35)
C_i	defined in equation (10)
$C(y)$	correlation function, dimensionless
D_i	defined in equation (11)
e	two-nucleon kinetic energy in their center of mass frame, GeV
F_i	defined in equation (12)
$g(\vec{r}_1,\vec{r}_2)$	defined in equation (25)
I_1	defined in equation (41)
I_2	defined in equation (42)
I_3	defined in equation (43)
I_4	defined in equation (44)
k_F	Fermi momentum wave number, fm^{-1}
M	nucleon mass, 938 MeV/c^2
N_i	defined in equation (9)
r_p	proton rms charge radius, fm
s	defined in equation (13)
$\tilde{t}$	average two-nucleon transition amplitude, MeV
t_o	defined in equation (6)
$t_{\beta j}$	two-nucleon transition operator for nucleons β and j, MeV
V_{opt}	optical potential operator, MeV
$W(\vec{x})$	optical potential, MeV

$\vec{x}$	relative position vector of projectile, fm
$\vec{y}$	two-nucleon relative position vector, fm
Z	nuclear proton number, dimensionless
$\vec{z}$	collection of constituent relative coordinates for target, fm
$\alpha(e)$	average ratio of real part to imaginary part of nucleon-nucleon scattering amplitude, dimensionless
β	defined in equation (23)
$\delta(\vec{x})$	Dirac delta function, dimensionless
Γ	defined in equation (47)
Γ_1	defined in equation (62)
γ	harmonic well charge density parameter, dimensionless
$\vec{\xi}$	defined in equation (50)
ρ	nuclear density, fm^{-3}
$\sigma(e)$	average nucleon-nucleon total cross section, fm^2 or mb
Ω_T	defined in equation (22)
Ω_P	defined in equation (48)
ω_T	defined in equation (34)

Subscripts:

C	charge
m	matter
P	projectile
p	proton
T	target
nc	Pauli correlations excluded
pc	Pauli correlations included

Arrows over symbols indicate vectors.

ANALYTIC OPTICAL POTENTIALS FOR NUCLEON-NUCLEUS AND NUCLEUS-NUCLEUS COLLISIONS INVOLVING LIGHT AND MEDIUM NUCLEI

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SUMMARY

Utilizing an optical model potential approximation to the exact nucleus-nucleus multiple-scattering series, optical potentials for nucleon-nucleus and nucleus-nucleus collisions are analytically derived. These expressions are applicable to light and medium cosmic ray nuclei as their single-particle density distributions are analytically determined, without approximation, from their actual harmonic well charge density distributions. Pauli correlation effects are included through the use of a simple gaussian function to replace the usual expression obtained in the infinite nuclear matter approximation.

INTRODUCTION

Microscopic analyses of optical potentials for use in nucleon-nucleus and nucleus-nucleus scatterings typically utilize Woods-Saxon parameterizations for the single-particle densities of the colliding systems. Although accurate as representations for the density distributions of most nuclei heavier than oxygen (ref. 1), they are only approximately correct for light and medium nuclei ($3 \leq Z \leq 8$). For these nuclei, a more exact representation for the actual nuclear density is the harmonic well distribution (ref. 1) based upon the nuclear shell model. In addition to being a more accurate density representation, the harmonic well distribution, unlike the Woods-Saxon one, allows a completely analytic expression for the optical potential to be determined. In this paper, optical potentials for nucleon-nucleus and nucleus-nucleus scattering, with and without Pauli correlations, will be analytically derived for light and medium cosmic ray nuclei.

NUCLEON-NUCLEUS POTENTIALS

For scattering between composite nuclear systems, the effective optical model potential operator (ref. 2) is

$$V_{opt} = \sum_{\beta j} t_{\beta j} \ , \tag{1}$$

where $t_{\beta j}$ is the transition operator for scattering between the β constituent of the target and the j constituent of the projectile. From references 3-5, equation (1) gives a generalized expression for the optical potential

$$W(\vec{x}) = A_P A_T \int d^3\vec{z}\ \rho_T(\vec{z}) \int d^3\vec{y}\ \rho_P(\vec{x} + \vec{y} + \vec{z})\ \tilde{t}(e,\vec{y}) \ , \tag{2}$$

where $\tilde{t}$ is the two-body transition amplitude averaged over the constituent types (ref. 3) and ρ_T and ρ_P are the target and projectile single-particle number (matter) densities.

NUCLEON-NUCLEUS POTENTIALS

Optical Potential Excluding Pauli Correlations

Since equation (2) was derived using simple unsymmetrized product wavefunctions (refs. 3, 5), it does not include Pauli correlation effects. If the incident projectile is elementary (i.e., a nucleon), then $A_P \equiv 1$ and the projectile single-particle density is

$$\rho_P(\vec{x} + \vec{y} + \vec{z}) = \delta(\vec{x} + \vec{y} + \vec{z}) \ , \tag{3}$$

so that equation (2) reduces to

$$W(\vec{x}) = A_T \int d^3\vec{z} \ \rho_T(\vec{z}) \ \tilde{t}(e, \vec{x} + \vec{z}) \ , \tag{4}$$

where

$$\tilde{t}(e, \vec{y}) = -t_o \exp[-y^2/2B(e)] \ , \tag{5}$$

and

$$t_o = (e/m)^{1/2} \ \sigma(e) \ [\alpha(e) + i] \ [2\pi \ B(e)]^{-3/2} \ . \tag{6}$$

In equation (6), e is the two-nucleon kinetic energy in their center of mass frame, and $\sigma(e)$, $\alpha(e)$, and $\beta(e)$ are the usual nucleon-nucleon scattering parameters (refs. 3-5).

