NASA Contractor Report 165837-1

MONTE CARLO STUDIES OF OCEAN WIND VECTOR MEASUREMENTS BY SCATT: OBJECTIVE CRITERIA AND MAXIMUM LIKELIHOOD ESTIMATES FOR REMOVAL OF ALIASES, AND EFFECTS OF CELL SIZE ON ACCURACY OF VECTOR WINDS

Willard J. Pierson and Robert E. Salfi

THE CITY COLLEGE OF THE CITY UNIVERSITY OF NEW YORK
CUNY Institute of Marine and Atmospheric Sciences
New York, NY 10031

Contract NAS1-15669
February 1982

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

ABSTRACT

The Scatterometer on the NOSS* is studied by means of Monte Carlo techniques so as to determine the effect of two additional antennas for alias (or ambiguity) removal by means of an objective criteria technique and a normalized maximum likelihood estimator. Cells nominally 10 km by 10 km, 10 km by 50 km and 50 km by 50 km are simulated for winds of 4, 8, 12 and 24 m/s and incidence angles of 29°, 39°, 47°, and 53.5° for 15° changes in direction.

The normalized maximum likelihood estimate (MLE) is correct a large part of the time, but the objective criterion technique is recommended as a reserve, and more quickly computed, procedure. Both methods for alias removal depend on the differences in the present model function at upwind and downwind.

For 10 km by 10 km cells, it is found that the MLE method introduces a correlation between wind speed errors and aspect angle (wind direction) errors that can be as high as 0.8 or 0.9 and that the wind direction errors are unacceptably large, compared to those obtained for the SASS for similar assumptions. These large errors will obscure any information about typical mesoscale wind fields at 10 km resolution.

Pooling the 10 km cells into 50 km cells (5 by 5) effectively samples the 2500 square kilometer area more uniformly and over a larger total area than using a single large cell. Also in areas of high wind gradients, pooling cells in various patterns can provide the required resolution. Thus, the 10 km resolution is still needed for these applications. Other scanning patterns might be an improvement.

---

*Acronym for National Oceanic Satellite System, which was proposed and then cancelled. The results of this research will be helpful in the design of any future remote sensing radar for measuring vector wind over the ocean.

TABLE OF CONTENTS

Preceding Page Blank

TABLE OF CONTENTS (CONT'D)

INTRODUCTION

The first measurements from space of radar backscatter in an attempt to determine the winds over the surface of the ocean were made from SKYLAB by means of an instrument called S 193, which was a combination scanning pencil beam, radar-radiometer and altimeter. The results of this investigation are summarized by Pierson et al. (1978) and by Polcyn et al. (1978). When the SKYLAB Earth Resources Experiment was designed, the strong effects of anisotropy as a function of wind direction relative to the pointing direction of the radar beam had not been realized. It was therefore necessary to assume that the wind direction was known, so that the value of the backscatter could be related to the speed of the wind.

Prior to the launch of SKYLAB and in the design stages of SEASAT, Jones, Schroeder et al. (1977) developed an aircraft maneuver that made it possible to study the variability of radar backscatter as a function of wind direction. This led to the design of the SASS on SEASAT in which two radar beams were used in a manner such that close-by points on the sea surface could be viewed from two different aspect angles, roughly ninety degrees apart. After the discovery of this particular backscatter feature, Pierson, Cardone and Greenwood (1974) illustrated how it would be possible to recover from one to four vector winds from these pairs of measurements. One of the vector winds obtained in this way is close to the correct wind. The remaining vector winds are called aliases. The theory for the analysis of the data that were obtained by the SASS is based upon these circle flight results and the concept described by Pierson, Cardone and Greenwood (1974).

The most difficult aspect of the problem of relating backscatter to the wind speed and direction has been the problem of finding a correct empirical relationship (the model function) between the measured backscatter values and the vector wind. The difficulty lies in part in the small scale (or mesoscale) variability of the winds in attempting to reduce the scatter that results when the winds and the backscatter values are compared. This basic problem, namely the determination of the model function, has been pursued through SKYLAB and the GOASEX I and II Workshops and finally, the JASIN Workshop.* The substantial improvement in the model function as a result of the JASIN Workshop will be reviewed below.

The effort to measure the winds over the ocean by means of a radar may be continued on some future Ocean Satellite System by means of an instrument called the SCATT. The SCATT, Scatterometer, has a number of important planned design improvements compared to the SASS on SEASAT. The most obvious of these is the addition of one more antenna for each polarization, which will permit six measurements to be made for a given area of the ocean surface. The purpose of this study is to compare some of the results from the SASS with predicted results from the SCATT by means of Monte Carlo simulations, so as to determine whether or not some of the so called aliases, present in the SASS measurements, can be removed from the SCATT measurements.

Also the effect of cell size on the sampling variability of the measurements and the communication noise and attitude errors in the estimation of the vector winds will be studied. It appears that the 10 km resolution cells introduce large sampling variability effects in the determination of the winds.†

---

*Born, G., J. Wilkerson, D. Lame et al. (1979); Barrick, D., J. Wilkerson et al (1980); Businger, J., R. Stewart et al. (1980).

†Pooling 25 of the 10 km cells will yield better data than that from one large cell for those situations where mesoscale turbulence affects the measurements.

THE MODEL FUNCTION

The basic assumption in determining the relationship between wind speed and backscatter is that the backscatter can be expressed as a function of only three variables, namely the magnitude of the wind, the wind direction relative to the pointing direction of the radar beam, and the incidence angle of the radar at the sea surface. This is expressed by Equation 1.

$$\sigma^{o} = \sigma^{o}(V, \chi, \theta) \tag{1}$$

There are other effects that might be involved so that it is possible that more accurate and more plentiful measurements might require a modification of this basic assumption. A possibility is that the state of the sea in terms of the large scale gravity waves, which are not in nearly instantaneous equilibrium with the local wind, may cause slight perturbations of the value of σ^{o}. If this were the case the backscatter would be a function of the large scale gravity wave spectrum, which is highly variable from one place to another over the oceans for a given wind speed and direction.

The results so far have indicated that Equation 1 can be expressed in a very simple form, as given by Equation 2, where backscatter is expressed in decibels and the logarithm of the wind speed is used.

$$\sigma^{o}_{db} = G(\chi, \theta) + H(\chi, \theta) \log_{10} V \tag{2}$$

In this form, when plotted as σ^{o}_{db} versus $\log_{10} V$, equation is that of a straight line whose slope is determined by $H(\chi, \theta)$ and whose intercept at the value corresponding to a wind speed of one meter per second (when $\log_{10} V$ is zero) is given by $G(\chi, \theta)$.

---

† See the extensive literature on two scale theories by various authors.

* In some studies, G. and H are defined slightly differently by factoring out a 10.

Moreover, all of the available data for the present time indicate that this function is even simpler than Equation 2, in that both G and H are even functions of the aspect angle, χ. Thus they can be represented by Equations 3 and 4, where it is noted that $\cos n\chi$ for any value of n can always be expressed as a polynomial in $\cos \chi$ as in various equations to follow.

$$G(\chi, \theta) = G(\cos \chi, \theta) \tag{3}$$

$$H(\chi, \theta) = H(\cos \chi, \theta) \tag{4}$$

There are many ways to express the results of Equation (2). Three different methods have been pursued up to the present time. Two of them will be described in considerable detail in what follows.

One method is to express G and H as functions of aspect angle and incidence angle in steps of 10 degrees for aspect angle, and two degrees for incidence angle with interpolation to get values in between.* Such a model, as does the one that follows, also assumes that the backscatter has maxima for aspect angles of zero degrees and 180 degrees and minima for aspect angles of 90° and 270°. When expressed in this form the result is a G-H table that is stored in the memory of the computer, and used in each stage of the calculation of the winds recovered from the spacecraft measurements. Because the function is an even function it need only be tabulated in ten degree steps from 0° to 180°.

A second method is quasi-analytical. It also uses the inverse relationship for Equation (2) with the objective in mind to compute the wind speed from the backscatter, and not the backscatter from the wind speed. The method is quasi-analytical in the sense that twelve of the functions that are required are given in tabular form in one degree steps for incidence angle instead of in terms of some polynomial fit.

*Wentz (1978)

The first step is to specify backscatter as a function of incidence angle for upwind, downwind and cross wind, and various additional functions of incidence angle so as to control the shape as a function of aspect angle. If the twelve functions that result as tables are combined with the shape functions, the result can then be used to calculate the wind speed for any value of backscatter, aspect angle and incidence angle.

Equation (2) has an inverse, and that inverse can be written as Equation (5).

$$\log_{10} V = A\,(\chi,\theta) + B\,(\chi,\theta)\,\sigma^{o}_{db} \tag{5}$$

In Equation (5) the relationship between A and G and H and B and G and H is given by Equations (6) and (7).

$$A\,(\chi,\theta) = G\,(\chi,\theta)/H\,(\chi,\theta) \tag{6}$$

$$B\,(\chi,\theta) = 1/H\,(\chi,\theta) \tag{7}$$

If these two functions can be found as a function of aspect angle and incidence angle, they are the equivalent of the functions given by Equation (2).

The details that follow are repeated from Pierson and Salfi (1978). They are given here, in order to highlight the differences between the G - H table approach and the quasi-analytical approach.

We define the log to the base 10 of the wind speed to be the variable Y in Equation (8).

$$\log_{10} V = Y \tag{8}$$

Then the upwind, downwind and crosswind values for Y, can be defined as functions of the incidence angle only, by means of Equations (9a), (9b), and (9c).

$$Y_u = A_u(\theta) + B_u(\theta) \quad \sigma^o_{db}(\theta) \tag{9a}$$

$$Y_D = A_D(\theta) + B(\theta) \quad \sigma^o_{db}(\theta) \tag{9b}$$

$$Y_c = A_c(\theta) + B_c(\theta) \quad \sigma^o_{db}(\theta) \tag{9c}$$

If the values for Y are to equal the values at two particular wind speeds, say 23.6 and 4.6 meters per second, the numerical values must be equal to those given by Equations (10a) and (10b).

$$Y_1 = \log_{10} 23.6 = 1.3729 \tag{10a}$$

$$Y_2 = \log_{10} 4.6 = 0.66276 \tag{10b}$$

Since values of the backscatter, for upwind, downwind, and crosswind as expressed for example by Equations (11a) and (11b) for upwind, are also required it is then possible to solve for the values of A_u and B_u for upwind,

$$\sigma^o_{1u} = \sigma^o\ (\theta,\ \chi = 0,\ 23.6) \qquad \text{(in db)} \tag{11a}$$

$$\sigma^o_{2u} = \sigma^o\ (\theta,\ \chi = 0,\ 4.6) \qquad \text{(in db)} \tag{11b}$$

and so on for σ^o_{1D}, σ^o_{2D}, σ^o_{1c} and σ^o_{2c} in db.

$$A_u = (Y_1\ \sigma^o_{2u} - Y_2\ \sigma^o_{1u})/(\sigma^o_{1u} - \sigma^o_{2u}) \text{(in db)} \tag{12a}$$

$$B_u = (Y_1 - Y_2)/(\sigma^o_{1u} - \sigma^o_{2u}) \qquad \text{(in db)} \tag{12b}$$

On the basis of the values for backscatter at upwind for two wind speeds, and the values of these two wind speeds it has been possible to determine the straight line for Y_u, for example, as in Equation (9a). Exactly the same procedure is followed for (9b) and (9c).

Equation (13) has the property that it will yield the upwind, downwind and crosswind values for Y, when the appropriate values for the aspect angle are substituted into it.

$$Y = Y_c - F(\sigma^o, \theta)\ F(\chi) - G^*(\sigma^o, \theta)(G^{**}(\chi, \theta)) \qquad (13)$$

The functions in Equation (13) are defined by Equations (14a), (14b) and (14c) and (15) through (18).

$$F\ (\sigma^o, \theta) = (Y_D - Y_u)/2 \qquad (14a)$$

$$G^*(\sigma\ , \theta) = Y_c - (Y_D + Y_u)/2 \qquad (14b)$$

$$F(\chi) = (\cos\ \chi)^3 = p^3 \qquad (14c)$$

$$G^{**}\ (\chi, \theta) = G_1(\chi) + D(\theta)\ H_1(\chi) + K(\theta)\ M(\chi) + E(\theta)\ P(\chi) \qquad (15)$$

$$+ K^*(\theta)\ Q(\chi)$$

$$G_1(\chi) = (\cos\ \chi)^2 = p^2 \qquad (16a)$$

$$H_1(\chi) = p^2 - p^4 \qquad (16b)$$

$$M\ (\chi) = p(H_1(\chi)) \qquad (16c)$$

$$P\ (\chi) = p^2 - 3\ p^4 + 2\ p^6 \qquad (16d)$$

$$Q\ (\chi) = p\ \ P(\chi) \qquad (16e)$$

For Vertical Polarization

$$D = K = 0 \; ; \; 19^{o} < \theta < 25^{o} \tag{17a}$$

$$D = 1.05 \; (\theta - 25^{o})/40^{o} \; ; \; 25^{o} < \theta < 66^{o} \tag{17b}$$

$$K = 0.50 \; (\theta - 25^{o})/40^{o} \; ; \; 25^{o} < \theta < 66^{o} \tag{17c}$$

$$E = K^{*} = 0 \; ; \; 19^{o} < \theta < 45^{o} \tag{17d}$$

$$E = 0.2857 \; (\theta - 45^{o})/20^{o} \; ; \; 45^{o} < \theta < 66^{o} \tag{17e}$$

$$K^{*} = 0.3143 \; (\theta - 45^{o})/20^{o} \; ; \; 45^{o} < \theta < 66^{o} \tag{17f}$$

For Horizontal Polarization

$$D = K = 0 \; ; \; 19^{o} < \theta < 25^{o} \tag{18a}$$

$$D = 0.83 \; (\theta - 25^{o})/40^{o} \; ; \; 25^{o} < \theta < 66^{o} \tag{18b}$$

$$K = 1.81 \; (\theta - 25^{o})/40^{o} \quad 25^{o} < \theta < 66^{o} \tag{18c}$$

$$E = K = 0 \; ; \; 19^{o} < \theta < 40^{o} \tag{18d}$$

$$E = 0.30 \; (\theta - 40^{o})/25^{o} \; ; \; 40^{o} < \theta < 66^{o} \tag{18e}$$

$$K^{*} = 1.40 \; (\theta - 40^{o})/25^{o} \; ; \; 40^{o} < \theta < 66^{o} \tag{18f}$$

The functions $D(\theta)$, $K(\theta)$, $E(\theta)$ and $K^*(\theta)$ are the shape functions. In particular, D and K make it possible to shift the curves of $\log_{10}V$ versus χ for a given θ, up and down at aspect angles of 45^o plus integer steps of 90^o. These shape functions should be a very important part of the determination of the model function. It is finally necessary to exhibit explicitly the functions A and B. These are given by Equations (19) and (20).

$$A = A_c (1 - G^{**}) - (A_D - A_u) F/2 + (A_D + A_u) G^{**}/2 \tag{19}$$

$$B = B_c (1 - G^{**}) - (B_D - B_u) F/2 + (B_D + B_u) G^{**}/2 \tag{20}$$

A major effort of the GOASEX I, GOASEX II and JASIN* Workshops has been that of determining this model function under the assumptions given above. Part of the difficulty in determining the model function was that the GOASEX I and GOASEX II Workshops did not provide very much data for either low incidence angles between 20 and 30 degrees or high incidence angles above 50 degrees. The JASIN Workshop provided additional data for these two important ranges of incidence angles, and it was possible to improve upon the model function on the basis of this new data.

The JASIN Workshop served two different purposes. One was to demonstrate that without prior knowledge of the actual measurements made during the JASIN Workshop, the wind speeds could be

---

*Born, G., J. Wilkerson, D. Lame et. al. (1979), Barrick, D., J. Wilkersen et. al. (1980), Businger, J., R. Stewart et. al. (1980).

recovered to within ± 2 meters per second for the nearest solution; for winds under 20 meters per second and the wind directions could be recovered within $\pm 20^{\circ}$ for the nearest solution. This was demonstrated by means of the model functions that were developed prior to the JASIN Workshop. Another very important purpose of the JASIN Workshop was to permit a further improvement in the function on the basis of the JASIN data, because it supplied new information for low and high incidence angles.

SAMPLING VARIABILITY EFFECTS FOR THE SASS AND THE SCATT

The design of an instrument to measure radar backscatter is very sophisticated. It is quite possible to measure the received power, which is the important parameter that varies in the radar equation for a given design and a given incidence angle, even when that received power is only 10 percent of the noise in the system. Signal-to-noise ratios of - 10 db are therefore quite acceptable in determining the radar backscattering cross section for a given area of the ocean surface.

To accomplish this goal, for each of the cells (or areas of the ocean) to be probed by the radar, two quantities are actually measured. They are the estimate of the power of the signal plus the noise and the power of the noise as in the following notation.

$$\hat{P}_{sn} \qquad \hat{N} \tag{21}$$

The received power is calculated simply by subtracting the estimate of the noise power from the estimate of the power in the signal plus the noise as in Equation (22).

$$\hat{P}_R = \hat{P}_{sn} - \hat{N} \tag{22}$$

It has been shown in the design of both the SASS and the SCATT that the average value of the estimate of the received power is the correct value as shown in Equation (23), where E is the expectation operator of probability theory,

$$E(\hat{P}_R) = P_R \tag{23}$$

and that the variance of the departure of the estimate from the true value can be put in the form of Equation (24).

$$E(\hat{P}_R - P_R)^2 = \mathrm{VAR}\ (\hat{P}_R) = \lambda' \left((P_R + N)^2 + \kappa N^2\right) \tag{24}$$

The parameters, λ' and κ, vary as a function of the radar cell being scanned. The parameter, λ', is a function of the product of the bandwidth and integration time for the received power and of the error in the measurement of the attitude of the spacecraft as shown by Pierson (1978).* One of the features of future spacecraft will be a very accurate attitude measuring system that will greatly reduce the value of λ', and thus make the variance of the estimate of the received power much smaller than it would have been for the identical SCATT on such a spacecraft, had it been installed on a spacecraft with the attitude controls of SEASAT.

It can also be shown from the design features of both the SCATT and the SASS that the probability density function for the received power measured (or estimated in a probabilistic sense) by the spacecraft is given by Equation (25), in which all quantities are in antilog form.

$$f(\hat{P}_R) = (2\pi)^{-\frac{1}{2}}\ (\mathrm{VAR}\ (\hat{P}_R))^{-\frac{1}{2}} \exp -\tfrac{1}{2} \frac{(\hat{P}_R - P_R)^2}{\mathrm{VAR}\ \hat{P}_R} \tag{25}$$

---

* The value of λ' takes into account that part of the variability of the backscatter measurement comes from attitude measurement errors and enters through other terms in the radar equation. See Appendix D.

By means of the radar equation, once P_R has been estimated, the backscatter can also be estimated as in Equation (26) in which R represents all of the other terms of the radar equation.

$$\hat{\sigma}^o = \hat{R} \ \hat{P}_R^* = R \ \hat{P}_R \tag{26}$$

From Equation (26), the probability density function for the estimate of $\hat{\sigma}^o$ can be derived, and this is given by Equation (27).

$$f(\sigma^o) = (2\pi)^{-\frac{1}{2}} \ (\mathrm{VAR} \ (\hat{\sigma}^o))^{-\frac{1}{2}} \ \exp -\tfrac{1}{2} \ \frac{(\hat{\sigma}^o - \sigma^o)^2}{\mathrm{VAR} \ (\hat{\sigma}^o)} \tag{27}$$

where

$$\mathrm{VAR} \ (\hat{\sigma}^o) = \lambda' \ ((\sigma^o + N \ R)^2 \ + \kappa \ N^2 \ R^2) \tag{28}$$

It follows that the estimate of backscatter for a given cell on the sea surface is a normally distributed random variable in antilog form with an expected value equal to the true value and a variance determined by the expected value.

For Monte Carlo purposes, it is therefore possible to write Equation (29). In this equation, the value of t is picked at random from a normal distribution with a zero mean and a unit standard deviation. The quantity that results from such a calculation will then be equal to the kind of value that would be measured for one of the radar beams at one of the cells scanned by either the SASS or the SCATT. Note that the value of the term, R, used in the radar equation in order to calculate the backscatter is needed in order to compute this quantity.*

$$\sigma^o = \sigma^o + t(\lambda'((\sigma^o + N \ R)^2 + \kappa \ N^2 \ R^2))^{\frac{1}{2}} \tag{29}$$

*See Appendix D

If the estimate of the backscatter is divided by the true backscatter as in Equation (30), the result is that the term following the t is the K_p design number that is calculated in the processing of the SASS data. One needs to know the value of the received power and the value of the noise and hence the reciprocal of the signal-to-noise ratio.

$$\frac{\hat{\sigma}^o}{\sigma^o} = 1 + t \left(\lambda' \left(1 + \frac{N}{P_R}\right)^2 + \kappa \frac{N^2}{P_R^2}\right)^{\frac{1}{2}} \tag{30}$$

$$K_p = \lambda' \left(\left(1 + \frac{N}{P_R}\right)^2 + \kappa \frac{N^2}{P_R^2}\right)^{\frac{1}{2}} \tag{31}$$

It can be noted in passing that there is no transformation of the form, $\sigma^o_{db} = 10 \log_{10} \sigma^o$, or of any other form, that will transform Equation (27) into the log normal distribution. Nor is it correct to use σ^o in either bels or decibels in (27) with some modification of the definition of the variance.

Another incorrect probability model for the sampling variability of the backscatter estimates is to assume that backscatter in decibels is normally distributed with an expected value given by the true value in decibels and a standard deviation expressed in decibels computed from K_p and $\hat{\sigma}^o$. This pdf is not the log normal distribution.*

---

*See Appendix B, these incorrect distributions do well for large backscatter values and poorly for small backscatter values as shown in that appendix.

SOURCES OF SCATTER IN COMPARING SASS WINDS WITH CONVENTIONAL WINDS

The workshops that have been carried out in order to determine the ability of the SASS on SEASAT to measure the winds over the ocean have successively improved upon the model function up to the point where the model function obtained from the JASIN data seems to be quite good.* It is nevertheless important to try to explain why the winds measured by the SASS do not agree exactly with the winds measured by an anemometer on a platform somewhere near the point where the winds were determined by the SASS. This particular problem has been ever-present in attempting to interpret the SASS results.

The usual procedure has been to pair up, or group, the measurements made by the SASS in such a way that a number of measurements from each beam (close to a measurement made by some meteorological method) are obtained and a wind is calculated from the backscatter data to be compared with the wind measured with an anemometer. When this is done, the speed of the wind from the anemometer measurement can be plotted on one axis, versus the speed of the wind from the backscatter measurement on another. For a second plot, the direction of the wind (the closest of the aliases involved) from the SASS measurement can be plotted against the direction of the wind from the meteorological measurement. The points that are obtained in this way do not in general fall on the 45^{o} line, which would indicate perfect agreement. They scatter about this line. For the earlier GOASEX I results, the scatter has a bias plus a slope, relative to the 45^{o} line, which was quite large. With progressive iterations of the same techniques, all of which involve changing the model function, the scatter has been reduced and the departure from the 45^{o} line for the cluster of points involved, has been made essentially zero.

---

* Schroeder et. al. (1981), submitted to JGR.

Not all of the scatter is actually an error, although some of it is. Results of the GOASEX II workshop attempted to identify those sources of scatter that could not strictly speaking be called errors.

The various sources of scatter are: (1) communication noise, (2) attitude errors, (3) effects of mesoscale turbulence, (4) non-coincidence of cells, (5) lack of exact space time coincidence, (6) an incorrect model function and (7) inaccurate wind observations.

The effects of communication noise and attitude errors, are treated in the preceeding equations, and they enter into the determination of the noise and of and of the parameters, λ' and κ, in for example, Equation (24). The received power that is estimated for each of the cells, is a random variable that does not represent the true value of the received power that would have been observed, in principle, had the averaging time been infinitely long, had a constant wind with a constant direction covered the entire cell, and had the theory been perfect. This is a sampling variability effect that is not removeable. The quantities that are "measured" by the SASS on SEASAT, are random variables. The quantities that will be measured by the SCATT in the future will also be random variables. Any theory that attempts to use the backscatter measurements to determine the wind speed at the cells scanned by these instruments must take this effect into account.

An additional complication arises because the wind over the area sampled by the instrument is variable in space at the moment when the sample is taken. When averaged over the area of the cell that is scanned, the average vector wind for that cell need not necessarily coincide with the average wind from an anemometer, as averaged over a time interval varying from 8 to 20 or 30 minutes, depending upon the platform. This effect of mesoscale turbulence is probably the least understood of all of the sources of scatter in this problem, and work in this area is continuing.* Some new results can be found in a recent issue of Radio Science in papers by Kropfli and Hilderbrand (1980), James (1980) and Doviak and Berger (1980). Reference is made in particular to Fig. 6 on page 303 of this issue. Although the average wind subtracted from this vector wind field is light, the eddies that are shown over this area, which is comparable in size to a cell to be scanned by the NOSS, illustrate the complexity of the problem of interpreting the wind measured by the SASS to be compared to the wind measured with an anemometer.

If it is conceded that at least in principle the average of the winds over a cell scanned by the SASS need not necessarily be equal to an anemometer average at a nearby point, then the fact that the two different cells scanned by the SASS that are paired, do not lie one on top of the other, takes on additional significance. The average of the winds over one of the cells need not necessarily be equal to the average of the winds over the second cell. The difference between the two winds could be much larger than the difference that would be computed from the gradient of the synoptic scale wind over the distances involved. If these two cells have sampled different areas of the ocean, as is typically the case, an additional source of scatter is introduced when the wind is computed from the SASS measurements and compared with a conventional wind measurement.

For the GOASEX I and II workshops, data from the National Data Buoy network were used. The winds reported by the data buoys, as averaged over slightly more than 8 minutes were reported once each hour. Since the spacecraft did not pass the data buoy exactly

---

*See Pierson (1982).

on the hour, the averages of these two winds were interpolated to the time of the spacecraft passage. Unfortunately, the fluctuations due to mesoscale turbulence in the winds probably overcame the synoptic scale trends that should have been present in hourly measurements to the extent that such an interpolation would still have a large contribution from mesoscale turbulence at the two different times when the anemometer made the measurement. The mesoscale properties of the atmosphere at the two times when the anemometer averages were taken need not have had anything at all to do with the mesoscale properties of the winds over the sea surface at the time of the spacecraft passage. Therefore, there would be an additional source of scatter in the data when comparing the anemometer averaged winds with the SASS winds. The essential feature of these three sources of scatter, namely items 3, 4, and 5, is that the averages for the anemometers may have been perfectly accurate within say, a half a meter per second and five degrees in direction for the location and time that they were made and that the SASS backscatter measurements may have been within the confines of sampling variability due to communication noise and attitude error, which would result in a very small theoretical difference between the measured winds and the estimated winds. Yet the pairs of values for the vector wind, one from the conventional data and one from the SASS data, could differ by a considerable amount in both speed and direction.

Finally, none of these conclusions would be valid if the model function that has been derived differs substantially from the correct model function that ought to be used in such a calculation. The model function that resulted from the JASIN workshop is the best yet obtained, but further improvements in the model function may still be needed.*

* See Schroeder et. al. (1982).

JASIN WORKSHOP RESULTS

The results of the JASIN workshop were a substantial improvement over both the GOASEX I and GOASEX II workshops. The major accomplishments were that the coincidence of the measurements was maintained and the effects of mesoscale turbulence were averaged out more thoroughly for the JASIN wind fields. The many different anemometers on the different platforms were all cross calibrated, one against the other and against a primary buoy. The remaining sources of scatter because of communication noise, attitude errors and an incorrect model function were still present as was the noncoincidence of the SEASAT pairs of cells. The scatter was reduced substantially by improving the model function for both low and high incidence angles.

For the analytical method described by Equations (8) through (20), the required table to describe backscatter, at upwind downwind and crosswind for two wind speeds as a function of incidence angle, is given in Table 1. In G - H table form, this would be SASS 1. There are considerable differences between vertical and horizontal polarization as a function of wind speed aspect angle and incidence angle. Downwind vertical polarization is greater than upwind for high winds and down wind is less than equal to upwind for low winds. For horizontal polarization downwind is always less than up wind for the range of speeds analysed.

The error structure for the wind speeds as determined by the JASIN data by means of the model function defined by Table 1 (SASS 1) is summarized in Table 2 as a function of wind speed ranges. Both vertical polarization, horizontal polarization and the results when all of the data for a given wind speed range are pooled are shown in this table. The "bottom line" is that for all combined data the standard deviation of the difference between the winds re-

TABLE 1. Upwind, downwind, and crosswind backscatter values (db) for two wind speeds and both polarizations.

SEA MODEL FUNCTION

	V-POL						H-POL					
	UPWIND		DOWNWIND		CROSSWIND		UPWIND		DOWNWIND		CROSSWIND	
THETA	S1UV	S2UV	S1DV	S2DV	S1CV	S2CV	S1UH	S2UH	S1DH	S2DH	S1CH	S2CH
19	3.5421	-2.7463	4.0155	-2.8126	.9304	-5.3261	3.4921	-2.5868	3.0423	-3.6083	.8176	-5.3003
20	3.0332	-3.9018	3.5354	-3.9780	.2704	-6.8961	2.9650	-3.8240	2.4395	-4.9674	.1406	-6.8686
21	2.4913	-5.1290	3.0167	-5.1181	-.3811	-8.4058	2.3845	-5.0224	1.7726	-6.2877	-.5780	-8.3897
22	1.9495	-6.2984	2.4980	-6.2582	-1.0386	-9.9155	1.8039	-6.2208	1.1056	-7.6080	-1.2966	-9.9107
23	1.3843	-7.3648	1.9498	-7.3444	-1.7120	-11.3061	1.1488	-7.3518	.3954	-8.8224	-2.0422	-11.3026
24	.8246	-8.4390	1.3803	-8.4306	-2.3787	-12.6914	.4968	-8.4801	-.3194	-10.0403	-2.7853	-12.6914
25	.2277	-9.4000	.7977	-9.4000	-3.0854	-13.8806	-.2093	-9.5371	-1.1093	-11.1271	-3.6060	-13.8606
26	-.3625	-10.2683	.2075	-10.4295	-3.7275	-15.0807	-.9879	-10.5246	-1.8634	-12.1823	-4.3596	-15.0970
27	-.9361	-11.1494	-.3681	-11.4274	-4.3479	-16.2336	-1.7423	-11.5058	-2.5995	-13.2279	-5.0986	-16.2907
28	-1.4993	-11.9570	-.9293	-12.3337	-4.9469	-17.3400	-2.4733	-12.4797	-3.3181	-14.2628	-5.9230	-17.4612
29	-2.0461	-12.7388	-1.4761	-13.3284	-5.5246	-18.4007	-3.1814	-13.4454	-4.0195	-15.2863	-6.5324	-18.6093
30	-2.5765	-13.4865	-2.0085	-14.2314	-6.0814	-19.4166	-3.8675	-14.4017	-4.7042	-16.2974	-7.2269	-19.7316
31	-3.0965	-14.2009	-2.5265	-15.1027	-6.6175	-20.3804	-4.5324	-15.3477	-5.3726	-17.2951	-7.9060	-20.8396
32	-3.6002	-14.8827	-3.0302	-15.9423	-7.1533	-21.3170	-5.1767	-16.2824	-6.0251	-18.2785	-8.5698	-21.9049
33	-4.0856	-15.5385	-3.5196	-16.7502	-7.6289	-22.2031	-5.8012	-17.2047	-6.6622	-19.2467	-9.2180	-22.9542
34	-4.5547	-16.1511	-3.9947	-17.5263	-8.1046	-23.0476	-6.4066	-18.1137	-7.2842	-20.1989	-9.8505	-23.9792
35	-5.0237	-16.7393	-4.4557	-18.2706	-8.5606	-23.8513	-6.9937	-19.0083	-7.8917	-21.1339	-10.4671	-24.9763
36	-5.4725	-17.2976	-4.9028	-18.9832	-8.9974	-24.6149	-7.5633	-19.8876	-8.4850	-22.0510	-11.0676	-25.9483
37	-5.9051	-17.8470	-5.3351	-19.6639	-9.4150	-25.3393	-8.1160	-20.7505	-9.0646	-22.9492	-11.6518	-26.8937
38	-6.3230	-18.3280	-5.7536	-20.3127	-9.8139	-26.0252	-8.6526	-21.5960	-9.6308	-23.8275	-12.2196	-27.8131
39	-6.7281	-18.8014	-6.1581	-20.9297	-10.1941	-26.6735	-9.1738	-22.4231	-10.1841	-24.6851	-12.7708	-28.7033
40	-7.1185	-19.2479	-6.5485	-21.5148	-10.5561	-27.2651	-9.6804	-23.2308	-10.7249	-25.5210	-13.3052	-29.5667
41	-7.4949	-19.6692	-6.9249	-22.0679	-10.9000	-27.8606	-10.1731	-24.0181	-11.2537	-26.3344	-13.8227	-30.4020
42	-7.8573	-20.0631	-7.2873	-22.5891	-11.2262	-28.4009	-10.6527	-24.7839	-11.7709	-27.1242	-14.3230	-31.2098
43	-8.2058	-20.4333	-7.6358	-23.0784	-11.5349	-28.9068	-11.1199	-25.5273	-12.2768	-27.8895	-14.8061	-31.9868
44	-8.5405	-20.7764	-7.9705	-23.5356	-11.8263	-29.3791	-11.5754	-26.2472	-12.7720	-28.6295	-15.2717	-32.7355
45	-8.8612	-21.1022	-8.2912	-23.9609	-12.1008	-29.8186	-12.0200	-26.9427	-13.2568	-29.3432	-15.7197	-33.4545
46	-9.1682	-21.4092	-8.5982	-24.3540	-12.3585	-30.2261	-12.4544	-27.6128	-13.7316	-30.0296	-16.1499	-34.1435
47	-9.4613	-21.7023	-8.8913	-24.7152	-12.5998	-30.6024	-12.8794	-28.2563	-14.1970	-30.6879	-16.5621	-34.8022
48	-9.7407	-21.9817	-9.1707	-25.0462	-12.8250	-30.9464	-13.2957	-28.8723	-14.6532	-31.3172	-16.9562	-35.4300
49	-10.0064	-22.2474	-9.4364	-25.3411	-13.0342	-31.2648	-13.7040	-29.4599	-15.1008	-31.9164	-17.3319	-36.0266
50	-10.2584	-22.4994	-9.6884	-25.6059	-13.2277	-31.5524	-14.1051	-30.0179	-15.5401	-32.4847	-17.6892	-36.5917
51	-10.4967	-22.7377	-9.9267	-25.8442	-13.4059	-31.8120	-14.4997	-30.5454	-15.9717	-33.0212	-18.0278	-37.1248
52	-10.7215	-22.9625	-10.1515	-26.0690	-13.5689	-32.0445	-14.8885	-31.0414	-16.3958	-33.5249	-18.3475	-37.6256
53	-10.9326	-23.1736	-10.3626	-26.2801	-13.7170	-32.2506	-15.2724	-31.5048	-16.8129	-33.9949	-18.6482	-38.0936
54	-11.1303	-23.3713	-10.5603	-26.4778	-13.8506	-32.4311	-15.6520	-31.9347	-17.2236	-34.4302	-18.9298	-38.5296
55	-11.3144	-23.5554	-10.7444	-26.6619	-13.9698	-32.5869	-16.0280	-32.3300	-17.6280	-34.8300	-19.1920	-38.9301
56	-11.4286	-23.6696	-10.8586	-26.7761	-14.0606	-32.6777	-16.2923	-32.6001	-17.8920	-35.1456	-19.4410	-39.1889
57	-11.5219	-23.7629	-10.9519	-26.8694	-14.1414	-32.7595	-16.5217	-32.8295	-18.1356	-35.4332	-19.6748	-39.4421
58	-11.5987	-23.8397	-11.0287	-26.9462	-14.2132	-32.8303	-16.7204	-33.0282	-18.3603	-35.6943	-19.8936	-39.6894
59	-11.6636	-23.9046	-11.0936	-27.0111	-14.2772	-32.8943	-16.8929	-33.2007	-18.5672	-35.9308	-20.0971	-39.9265
60	-11.7209	-23.9619	-11.1509	-27.0684	-14.3344	-32.9515	-17.0433	-33.3511	-18.7578	-36.1441	-20.2854	-40.1552
61	-11.7751	-24.0161	-11.2051	-27.1226	-14.3859	-33.0030	-17.1762	-33.4840	-18.9331	-36.3359	-20.4584	-40.3733
62	-11.8293	-24.0703	-11.2593	-27.1768	-14.4327	-33.0498	-17.2958	-33.6036	-19.0947	-36.5079	-20.6160	-40.5794
63	-11.8835	-24.1245	-11.3135	-27.2310	-14.4759	-33.0930	-17.4065	-33.7143	-19.2436	-36.6568	-20.7592	-40.7724
64	-11.9377	-24.1787	-11.3677	-27.2852	-14.5165	-33.1336	-17.5125	-33.8203	-19.3813	-36.7945	-20.8850	-40.9510
65	-11.9919	-24.2329	-11.4219	-27.3394	-14.5557	-33.1728	-17.6182	-33.9260	-19.5090	-36.9222	-20.9963	-41.1140
66	-12.0461	-24.2871	-11.4761	-27.3936	-14.5945	-33.2116	-17.7240	-34.0318	-19.6280	-37.0412	-21.0920	-41.2601

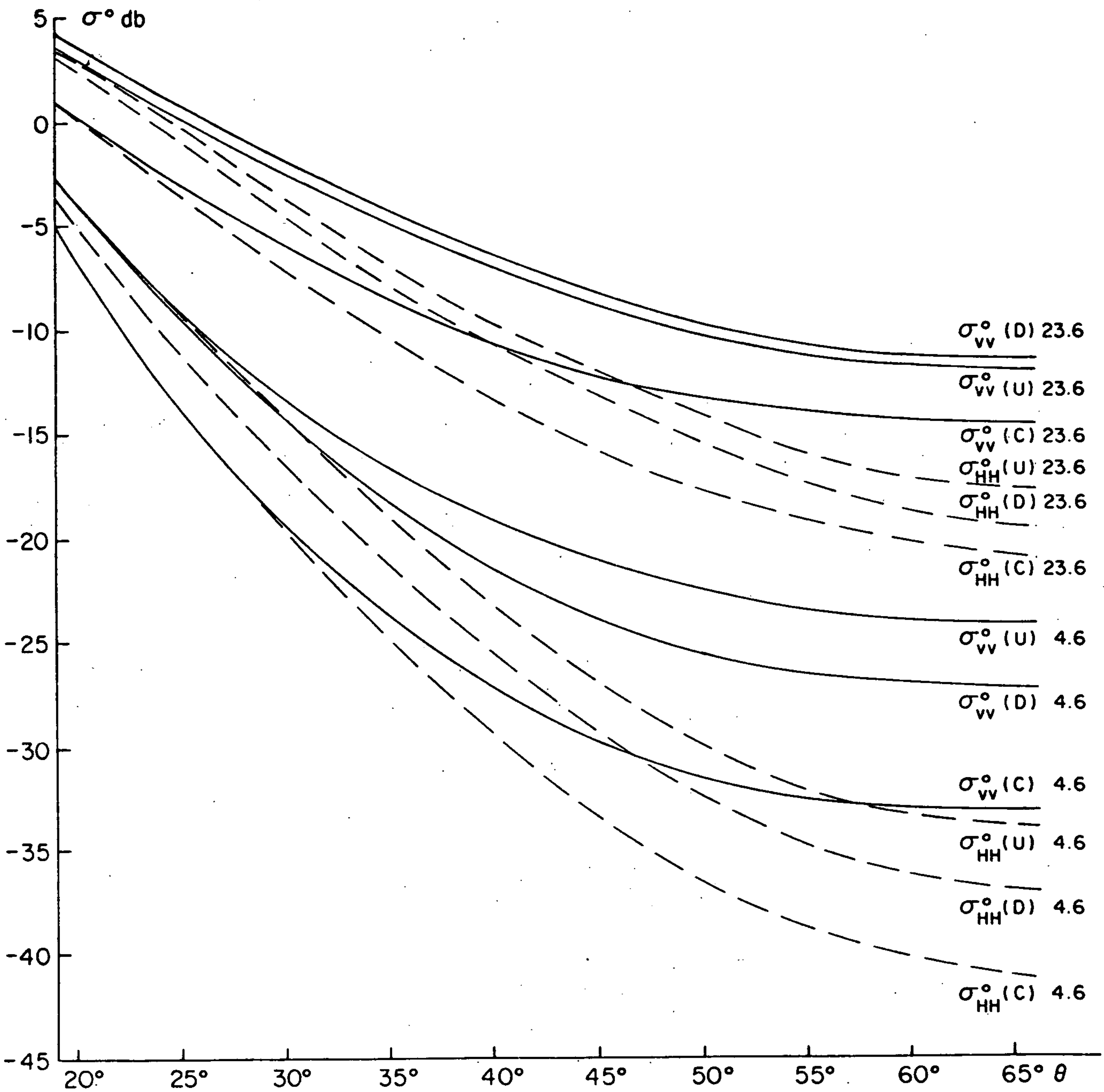


FIGURE 1 BACKSCATTER AS A FUNCTION OF INCIDENCE ANGLE FOR UPWIND, DOWNWIND, AND CROSSWIND FOR WIND SPEEDS OF 23.6 AND 4.6 ms^{-1}, AND FOR VERTICAL AND HORIZONTAL POLARIZATIONS.

TABLE 2. Statistical Results for the SASS-1 Model Function and the JASIN Comparison Data Set. The Bias is Statistically No Different from Zero for the Underlined Values. (From Schroeder et. al. (1982)).

RANGE M/S	SAMPLE SIZE (N) V POL	SAMPLE SIZE (N) H POL	SAMPLE SIZE (N) ALL	BIAS V POL	BIAS H POL	BIAS ALL	STANDARD DEV. V POL	STANDARD DEV. H POL	STANDARD DEV. ALL	(STANDARD DEV.)/$N^{1/2}$ V POL	(STANDARD DEV.)/$N^{1/2}$ H POL	(STANDARD DEV.)/$N^{1/2}$ ALL
4 - 6	49	30	66	- <u>0.06</u>	+ 0.28	+ <u>0.04</u>	1.23	1.30	1.26	0.17	0.24	0.14
6 - 8	86	93	208	- <u>0.01</u>	+ <u>0.02</u>	- <u>0.03</u>	1.23	1.24	1.17	0.13	0.13	0.08
8 - 10	75	70	153	+ <u>0.06</u>	- 0.17	- 0.12	1.38	1.24	1.19	0.16	0.15	0.10
10 - 12	49	53	115	+ <u>0.01</u>	+ <u>0.07</u>	- <u>0.08</u>	0.83	1.17	.92	0.11	0.16	0.11
12 - 14	48	40	103	- <u>0.06</u>	- <u>0.05</u>	- <u>0.01</u>	1.25	1.77	1.44	0.18	0.28	0.14
14 - 16	29	31	59	- <u>0.05</u>	+ 0.82	+ 0.23	1.48	1.80	1.91	0.27	0.32	0.23
ALL	336	317	704	- <u>0.01</u>	+ <u>0.08</u>	- <u>0.03</u>	1.23	1.39	1.27	0.07	0.08	0.05

ported from meteorological measurements in the JASIN array and the wind speeds calculated from this model function is 1.27 ms^{-1} and the bias is $- 0.03$ ms^{-1} (or minus three centimeters per second).

The 704 points that enter in this tabulation are not independent. However, if the errors were normally distributed errors for the wind speed, and if they had been independent the standard deviations could be divided by the square root of the sample size in order to obtain an estimate of the standard deviation of the mean. This has been done in the last three columns of Table 2. Such a calculation provides an estimate of the standard deviation of the mean that is too low, because the sample size is too high. However, even with this conservative interpretation, the biases that are tabulated lie mostly within less than one standard deviation of the mean. Those that do have been underlined in the appropriate columns. Of the 18 values for the bias that are tabulated, only five lie outside of ± one standard deviation as computed in this way, and of those five, four are within two standard deviations of the mean, and only one is more than two standard deviations from the mean. This is about the kind of result that could be expected under these circumstances, and the major conclusion is that the biases are statistically not different from zero. This is also true with reference to the average bias for each polarization and with reference to the combined bias for all of the data sets.

It must be reemphasized that the standard deviation should not be interpreted as the error of the SASS measurements. It is composed of at least three different effects. One is the errors in the anemometer averages of the wind. A second is that of the communication noise and the attitude error effects of the measurements of the backscatter. The third, and probably the most important reason for the difference between the SASS winds and the meteorological winds arises from the combined effects of mesoscale turbulence and the lack of coincidence of the paired SASS cells.

With reference to wind direction, for vertical polarization the root mean square scatter between the SASS wind directions and the meteorological wind directions was 16.2 degrees for vertical polarization and 17.8 degrees for horizontal polarization. This gross statistic does not reflect one of the more important features of the measurements. This feature is that for winds above 10 meters per second, and for the middle range of incidence angles from say 35 degrees to 55 degrees, the scatter averaged much less than this and almost equalled ± 10 degrees in direction*. This would be expected on the basis of the known properties of the variation of signal to noise and the nature of the antenna pattern.

Another way to summarize the statistical results shown in Table 2 is by means of Figures 2 and 3. These figures simply represent a graphical summary of the vertical polarization and horizontal polarization results for wind speed. A scatter diagram of the 336 values for vertical polarization, if plotted on a rectangular system and scale of this figure, would be so full of points that it would be virtually impossible to see them all. The procedure has simply been to plot the mid point of each range, as a point representing that mid point plus the bias, and then bars to represent this value plus or minus the standard deviation. The scatter of points for each of these polarizations would then be such that about two-thirds of them would lie between the dashed lines and would cluster around the 45° lines quite nicely. The fit of the SASS winds to the JASIN winds is therefore quite satisfactory.

---

*See Schroeder et. al. (1982).

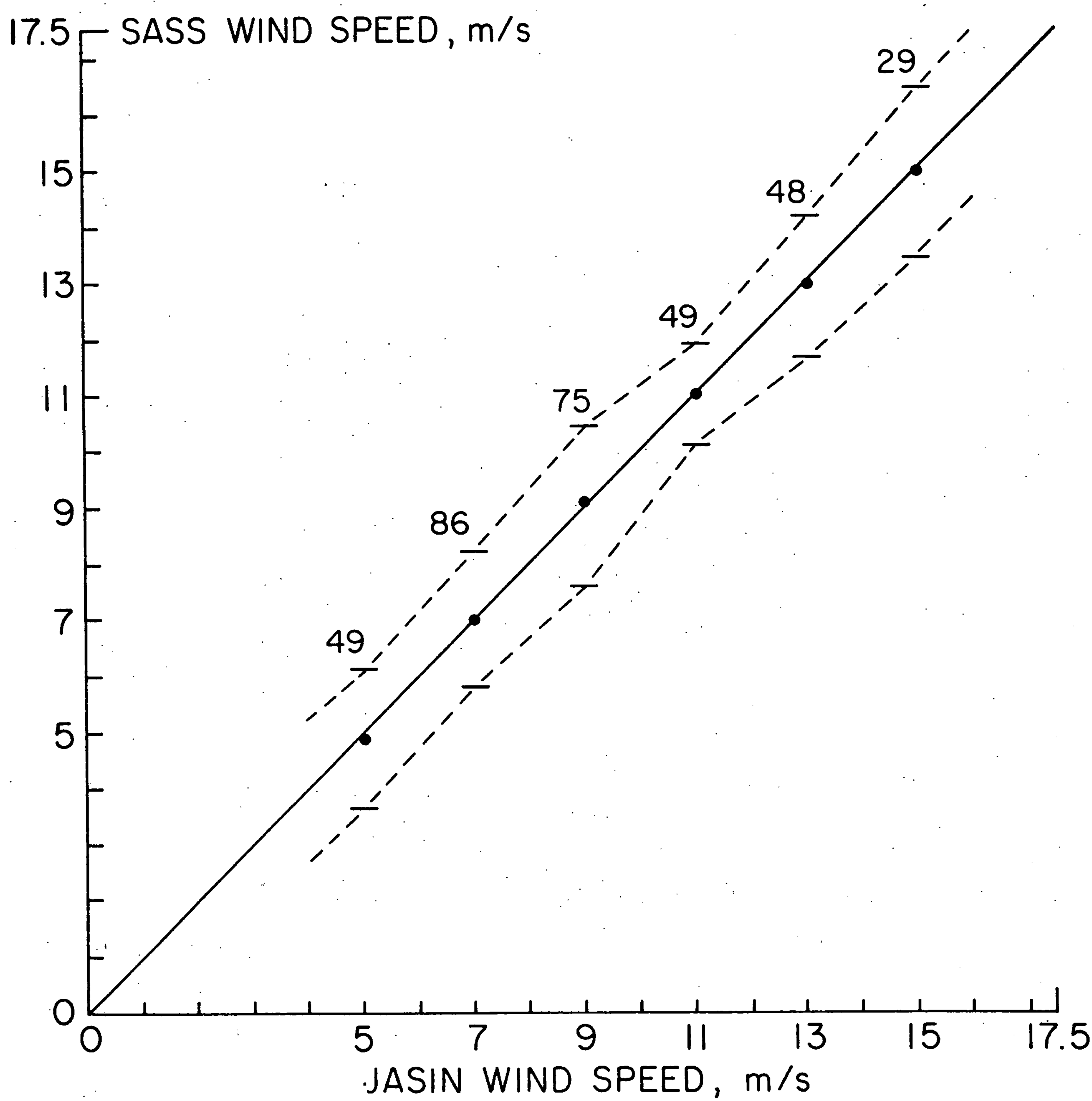


FIG. 2 JASIN WIND SPEED PLUS BIAS, PLUS AND MINUS STANDARD DEVIATION FOR THE SASS I MODEL FUNCTION FOR VERTICAL POLARIZATION. NUMBERS SHOW SAMPLE SIZE. SEE TABLE 2.