For light and medium target nuclei, the matter density is

$$\rho_T(\vec{r}) = N_T \ (C_T + D_T \ r^2) \ \exp(-F_T \ r^2) \ , \tag{7}$$

which results when the gaussian proton charge density is unfolded from the harmonic well charge distribution

$$\rho_c(\vec{r}) = \rho_o \ [1 + (\gamma \ r^2/a^2)] \ \exp(-r^2/a^2) \ , \tag{8}$$

using the general procedures given in references 4 and 5. In equation (7), the various parameters, written in terms of the charge density parameters, γ and a, are

$$N_T = \rho_o a^3/8s^3 \ , \tag{9}$$

$$C_T = 1 + (3\gamma/2) - (3\gamma a^2/8s^2) \ , \tag{10}$$

$$D_T = \gamma a^2/16s^4 \;, \tag{11}$$

$$F_T = 1/4s^2 \;, \tag{12}$$

where

$$s^2 = (a^2/4) - (r_p{}^2/6) \;, \tag{13}$$

and $r_p \approx 0.87$ fm (ref. 6) is the proton rms charge radius. Values for γ and a are tabulated in ref. 1. The normalization constant, ρ_0, is determined by requiring that

$$4\pi \int_0^\infty r^2 \; \rho_c(\vec{r}) \; dr \equiv 1 \;. \tag{14}$$

Incorporating equations (5) and (7) into equation (4) yields

$$W(\vec{x}) = -A_T \; N_T \; t_o \int d^3\vec{z} \; (C_T + D_T \; z^2) \; \exp(-F_T \; z^2) \tag{15}$$

$$\times \; \exp[-(\vec{x} + \vec{z})^2/2B(e)] \;,$$

which can be rewritten as

$$W(\vec{x}) = -2\pi \; A_T N_T \; t_o \; \exp\,[-x^2/2B(e)] \int_0^\infty z^2 dz \; (C_T + D_T \; z^2) \tag{16}$$

$$\times \; \exp[-(F_T + \frac{1}{2B}) \; z^2] \int_0^\pi \exp[-xz \cos\theta/B(e)] \; \sin\theta d\theta \;.$$

The angular integration is of the form

$$\int_0^\pi \exp(-q\cos\theta) \; \sin\theta d\theta = [\exp(q) - \exp(-q)]/q \;, \tag{17}$$

giving

$$W(\vec{x}) = -2\pi \; A_T N_T \; t_o \; \exp[-x^2/2B(e)] \; [B(e)/x] \tag{18}$$

$$\times \int_0^\infty zdz \; (C_T + D_T \; z^2) \; \exp\{-[F_T + (1/2B)]z^2\}$$

$$\times \; \{\exp[xz/B(e)] - \exp[-xz/B(e)]\} \;.$$

The z integration can be rewritten as

$$\int_0^\infty dz \; (C_T z + D_T \; z^3) \; \exp\{-[F_T + 1/2B(e)]z^2\} \; \{\exp[xz/B(e)] \tag{19}$$

$$-\exp[-xz/B(e)]\} = \int_{-\infty}^{\infty} dz\ (C_T z + D_T z^3)$$

$$\times\ \exp\{-[F_T + 1/2B(e)]z^2\}\ \exp[xz/B(e)]\ ,$$

which is of the general form (ref. 7)

$$\int_{-\infty}^{\infty} z^n \exp[-pz^2 + 2qz]\ dz = [(2^{1-n})/p](\pi/p)^{1/2} \qquad (20)$$

$$\times\ \frac{d^{n-1}}{dq^{n-1}} [q \exp(q^2/p)]\ .$$

Thus, the nucleon-nucleus optical potential, without Pauli correlations, is

$$W(\vec{x}) = A_T N_T\ (e/m)^{1/2}\ \sigma(e)\ [\alpha(e) + i]\ (\beta\Omega_T)^{3/2} \qquad (21)$$

$$\times\ [C_T + (3\Omega_T D_T/2) + D_T \beta^2 \Omega_T^2\ x^2]\ \exp(-\beta F_T \Omega_T x^2)\ ,$$

where

$$\Omega_T = (\beta + F_T)^{-1}\ , \qquad (22)$$

and

$$\beta = 1/2B(e)\ . \qquad (23)$$

NUCLEON-NUCLEUS POTENTIALS

Optical Potential Including Pauli Correlations

From ref. 8, Pauli correlation effects can be included by substituting

$$\rho_i(\vec{r}_1)\ \rho_i(\vec{r}_2) \rightarrow \rho_i(\vec{r}_1)\ \rho_i(\vec{r}_2)\ g(\vec{r}_1, \vec{r}_2)\ , \tag{24}$$

where

$$g(\vec{r}_1, \vec{r}_2) \approx 1 - C(|\vec{r}_1 - \vec{r}_2|)\ , \tag{25}$$

with

$$C(y) = 0.238\ \exp(-k_F^2\ y^2/5)\ . \tag{26}$$

In equation (25), $k_F = 1.36\ \mathrm{fm}^{-1}$ is the Fermi momentum wave number for spatially infinite nuclear matter. Incorporating the ansatz, equation (24), into equation (2) yields a generalized optical potential, including Pauli correlations, of

$$W(\vec{x}) = A_P A_T \int d^3\vec{z}\ \rho_T(\vec{z}) \int d^3\vec{y}\ \rho_P(\vec{x} + \vec{y} + \vec{z})\ \tilde{t}(e,\vec{y})\ [1 - C(y)]\ . \tag{27}$$