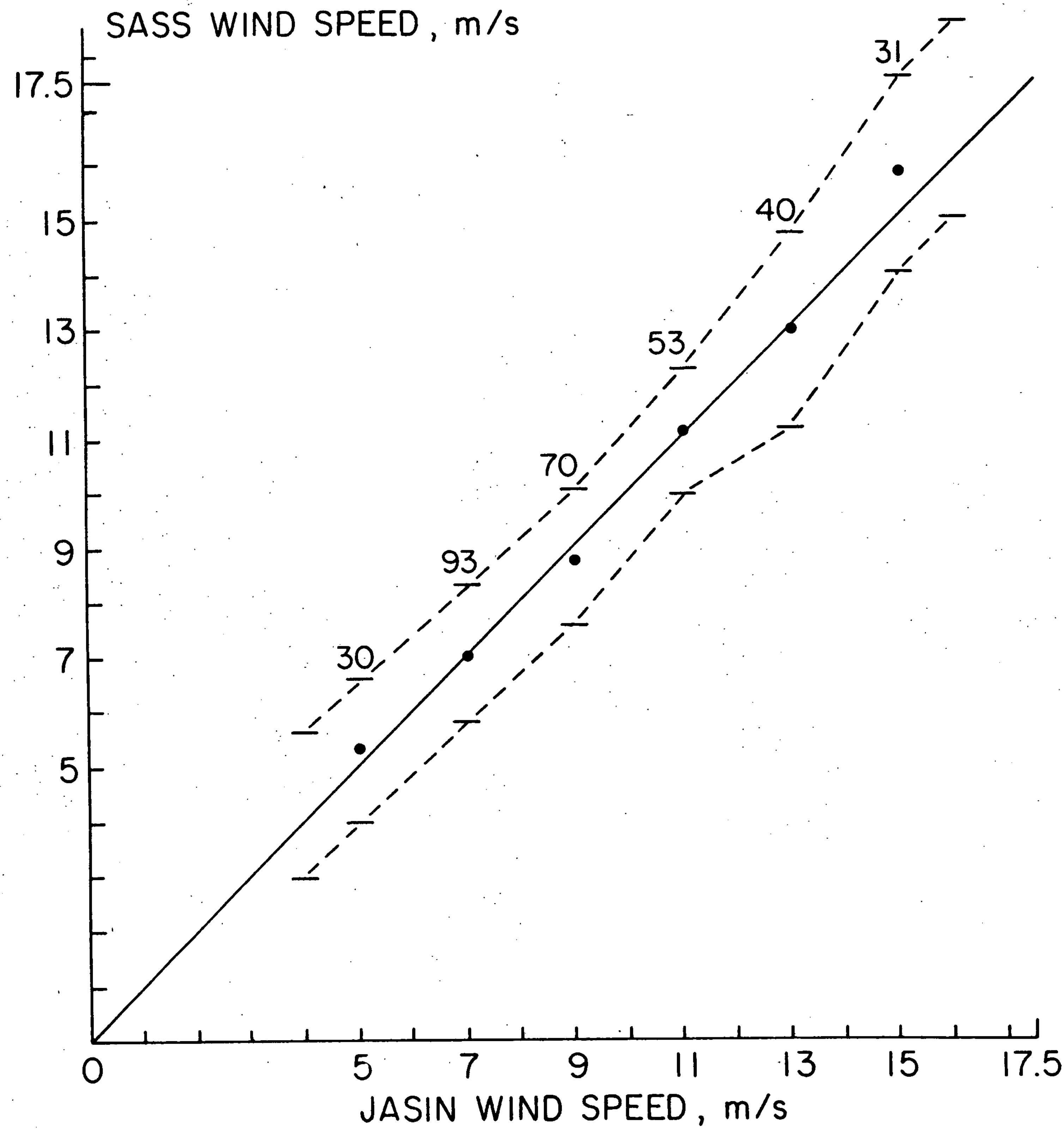


FIG. 3 JASIN WIND SPEED PLUS BIAS PLUS AND MINUS STANDARD DEVIATION FOR THE SASS I MODEL FUNCTION FOR HORIZONTAL POLARIZATION. NUMBERS SAMPLE SIZE. SEE TABLE 2.

THE THEORY FOR THE SASS VERTICAL POLARIZATION PAIRS

One of the modes for the SASS is to scan on both sides of the spacecraft and make measurements with vertical polarization only. This theory will be discussed in this section. An exactly similar theory would apply to measurements in the horizontal mode. There will be two measurements at close-by points, one from each antenna, such that the Azimuth angles of the radar beam will differ by approximately 90° for this pair of measurements.

The probability density function for this pair of measurements is given by Equation (32) where VAR $\hat{\sigma}^{o}$ is a function of σ^{o}.

$$f(\hat{\sigma}_1,\hat{\sigma}_2) = (2\pi)^{-1}(\text{VAR } \hat{\sigma}_1^{o}, \text{ VAR } \hat{\sigma}_2^{o})^{-\frac{1}{2}} \exp -\tfrac{1}{2} \left(\frac{(\hat{\sigma}_1^{o} - \sigma_1^{o})^2}{\text{VAR } \hat{\sigma}_1^{o}} + \frac{(\hat{\sigma}_2^{o} - \sigma_2^{o})^2}{\text{VAR } \hat{\sigma}_2} \right) \tag{32}$$

In Equation (32), the quantities available from this pair of SASS measurements, are the single observations of the random variables, $\hat{\sigma}_1^{o}$ and $\hat{\sigma}_2^{o}$. These two numbers are samples of size 1, each drawn at random from a normal population with an unknown mean, or expected value, and a variance that is determined from radar theory as a function of this unknown mean value.

Under the simplifying assumptions that mesoscale effects can be neglected and that the wind was exactly the same as the two cells that were scanned by SEASAT, and under the further assumption that a constant wind speed and direction will generate a unique value for the population parameters, σ_1^{o} and σ_2^{o}, then the question arises as to how far from these true population parameters the measurements (or the estimates) can scatter, due to the effects of sampling variability.

The problem that arises is that of determining the value of the population parameters $(\sigma_1^o$ and $\sigma_2^o)$ to be used in calculating the wind that generated the backscatter.

For many applications in statistical theory, a good way to proceed is to determine the maximum liklihood estimates of the unknown parameters. This is accomplished by finding the two values of the unknown population parameters that maximize Equation (32). In a maximum likelihood estimate, the random variables are known and are kept equal to those of the sample and the population parameters are varied in order to maximize the probability density function.

The likelihood function is the equivalent of the probability density function insofar as the process of determining a maximum likelihood estimate is concerned. The log of the likelihood function is defined by Equation (33).(See Mood et. al. (1963) for the theory.)

$$\ln L\,(\sigma_1^o,\sigma_2^o) = \ln f(\hat{\sigma}_1^o,\hat{\sigma}_2^o\ ;\ \sigma_1^o,\sigma_2^o) = -\ln 2\pi - \tfrac{1}{2}\ln \mathrm{VAR}\,\hat{\sigma}_1^o - \tfrac{1}{2}\ln \mathrm{VAR}\,\hat{\sigma}_2^o$$

$$-\tfrac{1}{2}\left(\frac{(\hat{\sigma}_1^o - \sigma_1^o)}{\mathrm{VAR}\,\hat{\sigma}_1^o} + \frac{(\hat{\sigma}_2^o - \sigma_2^o)^2}{\mathrm{VAR}\,\sigma_2^o}\right) \qquad (33)$$

To maximize the likelihood function in this extremely simple case (since it is really the sum of two different functions, and since there is no correlation between the two backscatter measurements), one obtains derivatives with respect to the unknown population parameters and sets the derivatives equal to zero. Values of the population parameters that make the derivatives equal to zero will clearly maximize both $\ln L$ and $f\,(\sigma_1^o,\sigma_2^o)$.

Since $\hat{\sigma}_1^o$ and $\hat{\sigma}_2^o$ are independent the maximum of (33) can be found quite simply.

$$\frac{\partial\, \ln L}{\partial \sigma_1^o} = 0 = - \quad (\sigma_1^o + N\, R)\lambda' + (\hat{\sigma}_1^o - \sigma_1^o)$$

$$+ \quad \frac{(\hat{\sigma}_1^o - \sigma_1^o)^2 \; (\sigma_1^o + N\, R)}{(\sigma_1^o + N\, R)^2 + \kappa\, N^2\, R^2} \tag{34}$$

This equation ought to be solved for σ_1^o as a function of $\hat{\sigma}_1^o$ and the various parameters. It seems hardly worth the trouble.

In Equation (34), if σ_1^o is set equal to $\hat{\sigma}_1^o$, then only the first term remains. It is negative. If on the other hand the last term of Equation (34) is omitted, one can solve for σ_1^o in terms of $\hat{\sigma}_1^o$ so as to obtain Equation (35).

$$\sigma_1^o \cong \frac{\hat{\sigma}_1^o - N\, R\, \lambda'}{1 + \lambda'} \tag{35}$$

For this situation, the value of Equation (34) on the right hand side is positive. Therefore, the maximum likelihood estimate is some value between $\hat{\sigma}_1^o$ and the result of Equation (35). The maximum likelihood estimate of the unknown population parameter, σ_1^o , is therefore bounded by these two approximations and is given by Equation (36).

$$\hat{\sigma}_1^o \left(\frac{1 - \lambda'(N/s)}{1 + \lambda'}\right) < \sigma_1^o < \hat{\sigma}_1 \tag{36}$$

For a typical cell of the SASS on SEASAT, one can quickly determine that λ' would be given by Equation (37), as computed from $\beta = 0.08$ and $\beta_c T_{sN} = 1556.8$ which are the effects of attitude error doppler cell bandwidth and averaging time as in Pierson (1978).

$$\lambda' = 7 \times 10^{-3} \tag{37}$$

For noise to signal ratios of 10 and 1, as given by Equation (38) and (39), the appropriate bounds on the likelihood estimate for the population parameter are thus obtained as examples.

$$N/s = 10 \quad 0.92\, \hat{\sigma}_1^o < \sigma_1^o < \hat{\sigma}_1^o \tag{38}$$

$$N/s = 1 \quad 0.986\, \hat{\sigma}_1^o < \sigma_1^o < \hat{\sigma}_1^o \tag{39}$$

This result is somewhat out of the ordinary. For most studies of sampling from a normal population in which the variance is not connected by a physical law to the mean, the unbiased estimate of the mean in terms of moments is also the maximum liklihood estimate of the mean. For this particular case, one notes from Equation (40), that the expected value of $\hat{\sigma}^o$ from Equations (27) is (40), and once the estimate of the mean is obtained, the variance can be computed directly from (28), as an estimate.

$$E(\hat{\sigma}_1^o) = \sigma_1^o \tag{40}$$

The implications of this particular analysis are that $\hat{\sigma}^o$ as measured by the SASS, when treated as a random variable, is an unbiased estimate of σ^o. The correctly derived likelihood estimate may be inappropriate for either the SASS or the SCATT because it is biased low. The maximum likelihood estimate has clearly not been used to compute vector winds by means of the G - H table method.

In attempting to obtain the maximum likelihood estimate, there have been variations on the above correctly derived theory. For example, in Equation (32), the variance of $\hat{\sigma}_1^o$ can be calculated by substituting $\hat{\sigma}_1^o$ into the equation for the variance (Eq. 28) and treating the variance as a constant that does not vary when attempting to obtain the maximum liklihood estimate. This is an incorrect step; however, if it is done, the incorrect maximum liklihood estimate then becomes identically equal to the expected value.

For pairs of SASS measurements 90^{o} apart, maximum liklihood estimates of the two backscatter values can in fact, be computed by maximizing (33) in terms of (34) and a similar equation for σ_2^o. These backscatter values would always be less than the measured values. Given these two maximum likelihood values, they could be used in exactly the same ways as the values of $\hat{\sigma}_1^o$ and $\hat{\sigma}_2^o$ in what follows. The wind speeds would always be somewhat less than those computed by the methods presently in use.

Such a procedure does not seem advisable because the MLE is biased low. Also, the model function has been determined empirically by means of procedures that do not use maximum likelihood estimates.

This analysis shows that the MLE can be found without any consideration of the wind speeds and directions that could have produced the backscatter.* However, there are solutions for the MLE that do not correspond to physically realizable values for a particular model function. These conditions need to be investigated also.

The V-χ plane search procedure

From the preceeding equations one way to obtain values for the wind speed and direction that could have generated the backscatter values that were estimated would be to consider the quadratic forms given by Equations (41a) and (41b) in which V and χ are the wind speed and direction relative to the pointing direction of the lead radar beam. Strictly speaking, the denominator should also be a function of wind speed and direction as in (41a). Equation (41a) is incomplete. The likelihood function should really be used and it involves additional terms. In (41b), the

---

*For pairs of measurements only.

problem is simplified by not varying the variance as a function of V and χ. The estimates of the variance are computed directly from the estimates, $\hat{\sigma}_1^o$ and $\hat{\sigma}_2^o$. The result will no longer be a maximum likelihood estimate, but a probability, in modified form similar to (32), can still be maximized.

$$Q^* = \frac{(\hat{\sigma}_1^o - \sigma_1^o\,(V,\chi))^2}{\mathrm{VAR}\;\hat{\sigma}_1^o\;(V,\chi)} + \frac{(\hat{\sigma}_2^o - \sigma_2^o\,(V,\,\chi - \pi/2))^2}{\mathrm{VAR}\;\hat{\sigma}_2^o\;(V,\chi - \pi/2)} \tag{41a}$$

$$Q = \frac{(\hat{\sigma}_{1db}^o - \sigma_1^o\,(V,\chi)_{db})^2}{(\widehat{\mathrm{VAR}}\;\hat{\sigma}_1^o)_{db}} + \frac{(\hat{\sigma}_{2db}^o - \sigma_2^o\,(V,\chi - \pi/2)_{db})^2}{(\widehat{\mathrm{VAR}}\;\hat{\sigma}_1^o)_{db}} \tag{41b}$$

The presently operational technique for determining the wind speeds and directions that go with pairs of SASS measurements is to construct a grid in the $V - \chi$ plane, substitute values of V and χ into the model function, with an appropriate shift of angle for the second term as indicated in this equation and calculate the values of Q as in (41b) using db*. The smallest values of Q in the plane determine the first approximation (as resolved to within whatever step is used in the wind speed and wind direction ranges) for the values for the wind speed and direction that could have generated values of the backscatter that correspond to the estimates that were obtained. The search has to be carried out over the entire $V - \chi$ plane in order to find local maxima (as many as four) and then a closer search by means of some kind of polynominal fit has to be made around the relatively coarse location of the maxima.

---

*See Appendix B.

The first term in the equation for Q will vanish if Equation (42) is satisfied by values of V and χ in this plane, and (43) will cause the second term to vanish for some other set of values of V and χ. One needs to find the values of aspect angle and wind speed that makes (42) and (43) an identity. There exists a continuous range of wind speeds and aspect angles that will satisfy Equation (42) for the estimated value of backscatter, and another continuous range of wind speeds and aspect angles that will satisfy Equation (43) where backscatter is in db.*

$$\hat{\sigma}_1^o = G(\theta_1,\chi) + H\,(\theta_1,\chi)\,\log_{10}V \tag{42}$$

$$\hat{\sigma}_2^o = G(\theta_2,\, \chi - \pi/2) + H\,(\theta_2,\, \chi - \pi/2)\,\log_{10}V \tag{43}$$

The simplest way to find those wind speeds and directions that satisfy (42) and (43), and that therefore make the respective terms in Equation (41b) vanish identically, is to invert Equations (42) and (43) to obtain Equations (44) and (45) where backscatter is in db.

$$\log_{10}V = \mathcal{A}(\chi,\, \theta_1) + \mathcal{B}(\chi,\theta)\,\hat{\sigma}_1^o \tag{44}$$

$$\log_{10}V = \mathcal{A}(\chi - \pi/2,\theta_2) + \mathcal{B}(\chi - \pi/2,\theta_2)\,\hat{\sigma}_2^o \tag{45}$$

In Equations (44) and (45) on the right hand side, everything is known except the aspect angle. As the aspect angle is varied, in (44) that value of the wind speed is obtained such that when these particular aspect angles and wind speeds are substituted into (42), the equation will be satisfied identically.

---

*See the definitions in Schroeder et. al. (1981). The G and H values used here absorb the factor of 10.

The pair of backscatter measurements must have corresponded to the same magnitude for the wind speed for whatever direction the wind had for the two cells. Therefore, the requirement is simply Equation (46), in which the right hand sides of Equations (44) and (45) have been set equal.

$$A(\chi,\theta_1) + B(\chi,\theta_1)\hat{\sigma}_1^o = A(\chi - \pi/2,\theta_2) + B(\chi - \pi/2,\theta_2)\ \hat{\sigma}_2 \quad (46)$$

In general for noise-free data and perfect measurements, which is the logical starting point for such an investigation, there will be from one to four values of the aspect angle, that will satisfy Equation (46). These angles are solved for more directly by simple algebraic techniques without a search of the entire $V - \chi$ plane in the quasi-analytical method that was described in the preceeding section.*

The solution to Equation (46) is often four values of aspect angle, one for each quadrant defined by, say, χ_1, χ_2, χ_3 and χ_4. For either of the two measured (estimated) values of backscatter, the wind speed that goes with each of these four aspect angles can be computed directly as shown by Equation (47).

$$\log_{10} V_q = A(\chi_q, \theta_1) + B(\chi_q, \theta_2)\hat{\sigma}_1^o \quad (47)$$

$$q = 1 \text{ to } 4$$

The equations that are solved analytically, by means of a computer program (or algorithm), so as to obtain these four angles are given in an appendix. It suffices to say at this time that these equations have been extended for the four solution case to treat the situation in which the difference in direction is given by $\pi/2 + \varepsilon$ instead of $\pi/2$.†

---

*The present SASS-1 algorithm searches the entire V-χ plane.

†See Appendix A.

The only basic difficulty that arises in the analytical approach is that which occurs when Equation (46) does not have any solutions or for the case in which it has two solutions close together and not a third. The lack of the solutions corresponds to the situation in which the sampling variability effects of the measurement have produced curves for (44) and (45) that do not intersect. This matter will be discussed in greater detail later.

Some examples of the theoretical curves involved for the noise-free (sampling variability absent) case, are shown in Figure 4 for pairs of vertically polarized backscatter measurements. Each of the three sets of graphs was generated by first calculating the two values of backscatter as given by Equations (42) and (43) for an incidence angle of 25°, a wind speed of 10 meters per second and aspect angles of 0°, 40°, and 80°. These two values for the backscatter were then substituted into Equations (44) and (45) and the aspect angle was varied in ten degree steps for convenience, so as to generate the graphs that are shown. The SASS-1 model function from the JASIN results as defined by Table 1 and Equations (5) through (20) was used.

For the top set of graphs in Figure 4, the input values were for a wind direction of 0° relative to the pointing direction of beam 1 and a wind speed of 10 meters per second. The upper curve is a graph of Equation (44). Clearly for a given measured value of backscatter, the wind that would generate that value of backscatter would be the lowest when looking upwind. It would have to be much higher when looking cross wind. This curve, therefore, has a minimum at 10 meters per second for $\chi = 0$, a maximum

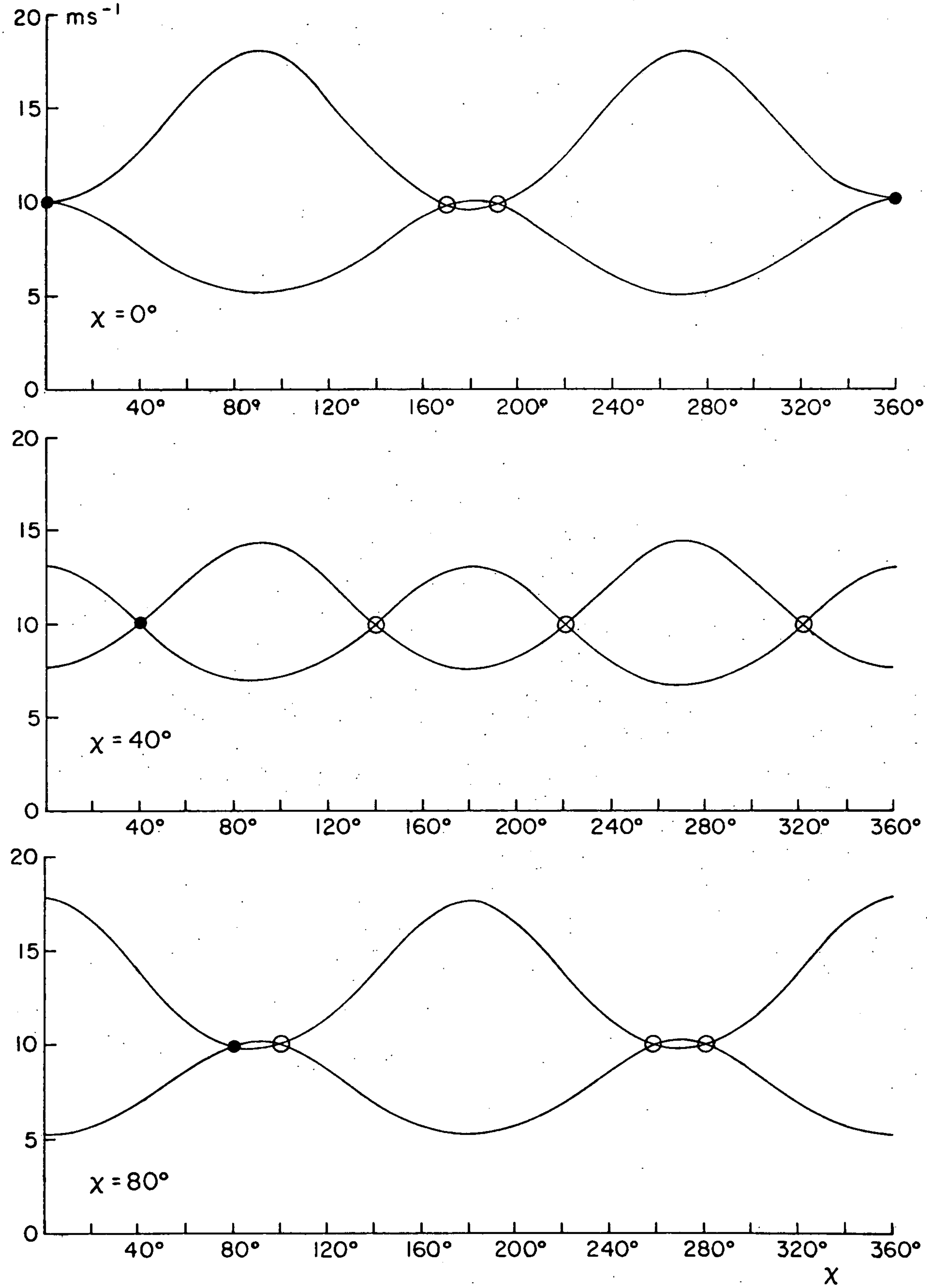


FIG. 4 EXAMPLES OF NOISE FREE SOLUTIONS FOR BEAMS 90° APART, VERTICAL POLARIZATION, AND A WIND SPEED OF 10 m/s FOR WIND DIRECTIONS OF 0°, 40°, AND 80°.

slightly above 18 meters per second at $\chi = 90^{\circ}$ and a second minimum at 180°, which is slightly below the 10 meters per second value. This is a consequence of the model function because downwind backscatter values are slightly stronger than upwind backscatter values for higher winds. The function that is graphed is an even function about 180°. The lower curve corresponds to the measurement by the second beam which is 90° away from the first beam, as indicated by Equation (45). If beam 1 was looking upwind and if beam 2 was exactly 90° from beam 1, then beam 2 would be looking crosswind. Therefore, for that value of backscatter, the wind would be the highest of those that would generate that backscatter. This curve then goes to a minimum at 90°, which is slightly higher than five meters per second. A maximum, which is identically the same as the value at 10 meters per second is at 180° because it is a crosswind value. Another minimum is at 270°, which is ever so slightly lower than the one at 90°, and finally, the curve returns to 10 meters per second at 360°.

The black dot represents the input wind speed and direction, and there are two circles at roughly, 168° and 192°. These correspond to the aliases that are a consequence of the nature of the measurements. They are not produced by errors in the measurements due to the SASS. They are a fundamental feature of the fact that only two measurements of backscatter have been obtained.

In the quasi-analytical method the points at which the two curves cross, if they cross, can be solved for immediately. There is no need to search the entire $V - \chi$ plane. The values of V and χ are typically found to within $\pm\ 0.01\ ms^{-1}$ and $\pm\ 0.01^{\circ}$ and listed to the nearest tenth of a meter per second and tenth of a degree.

The results for a wind direction of 40° clockwise from the pointing direction of beam 1 are illustrated in the second set of curves. One notes here that the maxima and minima of these curves fall exactly at the same place, and have the same relative shapes as the curves in the first set of curves. However, the curve corresponding to beam 1 has shifted downward, and the curve corresponding to beam 2 has shifted upward. The input conditions are those shown by the black dot, and the three clearly defined aliases are shown by the circled dots.

The third graph in Figure 4 shows more of the same. Here the input wind direction corresponded to 80^{o} from the pointing direction of the first beam. There are 3 aliases at roughly 100^{o}, 260^{o} and 280^{o}.

These curves have been drawn for the noise-free case. The actual estimates of the backscatter measured for the two cells will differ from the true values, and, therefore, it is not to be expected that they will recover the winds with 100% accuracy. However, all that can happen is that the two estimated values, $\hat{\sigma}_1^o$ and $\hat{\sigma}_2^o$, will differ from the true values in some unknown way, and the only thing that can be done is to graph the corresponding curves as in Figure 4 for these estimates. The points where the curves cross (if they cross) then determine the values of wind speed and direction estimated by the SASS.

If the values for the wind speed and wind direction that result from each of the points in the $V - \chi$ plane where the curves that have been graphed above intersect are substituted into Equations (42) and (43) on the right hand side the results will be identically equal on the left hand side to any desired degree of accuracy. This is true for each of the points shown in the figure- Therefore, Equation (41) is such that the value of Q is identically zero for each of these conditions. Moreover, the values of $\hat{\sigma}_1^o$ and $\hat{\sigma}_2^o$ used to estimate the variances in (32) are identically the same for all of the four solutions. The probability of observing this particular set of backscatter values and any of the solutions for wind speed and directions is therefore identically the same for all of the solutions shown in each part of Figure 4. The probabilities are for all solutions identically the same, no matter how they are computed, either correctly, or with the approximations that have been discussed above.

One of the reasons for the selection of the present SASS-1 algorithm that uses the $V - \chi$ search routine in order to determine the wind speed and direction from the SASS estimates of backscatter is that it provides a quantity called a relative probability for each of the solutions that are obtained. This particular algorithm was used for pairs of vertically polarized values, or horizontally polarized values, or mixed (one horizontal and one vertical) values in order to produce the results shown in Table 3. The relative probabilities that are tabulated are some function related in some way to some equation similar to Equation (32) as described in Wentz (1978).

As demonstrated conclusively above, these four relative probabilities ought to be identically the same number for all times for which four solutions are recovered with the algorithm. This is clearly not the case. They differ one from the other. These differences simply represent the lack of accuracy with which the wind speeds and directions were determined by the algorithm. Had they been determined to, say, several additional significant figures, it would have been possible to find wind speeds and directions, as given in the appropriate columns of the listing, such that the backscatter values that were estimated from the spacecraft measurements, would have caused the parenthesis in an equation similar to Equation (32) to vanish identically for all four solutions. The result would then be that the relative probabilities would be identically equal.

TABLE 3. Examples of Incorrectly Computed Relative Probabilities for Paired SASS cells.

REV= 432.16

	TIME		SPEEDS				DIRECTIONS				RELATIVE PROBABILITIES				LAT	LON	INC	REC
1	208:	5:21:59	9.1	9.5	9.1	9.6	53	154	233	334	4.613	4.521	4.485	4.591	55.6	353.7	25.6	195
2	208:	5:22: 1	10.0	10.7	11.0	11.1	66	143	250	315	5.029	5.183	5.624	5.530	55.9	353.5	25.4	195
3	208:	5:22: 6	11.7	12.1	11.6	12.1	60	146	240	327	4.750	4.623	4.540	4.620	56.2	353.2	25.6	195
4	208:	5:22: 8	11.9	12.7	13.0	13.1	65	143	249	315	5.047	5.186	5.632	5.512	56.3	353.1	25.4	195
5	208:	5:22:14	12.9	13.3	12.8	13.4	53	151	234	332	4.768	4.615	4.556	4.634	56.6	352.8	25.6	195
6	208:	5:22:16	12.6	13.8	13.7	13.5	51	159	229	331	5.008	5.426	5.683	5.194	56.7	352.6	25.4	195
7	208:	5:22: 6	10.3	10.8	10.6	10.9	75	132	256	310	4.345	4.535	4.288	4.228	56.4	353.9	30.3	195
8	208:	5:22:21	12.5	12.9	12.3	12.9	54	150	235	331	4.758	4.610	4.552	4.637	57.0	352.4	25.6	195
9	208:	5:22: 7	11.0	11.9	11.6	11.5	67	143	244	317	4.387	4.514	4.672	4.416	56.5	353.9	30.2	195
10	208:	5:22: 8	11.0	11.9	12.4	12.4	67	143	255	308	4.602	4.626	5.751	5.132	56.5	353.6	30.1	195
11	208:	5:22:23	11.4	12.2	12.6	12.6	64	143	247	315	5.066	5.176	5.709	5.481	57.0	352.2	25.4	195
12	208:	5:22:14	10.1	10.6	10.4	10.7	68	138	249	315	4.198	4.024	4.155	4.040	56.8	353.5	30.3	195
13	208:	5:22:29	13.5	13.9	13.4	14.1	49	153	231	335	4.816	4.655	4.598	4.733	57.4	351.9	25.6	195
14	208:	5:22:15	10.5	11.4	11.2	11.1	64	145	240	319	4.379	4.510	4.571	4.237	56.9	353.4	30.2	195
15	208:	5:22:16	10.5	11.4	11.9	11.9	64	146	250	312	4.594	4.832	5.595	5.147	56.9	353.3	30.1	195
16	208:	5:22:31	13.2	14.4	14.4	14.2	52	156	230	329	5.005	5.292	5.674	5.220	57.4	351.7	25.4	195
17	208:	5:22:22	12.0	12.7	12.3	12.8	58	147	239	325	4.147	4.014	4.025	3.995	57.2	353.1	30.3	195
18	208:	5:22:37	13.0	13.4	12.9	13.6	55	147	236	328	4.786	4.647	4.563	4.660	57.7	351.4	25.6	195
19	208:	5:22:22	12.3	13.6	13.3	12.8	56	154	231	326	4.319	4.457	4.543	4.238	57.3	353.0	30.2	195
20	208:	5:22:23	11.8	13.2	13.2	12.9	53	159	231	326	4.535	4.910	5.553	4.889	57.3	352.9	30.1	195
21	208:	5:22:14	10.9	11.9	11.6	11.8	53	154	233	329	3.729	3.581	3.825	3.651	57.1	354.2	34.7	195
22	208:	5:22:18	12.4	13.3	13.2	13.6	65	141	248	316	3.643	3.793	3.950	3.698	57.2	354.5	36.6	195
23	208:	5:22:38	13.4	14.8	14.7	14.4	49	159	226	332	4.980	5.458	5.778	5.203	57.8	351.3	25.4	195
24	208:	5:22:29	10.8	11.5	11.1	11.6	60	145	240	323	4.084	3.949	4.001	3.958	57.6	352.6	30.3	195
25	208:	5:22:15	11.6	13.5	13.1	12.4	48	166	219	334	3.850	4.048	4.246	3.913	57.1	354.1	34.6	195
26	208:	5:22:16	12.6	14.2	14.5	14.3	57	155	237	320	4.036	4.147	4.942	4.185	57.2	354.0	34.5	195
27	208:	5:22:20	15.6	16.1	17.5	.0	79	131	283	0	4.717	5.056	1.648	.000	57.3	354.3	36.3	195
28	208:	5:22:44	12.8	13.2	12.6	13.2	51	151	232	332	4.792	4.604	4.587	4.642	58.1	351.0	25.6	195
29	208:	5:22:30	11.0	12.2	12.1	11.7	58	151	232	324	4.338	4.417	4.517	4.198	57.6	352.5	30.2	195
30	208:	5:22:31	11.5	12.5	13.0	13.0	62	147	246	313	4.569	4.859	5.538	4.938	57.7	352.4	30.1	195
31	208:	5:22:22	10.4	11.2	11.1	11.4	64	143	245	318	3.723	3.604	3.851	3.556	57.5	353.8	34.7	195
32	208:	5:22:25	9.9	10.9	10.8	11.1	60	147	242	320	3.595	3.474	3.764	3.490	57.6	354.1	36.6	195
33	208:	5:22:46	12.3	13.4	13.5	13.5	57	149	237	322	4.928	5.226	5.624	5.307	58.2	350.8	25.4	195
34	208:	5:22:37	10.8	11.4	11.1	11.5	64	140	245	318	4.132	4.034	4.074	3.945	58.0	352.2	30.3	195
35	208:	5:22:22	10.5	11.9	11.7	11.5	62	148	238	319	3.803	3.687	4.298	3.710	57.5	353.7	34.6	195
36	208:	5:22:23	10.8	12.0	12.5	12.5	64	146	250	310	3.964	4.167	4.933	4.234	57.6	353.6	34.5	195
37	208:	5:22:27	9.0	10.7	10.6	10.1	46	168	220	330	3.774	4.419	5.002	4.091	57.7	353.9	36.3	195
38	208:	5:22:52	11.7	12.2	11.6	12.3	53	148	234	329	4.728	4.575	4.529	4.625	58.5	350.5	25.6	195
39	208:	5:22:36	10.6	11.4	11.1	11.2	66	140	243	315	4.403	4.596	4.712	4.320	58.0	352.1	30.2	195
40	208:	5:22:39	10.2	11.1	11.6	11.6	63	145	249	311	4.619	4.781	5.633	5.066	58.1	352.0	30.1	195
41	208:	5:22:29	9.4	10.0	10.1	10.3	70	136	253	310	3.706	3.796	4.012	3.683	57.9	353.3	34.7	195
42	208:	5:22:33	8.2	9.1	9.1	9.4	60	146	243	318	3.441	3.332	3.761	3.375	58.0	353.6	36.6	195
43	208:	5:22:54	11.2	12.2	12.3	12.3	55	150	235	323	4.911	5.213	5.621	5.186	58.6	350.3	25.4	195
44	208:	5:22:44	10.8	11.5	11.1	11.5	55	148	235	326	4.053	3.868	3.984	3.951	58.4	351.7	30.3	195
45	208:	5:22:30	12.2	14.3	14.0	12.9	44	170	211	337	3.905	4.390	4.659	3.913	57.9	353.2	34.6	195
46	208:	5:22:31	13.2	15.1	15.2	14.7	53	159	231	323	4.045	4.245	4.997	4.269	58.0	353.2	34.5	195
47	208:	5:22:35	13.5	15.4	15.7	15.5	56	155	237	318	3.964	4.015	4.930	4.186	58.1	353.5	36.3	195
48	208:	5:22:25	12.6	13.9	13.5	13.8	50	156	229	330	3.636	3.311	3.771	3.515	57.9	354.8	40.5	195
49	208:	5:22:22	18.5	18.6	19.9	.0	95	115	283	0	5.504	5.610	4.297	.000	57.7	354.9	40.4	195
50	208:	5:22:59	11.2	11.6	11.1	11.7	44	155	226	336	4.769	4.610	4.622	4.692	58.9	350.0	25.6	195
51	208:	5:22:45	10.7	11.9	11.7	11.4	56	151	230	325	4.249	4.383	4.506	4.182	58.4	351.6	30.2	195
52	208:	5:22:46	9.9	11.3	11.2	10.8	47	164	221	330	4.536	5.058	5.642	4.832	58.5	351.5	30.1	195
53	208:	5:22:37	10.1	11.0	10.8	11.1	61	144	242	319	3.672	3.573	3.809	3.537	58.3	352.9	34.7	195
54	208:	5:22:40	8.8	9.9	9.6	9.8	49	156	229	329	3.422	3.236	3.718	3.449	58.4	353.2	36.6	195

The argument has been advanced that, when more than two cells are used such as in a case in which all of the backscatter values within a one degree square are combined in a generalization of Equation (32) and such that perhaps eight terms would be involved in the sum, four from one beam and four from another, the relative probabilities would then have a meaning such that the highest relative probability would correspond to the most likely wind speed and direction at the sea surface. This has yet to be demonstrated with any data from SEASAT, especially for the single polarization cases. A convincing theoretical proof is also lacking. Before it can be demonstrated, it seems that it would be advisable to carry the calculations out to a sufficient accuracy such that differences would not exist in the relative probabilities for those conditions in the theory where it can be shown that the relative probabilities are identically equal.

The study of a similar problem for the SCATT to be described later shows that the maximum of the MLE (for six measurements) near the "true" wind is highly variable and is not a measure of how close the estimate is to the "true" wind. The material on pages 80 through 90 could perhaps be generalized to provide this proof.

MODES THREE AND FOUR FOR THE SASS

For two of the modes of the SASS, the antennas were operated on only one side of the spacecraft. A series of horizontally polarized measurements were made along one beam followed immediately by a series of vertically polarized measurements; then the radar was switched to the second beam, and the process was repeated. It is then possible to locate four sets of measurements closely spaced on the sea surface. One set consists of vertical polarization and horizontal polarization backscatter values for the first beam located near a second set of horizontal and vertical polarization values for the second beam. If the wind speed and direction does not change substantially, due to the lack of co-location of the pair of cells from beam 1 relative to the pair of cells from beam 2, one could consider this set of four measurements as a possible way to determine the wind speed and direction uniquely. Noise-free simulations of this case, in which it is assumed that the wind speed and direction were the same for all four measurements are shown in Figures 5 and 6. For Figure 5, the first three measurements for vertical polarization were identical to those made for the vertical polarization measurements in the preceeding section. They are graphed as the solid lines. The dashed lines represent the curves that would be obtained from the horizontally polarized measurements. The four values appear to have the potential of being able to eliminate some of the alias solutions under certain circumstances. For example, in Figure 5, the input wind speed was 10 meters per second from zero degrees relative to beam 1. The two aliases occur for vertical polarization. They also occur, as would be expected, when one of the horizontally polarized measurements (beam 2) is paired with a vertical polarization (beam 1).

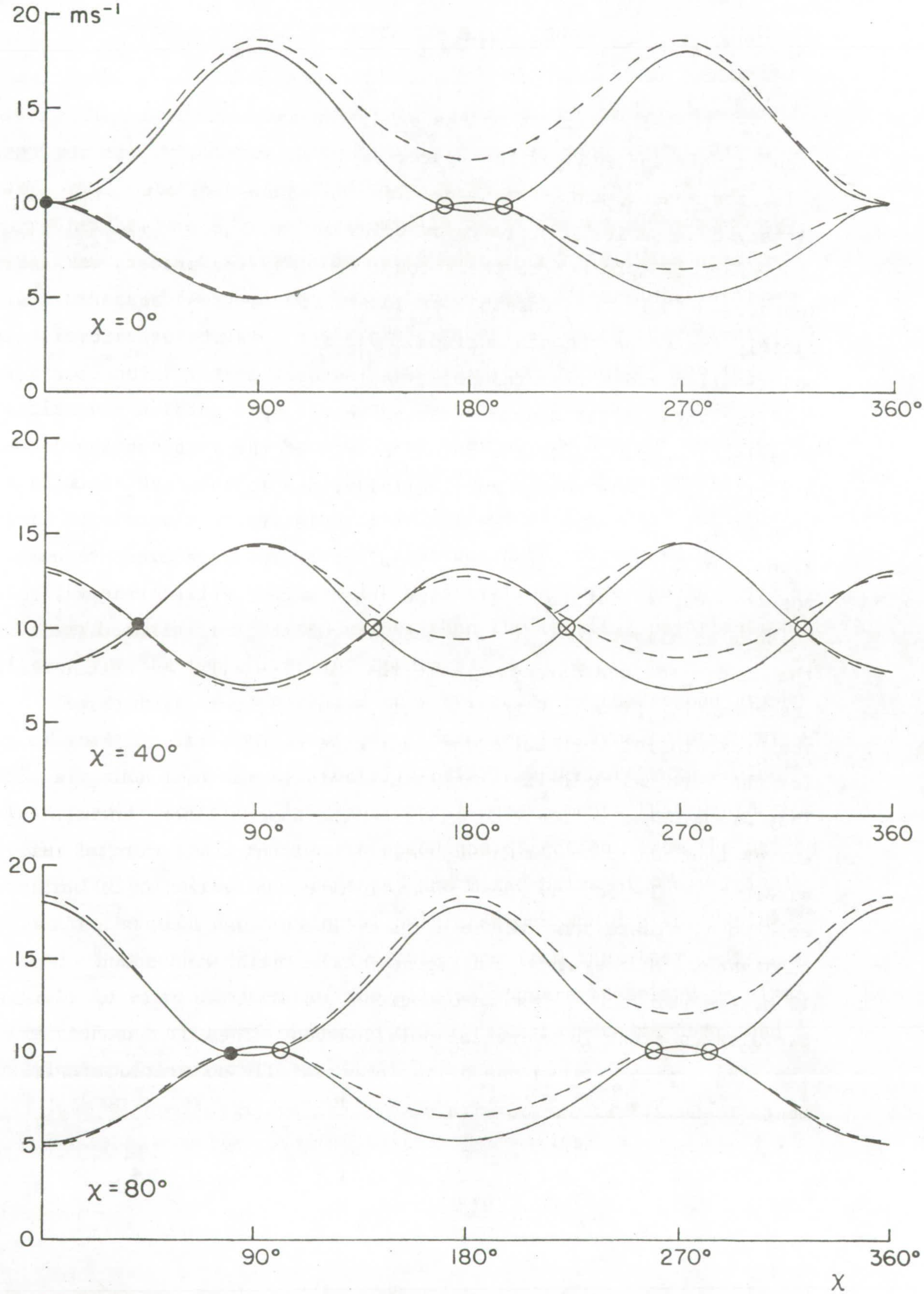


FIG. 5 EXAMPLES OF NOISE FREE SOLUTIONS FOR BEAMS 90° APART, VERTICAL (CONTINUOUS) AND HORIZONTAL (DASHED) POLARIZATION, AND A WIND SPEED OF 10 ms^{-1} FOR WIND DIRECTIONS OF 0°, 40° AND 80°.

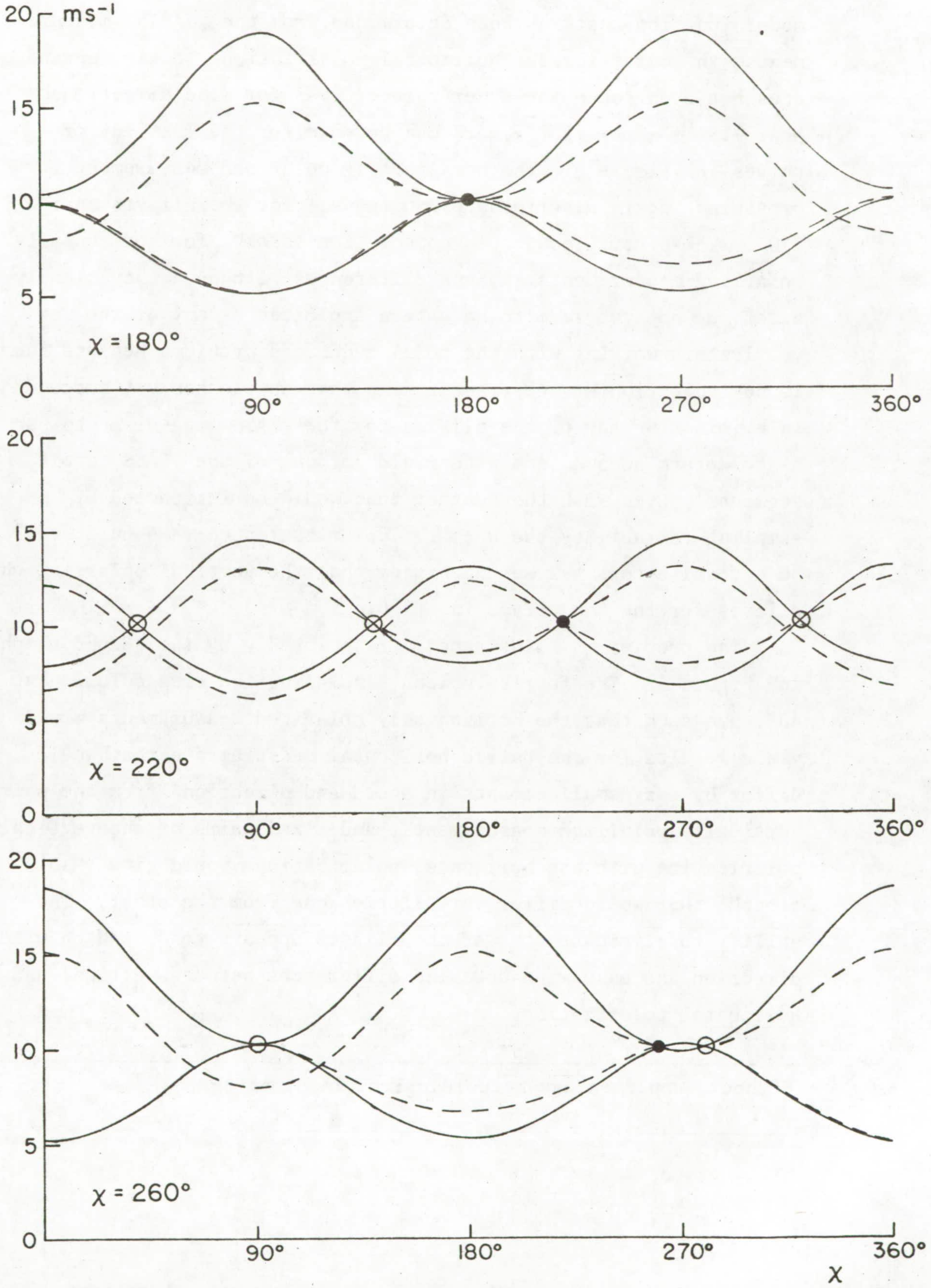


FIG. 6 EXAMPLES OF NOISE FREE SOLUTIONS FOR BEAMS 90° APART, VERTICAL (CONTINUOUS) AND HORIZONTAL (DASHED) POLARIZATION, AND A WIND SPEED OF 10 m/s FOR WIND DIRECTIONS OF 180°, 220°, AND 260°.

However, because of the difference between upwind and downwind for horizontal polarization as indicated by Figure 1 and the model function that has been determined from the JASIN measurements, the curve for the horizontal polarization looking upwind at 0° does not even touch the other three curves for wind directions near 180°. A similar remark can be made for the last set of curves in Figure 5. The horizontally polarized measurement for crosswind again discriminates quite well for the aliases near 270°. These correspond to "no solution cases" for horizontally polarized measurements. These differences, although they clearly exist, do not yet seem to have been exploited by any of the investigators working with the model functions problem because there is no indication at the present time that anyone has yet succeeded in eliminating any of the aliases for the SASS measurements, so as to determine a unique wind field for one of the SASS orbit segments.* Even with the scatter that would be introduced by sampling variability the beam 1H polarization curve should still be several meters per second higher than the vertical polarization aliases for the top curves in Figure 5.

The problem is illustrated more pointedly by the second graph in Figure 5. The four solutions, including the true solution at 40°, are such that the horizontally polarized measurements would yield results for the paired hozizontal measurements that would differ by very small amounts in speed and direction from the paired vertically polarized measurements, and mixed pairs of one vertical polarization with one horizontal polarization would give "solutions" that would differ very little one from the other. The ability to eliminate some of the aliases appears to depend on wind direction and on upwind-downwind differences between vertical and horizontal polarization.

*Without supplementary meteorological information.

To emphasize the amount of information that could be extracted from properly grouped mode three or mode four measurements, Figure 6 shows the results of using input wind speeds and directions for this mode that are 180° away from the input wind speeds and directions in Figure 5. The correct solution has been moved 180° from the correct solutions in the first figure. The horizontally polarized data may succeed in eliminating the possibility of a solution near 0°, for the first of the three sets of curves in Figure 6. The same kinds of problems arise for a true wind from 220°, as arose from a true wind from 40°. Again, the horizontally polarized data has the potential of eliminating one of the aliases at 90° for the last set of curves. The only remaining alias would be the one at 280°, to be compared with the one at 260° in this example.

When the numerous sources of error for the SASS, summarized in a preceeding section, are considered, it is not too surprising on the basis of these two figures that it has not yet been possible to determine ways, using modes three and four, to eliminate aliases.

In the analysis of the SASS data, conditions such as those illustrated by the first and third set of graphs in these two figures, often result in no solution at all in the quasi-analytical case and in poorly defined solutions in the $V - \chi$ search routine. This is caused by the fact that the estimates of the backscatter are too high for the upper of the curves and too low for the lower of the curves. This results in a situation for which no solutions exist in Equation 46. It is then necessary to calculate wind speeds at upwind and downwind, relative to one of the beams that are more nearly maximum likelihood estimates by means of a very complicated search routine. This search routine was described by Pierson and Salfi (1978). It can be simplified on the basis of the analysis given in the preceeding material.

A part of the problem of eliminating at least some of the aliases in past studies has been the lack of a reliable model function. With the JASIN model function, based on the kinds of graphs that would result for a wider range of wind speeds and and incidence angles, some, but not all, of the aliases might be eliminated by considerations such as these.

DESIGN FEATURES FOR THE SCATT ON THE NOSS

The new scatterometer to be flown on some future spacecraft, and presently under design, has many planned improvements over the SASS that was on SEASAT (Grantham (1980)). The design is still in the process of review. Eight features of the improved design are described below.

(1) Six antennas instead of four on each side of the spacecraft, three for each polarization, at angles of $\pm 45^{\circ}$, $\pm 65^{\circ}$, $- 115^{\circ}$ and $\pm 135^{\circ}$ to the spacecraft direction of travel.

(2) Higher resolution with cells 10 kilometers apart and virtually continuous coverage with simultaneous operation on each side for alternating vertical and horizontal polarization.

(3) Co-located cells by means of tuneable Doppler band pass filters to keep paired cells the same distance abeam of the spacecraft and provide a full swath for all latitudes.

(4) Improved antenna elevation angles to peak gain for the two new antennas so that the signal-to-noise ratio will be increased at low incidence angles.

(5) Greatly reduced attitude uncertainty errors.

(6) Improved low noise amplifier.

(7) A different inclination for the orbit.

(8) Improved measurement accuracy of the noise power by measuring it over the full I.F. bandwidth rather than individual doppler bandwidth.

Six antennas on each side. In addition to the four antennas at $\pm 45^{o}$ and $\pm 135^{o}$ to the spacecraft direction of travel, as there were on the SASS, there will be two more antennas, one for each polarization at $+ 65$ and $- 115^{o}$ to the forward direction of travel. Modes three and four of the SASS have thus been augmented by two additional measurements and new azimuth angles (one for each polarization) for each area of the sea surface scanned by the instrument. As will be shown, these additional measurements can frequently eliminate two of the three aliases in the four solution cases described above. (When there are four solutions, one is correct, and so this leaves three aliases.)

Higher Resolution. The 10 kilometer by 10 kilometer spacing of the cells that will be scanned by the SCATT will be virtually continuous over the measurement swath. Each alternate polarization measurement will be about 1.5 kilometers displaced from the preceeding polarization measurement. When grid points are pooled in 5 by 5 clusters nearly the same area (which will be 50 kilometers by 50 kilometers) will have been sampled six (x 25) different ways (three different azimuth angles and two different polarizations).

The vertical and horizontal polarization measurements will be interlaced and much more nearly coincident than those made by mode 3 or mode 4 of the SASS on SEASAT because of the shorter time interval between samples. Sources of scatter in the SEASAT data because the paired cells did not exactly cover the same ocean area will be greatly reduced. Mesoscale fluctuations would still be present for a 50 x 50 kilometer area, but the averages of the 25 measurements for each antenna and polarization will be representative of the same area for all six of the averages.

Also, 4 x 4 grid point clusters, 3 x 3 clusters, 2 x 2 clusters, and perhaps even other arrangements, can be produced if higher spatial resolution is needed at the expense of sampling variability. The combined "true" SCATT communication attitude and squint errors at high spatial resolution may become the dominant vector wind error source.

Co-located cells.* For the SASS, the cells in the overlap area moved in and out with latitude for the different beams. To measure the received power the range gates had to be longer than the duration of the return signa- after passing through the Doppler filters. Thus the signal plus the noise would have added to it an additional noise that was measured for each cell. The added noise effect produced the κ term in the variance of the received power. By tuning the Doppler filters so as to counter this effect, the cells can be kept the same distance abeam of the spacecraft at all latitudes and the range gates can be kept narrower. Typical values for κ in Equation (31) for the SASS ranged from 0.29 to 1.36 after averaging the noise. For the SCATT, the values will be 0.2 for all cells. This allows the cells for all beams to be co-located for the same area of the sea surface. The narrower range gates reduce the variance of the estimate of the estimate of the received power and will consequently improve the estimates of the wind speed.

Improved antenna elevation angles. The squared antenna gain pattern has a strong effect on the signal-to-noise ratio for a given cell. For the SASS, the antenna elevation angle was such that the K_p values increased toward the inner edge of the swath because of the falloff of the antenna gain toward nadir. A parametric study of the effect of varying the elevation angle for the two new antenna at $+ 65^o$ and $- 115^o$ yielded optimum values that produced more nearly constant values of K_p for the inner edge of the swath and reasonable results over the entire swath. The effect will be to decrease the variance of the estimate of the recieved power for low incidence angles and cause the sampling variability of the estimates of the vector winds to decrease.

Reduced attitude uncertainties. The attitude of SEASAT was telemetered to the earth as an intermittent measurement. The reported values has an uncertainty, or an unknown error, that was fairly large. The combination of all of the attitude error effects given a maximum error for σ of about 10%.

*See Appendix G.

This uncertainty can be interpreted in terms of an additional contribution to the variance of the received power which effectively greatly increased the value of λ' in Equation (24). (See Equations (41) to (45) of Pierson (1977)). The transformation involved placing all of the random errors of measurement into the variability of the received power instead of partly into other terms of the radar equation. This particular effect was especially important for high wind speeds, when the "effective" signal-to-noise ratio would have been quite a bit lower. Improved attitude measurements for the NOSS will effectively eliminate all but squint angle error in the calculation of the backscatter measured by the SCATT. The improvement is approximately a factor of two for the fore and aft antennas. For the SCATT, maximum errors for the fore and aft beams will be about 5%. For the middle beams, it will be about 10%.

Improved low noise amplifier. Improvements in amplifier design have made possible an amplifier with a noise figure of 4.5 db.

A Different Inclination for the Orbit. The SCATT has been designed for a different inclination of the orbit. This change had little or no, effect on the accuracy of the measurements.

Improved Noise Measurements. The received power is found by subtracting the noise from a measurement of the signal plus the noise. The estimation of the noise for the SCATT will be determined from a measurement over the total intermediate frequency bandwidth and thus its value will have less sampling variability.

Tradeoffs. Four of the last six effects described above reduce the variance of the estimate of the backscatter and thus, reduce the scatter of the vector wind estimates. The greatly improved colocation of the six cells (three for each polarization) to be used to compute the winds, will reduce the effects of mesoscale variability of some unknown amount.* The benefits because of fixed narrower range gates, improved noise measurements, better attitude measurements, better attitude measurements, an improved amplifier and adjustable Doppler filters have been partly traded off to get a higher spatial resolution (smaller cell size).

* At present.

All in all, for 5 x 5 and perhaps even smaller arrays, the variances of the estimates of backscatter for the SCATT will be smaller than for the SASS. Also the pooled areas for which the backscatter was measured, will be nearly coincident instead of being varying distances apart depending on factors beyond control as in the design of the SASS. These improvements provide an opportunity to eliminate some of the aliases present in the SASS wind vector recoveries, perhaps even to the point where only one wind speed and direction correct nearly all of the time will survive. The best way to do this need not necessarily be any of the ways developed to date for the analysis of SASS data.

A NOISE FREE ANALYSIS OF SCATT MEASUREMENTS

Backscatter measurements at 10 meters per second for a 25° incidence angle for the 45° and 135° beams.

Figure 7 shows graphs of backscatter versus Azimuth angle for vertical and horizontal polarization as predicted by the model function for 25° and 19° incidence angles for a wind speed of $10\ ms^{-1}$. The curves are similar for the $\pm 90^{\circ}$ range about 0° aspect angle. From 90° to 270° however, the backscatter values for horizontal polarization lie considerably below those for vertical polarization. This particular feature may be the property that will allow the elimination of one of the four aliases possible in the four solution case. The upper curves represent the measurements made by the new antenna at 65° to the spacecraft direction of travel.

Also shown on Figure 7 are five heavy black dots on the backscatter curves. They represent the measurements that would be made in the noise-free case for a perfect model function, if the lead beam was pointing upwind. For the black dots, the six measurements are for beam 1 V Pol at upwind, beam 2 V pol 20° behind upwind, beam 3 V Pol 90° behind upwind, beam 1 H pol upwind, beam 2 H pol, 20°, behind upwind (and equal to the corresponding V Pol value) and beam 3 H pol, 90° behind upwind.

For an alias to exist, six measurements should be possible somewhere else along these curves with the same angular spacing such that they would nearly equal the six measurements shown by the black dots. It is possible to come fairly close to four of the six measurements as shown by the open circles on the right hand side of these curves for the conditions assumed. With a very small amount of scatter, three of the vertical polarization measurements on the right side could nearly equal the three measurements on the left. The lowest horizontal polarization measurement could also nearly equal the horizontal polarization measurement on the left. However, the other two horizontal polarization measurements on the right are quite different from the two horizonal polarization measurements on the left. It is this difference, if not masked by sampling variability effects, that might make it possible to eliminate some of the aliases in the solutions to the vector winds that can be computed from these six measurements.

The black triangles on Figure 7 show six values that would have been measured at 20°, on the left hand side, and a candidate set of six values on the right hand side, as shown by open triangles that might correspond to an aliased wind direction.

The black diamonds and black squares are for 40° and 60°. For the 40° wind direction, two candidate aliases that would exist for V POL two antenna SASS measurements are shown as open diamonds marked 1's and 2's. Alias 2 is immediately eliminated by the new measurements because it differs by 2 db from the actual measurement. The additional measurements from the additional antennas on the SCATT inform one immediately as to whether they straddle a convex or a concave portion of the curves. This locates the true wind, to within 180°, even if the right hand side of the curve as plotted would be identical to the left hand side for horizontal polarization. This figure therefore shows that it ought to be possible to eliminate two of the three aliases in every four solution case, soley on the basis of the measurements made with one polarization, and that differences between vertical and horizontal polarization for upwind and downwind may make it possible to eliminate the third alias. The remaining value would then, of course, be the correct value. The question that needs to be answered by means of a Monte Carlo study is whether or not the sampling variability effects in the measurements are so large that this effect will be masked too often and the technique would not succeed.

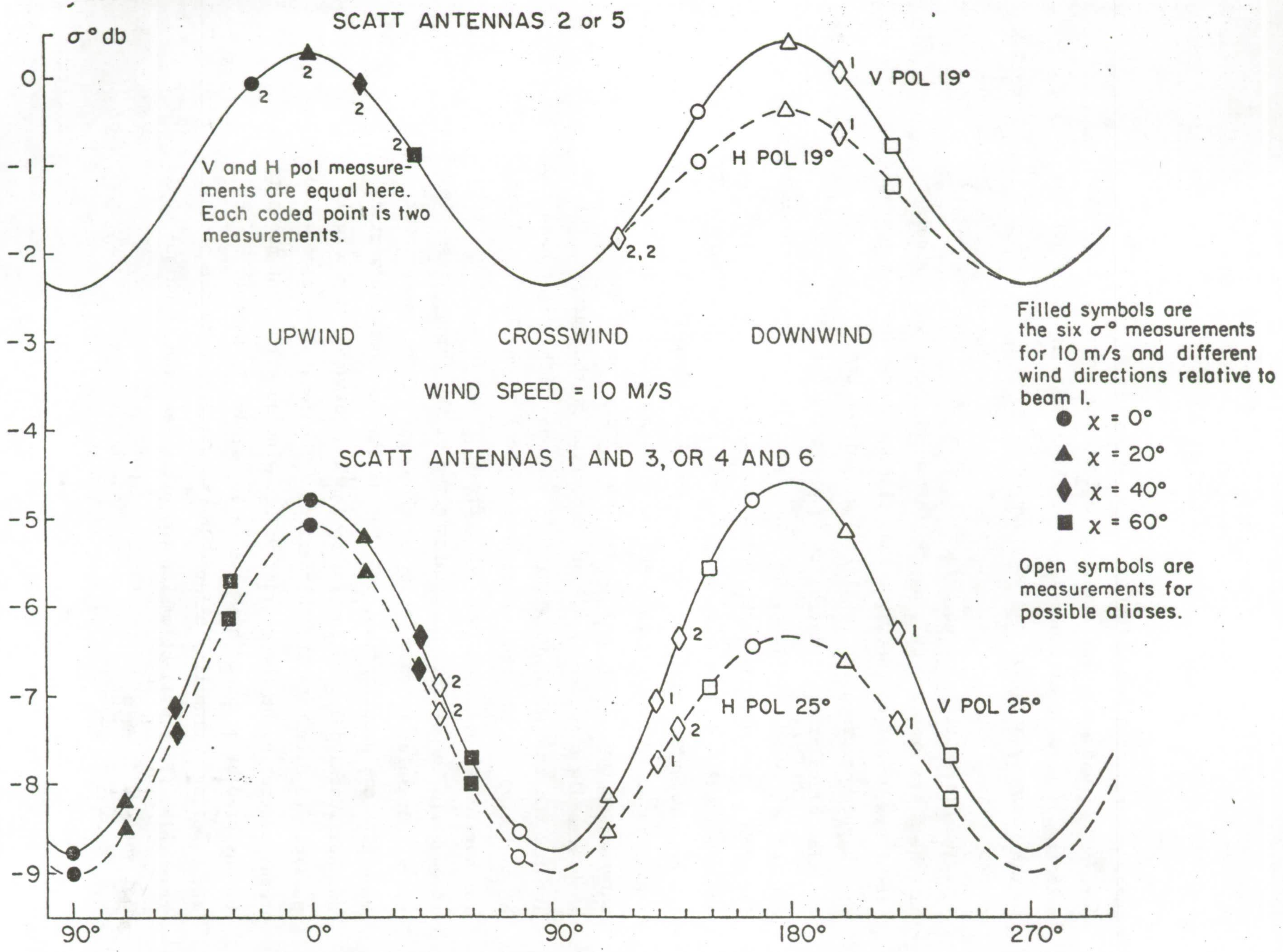


FIG. 7 BACKSCATTER (V AND H POL) AS A FUNCTION OF ASPECT ANGLE FOR 10 M/S WIND FOR INCIDENCE ANGLES OF 25° AND 19° AND MEASUREMENTS OF $\sigma°$ THAT WOULD BE MADE FOR DIFFERENT WIND DIRECTIONS.

Graphs for the six SCATT measurements

Figures 8A through 8H are similar to the graphs given in Figures 4, 5 and 6, except that the full curves have not always been plotted. The curves of wind speed versus aspect angle for input conditions, from 0° through 110° have been graphed in Figures 8A through 8D. Figures 8E through 8H continue the graphs in 20° steps to 340°. Only those portions of some of the horizontal polarization curves are plotted that differ from the vertical polarization curves near solutions and aliases.

In Figures 8 and 9, the curves for the 65° antenna are slightly in error. They were computed for the same incidence angle as the 45° and 135° antennas instead of for a lower angle. The corrected curves would be slightly different, in general in a way that would improve on alias removal capabilities and reinforce the conclusions drawn from the figures.

The top part of Figure 8A shows the graphs for the vertical polarization measurements, for an input, wind speed and direction of 10 ms^{-1} from 0°. The black dot shows the true value. The two vertical polarization measurements shown by the solid curves, intersect and yield aliases for the 90° apart pair, near 180° as shown by the two circled points. The third beam at 20° to the lead beam for vertical polarization passes in between these two aliases. Any calculation of a quantity such as Q, based upon these three measurements would be difficult and would yield a very fuzzy relationship with a wind speed and direction somewhere near the center of this intersecting set of these three curves. The curves for horizontal polarization are such that the one for the beam at 135° virtually coincides with the vertical polarization measurement. However, the curves for the lead beam at 45°, and the second beam at 65° both lie considerable above the points at which the vertical polarization curves cross. One way to discriminate between the possible aliases near 180° and a true wind near 0° would be to calculate the solutions for the vertical polarization paired beams at 45° and 135°, use the wind direction that result, calculate the wind speeds for the horizontally polarized measurements at these two directions and discover that they were much larger and differed from, in this case, $10\ ms^{-1}$ speeds by nearly $2\frac{1}{2}\ ms^{-1}$, except for one. If sampling variability effects did not reduce this difference then one would conclude that the correct value would be at the 0° wind direction.

The second set of curves in Figure 8A shows only portions of the full curves for simplicity in drafting. Portions of the three curves for wind speed versus aspect angle for the vertically polarized measurements are shown as solid curves. The input wind speed and direction was $10\ ms^{-1}$ from 10°. All three curves go through this point, as would the three curves for the horizontally polarized measurements. The three aliases for the measurements from the beams at 45° and 135°, as computed without any knowledge from any other measurement, are shown by the circled points. The curve for the measurement with the third vertically polarized antenna at 65° goes through the true value. It comes very, very close to the alias just beyond 180°. It is about 1 to 2 ms^{-1} higher for the other two aliases. It might be possible to eliminate two of these aliases just by looking at the difference between the values computed from the paired measurements and the value computed from the third antenna at these directions using vertical polarization only. This would be one way to eliminate the alias near 360°. Again however, the difference between horizontal and vertical polarization produces curves for horizontal polarization near 180° that are distinctly different from the 45° beam and the 65° beam, as shown by the dashed lines on this figure.

Discrimination against these two aliases near 180^{o} must therefore depend on differences in the model function between upwind and downwind for vertical and horizontal polarization.

At 20^{o}, as shown by the last set of curves in Figure 8A with the same interpretation, the third beam at 65^{o} for vertical polarization can almost surely eliminate two of the three aliases. The curve for horizontal polarization can then be used to eliminate the third, under the assumption that the sampling variability effects do not obliterate these differences.

The top part of Figure 8B, for a wind direction of 30^{o}, shows the full curves for the vertical polarization measurements. The beam at 65^{o} now produces a very large difference for the wind speed at the directions of two of the aliases for the paired for aft beam, and surely elimintes these two aliases. The curve for the third beam at 65^{o} still comes close to the third alias, and, thus, because of sampling variability effects cannot be considered to eliminate it. The horizontally polarized measurements again can be used as a discriminator for this third alias.

For 40^{o} and 50^{o} in Figure 8B, the third vertical polarization measurement at 65^{o} clearly eliminates two of the three possible aliases. The horizontally polarized measurements do not help very much in eliminating the third alias. For conditions similar to this, it may be only possible to pick two wind directions out of a possible four from the SCATT measurements. If any discrimination is to be made in an attempt to obtain the true solution, this information will have to come from the differences between the horizontally polarized and vertically polarized measurements with reference to the variations in the shapes of these curves.

Figures 8C through 8H show more of the same. If the true wind direction is sufficiently different from the directions, 0°, 90°, 180° or 270° relative to the pointing direction of the lead beam, then the third measurement with the new antenna for vertical polarization will make it possible to eliminate two of the three aliases. The remaining two solutions for vertically polarized pairs of measurements 180°, or so, apart will then be such that one of them will be the correct measurement. These two values will be roughly, but not exactly 180° apart. Differences between the vertically polarized curves and the horizontally polarized curves can then provide the possibility, in the absence of strong sampling variability, to eliminate the third alias. This will be more or less successful depending upon the values of K_p that actually result from the design of the SCATT.

As the true wind directions approach 0°, 90°, 180° or 270° the curves that are graphed take on a different form. The ability of the third beam at 65° for vertical polarization to eliminate two of the aliases decreases. However, the difference between the vertical and horizontal polarization then become large for two of the possible aliases and this will, in general, allow them to be eliminated. The remaining uncertainty will be for wind speeds and direction that are just a few degrees away from the true wind speed and direction as contrasted to the case for wind directions near 45°, 135°, 225°, and 315°.

To show the full details of the information contained in all six measurements the top set of graphs in Figure 8G shows all six curves. The essential information that is needed to interpret the SCATT measurements, is nevertheless contained in the abbreviated version of these graphs.

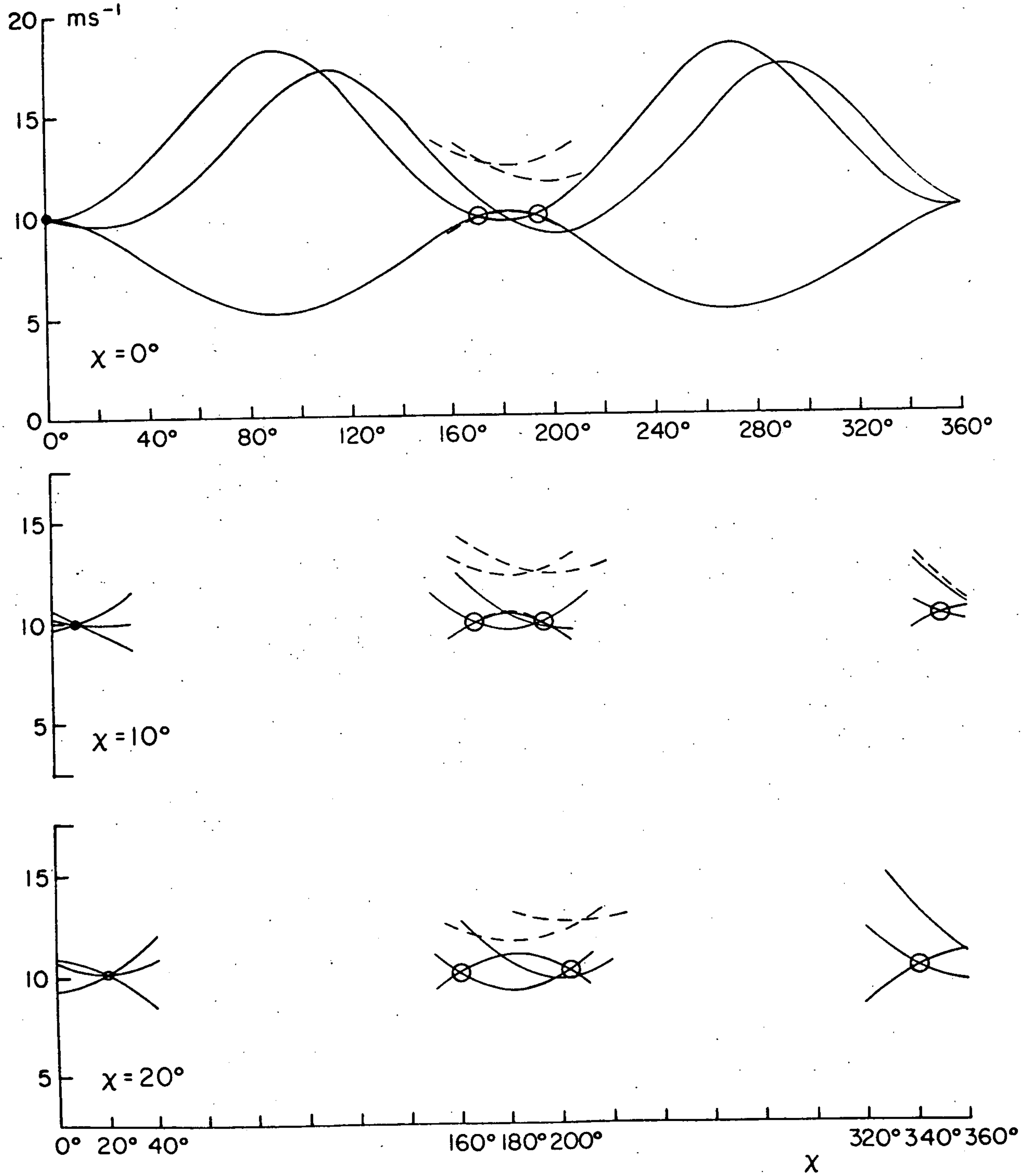


FIG. 8A EXAMPLES OF NOISE FREE SOLUTIONS FOR SIX CO-LOCATED BACKSCATTER MEASUREMENTS, THREE VERTICAL POLARIZATIONS (SOLID) AND THREE HORIZONTAL POLARIZATIONS (DASHED) FOR BEAMS 20° AND 90° APART, AN INCIDENCE ANGLE OF 25°, AND FOR A WIND SPEED OF 10 ms^{-1} WITH DIRECTIONS OF 0°, 10°, AND 20°. (SEE TEXT.)

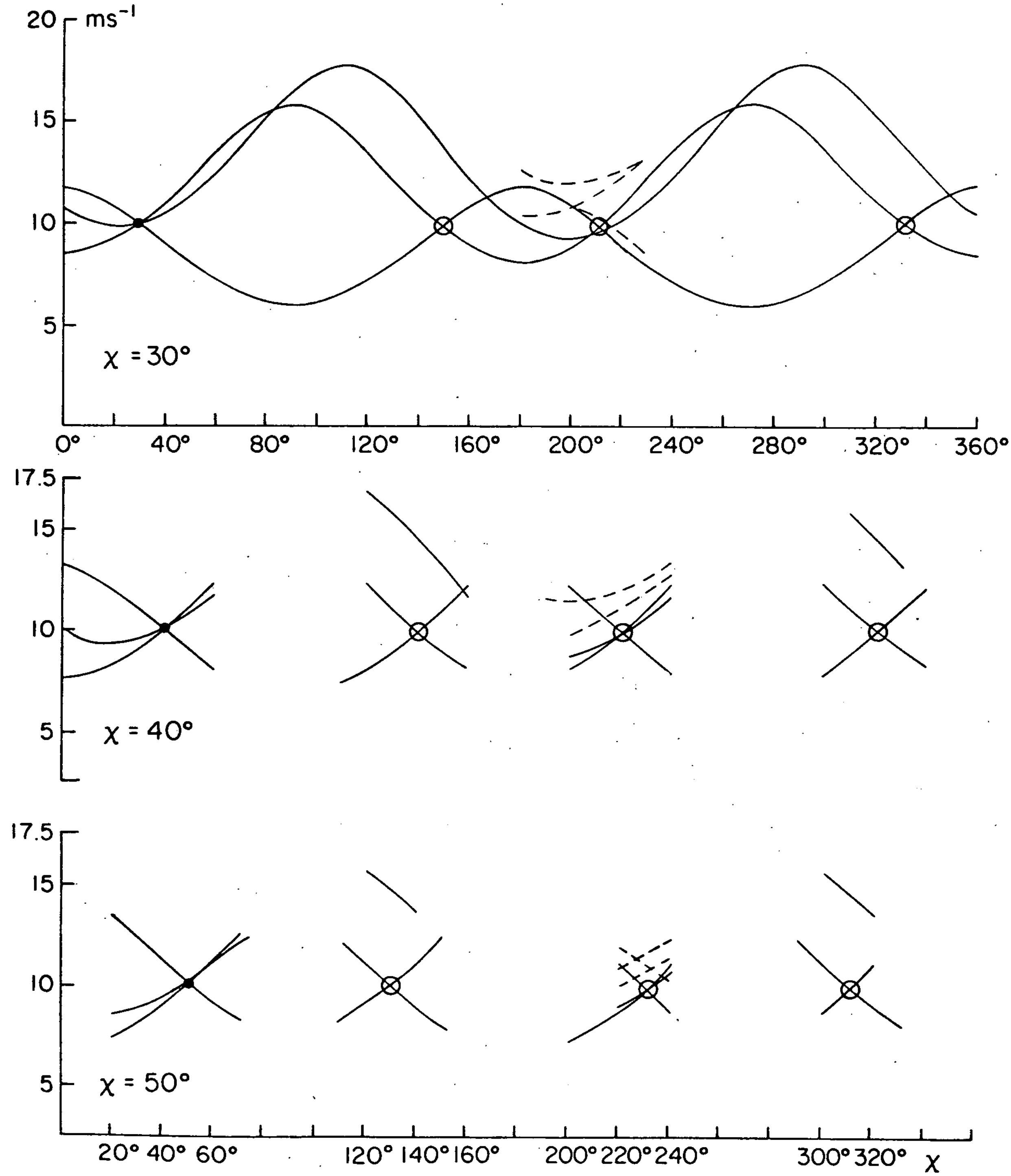


FIG. 8B SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 30°, 40°, AND 50°.

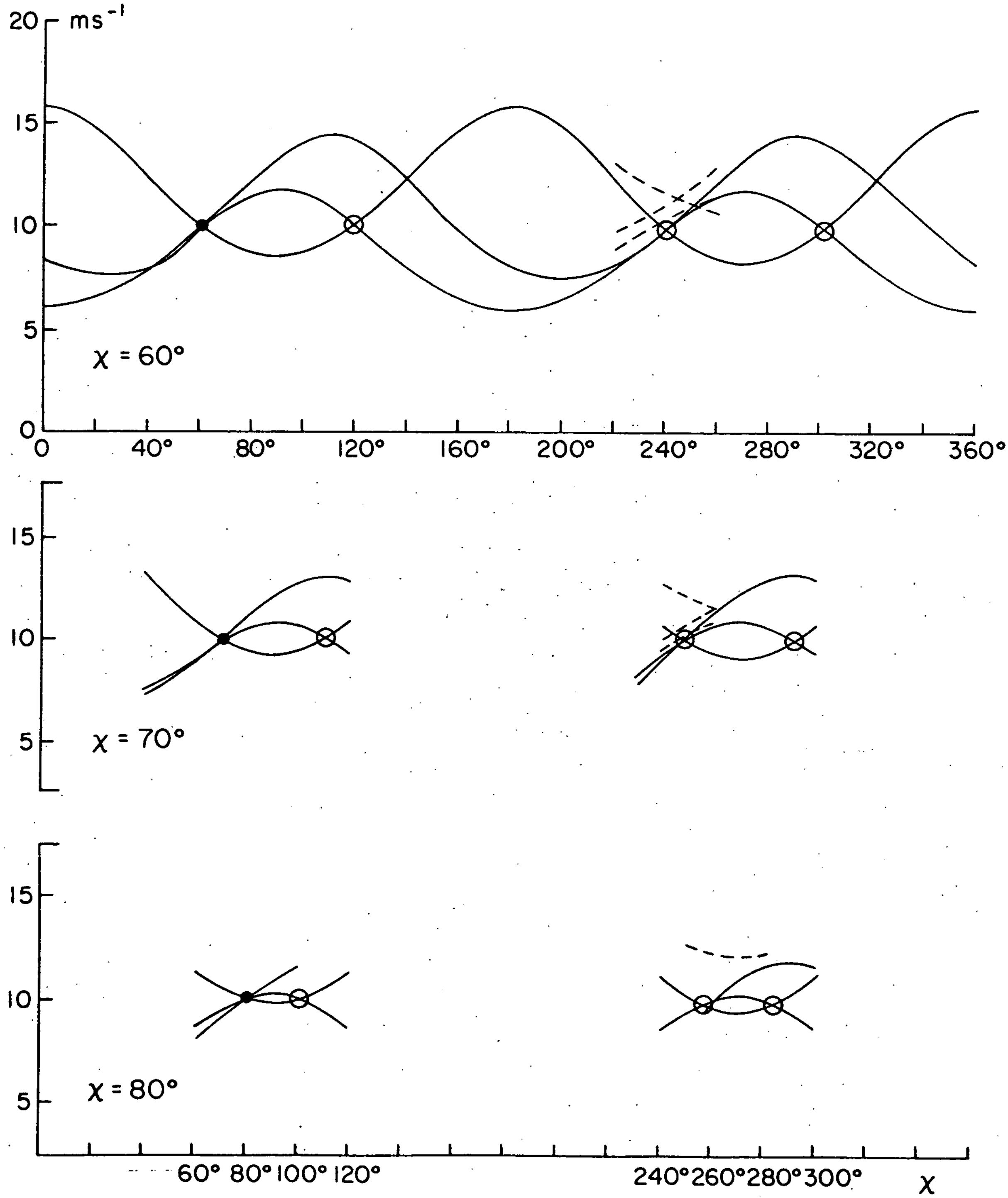


FIG. 8C SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 60°, 70°, AND 80°.

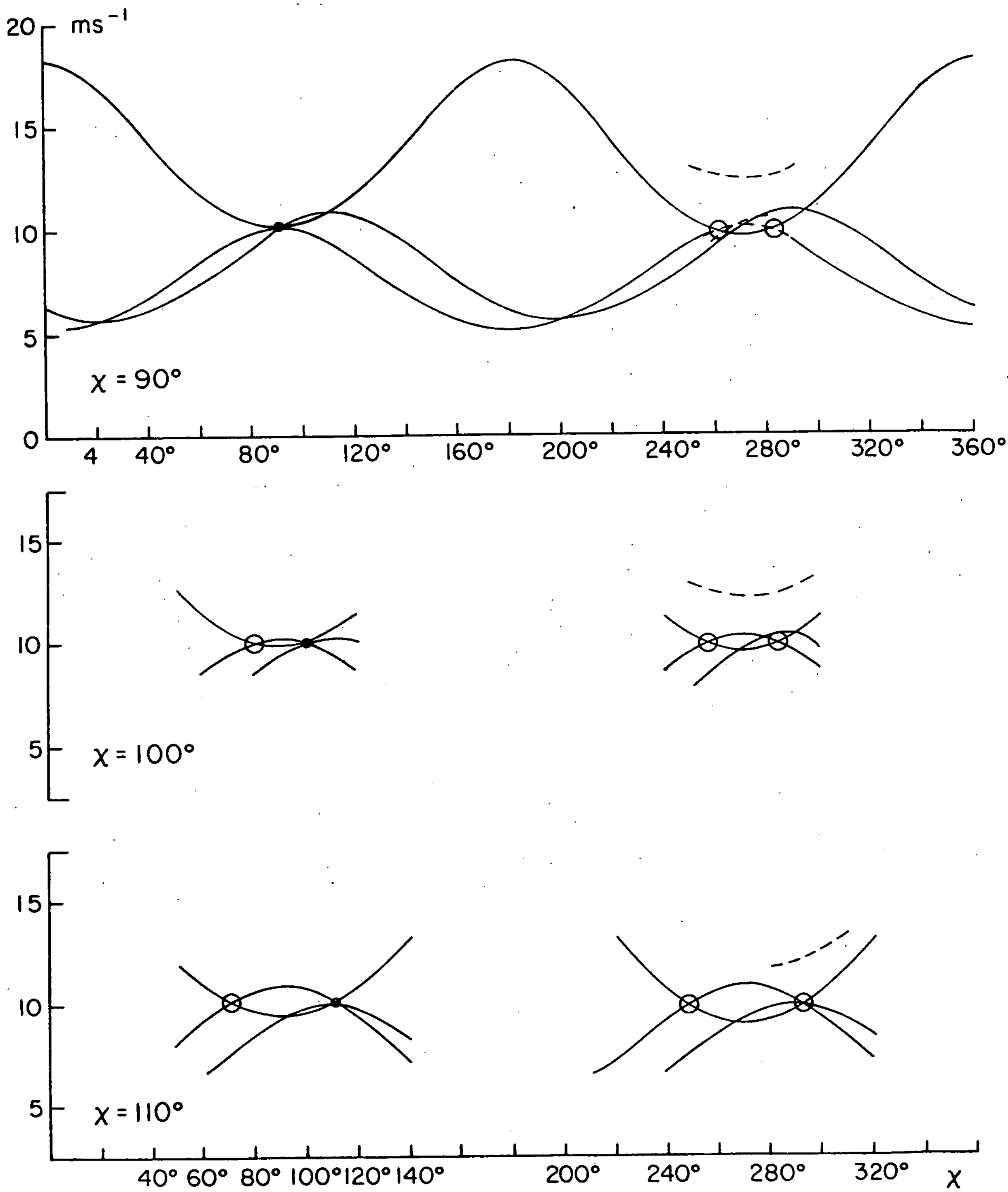


FIG. 8D SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 90°, 100°, AND 110°.

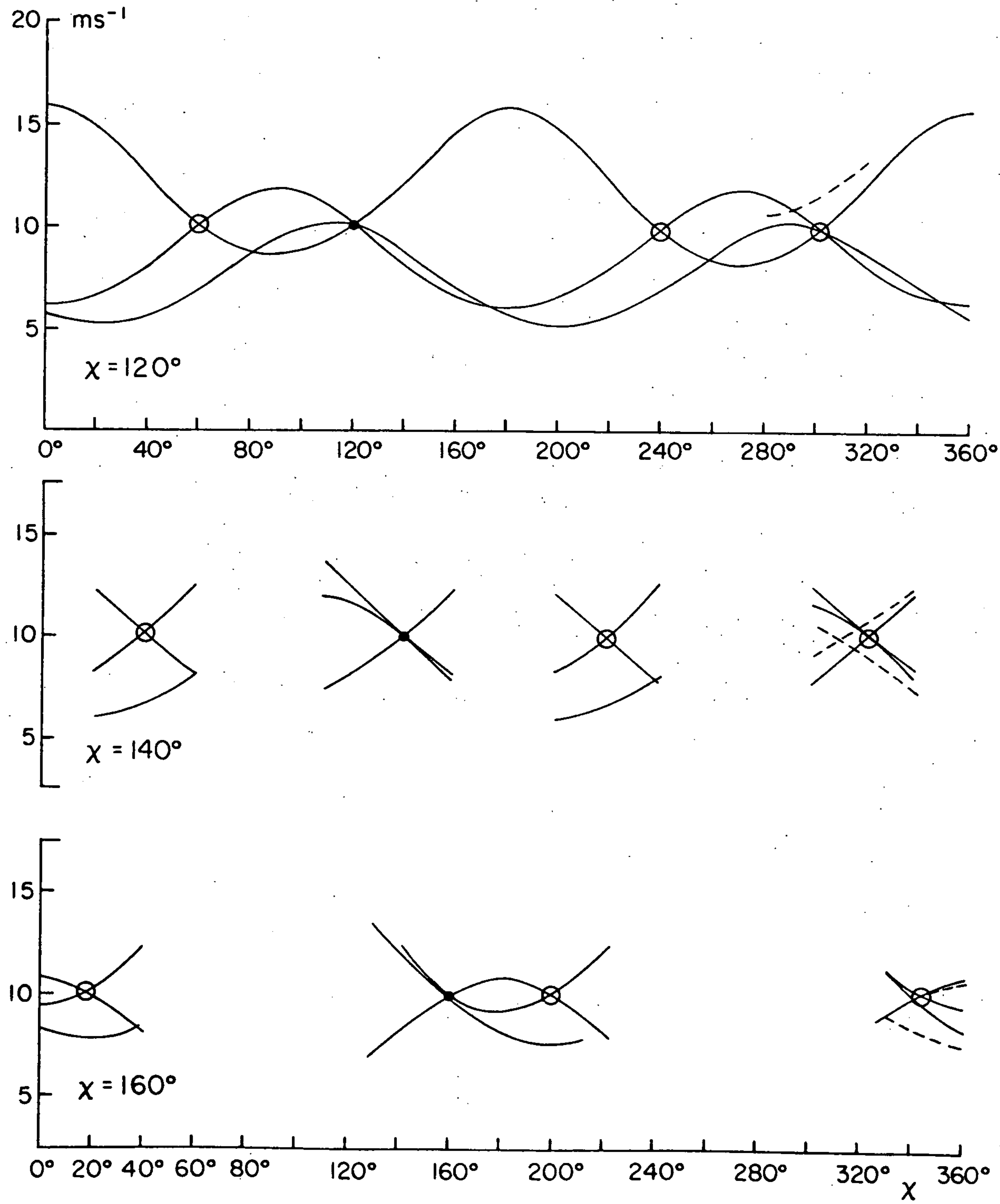


FIG. 8E SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 120°, 140°, AND 160°.

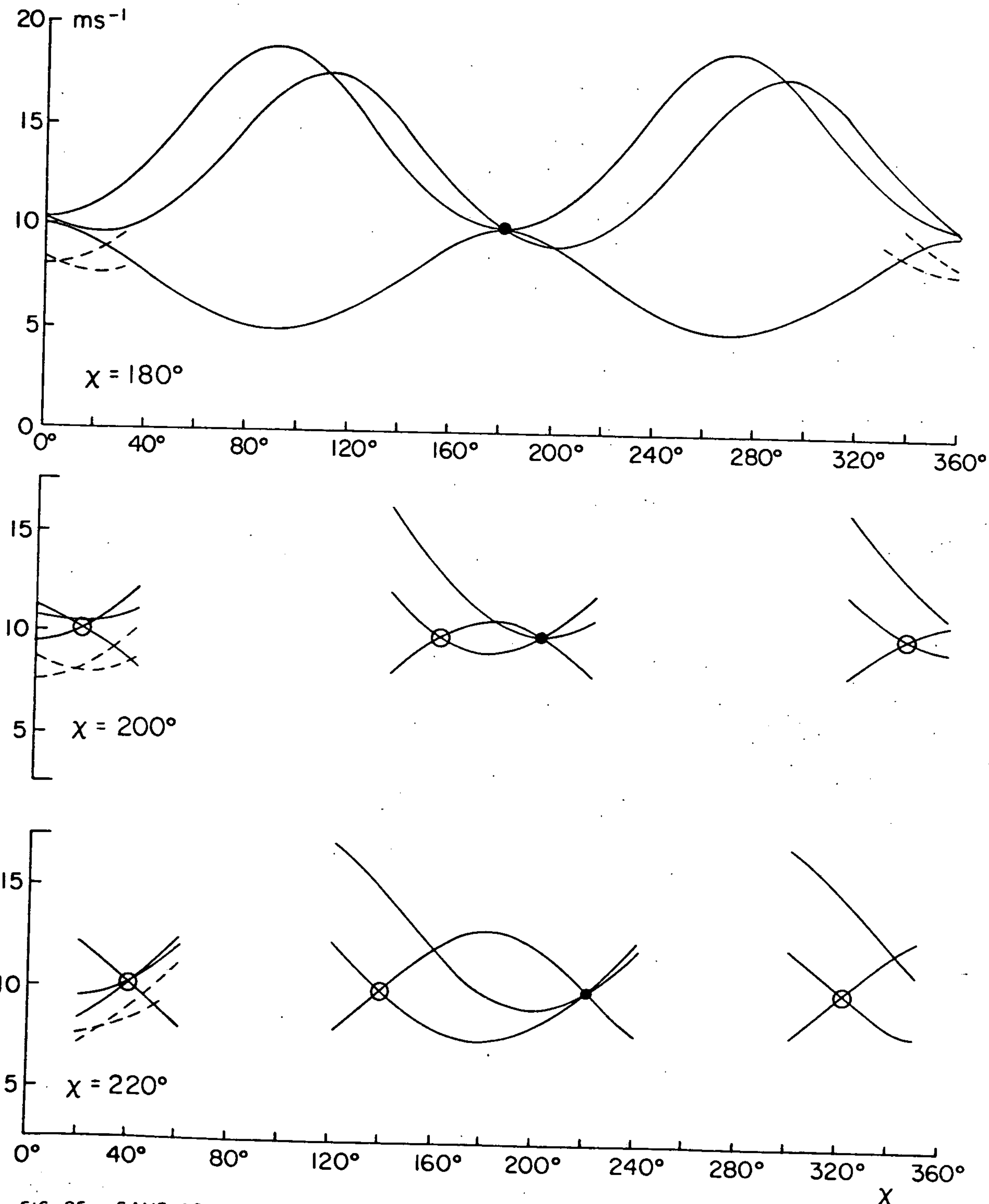


FIG. 8F SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 180°, 200°, AND 220°.

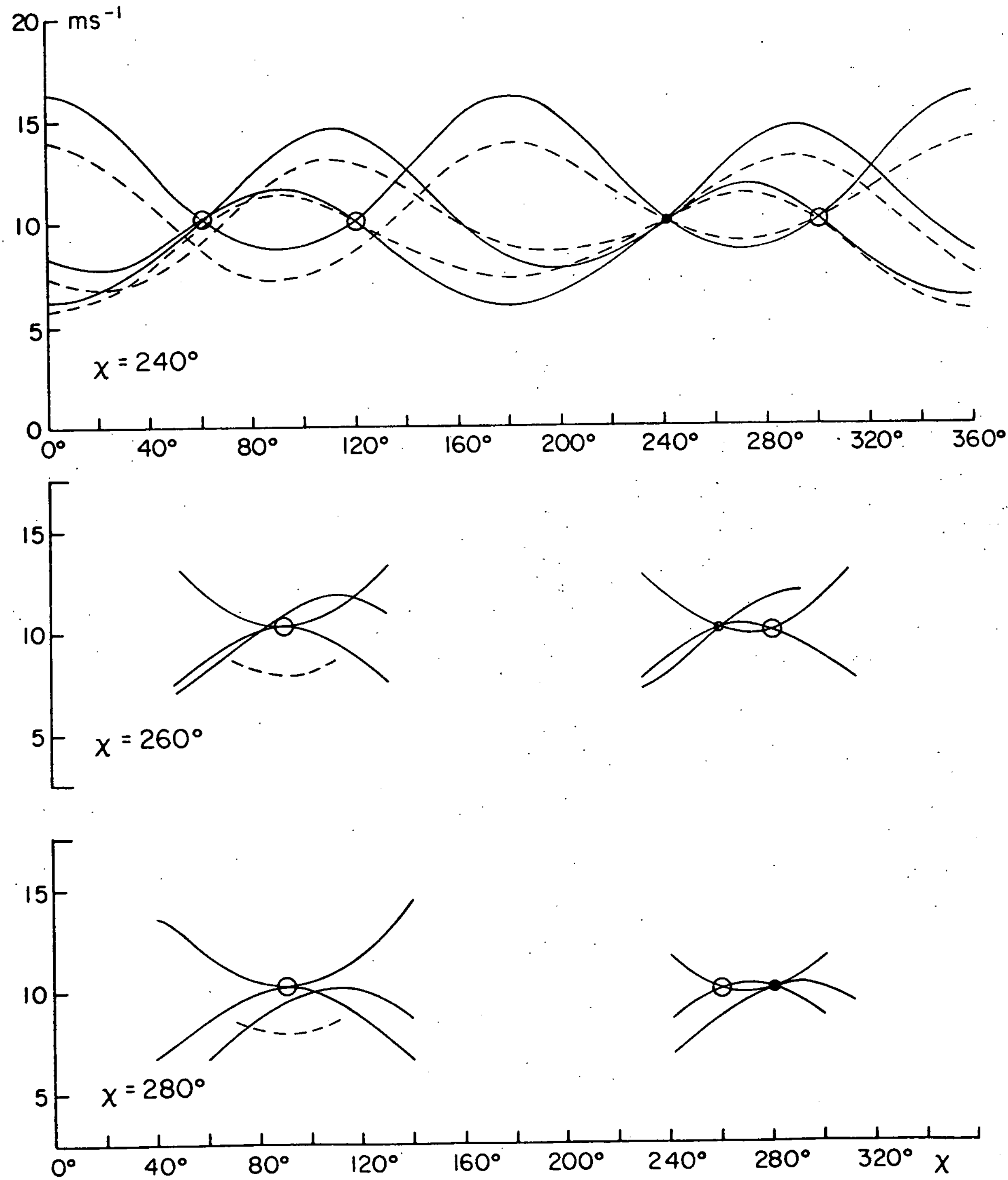


FIG. 8G SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 240°, 260°, AND 280°.

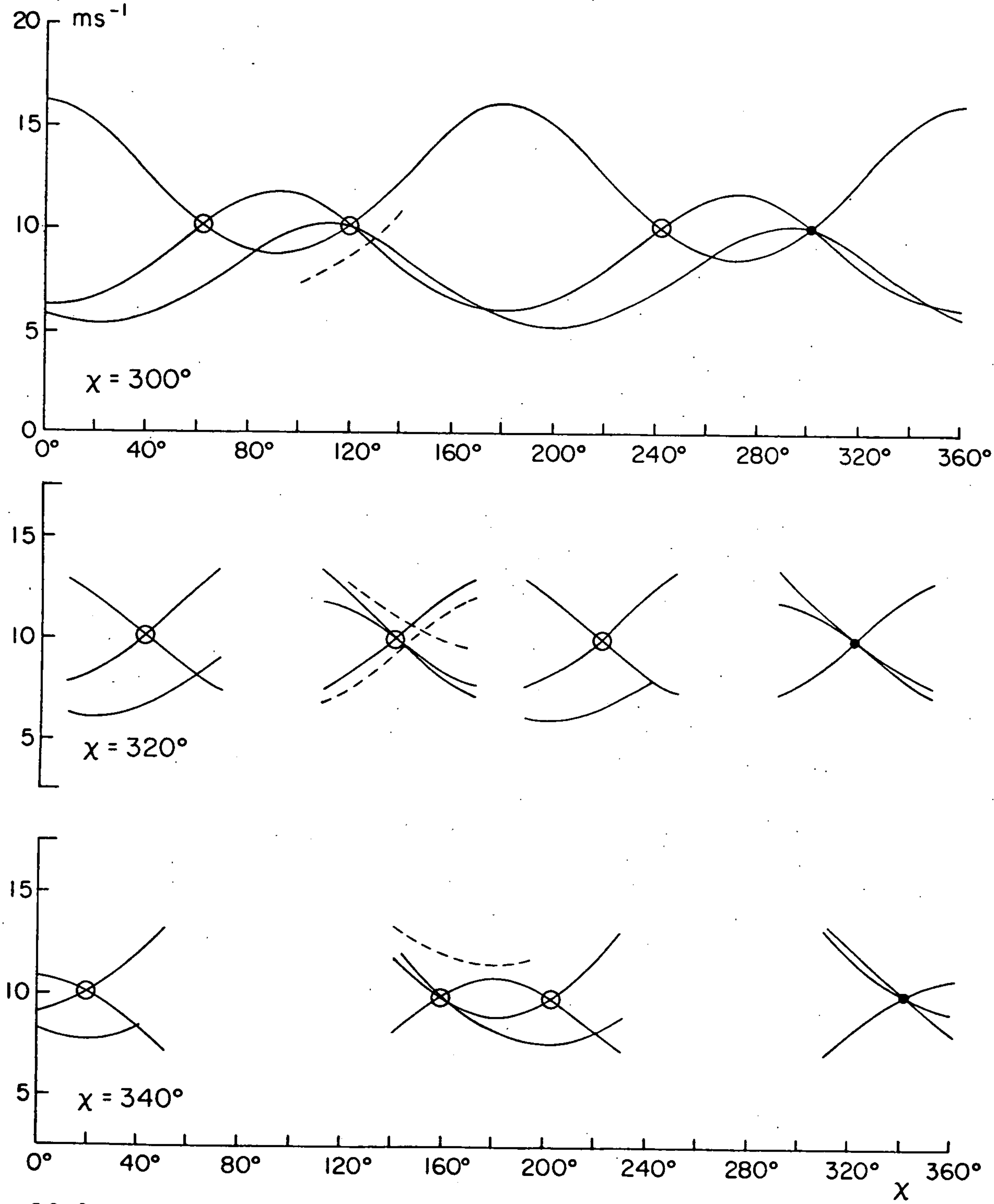


FIG. 8H SAME AS 8A EXCEPT FOR WIND DIRECTIONS OF 300°, 320°, AND 340°.

For higher incidence angles, upwind backscatter is stronger than downwind backscatter for low wind and both polarizations. whereas for high winds, downwind backscatter is higher for V pol than upwind backscatter. At 40° incidence angle the changeover occurs between $15\ ms^{-1}$ and $20\ ms^{-1}$. The changes in the graphs of wind speed versus aspect angle as the wind speed increases for the higher incidence angles provides a way to eliminate some of the aliases on the basis of the differences between vertical and horizontal polarization.

Figure 9A through 9E show the noise free graphs, or appropriate portions thereof, for an incidence angle of 40°, wind directions of 0, 180°, 40° and 220° and wind speeds of 5, 10, 15 and $20\ ms^{-1}$.

Figure 9A shows the curves for $5\ ms^{-1}$. There will be difficulty in eliminating any aliases near upwind for either the 0° or 90° beam. Near 45°, 135°, 225° and 315°, it may be possible to eliminate two of the three aliases.

For $10\ ms^{-1}$ in Figure 9B, it may be possible to determine upwind for either beam. However, the two aliases near the downwind case may not be removable consistantly. For angles near 45°, and so on, two of the aliases can probably be removed but one may remain 180° away from the correct direction.

For the higher winds, as shown in Figures 9C, 9D, and 9E, the differences between vertical and horizontal polarization seem to provide information that will frequently result in the elimination of that alias that is 180° away from the true wind and the third antenna will easily eliminate the other two.

Inferences based on the above

In the following sections of this report, an algorithm will be developed and tested to determine how many of the aliases can be removed by means of the six measurements made by the SCATT.

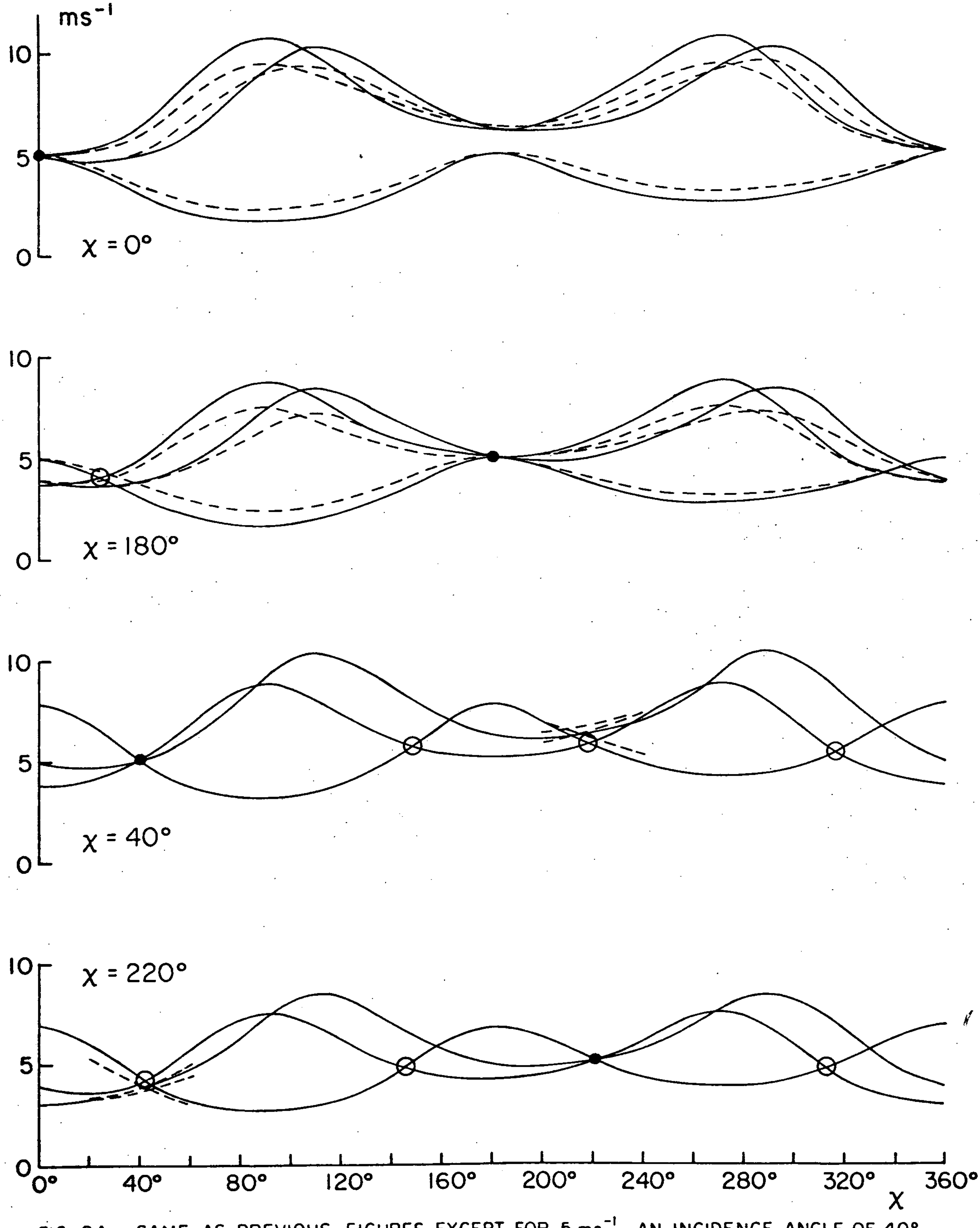


FIG. 9A SAME AS PREVIOUS FIGURES EXCEPT FOR 5 ms^{-1}, AN INCIDENCE ANGLE OF 40° AND WIND DIRECTIONS OF 0°, 180°, 40° AND 220°.