For an elementary projectile, this reduces to the nucleon-nucleus potential

$$W(\vec{x}) = A_T \int d^3\vec{z}\ \rho_T(\vec{z})\ \tilde{t}(e, \vec{x} + \vec{z})\ [1 - C(|\vec{x} + \vec{z}|)]\ , \tag{28}$$

which is separated into parts as

$$W(\vec{x}) = W_{nc}(\vec{x}) + W_{pc}(\vec{x})\ . \tag{29}$$

In equation (29),

$$W_{nc}(\vec{x}) = A_T \int d^3\vec{z}\ \rho_T(\vec{z})\ \tilde{t}(e, \vec{x} + \vec{z})\ , \tag{30}$$

which was the potential in equation (4) for no correlations (nc). Its solution is given by equation (21). The second part is

$$W_{pc}(\vec{x}) = A_T \int d^3\vec{z}\ \rho_T(\vec{z})\ \tilde{t}(e, \vec{x} + \vec{z})\ C(|\vec{x} + \vec{z}|)\ , \tag{31}$$

which is the Pauli correlation (pc) correction to the nucleon-nucleus optical potential.

Incorporating equations (5), (7), and (26) into equation (31) gives

$$W_{pc}(\vec{x}) = 0.476\ \pi\ A_T N_T t_o\ \exp\{-[(k_F^2/5) + (1/2B(e))]\ x^2\} \tag{32}$$

$$\times \int_0^\infty z^2 dz\ (C_T + D_T z^2)\ \exp\{-[F_T + (1/2B(e))$$

$$+ (k_F^2/5)]\ z^2\} \int_0^\pi \exp\{-[xz/B(e)) + (2k_F^2\ xz/5)]\ \cos\theta\}$$

$$\times \sin\theta d\theta\ .$$

The solution to equation (32) is obtained in the same manner as the solution to (21) since it is of the same general form. Hence, the Pauli correction to the potential is

$$W_{pc}(\vec{x}) = 0.238\ A_T N_T\ (e/m)^{1/2}\ \sigma(e)\ [\alpha(e) + i]\ (\beta\ \omega_T)^{3/2} \tag{33}$$

$$\times [C_T + (3\ \omega_T D_T/2) + B_1^2\ \omega_T^2\ D_T x^2]\ \exp(-B_1 \omega_T F_T x^2)\ ,$$

where

$$\omega_T = (F_T + B_1)^{-1}\ , \tag{34}$$

and

$$B_1 = \beta + (k_F^2/5)\ . \tag{35}$$

The nucleon-nucleus optical potential, with Pauli correlations, is

$$W(\vec{x}) = -A_T N_T\ (e/m)^{1/2}\ \sigma(e)\ [\alpha(e) + i]\ \beta^{3/2}\ \{\Omega_T{}^{3/2}\ [C_t \tag{36}$$

$$+ (3\ \Omega_T D_T/2) + D_T \beta^2 \Omega_T^2 x^2]\ \exp(-\beta F_T \Omega_T x^2)$$

$$- 0.238\ \omega_T{}^{3/2}\ [C_T + (3\ \omega_T D_T/2) + B_1^2\ \omega_T^2\ D_T\ x^2]$$

$$\times \exp(-B_1 \omega_t F_t\ x^2)\}\ .$$

NUCLEUS-NUCLEUS OPTICAL POTENTIALS

Optical Potential Excluding Pauli Correlations

From equation (2), the generalized expression for the nucleus-nucleus optical potential is

$$W(\vec{x}) = A_P A_T \int d^3\vec{z}\ \rho_T(\vec{z}) \int d^3\vec{y}\ \rho_P(\vec{x} + \vec{y} + \vec{z})\ \tilde{t}(e, \vec{y}) \ , \tag{37}$$

where, as before, $\tilde{t}$ is given by equations (5) and (6). For light and medium projectile nuclei, the matter density is

$$\rho_P(\vec{r}) = N_P\ (C_P + D_P\ r^2)\ \exp(-F_P\ r^2) \ , \tag{38}$$

where the parameters, N_P, C_P, D_P and F_P are also given by equations (9) through (12). The target nuclei matter density is given in equation (7).