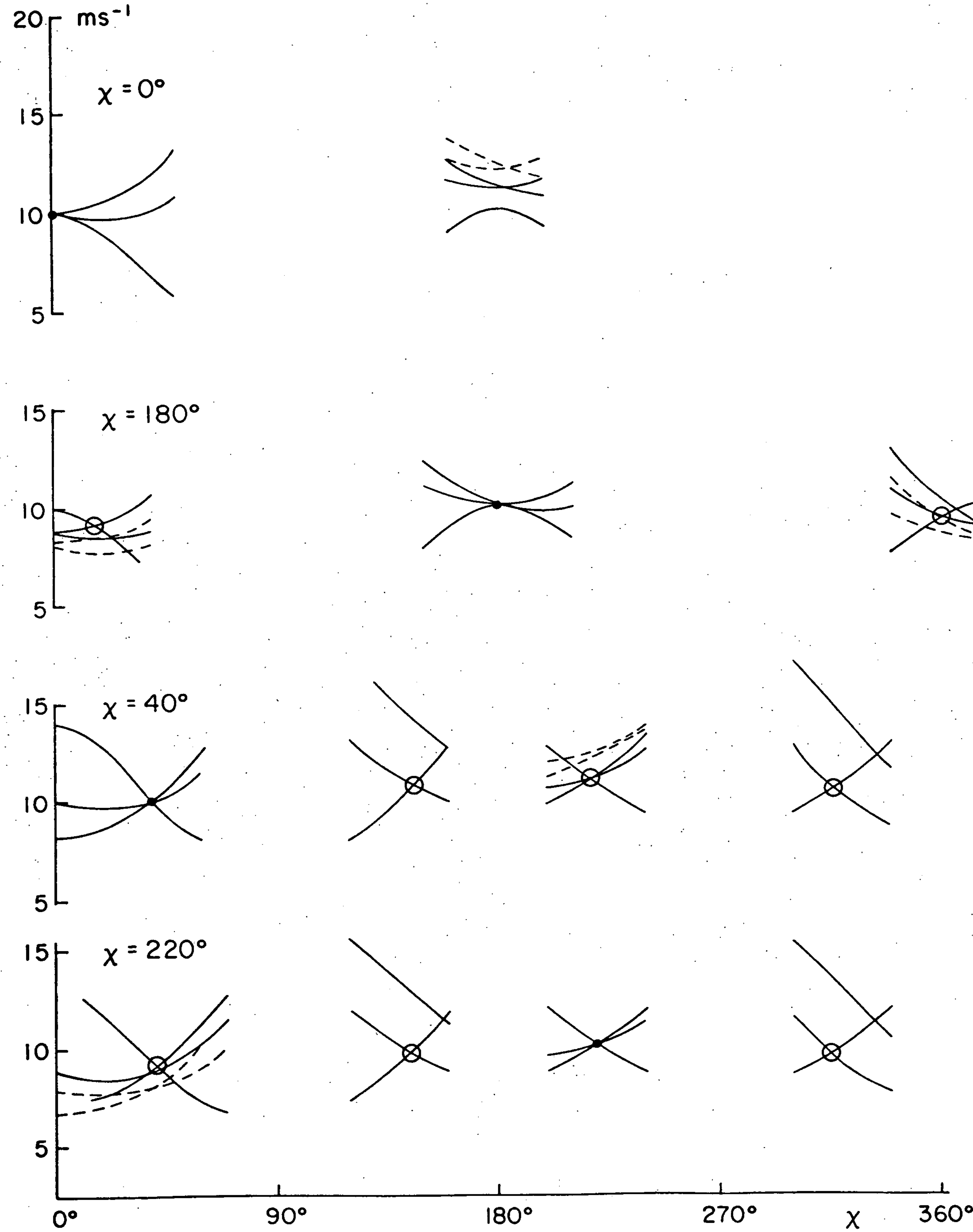


FIG. 9B SAME AS FIG. 9A EXCEPT FOR 10 MS^{-1}.

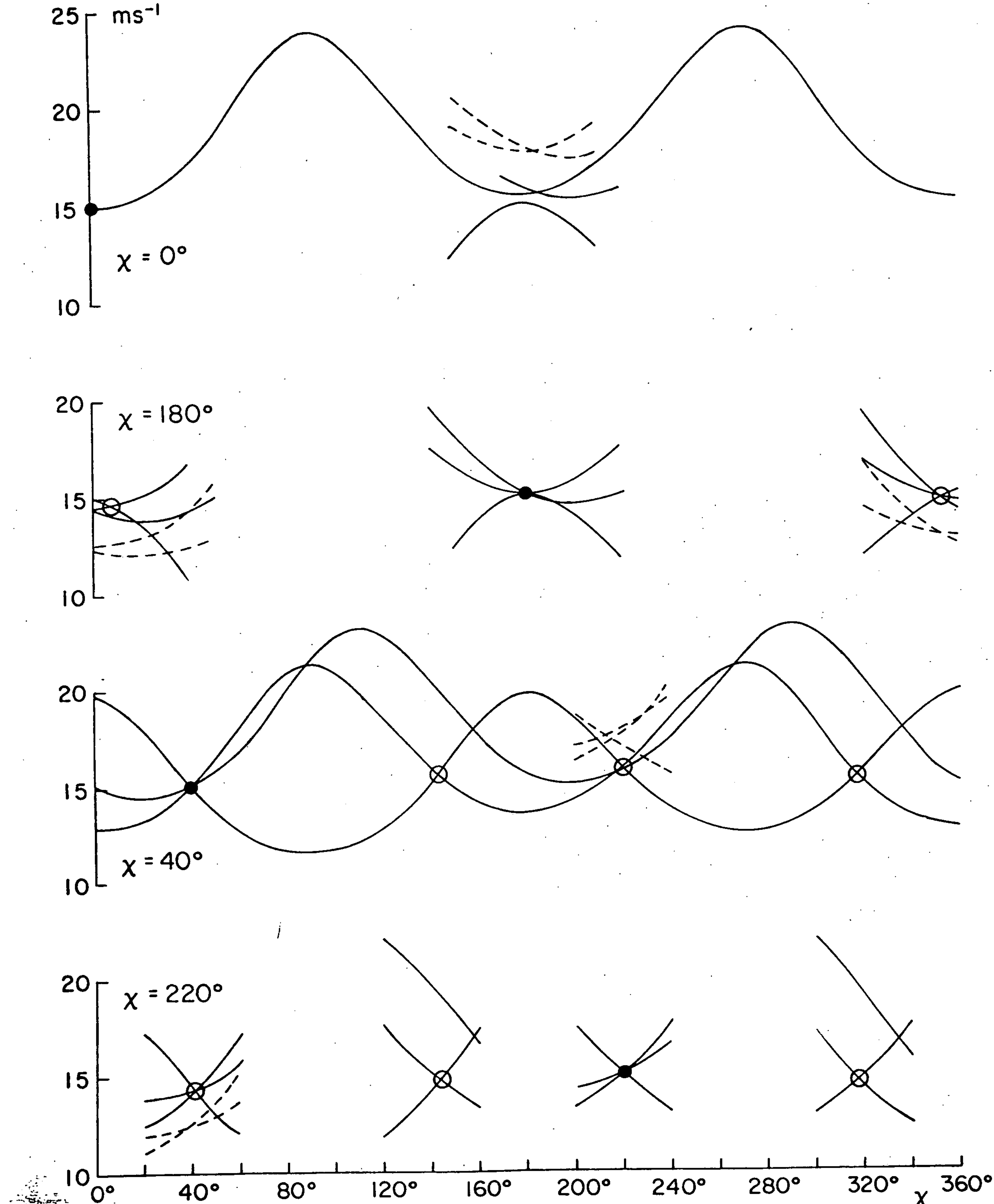


FIG. 9C SAME AS FIG. 9A EXCEPT FOR 15 ms^{-1}.

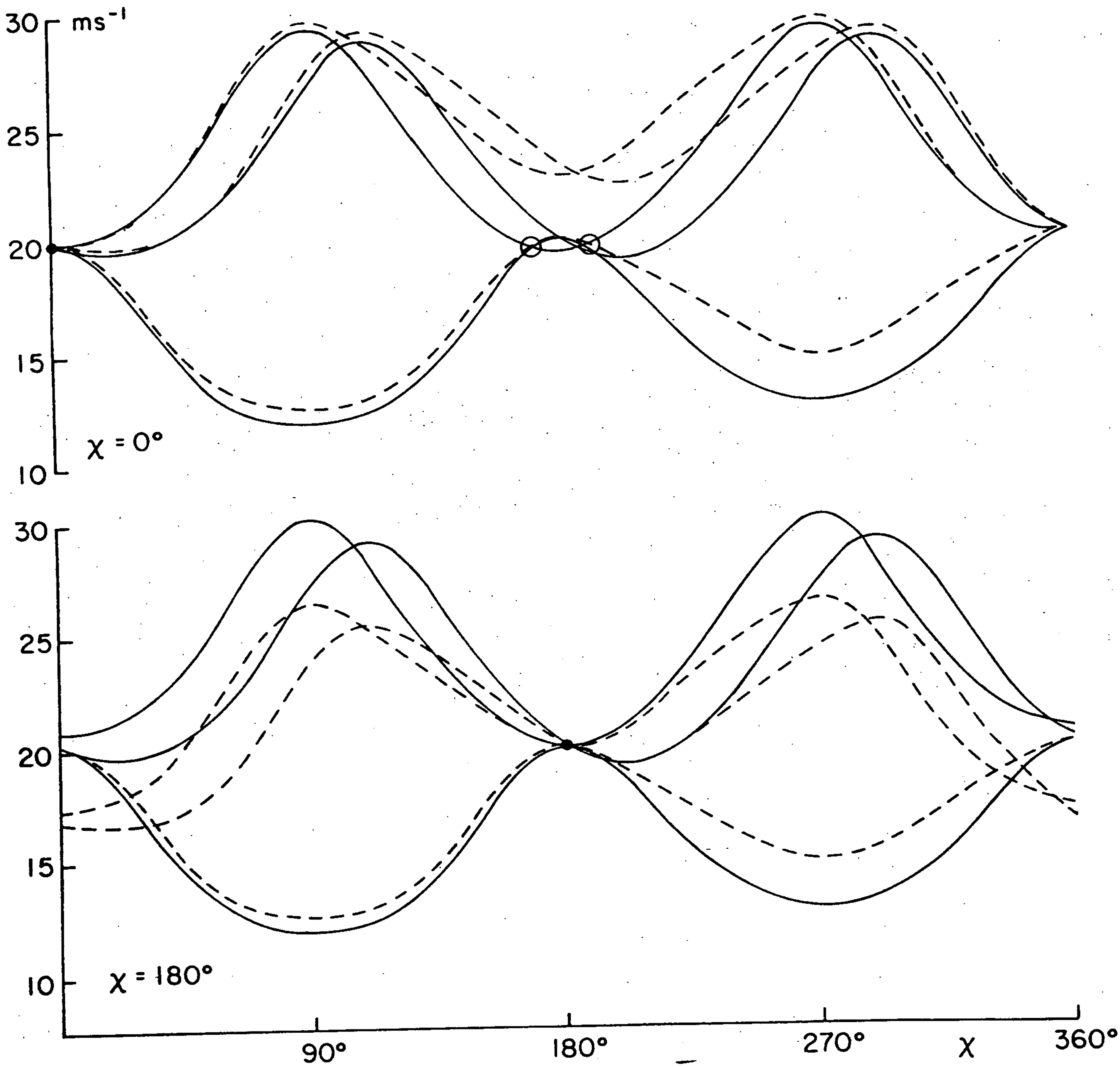


FIG. 9D SAME AS FIG. 9A EXCEPT 20 MS^{-1} AND χ =0° AND 180°.

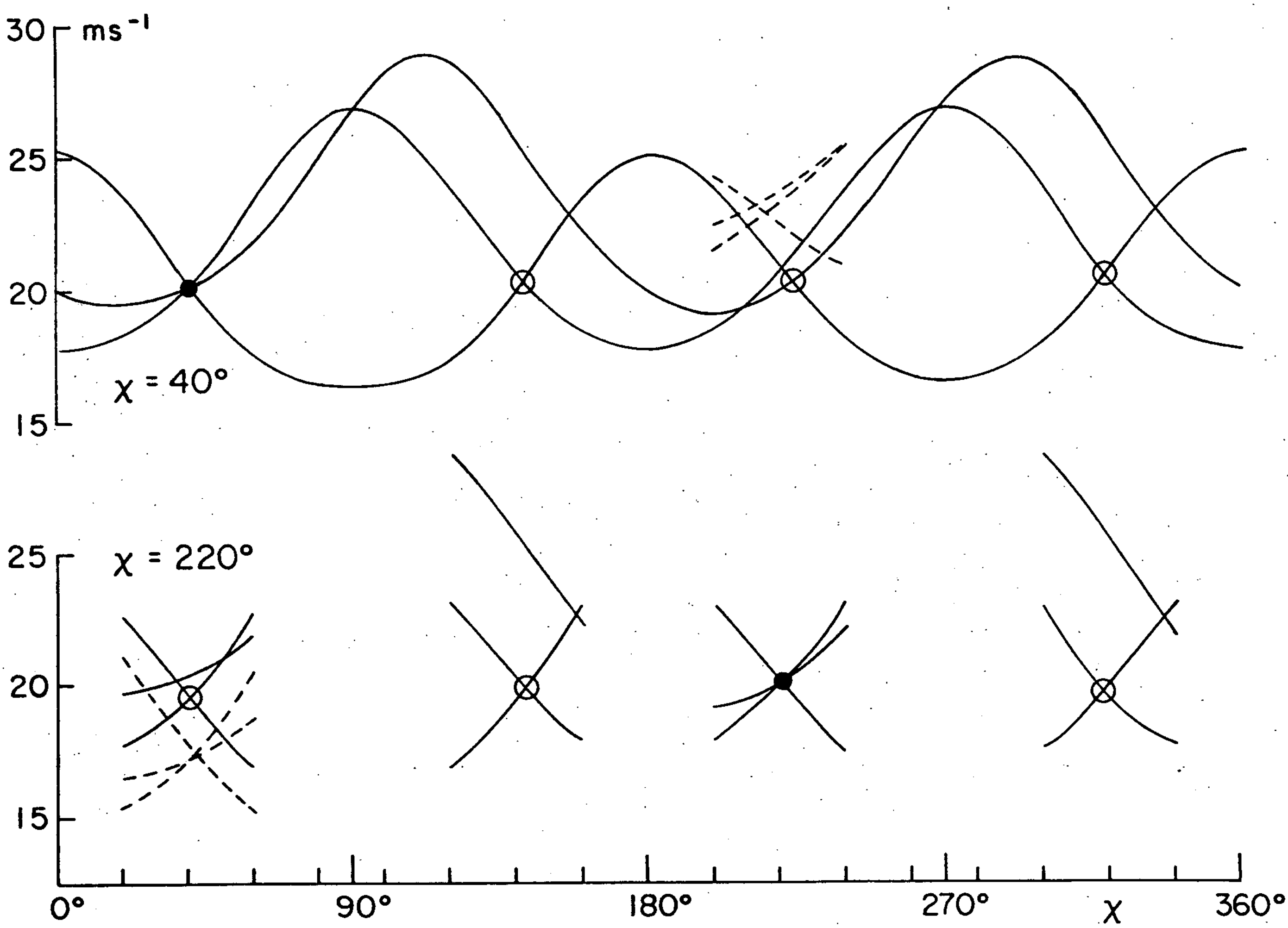


FIG. 9E SAME AS FIG. 9A EXCEPT 20 ms^{-1} AND χ = 40° AND 220°.

It will be tested by a Monte Carlo technique that will randomize the measurements based upon the equations of a preceeding section. Almost by inspection of Figure 1 it should be possible to predict when such a procedure will be successful.

It will nearly always eliminate two of the three aliases in four solution cases for high enough winds. The remaining two solutions will be nearly 180° apart.* This will greatly simplify the problem of using meteorological information to remove the third alias. The removal of the third alias solely on the basis of the data provided by the SCATT depends on there being a real difference between the curves for horizontal and vertical polarization at upwind and downwind. This real difference would exist, if Figure 1 is correct, for high winds because a backscatter value for upwind will be lower than that for downwind for vertical polarization and conversely, for high winds for horizontal polarization the upwind values are stronger than the downwind. These differences should be sufficiently large to permit the discrimination of the third alias if the signal to noise ratios are high enough.

For low winds and low incidence angles there also appears to be a difference between upwind and downwind for vertical and horizontal polarization. For vertical polarization the upwind and downwind values are nearly equal. The downwind H-pol are considerably smaller than the upwind values. If the signal to noise ratio is high enough at low incidence angles for light winds this should also permit the elimination of the third alias. For high incidence angles and light wind, the prediction can be hazarded that the techniques will not be successful because there is no strong difference between upwind and downwind for vertical and horizontal polarization. This remains to be tested by the Monte Carlo techniques. At high incidence angles and light winds the signal-to-noise ratio for horizontal polarization may be so low that these values will not have the ability to eliminate any aliases.

Ths Monte Carlo study that will be carried out, is done under the assumption that the model function is correct. The preceeding material on the JASIN experiment indicates that the

*Most of the time.

model function is becoming quite realistic. Nevertheless prudence dictates that continued efforts to improve upon the model function with the presently available data be made, and that plans to update the model function once more when the final design of the SCATT is completed and the instrument becomes part of an orbiting spacecraft.

AN ALGORITHM FOR THE SCATT

<u>Definitions</u> With beam 1 at 45^{o} to the direction of spacecraft travel and all wind directions referred to zero degrees relative to the pointing direction of beam 1, the direction of beam 2 on the right side will be at 65^{o}, or 20^{o} relative to beam 1, and beam 3 will be at 135^{o}, or 90^{o} relative to beam 1. Also with a 1 for H pol and 2 for V pol, the six measurements (estimates) obtained by the SCATT can be labeled as $\hat{\sigma}^{o}_{11}$, $\hat{\sigma}^{o}_{12}$, $\hat{\sigma}^{o}_{13}$, for beams 1, 2 and 3 H pol and as $\hat{\sigma}^{o}_{21}$, $\hat{\sigma}^{o}_{22}$ and $\hat{\sigma}^{o}_{23}$ for beams 1, 2, and 3 V pol.

For a known wind speed and direction at the location on the sea surface where the SCATT measurements were made, the theoretical values for the measured backscatter, where V_T and χ_T are the true wind speed and direction, are given by Equation (48) where the σ values are in decibels

$$\sigma^{o}_{11} = G(\chi_T, \theta_1) + H(\chi_T, \theta_1) \log_{10} V_T$$

$$\sigma^{o}_{21} = G(\chi_T, \theta_1) + H(\chi_T, \theta_1) \log_{10} V_T$$

$$\sigma^{o}_{12} = G(\chi_T - 20^{o}, \theta_2) + H(\chi_T - 20^{o}, \theta_2) \log_{10} V_T$$

$$\sigma^{o}_{22} = G(\chi_T - 20^{o}, \theta_2) + H(\chi_T - 20^{o}, \theta_2) \log_{10} V_T$$

$$\sigma^{o}_{13} = G(\chi_T - 90^{o}, \theta_1) + H(\chi_T - 90^{o}, \theta_1) \log_{10} V_T$$

$$\sigma^{o}_{23} = G(\chi_1 - 90^{o}, \theta_1) + H(\chi_T - 90^{o}, \theta_1) \log_{10} V_T \tag{48}$$

These six backscatter values in the actual situation are unknown. They are calculated from the appropriate model function for the appropriate incidence angles, which are (for simulation purposes) equal for beams 1 and 3 and slightly smaller for beam 2.

Associated with each σ^o_{ij} for each of the cells of the SCATT, the parameters that enter into the calculation of the variance of the measurement are known so that in antilog form, the values of $\hat{\sigma}^o_{ij}$ can be calculated from Equation (29) as in (49).

$$\hat{\sigma}^o_{ij} = \sigma^o_{ij} + t(\mathrm{VAR}\ \hat{\sigma}^o_{ij})^{\frac{1}{2}} \tag{49}$$

It is important to note that VAR $\hat{\sigma}^o_{ij}$ is calculated from σ^o_{ij}, not $\hat{\sigma}^o_{ij}$. The values of t (there are 6 different values for each simulation) are independent zero mean unit variance normally distributed random variables.

For a given Monte Carlo simulation and for the actual conditions, the knowns are the $\hat{\sigma}^o_{ij}$ and the unknowns are the σ^o_{ij} and the t_{ij} that perturbed the measured (estimated) values away from the true values.

A Quick Search for the Candidate True Wind Speed and Direction

The information available consists of six numbers, $\hat{\sigma}^o_{ij}$, and the incidence angles. To search the entire $V - \chi$ plane with an extension of either (41a) or (41b) or perhaps even a correct likelihood function, seems a rather inefficient way to start. Instead, let us use

$$\hat{\sigma}^o_{21} \text{ and } \hat{\sigma}^o_{23} \quad \text{(in db)}$$

to compute the wind speeds and directions that would be possible for this pair of measurements using the ideas expressed in the discussion of Figure 4.

There will be either four solutions, two solutions, or no solutions where the curves similar to those in Figure 4 cross four times, two times or not at all. If there are two solutions they will be close together. A third solution needs to be found about 180^o away from the average of the two solutions because of sampling variability effects. If there are no formal solutions because of sampling variability effects, two need to be found 180^o apart. The techniques described by Pierson and Salfi (1978) can be used. The three solution case as in Fig. 8A for zero degrees has zero chance of happening as soon as sampling variability enters.

If there are four solutions, they will be of the form

$$V_1, \chi_1; V_2, \chi_2; V_3, \chi_3; V_4, \chi_4$$

where

$$0 < \chi_1 < 90^o$$

$$90^o < \chi_2 < 180^o$$

$$180^o < \chi_3 < 270^o$$

$$270^o < \chi_4 < 360^o$$

The other cases occur, for example, as χ_1 approaches either zero or 90^o. As χ_1 approaches zero, χ_4 approaches 360^o. As χ_1 approaches 90^o, χ_2 approaches 90^o. Similarly as χ_3 approaches 180^o, χ_2 approaches 180^o, and as χ_3 approaches 270^o, χ_4 approaches 270^o.

The various special cases, other than the most frequent four solution case, are given below where the limiting angle is repeated for each quadrant. The "Y" type solutions are

$$0^o, \chi_2, \chi_3, 0^o$$

$$90^o, 90^o, \chi_3, \chi_4$$

$$\chi_1, 180^o, 180^o, \chi_4$$

$$\chi_1, \chi_2, 270^o, 270^o$$

and the 180° apart solutions are

$$0^{\circ},\ 180^{\circ},\ 180^{\circ},\ 0^{\circ}$$

$$90^{\circ},\ 90^{\circ},\ 270^{\circ},\ 270^{\circ}.$$

If there are four solutions, the next step is to compute the wind speed for beam 2, V pol from

$$\log_{10} V_{22k} = A(\chi - 20^{\circ}, \theta_2) + B(\chi - 20^{\circ}, \theta_2)\, \sigma^{o}_{22}$$
$$\text{for } \chi_1, \chi_2, \chi_3, \chi_4. \tag{50}$$

The differences $|V_{2,\,13,\,i} - V_{2,2i}|$ for i equal to 1 to 4 tell how far the curve for beam 2, V pol, is from the wind speed for the four solution V pol case. The largest two of the absolute values can be discarded as yielding not very probable solutions.

Typically the two solutions that survive will be nearly 180° apart so that the two survivors will usually be either

$$V_1, \chi_1 \text{ and } V_3, \chi_3$$

or

$$V_2, \chi_2 \text{ and } V_4, \chi_4$$

The other possibilities could also occur and are carried forward.

For the two surviving wind directions, the next step is to calculate the wind speeds at either χ_1 and χ_3 or χ_2 and χ_4 (or whatever) for each H pol measurement.

$$\log_{10} V_{11k} = A(\chi, \theta_1) + B(\chi, \theta_1)\, \hat{\sigma}^{o}_{11}$$

$$\log_{10} V_{12k} = A(\chi - 20^{\circ}, \theta_2) + B(\chi - 20^{\circ}, \theta_2)\, \hat{\sigma}^{o}_{12}$$

$$\log_{10} V_{13k} = A(\chi - 90^{\circ}, \theta_1) + B(\chi - 90^{\circ}, \theta_1)\, \hat{\sigma}^{o}_{13} \tag{51}$$

The six differences in sets of three of the form

$$\Delta V_k = |V_{2,13k} - V_{ijk}| \tag{52}$$

for k equals 1 or 3, or 2 or 4 (or whatever) are such that one of each three will be close to the V pol wind. This third ΔV_k is discarded. Of the two remaining values at the two candidate directions, either one set of two will be clearly larger than the other set of two or the results will be indecisive. A decision can be made on the basis of the results of (52) such that either one or the other of the four V pol solutions emerges as the winner or no superior solution of the two possibilities emerges.

The chosen V pol solution or the two solutions usually 180^o apart can be then used in further calculations to obtain an improved result.

If there are three solutions for the V pol zero and 90^o beams, Figures 8 and 9 show that the 20^o beam for V pol provides little discrimination among the three solutions. For the three possible directions, χ_1, χ_2, χ_3, and each H pol measurement, the three wind speeds can be found.

For each of three sets of three solutions, one absolute value of the speed differences will be a minimum to be discarded. The differences between the V pol solutions and the two corresponding H pol solutions can then be compared. If the vertical leg of the "Y" type solution has minimum absolute differences the vertical leg is the candidate solution, if not one or the other of the "V" part of the "Y" may have a clear absolute minimum; in which case it is the candidate. Otherwise both are considered.

If there are only two solutions for the V pol zero and 90^o beams, the three H pol solutions can be used, and usually one or the other candidate V pol solutions will be selected. If not, both are candidates.

A TRUE MAXIMUM LIKELIHOOD ESTIMATE OF V AND χ

For a given SCATT measurement, the six measurements (estimates) $\hat{\sigma}^o_{ij}$ as defined above are available. Also there are six unknown values of σ^o_{ij} as would be generated by Equation (48) for some unknown true wind speed, V_T, and wind direction, χ_T. The likelihood function is the product of the probability density functions for each of the random variables $\hat{\sigma}^o_{ij}$ as in Equation (53), where the backscatter is in antilog form

$$L(\sigma^o_{ij};\hat{\sigma}^o_{ij}) = \prod_{ij} \frac{1}{(2\pi)^3} \frac{\exp(-\frac{1}{2}(\hat{\sigma}^o_{ij}-\sigma^o_{ij})^2 / \mathrm{VAR}\ \hat{\sigma}^o_{ij})}{(\mathrm{VAR}\ \hat{\sigma}^o_{ij})^{\frac{1}{2}}} \tag{53}$$

and where

$$\mathrm{VAR}\ \hat{\sigma}^o_{ij}2 = (A + \lambda)\sigma^o_{ij}2 + 2\lambda\ NR\,\sigma^o_{ij}2 + 1.2\ \lambda(NR)^2 \tag{54}$$

The values of A, λ and NR are a function of the incidence angle, the beam number and the cell being scanned. Their specific values are given in an appendix along with an analysis of the relationship between (54) and (28).

The likelihood function is a product in these circumstances of six probability density functions. It is not a probability until integrated over some region of the six dimensional space defined by the $\hat{\sigma}^o_{ij}$. For a likelihood function, the variables are the σ^o_{ij} <u>NOT</u> the $\hat{\sigma}^o_{ij}$. For the given set of measurements the $\hat{\sigma}^o_{ij}$ are known, as samples of size one from six different probability density functions.

The σ^o_{ij} are unknown, but they are connected by the physics of the problem via the model function in that all six values are related to the same unique (assumed) values of V_T and χ_T as in (48). The problem is to estimate the best values of $\hat{V}_T$ and $\hat{\chi}_T$ given the sample, $\hat{\sigma}^o_{ij}$.

Given then a trial value of $\hat{V}_T$ and $\hat{\chi}_T$ the six values of σ^o_{ij} can be found from the model function and substituted into (54). A value of $L(\hat{\sigma}^o_{ij}, \sigma^o_{ij})$ is the result. A search over all possible values of $\hat{V}_T$ and $\hat{\chi}_T$ will yield local maxima of $L(\hat{\sigma}^o_{ij}, \sigma^o_{ij})$. The maximum of these local maxima is the maximum likelihood estimate of V_T and χ_T and of each of the σ^o_{ij}, subject to the constraint of Equation (48).

When the values of the exponential terms are near 1 in (53), the value of L is determined by the variances in the denominator of (53). These are usually very much less than one and being in the denominator, the result will be a large numerical value for L. The likelihood function can be scaled by any arbitrary constant and the local maxima will be preserved.

Let this constant be

$$K(\hat{\sigma}^o_{ij}) = \prod_{ij} \frac{1}{(2\pi)^3} \frac{1}{(\hat{VAR}\ \hat{\sigma}^o_{ij})^{\frac{1}{2}}} \tag{55}$$

where

$$\hat{VAR}\ \hat{\sigma}^o_{ij} = (A + \lambda)\hat{\sigma}^{o2} + 2\lambda N R\ \hat{\sigma}^o_{ij} + 1.2\lambda(NR)^2 \tag{56}$$

Then

$$L'(\sigma^o_{ij}, \hat{\sigma}^o_{ij}) = L(\sigma^o_{ij}, \hat{\sigma}^o_{ij}) \Big/ K(\hat{\sigma}^o_{ij})$$

$$= \prod_{ij} \frac{\exp(-\frac{1}{2}((\hat{\sigma}^o_{ij} - \sigma^o_{ij})^2 \Big/ \mathrm{VAR}\ \hat{\sigma}^o_{ij}))\ (\hat{VAR}\ \hat{\sigma}^o_{ij})^{\frac{1}{2}}}{(\mathrm{VAR}\ \hat{\sigma}^o_{ij})^{\frac{1}{2}}} \tag{57}$$

It can be noted that L' is equal to one if the $\hat{\sigma}^o_{ij}$ are identically equal to the σ^o_{ij}.

The search for a local maximum is facilitated by working with the natural log of the likelihood function as in (58) because a product of exponentials is involved in the function, L'.*

$$\ln L' = \sum_{ij} \left(-\tfrac{1}{2} \left((\hat{\sigma}^{o}_{ij} - \sigma^{o}_{ij}(V,\chi))^2 / \mathrm{VAR}\, \hat{\sigma}^{o}_{ij}(V,\chi)\right)\right)$$

$$+ \sum_{ij} \left(\tfrac{1}{2} \ln \widehat{\mathrm{VAR}}\, \hat{\sigma}^{o}_{ij}\right)$$

$$- \sum_{ij} \left(\tfrac{1}{2} \ln \mathrm{VAR}\, \hat{\sigma}^{o}_{ij}(V,\chi)\right) \tag{58}$$

If for a given V and χ, the values of σ^{o}_{ij} were identically equal to the values of $\hat{\sigma}^{o}_{ij}$, then $\ln L'$ would be identically zero. The sum of the last 12 terms in (58) can be either positive or negative, but, whatever that sum is, it will be slowly varying in a neighborhood of the V-χ plane. The first six terms become rapidly negative as soon as $\hat{\sigma}^{o}_{ij}$ departs slightly from $\sigma^{o}_{ij}(V,\chi)$. For (58), and hereafter, the notation for $\hat{V}_T$ and $\hat{\chi}_T$ is simplified to V and χ.

Equation (57) will be called the normalized likelihood function since it varies from one (or slightly more than one) to zero. Equation (58) will be called the logarithmic normalized likelihood function. It varies from ever so slightly positive to minus infinity and proved to be the best function to use in searching for maxima in the V,χ plane during Monte Carlo simulations.

A Simplification for Computational Purposes

It is convenient to make a change of variables in order to simplify the calculation of Equation (58). Let

$$\sigma^{o}_{ij}(V,\chi) = \hat{\sigma}^{o}_{ij} + \delta_{ij}(V,\chi) \tag{59}$$

---

*The SASS-1 vector wind recovery algorithm is different from the one described by Wentz (1978) and from equation (58). For pairs of cells as in Seasat, the differences would rarely matter, but for the SCATT they could be appreciable.

which is simply a constant $(\hat{\sigma}_{ij})$ for each term plus the variability for a particular wind speed and direction for a given set of six estimates.

Then

$$\mathrm{VAR}(\sigma^o_{ij}(V,\chi)) = \mathrm{VAR}(\hat{\sigma}^o_{ij}(V,\chi)) + \alpha(\delta_{ij}(V,\chi))^2 + (2\,\alpha\,\hat{\sigma}^o_{ij} + \beta)\;\delta_{ij}(V,\chi) \tag{60}$$

where α and β can be found from Equation D3, and

$$ln\ L' = -\tfrac{1}{2}\sum_{ij}\left[\frac{(\delta_{ij}(V,\chi))^2}{\mathrm{VAR}(\hat{\sigma}^o_{ij}) + \alpha(\delta_{ij}(V,\chi))^2 + (\alpha\,\hat{\sigma}^o_{ij} + \beta)\;\delta_{ij}(V,\chi)}\right]$$

$$-\tfrac{1}{2}\sum_{ij} ln\left[1 + \frac{\alpha(\delta_{ij}(V,\chi))^2 + (2\,\alpha\,\sigma_{ij} + \beta)\;\delta_{ij}(V,\chi)}{\mathrm{VAR}(\hat{\sigma}^o_{ij})}\right] \tag{61}$$

In this form, the only quantities that vary as a function of V and χ are the $\delta_{ij}(V,\chi)$. All of the other quantities in the twelve terms of the sum are known constants as a function of α, β, γ and the six estimates of the $\hat{\sigma}^o_{ij}$ obtained by the radar. These constants can be evaluated just once for a given set of six measurements (estimates). The six values for the $\delta_{ij}(V,\chi)$ and found from

$$\delta_{ij}(V,\chi) = \sigma^o_{ij}(V,\chi) - \hat{\sigma}^o_{ij} \tag{62}$$

and then $ln\ L'$ can be quickly computed.*

---

*See Appendix C

Interpretation

Equations (58) and (61) are extremely difficult to interpret from both probabilistic and physical points of view because of the interposition of the physical law defining the acceptable sets of six values of backscatter by the model function. In a Monte Carlo simulation, the true values of V and χ are known. If either (58), or (61), is evaluated at the true value of the input wind speed and direction, the result is quite instructive. The value of the function reduces to Equation (63) where the σ^o_{ij} are the "true" values of the backscatter from the model function and the t_{ij} are the six unit variance zero mean random variables from a normal distribution used in the simulation.

$$ln\, L' = \tfrac{1}{2} \sum_{ij} t^2_{ij} + \tfrac{1}{2} \sum_{ij} ln \left[1 + \alpha\, t^2_{ij} + \frac{(2\,\sigma^o_{ij} + \beta)}{(\mathrm{VAR}\ \sigma^o_{ij})^{\frac{1}{2}}}\, t_{ij} \right] \tag{63}$$

From Table B1 in Appendix B, the value of the second term in (63) for S/N = 10 db is

$$\tfrac{1}{2}\, \ell n\ (1 + 1.13 \times 10^{-5}\, t^2 + 0.1158t)$$

and for S/N = - 12 db, it is

$$\tfrac{1}{2}\, \ell n\ (1 + 1.13 \times 10^{-5}\, t^2 + 0.0097t)$$

For t = 2,* the first expression equals 0.104 and for the second it equals 0.0096 so that the second correction is small, but not negligible for high signal to noise.

The first term in (63) is half the sum of the squares of six values from a zero mean unit variance normal distribution. Without the one half,this is the Chi square variable with 6 degrees of freedom.

---

*See Equation (29).

Some of the values from the cumulative Chi Square distribution for 6 degrees of freedom are given by Table 4.

The top row is the probability of a Chi Square less than the value in the second row. The last row is approximately the value of the normalized likelihood function.

Table 4. The Relationship Between Chi Square and the Normalized Likelihood Function.

P	0.05	0.10	0.25	0.5	0.75	0.90	0.95
Chi Sq.	1.64	2.20	3.45	5.35	7.84	10.6	12.6
Chi Sq./2	0.82	1.10	1.73	2.68	3.92	5.3	6.3
exp -(Chi Sq./2)	0.42	0.33	0.18	0.068	0.020	0.005	0.002

In the search for the maximum value of the normalized likelihood function, the values of V and χ that yield the maximum are not the input values for the Monte Carlo simulation. Therefore, the values of the normalized likelihood function will be greater than the values derivable from the Chi Square value generated by the six Monte Carloed values of t. The third row and the fourth row of Table 4 are equally useful values for the maximization of the likelihood. For some wind speed directions and incidence angles, it might be expected that these values from a Monte Carlo simulation might approach a Chi Square distribution.

Consider Figure 8G in terms of the evaluation of either Equation (61) or (57). At 240^o, there are four curves with a positive slope going through the point for the true wind speed and direction. If the normalized likelihood function is evaluated for these four terms only, the result would be a ridge-like function with a ridge running from under 10 m/s at 220^o to over 10 m/s at 260^o. A similar ridge, perhaps not as high, would occur near 60^o.

Small perturbations, such as the random fluctuations caused by communication noise, can cause the point where these four curves form a maximum along the ridge to vary over a considerable range of correlated directions and speeds. The other two curves with the negative slope near 10 m/s and 240^{o} cross the ridge formed by the four curves at a sharper angle. Fluctuations in the measurement of backscatter for these two curves will cause them to cross the ridge formed by the other four curves at varying locations. It should be expected that the maxima for Monte Carloed values of the normalized likelihood function will lie along the ridge formed by the four curves in this figure that pass through the "true value".

Also as shown in Figure 8G, consider the possibility that the four Monte Carloed values of the backscatter for V and H polarization for beams 1 and 2 are all greater than the true value. Also the values for beam 3 could both be less than the true value. The result could easily be that all six curves would pass very near each other at a point corresponding to some speed and direction other than the "true" one. The normalized likelihood function could be close to one and yet the speed and direction could differ from the true speed and direction.

Other combinations of random effects could cause the six curves to shift in such a way that the normalized likelihood function would be a maximum at a point close to 10 m/s and 240^{o} but at the same time be low compared to the value obtained by the preceeding analysis. There is no reason to expect that the values of the normalized likelihood functions from one set of six measurements to another set of six will be correlated with how close the resulting wind speeds and directions are to the true wind speed and direction. Both of these features of the maximum likelihood estimates will be illustrated in the results to follow.

Also in Fig. 8B, four of the six curves come close together at 60^{o}. If the estimates at 60^{o} for these four curves were all close to the true value, and if the remaining two were both high, by chance, the speed and direction at 60^o near 10 m/s could produce a higher value for the likelihood function than the value for 10 m/s and 240^o.

Averages over Cells

The NOSS will scan rows of cells abeam of the spacecraft such that every ten kilometers there will be six measurements all close together for the six antennas. The incidence angles will be equal for these rows*. Rows at slightly larger or smaller incidence angles will also exist such that a 50 km on a side square will contain 5 rows of 5 sets of six measurements of backscatter at 10 km resolution. Each row of 5 will have 30 measurements and each 50 km square will have 150 measurements.

A row of 5 at the same incidence angles could possible be averaged for each beam's backscatter values to reduce the 30 measurements to 6, and then the 6 averaged values could be used in (61), with the equations for the variance corrected, to get the maxima of the normalized likelihood function.

Five Monte Carloed values of $\hat{\sigma}^o$ for the same wind speed and direction and the same aspect and incidence angles would be given by Equation (64).

$$\hat{\sigma}^o_p = \sigma^o(V,\chi) + t_p \left(\text{VAR } \sigma^o(V,\chi)\right)^{\frac{1}{2}}$$

$$\text{for } p = 1 \text{ to } 5 \tag{64}$$

The average of the five values is given by

$$\hat{\bar{\sigma}}^o = \frac{1}{5}\,\Sigma\,\hat{\sigma}^o_j = \sigma^o(V,\chi) + \frac{\Sigma t_p}{5}\left(\text{VAR } \sigma^o(V,\chi)\right)^{\frac{1}{2}} \tag{65}$$

---

*For each beam, along a row of successive cells.

The expected value of $\hat{\sigma}^o$ is

$$E(\hat{\sigma}^o) = \sigma^o(V,\chi) \tag{66}$$

and the variance of $\hat{\sigma}^o$ is

$$E\left(\hat{\sigma}^o - \sigma^o(V,\chi)\right)^2 = \frac{\text{VAR } \sigma^o(V,\chi)}{5} \tag{67}$$

Therefore to Monte Carlo an average by 5's, it is simply necessary to divide the constants in (54) by 5 and use the resulting new constants to calculate the terms in (58) or (61).

Incidently, this average by 5's of the backscatter values is not the maximum likelihood estimate that can be obtained from this sample of 5. There may be a better way to approach this average, to be the subject of further investigation.

As a first approximation to the 50 km square, 25 cell averages, the cross track variation of backscatter caused by changes in the incidence angle can be neglected.* Averages over 25 cells can then be simulated by using 25 instead of 5 in Equation (67). In appendix B, the effect of reducing the standard deviation by a factor of 5 for the functions that are illustrated would eliminate a lot of negative values of backscatter for low signal to noise ratios.

Whether or not these averages are justifiable on the basis of the effects of mesoscale turbulence and the variability of the winds from one 10 km by 10 km cell to another for the 25 such cells in the 2500 square kilometer standard areas for the SCATT needs to be investigated.

As an alternative to averaging five values of backscatter for each beam along a row and then searching for the maximum of (58) or (61), these equations could be extended to represent the sum of 90 (Equation (58)) or 60 (Equation (61)) terms one for each backscatter measurement (estimate) and the maxima could be found. The dominant terms in (63) would then be Chi Square variables with 30 degrees of freedom.

---

* By making some assumptions about the nature of the backscatter, the variation in incidence angle can be corrected.

Table 5 illustrates the variability that would occur under such circumstances if the likelihood function could be evaluated for the "true" wind speed and direction. From Table 5, about one half of the values of the logarithm of the normalized likelihood function will fall between - 14.25 and - 17.4, and ninety percent will fall between - 9.25 and - 21.9 because the slight increase as the maximum is found will be dominated by the scatter in the measurements caused by communication noise and attitude error effects. The corresponding values for the 150 measurements made at 25 cells would center around a Chi Square value of 150. The values of the logarithm of the normalized likelihood function would be near - 75 and the anti-logs would be very small indeed.

A search procedure to maximize the normalized likelihood function for either five or twenty-five cells seems to be a highly inefficient procedure. Others ways to recover winds needs to be sought. Such a search procedure would not yield the same results as this study. The use of 30 values from a row of 5 10 by 10 km cells might produce better results.* +

There are other advantages to the 10 kilometer resolution compared to coarser resolutions that are addressed in Appendix G. Effectively the sea surface is sampled more uniformly over a more nearly square area for pooled data at 50 km resolution than it would be at a 50 km resolution.

---

*See Appendix C

+See Appendix F

Table 5 The Relationship Between Chi Square and the Normalized Likelihood Function for Five Ten Kilometer Resolution Cells at the Same Incidence Angle.

P	0.01	0.025	0.05	0.100	0.25	0.50	0.75	0.90	0.95	0.975	0.99
Chi Sq.	.15	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9
(Chi Sq.)/2	7.5	8.4	9.25	10.3	14.25	14.65	17.4	20.15	21.9	23.5	25.45
exp(-(Chi Sq.)/2 (x 10^{-7})*	5531	2249	961.1	336.3	6.48	4.34	0.278	0.018	0.003	0.000	0.000

*((i.e.) multiply values in last row by 10^{-7}.

SELECTION BY OBJECTIVE CRITERIA AND THEIR CORRESPONDING MLE VALUES

Every twelve hours there will be about two million 10 km cells sampled by the SCATT. This will involve processing twelve million backscatter measurements. It is important to find a quick and efficient way to determine the winds from sets of six backscatter measurements. Two possible ways have been described, one involving a quick search and the other involving maximizing a likelihood function by means of a search over the V-χ plane. The quick search can be used to find the starting point for the V-χ plane search for the maxima of the likelihood function.

There are 9 different decisions that can be made for each set of 6 co-located backscatter measurements. An example of each will be given along with the corresponding estimates obtained from the likelihood functions.

Class 1 (For V Pol Pair Solutions)

The input wind speed and direction for this example was 12 m/s and 15^{o}. In Table 6, the computations that result in a typical Class 1 decision for a Monte Carlo simulation are shown.

The first 16 numbers show the wind speeds (first column) and wind directions (second column) for the four vertical polarization solutions for beams 1 and 3. There is one solution for each quadrant. The third column s the wind speed calculated for beam 2 at the wind direction given in the second column. The last column is the difference between the third column and the first column. Two of these differences are very large and eliminate these two V pol pair solutions from further consideration.

The next two rows of numbers repeat the two candidate speeds and directions that survived the first two steps, and the differences between the speeds for the V pol pairs and the speeds for beam 2 V pol at the same direction.

Wind speeds that result from the three H pol measurements for the two known surviving V pol pair wind directions follow. The three speeds and the absolute values of the differences between these speeds and the corresponding V pol speed are tabulated in two rows.

Table 6a Example for a Class 1 Decision and the Corresponding Maximum Likelihood Decision: input 12 m/s, 15°.

SPEED	DIR.	BM 3	DIFF.	
12.07	17.99	11.99	-.08	
12.68	168.77	13.78	1.10	- eliminated
12.69	191.61	12.13	-.56	
12.11	340.99	14.53	2.42	- eliminated

CANDIDATE			H POL 1		H POL 2		H POL 3	
12.07	17.99	.08	11.71	.35	12.24	.17	12.02	.04
12.69	191.61	.56	13.92	1.23	14.72	2.03	12.58	~~.10~~

CLASS 1 12.1 18.0 Selected
corresponding MLE values.

MLE	12.0	18.5	.796605+00
MLE	13.2	187.5	.515984-03
MLE	13.2	187.5	.515982-03
MLE	12.0	18.5	.796605+00

MLE 12.0 18.5 .796605+00 Selected solution from MLE.

Table 6b Example for a Class 2 Decision

SPEED	DIR.	BM 3	DIFF.	
11.84	18.53	12.06	.22	
12.47	168.35	13.90	1.43	- eliminated
12.48	192.06	12.17	-.30	
11.88	340.40	14.68	2.80	- eliminated

CANDIDATES								
11.84	18.53	.22	11.84		12.15	.32	11.91	.07
12.48	192.06	.30	14.05	1.57	14.62	2.14	12.50	

CLASS 2 11.8 18.5 No Decision
12.5 192.1
corresponding MLE values.

MLE	11.9	18.5	.875611+00
MLE	13.1	187.5	.259304-03
MLE	13.1	187.5	.259304-03
MLE	11.9	18.5	.875612+00

MLE 11.9 18.5 .875612+00 Selected solution from MLE.

The smallest difference for the H pol solutions in each row is then eliminated from consideration. If all three of the remaining differences in one row (one V pol beam 2 and 2 H pol values) are less than all of the remaining differences in the other row, then the V pol solution for that row is uniquely selected as the correct wind speed and direction.

For the example selected, the chosen wind speed and direction differs by 0.1 m/s and 3.0 degrees from the input speed and direction used and is the correct choice.

For these Monte Carlo simulations all four of the original V-pol pair solutions were used as the starting point for a search of the V-χ plane. Operationally, either only the chosen solution or the V pol pair could be used (with some chance of an error).

The MLE search yielded only two solutions which were found to the nearest 0.1 m/s and 0.5 degrees, and five values for the MLE are reported. The first four are the result from starting the search at each of the four V pol solutions.* The last value in the selected MLE solution. One was quite close to the Class 1 Choice (with somewhat improved accuracy for speed). The other was about 180° away. The values of the normalized likelihood function for these two maxima were 0.7966 and 0.00516.

Class 2, (Four V Pol Pair Solutions)

Table 6 also shows the calculations made in order to determine a class 2 solution. For an input wind of 12 m/s and 15°. The calculations proceed as in the previous example except that the test for a Class 1 solution fails because the beam 2 V pol difference and the 2 H pol differences for either candidate solution are not all less than those for the other candidate solution. Since the test for a Class 1 solution fails, the two H pol differences are considered. If both of these H pol differences for one candidate speed and direction are less than the differences for the other candidate, than the winner is listed as the most probable of the two candidate values and the second one is listed as the next possible solution. The two that result in this example are about 180° apart.

*i.e. The search was started from all four speeds and directions and found only two maxima in the entire plane.

The four original V pol pair solutions are then used as the starting point for a search for the MLE solutions. Again only two are found. One is correct and has an MLE of 0.8756 compared to 0.00257 for the other.

<u>Class 3, (Four V pol Pair Solutions)</u>

Table 7 shows the calculations for a Class 3 solution. The test for the 3 differences for the two candidate directions is not passed, nor is the test for the H-pol differences. All four of the original V-pol pair solutions are listed and no decision is made.

The MLE search again in this example only yields 2 solutions The one with the highest MLE value is correct.

<u>Class 4, (3 V Pol Pair Solutions)</u>

Table 8 illustrates the results of calculation that yields a Class 4 unique solution. The input was 12 m/s and 165°. The beam 2 V pol measurement is not particularly useful for the conditions as shown in Figures 8A, 8D, 8F, 9B, 9C, and 9B. One of the three V pol solutions will be at 0°, 90°, 180° or 270°. The table shows these three solutions in the first two columns. The speeds for the 3 H pol measurements for the 3 directions in the second column and the absolute values of the differences between these speeds and the V pol speeds are shown. For each row the smallest value of the differences is eliminated.

If for the remaining differences, the values for the base of the "Y" pattern are both less than all four values for the "V" part of the "Y" then this solution is picked as a unique Class 4 solution*

All three values for the V pol pair solutions are then used as a starting point for the MLE search. The example shows that only two maxima exist in the V-χ plane. The one at 11.7 m/s and 171° is selected.

---

*See discussion, page 77.

TABLE 7 Example of a Class 3 Decision and the Corresponding Maximum Likelihood Decision, input 12 m/s, 15°*.

SPEED	DIR.	BM 3	DIFF.	
12.46	12.95	11.74	-.72	- eliminated
13.00	178.84	12.51	-.48	
13.00	181.18	12.35	-.65	
12.47	346.49	13.49	1.02	- eliminated

CANDIDATES			BEAM 1		BEAM 2		BEAM 3	
13.00	178.84	.48	14.02	1.02	15.10	2.11	11.85	1.14
13.00	181.18	.65	14.02	1.02	14.96	1.96	11.85	1.14

SELECTED	2	3	eliminated
CLASS 3	12.5	12.9	no tests passed, all four still possible
	13.0	178.8	
	13.0	181.2	
	12.5	346.5	

CORRESPONDING MLE VALUES

MLE	12.1	12.0	.270158+00	
MLE	13.3	182.5	.123335-03	
MLE	13.3	182.5	.123334-03	
MLE	12.1	12.0	.270158+00	
MLE	12.1	12.0	.270158+00	Selected solution from MLE.

*Note that two of the Class 3 decisions could be selected as more probable than the other two without the aid of the MLE. Both would have been incorrect.

TABLE 8 Example of a Class 4 Decision and the Corresponding Maximum Likelihood Decisions, input 12 m/s, 165°.

SPEED	DIR.	H POL BEAM 1		H POL BEAM 2		H POL BEAM 3	
11.33	8.88	9.72	1.61	8.47	2.86	11.87	~~.54~~
11.34	350.88	9.72	1.61	9.40	1.93	11.90	~~.56~~
11.88	180.00	11.71	.17	10.77	1.11	12.06	~~.18~~

CLASS 4 11.9 180.0 SELECTED SOLUTION

CORRESPONDING MLE SOLUTIONS

MLE 11.0 345.0 .584267-03

MLE 11.0 345.0 .584266-03

MLE 11.7 171.0 .227553+00

MLE 11.7 171.0 .227553+00 SELECTED SOLUTION FROM MLE

Class 5, (3 V Pol Pair Solutions)

Table 9 shows a Class 5 example. The smallest difference in each row is eliminated. The input speed and direction were 12 m/s and 0. The two differences for one of the two legs of the "V" part of the "Y" are less than all of the other differences for the other branch of the "V" and for the base of the "Y".

This candidate solution is selected as a unique Class 5 solution.

The MLE solutions are found by starting from the three V pol solutions. There are only two and the one with the largest MLE is correct but 13° away from the input value.

Class 6, (3 V Pol Pair Solutions)

Table 9 also shows a Class 6 example. The input was 12 m/s at 0°. The differences for the "V" part of the "Y" are less than the differences for the base of the "Y", but those for either side of the "V" are not both less than those for the other side of the "V". The two solutions for the "V" are listed as Class 6 solutions without choosing which is correct.

The MLE solutions are found as before. There are two that survive and the one with the largest MLE is 0.1 m/s too low and -3° away from zero degrees.

Class 7, (3 V Pol Pair Solution)

Table 9 also shows a Class 7 example. None of the requirements for Class 4, 5 or 6 solutions are met. All three of the V-Pol solutions are listed as a Class 7 solution.

The MLE solutions show that there are only two maxima in the V-χ field, and the selected solution is close to the input speed and direction of 12 m/s and 15°.

TABLE 9 Examples of Class 5, 6 and 7 Decisions and the Corresponding Maximum Likelihood Decisions, input, 12 m/s, 0^{o}.

CLASS 5	SPEED	DIR.	H POL 1		H POL 2		H POL 3	
	11.79	13.67	12.23	.44	11.62	.17	12.26	.47
	11.81	345.78	12.24	.43	13.46	1.65	12.36	.55
	12.38	180.00	14.37	1.99	14.48	2.10	12.73	.35

CLASS 5 11.8 13.7 SELECTED SOLUTION

CORRESPONDING MLE SOLUTIONS

MLE 12.0 13.0 .495509+00

MLE 12.0 13.0 .495509+00

MLE 13.1 183.5 .371943-03

MLE 12.0 13.0 .495509+00 SELECTED SOLUTION FROM MLE

CLASS 6	SPEED	DIR.	H POL 1		H POL 2		H POL 3	
	11.93	10.49	11.86	.07	11.30	.62	11.48	.45
	11.94	349.18	11.87	.07	12.66	.73	11.52	.41
	12.44	180.00	14.03	1.59	14.05	1.60	11.74	.70

CLASS 6 11.9 10.5
11.9 349.2 SELECTED SOLUTIONS

CORRESPONDING MLE SOLUTIONS

MLE 11.9 357.0 .731221+00

MLE 11.9 357.0 .731221+00

MLE 12.8 180.5 .251710-03

MLE 11.9 357.0 .731221+00 SELECTED SOLUTION FROM MLE

CLASS 7	SPEED	DIR.	H POL 1		H POL 2		H POL 3	
	11.89	11.61	11.65	.24	12.36	.47	12.10	.21
	11.90	347.98	11.66	.24	13.97	2.07	12.16	.26
	12.42	180.00	13.79	1.36	15.27	2.84	12.44	.01

CLASS 7 11.9 11.6
11.9 348.0 NO DECISION
12.4 180.0

CORRESPONDING MLE SOLUTIONS

MLE 11.9 13.5 .653080+00

MLE 11.9 13.5 .653080+00

MLE 13.1 185.5 .557492-04

MLE 11.9 13.5 .653080+00 SELECTED SOLUTION FROM MLE

Class 8, (2 V Pol Pair Solutions)

Table 10 shows two Class 8 examples. The input was 12 m/s from 0^{o}. There are two V Pol solutions 180 degrees apart. The 3H Pol measurements yield the values and differences that are tabulated and the smallest differences in each row is excluded. If both remaining differences for one solution are less tha those for the other, this V Pol solution is listed as a unique Class 8 solution. The Class 8 choices are both correct.

The MLE solutions show that there are only two maxima in the V-χ plane for each example. In the first example, the MLE choice is correct. In the second, it is incorrect.

Class 9, (2 V Pol Pair Solutions)

Table 10 also shows a Class 9 example. The four differences involved in the decision do not satisfy the requirement for a Class 8 unique solution. Both V Pol results are listed as a Class 9 result.

There are two MLE solutions. The maximum MLE is correct, and the wind is 0.2 m/s too high and - 7^{o} from zero degrees.

Summary Description of the Classes for the Objective SCATT Alias Removal Algorithm.

The examples of the nine possible classes and the calculations that are made to select each class have been described above. These classes are summarized below.

Vertical polarization backscatter values for beams 1 and 3 are used to find vector winds from these two measurements. This will yield either 4,3 or 2 solutions. The vertical polarization value for beam 2 and the three horizontal polarization measurements are then used to decide which one of the 4, 3 or 2 solutions is the correct wind if possible. The decision that is made falls uniquely into one of 9 possible classes for each Monte Carloed set of 6 backscatter measurements. The 9 classes are defined as follows:

TABLE 10 Examples of Class 8 and Class 9 Decisions and the Corresponding Maximum Likelihood Decisions, input 12 m/s. from 0^o.

CLASS 8

SPEED	DIR	H POL 1		H POL 2		H POL 3	
11.95	.00	12.37	.43	11.72	.23	11.45	.50
12.48	180.00	14.66	2.18	13.98	1.50	11.45	1.03

CLASS 8 11.9 .0 SELECTED SOLUTION

CORRESPONDING MLE SOLUTIONS

MLE 11.9 358.5 .402803+00

MLE 13.0 179.5 .126505-05

MLE 11.9 358.5 .402803+00 SELECTED SOLUTION FROM MLE

SPEED	DIR.	H POL 1		H POL 2		H POL 3	
12.17	.00	11.39	.78	11.58	.59	12.30	.14
12.63	180.00	13.59	.97	13.83	1.20	12.30	.32

CLASS 8 12.2 .0 SELECTED SOLUTION

CORRESPONDING MLE SOLUTIONS

MLE 12.0 9.5 .348446-01

MLE 13.0 183.5 .368756-01

MLE 13.0 183.5 .368756-01 SELECTED SOLUTION FROM MLE (INCORRECT)

CLASS 9

SPEED	DIR.	H POL 1		H POL 2		H POL 3	
11.88	.00	11.79	.09	11.56	.32	13.74	1.86
12.43	180.00	14.03	1.60	13.81	1.38	13.74	1.31

CLASS 9 11.9 .0 NO DECISION

12.4 180.0

CORRESPONDING MLE SOLUTIONS

MLE 12.2 353.0 .196311-01

MLE 13.0 178.0 .186137-03

MLE 12.2 353.0 .196311-01 SELECTED SOLUTION FROM MLE

Four solutions: If vertical polarization, beams 1 and 3 gave 4 solutions.

CLASS 1 A single vector wind is picked uniquely.

CLASS 2 The correct wind is one of two vector winds listed, and the first one listed will usually be the correct wind.

CLASS 3 No decision among the four solutions is made

Three solutions: If vertical polarization, beams 1 and 3 gave 3 solutions, one of them will be at 0^o, 90^o, 180^o, or 270^o relative to beam 1; the other two will be at $180^o \pm \delta$, $270^o \pm \delta$, $0^o \pm \delta$ and $90^o \pm \delta$ relative to beam 1.

CLASS 4 The 0, 90^o, 180^o or 270^o direction and the corresponding speed is picked uniquely.

CLASS 5 One of the $180^o \pm \delta$, $270^o \pm \delta$, $0^o \pm \delta$ or $90^o \pm \delta$ directions and the corresponding speed is picked uniquely.

CLASS 6 Both of the wind vectors, not at upwind downwind or cross-wind, are picked and no decision is made as to which is correct.

CLASS 7 No decision can be made and all three wind vectors are listed.

Two solutions: If vertical polarization, beams 1 and 3 gave 2 solutions, the two solutions will either be at 0^o and 180^o or at 90^o and 270^o.

CLASS 8 One of the two solutions is picked uniquely.

CLASS 9 No decision can be made and both solutions are listed.

Discussion

The selection of candidate solutions by means of these objective criteria is accomplished by means of direct uniquely defined calculations that do not involve any search in the V-χ plane for MLE estimates. Of the 9 classes, Classes 1, 4, 5 and 8 yield a unique wind speed and direction (which may or may not actually be correct). Classes 2, 6, and 9 yield two wind speeds and directions. For Class 2, the first solution listed may prove to be the correct one more often than the second. Class 7 yields three wind speeds and directions, and Class 3 yields four wind speeds and directions.

If for a given area of the ocean, there are enough Class 1, 4, 5, and 8 solutions that are correct, and if then one of the two solutions for Classes 2, 6 and 9 is correct, the unique solutions can be used to pick the correct solutions from the pairs. The resulting field of unique winds can then be used to select the correct solutions from the remaining triplets and quadruplets so as to determine a complete wind field. It is concievable that all this can be done without the additional complexity of computing any MLE's.

The Monte Carlo simulations that follow indicate that the MLE solutions are correct more often than the objective results and have less scatter. This may prove to be an artifact of the Monte Carlo procedure. The objective technique should be kept as an independent reserve procedure because it <u>might</u> prove to be less sensitive to an exactly correct model function than the MLE method.

THE MONTE CARLO SIMULATIONS

Description

Twenty-eight Monte Carlo NOSS simulations for both the Objective Criterion technique and Maximum Likelihood Estimations were computed for various combinations of winds of 4, 8, 12, and 24 meters per second, for incidence angles of 29°, 39°, 49° and 53.5° and for 1 cell, 5 cell and 25 cell averages. For two simulations an additional random error of $\pm$ 0.7 db was added.* The wind direction was varied in 15° steps from zero to 345°. Each speed and direction was done for 50 cases. The result was a total of 33,600 simulations from which inferences of the results of 12 million SCATT measurements per day can be made for the assumptions of the model.

Also a run was made for one degree wind direction changes from zero to 47° for a 10 km cell at a 39° incidence angle for a wind speed of 8 m/s. This run shows the variability of the coarser 15° resolution data.†

For each wind speed, wind direction, incidence angle and number of cells averaged, two tables were prepared, one to summarize the results of the objective criterion method and the other to summarize the MLE results. Also the results of each MLE set of 50 values was summarized as a scatter plot and in terms of regression equations and correlation coefficients. All of these results are given in a large separate data appendix. Explanations of the various tables that were generated will be given, and examples will be discussed in the following pages. Then various summaries will be given, followed by some conclusions about the winds that will be measured by the SCATT on the NOSS.

The computations consist of 28 combinations of the various parameters of the system. A check mark in the following table shows the values of the parameters that were selected.

---

* See page 233

† See Appendix E

TABLE 11 The Parameters of the Simulation
(An asterisk shows an additional run with ± 0.7 db added random error)

ND SPEED(m/s)	4			8			12			24		
MBER OF CELLS	1	5	25	1	5	25	1	5	25	1	5	25
CIDENCE ANGLE												
29^o	√	√		√			√			√		
39^o	√	√		√*			√*			√		
47^o	√	√	√	√	√		√	√		√		
53.5^o		√	√		√	√		√	√		√	√

The Basic Computational Set

The basic computational set consists of an input wind speed, a chosen incidence angle and the number of cells averaged. Fifty simulations were done for each 15^o change in wind direction. Tables 12A through 12L show the calculations for 1 cell, 12 m/s and an incidence angle of 29^o for beam 1. A data appendix repeats this table and the 27 others similar in form. (Appendix H, given separately).

Table 12A is for 0^o and 15^o. The top two are for 0^o. For the selection by objective criteria method for an input of 12 m/s and 0^o, there were 1 class 1 solution, 29 class 2 solutions, 6 class 4 solutions and 14 class 8 solutions. Classes 1, 4 and 8 are unique. Class 2 is treated as unique, and needs careful interpretation.

The class 1 choice was within $\pm 45^o$ of the input and was correct as shown by the 1 (1). The first choice for all 29 class 2 solutions was correct as shown by the 29(29). Similarly, all class 4 solutions were correct and all 14 class 8 solutions were correct.

The average wind speed, the average direction, the RMS speed variation and the RMS direction variation are given below the number of cases relative to the input, speed and direction. Classes 4 and 8 were exactly at 0^o.

The MLE search was started (probably expensively) at all solutions that came from the V Pol pairs. All solutions were correct (i.e. within $\pm 45^o$) of the wind direction.* Both the objective method and the MLE method found 50 correct solutions. Near 0^o, the RMS speed and direction errors are smaller for the MLE solutions than for the objective method by 3^o and 0.1 m/s.

At 15^o, the objective method has 3 class 3 solutions. It could not decide on a "correct" speed and direction. Forty-seven out of 50 were unique and correct, with the first choice for class 2 correct every time. One of the four solutions of each of the class 3 cases was correct.

At 15^o, the MLE method found a unique correct solution (even the 3 class 3 cases) for all but the set of data that corresponded to the class 8 objective method. For this case, the MLE solution was not correct. (A secondary maximum might have been correct, but this was not checked). The RMS statistics are again somewhat better for the MLE.

For Table 12B, at 45^o, there are only four solution cases for the objective method. For 13 cases no decision could be made. For the remaining 37, 5 were unique and the first choice for a class 2 decision was correct for 31 out of 32 occurrences. In the sums on the far right, the first choice for a class 2 solution is treated as unique. Thus there are shown 36 correct out of 37. This interpretation is not the only one that could be made. In a field of wind vectors, the second choice will be correct and would obviously be selected. The 13 class 3 cases do not represent errors. They represent "don't know" for the objective method.

---

*As a yes or no criterion, the errors are actually much smaller.

For 45°, the MLE method missed 7 out of 50, consisting of 2 for conditions that correspond to a class 2 objective case and 5 that correspond to a class 3 case. When the objective method could not decide, the MLE missed 40% of the time.

The tables continue with the final one at 345°. There were 734 out of 1200 uniquely selected winds from the objective method of which 700 were correct. Of the 34 incorrect unique choices, 25 second choices for class 2 were correct. There were no class 2 pairs for which neither wind was correct. The MLE method picked 1137 vector winds correctly.

TABLE 12A

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3) 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI= 0

CLASS	1*	2A* (# 4 SOL(30))	2B	3	4*	5* (# 3 SOL(6))	6	7	8* (# 2 SOL(14))	9	UNIQUE	CORRECT
# SOL	1(1)	29(29)	0(29,0)	-	6(6)	-	-	-	14(14)	-	50(50)	50(50)
V BAR	11.9	12.1	-	-	12.2	-	-	-	11.9	-		12.1
X BAR	12.6	-2.2	-	-	-	-	-	-	-	-		-1.0
V RMS	.1	.3	-	-	.3	-	-	-	.2	-		.3
X RMS	12.6	10.7	-	-		-	-	-	-	-		8.4

MAXIMUM LIKELIHOOD ESTIMATES CHI= 0

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	1(1)	29(29)	-	6(6)	-	-	-	14(14)	-	50(50)
V BAR	11.8	12.0	-	12.0	-	-	-	12.0	-	12.0
X BAR	12.0	-.3	-	-2.1	-	-	-	-1.1	-	-.5
V RMS	.2	.2	-	.1	-	-	-	.2	-	.2
X RMS	12.0	5.9	-	3.6	-	-	-	3.4	-	5.3

RHO= -.271

SELECTION BY OBJECTIVE CRITERIA CHI= 15

CLASS	1*	2A* (# 4 SOL(49))	2B	3	4*	5* (# 3 SOL(0))	6	7	8* (# 2 SOL(1))	9	UNIQUE	CORRECT
# SOL	20(20)	26(26)	0(26,0)	3(3)	-	-	-	-	1(1)	-	47(50)	47(97)
V BAR	12.0	11.9	-	12.3	-	-	-	-	12.0	-		12.0
X BAR	15.9	13.7	-	18.5	-	-	-	-	-	-		14.3
V RMS	.2	.4	-	.5	-	-	-	-	-	-		.3
X RMS	4.0	5.8	-	4.7	-	-	-	-	15.0	-		5.5

MAXIMUM LIKELIHOOD ESTIMATES CHI= 15

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	20(20)	26(26)	3(3)	-	-	-	-	0(1)	-	49(50)
V BAR	11.9	12.0	11.9	-	-	-	-	-	-	11.9
X BAR	15.5	13.5	16.7	-	-	-	-	-	-	14.6
V RMS	.2	.2	.2	-	-	-	-	-	-	.2
X RMS	3.6	4.8	4.1	-	-	-	-	-	-	4.3

RHO= -.283

TABLE 12 B

| CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI= 30

\# 4 SOL(50)   # 3 SOL(0)   # 2 SOL(0)

CLASS	1•	2A•	2B	3	4•	5•	6	7	8•	9	UNIQUE	CORRECT
# SOL	21(21)	21(22)	1(22,0)	7(7)	-	-	-	-	-	-	43(50)	42(43)
V BAR	12.1	11.8	12.6	12.1	-	-	-	-	-	-		12.0
X BAR	29.2	30.5	31.4	31.5	-	-	-	-	-	-		29.8
V RMS	.2	.4	.6	.4	-	-	-	-	-	-		.3
X RMS	2.3	3.4	1.4	4.3	-	-	-	-	-	-		2.9

MAXIMUM LIKELIHOOD ESTIMATES CHI= 30   RHO= .188

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	21(21)	22(22)	6(7)	-	-	-	-	-	-	49(50)
V BAR	11.9	12.1	12.0	-	-	-	-	-	-	12.0
X BAR	29.5	29.7	32.2	-	-	-	-	-	-	29.9
V RMS	.2	.3	.1	-	-	-	-	-	-	.2
X RMS	1.6	2.5	3.0	-	-	-	-	-	-	2.3

SELECTION BY OBJECTIVE CRITERIA CHI= 45

\# 4 SOL(50)   # 3 SOL(0)   # 2 SOL(0)

CLASS	1•	2A•	2B	3	4•	5•	6	7	8•	9	UNIQUE	CORRECT
# SOL	5(5)	31(32)	1(32,0)	13(13)	-	-	-	-	-	-	37(50)	36(37)
V BAR	11.9	11.4	12.5	11.8	-	-	-	-	-	-		11.5
X BAR	43.1	45.1	46.4	43.6	-	-	-	-	-	-		44.8
V RMS	.4	.7	.5	.6	-	-	-	-	-	-		.6
X RMS	2.3	2.8	1.4	3.2	-	-	-	-	-	-		2.7

MAXIMUM LIKELIHOOD ESTIMATES CHI= 45   RHO= .111

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	5(5)	30(32)	8(13)	-	-	-	-	-	-	43(50)
V BAR	12.0	11.9	12.0	-	-	-	-	-	-	11.9
X BAR	44.5	44.6	45.6	-	-	-	-	-	-	44.8
V RMS	.2	.3	.3	-	-	-	-	-	-	.3
X RMS	2.2	2.0	1.4	-	-	-	-	-	-	1.9

TABLE 12 C

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3| 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI= 60

4 SOL(50) # 3 SOL(0) # 2 SOL(0)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	3(3)	29(29)	0(29,0)	18(18)	-	-	-	-	-	-	32(50)	32(32)
V BAR	11.8	11.8	-	11.7	-	-	-	-	-	-		11.8
X BAR	60.1	60.9	-	58.1	-	-	-	-	-	-		60.8
V RMS	.3	.4	-	.4	-	-	-	-	-	-		.4
X RMS	.6	2.8	-	4.0	-	-	-	-	-	-		2.6

MAXIMUM LIKELIHOOD ESTIMATES CHI= 60

RHO= .272

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	3(3)	29(29)	17(18)	-	-	-	-	-	-	49(50)
V BAR	11.8	12.1	11.8	-	-	-	-	-	-	12.0
X BAR	60.5	59.9	60.1	-	-	-	-	-	-	60.0
V RMS	.2	.2	.2	-	-	-	-	-	-	.2
X RMS	.9	1.9	2.8	-	-	-	-	-	-	2.2

SELECTION BY OBJECTIVE CRITERIA CHI= 75

4 SOL(43) # 3 SOL(3) # 2 SOL(4)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	-	22(22)	0(22,0)	21(21)	3(3)	-	-	-	1(1)	3(3)	26(50)	26(26)
V BAR	-	11.6	-	11.8	11.9	-	-	-	11.4	11.9		11.6
X BAR	-	80.4	-	72.9	90.0	-	-	-	90.0	90.0		81.9
V RMS	-	.5	-	.4	.5	-	-	-	.6	.5		.5
X RMS	-	12.1	-	6.1	15.0	-	-	-	15.0	15.0		12.6

MAXIMUM LIKELIHOOD ESTIMATES CHI= 75

RHO= .462

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	-	22(22)	21(21)	2(3)	-	-	-	1(1)	3(3)	49(50)
V BAR	-	12.0	11.9	11.9	-	-	-	11.9	12.1	11.9
X BAR	-	76.4	75.5	79.3	-	-	-	76.0	78.8	76.3
V RMS	-	.3	.3	.2	-	-	-	.1	.3	.3
X RMS	-	9.5	3.0	4.4	-	-	-	1.0	5.8	6.8

TABLE 12 D

DATE 010481 PAGE 47

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI= 90

4 SOL(22) # 3 SOL(9) # 2 SOL(19)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	1(1)	12(12)	0(12,0)	9(9)	9(9)	-	-	-	7(7)	12(12)	29(50)	29(29)
V BAR	11.8	11.7	-	11.7	11.8	-	-	-	11.6	11.8		11.7
X BAR	100.1	101.9	-	83.2	90.0	-	-	-	90.0	90.0		95.3
V RMS	.2	.3	-	.4	.4	-	-	-	.6	.4		.4
X RMS	10.1	13.0	-	9.4	-	-	-	-	-	-		8.6

MAXIMUM LIKELIHOOD ESTIMATES CHI= 90 RHO= .393

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	1(1)	12(12)	9(9)	9(9)	-	-	-	7(7)	12(12)	50(50)
V BAR	11.7	11.8	11.6	11.8	-	-	-	11.8	11.8	11.8
X BAR	90.5	91.2	89.7	93.4	-	-	-	88.6	91.0	90.9
V RMS	.3	.3	.4	.3	-	-	-	.4	.4	.4
X RMS	.5	6.0	5.8	3.9	-	-	-	5.3	3.8	5.0

SELECTION BY OBJECTIVE CRITERIA CHI=105

4 SOL(46) # 3 SOL(2) # 2 SOL(2)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	-	16(17)	1(17,0)	29(29)	2(2)	-	-	-	1(1)	1(1)	20(50)	19(20)
V BAR	-	12.0	12.3	11.7	12.1	-	-	-	12.1	11.7		12.0
X BAR	-	106.1	104.7	108.2	90.0	-	-	-	90.0	90.0		103.5
V RMS	-	.3	.3	.5	.5	-	-	-	.1	.3		.3
X RMS	-	4.2	.3	6.2	15.0	-	-	-	15.0	16.0		7.1

MAXIMUM LIKELIHOOD ESTIMATES CHI= 105 RHO= .091

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	-	16(17)	28(29)	2(2)	-	-	-	1(1)	1(1)	48(50)
V BAR	-	12.0	11.8	12.0	-	-	-	12.1	11.8	11.9
X BAR	-	103.8	106.0	97.0	-	-	-	91.5	98.0	104.4
V RMS	-	.2	.3	.5	-	-	-	.1	.2	.3
X RMS	-	6.3	6.2	8.5	-	-	-	13.5	7.0	6.6

TABLE 12 E

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=120

4 SOL(50) # 3 SOL(0) # 2 SOL(0)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	1(1)	12(12)	0(12.0)	37(37)	-	-	-	-	-	-	13(50)	13(13)
V BAR	11.7	11.8	-	11.6	-	-	-	-	-	-		11.8
X BAR	119.7	121.2	-	120.0	-	-	-	-	-	-		121.1
V RMS	.3	.3	-	.5	-	-	-	-	-	-		.3
X RMS	.3	3.1	-	2.9	-	-	-	-	-	-		3.0

MAXIMUM LIKELIHOOD ESTIMATES CHI= 120 RHO= .038

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	1(1)	12(12)	37(37)	-	-	-	-	-	-	50(50)
V BAR	11.8	11.8	11.8	-	-	-	-	-	-	11.8
X BAR	117.5	120.4	120.0	-	-	-	-	-	-	120.0
V RMS	.2	.3	.3	-	-	-	-	-	-	.3
X RMS	2.5	2.9	2.2	-	-	-	-	-	-	2.4

SELECTION BY OBJECTIVE CRITERIA CHI=135

4 SOL(50) # 3 SOL(0) # 2 SOL(0)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	6(6)	15(16)	1(16.0)	28(28)	-	-	-	-	-	-	22(50)	21([illegible]
V BAR	12.0	12.0	11.8	11.6	-	-	-	-	-	-		12.0
X BAR	134.5	136.5	134.2	134.7	-	-	-	-	-	-		135.9
V RMS	.3	.2	.2	.5	-	-	-	-	-	-		.3
X RMS	2.2	2.6	.8	3.0	-	-	-	-	-	-		2.5

MAXIMUM LIKELIHOOD ESTIMATES CHI= 135 RHO= .066

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	6(6)	15(16)	25(28)	-	-	-	-	-	-	46(50)
V BAR	12.0	12.0	11.9	-	-	-	-	-	-	11.9
X BAR	134.6	135.7	134.9	-	-	-	-	-	-	135.1
V RMS	.2	.2	.3	-	-	-	-	-	-	.2
X RMS	1.7	2.0	2.2	-	-	-	-	-	-	2.1

TABLE 12 F

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=150

CLASS	1*	2A* (# 4 SOL(50))	2B	3	4*	5* (# 3 SOL(0))	6	7	8* (# 2 SOL(0))	9	UNIQUE	CORRECT
# SOL	5(5)	25(27)	2(27.0)	18(18)	-	-	-	-	-	-	32(50)	30(32)
V BAR	11.7	11.9	11.8	11.8	-	-	-	-	-	-		11.9
X BAR	150.2	151.3	145.6	150.6	-	-	-	-	-	-		151.1
V RMS	.3	.3	.3	.5	-	-	-	-	-	-		.3
X RMS	1.7	4.4	4.7	2.7	-	-						4.1

MAXIMUM LIKELIHOOD ESTIMATES CHI= 150

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	5(5)	26(27)	15(18)	-	-	-	-	-	-	46(50)
V BAR	11.8	12.0	11.9	-	-	-	-	-	-	11.9
X BAR	150.4	149.8	150.7	-	-	-	-	-	-	150.2
V RMS	.2	.2	.3	-	-	-	-	-	-	.3
X RMS	1.6	2.8	2.3	-	-	-	-	-	-	2.5

RHO= -.287

SELECTION BY OBJECTIVE CRITERIA CHI=165

CLASS	1*	2A* (# 4 SOL(38))	2B	3	4*	5* (# 3 SOL(4))	6	7	8* (# 2 SOL(8))	9	UNIQUE	CORRECT
# SOL	6(6)	21(22)	1(22.0)	10(10)	-	1(1)	3(3)	-	6(6)	2(2)	35(50)	34([illegible])
V BAR	11.9	12.1	11.7	12.1	-	12.3	12.1	-	11.9	12.2		12.0
X BAR	162.7	167.8	160.7	161.0	-	175.8	175.2	-	180.0	180.0		169.3
V RMS	.3	.4	.3	.3	-	.3	.2	-	.4	.4		.4
X RMS	3.3	8.3	4.3	4.9	-	10.8	10.2	-	15.0	15.0		9.4

MAXIMUM LIKELIHOOD ESTIMATES CHI= 165

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	6(6)	21(22)	8(10)	-	1(1)	3(3)	-	6(6)	2(2)	47(50)
V BAR	11.9	12.0	12.0	-	11.9	12.2	-	11.8	11.9	12.0
X BAR	165.3	165.6	164.5	-	167.0	166.3	-	170.8	166.3	166.1
V RMS	.3	.3	.2	-	.1	.3	-	.3	.3	.3
X RMS	2.3	4.0	1.3	-	2.0	2.1	-	6.0	2.2	3.7

RHO= -.438

TABLE 12 G

I CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=180

		# 4 SOL(21)				# 3 SOL(4)			# 2 SOL(25)			
CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SCL	1(1)	12(12)	0(12,0)	8(8)	-	2(2)	2(2)	-	23(23)	2(2)	38(50)	38(38)
V BAR	11.9	12.2	-	12.2	-	12.3	12.1	-	12.2	12.1		12.2
X BAR	188.2	182.1	-	164.9	-	174.1	177.1	-	180.0	180.0		180.6
V RMS	.1	.4	-	.5	-	.5	.2	-	.5	.1		.5
X RMS	8.2	11.5	-	15.5	-	6.0	3.0	-	-	-		6.7

MAXIMUM LIKELIHOOD ESTIMATES CHI= 180

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= -.370
# SOL	1(1)	12(12)	6(8)	-	2(2)	2(2)	-	23(23)	2(2)	48(50)	
V BAR	11.9	12.1	12.2	-	12.2	12.2	-	12.1	12.1	12.1	
X BAR	190.0	179.0	182.9	-	172.5	175.0	-	178.9	178.0	179.2	
V RMS	.1	.3	.3	-	.3	.2	-	.3	.2	.3	
X RMS	10.0	8.2	11.4	-	8.5	5.4	-	4.6	2.5	7.0	

SELECTION BY OBJECTIVE CRITERIA CHI=195

		# 4 SOL(44)				# 3 SOL(1)			# 2 SOL(5)			
CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SCL	24(24)	14(14)	0(14,0)	6(6)	-	-	-	1(1)	3(4)	1(1)	42(50)	41[illegible]
V BAR	11.9	12.1	-	12.0	-	-	-	11.8	12.0	12.0		12.0
X BAR	195.8	196.0	-	201.2	-	-	-	185.0	180.0	180.0		194.7
V RMS	.2	.3	-	.4	-	-	-	.2	.3	-		.3
X RMS	5.0	3.8	-	7.0	-	-	-	10.0	15.0	15.0		6.0

MAXIMUM LIKELIHOOD ESTIMATES CHI= 195

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= -.206
# SOL	24(24)	14(14)	6(6)	-	-	-	1(1)	4(4)	1(1)	50(50)	
V BAR	12.0	12.0	12.0	-	-	-	12.3	12.2	12.4	12.0	
X BAR	196.7	195.6	198.9	-	-	-	190.0	186.5	186.5	195.5	
V RMS	.2	.2	.2	-	-	-	.3	.3	.4	.2	
X RMS	4.1	3.2	5.2	-	-	-	5.0	9.6	8.5	4.9	

TABLE 12 H

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=210

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
		# 4 SOL(50)				# 3 SOL(0)			# 2 SOL(0)			
# SOL	14(14)	21(22)	1(22,0)	14(14)	-	-	-	-	-	-	36(50)	35(36)
V BAR	12.1	12.0	11.0	11.8	-	-	-	-	-	-		12.0
X BAR	211.3	210.5	215.8	210.0	-	-	-	-	-	-		210.8
V RMS	.3	.3	1.0	.4	-	-	-	-	-	-		.3
X RMS	2.7	2.3	5.8	3.7	-	-	-	-	-	-		2.5

MAXIMUM LIKELIHOOD ESTIMATES CHI= 210

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	14(14)	21(22)	13(14)	-	-	-	-	-	-	48(50)
V BAR	12.1	11.9	12.0	-	-	-	-	-	-	12.0
X BAR	211.1	210.9	210.0	-	-	-	-	-	-	210.7
V RMS	.2	.3	.2	-	-	-	-	-	-	.2
X RMS	2.5	2.5	1.6	-	-	-	-	-	-	2.3

RHO= .029

SELECTION BY OBJECTIVE CRITERIA CHI=225

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
		# 4 SOL(50)				# 3 SOL(0)			# 2 SOL(0)			
# SOL	11(11)	20(21)	1(21,0)	18(18)	-	-	-	-	-	-	32(50)	31(32)
V BAR	11.9	12.2	11.6	11.9	-	-	-	-	-	-		1[illegible]
X BAR	226.1	224.6	228.1	224.0	-	-	-	-	-	-		225.1
V RMS	.3	.4	.4	.5	-	-	-	-	-	-		.4
X RMS	4.0	2.0	3.1	3.8	-	-	-	-	-	-		2.9

MAXIMUM LIKELIHOOD ESTIMATES CHI= 225

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	11(11)	20(21)	18(18)	-	-	-	-	-	-	49(50)
V BAR	11.9	11.9	12.0	-	-	-	-	-	-	11.9
X BAR	225.0	225.6	224.4	-	-	-	-	-	-	225.0
V RMS	.2	.3	.3	-	-	-	-	-	-	.3
X RMS	2.7	2.0	2.7	-	-	-	-	-	-	2.4

RHO= .191

TABLE 12 I

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3I 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=240

		# 4 SOL(50)				# 3 SOL(0)			# 2 SOL(0)			
CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	4(4)	22(24)	2(24,0)	22(22)	-	-	-	-	-	-	28(50)	26(28)
V BAR	11.8	12.2	11.3	11.8	-	-	-	-	-	-		12.1
X BAR	240.7	239.9	244.1	240.3	-	-	-	-	-	-		240.0
V RMS	.3	.4	.8	.3	-	-	-	-	-	-		.3
X RMS	3.5	3.1	5.5	4.8	-	-	-	-	-	-		3.2

MAXIMUM LIKELIHOOD ESTIMATES CHI= 240 RHO= .030

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	4(4)	22(24)	21(22)	-	-	-	-	-	-	47(50)
V BAR	11.7	12.0	12.0	-	-	-	-	-	-	12.0
X BAR	238.6	240.8	239.1	-	-	-	-	-	-	239.9
V RMS	.3	.2	.3	-	-	-	-	-	-	.3
X RMS	3.2	2.4	2.6	-	-	-	-	-	-	2.5

SELECTION BY OBJECTIVE CRITERIA CHI=255

		# 4 SOL(44)				# 3 SOL(1)			# 2 SOL(5)			
CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	2(2)	18(18)	0(18,0)	24(24)	0(1)	-	-	-	-	5(5)	21(50)	20(21)
V BAR	12.2	12.2	-	12.0	-	-	-	-	-	12.0		12.[illegible]
X BAR	260.2	252.8	-	255.9	-	-	-	-	-	270.0		253.5
V RMS	.3	.3	-	.5	-	-	-	-	-	.5		.3
X RMS	6.3	4.6	-	3.8	-	-	-	-	-	15.0		4.8

MAXIMUM LIKELIHOOD ESTIMATES CHI= 255 RHO= .535

CLASS	1	2	3	4	5	6	7	8	9	TOTALS
# SOL	2(2)	18(18)	22(24)	1(1)	-	-	-	-	5(5)	48(50)
V BAR	12.2	12.0	12.0	12.0	-	-	-	-	12.1	12.1
X BAR	256.8	253.8	255.7	253.0	-	-	-	-	258.9	255.3
V RMS	.4	.2	.3	-	-	-	-	-	.2	.3
X RMS	1.8	3.9	7.3	2.0	-	-	-	-	6.1	5.8

TABLE 12 J

1 CELL DATE 010481 PAGE 6

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=270

CLASS	1*	# 4 SOL(22) 2A*	2B	3	4*	# 3 SOL(4) 5*	6	7	# 2 SOL(24) 8*	9	UNIQUE	CORRECT
# SOL	-	5(6)	1(6.0)	16(16)	0(4)	-	-	-	12(12)	12(12)	22(50)	17(22)
V BAR	-	11.8	10.9	11.9	-	-	-	-	12.2	11.7		12.1
X BAR	-	266.0	274.0	283.6	-	-	-	-	270.0	270.0		268.8
V RMS	-	.4	1.1	.3	-	-	-	-	.5	.5		.5
X RMS	-	11.3	4.0	14.3	-	-	-	-	-	-		6.1

MAXIMUM LIKELIHOOD ESTIMATES CHI= 270

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= .359
# SOL	-	5(6)	16(16)	3(4)	-	-	-	12(12)	12(12)	48(50)	
V BAR	-	11.8	11.9	12.0	-	-	-	12.0	11.8	11.9	
X BAR	-	270.8	275.2	266.3	-	-	-	273.0	268.7	272.0	
V RMS	-	.4	.2	.4	-	-	-	.4	.3	.3	
X RMS	-	8.9	10.1	7.3	-	-	-	4.8	5.0	7.6	

SELECTION BY OBJECTIVE CRITERIA CHI=285

CLASS	1*	# 4 SOL(44) 2A*	2B	3	4*	# 3 SOL(2) 5*	6	7	# 2 SOL(4) 8*	9	UNIQUE	CORRECT
# SOL	1(1)	19(19)	0(19.0)	24(24)	0(2)	-	-	-	2(2)	2(2)	24(50)	22(24)
V BAR	12.1	11.9	-	11.9	-	-	-	-	12.4	12.2		12.[illegible]
X BAR	281.3	284.1	-	285.8	-	-	-	-	270.0	270.0		282.7
V RMS	.1	.2	-	.4	-	-	-	-	.4	.3		.3
X RMS	3.7	3.0	-	4.1	-	-	-	-	15.0	15.0		5.4

MAXIMUM LIKELIHOOD ESTIMATES CHI= 285

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= .032
# SOL	1(1)	19(19)	21(24)	2(2)	-	-	-	2(2)	2(2)	47(50)	
V BAR	12.1	11.9	12.1	12.0	-	-	-	12.2	11.9	12.0	
X BAR	283.5	285.9	283.0	280.0	-	-	-	280.7	278.7	283.8	
V RMS	.1	.2	.2	.1	-	-	-	.3	.2	.2	
X RMS	1.5	3.0	7.5	7.4	-	-	-	4.6	8.2	5.9	

TABLE 12 K

CELL	DATE 010481	PAGE

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=300

4 SOL(50) · # 3 SOL(0) · # 2 SOL(0)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	0(1)	14(18)	4(18.0)	31(31)	-	-	-	-	-	-	19(50)	19(19)
V BAR	-	11.9	11.7	12.1	-	-	-	-	-	-		11.9
X BAR	-	299.4	305.6	299.8	-	-	-	-	-	-		299.4
V RMS	-	.4	.5	.4	-	-	-	-	-	-		.4
X RMS	-	3.5	6.2	3.3	-	-	-	-	-	-		3.5

MAXIMUM LIKELIHOOD ESTIMATES CHI= 300

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= .510
# SOL	0(1)	14(18)	28(31)	-	-	-	-	-	-	42(50)	
V BAR	-	11.9	12.0	-	-	-	-	-	-	12.0	
X BAR	-	300.4	299.7	-	-	-	-	-	-	300.0	
V RMS	-	.3	.3	-	-	-	-	-	-	.3	
X RMS	-	2.9	2.5	-	-	-	-	-	-	2.6	

SELECTION BY OBJECTIVE CRITERIA CHI=315

4 SOL(50) · # 3 SOL(0) · # 2 SOL(0)

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
# SOL	2(2)	13(19)	6(19.0)	29(29)	-	-	-	-	-	-	21(50)	15([illegible]
V BAR	12.2	12.0	12.1	12.0	-	-	-	-	-	-		12.0
X BAR	315.0	313.8	318.9	315.6	-	-	-	-	-	-		313.9
V RMS	.2	.4	.2	.4	-	-	-	-	-	-		.4
X RMS	2.2	2.6	4.8	1.9	-	-	-	-	-	-		2.5

MAXIMUM LIKELIHOOD ESTIMATES CHI= 315

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= .450
# SOL	2(2)	13(19)	21(29)	-	-	-	-	-	-	36(50)	
V BAR	12.0	12.1	11.9	-	-	-	-	-	-	12.0	
X BAR	314.7	314.7	315.2	-	-	-	-	-	-	315.0	
V RMS	.1	.3	.3	-	-	-	-	-	-	.3	
X RMS	1.8	1.8	1.9	-	-	-	-	-	-	1.8	

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA CHI=330

CLASS	1*	# 4 SOL(50) 2A*	2B	3	4*	# 3 SOL(0) 5*	6	7	# 2 SOL(0) 8*	9	UNIQUE	CORRECT
# SOL	4(4)	29(29)	0(29,0)	17(17)	-	-	-	-	-	-	33(50)	33(33)
V BAR	12.2	12.1	-	12.1	-	-	-	-	-	-		12.1
X BAR	331.8	328.2	-	330.0	-	-	-	-	-	-		328.6
V RMS	.4	.4	-	.4	-	-	-	-	-	-		.4
X RMS	4.1	3.6	-	5.7	-	-	-	-	-	-		3.7

MAXIMUM LIKELIHOOD ESTIMATES CHI= 330

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= -.382
# SOL	4(4)	29(29)	16(17)	-	-	-	-	-	-	49(50)	
V BAR	12.1	12.2	12.1	-	-	-	-	-	-	12.1	
X BAR	330.7	329.7	329.7	-	-	-	-	-	-	329.8	
V RMS	.3	.3	.3	-	-	-	-	-	-	.3	
X RMS	2.8	2.3	3.1	-	-	-	-	-	-	2.6	

SELECTION BY OBJECTIVE CRITERIA CHI=345

CLASS	1*	# 4 SOL(48) 2A*	2B	3	4*	# 3 SOL(2) 5*	6	7	# 2 SOL(0) 8*	9	UNIQUE	CORRECT
# SOL	4(4)	25(28)	3(28,0)	16(16)	-	-	0(1)	1(1)	-	-	32(50)	29(32)
V BAR	12.1	11.9	12.5	12.1	-	-	-	11.6	-	-		12.0
X BAR	348.9	342.2	349.8	344.7	-	-	-	360.0	-	-		343.1
V RMS	.3	.4	.6	.5	-	-	-	.4	-	-		.4
X RMS	4.7	4.7	6.4	5.7	-	-	-	15.0	-	-		4.7

MAXIMUM LIKELIHOOD ESTIMATES CHI= 345

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	RHO= -.360
# SOL	4(4)	27(28)	16(16)	-	-	1(1)	1(1)	-	-	49(50)	
V BAR	12.0	12.0	12.1	-	-	12.1	11.7	-	-	12.0	
X BAR	346.9	344.4	343.7	-	-	347.5	350.5	-	-	344.6	
V RMS	.1	.2	.3	-	-	.1	.3	-	-	.2	
X RMS	2.9	3.2	4.1	-	-	2.5	5.5	-	-	3.6	

DATE.01048I PAGE

1 CELL

GRAND AVERAGES

OBJECTIVE V BAR 12.0 V RMS .4 X RMS 5.8 # COR(# UNQ, TOTAL) 700(734,1200)
MLE V BAR 12.0 V RMS .3 X RMS 4.4 # COR(TOTAL) 1137(1200)

TOTAL ATTEMPTS 1200 NUMBER OF FAILURES 0

Summary Tables

The results of each of the 28 Monte Carlo simulations are summarized in the 28 tables that follow. They are tables 13 through 40 as indexed below.

Table 13, 29^o, 4 m/s, 1 cell
14, 29^o, 4 m/s, 5 cell
15, 29^o, 8 m/s, 1 cell
16, 29^o, 12 m/s, 1 cell
17, 29^o, 24 m/s, 1 cell
18, 39^o, 4 m/s, 1 cell
19, 29^o, 4 m/s, 5 cells
20, 39^o, 8 m/s, 1 cell
21, 39^o, 8 m/s, 1 cell ± 0.7 db
22, 39^o, 12 m/s, 1 cell
23, 39^o, 12 m/s, 1 cell ± 0.7 db
24, 39^o, 24 m/s, 1 cell
25, 47^o, 4 m/s, 1 cell
26, 47^o, 4 m/s, 5 cells
27, 47^o, 4 m/s, 25 cells
28, 47^o, 8 m/s, 1 cell
29, 47^o, 8 m/s, 5 cells
30, 47^o, 12 m/s, 1 cell
31, 47^o, 12 m/s, 5 cells
32, 47^o, 24 m/s, 1 cell
33, 53.5^o, 4 m/s, 5 cells
34, 53.5^o, 4 m/s, 25 cells
35, 53.5^o, 8 m/s, 5 cells
36, 53.5^o, 8 m/s, 25 cells
37, 53.5^o, 12 m/s, 5 cells
38, 53.5^o, 12 m/s, 25 cells
39, 53.5^o, 24 m/s, 5 cells
40, 53.5^o, 24 m/s, 25 cells

TABLE 13 29°, 4 M/S, 1 CELL

1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	9(9)	0(9, 0)	4(4)	-	2(2)	6(6)	3(3)	22(22)	4(4)	33(50)	33(33)
X= 15	-	16(16)	0(16, 0)	6(6)	-	-	5(5)	10(10)	6(7)	6(6)	23(50)	22(23)
X= 30	7(7)	22(23)	1(23, 0)	20(20)	-	-	-	-	-	-	30(50)	29(30)
X= 45	-	16(17)	1(17, 0)	33(33)	-	-	-	-	-	-	17(50)	16(17)
X= 60	2(2)	8(9)	1(9, 0)	28(28)	-	-	-	6(6)	0(2)	3(3)	13(50)	10(13)
X= 75	-	5(5)	0(5, 0)	13(13)	-	-	1(1)	16(16)	1(2)	13(13)	7(50)	6(7)
X= 90	-	3(3)	0(3, 0)	11(11)	-	1(1)	2(2)	6(6)	12(13)	14(14)	17(50)	16(17)
X=105	-	7(7)	0(7, 0)	10(10)	-	-	3(3)	13(13)	7(7)	10(10)	14(50)	14(14)
X=120	-	8(12)	2(12, 2)	35(35)	-	-	-	2(2)	-	1(1)	12(50)	8(12)
X=135	0(1)	3(6)	2(6, 1)	43(43)	-	-	-	-	-	-	7(50)	3(7)
X=150	1(1)	7(11)	3(11, 1)	37(37)	-	-	-	1(1)	-	-	12(50)	8(12)
X=165	2(2)	9(11)	0(11, 2)	24(24)	4(4)	-	-	4(4)	2(2)	3(3)	19(50)	17(19)
X=180	-	4(4)	0(4, 0)	20(20)	4(4)	-	-	7(7)	7(7)	8(8)	15(50)	15(15)
X=195	3(3)	10(11)	1(11, 0)	22(22)	3(3)	-	-	5(5)	1(1)	5(5)	18(50)	17(18)
X=210	3(3)	13(14)	1(14, 0)	33(33)	-	-	-	-	-	-	17(50)	16(17)
X=225	1(1)	14(17)	2(17, 1)	32(32)	-	-	-	-	-	-	18(50)	15(18)
X=240	0(1)	12(17)	4(17, 1)	31(31)	-	-	-	-	-	1(1)	18(50)	12(18)
X=255	0(1)	3(4)	0(4, 1)	31(31)	1(1)	-	-	4(4)	3(3)	6(6)	9(50)	7(9)
X=270	1(1)	1(3)	3(3, -1)	21(21)	2(2)	-	-	9(9)	3(3)	11(11)	9(50)	7(9)
X=285	-	5(8)	4(8, -1)	28(28)	1(1)	-	-	6(6)	1(1)	6(6)	10(50)	7(10)
X=300	2(2)	8(11)	2(11, 1)	36(36)	-	-	-	1(1)	-	-	13(50)	10(13)
X=315	1(3)	7(12)	5(12, 0)	35(35)	-	-	-	-	-	-	15(50)	8(15)
X=330	1(1)	5(6)	1(6, 0)	40(40)	-	-	-	1(1)	0(1)	1(1)	8(50)	6(8)
X=345	1(1)	12(12)	0(12, 0)	11(11)	0(1)	1(1)	3(3)	8(8)	4(5)	8(8)	20(50)	18(20)
TOTAL	25(30)	207(248)	33(248, 8)	604(604)	15(16)	4(4)	20(20)	102(102)	69(76)	100(100)	374(1200)	320(374)

1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	9(9)	4(4)	-	2(2)	6(6)	3(3)	22(22)	3(4)	49(50)	RHO= -.661
X= 15	-	16(16)	6(6)	-	-	5(5)	10(10)	6(7)	5(6)	48(50)	RHO= -.052
X= 30	7(7)	23(23)	20(20)	-	-	-	-	-	-	50(50)	RHO= .513
X= 45	-	11(17)	24(33)	-	-	-	-	-	-	35(50)	RHO= .435
X= 60	2(2)	7(9)	18(28)	-	-	-	6(6)	2(2)	3(3)	38(50)	RHO= .342
X= 75	-	5(5)	10(13)	-	-	1(1)	12(16)	1(2)	13(13)	42(50)	RHO= .443
X= 90	-	3(3)	11(11)	-	1(1)	2(2)	6(6)	13(13)	12(14)	48(50)	RHO= .416
X=105	-	7(7)	10(10)	-	-	3(3)	13(13)	7(7)	10(10)	50(50)	RHO= .488
X=120	-	9(12)	29(35)	-	-	-	1(2)	-	0(1)	39(50)	RHO= .671
X=135	0(1)	3(6)	34(43)	-	-	-	-	-	-	37(50)	RHO= -.086
X=150	1(1)	4(11)	30(37)	-	-	-	1(1)	-	-	36(50)	RHO= -.458
X=165	2(2)	9(11)	22(24)	4(4)	-	-	4(4)	1(2)	2(3)	44(50)	RHO= -.608
X=180	-	4(4)	18(20)	4(4)	-	-	7(7)	7(7)	7(8)	47(50)	RHO= -.445
X=195	3(3)	11(11)	22(22)	3(3)	-	-	3(5)	1(1)	5(5)	48(50)	RHO= -.542
X=210	3(3)	13(14)	29(33)	-	-	-	-	-	-	45(50)	RHO= .406
X=225	1(1)	14(17)	25(32)	-	-	-	-	-	-	40(50)	RHO= .418
X=240	0(1)	12(17)	24(31)	-	-	-	-	-	1(1)	37(50)	RHO= .374
X=255	0(1)	3(4)	21(31)	1(1)	-	-	4(4)	3(3)	3(6)	35(50)	RHO= .584
X=270	1(1)	0(3)	16(21)	2(2)	-	-	7(9)	3(3)	9(11)	38(50)	RHO= .604
X=285	-	5(8)	20(28)	1(1)	-	-	4(6)	1(1)	3(6)	34(50)	RHO= .565
X=300	2(2)	8(11)	20(36)	-	-	-	1(1)	-	-	31(50)	RHO= .247
X=315	1(3)	8(12)	28(35)	-	-	-	-	-	-	37(50)	RHO= -.412
X=330	1(1)	6(6)	34(40)	-	-	-	1(1)	1(1)	1(1)	44(50)	RHO= -.581
X=345	1(1)	12(12)	9(11)	0(1)	1(1)	3(3)	8(8)	5(5)	8(8)	47(50)	RHO= -.634
TOTAL	25(30)	202(248)	484(604)	15(16)	4(4)	20(20)	91(102)	73(76)	85(100)	999(1200)	