With these assumptions, the optical potential is

$$W(\vec{x}) = -A_P A_T N_T N_P t_o \int d^3z\ (C_T + D_T z^2)\ \exp(-F_T z^2) \tag{39}$$

$$\times \int d^3\vec{y}\ [C_P + D_P\ (\vec{x} + \vec{y} + \vec{z})^2]\ \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2]$$

$$\times \exp[-y^2/2B(e)] \ .$$

Expanding the sums yields

$$W\ (\vec{x}) = -A_P A_T N_P N_T t_o\ \{C_T C_P I_1 + D_T C_P I_2 + C_T D_P I_3 + D_T D_P I_4\} \ , \tag{40}$$

where

$$I_1 = \int d^3\vec{z}\ \exp(-F_T z^2) \int d^3\vec{y}\ \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2] \tag{41}$$

$$\times \exp(-y^2/2B(e)] \ ,$$

$$I_2 = \int z^2 d^3\vec{z}\ \exp(-F_T z^2) \int d^3\vec{y}\ \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2] \tag{42}$$

$$\times \exp[-y^2/2B(e)] \ ,$$

$$I_3 = \int d^3\vec{z}\ \exp(-F_T z^2) \int d^3\vec{y}\ (\vec{x} + \vec{y} + \vec{z})^2 \tag{43}$$

$$\times \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2]\ \exp[-y^2/2B(e)]\ ,$$

$$I_4 = \int z^2 d^3\vec{z}\ \exp(-F_T z^2)\ \int d^3\vec{y}\ (\vec{x} + \vec{y} + \vec{z})^2 \tag{44}$$

$$\times \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2]\ \exp[-y^2/2B(e)]\ .$$

Evaluation of I_1 and I_2 is straightforward since the angular dependencies are given by equation (17) and the spatial integrations are of the general form in equation (20). Thus, merely quoting the results, we have

$$I_1 = \pi^3\ \Gamma^{3/2}\ \exp(-\beta\Gamma F_P F_T x^2)\ , \tag{45}$$

and

$$I_2 = \pi^3\ \Gamma^{5/2}\ (1.5\ \Omega_P^{-1} + \beta^2 F_P{}^2 \Gamma\ x^2)\ \exp(-\beta\Gamma F_P F_T\ x^2)\ , \tag{46}$$

where Ω_T is defined in equation (22) and

$$\Gamma = (\beta F_P + \beta F_T + F_T F_P)^{-1}\ , \tag{47}$$

$$\Omega_P = (\beta + F_P)^{-1}\ . \tag{48}$$

The integral over $\vec{y}$ in I_3 and I_4 can be written as

$$\int d^3\vec{y}\ (\vec{x} + \vec{y} + \vec{z})^2\ \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2]\ \exp(-\beta y^2) \tag{49}$$

$$= \int d^3\vec{y}\ (\xi^2 + 2\xi y\cos\theta + y^2)\ \exp[-F_P\ (\vec{\xi} + \vec{y})^2]$$

$$\times \exp(-\beta y^2)\ ,$$

where

$$\vec{\xi} = \vec{x} + \vec{z}\ . \tag{50}$$

Two of the the three angular integrations in equation (49) are of the type whose solution is given by equation (17). The remaining angular integration is of the form

$$\int_0^\pi e^{-q\cos\theta}\ \cos\theta\ \sin\theta d\theta = -q^{-1}\ [\exp(q) + \exp(-q)] \tag{51}$$

$$+ q^{-2} [\exp(q) - \exp(-q)] .$$

Hence, equation (49) becomes

$$\int d^3\vec{y}\ (\vec{x} + \vec{y} + \vec{z})^2 \exp[-F_P\ (\vec{x} + \vec{y} + \vec{z})^2] \exp(-\beta y^2) \tag{52}$$

$$= \pi^{3/2}\ \Omega_P^{5/2}\ [1.5 + \Omega_P\beta^2\ (\vec{x} + \vec{z})^2] \exp[-\beta\Omega_P F_P\ (\vec{x} + \vec{z})^2 .$$

Inserting equation (52) into I_3 and I_4 and using equations (17), (20), and (51) to evaluate the resulting integral forms gives

$$I_3 = \pi^3\ \Gamma^{5/2}\ (1.5\ \Omega_T^{-1} + \beta^2 F_T^2\ \Gamma\ x^2) \exp(-\beta F_T F_P \Gamma\ x^2) , \tag{53}$$

and

$$I_4 = \pi^3\ \Gamma^{5/2}\ \{2.25 + 3.75\beta^2\Gamma + [1.5\beta^2\Gamma\ (F_P + F_T) \tag{54}$$

$$- 5\beta^3 F_P F_T \Gamma^2]\ x^2 + \beta^4 F_P^2 F_T^2 \Gamma^3\ x^4\} \exp(-\beta F_T F_P \Gamma\ x^2) .$$

Upon combining these integrals, the nucleus-nucleus optical potential, without Pauli correlations, is

$$W(\vec{x}) = -A_P A_T N_P N_T\ (e/m)^{1/2}\ \sigma(e)\ [\alpha(e) + i]\ (\pi\beta\Gamma)^{3/2} \tag{55}$$

$$\times \Big(C_T C_P + D_T C_P \Gamma\ (1.5\ \Omega_P^{-1} + \beta^2 F_P^2 \Gamma\ x^2) + C_T D_P \Gamma$$

$$\times\ (1.5\ \Omega_T^{-1} + \beta^2 F_T^2 \Gamma\ x^2) + D_P D_T \Gamma\ \{2.25 + 3.75\beta^2\Gamma$$

$$+ [1.5\beta^2\Gamma\ (F_P + F_T) - 5\beta^3 F_P F_T \Gamma^2]\ x^2 + \beta^4 F_P^2 F_T^2 \Gamma^3\ x^4\} \Big)$$

$$\times \exp(-F_P F_T \Gamma\beta\ x^2) .$$

NUCLEUS-NUCLEUS OPTICAL POTENTIALS

Optical Potential Including Pauli Correlations

From equation (27), the nucleus-nucleus optical potential, including Pauli correlations, is

$$W(\vec{x}) = A_P A_T \int d^3\vec{z}\, \rho_T(\vec{z}) \int d^3\vec{y}\, \rho_P(\vec{x} + \vec{y} + \vec{z})\, \tilde{t}(e, \vec{y}) \qquad (56)$$

$$\times\ [1 - C(y)] \ ,$$

with $C(y)$ given by equation (27), ρ_T by equation (7), ρ_P by equation (38), and $\tilde{t}(e,y)$ by equation (5). Separating the potential as in equation (29), we find

$$W(\vec{x}) = W_{nc}(\vec{x}) + W_{pc}(\vec{x}) \ , \qquad (57)$$

where $W_{nc}(x)$ is

$$W_{nc}(\vec{x}) = A_P A_T \int d^3\vec{z}\, \rho_T(\vec{z}) \int d^3\vec{y}\, \rho_P(\vec{x} + \vec{y} + \vec{z})\, \tilde{t}(e, \vec{y}) \ , \qquad (58)$$

whose solution is given in equation (55), and $W_{pc}(x)$ is

$$W_{pc}(\vec{x}) = -A_P A_T \int d^3\vec{z}\, \rho_T(\vec{z}) \int d^3\vec{y}\, \rho_P(\vec{x} + \vec{y} + \vec{z}) \qquad (59)$$

$$\times\ \tilde{t}(e, \vec{y})\, C(y) \ .$$

Inserting equations (5), (7), (27), and (38) into equation (59) yields

$$W_{pc}(\vec{x}) = 0.238\, A_P A_T N_P N_T t_o \int d^3\vec{z}\, (C_T + D_T z^2) \exp(-F_T z^2) \qquad (60)$$

$$\times \int d^3\vec{y}\, [C_P + D_P (\vec{x} + \vec{y} + \vec{z})^2] \exp[-F_P (\vec{x} + \vec{y} + \vec{z})^2]$$

$$\times \exp(-\beta y^2) \exp(-k_F^2 y^2/5) \ .$$

This expression can be put into the same form as equation (39) by using equation (35) to combine the exponentials in y. Hence, the solution, which is of the form given by equation (55), is

$$W_{pc}(\vec{x}) = 0.238\, A_P A_T N_P N_T (e/m)^{1/2} [\alpha(e) + i] (\pi B_1 \Gamma_1)^{3/2} \qquad (61)$$

$$\times\ C_T C_P + D_T C_P \Gamma_1\ (1.5\ \omega_P^{-1} + B_1{}^2 F_P{}^2 \Gamma_1\ x^2)$$

$$\times\ C_T D_P \Gamma_1\ (1.5\ \omega_T^{-1} + B_1{}^2 F_T^2 \Gamma_1\ x^2) + D_T D_P \Gamma_1$$

$$\times\ \{2.25 + 3.75\ B_1{}^2 \Gamma_1 + [1.5\ B_1{}^2 \Gamma_1\ (F_P + F_T)$$

$$-\ 5B_1{}^3\ F_P F_T \Gamma_1{}^2]\ x^2 + B_1{}^4 F_P^2 F_T^2 \Gamma_1^3\ x^4\}$$

$$\times\ \exp(-F_P F_T \Gamma_1 B_1\ x^2)$$

where

$$\Gamma_1 = (B_1 F_P + B_1 F_T + F_T F_P)^{-1}\ . \qquad (62)$$

Combining the results in equations (55) and (61) gives a nucleus-nucleus optical potential, including Pauli correlations, of

$$W(\vec{x}) = -A_P A_T N_P N_T\ (e/m)^{1/2}\ \sigma(e)\ [\alpha(e) + i]\ \pi^{3/2} \Bigg[\beta^{3/2} \Gamma^{3/2} \qquad (63)$$

$$\times \Big(C_T C_P + D_T C_P \Gamma\ (1.5\ \Omega_P{}^{-1} + \beta^2 F_P{}^2 \Gamma\ x^2) + C_T D_P \Gamma$$

$$\times\ (1.5 \Omega_T^{-1} + \beta^2 F_T{}^2 \Gamma\ x^2)$$

$$+\ D_P D_T \Gamma\ \{2.25 + 3.75 \beta^2 \Gamma + [1.5 \beta^2 \Gamma (F_P + F_T) - 5\ \beta^3 F_P F_T \Gamma^2] x^2$$

$$+\ \beta^4 F_P F_T{}^2 \Gamma^3\ x^4\} \Big)\ \exp(-F_P F_T \Gamma \beta x^2) - 0.238\ B_1^{3/2} \Gamma_1^{3/2}$$

$$\times \Big(C_T C_P + D_T C_P \Gamma_1\ (1.5\ \omega_P^{-1} + B_1{}^2 F_P{}^2 \Gamma_1 x^2)$$

$$+\ C_T D_P \Gamma_1\ (1.5\ \omega_T^{-1} + B_1{}^2 F_T{}^2 \Gamma_1 x^2) + D_P D_T \Gamma_1$$

$$\times\ \{2.25 + 3.75\ B_1{}^2 \Gamma_1 + [1.5\ B_1{}^2 \Gamma_1\ (F_P + F_T) - 5B_1{}^3 F_P F_T \Gamma_1{}^2] x^2$$

$$+\ B_1{}^4 F_P{}^2 F_T{}^2 \Gamma_1{}^3\ x^4\} \Big)\ \exp\ (-F_P F_T \Gamma_1 B_1 x^2) \Bigg]\ .$$