TABLE 14 29°, 4 M/S, 5 CELLS

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	2(2)	22(22)	2(2)	24(24)	-	26(50)	26(26)
X= 15	4(4)	19(19)	0(19, 0)	-	-	4(4)	3(3)	18(18)	2(2)	-	29(50)	29(29)
X= 30	25(25)	22(22)	0(22, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X= 45	2(2)	40(40)	0(40, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X= 60	4(4)	31(32)	1(32, 0)	14(14)	-	-	-	-	-	-	36(50)	35(36)
X= 75	4(4)	2(3)	0(3, 1)	11(11)	-	-	-	25(25)	0(1)	6(6)	8(50)	6(8)
X= 90	-	-	-	-	-	4(4)	3(3)	8(8)	27(27)	8(8)	31(50)	31(31)
X=105	17(17)	4(4)	0(4, 0)	3(3)	-	-	2(2)	21(21)	1(1)	2(2)	22(50)	22(22)
X=120	5(5)	19(19)	0(19, 0)	26(26)	-	-	-	-	-	-	24(50)	24(24)
X=135	3(3)	14(15)	1(15, 0)	32(32)	-	-	-	-	-	-	18(50)	17(18)
X=150	12(12)	9(11)	2(11, 0)	27(27)	-	-	-	-	-	-	23(50)	21(23)
X=165	10(10)	6(6)	0(6, 0)	24(24)	7(7)	-	-	3(3)	-	-	23(50)	23(23)
X=180	1(1)	24(24)	0(24, 0)	5(5)	5(5)	-	-	11(11)	3(3)	1(1)	33(50)	33(33)
X=195	6(6)	28(28)	0(28, 0)	10(10)	1(1)	-	-	5(5)	-	-	35(50)	35(35)
X=210	21(21)	14(15)	1(15, 0)	14(14)	-	-	-	-	-	-	36(50)	35(36)
X=225	7(7)	23(25)	2(25, 0)	18(18)	-	-	-	-	-	-	32(50)	30(32)
X=240	6(7)	22(24)	2(24, 0)	19(19)	-	-	-	-	-	-	31(50)	28(31)
X=255	3(3)	19(19)	0(19, 0)	20(20)	1(1)	-	-	6(6)	1(1)	-	24(50)	24(24)
X=270	4(4)	3(4)	2(4, -1)	13(13)	11(11)	-	-	11(11)	6(6)	1(1)	25(50)	24(25)
X=285	11(11)	13(14)	1(14, 0)	18(18)	4(4)	-	-	3(3)	-	-	29(50)	28(29)
X=300	3(3)	11(14)	3(14, 0)	33(33)	-	-	-	-	-	-	17(50)	14(17)
X=315	8(9)	5(5)	0(5, 0)	36(36)	-	-	-	-	-	-	14(50)	13(14)
X=330	10(10)	13(14)	1(14, 0)	26(26)	-	-	-	-	-	-	24(50)	23(24)
X=345	8(8)	11(11)	0(11, 0)	6(6)	-	-	1(1)	23(23)	1(1)	-	20(50)	20(20)
TOTAL	174(176)	352(368)	16(368, 0)	366(366)	29(29)	10(10)	31(31)	136(136)	65(66)	18(18)	649(1200)	630(649)

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	2(2)	22(22)	2(2)	24(24)	-	50(50)	RHO= -.617
X= 15	4(4)	19(19)	-	-	4(4)	3(3)	18(18)	2(2)	-	50(50)	RHO= .200
X= 30	25(25)	22(22)	3(3)	-	-	-	-	-	-	50(50)	RHO= .577
X= 45	2(2)	39(40)	7(8)	-	-	-	-	-	-	48(50)	RHO= .440
X= 60	4(4)	30(32)	13(14)	-	-	-	-	-	-	47(50)	RHO= .225
X= 75	4(4)	3(3)	11(11)	-	-	-	25(25)	0(1)	6(6)	49(50)	RHO= .310
X= 90	-	-	-	-	4(4)	3(3)	8(8)	27(27)	8(8)	50(50)	RHO= .567
X=105	17(17)	4(4)	3(3)	-	-	2(2)	21(21)	1(1)	2(2)	50(50)	RHO= .692
X=120	5(5)	19(19)	25(26)	-	-	-	-	-	-	49(50)	RHO= .233
X=135	3(3)	15(15)	31(32)	-	-	-	-	-	-	49(50)	RHO= -.384
X=150	12(12)	11(11)	27(27)	-	-	-	-	-	-	50(50)	RHO= -.632
X=165	10(10)	6(6)	24(24)	7(7)	-	-	3(3)	-	-	50(50)	RHO= -.750
X=180	1(1)	24(24)	5(5)	5(5)	-	-	11(11)	3(3)	1(1)	50(50)	RHO= -.624
X=195	6(6)	28(28)	10(10)	1(1)	-	-	5(5)	-	-	50(50)	RHO= -.066
X=210	21(21)	15(15)	14(14)	-	-	-	-	-	-	50(50)	RHO= .397
X=225	7(7)	25(25)	15(18)	-	-	-	-	-	-	47(50)	RHO= .602
X=240	7(7)	23(24)	17(19)	-	-	-	-	-	-	47(50)	RHO= .323
X=255	3(3)	19(19)	18(20)	1(1)	-	-	5(6)	1(1)	-	47(50)	RHO= .259
X=270	4(4)	4(4)	13(13)	11(11)	-	-	11(11)	6(6)	1(1)	50(50)	RHO= .592
X=285	11(11)	13(14)	18(18)	4(4)	-	-	3(3)	-	-	49(50)	RHO= .518
X=300	3(3)	11(14)	27(33)	-	-	-	-	-	-	41(50)	RHO= -.315
X=315	8(9)	5(5)	31(36)	-	-	-	-	-	-	44(50)	RHO= -.737
X=330	10(10)	14(14)	25(26)	-	-	-	-	-	-	49(50)	RHO= -.633
X=345	8(8)	11(11)	6(6)	-	-	1(1)	23(23)	1(1)	-	50(50)	RHO= -.808
TOTAL	175(176)	360(368)	343(366)	29(29)	10(10)	31(31)	135(136)	65(66)	18(18)	1166(1200)	

TABLE 15 29^{o}, 8 M/S, 1 CELL

1 CELL

WIND SPEED 8.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	15(15)	0(15, 0)	1(1)	-	-	4(4)	1(1)	29(29)	-	44(50)	44(44)
X= 15	16(16)	19(19)	0(19, 0)	2(2)	-	1(1)	2(2)	6(6)	3(3)	1(1)	39(50)	39(39)
X= 30	21(21)	26(26)	0(26, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X= 45	2(3)	30(31)	1(31, 0)	16(16)	-	-	-	-	-	-	34(50)	32(34)
X= 60	0(1)	28(29)	1(29, 0)	20(20)	-	-	-	-	-	-	30(50)	28(30)
X= 75	2(2)	8(8)	0(8, 0)	23(23)	-	-	-	4(4)	0(1)	12(12)	11(50)	10(11)
X= 90	-	5(5)	0(5, 0)	8(8)	-	-	2(2)	10(10)	6(6)	19(19)	11(50)	11(11)
X=105	4(4)	15(15)	0(15, 0)	19(19)	-	-	-	7(7)	3(3)	2(2)	22(50)	22(22)
X=120	1(1)	16(16)	0(16, 0)	33(33)	-	-	-	-	-	-	17(50)	17(17)
X=135	2(2)	15(17)	2(17, 0)	31(31)	-	-	-	-	-	-	19(50)	17(19)
X=150	5(5)	17(20)	3(20, 0)	25(25)	-	-	-	-	-	-	25(50)	22(25)
X=165	6(6)	20(22)	2(22, 0)	16(16)	4(4)	-	-	1(1)	1(1)	-	33(50)	31(33)
X=180	3(3)	24(25)	1(25, 0)	2(2)	6(6)	-	-	4(4)	8(8)	2(2)	42(50)	41(42)
X=195	13(13)	20(22)	2(22, 0)	13(13)	1(1)	-	-	-	-	1(1)	36(50)	34(36)
X=210	12(12)	24(25)	1(25, 0)	13(13)	-	-	-	-	-	-	37(50)	36(37)
X=225	10(10)	16(22)	5(22, 1)	18(18)	-	-	-	-	-	-	32(50)	26(32)
X=240	2(2)	22(26)	4(26, 0)	22(22)	-	-	-	-	-	-	28(50)	24(28)
X=255	-	24(24)	1(24, -1)	21(21)	-	-	-	4(4)	1(1)	-	25(50)	25(25)
X=270	1(1)	5(6)	1(6, 0)	17(17)	1(1)	-	-	9(9)	7(7)	9(9)	15(50)	14(15)
X=285	1(1)	10(11)	2(11, -1)	31(31)	-	-	-	3(3)	-	4(4)	12(50)	11(12)
X=300	2(3)	9(13)	3(13, 1)	34(34)	-	-	-	-	-	-	16(50)	11(16)
X=315	3(3)	11(13)	2(13, 0)	34(34)	-	-	-	-	-	-	16(50)	14(16)
X=330	5(5)	19(22)	3(22, 0)	23(23)	-	-	-	-	-	-	27(50)	24(27)
X=345	9(9)	11(12)	1(12, 0)	12(12)	0(2)	-	-	3(3)	2(3)	9(9)	26(50)	22(26)
TOTAL	120(123)	409(444)	35(444, 0)	437(437)	12(14)	1(1)	8(8)	52(52)	60(62)	59(59)	644(1200)	602(644)

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	15(15)	1(1)	-	-	4(4)	1(1)	29(29)	-	50(50)	RHO= -.488
X= 15	16(16)	19(19)	2(2)	-	1(1)	2(2)	6(6)	3(3)	1(1)	50(50)	RHO= .011
X= 30	21(21)	26(26)	3(3)	-	-	-	-	-	-	50(50)	RHO= .417
X= 45	2(3)	26(31)	13(16)	-	-	-	-	-	-	41(50)	RHO= .468
X= 60	0(1)	28(29)	19(20)	-	-	-	-	-	-	47(50)	RHO= .244
X= 75	2(2)	8(8)	23(23)	-	-	-	4(4)	0(1)	10(12)	47(50)	RHO= .263
X= 90	-	5(5)	8(8)	-	-	2(2)	9(10)	6(6)	19(19)	49(50)	RHO= .389
X=105	4(4)	15(15)	17(19)	-	-	-	7(7)	3(3)	1(2)	47(50)	RHO= .318
X=120	1(1)	16(16)	30(33)	-	-	-	-	-	-	47(50)	RHO= .184
X=135	2(2)	15(17)	26(31)	-	-	-	-	-	-	43(50)	RHO= .092
X=150	5(5)	17(20)	22(25)	-	-	-	-	-	-	44(50)	RHO= -.498
X=165	6(6)	20(22)	16(16)	4(4)	-	-	1(1)	0(1)	-	47(50)	RHO= -.377
X=180	3(3)	24(25)	2(2)	6(6)	-	-	4(4)	8(8)	0(2)	47(50)	RHO= -.466
X=195	13(13)	22(22)	13(13)	1(1)	-	-	-	-	1(1)	50(50)	RHO= -.135
X=210	12(12)	22(25)	12(13)	-	-	-	-	-	-	46(50)	RHO= .171
X=225	10(10)	18(22)	15(18)	-	-	-	-	-	-	43(50)	RHO= .422
X=240	2(2)	24(26)	19(22)	-	-	-	-	-	-	45(50)	RHO= .402
X=255	-	23(24)	20(21)	-	-	-	3(4)	1(1)	-	47(50)	RHO= .508
X=270	1(1)	5(6)	15(17)	1(1)	-	-	7(9)	7(7)	9(9)	45(50)	RHO= .621
X=285	1(1)	10(11)	29(31)	-	-	-	3(3)	-	3(4)	46(50)	RHO= .579
X=300	2(3)	10(13)	30(34)	-	-	-	-	-	-	42(50)	RHO= .147
X=315	3(3)	11(13)	29(34)	-	-	-	-	-	-	43(50)	RHO= -.488
X=330	5(5)	17(22)	21(23)	-	-	-	-	-	-	43(50)	RHO= -.359
X=345	9(9)	11(12)	11(12)	2(2)	-	-	3(3)	3(3)	9(9)	48(50)	RHO= -.629
TOTAL	120(123)	407(444)	396(437)	14(14)	1(1)	8(8)	48(52)	60(62)	53(59)	1107(1200)	

TABLE 16 29^{o}, 12 M/S, 1 CELL

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	1(1)	29(29)	0(29, 0)	-	6(6)	-	-	-	14(14)	-	50(50)	50(50)
X= 15	20(20)	26(26)	0(26, 0)	3(3)	-	-	-	-	1(1)	-	47(50)	47(47)
X= 30	21(21)	21(22)	1(22, 0)	7(7)	-	-	-	-	-	-	43(50)	42(43)
X= 45	5(5)	31(32)	1(32, 0)	13(13)	-	-	-	-	-	-	37(50)	36(37)
X= 60	3(3)	29(29)	0(29, 0)	18(18)	-	-	-	-	-	-	32(50)	32(32)
X= 75	-	22(22)	0(22, 0)	21(21)	3(3)	-	-	-	1(1)	3(3)	26(50)	26(26)
X= 90	1(1)	12(12)	0(12, 0)	9(9)	9(9)	-	-	-	7(7)	12(12)	29(50)	29(29)
X=105	-	16(17)	1(17, 0)	29(29)	2(2)	-	-	-	1(1)	1(1)	20(50)	19(20)
X=120	1(1)	12(12)	0(12, 0)	37(37)	-	-	-	-	-	-	13(50)	13(13)
X=135	6(6)	15(16)	1(16, 0)	28(28)	-	-	-	-	-	-	22(50)	21(22)
X=150	5(5)	25(27)	2(27, 0)	18(18)	-	-	-	-	-	-	32(50)	30(32)
X=165	6(6)	21(22)	1(22, 0)	10(10)	-	1(1)	3(3)	-	6(6)	2(2)	35(50)	34(35)
X=180	1(1)	12(12)	0(12, 0)	8(8)	-	2(2)	2(2)	-	23(23)	2(2)	38(50)	38(38)
X=195	24(24)	14(14)	0(14, 0)	6(6)	-	-	-	1(1)	3(4)	1(1)	42(50)	41(42)
X=210	14(14)	21(22)	1(22, 0)	14(14)	-	-	-	-	-	-	36(50)	35(36)
X=225	11(11)	20(21)	1(21, 0)	18(18)	-	-	-	-	-	-	32(50)	31(32)
X=240	4(4)	22(24)	2(24, 0)	22(22)	-	-	-	-	-	-	28(50)	26(28)
X=255	2(2)	18(18)	0(18, 0)	24(24)	0(1)	-	-	-	-	5(5)	21(50)	20(21)
X=270	-	5(6)	1(6, 0)	16(16)	0(4)	-	-	-	12(12)	12(12)	22(50)	17(22)
X=285	1(1)	19(19)	0(19, 0)	24(24)	0(2)	-	-	-	2(2)	2(2)	24(50)	22(24)
X=300	0(1)	14(18)	4(18, 0)	31(31)	-	-	-	-	-	-	19(50)	14(19)
X=315	2(2)	13(19)	6(19, 0)	29(29)	-	-	-	-	-	-	21(50)	15(21)
X=330	4(4)	29(29)	0(29, 0)	17(17)	-	-	-	-	-	-	33(50)	33(33)
X=345	4(4)	25(28)	3(28, 0)	16(16)	-	-	0(1)	1(1)	-	-	32(50)	29(32)
TOTAL	136(137)	471(496)	25(496, 0)	418(418)	20(27)	3(3)	5(6)	2(2)	70(71)	40(40)	734(1200)	700(734)

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	1(1)	29(29)	-	6(6)	-	-	-	14(14)	-	50(50)	RHO= -.271
X= 15	20(20)	26(26)	3(3)	-	-	-	-	0(1)	-	49(50)	RHO= -.283
X= 30	21(21)	22(22)	6(7)	-	-	-	-	-	-	49(50)	RHO= .188
X= 45	5(5)	30(32)	8(13)	-	-	-	-	-	-	43(50)	RHO= .111
X= 60	3(3)	29(29)	17(18)	-	-	-	-	-	-	49(50)	RHO= .272
X= 75	-	22(22)	21(21)	2(3)	-	-	-	1(1)	3(3)	49(50)	RHO= .462
X= 90	1(1)	12(12)	9(9)	9(9)	-	-	-	7(7)	12(12)	50(50)	RHO= .393
X=105	-	16(17)	28(29)	2(2)	-	-	-	1(1)	1(1)	48(50)	RHO= .091
X=120	1(1)	12(12)	37(37)	-	-	-	-	-	-	50(50)	RHO= .038
X=135	6(6)	15(16)	25(28)	-	-	-	-	-	-	46(50)	RHO= .066
X=150	5(5)	26(27)	15(18)	-	-	-	-	-	-	46(50)	RHO= -.287
X=165	6(6)	21(22)	8(10)	-	1(1)	3(3)	-	6(6)	2(2)	47(50)	RHO= -.438
X=180	1(1)	12(12)	6(8)	-	2(2)	2(2)	-	23(23)	2(2)	48(50)	RHO= -.370
X=195	24(24)	14(14)	6(6)	-	-	-	1(1)	4(4)	1(1)	50(50)	RHO= -.206
X=210	14(14)	21(22)	13(14)	-	-	-	-	-	-	48(50)	RHO= .029
X=225	11(11)	20(21)	18(18)	-	-	-	-	-	-	49(50)	RHO= .191
X=240	4(4)	22(24)	21(22)	-	-	-	-	-	-	47(50)	RHO= .030
X=255	2(2)	18(18)	22(24)	1(1)	-	-	-	-	5(5)	48(50)	RHO= .535
X=270	-	5(6)	16(16)	3(4)	-	-	-	12(12)	12(12)	48(50)	RHO= .359
X=285	1(1)	19(19)	21(24)	2(2)	-	-	-	2(2)	2(2)	47(50)	RHO= .032
X=300	0(1)	14(18)	28(31)	-	-	-	-	-	-	42(50)	RHO= -.510
X=315	2(2)	13(19)	21(29)	-	-	-	-	-	-	36(50)	RHO= -.450
X=330	4(4)	29(29)	16(17)	-	-	-	-	-	-	49(50)	RHO= -.382
X=345	4(4)	27(28)	16(16)	-	-	1(1)	1(1)	-	-	49(50)	RHO= -.360
TOTAL	136(137)	-474(496)	381(418)	25(27)	3(3)	6(6)	2(2)	70(71)	40(40)	1137(1200)	

TABLE 17 29°, 24 M/S, 1 CELL

1 CELL

WIND SPEED 24.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	10(10)	4(4)	1(4, -1)	16(16)	16(16)	-	-	3(3)	1(1)	-	31(50)	31(31)
X= 15	27(27)	10(10)	0(10, 0)	9(9)	4(4)	-	-	-	-	-	41(50)	41(41)
X= 30	23(23)	20(20)	0(20, 0)	7(7)	-	-	-	-	-	-	43(50)	43(43)
X= 45	5(5)	33(33)	0(33, 0)	12(12)	-	-	-	-	-	-	38(50)	38(38)
X= 60	3(3)	31(31)	0(31, 0)	16(16)	-	-	-	-	-	-	34(50)	34(34)
X= 75	-	14(15)	1(15, 0)	29(29)	6(6)	-	-	-	-	-	21(50)	20(21)
X= 90	3(3)	10(10)	5(10, -5)	16(16)	21(21)	-	-	-	-	-	34(50)	34(34)
X=105	9(9)	11(11)	0(11, 0)	26(26)	4(4)	-	-	-	-	-	24(50)	24(24)
X=120	3(3)	10(11)	1(11, 0)	36(36)	-	-	-	-	-	-	14(50)	13(14)
X=135	2(3)	18(20)	2(20, 0)	27(27)	-	-	-	-	-	-	23(50)	20(23)
X=150	10(10)	31(31)	0(31, 0)	9(9)	-	-	-	-	-	-	41(50)	41(41)
X=165	9(9)	10(11)	0(11, 1)	3(3)	-	10(10)	13(13)	2(2)	1(1)	1(1)	31(50)	30(31)
X=180	1(1)	4(5)	0(5, 1)	1(1)	-	1(1)	17(17)	1(1)	22(23)	1(1)	30(50)	28(30)
X=195	14(14)	4(4)	0(4, 0)	1(1)	-	3(3)	6(6)	19(19)	1(1)	2(2)	22(50)	22(22)
X=210	20(20)	16(19)	3(19, 0)	11(11)	-	-	-	-	-	-	39(50)	36(39)
X=225	26(26)	20(20)	0(20, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=240	13(13)	19(20)	1(20, 0)	17(17)	-	-	-	-	-	-	33(50)	32(33)
X=255	1(1)	9(9)	0(9, 0)	11(11)	0(28)	-	-	-	-	1(1)	38(50)	10(38)
X=270	-	-	-	1(1)	0(26)	-	-	-	11(11)	12(12)	37(50)	11(37)
X=285	7(7)	5(5)	0(5, 0)	11(11)	0(25)	-	-	-	1(1)	1(1)	38(50)	13(38)
X=300	3(3)	18(19)	1(19, 0)	28(28)	-	-	-	-	-	-	22(50)	21(22)
X=315	2(2)	19(19)	0(19, 0)	29(29)	-	-	-	-	-	-	21(50)	21(21)
X=330	13(15)	23(24)	1(24, 0)	11(11)	-	-	-	-	-	-	39(50)	36(39)
X=345	8(8)	29(29)	3(29, -3)	10(10)	-	-	-	2(2)	1(1)	-	38(50)	38(38)
TOTAL	212(215)	368(380)	19(380, -7)	341(341)	51(130)	14(14)	36(36)	27(27)	38(39)	18(18)	778(1200)	683(778)

1 CELL

WIND SPEED 24.0 THETA BEAMS 1,2,3: 29.0 22.0 29.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	10(10)	4(4)	16(16)	16(16)	-	-	2(3)	1(1)	-	49(50)	RHO= -.438
X= 15	27(27)	10(10)	9(9)	4(4)	-	-	-	-	-	50(50)	RHO= -.590
X= 30	23(23)	20(20)	7(7)	-	-	-	-	-	-	50(50)	RHO= -.068
X= 45	5(5)	33(33)	10(12)	-	-	-	-	-	-	48(50)	RHO= .387
X= 60	3(3)	31(31)	15(16)	-	-	-	-	-	-	49(50)	RHO= .169
X= 75	-	15(15)	29(29)	6(6)	-	-	-	-	-	50(50)	RHO= .533
X= 90	3(3)	10(10)	16(16)	21(21)	-	-	-	-	-	50(50)	RHO= .054
X=105	9(9)	11(11)	25(26)	4(4)	-	-	-	-	-	49(50)	RHO= -.097
X=120	3(3)	10(11)	35(36)	-	-	-	-	-	-	48(50)	RHO= -.165
X=135	2(3)	17(20)	26(27)	-	-	-	-	-	-	45(50)	RHO= -.152
X=150	10(10)	30(31)	9(9)	-	-	-	-	-	-	49(50)	RHO= -.066
X=165	9(9)	11(11)	3(3)	-	10(10)	13(13)	2(2)	1(1)	1(1)	50(50)	RHO= -.651
X=180	1(1)	5(5)	0(1)	-	1(1)	17(17)	1(1)	22(23)	1(1)	48(50)	RHO= -.166
X=195	14(14)	4(4)	0(1)	-	3(3)	6(6)	19(19)	1(1)	2(2)	49(50)	RHO= -.120
X=210	20(20)	17(19)	8(11)	-	-	-	-	-	-	45(50)	RHO= .006
X=225	26(26)	19(20)	4(4)	-	-	-	-	-	-	49(50)	RHO= .210
X=240	13(13)	19(20)	17(17)	-	-	-	-	-	-	49(50)	RHO= .051
X=255	1(1)	9(9)	10(11)	13(28)	-	-	-	-	1(1)	34(50)	RHO= .580
X=270	-	-	1(1)	13(26)	-	-	-	11(11)	12(12)	37(50)	RHO= .204
X=285	7(7)	5(5)	9(11)	14(25)	-	-	-	1(1)	1(1)	37(50)	RHO= .112
X=300	3(3)	18(19)	25(28)	-	-	-	-	-	-	46(50)	RHO= -.463
X=315	2(2)	19(19)	24(29)	-	-	-	-	-	-	45(50)	RHO= -.298
X=330	14(15)	24(24)	10(11)	-	-	-	-	-	-	48(50)	RHO= -.075
X=345	8(8)	29(29)	10(10)	-	-	-	2(2)	1(1)	-	50(50)	RHO= -.479
TOTAL	213(215)	370(380)	318(341)	91(130)	14(14)	36(36)	26(27)	38(39)	18(18)	1124(1200)	

TABLE 18 39°, 4 M/S, 1 CELL

1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	2(2)	0(2)	1(1)	1(1)	16(18)	2(3)	23(23)	6(50)	3(6)
X= 15	-	1(1)	0(1, 0)	4(4)	0(2)	-	1(1)	24(27)	6(7)	8(8)	10(50)	7(10)
X= 30	1(1)	18(18)	0(18, 0)	26(26)	-	-	-	5(5)	-	-	19(50)	19(19)
X= 45	-	11(12)	1(12, 0)	38(38)	-	-	-	-	-	-	12(50)	11(12)
X= 60	1(1)	2(2)	0(2, 0)	32(33)	0(2)	-	-	6(6)	0(4)	1(2)	9(50)	3(9)
X= 75	0(1)	0(1)	1(1, 0)	3(3)	-	-	-	20(22)	1(4)	17(19)	6(50)	1(6)
X= 90	-	-	-	-	-	2(2)	2(2)	10(14)	17(19)	11(13)	21(50)	19(21)
X=105	1(1)	1(1)	0(1, 0)	1(1)	-	2(2)	8(8)	14(15)	13(14)	7(8)	18(50)	17(18)
X=120	3(5)	5(5)	0(5, 0)	19(19)	-	-	2(2)	16(16)	-	2(3)	10(50)	8(10)
X=135	-	3(7)	2(7, 2)	39(40)	-	-	-	3(3)	-	-	7(50)	3(7)
X=150	1(1)	2(6)	3(6, 1)	38(39)	-	-	-	4(4)	-	-	7(50)	3(7)
X=165	2(2)	5(8)	2(8, 1)	23(25)	1(1)	-	-	10(10)	-	4(4)	11(50)	8(11)
X=180	-	2(4)	2(4, 0)	12(19)	1(1)	-	0(1)	17(17)	0(1)	5(7)	6(50)	3(6)
X=195	-	0(2)	2(2, 0)	23(33)	-	-	-	11(11)	-	3(4)	2(50)	0(2)
X=210	-	1(3)	2(3, 0)	43(47)	-	-	-	-	-	-	3(50)	1(3)
X=225	-	3(4)	1(4, 0)	41(46)	-	-	-	-	-	-	4(50)	3(4)
X=240	0(1)	0(6)	6(6, 0)	32(40)	-	-	-	0(1)	1(1)	1(1)	8(50)	1(8)
X=255	0(1)	0(6)	4(6, 2)	23(25)	1(1)	-	-	10(11)	1(2)	4(4)	10(50)	2(10)
X=270	1(1)	0(3)	3(3, 0)	19(20)	1(1)	-	-	15(18)	0(2)	5(5)	7(50)	2(7)
X=285	2(3)	1(5)	4(5, 0)	26(27)	-	-	0(1)	9(10)	0(1)	3(3)	9(50)	3(9)
X=300	0(2)	2(4)	2(4, 0)	39(41)	-	-	-	3(3)	-	-	6(50)	2(6)
X=315	3(3)	5(9)	4(9, 0)	37(38)	-	-	-	-	-	-	12(50)	8(12)
X=330	1(1)	0(1)	1(1, 0)	29(32)	-	-	-	12(13)	-	3(3)	2(50)	1(2)
X=345	-	1(1)	0(1, 0)	4(7)	-	1(1)	4(4)	20(23)	1(1)	12(13)	3(50)	3(3)
TOTAL	16(24)	63(109)	40(109, 6)	553(605)	4(10)	6(6)	18(20)	225(247)	42(59)	109(120)	208(1200)	131(208)

1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	2(2)	2(2)	1(1)	1(1)	18(18)	3(3)	23(23)	50(50)	RHO= -.728
X= 15	-	1(1)	3(4)	2(2)	-	1(1)	26(27)	7(7)	8(8)	48(50)	RHO= -.032
X= 30	1(1)	18(18)	23(26)	-	-	-	5(5)	-	-	47(50)	RHO= .581
X= 45	-	6(12)	23(38)	-	-	-	-	-	-	29(50)	RHO= .558
X= 60	0(1)	1(2)	11(33)	2(2)	-	-	5(6)	3(4)	1(2)	23(50)	RHO= .651
X= 75	0(1)	0(1)	1(3)	-	-	-	17(22)	4(4)	17(19)	39(50)	RHO= .464
X= 90	-	-	-	-	2(2)	2(2)	14(14)	19(19)	13(13)	50(50)	RHO= .553
X=105	1(1)	1(1)	1(1)	-	2(2)	8(8)	15(15)	14(14)	8(8)	50(50)	RHO= .499
X=120	3(5)	5(5)	14(19)	-	-	2(2)	12(16)	-	3(3)	39(50)	RHO= .797
X=135	-	4(7)	32(40)	-	-	-	0(3)	-	-	36(50)	RHO= -.082
X=150	1(1)	4(6)	26(39)	-	-	-	4(4)	-	-	35(50)	RHO= -.588
X=165	2(2)	6(8)	19(25)	0(1)	-	-	4(10)	-	0(4)	31(50)	RHO= -.864
X=180	-	4(4)	12(19)	1(1)	-	1(1)	15(17)	1(1)	4(7)	38(50)	RHO= -.550
X=195	-	1(2)	29(33)	-	-	-	7(11)	-	4(4)	41(50)	RHO= -.555
X=210	-	3(3)	42(47)	-	-	-	-	-	-	45(50)	RHO= -.035
X=225	-	4(4)	30(46)	-	-	-	-	-	-	34(50)	RHO= .633
X=240	0(1)	3(6)	28(40)	-	-	-	0(1)	0(1)	1(1)	32(50)	RHO= .717
X=255	1(1)	1(6)	18(25)	0(1)	-	-	5(11)	1(2)	1(4)	27(50)	RHO= .659
X=270	1(1)	2(3)	14(20)	1(1)	-	-	5(18)	0(2)	0(5)	23(50)	RHO= .546
X=285	2(3)	1(5)	18(27)	-	-	0(1)	8(10)	0(1)	0(3)	29(50)	RHO= .183
X=300	0(2)	3(4)	30(41)	-	-	-	3(3)	-	-	36(50)	RHO= .275
X=315	3(3)	5(9)	28(38)	-	-	-	-	-	-	36(50)	RHO= -.643
X=330	1(1)	1(1)	26(32)	-	-	-	13(13)	-	3(3)	44(50)	RHO= -.757
X=345	-	1(1)	3(7)	-	1(1)	4(4)	23(23)	1(1)	13(13)	46(50)	RHO= -.755
TOTAL	16(24)	75(109)	433(605)	8(10)	6(6)	19(20)	199(247)	53(59)	99(120)	908(1200)	

TABLE 19 39°, 4 M/S, 5 CELLS

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	-	1(1)	26(26)	3(3)	20(20)	3(50)	3(3)
X= 15	-	-	-	-	-	1(1)	5(5)	42(42)	1(1)	1(1)	2(50)	2(2)
X= 30	13(13)	10(10)	0(10, 0)	27(27)	-	-	-	-	-	-	23(50)	23(23)
X= 45	1(1)	24(24)	0(24, 0)	25(25)	-	-	-	-	-	-	25(50)	25(25)
X= 60	6(6)	7(8)	1(8, 0)	33(33)	-	-	-	3(3)	-	-	14(50)	13(14)
X= 75	-	-	-	1(1)	0(4)	-	-	34(34)	0(7)	4(4)	11(50)	0(11)
X= 90	-	-	-	-	-	4(4)	10(10)	3(3)	26(26)	6(7)	30(50)	30(30)
X=105	2(2)	-	-	-	-	13(13)	8(8)	23(23)	3(3)	1(1)	18(50)	18(18)
X=120	17(17)	3(3)	0(3, 0)	22(22)	-	-	-	8(8)	-	-	20(50)	20(20)
X=135	3(3)	7(11)	4(11, 0)	36(36)	-	-	-	-	-	-	14(50)	10(14)
X=150	16(16)	4(5)	1(5, 0)	29(29)	-	-	-	-	-	-	21(50)	20(21)
X=165	6(6)	4(4)	0(4, 0)	29(29)	2(2)	-	-	9(9)	-	-	12(50)	12(12)
X=180	-	8(8)	0(8, 0)	23(23)	2(2)	-	-	16(16)	-	1(1)	10(50)	10(10)
X=195	-	1(3)	2(3, 0)	36(36)	1(1)	-	-	10(10)	-	-	4(50)	2(4)
X=210	6(6)	7(14)	7(14, 0)	30(30)	-	-	-	-	-	-	20(50)	13(20)
X=225	1(1)	5(10)	5(10, 0)	39(39)	-	-	-	-	-	-	11(50)	6(11)
X=240	-	8(11)	3(11, 0)	39(39)	-	-	-	-	-	-	11(50)	8(11)
X=255	0(1)	8(12)	5(12, -1)	22(22)	1(1)	-	-	14(14)	-	-	14(50)	9(14)
X=270	1(1)	1(1)	1(1, -1)	20(20)	3(3)	-	-	24(24)	1(1)	-	6(50)	6(6)
X=285	13(14)	3(4)	1(4, 0)	25(25)	2(2)	-	-	5(5)	-	-	20(50)	18(20)
X=300	2(4)	7(12)	5(12, 0)	34(34)	-	-	-	-	-	-	16(50)	9(16)
X=315	13(13)	4(5)	1(5, 0)	32(32)	-	-	-	-	-	-	18(50)	17(18)
X=330	22(22)	2(4)	2(4, 0)	22(22)	-	-	-	2(2)	-	-	26(50)	24(26)
X=345	-	-	-	1(1)	-	-	1(1)	44(44)	0(1)	3(3)	1(50)	0(1)
TOTAL	122(126)	113(149)	38(149, -2)	525(525)	11(15)	18(18)	25(25)	263(263)	34(42)	36(37)	350(1200)	298(350)

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	-	1(1)	25(26)	3(3)	20(20)	49(50)	RHO= -.798
X= 15	-	-	-	-	1(1)	5(5)	42(42)	1(1)	1(1)	50(50)	RHO= .074
X= 30	13(13)	10(10)	27(27)	-	-	-	-	-	-	50(50)	RHO= .619
X= 45	1(1)	22(24)	23(25)	-	-	-	-	-	-	46(50)	RHO= .585
X= 60	6(6)	8(8)	24(33)	-	-	-	2(3)	-	-	40(50)	RHO= .438
X= 75	-	-	0(1)	4(4)	-	-	31(34)	7(7)	4(4)	46(50)	RHO= .505
X= 90	-	-	-	-	4(4)	10(10)	3(3)	26(26)	7(7)	50(50)	RHO= .699
X=105	2(2)	-	-	-	13(13)	8(8)	23(23)	3(3)	1(1)	50(50)	RHO= .756
X=120	17(17)	3(3)	22(22)	-	-	-	8(8)	-	-	50(50)	RHO= .240
X=135	3(3)	10(11)	35(36)	-	-	-	-	-	-	48(50)	RHO= -.395
X=150	16(16)	5(5)	29(29)	-	-	-	-	-	-	50(50)	RHO= -.582
X=165	6(6)	4(4)	28(29)	2(2)	-	-	8(9)	-	-	48(50)	RHO= -.841
X=180	-	7(8)	23(23)	2(2)	-	-	14(16)	-	1(1)	47(50)	RHO= -.775
X=195	-	3(3)	36(36)	1(1)	-	-	9(10)	-	-	49(50)	RHO= -.355
X=210	6(6)	14(14)	30(30)	-	-	-	-	-	-	50(50)	RHO= .503
X=225	1(1)	10(10)	36(39)	-	-	-	-	-	-	47(50)	RHO= .726
X=240	-	8(11)	32(39)	-	-	-	-	-	-	40(50)	RHO= .680
X=255	0(1)	10(12)	21(22)	1(1)	-	-	9(14)	-	-	41(50)	RHO= .478
X=270	1(1)	1(1)	15(20)	2(3)	-	-	14(24)	0(1)	-	33(50)	RHO= .617
X=285	13(14)	4(4)	22(25)	2(2)	-	-	5(5)	-	-	46(50)	RHO= .589
X=300	2(4)	8(12)	20(34)	-	-	-	-	-	-	30(50)	RHO= -.378
X=315	13(13)	5(5)	29(32)	-	-	-	-	-	-	47(50)	RHO= -.832
X=330	22(22)	4(4)	20(22)	-	-	-	2(2)	-	-	48(50)	RHO= -.801
X=345	-	-	1(1)	-	-	1(1)	44(44)	1(1)	3(3)	50(50)	RHO= -.874
TOTAL	122(126)	136(149)	473(525)	14(15)	18(18)	25(25)	239(263)	41(42)	37(37)	1105(1200)	

TABLE 20 39°, 8 M/S, 1 CELL

1 CELL

WIND SPEED 8.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	1(1)	0(1, 0)	-	-	1(1)	11(11)	8(8)	25(25)	4(4)	27(50)	27(27)
X= 15	1(1)	3(3)	0(3, 0)	-	-	1(1)	3(3)	39(39)	2(2)	1(1)	7(50)	7(7)
X= 30	20(20)	17(18)	1(18, 0)	12(12)	-	-	-	-	-	-	38(50)	37(38)
X= 45	2(2)	33(34)	1(34, 0)	14(14)	-	-	-	-	-	-	36(50)	35(36)
X= 60	5(5)	24(26)	2(26, 0)	19(19)	-	-	-	-	-	-	31(50)	29(31)
X= 75	-	2(2)	0(2, 0)	11(11)	0(1)	-	-	24(24)	1(4)	8(8)	7(50)	3(7)
X= 90	-	-	-	-	-	2(2)	7(7)	15(15)	12(13)	13(13)	15(50)	14(15)
X=105	4(4)	-	-	3(3)	-	1(1)	1(1)	36(36)	5(5)	-	10(50)	10(10)
X=120	6(6)	20(20)	0(20, 0)	23(23)	-	-	-	1(1)	-	-	26(50)	26(26)
X=135	8(8)	8(11)	3(11, 0)	31(31)	-	-	-	-	-	-	19(50)	16(19)
X=150	18(18)	3(5)	2(5, 0)	27(27)	-	-	-	-	-	-	23(50)	21(23)
X=165	10(10)	10(11)	1(11, 0)	24(24)	-	-	-	5(5)	-	-	21(50)	20(21)
X=180	4(4)	11(13)	2(13, 0)	13(13)	1(1)	-	-	19(19)	-	-	18(50)	16(18)
X=195	6(6)	16(20)	4(20, 0)	22(22)	1(1)	-	-	1(1)	-	-	27(50)	23(27)
X=210	6(6)	15(18)	3(18, 0)	26(26)	-	-	-	-	-	-	24(50)	21(24)
X=225	5(5)	13(19)	5(19, 1)	26(26)	-	-	-	-	-	-	24(50)	18(24)
X=240	1(1)	15(19)	4(19, 0)	30(30)	-	-	-	-	-	-	20(50)	16(20)
X=255	5(6)	12(13)	2(13, -1)	27(27)	1(1)	-	-	3(3)	-	-	20(50)	18(20)
X=270	3(3)	2(2)	1(2, -1)	19(19)	2(2)	-	-	24(24)	-	-	7(50)	7(7)
X=285	7(7)	9(9)	0(9, 0)	30(30)	1(1)	-	-	3(3)	-	-	17(50)	17(17)
X=300	3(3)	12(16)	4(16, 0)	31(31)	-	-	-	-	-	-	19(50)	15(19)
X=315	19(19)	8(8)	0(8, 0)	23(23)	-	-	-	-	-	-	27(50)	27(27)
X=330	15(15)	4(5)	1(5, 0)	30(30)	-	-	-	-	-	-	20(50)	19(20)
X=345	-	2(2)	0(2, 0)	5(5)	0(5)	1(1)	1(1)	25(25)	1(2)	9(9)	10(50)	4(10)
TOTAL	148(149)	240(275)	36(275, -1)	446(446)	6(12)	6(6)	23(23)	203(203)	46(51)	35(35)	493(1200)	446(493)

1 CELL

WIND SPEED 8.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	1(1)	-	-	1(1)	11(11)	8(8)	25(25)	4(4)	50(50)	RHO= -.597
X= 15	1(1)	3(3)	-	-	1(1)	3(3)	39(39)	2(2)	1(1)	50(50)	RHO= -.389
X= 30	20(20)	17(18)	11(12)	-	-	-	-	-	-	48(50)	RHO= .301
X= 45	2(2)	28(34)	11(14)	-	-	-	-	-	-	41(50)	RHO= .544
X= 60	5(5)	24(26)	17(19)	-	-	-	-	-	-	46(50)	RHO= .427
X= 75	-	2(2)	10(11)	1(1)	-	-	23(24)	3(4)	8(8)	47(50)	RHO= .509
X= 90	-	-	-	-	2(2)	7(7)	15(15)	13(13)	13(13)	50(50)	RHO= .446
X=105	4(4)	-	3(3)	-	1(1)	1(1)	36(36)	5(5)	-	50(50)	RHO= .183
X=120	6(6)	20(20)	21(23)	-	-	-	0(1)	-	-	47(50)	RHO= .098
X=135	8(8)	9(11)	30(31)	-	-	-	-	-	-	47(50)	RHO= .039
X=150	18(18)	4(5)	27(27)	-	-	-	-	-	-	49(50)	RHO= -.569
X=165	10(10)	10(11)	24(24)	-	-	-	3(5)	-	-	47(50)	RHO= -.392
X=180	4(4)	12(13)	12(13)	1(1)	-	-	16(19)	-	-	45(50)	RHO= -.572
X=195	6(6)	20(20)	19(22)	1(1)	-	-	1(1)	-	-	47(50)	RHO= -.425
X=210	6(6)	18(18)	21(26)	-	-	-	-	-	-	45(50)	RHO= .174
X=225	5(5)	18(19)	22(26)	-	-	-	-	-	-	45(50)	RHO= .521
X=240	1(1)	15(19)	24(30)	-	-	-	-	-	-	40(50)	RHO= .581
X=255	5(6)	12(13)	20(27)	1(1)	-	-	1(3)	-	-	39(50)	RHO= .624
X=270	3(3)	2(2)	16(19)	2(2)	-	-	17(24)	-	-	40(50)	RHO= .724
X=285	7(7)	9(9)	26(30)	1(1)	-	-	2(3)	-	-	45(50)	RHO= .510
X=300	3(3)	12(16)	26(31)	-	-	-	-	-	-	41(50)	RHO= .009
X=315	19(19)	8(8)	20(23)	-	-	-	-	-	-	47(50)	RHO= -.625
X=330	15(15)	5(5)	28(30)	-	-	-	-	-	-	48(50)	RHO= -.409
X=345	-	2(2)	4(5)	5(5)	1(1)	1(1)	25(25)	2(2)	9(9)	49(50)	RHO= -.666
TOTAL	148(149)	251(275)	392(446)	12(12)	6(6)	23(23)	186(203)	50(51)	35(35)	1103(1200)	

TABLE 21 39°, 8 M/S, 1 CELL ± 0.7 DB

1 CELL WITH 0.7 DB ERROR

WIND SPEED 8.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	4(4)	0(4, 0)	2(2)	-	1(1)	3(3)	11(11)	20(20)	9(9)	25(50)	25(25)
X= 15	-	6(6)	0(6, 0)	3(3)	-	-	6(6)	20(20)	5(6)	9(9)	12(50)	11(12)
X= 30	4(4)	17(20)	3(20, 0)	24(24)	-	-	-	2(2)	-	-	24(50)	21(24)
X= 45	0(1)	20(23)	3(23, 0)	26(26)	-	-	-	-	-	-	24(50)	20(24)
X= 60	1(1)	10(11)	0(11, 1)	36(36)	-	-	-	2(2)	-	-	12(50)	11(12)
X= 75	-	-	-	20(20)	-	-	-	14(14)	4(5)	11(11)	5(50)	4(5)
X= 90	1(1)	1(1)	0(1, 0)	4(4)	-	-	1(1)	19(19)	6(7)	17(17)	9(50)	8(9)
X=105	2(2)	4(4)	0(4, 0)	6(6)	-	-	3(3)	20(20)	5(5)	10(10)	11(50)	11(11)
X=120	2(2)	12(14)	0(14, 2)	28(28)	-	-	-	5(5)	-	1(1)	16(50)	14(16)
X=135	-	9(14)	3(14, 2)	36(36)	-	-	-	-	-	-	14(50)	9(14)
X=150	2(2)	8(11)	2(11, 1)	36(36)	-	-	-	1(1)	-	-	13(50)	10(13)
X=165	2(2)	4(9)	5(9, 0)	30(30)	2(2)	-	-	7(7)	-	-	13(50)	8(13)
X=180	2(2)	4(11)	7(11, 0)	15(15)	2(2)	-	-	15(15)	1(1)	4(4)	16(50)	9(16)
X=195	1(1)	6(15)	9(15, 0)	27(27)	1(1)	-	-	6(6)	-	-	17(50)	8(17)
X=210	1(2)	9(16)	5(16, 2)	31(31)	-	-	-	1(1)	-	-	18(50)	10(18)
X=225	0(1)	8(15)	7(15, 0)	34(34)	-	-	-	-	-	-	16(50)	8(16)
X=240	-	5(9)	4(9, 0)	40(40)	1(1)	-	-	-	-	-	10(50)	6(10)
X=255	0(1)	7(7)	0(7, 0)	32(32)	-	-	-	9(9)	-	1(1)	8(50)	7(8)
X=270	1(1)	2(3)	2(3, -1)	22(22)	-	-	-	21(21)	-	3(3)	4(50)	3(4)
X=285	3(3)	3(5)	3(5, -1)	34(34)	1(1)	-	-	4(4)	-	3(3)	9(50)	7(9)
X=300	1(1)	3(8)	5(8, 0)	41(41)	-	-	-	-	-	-	9(50)	4(9)
X=315	6(6)	10(11)	1(11, 0)	33(33)	-	-	-	-	-	-	17(50)	16(17)
X=330	2(2)	4(5)	1(5, 0)	39(39)	0(1)	-	-	2(2)	-	1(1)	8(50)	6(8)
X=345	-	2(3)	1(3, 0)	10(10)	0(2)	1(1)	3(3)	13(13)	2(7)	11(11)	13(50)	5(13)
TOTAL	31(35)	158(225)	61(225, 6)	609(609)	7(10)	2(2)	16(16)	172(172)	43(51)	80(80)	323(1200)	241(323)

1 CELL WITH 7/10'S DB ERROR

WIND SPEED 8.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	4(4)	2(2)	-	1(1)	3(3)	11(11)	20(20)	9(9)	50(50)	RHO= -.455
X= 15	-	6(6)	1(3)	-	-	6(6)	16(20)	6(6)	9(9)	44(50)	RHO= -.545
X= 30	4(4)	15(20)	17(24)	-	-	-	1(2)	-	-	37(50)	RHO= -.053
X= 45	0(1)	16(23)	14(26)	-	-	-	-	-	-	30(50)	RHO= .353
X= 60	1(1)	10(11)	27(36)	-	-	-	2(2)	-	-	40(50)	RHO= .546
X= 75	-	-	17(20)	-	-	-	14(14)	5(5)	9(11)	45(50)	RHO= .437
X= 90	1(1)	1(1)	3(4)	-	-	1(1)	18(19)	7(7)	17(17)	48(50)	RHO= .374
X=105	2(2)	4(4)	6(6)	-	-	3(3)	17(20)	5(5)	10(10)	47(50)	RHO= -.054
X=120	2(2)	14(14)	26(28)	-	-	-	5(5)	-	0(1)	47(50)	RHO= -.062
X=135	-	11(14)	25(36)	-	-	-	-	-	-	36(50)	RHO= .203
X=150	2(2)	9(11)	30(36)	-	-	-	1(1)	-	-	42(50)	RHO= -.342
X=165	2(2)	5(9)	26(30)	1(2)	-	-	1(7)	-	-	35(50)	RHO= -.245
X=180	2(2)	4(11)	12(15)	1(2)	-	-	12(15)	1(1)	0(4)	32(50)	RHO= -.395
X=195	1(1)	8(15)	20(27)	1(1)	-	-	5(6)	-	-	35(50)	RHO= -.403
X=210	1(2)	13(16)	20(31)	-	-	-	1(1)	-	-	35(50)	RHO= -.186
X=225	0(1)	10(15)	22(34)	-	-	-	-	-	-	32(50)	RHO= .467
X=240	-	5(9)	26(40)	1(1)	-	-	-	-	-	32(50)	RHO= .197
X=255	0(1)	7(7)	18(32)	-	-	-	7(9)	-	0(1)	32(50)	RHO= .571
X=270	1(1)	2(3)	10(22)	-	-	-	11(21)	-	1(3)	25(50)	RHO= .538
X=285	3(3)	2(5)	22(34)	1(1)	-	-	3(4)	-	1(3)	32(50)	RHO= .480
X=300	1(1)	3(8)	29(41)	-	-	-	-	-	-	33(50)	RHO= .016
X=315	6(6)	10(11)	26(33)	-	-	-	-	-	-	42(50)	RHO= -.580
X=330	2(2)	4(5)	29(39)	1(1)	-	-	1(2)	-	1(1)	38(50)	RHO= -.096
X=345	-	3(3)	9(10)	0(2)	1(1)	3(3)	12(13)	6(7)	11(11)	45(50)	RHO= -.549
TOTAL	31(35)	166(225)	437(609)	6(10)	2(2)	16(16)	138(172)	50(51)	68(80)	914(1200)	

TABLE 22 39°, 12 M/S, 1 CELL

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1•	2A•	2B	3	4•	5•	6	7	8•	9	UNIQUE	CORRECT
X= 0	-	3(3)	0(3, 0)	-	-	6(6)	20(20)	-	20(20)	1(1)	29(50)	29(29)
X= 15	6(6)	24(24)	0(24, 0)	6(6)	-	2(2)	3(3)	8(8)	1(1)	-	33(50)	33(33)
X= 30	24(24)	17(18)	1(18, 0)	8(8)	-	-	-	-	-	-	42(50)	41(42)
X= 45	5(5)	36(37)	1(37, 0)	8(8)	-	-	-	-	-	-	42(50)	41(42)
X= 60	2(2)	35(35)	0(35, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X= 75	7(7)	6(6)	0(6, 0)	14(14)	-	-	-	19(19)	-	4(4)	13(50)	13(13)
X= 90	-	-	-	4(4)	-	1(1)	1(1)	16(16)	18(18)	10(10)	19(50)	19(19)
X=105	10(10)	7(7)	0(7, 0)	16(16)	-	-	-	14(14)	3(3)	-	20(50)	20(20)
X=120	2(2)	26(26)	0(26, 0)	22(22)	-	-	-	-	-	-	28(50)	28(28)
X=135	14(14)	16(16)	0(16, 0)	20(20)	-	-	-	-	-	-	30(50)	30(30)
X=150	22(22)	12(13)	1(13, 0)	15(15)	-	-	-	-	-	-	35(50)	34(35)
X=165	16(16)	8(9)	1(9, 0)	19(19)	4(4)	-	-	2(2)	-	-	29(50)	28(29)
X=180	5(5)	6(7)	1(7, 0)	13(13)	8(8)	-	-	10(10)	7(7)	-	27(50)	26(27)
X=195	21(21)	10(11)	1(11, 0)	14(14)	1(1)	-	-	3(3)	-	-	33(50)	32(33)
X=210	16(16)	17(18)	1(18, 0)	16(16)	-	-	-	-	-	-	34(50)	33(34)
X=225	12(12)	20(21)	1(21, 0)	17(17)	-	-	-	-	-	-	33(50)	32(33)
X=240	3(3)	25(27)	2(27, 0)	20(20)	-	-	-	-	-	-	30(50)	28(30)
X=255	3(3)	25(26)	2(26, -1)	19(19)	-	-	-	2(2)	-	-	29(50)	28(29)
X=270	4(4)	3(3)	1(3, -1)	21(21)	5(5)	-	-	9(9)	5(5)	3(3)	17(50)	17(17)
X=285	16(16)	9(10)	2(10, -1)	22(22)	-	-	-	2(2)	-	-	26(50)	25(26)
X=300	5(6)	24(27)	3(27, 0)	17(17)	-	-	-	-	-	-	33(50)	29(33)
X=315	13(13)	8(10)	2(10, 0)	27(27)	-	-	-	-	-	-	23(50)	21(23)
X=330	11(11)	16(17)	1(17, 0)	22(22)	-	-	-	-	-	-	28(50)	27(28)
X=345	6(6)	12(14)	2(14, 0)	12(12)	0(3)	1(1)	1(1)	13(13)	-	-	24(50)	19(24)
TOTAL	223(224)	365(365)	23(365, -3)	365(365)	18(21)	10(10)	25(25)	98(98)	54(54)	18(18)	694(1200)	670(694)