EXAMPLES

To illustrate these results, the real and imaginary parts of the optical potentials for proton-carbon and carbon-carbon scattering are plotted in figures 1 through 4. The incident projectile kinetic energy is 200 MeV/nucleon. Values for the carbon charge density parameters, a and γ, were taken from reference 1. Values for the nucleon-nucleon scattering parameters, $\alpha(e)$, $B(e)$, and $\sigma(e)$ were taken from the compilations in references 9 and 10.

In figure 1 real parts of the proton-carbon optical potential are plotted, with and without Pauli correlations, as determined from equations (21) and (36). Note that the Pauli correlations reduce both the depth (strength) and width (range) of the interaction. Similar results are shown in figure 2 for the imaginary parts of the proton-carbon potential. The strength (depth) of the imaginary part of the potential is also greater than that for the real part.

Figures 3 and 4 display comparable results for carbon-carbon scattering. Again, the increased strength of the imaginary part of the interaction to the real part is evident, as is the reduced depth and width when Pauli effects are included. Note also that the well depth is much greater than in the proton-carbon interaction. This is indicative of the increased number of possible interactions in nucleus-nucleus collisions.

CONCLUDING REMARKS

In summary, analytic expressions for nucleon-nucleus and nucleus-nucleus optical potentials, applicable to light and medium cosmic ray nuclei, have been determined. Pauli correlation effects were included in an approximate, yet analytic, manner and resulted in a reduction of both the strength and range of the overall interaction. The use of these expressions, at energies below 100-200 MeV/nucleon, should be carefully monitored since the increased importance of spin-dependent effects (which are ignored here) with decreasing energy, may render these expressions only crudely approximate at best. Above 400 MeV/nucleon, the results herein may be considered quite accurate for most calculations.

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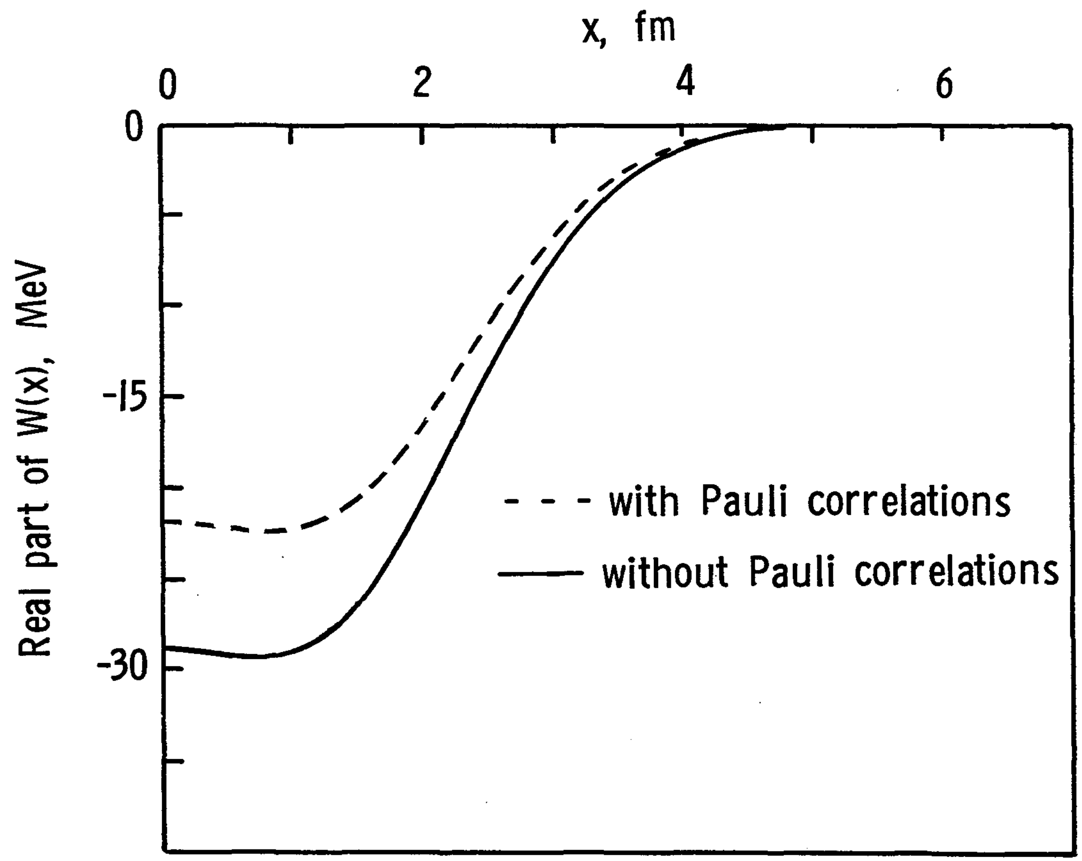

Figure 1. The real part of the proton-carbon optical potential at 200 MeV incident proton kinetic energy.

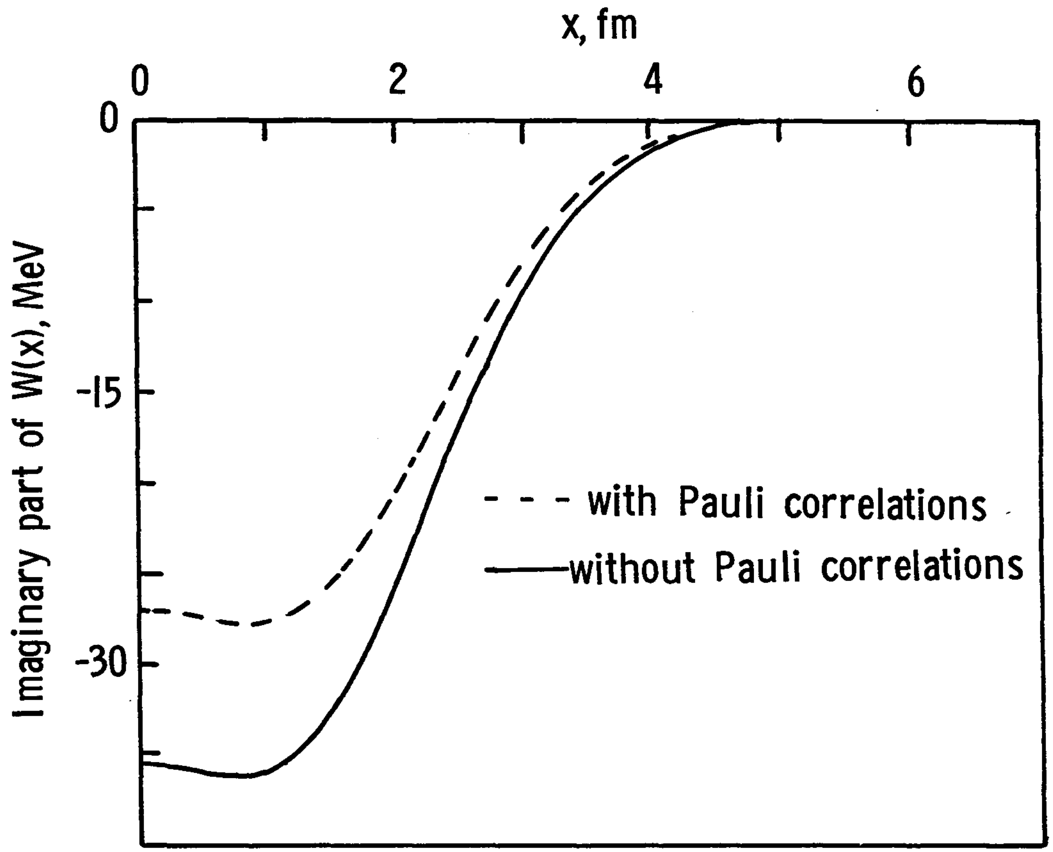

Figure 2. Imaginary part of the proton-carbon optical potential at 200 MeV incident proton kinetic energy.