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	3(3)	-	-	6(6)	20(20)	-	20(20)	1(1)	50(50)	RHO= -.397
X= 15	6(6)	24(24)	6(6)	-	2(2)	3(3)	8(8)	0(1)	-	49(50)	RHO= -.568
X= 30	24(24)	18(18)	7(8)	-	-	-	-	-	-	49(50)	RHO= -.033
X= 45	5(5)	35(37)	5(6)	-	-	-	-	-	-	45(50)	RHO= .115
X= 60	2(2)	35(35)	13(13)	-	-	-	-	-	-	50(50)	RHO= .422
X= 75	7(7)	6(6)	14(14)	-	-	-	19(19)	-	4(4)	50(50)	RHO= .476
X= 90	-	-	2(4)	-	1(1)	1(1)	16(16)	18(18)	10(10)	48(50)	RHO= .385
X=105	10(10)	7(7)	13(16)	-	-	-	14(14)	3(3)	-	47(50)	RHO= -.227
X=120	2(2)	26(26)	22(22)	-	-	-	-	-	-	50(50)	RHO= -.137
X=135	14(14)	16(16)	19(20)	-	-	-	-	-	-	49(50)	RHO= .072
X=150	22(22)	13(13)	15(15)	-	-	-	-	-	-	50(50)	RHO= -.166
X=165	16(16)	8(9)	18(19)	4(4)	-	-	1(2)	-	-	47(50)	RHO= -.391
X=180	5(5)	6(7)	10(13)	8(8)	-	-	10(10)	7(7)	-	46(50)	RHO= -.386
X=195	21(21)	11(11)	14(14)	1(1)	-	-	2(3)	-	-	49(50)	RHO= -.301
X=210	16(16)	18(18)	14(16)	-	-	-	-	-	-	46(50)	RHO= -.064
X=225	12(12)	21(21)	17(17)	-	-	-	-	-	-	50(50)	RHO= .280
X=240	3(3)	25(27)	19(20)	-	-	-	-	-	-	47(50)	RHO= .134
X=255	3(3)	25(26)	17(19)	-	-	-	2(2)	-	-	47(50)	RHO= .508
X=270	4(4)	3(3)	20(21)	5(5)	-	-	7(9)	4(5)	3(3)	46(50)	RHO= .440
X=285	16(16)	9(10)	22(22)	-	-	-	2(2)	-	-	49(50)	RHO= -.074
X=300	5(6)	24(27)	14(17)	-	-	-	-	-	-	43(50)	RHO= -.607
X=315	13(13)	6(10)	27(27)	-	-	-	-	-	-	48(50)	RHO= -.464
X=330	11(11)	17(17)	21(22)	-	-	-	-	-	-	49(50)	RHO= -.367
X=345	6(6)	14(14)	12(12)	3(3)	1(1)	1(1)	12(13)	-	-	49(50)	RHO= -.469
TOTAL	223(224)	372(385)	341(365)	21(21)	10(10)	25(25)	93(98)	52(54)	18(18)	1155(1200)	

TABLE 23 39°, 12 M/S, 1 CELL ± 0.7 DB

1 CELL WITH 7/10'S DB ERROR

WIND SPEED 12.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	12(12)	0(12, 0)	6(6)	0(1)	1(1)	1(1)	6(6)	14(14)	9(9)	28(50)	27(28)
X= 15	2(2)	15(16)	1(16, 0)	14(14)	-	1(1)	2(2)	8(8)	3(5)	2(2)	24(50)	21(24)
X= 30	4(7)	15(19)	4(19, 0)	24(24)	-	-	-	-	-	-	26(50)	19(26)
X= 45	-	15(16)	1(16, 0)	34(34)	-	-	-	-	-	-	16(50)	15(16)
X= 60	1(1)	16(16)	0(16, 0)	31(31)	-	-	-	2(2)	-	-	17(50)	17(17)
X= 75	-	5(7)	1(7, 1)	19(19)	-	-	-	8(8)	3(4)	12(12)	11(50)	8(11)
X= 90	-	3(4)	0(4, 1)	10(10)	-	1(1)	-	13(13)	7(8)	14(14)	13(50)	11(13)
X=105	1(1)	5(5)	0(5, 0)	25(25)	-	-	1(1)	10(10)	5(5)	3(3)	11(50)	11(11)
X=120	1(1)	8(10)	0(10, 2)	37(37)	-	-	-	2(2)	-	-	11(50)	9(11)
X=135	3(4)	8(8)	0(8, 0)	38(38)	-	-	-	-	-	-	12(50)	11(12)
X=150	4(4)	6(9)	2(9, 1)	35(35)	1(1)	-	-	-	-	1(1)	14(50)	11(14)
X=165	1(1)	10(10)	0(10, 0)	22(22)	4(4)	-	-	3(3)	4(4)	6(6)	19(50)	19(19)
X=180	3(3)	3(6)	3(6, 0)	17(17)	3(3)	-	-	5(5)	7(8)	8(8)	20(50)	16(20)
X=195	6(7)	5(7)	2(7, 0)	26(26)	2(2)	-	0(1)	2(2)	2(3)	2(2)	19(50)	15(19)
X=210	1(2)	16(22)	4(22, 2)	26(26)	-	-	-	-	-	-	24(50)	17(24)
X=225	1(2)	10(13)	2(13, 1)	34(34)	-	-	-	1(1)	-	-	15(50)	11(15)
X=240	1(2)	10(15)	4(15, 1)	31(31)	-	-	-	1(1)	-	1(1)	17(50)	11(17)
X=255	-	8(10)	2(10, 0)	29(29)	1(1)	-	-	5(5)	-	5(5)	11(50)	9(11)
X=270	1(1)	5(6)	2(6, -1)	18(18)	1(1)	-	-	11(11)	-	13(13)	8(50)	7(8)
X=285	2(2)	7(9)	3(9, -1)	29(29)	-	-	-	4(4)	1(1)	5(5)	12(50)	10(12)
X=300	-	6(11)	5(11, 0)	38(38)	-	-	-	1(1)	-	-	11(50)	6(11)
X=315	1(2)	7(9)	2(9, 0)	39(39)	-	-	-	-	-	-	11(50)	8(11)
X=330	3(3)	6(9)	2(9, 1)	34(34)	-	-	-	1(1)	0(2)	1(1)	14(50)	9(14)
X=345	2(2)	4(7)	3(7, 0)	20(20)	0(1)	1(1)	-	7(7)	4(8)	4(4)	19(50)	11(19)
TOTAL	38(47)	205(256)	43(256, 8)	636(636)	12(14)	4(4)	4(5)	90(90)	50(62)	86(86)	383(1200)	309(383)

EOF:40

1 CELL WITH 0.7DB ERROR

WIND SPEED 12.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	12(12)	5(6)	0(1)	1(1)	1(1)	6(6)	14(14)	8(9)	47(50)	RHO= -.469
X= 15	2(2)	15(16)	13(14)	-	1(1)	2(2)	7(8)	4(5)	2(2)	46(50)	RHO= -.557
X= 30	4(7)	14(19)	16(24)	-	-	-	-	-	-	34(50)	RHO= -.375
X= 45	-	15(16)	23(34)	-	-	-	-	-	-	38(50)	RHO= -.077
X= 60	1(1)	16(16)	21(31)	-	-	-	2(2)	-	-	40(50)	RHO= .515
X= 75	-	5(7)	17(19)	-	-	-	8(8)	3(4)	11(12)	44(50)	RHO= .454
X= 90	-	3(4)	6(10)	-	1(1)	-	12(13)	7(8)	13(14)	42(50)	RHO= .078
X=105	1(1)	5(5)	22(25)	-	-	1(1)	8(10)	5(5)	2(3)	44(50)	RHO= -.112
X=120	1(1)	9(10)	35(37)	-	-	-	2(2)	-	-	47(50)	RHO= .142
X=135	3(4)	7(8)	28(38)	-	-	-	-	-	-	38(50)	RHO= .037
X=150	4(4)	5(9)	26(35)	0(1)	-	-	-	-	1(1)	36(50)	RHO= -.134
X=165	1(1)	9(10)	14(22)	4(4)	-	-	3(3)	3(4)	2(6)	36(50)	RHO= -.569
X=180	3(3)	3(6)	8(17)	3(3)	-	-	4(5)	7(8)	4(8)	32(50)	RHO= -.271
X=195	6(7)	5(7)	21(26)	2(2)	-	0(1)	2(2)	2(3)	1(2)	39(50)	RHO= -.411
X=210	1(2)	19(22)	17(26)	-	-	-	-	-	-	37(50)	RHO= -.358
X=225	1(2)	12(13)	24(34)	-	-	-	0(1)	-	-	37(50)	RHO= .039
X=240	2(2)	11(15)	22(31)	-	-	-	0(1)	-	1(1)	36(50)	RHO= .183
X=255	-	9(10)	19(29)	1(1)	-	-	4(5)	-	3(5)	36(50)	RHO= .545
X=270	1(1)	5(6)	9(18)	1(1)	-	-	8(11)	-	4(13)	28(50)	RHO= .292
X=285	2(2)	7(9)	20(29)	-	-	-	2(4)	1(1)	5(5)	37(50)	RHO= -.135
X=300	-	6(11)	28(38)	-	-	-	1(1)	-	-	35(50)	RHO= -.153
X=315	1(2)	9(9)	28(39)	-	-	-	-	-	-	38(50)	RHO= -.480
X=330	3(3)	8(9)	26(34)	-	-	-	1(1)	2(2)	1(1)	41(50)	RHO= -.279
X=345	2(2)	6(7)	15(20)	0(1)	1(1)	-	5(7)	5(8)	3(4)	37(50)	RHO= -.330
TOTAL	39(47)	215(256)	463(636)	11(14)	4(4)	4(5)	75(90)	53(62)	61(86)	925(1200)	

EOF:40
0:

TABLE 24 39^{o}, 24 M/S, 1 CELL

1 CELL

WIND SPEED 24.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	7(7)	7(7)	3(7, -3)	14(14)	19(19)	-	-	2(2)	1(1)	-	34(50)	34(34)
X= 15	21(21)	13(13)	1(13, -1)	13(13)	3(3)	-	-	-	-	-	37(50)	37(37)
X= 30	27(27)	16(16)	0(16, 0)	7(7)	-	-	-	-	-	-	43(50)	43(43)
X= 45	5(5)	42(42)	0(42, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X= 60	4(4)	39(39)	0(39, 0)	7(7)	-	-	-	-	-	-	43(50)	43(43)
X= 75	1(1)	23(23)	0(23, 0)	20(20)	6(6)	-	-	-	-	-	30(50)	30(30)
X= 90	3(3)	7(7)	5(7, -5)	19(19)	21(21)	-	-	-	-	-	31(50)	31(31)
X=105	14(14)	12(12)	0(12, 0)	20(20)	4(4)	-	-	-	-	-	30(50)	30(30)
X=120	5(5)	13(13)	0(13, 0)	32(32)	-	-	-	-	-	-	18(50)	18(18)
X=135	9(9)	29(30)	1(30, 0)	11(11)	-	-	-	-	-	-	39(50)	38(39)
X=150	15(15)	33(33)	0(33, 0)	2(2)	-	-	-	-	-	-	48(50)	48(48)
X=165	10(10)	7(9)	0(9, 2)	2(2)	-	14(14)	13(13)	1(1)	1(1)	-	34(50)	32(34)
X=180	-	4(5)	0(5, 1)	1(1)	-	3(3)	15(15)	-	24(24)	2(2)	32(50)	31(32)
X=195	13(13)	4(4)	0(4, 0)	1(1)	-	9(9)	11(11)	9(9)	2(2)	1(1)	28(50)	28(28)
X=210	24(24)	15(17)	2(17, 0)	9(9)	-	-	-	-	-	-	41(50)	39(41)
X=225	29(29)	19(19)	0(19, 0)	2(2)	-	-	-	-	-	-	48(50)	48(48)
X=240	12(12)	31(32)	1(32, 0)	6(6)	-	-	-	-	-	-	44(50)	43(44)
X=255	-	9(9)	0(9, 0)	9(9)	0(29)	-	-	0(3)	-	-	38(50)	9(38)
X=270	-	-	-	-	0(27)	-	-	-	13(13)	10(10)	40(50)	13(40)
X=285	8(8)	5(5)	0(5, 0)	8(8)	0(28)	-	-	-	-	1(1)	41(50)	13(41)
X=300	6(6)	27(27)	0(27, 0)	17(17)	-	-	-	-	-	-	33(50)	33(33)
X=315	8(8)	28(28)	0(28, 0)	14(14)	-	-	-	-	-	-	36(50)	36(36)
X=330	4(5)	34(35)	0(35, 1)	10(10)	-	-	-	-	-	-	40(50)	38(40)
X=345	12(12)	27(27)	4(27, -4)	7(7)	1(1)	-	-	3(3)	-	-	40(50)	40(40)
TOTAL	237(238)	444(452)	17(452, -9)	234(234)	54(138)	26(26)	39(39)	15(18)	41(41)	14(14)	895(1200)	802(895)

1 CELL

WIND SPEED 24.0 THETA BEAMS 1,2,3: 39.0 30.5 39.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	7(7)	7(7)	14(14)	19(19)	-	-	2(2)	1(1)	-	50(50)	RHO= -.509
X= 15	21(21)	13(13)	13(13)	3(3)	-	-	-	-	-	50(50)	RHO= -.736
X= 30	27(27)	16(16)	7(7)	-	-	-	-	-	-	50(50)	RHO= -.327
X= 45	5(5)	42(42)	3(3)	-	-	-	-	-	-	50(50)	RHO= .327
X= 60	4(4)	39(39)	6(7)	-	-	-	-	-	-	49(50)	RHO= .350
X= 75	1(1)	23(23)	20(20)	6(6)	-	-	-	-	-	50(50)	RHO= .691
X= 90	3(3)	7(7)	19(19)	21(21)	-	-	-	-	-	50(50)	RHO= .134
X=105	14(14)	12(12)	19(20)	4(4)	-	-	-	-	-	49(50)	RHO= -.211
X=120	5(5)	13(13)	32(32)	-	-	-	-	-	-	50(50)	RHO= -.367
X=135	9(9)	30(30)	11(11)	-	-	-	-	-	-	50(50)	RHO= -.174
X=150	15(15)	33(33)	2(2)	-	-	-	-	-	-	50(50)	RHO= .036
X=165	10(10)	9(9)	2(2)	-	14(14)	13(13)	1(1)	1(1)	-	50(50)	RHO= -.618
X=180	-	5(5)	1(1)	-	3(3)	15(15)	-	24(24)	2(2)	50(50)	RHO= -.269
X=195	13(13)	4(4)	1(1)	-	9(9)	11(11)	9(9)	2(2)	1(1)	50(50)	RHO= -.335
X=210	24(24)	17(17)	9(9)	-	-	-	-	-	-	50(50)	RHO= -.074
X=225	29(29)	19(19)	2(2)	-	-	-	-	-	-	50(50)	RHO= .130
X=240	12(12)	31(32)	6(6)	-	-	-	-	-	-	49(50)	RHO= .040
X=255	-	9(9)	8(9)	12(29)	-	-	1(3)	-	-	30(50)	RHO= .562
X=270	-	-	-	13(27)	-	-	-	13(13)	10(10)	36(50)	RHO= .166
X=285	8(8)	5(5)	7(8)	14(28)	-	-	-	-	1(1)	35(50)	RHO= .093
X=300	6(6)	27(27)	15(17)	-	-	-	-	-	-	48(50)	RHO= -.524
X=315	8(8)	28(28)	14(14)	-	-	-	-	-	-	50(50)	RHO= -.354
X=330	4(5)	35(35)	10(10)	-	-	-	-	-	-	49(50)	RHO= .010
X=345	12(12)	27(27)	7(7)	1(1)	-	-	3(3)	-	-	50(50)	RHO= -.441
TOTAL	237(238)	451(452)	228(234)	93(138)	26(26)	39(39)	16(18)	41(41)	14(14)	1145(1200)	

TABLE 25 47^{O}, 4 M/S, 1 CELL

1
1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	1(5)	-	-	-	5(19)	0(1)	7(25)	1(50)	0(1)
X= 15	-	1(1)	0(1, 0)	0(4)	0(1)	-	1(1)	9(18)	3(4)	5(21)	6(50)	4(6)
X= 30	-	2(2)	0(2, 0)	19(31)	-	-	1(1)	7(14)	-	2(2)	2(50)	2(2)
X= 45	1(2)	8(8)	0(8, 0)	29(33)	-	-	-	5(6)	-	1(1)	10(50)	9(10)
X= 60	-	2(2)	0(2, 0)	19(21)	0(1)	-	-	10(11)	0(5)	9(10)	8(50)	2(8)
X= 75	-	0(1)	0(1, 1)	5(6)	0(1)	-	-	9(14)	-	18(28)	2(50)	0(2)
X= 90	-	-	-	1(1)	-	1(1)	1(1)	7(15)	2(6)	7(26)	7(50)	3(7)
X=105	-	-	-	2(6)	-	-	-	4(10)	3(4)	8(30)	4(50)	3(4)
X=120	-	1(2)	1(2, 0)	12(19)	-	-	-	6(12)	2(2)	12(15)	4(50)	3(4)
X=135	-	-	-	21(38)	-	-	-	5(7)	-	5(5)	0(50)	-
X=150	-	0(1)	0(1, 1)	21(42)	-	-	-	-	-	7(7)	1(50)	0(1)
X=165	1(1)	0(2)	2(2, 0)	9(27)	1(1)	-	-	5(11)	0(1)	2(7)	5(50)	2(5)
X=180	-	0(2)	1(2, 1)	6(19)	-	-	-	6(18)	-	2(11)	2(50)	0(2)
X=195	-	-	-	10(30)	-	-	-	3(11)	1(1)	2(8)	1(50)	1(1)
X=210	-	0(2)	2(2, 0)	22(41)	-	-	-	5(5)	-	1(2)	2(50)	0(2)
X=225	-	-	-	16(43)	-	-	-	1(1)	-	3(6)	0(50)	-
X=240	-	0(5)	5(5, 0)	14(28)	-	-	-	4(7)	1(1)	5(9)	6(50)	1(6)
X=255	-	0(2)	2(2, 0)	12(27)	-	-	-	5(11)	-	5(10)	2(50)	0(2)
X=270	-	-	-	7(19)	-	-	-	9(11)	2(2)	10(18)	2(50)	2(2)
X=285	-	0(1)	0(1, 1)	17(34)	1(1)	-	-	3(4)	1(1)	1(9)	3(50)	2(3)
X=300	-	1(3)	2(3, 0)	20(41)	-	-	-	2(2)	-	2(4)	3(50)	1(3)
X=315	-	2(2)	0(2, 0)	22(43)	-	-	-	2(2)	-	2(3)	2(50)	2(2)
X=330	1(1)	1(1)	0(1, 0)	16(28)	-	1(1)	-	7(14)	-	4(5)	3(50)	3(3)
X=345	-	0(1)	1(1, 0)	3(5)	0(1)	-	-	9(15)	5(6)	7(22)	8(50)	5(8)
TOTAL	3(4)	18(38)	16(38, 4)	304(591)	2(6)	2(2)	3(3)	128(238)	20(34)	127(284)	84(1200)	45(84)

1 CELL

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	3(5)	-	-	-	17(19)	1(1)	25(25)	46(50)	RHO= -.705
X= 15	-	0(1)	0(4)	1(1)	-	1(1)	15(18)	4(4)	21(21)	42(50)	RHO= -.272
X= 30	-	2(2)	22(31)	-	-	1(1)	11(14)	-	2(2)	38(50)	RHO= .063
X= 45	0(2)	5(8)	8(33)	-	-	-	1(6)	-	0(1)	14(50)	RHO= .442
X= 60	-	2(2)	2(21)	0(1)	-	-	3(11)	5(5)	8(10)	20(50)	RHO= .815
X= 75	-	0(1)	2(6)	1(1)	-	-	8(14)	-	21(28)	32(50)	RHO= .596
X= 90	-	-	0(1)	-	1(1)	1(1)	14(15)	6(6)	26(26)	48(50)	RHO= .605
X=105	-	-	6(6)	-	-	-	10(10)	4(4)	30(30)	50(50)	RHO= .353
X=120	-	1(2)	15(19)	-	-	-	8(12)	2(2)	12(15)	38(50)	RHO= .584
X=135	-	-	22(38)	-	-	-	2(7)	-	1(5)	25(50)	RHO= .097
X=150	-	1(1)	20(42)	-	-	-	-	-	2(7)	23(50)	RHO= -.702
X=165	1(1)	2(2)	15(27)	1(1)	-	-	1(11)	0(1)	0(7)	20(50)	RHO= -.828
X=180	-	1(2)	11(19)	-	-	-	7(18)	-	0(11)	19(50)	RHO= -.665
X=195	-	-	21(30)	-	-	-	6(11)	1(1)	2(8)	30(50)	RHO= -.435
X=210	-	1(2)	23(41)	-	-	-	3(5)	-	1(2)	28(50)	RHO= .174
X=225	-	-	19(43)	-	-	-	1(1)	-	2(6)	22(50)	RHO= .655
X=240	-	1(5)	9(28)	-	-	-	4(7)	0(1)	4(9)	18(50)	RHO= .776
X=255	-	0(2)	15(27)	-	-	-	5(11)	-	0(10)	20(50)	RHO= .519
X=270	-	-	10(19)	-	-	-	8(11)	0(2)	4(18)	22(50)	RHO= .150
X=285	-	0(1)	14(34)	0(1)	-	-	3(4)	0(1)	2(9)	19(50)	RHO= -.088
X=300	-	1(3)	24(41)	-	-	-	1(2)	-	2(4)	28(50)	RHO= -.249
X=315	-	2(2)	35(43)	-	-	-	2(2)	-	3(3)	42(50)	RHO= -.596
X=330	1(1)	1(1)	23(28)	-	1(1)	-	14(14)	-	5(5)	45(50)	RHO= -.879
X=345	-	1(1)	1(5)	1(1)	-	-	15(15)	6(6)	22(22)	46(50)	RHO= -.897
TOTAL	2(4)	21(38)	320(591)	4(6)	2(2)	3(3)	159(238)	29(34)	195(284)	735(1200)	

TABLE 26 47°, 4 M/S, 5 CELLS

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	0(1)	-	3(3)	16(23)	2(3)	17(20)	4(50)	2(4)
X= 15	-	-	-	-	0(2)	-	3(3)	29(32)	5(5)	3(8)	7(50)	5(7)
X= 30	0(1)	4(4)	0(4, 0)	35(39)	-	-	-	6(6)	-	-	5(50)	4(5)
X= 45	-	14(14)	0(14, 0)	34(36)	-	-	-	-	-	-	14(50)	14(14)
X= 60	-	1(3)	2(3, 0)	30(30)	0(3)	-	-	14(14)	-	-	6(50)	1(6)
X= 75	-	-	-	1(1)	0(9)	-	1(1)	20(22)	0(7)	8(10)	16(50)	0(16)
X= 90	-	-	-	-	-	-	5(5)	7(10)	7(9)	19(26)	9(50)	7(9)
X=105	2(2)	-	-	1(1)	-	10(10)	5(5)	9(15)	8(8)	5(9)	20(50)	20(20)
X=120	5(5)	2(2)	0(2, 0)	12(17)	-	1(1)	1(1)	18(22)	-	1(2)	8(50)	8(8)
X=135	1(3)	2(5)	2(5, 1)	38(42)	-	-	-	-	-	-	8(50)	3(8)
X=150	4(4)	1(3)	1(3, 1)	32(43)	-	-	-	-	-	-	7(50)	5(7)
X=165	-	1(3)	1(3, 1)	24(33)	1(1)	-	-	5(9)	-	1(4)	4(50)	2(4)
X=180	-	1(3)	2(3, 0)	20(28)	1(1)	-	-	7(13)	1(1)	3(4)	5(50)	3(5)
X=195	-	1(2)	1(2, 0)	23(32)	1(1)	-	-	5(10)	-	4(5)	3(50)	2(3)
X=210	0(1)	0(5)	5(5, 0)	36(43)	1(1)	-	-	-	-	-	7(50)	1(7)
X=225	-	1(7)	6(7, 0)	25(43)	-	-	-	-	-	-	7(50)	1(7)
X=240	-	0(2)	2(2, 0)	34(46)	1(1)	-	-	1(1)	-	-	3(50)	1(3)
X=255	1(1)	1(2)	1(2, 0)	18(25)	2(2)	-	-	11(18)	-	1(2)	5(50)	4(5)
X=270	-	2(2)	1(2, -1)	13(20)	1(1)	-	-	15(19)	1(2)	4(6)	5(50)	4(5)
X=285	0(1)	4(6)	2(6, 0)	23(28)	2(2)	-	-	8(11)	1(1)	1(1)	10(50)	7(10)
X=300	2(3)	1(2)	1(2, 0)	38(45)	-	-	-	-	-	-	5(50)	3(5)
X=315	3(3)	0(2)	2(2, 0)	32(43)	-	-	-	2(2)	-	-	5(50)	3(5)
X=330	6(6)	2(2)	0(2, 0)	22(25)	-	1(1)	1(1)	14(15)	-	-	9(50)	9(9)
X=345	-	-	-	1(2)	-	-	6(6)	20(30)	1(3)	6(9)	3(50)	1(3)
TOTAL	24(30)	38(69)	29(69, 2)	492(622)	10(25)	12(12)	25(25)	207(272)	26(39)	73(106)	175(1200)	110(175)

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	1(1)	-	3(3)	22(23)	3(3)	20(20)	49(50)	RHO= -.868
X= 15	-	-	-	2(2)	-	3(3)	30(32)	5(5)	8(8)	48(50)	RHO= -.539
X= 30	0(1)	3(4)	28(39)	-	-	-	5(6)	-	-	36(50)	RHO= .808
X= 45	-	10(14)	20(36)	-	-	-	-	-	-	30(50)	RHO= .782
X= 60	-	0(3)	14(30)	1(3)	-	-	8(14)	-	-	23(50)	RHO= .766
X= 75	-	-	0(1)	9(9)	-	1(1)	19(22)	7(7)	9(10)	45(50)	RHO= .684
X= 90	-	-	-	-	-	5(5)	10(10)	9(9)	26(26)	50(50)	RHO= .704
X=105	2(2)	-	1(1)	-	10(10)	5(5)	15(15)	8(8)	9(9)	50(50)	RHO= .697
X=120	5(5)	2(2)	17(17)	-	1(1)	1(1)	22(22)	-	2(2)	50(50)	RHO= .827
X=135	1(3)	2(5)	31(42)	-	-	-	-	-	-	34(50)	RHO= -.438
X=150	4(4)	1(3)	38(43)	-	-	-	-	-	-	43(50)	RHO= -.639
X=165	-	2(3)	26(33)	0(1)	-	-	2(9)	-	0(4)	30(50)	RHO= -.703
X=180	-	2(3)	21(28)	1(1)	-	-	8(13)	1(1)	1(4)	34(50)	RHO= -.657
X=195	-	1(2)	19(32)	0(1)	-	-	3(10)	-	2(5)	25(50)	RHO= -.555
X=210	1(1)	4(5)	24(43)	1(1)	-	-	-	-	-	30(50)	RHO= .118
X=225	-	5(7)	29(43)	-	-	-	-	-	-	34(50)	RHO= .774
X=240	-	1(2)	31(46)	0(1)	-	-	0(1)	-	-	32(50)	RHO= .789
X=255	1(1)	1(2)	18(25)	1(2)	-	-	3(18)	-	0(2)	24(50)	RHO= .712
X=270	-	2(2)	17(20)	1(1)	-	-	9(19)	0(2)	2(6)	31(50)	RHO= .628
X=285	0(1)	6(6)	20(28)	1(2)	-	-	4(11)	0(1)	1(1)	32(50)	RHO= .428
X=300	2(3)	1(2)	27(45)	-	-	-	-	-	-	30(50)	RHO= .212
X=315	3(3)	2(2)	40(43)	-	-	-	2(2)	-	-	47(50)	RHO= -.889
X=330	6(6)	1(2)	20(25)	-	1(1)	1(1)	15(15)	-	-	44(50)	RHO= -.876
X=345	-	-	1(2)	-	-	6(6)	30(30)	3(3)	9(9)	49(50)	RHO= -.901
TOTAL	25(30)	46(69)	442(622)	18(25)	12(12)	25(25)	207(272)	36(39)	89(106)	900(1200)	

TABLE 27 47^{o}, 4 M/S, 25 CELLS

25 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	-	3(3)	19(20)	7(7)	20(20)	7(50)	7(7)
X= 15	-	-	-	-	-	2(2)	3(3)	43(44)	-	1(1)	2(50)	2(2)
X= 30	1(1)	5(8)	3(8, 0)	41(41)	-	-	-	-	-	-	9(50)	6(9)
X= 45	4(4)	23(23)	0(23, 0)	23(23)	-	-	-	-	-	-	27(50)	27(27)
X= 60	6(6)	2(2)	0(2, 0)	42(42)	-	-	-	-	-	-	8(50)	8(8)
X= 75	-	-	-	-	0(4)	1(1)	-	31(33)	1(6)	5(6)	11(50)	2(11)
X= 90	-	-	-	-	-	6(6)	8(8)	3(4)	21(21)	7(11)	27(50)	27(27)
X=105	-	-	-	-	-	24(24)	11(11)	10(12)	3(3)	-	27(50)	27(27)
X=120	14(14)	1(1)	0(1, 0)	9(9)	-	2(2)	-	24(24)	-	-	17(50)	17(17)
X=135	2(2)	5(7)	2(7, 0)	41(41)	-	-	-	-	-	-	9(50)	7(9)
X=150	14(14)	1(1)	0(1, 0)	35(35)	-	-	-	-	-	-	15(50)	15(15)
X=165	5(5)	2(6)	4(6, 0)	33(36)	1(1)	-	-	2(2)	-	-	12(50)	8(12)
X=180	1(1)	3(6)	3(6, 0)	22(24)	2(2)	-	0(1)	15(16)	-	-	9(50)	6(9)
X=195	-	2(4)	2(4, 0)	40(41)	1(1)	-	-	4(4)	-	-	5(50)	3(5)
X=210	-	0(9)	9(9, 0)	41(41)	-	-	-	-	-	-	9(50)	0(9)
X=225	1(2)	1(5)	4(5, 0)	43(43)	-	-	-	-	-	-	7(50)	2(7)
X=240	-	0(7)	7(7, 0)	42(43)	-	-	-	-	-	-	7(50)	0(7)
X=255	0(1)	3(6)	3(6, 0)	36(38)	1(1)	-	-	4(4)	-	-	8(50)	4(8)
X=270	-	1(2)	1(2, 0)	16(23)	6(6)	-	-	17(19)	-	-	8(50)	7(8)
X=285	9(9)	0(3)	3(3, 0)	31(34)	1(1)	-	-	2(3)	-	-	13(50)	10(13)
X=300	3(4)	5(10)	5(10, 0)	35(36)	-	-	-	-	-	-	14(50)	8(14)
X=315	24(24)	-	-	26(26)	-	-	-	-	-	-	24(50)	24(24)
X=330	18(18)	-	-	15(17)	-	1(1)	-	13(14)	-	-	19(50)	19(19)
X=345	-	-	-	-	0(1)	4(4)	14(14)	22(23)	4(4)	4(4)	9(50)	8(9)
TOTAL	102(105)	54(100)	46(100, 0)	571(593)	12(17)	40(40)	39(40)	209(222)	36(41)	37(42)	303(1200)	244(303)

25 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	-	3(3)	20(20)	7(7)	20(20)	50(50)	RHO= -.916
X= 15	-	-	-	-	2(2)	3(3)	44(44)	-	1(1)	50(50)	RHO= -.687
X= 30	1(1)	5(8)	30(41)	-	-	-	-	-	-	36(50)	RHO= .632
X= 45	2(4)	21(23)	16(23)	-	-	-	-	-	-	39(50)	RHO= .775
X= 60	4(6)	2(2)	36(42)	-	-	-	-	-	-	42(50)	RHO= .696
X= 75	-	-	-	2(4)	1(1)	-	32(33)	6(6)	6(6)	47(50)	RHO= .744
X= 90	-	-	-	-	6(6)	8(8)	4(4)	21(21)	11(11)	50(50)	RHO= .714
X=105	-	-	-	-	24(24)	11(11)	12(12)	3(3)	-	50(50)	RHO= .585
X=120	14(14)	1(1)	9(9)	-	2(2)	-	24(24)	-	-	50(50)	RHO= .048
X=135	2(2)	5(7)	38(41)	-	-	-	-	-	-	45(50)	RHO= -.007
X=150	14(14)	1(1)	35(35)	-	-	-	-	-	-	50(50)	RHO= -.830
X=165	5(5)	6(6)	36(36)	1(1)	-	-	1(2)	-	-	49(50)	RHO= -.904
X=180	1(1)	5(6)	20(24)	1(2)	-	0(1)	10(16)	-	-	37(50)	RHO= -.921
X=195	-	2(4)	28(41)	0(1)	-	-	2(4)	-	-	32(50)	RHO= -.545
X=210	-	7(9)	29(41)	-	-	-	-	-	-	36(50)	RHO= .342
X=225	2(2)	3(5)	34(43)	-	-	-	-	-	-	39(50)	RHO= .853
X=240	-	4(7)	35(43)	-	-	-	-	-	-	39(50)	RHO= .860
X=255	0(1)	5(6)	31(38)	0(1)	-	-	1(4)	-	-	37(50)	RHO= .848
X=270	-	2(2)	22(23)	6(6)	-	-	14(19)	-	-	44(50)	RHO= .780
X=285	9(9)	1(3)	29(34)	1(1)	-	-	2(3)	-	-	42(50)	RHO= .721
X=300	3(4)	9(10)	24(36)	-	-	-	-	-	-	36(50)	RHO= -.259
X=315	24(24)	-	26(26)	-	-	-	-	-	-	50(50)	RHO= -.838
X=330	18(18)	-	17(17)	-	1(1)	-	14(14)	-	-	50(50)	RHO= -.874
X=345	-	-	-	1(1)	4(4)	14(14)	23(23)	4(4)	4(4)	50(50)	RHO= -.904
TOTAL	99(105)	79(100)	495(593)	12(17)	40(40)	39(40)	203(222)	41(41)	42(42)	1050(1200)	

TABLE 28 47°, 8 M/S, 1 CELL

1 CELL

WIND SPEED 8.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	1(1)	0(2)	-	5(5)	25(25)	3(3)	14(14)	5(50)	3(5)
X= 15	-	1(1)	0(1, 0)	5(5)	-	1(1)	8(8)	32(32)	1(1)	2(2)	3(50)	3(3)
X= 30	-	11(16)	5(16, 0)	34(34)	-	-	-	-	-	-	16(50)	11(16)
X= 45	-	19(19)	0(19, 0)	31(31)	-	-	-	-	-	-	19(50)	19(19)
X= 60	3(3)	15(15)	0(15, 0)	30(30)	-	-	-	2(2)	-	-	18(50)	18(18)
X= 75	-	1(1)	0(1, 0)	6(7)	0(2)	-	-	26(26)	2(7)	7(7)	10(50)	3(10)
X= 90	-	-	-	-	0(1)	1(1)	3(3)	12(12)	16(19)	14(14)	21(50)	17(21)
X=105	2(2)	1(1)	0(1, 0)	5(5)	0(2)	2(2)	2(2)	28(31)	3(3)	2(2)	10(50)	8(10)
X=120	4(4)	13(13)	0(13, 0)	26(27)	-	-	-	6(6)	-	-	17(50)	17(17)
X=135	4(6)	7(8)	1(8, 0)	36(36)	-	-	-	-	-	-	14(50)	11(14)
X=150	8(8)	3(3)	0(3, 0)	39(39)	-	-	-	-	-	-	11(50)	11(11)
X=165	2(2)	1(3)	2(3, 0)	26(30)	-	-	-	12(13)	1(1)	1(1)	6(50)	4(6)
X=180	3(3)	1(2)	1(2, 0)	20(23)	-	-	0(1)	16(18)	-	3(3)	5(50)	4(5)
X=195	-	2(7)	5(7, 0)	35(35)	1(1)	-	0(1)	5(5)	-	1(1)	8(50)	3(8)
X=210	-	6(10)	4(10, 0)	38(40)	-	-	-	-	-	-	10(50)	6(10)
X=225	-	7(9)	2(9, 0)	40(41)	-	-	-	-	-	-	9(50)	7(9)
X=240	-	3(4)	1(4, 0)	41(44)	-	-	-	2(2)	-	-	4(50)	3(4)
X=255	-	3(4)	2(4, -1)	36(36)	-	-	-	9(9)	1(1)	-	5(50)	4(5)
X=270	1(1)	1(2)	1(2, 0)	24(24)	1(1)	-	-	13(15)	3(3)	4(4)	7(50)	6(7)
X=285	3(3)	6(7)	2(7, -1)	28(28)	1(1)	-	-	11(11)	-	-	11(50)	10(11)
X=300	6(7)	5(9)	4(9, 0)	33(34)	-	-	-	-	-	-	16(50)	11(16)
X=315	5(6)	4(4)	0(4, 0)	39(40)	-	-	-	-	-	-	10(50)	9(10)
X=330	3(4)	5(7)	2(7, 0)	29(31)	0(1)	-	-	7(7)	-	-	12(50)	8(12)
X=345	-	1(2)	1(2, 0)	4(5)	0(1)	-	3(3)	31(33)	1(2)	4(4)	5(50)	2(5)
TOTAL	44(49)	116(147)	33(147, -2)	606(626)	3(12)	4(4)	21(23)	237(247)	31(40)	52(52)	252(1200)	198(252)

1 CELL

WIND SPEED 8.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	1(1)	2(2)	-	5(5)	25(25)	3(3)	14(14)	50(50)	RHO= -.532
X= 15	-	1(1)	5(5)	-	1(1)	8(8)	30(32)	1(1)	2(2)	48(50)	RHO= -.709
X= 30	-	9(16)	26(34)	-	-	-	-	-	-	35(50)	RHO= .156
X= 45	-	19(19)	18(31)	-	-	-	-	-	-	37(50)	RHO= .222
X= 60	3(3)	11(15)	19(30)	-	-	-	1(2)	-	-	34(50)	RHO= .470
X= 75	-	0(1)	1(7)	2(2)	-	-	25(26)	7(7)	7(7)	42(50)	RHO= .630
X= 90	-	-	-	1(1)	1(1)	3(3)	8(12)	19(19)	14(14)	46(50)	RHO= .677
X=105	2(2)	1(1)	5(5)	2(2)	2(2)	2(2)	31(31)	3(3)	2(2)	50(50)	RHO= .308
X=120	4(4)	11(13)	23(27)	-	-	-	5(6)	-	-	43(50)	RHO= -.018
X=135	4(6)	8(8)	27(36)	-	-	-	-	-	-	39(50)	RHO= -.180
X=150	8(8)	3(3)	35(39)	-	-	-	-	-	-	46(50)	RHO= -.531
X=165	2(2)	2(3)	28(30)	-	-	-	5(13)	0(1)	0(1)	37(50)	RHO= -.725
X=180	2(3)	1(2)	18(23)	-	-	1(1)	11(18)	-	2(3)	35(50)	RHO= -.456
X=195	-	4(7)	28(35)	1(1)	-	0(1)	4(5)	-	0(1)	37(50)	RHO= -.581
X=210	-	7(10)	28(40)	-	-	-	-	-	-	35(50)	RHO= -.340
X=225	-	5(9)	36(41)	-	-	-	-	-	-	41(50)	RHO= .604
X=240	-	4(4)	34(44)	-	-	-	0(2)	-	-	38(50)	RHO= .528
X=255	-	3(4)	26(36)	-	-	-	2(9)	0(1)	-	31(50)	RHO= .820
X=270	1(1)	2(2)	18(24)	1(1)	-	-	5(15)	0(3)	0(4)	27(50)	RHO= .704
X=285	3(3)	6(7)	23(28)	0(1)	-	-	10(11)	-	-	42(50)	RHO= .349
X=300	6(7)	5(9)	24(34)	-	-	-	-	-	-	35(50)	RHO= -.490
X=315	5(6)	4(4)	37(40)	-	-	-	-	-	-	46(50)	RHO= -.700
X=330	4(4)	7(7)	29(31)	1(1)	-	-	7(7)	-	-	48(50)	RHO= -.724
X=345	-	2(2)	3(5)	1(1)	-	3(3)	30(33)	2(2)	4(4)	45(50)	RHO= -.727
TOTAL	44(49)	115(147)	492(626)	11(12)	4(4)	22(23)	199(247)	35(40)	45(52)	967(1200)	

TABLE 29 47°, 8 M/S, 5 CELLS

5 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	2(2)	20(20)	9(9)	12(12)	7(7)	14(50)	14(14)
X= 15	-	-	-	-	-	2(2)	5(5)	41(41)	2(2)	-	4(50)	4(4)
X= 30	8(8)	15(17)	2(17, 0)	25(25)	-	-	-	-	-	-	25(50)	23(25)
X= 45	3(3)	35(35)	0(35, 0)	12(12)	-	-	-	-	-	-	38(50)	38(38)
X= 60	15(15)	22(22)	0(22, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X= 75	-	-	-	-	0(7)	-	-	41(41)	0(1)	1(1)	8(50)	0(8)
X= 90	-	-	-	-	-	6(6)	9(9)	7(7)	27(27)	1(1)	33(50)	33(33)
X=105	1(1)	-	-	-	-	4(4)	3(3)	42(42)	-	-	5(50)	5(5)
X=120	29(29)	4(4)	0(4, 0)	16(16)	-	-	-	1(1)	-	-	33(50)	33(33)
X=135	15(15)	10(12)	2(12, 0)	23(23)	-	-	-	-	-	-	27(50)	25(27)
X=150	33(33)	1(2)	1(2, 0)	15(15)	-	-	-	-	-	-	35(50)	34(35)
X=165	24(24)	4(4)	0(4, 0)	21(21)	-	-	-	1(1)	-	-	28(50)	28(28)
X=180	-	6(6)	0(6, 0)	20(20)	9(9)	-	-	15(15)	-	-	15(50)	15(15)
X=195	2(2)	3(7)	4(7, 0)	38(38)	1(1)	-	-	2(2)	-	-	10(50)	6(10)
X=210	3(3)	4(8)	4(8, 0)	39(39)	-	-	-	-	-	-	11(50)	7(11)
X=225	2(2)	20(22)	2(22, 0)	26(26)	-	-	-	-	-	-	24(50)	22(24)
X=240	2(2)	17(21)	4(21, 0)	27(27)	-	-	-	-	-	-	23(50)	19(23)
X=255	2(3)	6(6)	0(6, 0)	41(41)	-	-	-	-	-	-	9(50)	8(9)
X=270	-	1(1)	1(1, -1)	22(22)	10(10)	-	-	17(17)	-	-	11(50)	11(11)
X=285	18(19)	5(7)	2(7, 0)	24(24)	-	-	-	-	-	-	26(50)	23(26)
X=300	14(14)	11(13)	2(13, 0)	23(23)	-	-	-	-	-	-	27(50)	25(27)
X=315	24(24)	3(4)	1(4, 0)	22(22)	-	-	-	-	-	-	28(50)	27(28)
X=330	22(23)	1(2)	1(2, 0)	25(25)	-	-	-	-	-	-	25(50)	23(25)
X=345	-	-	-	-	0(2)	1(1)	2(2)	44(44)	-	1(1)	3(50)	1(3)
TOTAL	217(220)	168(193)	26(193, -1)	432(432)	20(29)	15(15)	39(39)	220(220)	41(42)	10(10)	499(1200)	461(499)

5 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	2(2)	20(20)	9(9)	12(12)	7(7)	50(50)	RHO= -.667
X= 15	-	-	-	-	2(2)	5(5)	41(41)	2(2)	-	50(50)	RHO= -.548
X= 30	8(8)	16(17)	21(25)	-	-	-	-	-	-	45(50)	RHO= .025
X= 45	3(3)	34(35)	9(12)	-	-	-	-	-	-	46(50)	RHO= .556
X= 60	14(15)	22(22)	12(13)	-	-	-	-	-	-	48(50)	RHO= .407
X= 75	-	-	-	7(7)	-	-	39(41)	1(1)	1(1)	48(50)	RHO= .807
X= 90	-	-	-	-	6(6)	9(9)	7(7)	27(27)	1(1)	50(50)	RHO= .556
X=105	1(1)	-	-	-	4(4)	3(3)	42(42)	-	-	50(50)	RHO= .442
X=120	29(29)	4(4)	16(16)	-	-	-	1(1)	-	-	50(50)	RHO= -.213
X=135	15(15)	12(12)	22(23)	-	-	-	-	-	-	49(50)	RHO= -.389
X=150	33(33)	2(2)	15(15)	-	-	-	-	-	-	50(50)	RHO= -.264
X=165	24(24)	4(4)	21(21)	-	-	-	1(1)	-	-	50(50)	RHO= -.750
X=180	-	6(6)	19(20)	9(9)	-	-	14(15)	-	-	48(50)	RHO= -.732
X=195	2(2)	7(7)	35(38)	0(1)	-	-	2(2)	-	-	46(50)	RHO= -.627
X=210	3(3)	6(8)	33(39)	-	-	-	-	-	-	42(50)	RHO= .071
X=225	2(2)	22(22)	25(26)	-	-	-	-	-	-	49(50)	RHO= .457
X=240	2(2)	20(21)	26(27)	-	-	-	-	-	-	48(50)	RHO= .342
X=255	2(3)	6(6)	39(41)	-	-	-	-	-	-	47(50)	RHO= .724
X=270	-	1(1)	22(22)	9(10)	-	-	13(17)	-	-	45(50)	RHO= .556
X=285	18(19)	6(7)	22(24)	-	-	-	-	-	-	46(50)	RHO= .429
X=300	14(14)	12(13)	21(23)	-	-	-	-	-	-	47(50)	RHO= -.702
X=315	24(24)	4(4)	22(22)	-	-	-	-	-	-	50(50)	RHO= -.726
X=330	23(23)	2(2)	25(25)	-	-	-	-	-	-	50(50)	RHO= -.650
X=345	-	-	-	2(2)	1(1)	2(2)	44(44)	-	1(1)	50(50)	RHO= -.824
TOTAL	217(220)	186(193)	405(432)	27(29)	15(15)	39(39)	213(220)	42(42)	10(10)	1154(1200)	

TABLE 30 47°, 12 M/S, 1 CELL

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2D	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	2(2)	0(2, .0)	-	-	1(1)	10(10)	8(8)	21(22)	7(7)	25(50)	24(25)
X= 15	1(1)	13(13)	0(13, 0)	3(3)	-	2(2)	9(9)	19(19)	-	3(3)	16(50)	16(16)
X= 30	12(12)	19(19)	0(19, 0)	19(19)	-	-	-	-	-	-	31(50)	31(31)
X= 45	1(1)	34(34)	0(34, 0)	15(15)	-	-	-	-	-	-	35(50)	35(35)
X= 60	13(13)	19(20)	1(20, 0)	15(15)	-	-	-	2(2)	-	-	33(50)	32(33)
X= 75	0(1)	-	-	14(14)	0(1)	-	-	21(21)	0(2)	11(11)	4(50)	0(4)
X= 90	-	-	-	2(2)	-	-	3(3)	21(21)	13(13)	11(11)	13(50)	13(13)
X=105	10(10)	1(1)	0(1, 0)	5(5)	-	-	1(1)	28(28)	2(2)	3(3)	13(50)	13(13)
X=120	4(4)	20(21)	1(21, 0)	24(24)	-	-	-	1(1)	-	-	25(50)	24(25)
X=135	9(9)	10(14)	4(14, 0)	27(27)	-	-	-	-	-	-	23(50)	19(23)
X=150	15(15)	5(8)	3(8, 0)	27(27)	-	-	-	-	-	-	23(50)	20(23)
X=165	6(6)	13(14)	1(14, 0)	25(25)	-	-	-	5(5)	-	-	20(50)	19(20)
X=180	2(2)	6(6)	0(6, 0)	14(14)	7(7)	-	0(1)	15(15)	3(3)	2(2)	18(50)	18(18)
X=195	-	12(13)	1(13, 0)	28(28)	-	-	0(2)	7(7)	-	-	13(50)	12(13)
X=210	6(7)	12(16)	4(16, 0)	27(27)	-	-	-	-	-	-	23(50)	18(23)
X=225	2(2)	26(28)	2(28, 0)	20(20)	-	-	-	-	-	-	30(50)	28(30)
X=240	2(4)	22(24)	2(24, 0)	22(22)	-	-	-	-	-	-	28(50)	24(28)
X=255	3(4)	14(15)	2(15, -1)	21(21)	3(3)	-	-	6(6)	-	1(1)	22(50)	20(22)
X=270	0(1)	1(1)	0(1, 0)	16(16)	13(13)	-	-	15(15)	1(3)	1(1)	18(50)	15(18)
X=285	7(7)	14(21)	8(21, -1)	19(19)	-	-	-	2(2)	-	1(1)	28(50)	21(28)
X=300	4(4)	17(18)	1(18, 0)	28(28)	-	-	-	-	-	-	22(50)	21(22)
X=315	21(21)	12(13)	1(13, 0)	16(16)	-	-	-	-	-	-	34(50)	33(34)
X=330	9(9)	2(4)	2(4, 0)	37(37)	-	-	-	-	-	-	13(50)	11(13)
X=345	2(2)	7(8)	0(8, 1)	7(7)	0(1)	1(1)	-	20(20)	4(4)	7(7)	16(50)	14(16)
TOTAL	129(135)	281(313)	33(313, -1)	431(431)	23(25)	4(4)	23(26)	170(170)	44(49)	47(47)	526(1200)	481(526)

1 CELL

WIND SPEED 12.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	2(2)	-	-	1(1)	10(10)	8(8)	22(22)	7(7)	50(50)	RHO= -.549
X= 15	1(1)	13(13)	3(3)	-	2(2)	9(9)	18(19)	-	3(3)	49(50)	RHO= -.634
X= 30	12(12)	19(19)	19(19)	-	-	-	-	-	-	50(50)	RHO= -.217
X= 45	1(1)	34(34)	13(15)	-	-	-	-	-	-	48(50)	RHO= .198
X= 60	13(13)	18(20)	14(15)	-	-	-	2(2)	-	-	47(50)	RHO= .577
X= 75	0(1)	-	10(14)	0(1)	-	-	18(21)	2(2)	11(11)	41(50)	RHO= .731
X= 90	-	-	2(2)	-	-	3(3)	20(21)	13(13)	11(11)	49(50)	RHO= .644
X=105	10(10)	1(1)	4(5)	-	-	1(1)	28(28)	2(2)	3(3)	49(50)	RHO= .187
X=120	4(4)	21(21)	21(24)	-	-	-	1(1)	-	-	47(50)	RHO= -.366
X=135	9(9)	12(14)	26(27)	-	-	-	-	-	-	47(50)	RHO= -.326
X=150	15(15)	6(8)	26(27)	-	-	-	-	-	-	47(50)	RHO= -.279
X=165	6(6)	13(14)	21(25)	-	-	-	4(5)	-	-	44(50)	RHO= -.465
X=180	2(2)	6(6)	12(14)	7(7)	-	0(1)	9(15)	2(3)	1(2)	39(50)	RHO= -.524
X=195	-	13(13)	26(28)	-	-	1(2)	7(7)	-	-	47(50)	RHO= -.510
X=210	6(7)	11(16)	20(27)	-	-	-	-	-	-	37(50)	RHO= -.297
X=225	2(2)	26(28)	16(20)	-	-	-	-	-	-	44(50)	RHO= .463
X=240	2(4)	22(24)	18(22)	-	-	-	-	-	-	42(50)	RHO= .646
X=255	3(4)	15(15)	16(21)	3(3)	-	-	3(6)	-	0(1)	40(50)	RHO= .815
X=270	0(1)	1(1)	14(16)	12(13)	-	-	10(15)	1(3)	0(1)	38(50)	RHO= .729
X=285	7(7)	16(21)	17(19)	-	-	-	1(2)	-	1(1)	42(50)	RHO= -.010
X=300	4(4)	17(18)	22(28)	-	-	-	-	-	-	43(50)	RHO= -.474
X=315	21(21)	11(13)	15(16)	-	-	-	-	-	-	47(50)	RHO= -.666
X=330	9(9)	4(4)	35(37)	-	-	-	-	-	-	48(50)	RHO= -.607
X=345	2(2)	7(8)	6(7)	0(1)	1(1)	-	20(20)	4(4)	7(7)	47(50)	RHO= -.737
TOTAL	129(135)	288(313)	376(431)	22(25)	4(4)	24(26)	149(170)	46(49)	44(47)	1082(1200)	