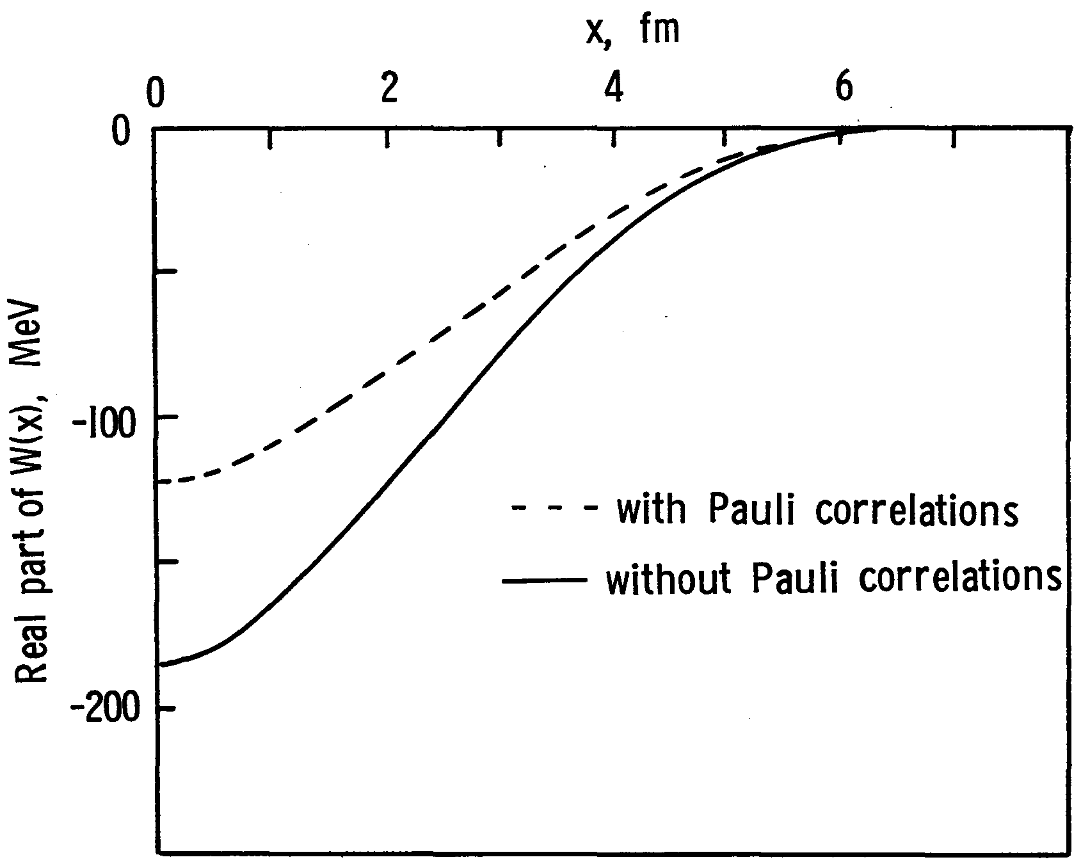

Figure 3. Real part of the carbon-carbon optical potential at 200 MeV/ nucleon incident kinetic energy.

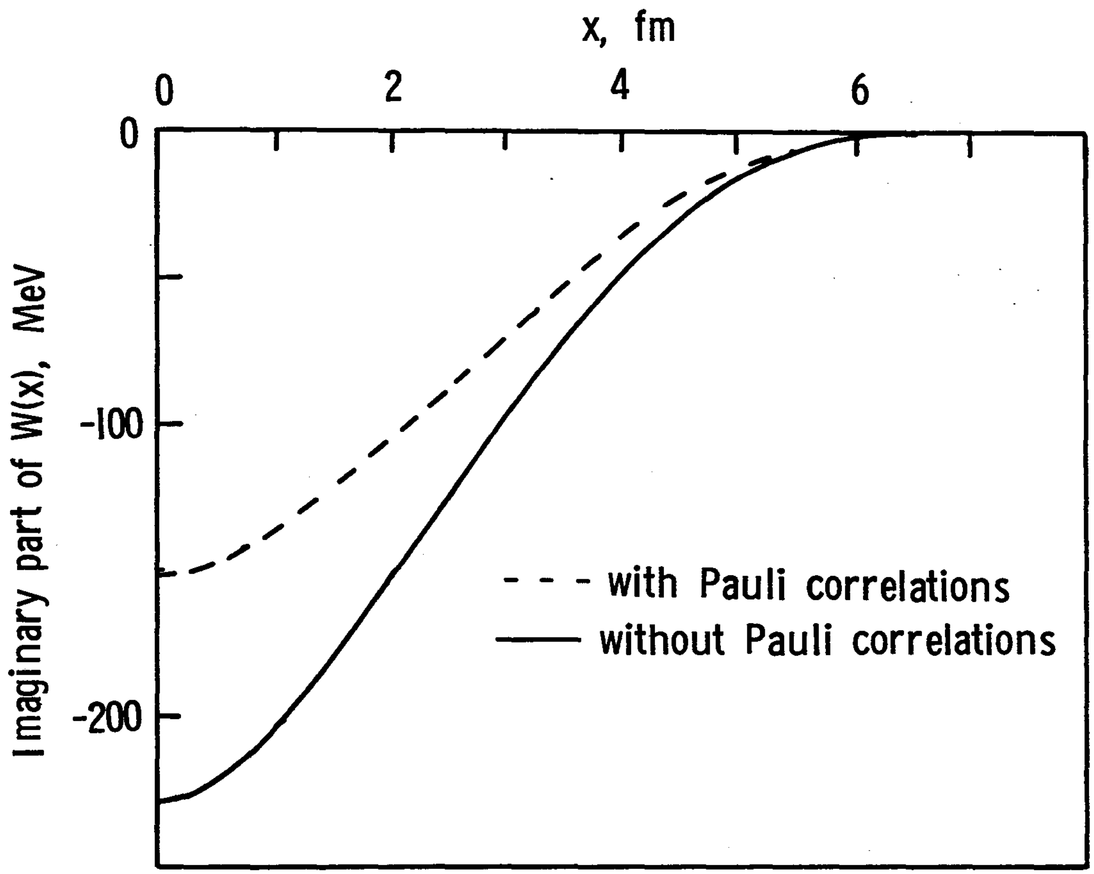

Figure 4. Imaginary part of the carbon-carbon optical potential at 200 MeV/ nucleon incident kinetic energy.

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16. Abstract

Utilizing an optical model potential approximation to the exact nucleus-nucleus multiple-scattering series, optical potentials for nucleon-nucleus and nucleus-nucleus collisions are analytically derived. These expressions are applicable to light and medium cosmic ray nuclei as their single-particle density distributions are analytically determined, without approximation, from their actual harmonic well charge density distributions. Pauli correlation effects are included through the use of a simple gaussian function to replace the usual expression obtained in the infinite nuclear matter approximation.

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