TABLE 31 47^{o}, 12 M/S, 5 CELLS

5 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	3(3)	19(19)	2(2)	26(26)	-	29(50)	29(29)
X= 15	-	10(10)	0(10, 0)	2(2)	-	9(9)	9(9)	20(20)	-	-	19(50)	19(19)
X= 30	34(34)	13(13)	0(13, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X= 45	6(6)	42(42)	0(42, 0)	2(2)	-	-	-	-	-	-	48(50)	48(48)
X= 60	20(20)	30(30)	0(30, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 75	7(7)	1(1)	0(1, 0)	2(2)	-	-	-	40(40)	-	-	8(50)	8(8)
X= 90	-	-	-	-	-	4(4)	8(8)	13(13)	20(20)	5(5)	24(50)	24(24)
X=105	3(3)	-	-	-	-	-	-	46(46)	-	1(1)	3(50)	3(3)
X=120	27(27)	17(17)	0(17, 0)	6(6)	-	-	-	-	-	-	44(50)	44(44)
X=135	34(34)	7(7)	0(7, 0)	9(9)	-	-	-	-	-	-	41(50)	41(41)
X=150	39(39)	1(1)	0(1, 0)	10(10)	-	-	-	-	-	-	40(50)	40(40)
X=165	35(35)	1(1)	0(1, 0)	14(14)	-	-	-	-	-	-	36(50)	36(36)
X=180	2(2)	20(20)	1(20, -1)	8(8)	13(13)	-	-	7(7)	-	-	35(50)	35(35)
X=195	11(11)	26(30)	4(30, 0)	9(9)	-	-	-	-	-	-	41(50)	37(41)
X=210	18(18)	19(19)	0(19, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X=225	21(21)	26(26)	0(26, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X=240	9(9)	37(37)	0(37, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=255	28(29)	6(6)	0(6, 0)	14(14)	1(1)	-	-	-	-	-	36(50)	35(36)
X=270	1(1)	2(2)	2(2, -2)	18(18)	23(23)	-	-	6(6)	-	-	26(50)	26(26)
X=285	36(36)	6(6)	0(6, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X=300	18(18)	20(20)	0(20, 0)	12(12)	-	-	-	-	-	-	38(50)	38(38)
X=315	42(42)	4(4)	0(4, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=330	30(30)	3(3)	0(3, 0)	17(17)	-	-	-	-	-	-	33(50)	33(33)
X=345	1(1)	1(1)	0(1, 0)	3(3)	-	3(3)	4(4)	36(36)	0(1)	1(1)	6(50)	5(6)
TOTAL	422(423)	292(296)	7(296, -3)	161(161)	37(37)	19(19)	40(40)	170(170)	46(47)	7(7)	822(1200)	816(822)

5 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	3(3)	19(19)	2(2)	26(26)	-	50(50)	RHO= -.730
X= 15	-	10(10)	2(2)	-	9(9)	9(9)	20(20)	-	-	50(50)	RHO= -.345
X= 30	34(34)	13(13)	3(3)	-	-	-	-	-	-	50(50)	RHO= -.064
X= 45	6(6)	42(42)	1(2)	-	-	-	-	-	-	49(50)	RHO= .232
X= 60	20(20)	30(30)	-	-	-	-	-	-	-	50(50)	RHO= .713
X= 75	7(7)	1(1)	2(2)	-	-	-	40(40)	-	-	50(50)	RHO= .640
X= 90	-	-	-	-	4(4)	8(8)	13(13)	20(20)	5(5)	50(50)	RHO= .706
X=105	3(3)	-	-	-	-	-	46(46)	-	1(1)	50(50)	RHO= .070
X=120	27(27)	17(17)	6(6)	-	-	-	-	-	-	50(50)	RHO= -.333
X=135	34(34)	7(7)	9(9)	-	-	-	-	-	-	50(50)	RHO= -.172
X=150	39(39)	1(1)	10(10)	-	-	-	-	-	-	50(50)	RHO= -.350
X=165	35(35)	1(1)	14(14)	-	-	-	-	-	-	50(50)	RHO= -.524
X=180	2(2)	20(20)	8(8)	13(13)	-	-	7(7)	-	-	50(50)	RHO= -.721
X=195	11(11)	30(30)	9(9)	-	-	-	-	-	-	50(50)	RHO= -.213
X=210	18(18)	19(19)	13(13)	-	-	-	-	-	-	50(50)	RHO= -.024
X=225	21(21)	26(26)	3(3)	-	-	-	-	-	-	50(50)	RHO= .454
X=240	9(9)	37(37)	4(4)	-	-	-	-	-	-	50(50)	RHO= .483
X=255	28(29)	6(6)	13(14)	1(1)	-	-	-	-	-	48(50)	RHO= .781
X=270	1(1)	2(2)	18(18)	23(23)	-	-	6(6)	-	-	50(50)	RHO= .468
X=285	36(36)	6(6)	8(8)	-	-	-	-	-	-	50(50)	RHO= -.056
X=300	18(18)	20(20)	12(12)	-	-	-	-	-	-	50(50)	RHO= -.546
X=315	42(42)	4(4)	4(4)	-	-	-	-	-	-	50(50)	RHO= -.626
X=330	30(30)	3(3)	17(17)	-	-	-	-	-	-	50(50)	RHO= -.569
X=345	1(1)	1(1)	3(3)	-	3(3)	4(4)	36(36)	1(1)	1(1)	50(50)	RHO= -.588
TOTAL	422(423)	296(296)	159(161)	37(37)	19(19)	40(40)	170(170)	47(47)	7(7)	1197(1200)	

TABLE 32 47^{o}, 24 M/S, 1 CELL

1 CELLS

WIND SPEED 24.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	3(3)	16(16)	3(16, -3)	9(9)	22(22)	-	-	-	-	-	41(50)	41(41)
X= 15	14(14)	20(20)	2(20, -2)	11(11)	5(5)	-	-	-	-	-	39(50)	39(39)
X= 30	23(23)	21(21)	0(21, 0)	6(6)	-	-	-	-	-	-	44(50)	44(44)
X= 45	-	46(46)	0(46, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X= 60	1(1)	45(45)	0(45, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X= 75	-	30(30)	0(30, 0)	13(13)	3(3)	-	-	3(3)	-	1(1)	33(50)	33(33)
X= 90	4(4)	9(9)	5(9, -5)	14(14)	21(21)	-	-	-	-	2(2)	34(50)	34(34)
X=105	14(14)	13(13)	0(13, 0)	17(17)	5(5)	-	-	-	-	1(1)	32(50)	32(32)
X=120	8(8)	24(24)	0(24, 0)	18(18)	-	-	-	-	-	-	32(50)	32(32)
X=135	10(10)	27(27)	0(27, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X=150	7(7)	35(35)	0(35, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X=165	3(3)	5(8)	0(8, 3)	2(2)	-	12(12)	19(19)	1(1)	5(5)	-	28(50)	25(28)
X=180	1(1)	3(3)	0(3, 0)	-	-	6(6)	18(18)	1(1)	21(21)	-	31(50)	31(31)
X=195	8(8)	6(6)	0(6, 0)	2(2)	-	4(4)	18(18)	8(8)	4(4)	-	22(50)	22(22)
X=210	28(28)	19(20)	1(20, 0)	2(2)	-	-	-	-	-	-	48(50)	47(48)
X=225	29(29)	16(16)	0(16, 0)	5(5)	-	-	-	-	-	-	45(50)	45(45)
X=240	9(9)	36(37)	1(37, 0)	4(4)	-	-	-	-	-	-	46(50)	45(46)
X=255	1(1)	12(12)	0(12, 0)	5(5)	0(18)	-	-	0(10)	2(2)	2(2)	33(50)	15(33)
X=270	-	-	-	1(1)	0(22)	-	-	0(2)	14(14)	11(11)	36(50)	14(36)
X=285	10(10)	2(3)	1(3, 0)	9(9)	0(16)	-	-	0(11)	1(1)	-	30(50)	13(30)
X=300	8(8)	24(25)	1(25, 0)	17(17)	-	-	-	-	-	-	33(50)	32(33)
X=315	7(7)	35(36)	1(36, 0)	7(7)	-	-	-	-	-	-	43(50)	42(43)
X=330	8(8)	29(29)	0(29, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X=345	4(4)	24(24)	1(24, -1)	14(14)	4(4)	-	0(1)	3(3)	-	-	32(50)	32(32)
TOTAL	200(200)	497(505)	16(505, -8)	198(198)	60(116)	22(22)	55(56)	16(39)	47(47)	17(17)	890(1200)	826(890)

1 CELL

WIND SPEED 24.0 THETA BEAMS 1,2,3: 47.0 37.9 47.0

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	3(3)	16(16)	9(9)	22(22)	-	-	-	-	-	50(50)	RHO= -.526
X= 15	14(14)	20(20)	11(11)	5(5)	-	-	-	-	-	50(50)	RHO= -.627
X= 30	23(23)	20(21)	6(6)	-	-	-	-	-	-	49(50)	RHO= -.485
X= 45	-	46(46)	4(4)	-	-	-	-	-	-	50(50)	RHO= -.039
X= 60	1(1)	45(45)	4(4)	-	-	-	-	-	-	50(50)	RHO= .571
X= 75	-	30(30)	13(13)	3(3)	-	-	3(3)	-	0(1)	49(50)	RHO= .616
X= 90	4(4)	9(9)	14(14)	21(21)	-	-	-	-	1(2)	49(50)	RHO= .482
X=105	14(14)	13(13)	16(17)	5(5)	-	-	-	-	1(1)	49(50)	RHO= -.320
X=120	8(8)	24(24)	18(18)	-	-	-	-	-	-	50(50)	RHO= -.382
X=135	10(10)	27(27)	13(13)	-	-	-	-	-	-	50(50)	RHO= -.311
X=150	7(7)	35(35)	8(8)	-	-	-	-	-	-	50(50)	RHO= .027
X=165	3(3)	8(8)	2(2)	-	12(12)	19(19)	1(1)	5(5)	-	50(50)	RHO= -.390
X=180	1(1)	3(3)	-	-	6(6)	18(18)	1(1)	21(21)	-	50(50)	RHO= -.461
X=195	8(8)	6(6)	2(2)	-	4(4)	18(18)	8(8)	4(4)	-	50(50)	RHO= -.472
X=210	28(28)	18(20)	2(2)	-	-	-	-	-	-	48(50)	RHO= -.055
X=225	29(29)	16(16)	5(5)	-	-	-	-	-	-	50(50)	RHO= .214
X=240	9(9)	36(37)	4(4)	-	-	-	-	-	-	49(50)	RHO= .467
X=255	1(1)	12(12)	3(5)	5(18)	-	-	5(10)	2(2)	2(2)	30(50)	RHO= .738
X=270	-	-	1(1)	12(22)	-	-	1(2)	14(14)	10(11)	38(50)	RHO= .357
X=285	10(10)	2(3)	7(9)	9(16)	-	-	4(11)	1(1)	-	33(50)	RHO= .306
X=300	8(8)	24(25)	16(17)	-	-	-	-	-	-	48(50)	RHO= -.306
X=315	7(7)	35(36)	7(7)	-	-	-	-	-	-	49(50)	RHO= -.438
X=330	8(8)	29(29)	13(13)	-	-	-	-	-	-	50(50)	RHO= -.140
X=345	4(4)	24(24)	14(14)	4(4)	-	1(1)	3(3)	-	-	50(50)	RHO= -.506
TOTAL	200(200)	498(505)	192(198)	86(116)	22(22)	56(56)	26(39)	47(47)	14(17)	1141(1200)	

TABLE 33 53.5°, 4 M/S, 5 CELLS

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	1(3)	-	-	-	4(23)	0(1)	6(23)	1(50)	0(1)
X= 15	-	-	-	1(5)	0(1)	-	-	10(20)	3(3)	6(21)	4(50)	3(4)
X= 30	-	2(2)	0(2, 0)	12(35)	-	-	-	4(11)	-	2(2)	2(50)	2(2)
X= 45	-	12(12)	0(12, 0)	31(35)	-	-	-	2(3)	-	-	12(50)	12(12)
X= 60	-	2(2)	0(2, 0)	20(24)	-	-	-	12(12)	0(3)	8(9)	5(50)	2(5)
X= 75	-	-	-	4(5)	0(1)	-	-	9(17)	-	17(27)	1(50)	0(1)
X= 90	-	-	-	-	-	-	2(2)	4(16)	1(4)	5(28)	4(50)	1(4)
X=105	-	-	-	2(3)	-	-	-	3(14)	1(2)	8(31)	2(50)	1(2)
X=120	-	1(2)	1(2, 0)	8(18)	-	-	-	6(14)	1(2)	8(14)	4(50)	2(4)
X=135	-	0(1)	0(1, 1)	15(39)	-	-	-	4(6)	-	3(4)	1(50)	0(1)
X=150	-	-	-	19(44)	-	-	-	2(2)	-	2(4)	0(50)	-
X=165	-	-	-	7(30)	-	-	-	5(12)	-	2(8)	0(50)	-
X=180	-	-	-	7(21)	-	-	-	4(18)	-	1(11)	0(50)	-
X=195	-	1(2)	2(2, -1)	6(29)	-	-	-	3(11)	1(1)	2(7)	3(50)	2(3)
X=210	-	0(2)	2(2, 0)	20(41)	-	-	-	5(6)	-	1(1)	2(50)	0(2)
X=225	0(1)	0(4)	4(4, 0)	9(38)	-	-	-	2(4)	-	1(3)	5(50)	0(5)
X=240	-	0(6)	6(6, 0)	9(27)	-	-	-	3(8)	1(1)	4(8)	7(50)	1(7)
X=255	-	0(1)	1(1, 0)	10(28)	-	-	-	4(13)	-	3(8)	1(50)	0(1)
X=270	-	-	-	7(19)	-	-	-	7(11)	1(1)	10(19)	1(50)	1(1)
X=285	-	1(2)	0(2, 1)	12(34)	-	-	-	2(4)	1(1)	1(9)	3(50)	2(3)
X=300	0(1)	1(2)	1(2, 0)	17(41)	-	-	-	2(3)	-	2(3)	3(50)	1(3)
X=315	-	0(1)	1(1, 0)	17(45)	-	-	-	2(3)	-	1(1)	1(50)	0(1)
X=330	1(1)	1(1)	0(1, 0)	13(28)	-	1(1)	1(1)	5(14)	0(1)	3(3)	4(50)	3(4)
X=345	-	0(1)	1(1, 0)	3(5)	-	-	-	9(16)	2(3)	6(25)	4(50)	2(4)
TOTAL	1(3)	21(41)	19(41, 1)	250(597)	0(2)	1(1)	3(3)	113(261)	12(23)	102(269)	70(1200)	35(70)

5 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	0(3)	-	-	-	17(23)	1(1)	23(23)	41(50)	RHO= -.768
X= 15	-	-	5(5)	0(1)	-	-	14(20)	3(3)	21(21)	43(50)	RHO= -.359
X= 30	-	2(2)	24(35)	-	-	-	8(11)	-	1(2)	35(50)	RHO= .562
X= 45	-	12(12)	29(35)	-	-	-	2(3)	-	-	43(50)	RHO= .779
X= 60	-	1(2)	11(24)	-	-	-	10(12)	2(3)	9(9)	33(50)	RHO= .891
X= 75	-	-	0(5)	1(1)	-	-	12(17)	-	26(27)	39(50)	RHO= .737
X= 90	-	-	-	-	-	2(2)	16(16)	4(4)	28(28)	50(50)	RHO= .603
X=105	-	-	3(3)	-	-	-	14(14)	2(2)	31(31)	50(50)	RHO= .345
X=120	-	2(2)	15(18)	-	-	-	10(14)	2(2)	12(14)	41(50)	RHO= .600
X=135	-	0(1)	24(39)	-	-	-	1(6)	-	1(4)	26(50)	RHO= .112
X=150	-	-	23(44)	-	-	-	1(2)	-	1(4)	25(50)	RHO= -.609
X=165	-	-	17(30)	-	-	-	1(12)	-	1(8)	19(50)	RHO= -.788
X=180	-	-	14(21)	-	-	-	5(18)	-	0(11)	19(50)	RHO= -.699
X=195	-	1(2)	16(29)	-	-	-	2(11)	0(1)	3(7)	22(50)	RHO= -.521
X=210	-	1(2)	19(41)	-	-	-	1(6)	-	0(1)	21(50)	RHO= -.276
X=225	0(1)	2(4)	19(38)	-	-	-	1(4)	-	3(3)	25(50)	RHO= .653
X=240	-	1(6)	10(27)	-	-	-	2(8)	0(1)	1(8)	14(50)	RHO= .825
X=255	-	1(1)	18(28)	-	-	-	5(13)	-	0(8)	24(50)	RHO= .773
X=270	-	-	11(19)	-	-	-	7(11)	0(1)	1(19)	19(50)	RHO= .310
X=285	-	0(2)	15(34)	-	-	-	2(4)	0(1)	2(9)	19(50)	RHO= -.446
X=300	0(1)	1(2)	27(41)	-	-	-	1(3)	-	2(3)	31(50)	RHO= -.311
X=315	-	1(1)	37(45)	-	-	-	3(3)	-	1(1)	42(50)	RHO= -.641
X=330	1(1)	1(1)	25(28)	-	1(1)	1(1)	14(14)	1(1)	3(3)	47(50)	RHO= -.894
X=345	-	1(1)	1(5)	-	-	-	16(16)	3(3)	25(25)	46(50)	RHO= -.905
TOTAL	1(3)	27(41)	363(597)	1(2)	1(1)	3(3)	165(261)	18(23)	195(269)	774(1200)	

TABLE 34 53.5^{O}, 4 M/S, 25 CELLS

25 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	0(3)	1(1)	2(2)	11(21)	1(1)	15(22)	5(50)	2(5)
X= 15	-	-	-	-	0(4)	-	2(2)	21(31)	2(2)	2(11)	6(50)	2(6)
X= 30	2(2)	3(3)	0(3, 0)	35(42)	-	-	-	3(3)	-	-	5(50)	5(5)
X= 45	1(1)	7(7)	0(7, 0)	37(42)	-	-	-	-	-	-	8(50)	8(8)
X= 60	1(1)	2(3)	1(3, 0)	34(35)	0(2)	-	-	9(9)	-	-	6(50)	3(6)
X= 75	-	-	-	1(1)	0(10)	-	1(1)	20(25)	0(5)	5(8)	15(50)	0(15)
X= 90	-	-	-	-	-	-	2(2)	5(12)	4(5)	15(31)	5(50)	4(5)
X=105	2(2)	-	-	-	-	5(5)	4(4)	14(23)	4(4)	6(12)	11(50)	11(11)
X=120	4(4)	-	-	14(20)	-	-	-	17(26)	-	-	4(50)	4(4)
X=135	1(1)	3(4)	1(4, 0)	37(45)	-	-	-	-	-	-	5(50)	4(5)
X=150	2(2)	3(5)	1(5, 1)	29(43)	-	-	-	-	-	-	7(50)	5(7)
X=165	1(1)	1(4)	2(4, 1)	21(33)	1(1)	-	0(1)	2(9)	-	0(1)	6(50)	3(6)
X=180	-	3(4)	3(4, -2)	15(29)	1(1)	-	-	4(10)	1(1)	2(5)	6(50)	5(6)
X=195	-	0(1)	1(1, 0)	15(33)	1(1)	-	-	3(10)	-	3(5)	2(50)	1(2)
X=210	-	0(4)	4(4, 0)	34(45)	-	-	-	1(1)	-	-	4(50)	0(4)
X=225	-	0(7)	7(7, 0)	21(43)	-	-	-	-	-	-	7(50)	0(7)
X=240	-	0(2)	2(2, 0)	29(47)	-	-	-	1(1)	-	-	2(50)	0(2)
X=255	1(1)	0(2)	2(2, 0)	13(25)	1(1)	-	-	10(19)	-	0(2)	4(50)	2(4)
X=270	-	1(1)	1(1, -1)	11(21)	1(1)	-	-	10(19)	-	4(8)	2(50)	2(2)
X=285	-	5(8)	3(8, 0)	20(30)	-	-	-	6(11)	-	1(1)	8(50)	5(8)
X=300	2(2)	2(3)	1(3, 0)	31(45)	-	-	-	-	-	-	5(50)	4(5)
X=315	2(2)	-	-	29(47)	-	-	-	1(1)	-	-	2(50)	2(2)
X=330	2(2)	-	-	22(31)	-	3(3)	2(2)	10(12)	-	-	5(50)	5(5)
X=345	-	-	-	0(1)	0(1)	-	5(5)	13(32)	1(2)	4(9)	3(50)	1(3)
TOTAL	21(21)	30(58)	29(58, -1)	448(658)	5(25)	9(9)	18(19)	161(275)	13(20)	57(115)	133(1200)	78(133)

25 CELLS

WIND SPEED 4.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	3(3)	1(1)	2(2)	21(21)	1(1)	22(22)	50(50)	RHO= -.911
X= 15	-	-	-	4(4)	-	2(2)	30(31)	2(2)	11(11)	49(50)	RHO= -.528
X= 30	2(2)	2(3)	40(42)	-	-	-	2(3)	-	-	46(50)	RHO= .735
X= 45	1(1)	7(7)	41(42)	-	-	-	-	-	-	49(50)	RHO= .835
X= 60	1(1)	0(3)	27(35)	2(2)	-	-	9(9)	-	-	39(50)	RHO= .852
X= 75	-	-	0(1)	10(10)	-	1(1)	22(25)	5(5)	7(8)	45(50)	RHO= .773
X= 90	-	-	-	-	-	2(2)	12(12)	5(5)	31(31)	50(50)	RHO= .734
X=105	2(2)	-	-	-	5(5)	4(4)	23(23)	4(4)	12(12)	50(50)	RHO= .702
X=120	4(4)	-	20(20)	-	-	-	26(26)	-	-	50(50)	RHO= .805
X=135	1(1)	3(4)	32(45)	-	-	-	-	-	-	36(50)	RHO= -.496
X=150	2(2)	4(5)	38(43)	-	-	-	-	-	-	44(50)	RHO= -.704
X=165	1(1)	3(4)	28(33)	0(1)	-	0(1)	1(9)	-	0(1)	33(50)	RHO= -.847
X=180	-	4(4)	20(29)	1(1)	-	-	7(10)	1(1)	1(5)	34(50)	RHO= -.807
X=195	-	1(1)	28(33)	1(1)	-	-	9(10)	-	4(5)	43(50)	RHO= -.475
X=210	-	2(4)	37(45)	-	-	-	0(1)	-	-	39(50)	RHO= -.594
X=225	-	2(7)	24(43)	-	-	-	-	-	-	26(50)	RHO= .667
X=240	-	1(2)	35(47)	-	-	-	0(1)	-	-	36(50)	RHO= .814
X=255	1(1)	2(2)	19(25)	0(1)	-	-	2(19)	-	0(2)	24(50)	RHO= .740
X=270	-	1(1)	18(21)	1(1)	-	-	8(19)	-	2(8)	30(50)	RHO= .640
X=285	-	8(8)	22(30)	-	-	-	4(11)	-	0(1)	34(50)	RHO= -.110
X=300	2(2)	3(3)	28(45)	-	-	-	-	-	-	33(50)	RHO= -.392
X=315	2(2)	-	44(47)	-	-	-	1(1)	-	-	47(50)	RHO= -.887
X=330	2(2)	-	26(31)	-	3(3)	2(2)	12(12)	-	-	45(50)	RHO= -.896
X=345	-	-	1(1)	1(1)	-	5(5)	31(32)	2(2)	9(9)	49(50)	RHO= -.929
TOTAL	21(21)	43(58)	528(658)	23(25)	9(9)	18(19)	220(275)	20(20)	99(115)	981(1200)	

TABLE 35 53.5°, 8 M/S, 5 CELLS

5 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	0(2)	1(1)	1(1)	28(30)	3(3)	13(13)	6(50)	4(6)
X= 15	-	-	-	5(5)	-	1(1)	8(8)	34(35)	-	1(1)	1(50)	1(1)
X= 30	-	6(10)	4(10, 0)	39(40)	-	-	-	-	-	-	10(50)	6(10)
X= 45	5(5)	20(20)	0(20, 0)	25(25)	-	-	-	-	-	-	25(50)	25(25)
X= 60	7(7)	9(9)	0(9, 0)	33(34)	-	-	-	-	-	-	16(50)	16(16)
X= 75	-	1(1)	0(1, 0)	5(6)	-	-	-	32(33)	1(4)	6(6)	5(50)	2(5)
X= 90	-	-	-	-	0(1)	-	3(3)	15(15)	10(12)	18(19)	13(50)	10(13)
X=105	1(1)	-	-	5(6)	0(1)	2(2)	1(1)	31(35)	1(1)	3(3)	5(50)	4(5)
X=120	6(6)	9(9)	0(9, 0)	28(30)	-	-	-	5(5)	-	-	15(50)	15(15)
X=135	4(4)	13(16)	3(16, 0)	29(30)	-	-	-	-	-	-	20(50)	17(20)
X=150	12(12)	2(3)	1(3, 0)	34(35)	-	-	-	-	-	-	15(50)	14(15)
X=165	1(1)	3(4)	4(4, -3)	29(34)	-	-	0(1)	8(9)	-	1(1)	5(50)	4(5)
X=180	-	1(2)	2(2, -1)	21(26)	-	-	0(3)	15(17)	-	2(2)	2(50)	1(2)
X=195	-	3(4)	3(4, -2)	35(39)	-	-	0(1)	5(5)	-	1(1)	4(50)	3(4)
X=210	-	5(8)	3(8, 0)	40(42)	-	-	-	-	-	-	8(50)	5(8)
X=225	2(3)	2(3)	1(3, 0)	35(44)	-	-	-	-	-	-	6(50)	4(6)
X=240	0(1)	4(6)	2(6, 0)	39(43)	-	-	-	-	-	-	7(50)	4(7)
X=255	-	2(4)	2(4, 0)	35(37)	1(1)	0(1)	-	7(7)	-	-	6(50)	3(6)
X=270	-	3(4)	2(4, -1)	21(23)	-	-	-	14(17)	2(2)	4(4)	6(50)	5(6)
X=285	2(2)	5(5)	0(5, 0)	36(38)	-	-	-	5(5)	-	-	7(50)	7(7)
X=300	5(6)	8(12)	4(12, 0)	30(32)	-	-	-	-	-	-	18(50)	13(18)
X=315	5(5)	2(3)	1(3, 0)	39(42)	-	-	-	-	-	-	8(50)	7(8)
X=330	4(4)	4(7)	3(7, 0)	27(32)	-	-	-	7(7)	-	-	11(50)	8(11)
X=345	-	-	-	3(3)	0(5)	-	1(1)	34(39)	1(1)	1(1)	6(50)	1(6)
TOTAL	54(57)	102(130)	35(130, -7)	593(646)	1(10)	4(5)	14(19)	240(259)	18(23)	50(51)	225(1200)	179(225)

5 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	2(2)	1(1)	1(1)	29(30)	3(3)	13(13)	49(50)	RHO= -.757
X= 15	-	-	5(5)	-	1(1)	8(8)	33(35)	-	1(1)	48(50)	RHO= -.807
X= 30	-	5(10)	26(40)	-	-	-	-	-	-	31(50)	RHO= .214
X= 45	5(5)	17(20)	17(25)	-	-	-	-	-	-	39(50)	RHO= .361
X= 60	5(7)	9(9)	30(34)	-	-	-	-	-	-	44(50)	RHO= .695
X= 75	-	1(1)	2(6)	-	-	-	30(33)	4(4)	6(6)	43(50)	RHO= .738
X= 90	-	-	-	0(1)	-	3(3)	14(15)	12(12)	19(19)	48(50)	RHO= .797
X=105	1(1)	-	6(6)	1(1)	2(2)	1(1)	35(35)	1(1)	3(3)	50(50)	RHO= .351
X=120	6(6)	9(9)	27(30)	-	-	-	4(5)	-	-	46(50)	RHO= -.221
X=135	4(4)	15(16)	25(30)	-	-	-	-	-	-	44(50)	RHO= -.383
X=150	12(12)	3(3)	34(35)	-	-	-	-	-	-	49(50)	RHO= -.639
X=165	1(1)	4(4)	31(34)	-	-	0(1)	5(9)	-	0(1)	41(50)	RHO= -.764
X=180	-	2(2)	21(26)	-	-	1(3)	11(17)	-	0(2)	35(50)	RHO= -.691
X=195	-	3(4)	19(39)	-	-	0(1)	2(5)	-	0(1)	24(50)	RHO= -.736
X=210	-	6(8)	27(42)	-	-	-	-	-	-	33(50)	RHO= -.310
X=225	2(3)	2(3)	38(44)	-	-	-	-	-	-	42(50)	RHO= .685
X=240	0(1)	5(6)	35(43)	-	-	-	-	-	-	40(50)	RHO= .724
X=255	-	3(4)	28(37)	0(1)	0(1)	-	2(7)	-	-	33(50)	RHO= .882
X=270	-	3(4)	22(23)	-	-	-	7(17)	0(2)	0(4)	32(50)	RHO= .779
X=285	2(2)	5(5)	32(38)	-	-	-	4(5)	-	-	43(50)	RHO= .455
X=300	5(6)	9(12)	27(32)	-	-	-	-	-	-	41(50)	RHO= -.742
X=315	5(5)	3(3)	41(42)	-	-	-	-	-	-	49(50)	RHO= -.783
X=330	4(4)	7(7)	31(32)	-	-	-	7(7)	-	-	49(50)	RHO= -.825
X=345	-	-	2(3)	5(5)	-	1(1)	36(39)	1(1)	1(1)	46(50)	RHO= -.783
TOTAL	52(57)	111(130)	526(646)	8(10)	4(5)	15(19)	219(259)	21(23)	43(51)	999(1200)	

TABLE 36 53.5^{o}, 8 M/S, 25 CELLS

25 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	--	9(9)	13(13)	11(11)	17(17)	11(50)	11(11)
X= 15	-	-	-	-	-	6(6)	16(16)	28(28)	-	-	6(50)	6(6)
X= 30	-	17(18)	1(18, 0)	32(32)	-	-	-	-	-	-	18(50)	17(18)
X= 45	13(13)	28(29)	1(29, 0)	8(8)	-	..	-	-	-	-	42(50)	41(42)
X= 60	23(23)	14(14)	0(14, 0)	13(13)	-	-	-	-	-	-	37(50)	37(37)
X= 75	-	-	-	1(1)	0(2)	-	-	46(46)	0(1)	-	3(50)	0(3)
X= 90	-	-	-	-	-	3(3)	10(10)	7(7)	18(19)	11(11)	22(50)	21(22)
X=105	-	-	-	-	-	7(7)	1(1)	42(42)	-	-	7(50)	7(7)
X=120	27(27)	6(6)	0(6, 0)	17(17)	-	-	-	-	-	-	33(50)	33(33)
X=135	13(13)	13(14)	1(14, 0)	23(23)	-	-	-	-	-	-	27(50)	26(27)
X=150	34(34)	-	-	16(16)	-	-	-	-	-	-	34(50)	34(34)
X=165	12(12)	2(3)	3(3, -2)	35(35)	-	-	-	-	-	-	15(50)	14(15)
X=180	1(1)	1(2)	2(2, -1)	28(28)	-	-	0(2)	17(17)	-	-	3(50)	2(3)
X=195	-	1(7)	6(7, 0)	43(43)	-	-	-	-	-	-	7(50)	1(7)
X=210	1(2)	6(10)	4(10, 0)	38(38)	-	-	-	-	-	-	12(50)	7(12)
X=225	4(4)	9(11)	2(11, 0)	35(35)	-	-	-	-	-	-	15(50)	13(15)
X=240	1(1)	7(10)	3(10, 0)	39(39)	-	-	-	-	-	-	11(50)	8(11)
X=255	8(8)	4(4)	0(4, 0)	38(38)	-	-	-	-	-	-	12(50)	12(12)
X=270	-	0(1)	1(1, 0)	24(25)	3(3)	-	-	21(21)	-	-	4(50)	3(4)
X=285	12(12)	6(6)	0(6, 0)	32(32)	-	-	-	-	-	-	18(50)	18(18)
X=300	13(13)	7(10)	3(10, 0)	27(27)	-	-	-	-	-	-	23(50)	20(23)
X=315	28(28)	-	-	22(22)	-	-	-	-	-	-	28(50)	28(28)
X=330	20(20)	1(4)	3(4, 0)	26(26)	-	-	-	-	-	-	24(50)	21(24)
X=345	-	-	-	-	0(3)	5(5)	2(2)	39(39)	-	1(1)	8(50)	5(8)
TOTAL	210(211)	122(149)	30(149, -3)	497(498)	3(8)	21(21)	38(40)	213(213)	29(31)	29(29)	420(1200)	385(420)

EOF:40

25 CELLS

WIND SPEED 8.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	-	9(9)	13(13)	11(11)	17(17)	50(50)	RHO= -.823
X= 15	-	-	-	-	6(6)	16(16)	28(28)	-	-	50(50)	RHO= -.567
X= 30	-	18(18)	30(32)	-	-	-	-	-	-	48(50)	RHO= .215
X= 45	13(13)	28(29)	6(8)	-	-	-	-	-	-	47(50)	RHO= .718
X= 60	23(23)	14(14)	13(13)	-	-	-	-	-	-	50(50)	RHO= .716
X= 75	-	-	0(1)	2(2)	-	-	43(46)	1(1)	-	46(50)	RHO= .830
X= 90	-	-	-	-	3(3)	10(10)	7(7)	19(19)	11(11)	50(50)	RHO= .815
X=105	-	-	-	-	7(7)	1(1)	42(42)	-	-	50(50)	RHO= .306
X=120	27(27)	6(6)	17(17)	-	-	-	-	-	-	50(50)	RHO= -.437
X=135	13(13)	14(14)	23(23)	-	-	-	-	-	-	50(50)	RHO= -.365
X=150	34(34)	-	16(16)	-	-	-	-	-	-	50(50)	RHO= -.698
X=165	12(12)	3(3)	35(35)	-	-	-	-	-	-	50(50)	RHO= -.715
X=180	1(1)	2(2)	27(28)	-	-	2(2)	15(17)	-	-	47(50)	RHO= -.869
X=195	-	2(7)	25(43)	-	-	-	-	-	-	27(50)	RHO= -.702
X=210	1(2)	7(10)	34(38)	-	-	-	-	-	-	42(50)	RHO= .277
X=225	4(4)	10(11)	31(35)	-	-	-	-	-	-	45(50)	RHO= .744
X=240	1(1)	9(10)	38(39)	-	-	-	-	-	-	48(50)	RHO= .807
X=255	8(8)	4(4)	35(38)	-	-	-	-	-	-	47(50)	RHO= .817
X=270	-	1(1)	25(25)	3(3)	-	-	21(21)	-	-	50(50)	RHO= .765
X=285	12(12)	6(6)	32(32)	-	-	-	-	-	-	50(50)	RHO= .520
X=300	13(13)	10(10)	26(27)	-	-	-	-	-	-	49(50)	RHO= -.482
X=315	28(28)	-	22(22)	-	-	-	-	-	-	50(50)	RHO= -.761
X=330	20(20)	4(4)	26(26)	-	-	-	-	-	-	50(50)	RHO= -.748
X=345	-	-	-	3(3)	5(5)	2(2)	39(39)	-	1(1)	50(50)	RHO= -.766
TOTAL	210(211)	138(149)	461(498)	8(8)	21(21)	40(40)	208(213)	31(31)	29(29)	1146(1200)	

EOF:40
O:

TABLE 37 53.5^{o}, 12 M/S, 5 CELLS

5 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	1(1)	16(16)	4(4)	24(24)	5(5)	25(50)	25(25)
X= 15	1(1)	15(15)	0(15, 0)	-	-	7(7)	11(11)	15(15)	-	1(1)	23(50)	23(23)
X= 30	5(5)	34(34)	0(34, 0)	11(11)	-	-	-	-	-	-	39(50)	39(39)
X= 45	-	42(42)	0(42, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X= 60	17(17)	26(26)	0(26, 0)	7(7)	-	-	-	-	-	-	43(50)	43(43)
X= 75	-	1(1)	0(1, 0)	13(13)	-	-	-	29(29)	0(1)	6(6)	2(50)	1(2)
X= 90	-	-	-	-	-	3(3)	5(5)	16(16)	16(16)	10(10)	19(50)	19(19)
X=105	8(8)	-	-	1(1)	-	-	2(2)	38(38)	-	1(1)	8(50)	8(8)
X=120	11(11)	18(16)	0(18, 0)	21(21)	-	-	-	-	-	-	29(50)	29(29)
X=135	14(14)	10(11)	1(11, 0)	25(25)	-	-	-	-	-	-	25(50)	24(25)
X=150	24(24)	3(5)	2(5, 0)	21(21)	-	-	-	-	-	-	29(50)	27(29)
X=165	7(7)	4(5)	2(5, -1)	38(38)	-	-	-	-	-	-	12(50)	11(12)
X=180	-	5(6)	2(6, -1)	16(16)	8(8)	-	0(1)	17(17)	2(2)	-	16(50)	15(16)
X=195	-	14(14)	0(14, 0)	32(32)	-	0(1)	-	3(3)	-	-	15(50)	14(15)
X=210	1(1)	16(18)	2(18, 0)	31(31)	-	-	-	-	-	-	19(50)	17(19)
X=225	0(1)	29(31)	2(31, 0)	18(18)	-	-	-	-	-	-	32(50)	29(32)
X=240	7(8)	22(23)	1(23, 0)	19(19)	-	-	-	-	-	-	31(50)	29(31)
X=255	6(6)	16(18)	3(16, -1)	21(21)	1(1)	-	-	4(4)	-	-	25(50)	23(25)
X=270	-	3(3)	1(3, -1)	16(16)	12(12)	0(1)	0(1)	17(17)	-	-	16(50)	15(16)
X=285	18(18)	6(11)	5(11, 0)	20(20)	-	-	-	1(1)	-	-	29(50)	24(29)
X=300	12(12)	14(15)	1(15, 0)	23(23)	-	-	-	-	-	-	27(50)	26(27)
X=315	29(29)	5(6)	1(6, 0)	15(15)	-	-	-	-	-	-	35(50)	34(35)
X=330	16(16)	4(7)	3(7, 0)	27(27)	-	-	-	-	-	-	23(50)	20(23)
X=345	1(1)	5(5)	0(5, 0)	5(5)	0(1)	6(6)	5(5)	20(20)	5(5)	2(2)	18(50)	17(18)
TOTAL	177(179)	292(314)	26(314, -4)	368(388)	21(22)	17(19)	39(41)	164(164)	47(48)	25(25)	582(1200)	554(582)

5 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	1(1)	16(16)	4(4)	24(24)	5(5)	50(50)	RHO= -.668
X= 15	1(1)	15(15)	-	-	7(7)	11(11)	15(15)	-	1(1)	50(50)	RHO= -.576
X= 30	5(5)	34(34)	11(11)	-	-	-	-	-	-	50(50)	RHO= -.226
X= 45	-	42(42)	6(6)	-	-	-	-	-	-	50(50)	RHO= .285
X= 60	16(17)	26(26)	7(7)	-	-	-	-	-	-	49(50)	RHO= .668
X= 75	-	1(1)	9(13)	-	-	-	28(29)	1(1)	6(6)	45(50)	RHO= .822
X= 90	-	-	-	-	3(3)	5(5)	16(16)	16(16)	10(10)	50(50)	RHO= .794
X=105	8(8)	-	1(1)	-	-	2(2)	36(38)	-	1(1)	50(50)	RHO= .357
X=120	11(11)	17(18)	20(21)	-	-	-	-	-	-	48(50)	RHO= -.449
X=135	15(14)	11(11)	25(25)	-	-	-	-	-	-	50(50)	RHO= -.397
X=150	24(24)	4(5)	21(21)	-	-	-	-	-	-	49(50)	RHO= -.462
X=165	7(7)	5(5)	38(38)	-	-	-	-	-	-	50(50)	RHO= -.716
X=180	-	6(6)	15(16)	7(8)	-	0(1)	15(17)	2(2)	-	45(50)	RHO= -.677
X=195	-	14(14)	30(32)	-	0(1)	-	3(3)	-	-	47(50)	RHO= -.658
X=210	1(1)	16(16)	28(31)	-	-	-	-	-	-	45(50)	RHO= .157
X=225	0(1)	30(31)	16(18)	-	-	-	-	-	-	46(50)	RHO= .601
X=240	8(8)	23(23)	18(19)	-	-	-	-	-	-	49(50)	RHO= .755
X=255	6(6)	17(18)	17(21)	1(1)	-	-	3(4)	-	-	44(50)	RHO= .784
X=270	-	2(3)	14(16)	12(12)	0(1)	0(1)	12(17)	-	-	40(50)	RHO= .793
X=285	18(18)	6(11)	19(20)	-	-	-	1(1)	-	-	46(50)	RHO= .039
X=300	12(12)	15(15)	21(23)	-	-	-	-	-	-	48(50)	RHO= -.742
X=315	29(29)	6(6)	15(15)	-	-	-	-	-	-	50(50)	RHO= -.743
X=330	16(16)	7(7)	27(27)	-	-	-	-	-	-	50(50)	RHO= -.690
X=345	1(1)	5(5)	9(9)	1(1)	6(6)	5(5)	20(20)	5(5)	2(2)	49(50)	RHO= -.812
TOTAL	177(179)	304(314)	364(388)	21(22)	17(19)	39(41)	155(164)	48(48)	25(25)	1150(1200)	

TABLE 38 53.5°, 12 M/S, 25 CELLS

25 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	-	-	-	-	-	3(3)	28(28)	-	19(19)	-	22(50)	22(22)
X= 15	1(1)	3(3)	(3, 0)	-	-	20(21)	17(17)	9(9)	-	-	24(50)	24(24)
X= 30	7(7)	43(43)	4(47, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 45	1(1)	45(49)	1(49, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 60	29(29)	20(20)	1(21, 0)	1(1)	-	-	-	-	-	-	49(50)	49(49)
X= 75	5(5)	1(1)	0(1, 0)	1(1)	-	-	-	43(43)	-	-	6(50)	6(6)
X= 90	-	-	-	-	-	7(7)	12(12)	6(6)	25(25)	-	32(50)	32(32)
X=105	1(1)	-	-	-	-	-	-	49(49)	-	-	1(50)	1(1)
X=120	36(36)	9(9)	0(9, 0)	5(5)	-	-	-	-	-	-	45(50)	45(45)
X=135	40(40)	6(6)	0(6, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=150	46(46)	-	-	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=165	40(40)	2(2)	1(2, -1)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X=180	3(3)	8(8)	1(8, -1)	15(15)	15(15)	-	-	9(9)	-	-	26(50)	26(26)
X=195	3(3)	31(31)	0(31, 0)	16(16)	-	-	-	-	-	-	34(50)	34(34)
X=210	4(4)	35(36)	4(36, 0)	10(10)	-	-	-	-	-	-	40(50)	38(40)
X=225	2(2)	46(46)	0(46, 0)	2(2)	-	-	-	-	-	-	48(50)	48(48)
X=240	15(15)	30(30)	0(30, 0)	5(5)	-	-	-	-	-	-	45(50)	45(45)
X=255	33(33)	9(9)	0(9, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X=270	-	1(1)	1(1, -1)	21(21)	28(28)	-	-	-	-	-	29(50)	29(29)
X=285	40(40)	2(2)	0(2, 0)	8(8)	-	-	-	-	-	-	42(50)	42(42)
X=300	36(36)	10(10)	0(10, 0)	4(4)	-	-	-	-	-	-	46(50)	46(46)
X=315	46(46)	1(1)	0(1, 0)	3(3)	-	-	-	-	-	-	47(50)	47(47)
X=330	41(41)	3(3)	(3, 0)	6(6)	-	-	-	-	-	-	44(50)	44(44)
X=345	-	-	-	-	-	11(11)	13(13)	26(26)	-	-	11(50)	11(11)
TOTAL	429(429)	306(310)	7(310, -3)	121(121)	43(43)	41(41)	70(70)	142(142)	44(44)	-	857(1250)	863(857)

25 CELLS

WIND SPEED 12.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	-	-	-	-	3(3)	28(28)	-	19(19)	-	50(50)	RHO= -.749
X= 15	1(1)	3(3)	-	-	20(20)	17(17)	9(9)	-	-	50(50)	RHO= -.505
X= 30	7(7)	43(43)	-	-	-	-	-	-	-	50(50)	RHO= -.240
X= 45	1(1)	49(49)	-	-	-	-	-	-	-	50(50)	RHO= .468
X= 60	29(29)	20(20)	1(1)	-	-	-	-	-	-	50(50)	RHO= .635
X= 75	5(5)	1(1)	1(1)	-	-	-	43(43)	-	-	50(50)	RHO= .847
X= 90	-	-	-	-	7(7)	12(12)	6(6)	25(25)	-	50(50)	RHO= .615
X=105	1(1)	-	-	-	-	-	49(49)	-	-	50(50)	RHO= .294
X=120	36(36)	9(9)	5(5)	-	-	-	-	-	-	50(50)	RHO= -.517
X=135	40(40)	6(6)	4(4)	-	-	-	-	-	-	50(50)	RHO= -.582
X=150	46(46)	-	4(4)	-	-	-	-	-	-	50(50)	RHO= -.242
X=165	40(40)	2(2)	8(8)	-	-	-	-	-	-	50(50)	RHO= -.652
X=180	3(3)	8(8)	15(15)	15(15)	-	-	9(9)	-	-	50(50)	RHO= -.821
X=195	3(3)	31(31)	16(16)	-	-	-	-	-	-	50(50)	RHO= -.515
X=210	4(4)	35(36)	10(10)	-	-	-	-	-	-	49(50)	RHO= .012
X=225	2(2)	46(46)	2(2)	-	-	-	-	-	-	50(50)	RHO= .587
X=240	15(15)	30(30)	5(5)	-	-	-	-	-	-	50(50)	RHO= .571
X=255	33(33)	9(9)	8(8)	-	-	-	-	-	-	50(50)	RHO= .743
X=270	-	1(1)	21(21)	28(28)	-	-	-	-	-	50(50)	RHO= .692
X=285	40(40)	2(2)	8(8)	-	-	-	-	-	-	50(50)	RHO= .229
X=300	36(36)	10(10)	4(4)	-	-	-	-	-	-	50(50)	RHO= -.793
X=315	46(46)	1(1)	3(3)	-	-	-	-	-	-	50(50)	RHO= -.743
X=330	41(41)	3(3)	6(6)	-	-	-	-	-	-	50(50)	RHO= -.631
X=345	-	-	-	-	11(11)	13(13)	26(26)	-	-	50(50)	RHO= -.751
TOTAL	429(429)	309(310)	121(121)	43(43)	41(41)	70(70)	142(142)	44(44)	-	1199(1200)	

TABLE 39 53.5°, 24 M/S, 5 CELLS

5 CELLS

WIND SPEED 24.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	2(2)	13(13)	1(13, -1)	12(12)	23(23)	-	-	-	-	-	38(50)	38(38)
X= 15	15(15)	21(21)	5(21, -5)	14(14)	-	-	-	-	-	-	36(50)	36(36)
X= 30	34(34)	16(16)	0(16, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 45	7(7)	43(43)	0(43, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 60	1(1)	49(49)	0(49, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 75	1(1)	46(46)	0(46, 0)	1(1)	-	-	-	2(2)	-	-	47(50)	47(47)
X= 90	-	8(8)	5(8, -5)	19(19)	23(23)	-	-	-	-	-	31(50)	31(31)
X=105	36(36)	5(5)	0(5, 0)	7(7)	2(2)	-	-	-	-	-	43(50)	43(43)
X=120	17(17)	28(28)	0(28, 0)	5(5)	-	-	-	-	-	-	45(50)	45(45)
X=135	22(22)	27(27)	0(27, 0)	1(1)	-	-	-	-	-	-	49(50)	49(49)
X=150	11(11)	39(39)	0(39, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=165	6(6)	3(3)	0(3, 0)	1(1)	-	21(21)	18(18)	-	1(1)	-	31(50)	31(31)
X=180	-	-	-	-	-	10(10)	19(19)	-	21(21)	-	31(50)	31(31)
X=195	5(5)	1(1)	0(1, 0)	1(1)	-	17(17)	26(26)	-	-	-	23(50)	23(23)
X=210	37(37)	13(13)	0(13, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=225	39(39)	11(11)	0(11, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=240	6(6)	44(44)	0(44, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=255	1(1)	7(7)	0(7, 0)	3(3)	-	-	-	0(39)	-	-	8(50)	8(8)
X=270	-	-	-	-	0(13)	-	-	0(12)	20(20)	5(5)	33(50)	20(33)
X=285	7(7)	-	-	5(5)	0(3)	-	-	0(34)	1(1)	-	11(50)	8(11)
X=300	15(15)	25(25)	0(25, 0)	10(10)	-	-	-	-	-	-	40(50)	40(40)
X=315	10(10)	39(39)	0(39, 0)	1(1)	-	-	-	-	-	-	49(50)	49(49)
X=330	17(17)	27(27)	0(27, 0)	6(6)	-	-	-	-	-	-	44(50)	44(44)
X=345	15(15)	19(19)	3(19, -3)	13(13)	3(3)	-	-	-	-	-	37(50)	37(37)
TOTAL	304(304)	484(484)	14(469,-14)	99(99)	51(67)	48(48)	63(63)	2(87)	43(43)	5(5)	946(1200)	930(946)

5 CELLS

WIND SPEED 24.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	2(2)	13(13)	12(12)	23(23)	-	-	-	-	-	50(50)	RHO= -.524
X= 15	15(15)	21(21)	14(14)	-	-	-	-	-	-	50(50)	RHO= -.759
X= 30	34(34)	16(16)	-	-	-	-	-	-	-	50(50)	RHO= -.515
X= 45	7(7)	43(43)	-	-	-	-	-	-	-	50(50)	RHO= .079
X= 60	1(1)	49(49)	-	-	-	-	-	-	-	50(50)	RHO= .648
X= 75	1(1)	46(46)	1(1)	-	-	-	2(2)	-	-	50(50)	RHO= .707
X= 90	-	8(8)	19(19)	23(23)	-	-	-	-	-	50(50)	RHO= .550
X=105	36(36)	5(5)	7(7)	2(2)	-	-	-	-	-	50(50)	RHO= -.197
X=120	17(17)	28(28)	5(5)	-	-	-	-	-	-	50(50)	RHO= -.537
X=135	22(22)	27(27)	1(1)	-	-	-	-	-	-	50(50)	RHO= -.499
X=150	11(11)	39(39)	-	-	-	-	-	-	-	50(50)	RHO= -.164
X=165	6(6)	3(3)	1(1)	-	21(21)	18(18)	-	1(1)	-	50(50)	RHO= -.310
X=180	-	-	-	-	10(10)	19(19)	-	21(21)	-	50(50)	RHO= -.622
X=195	5(5)	1(1)	1(1)	-	17(17)	26(26)	-	-	-	50(50)	RHO= -.532
X=210	37(37)	13(13)	-	-	-	-	-	-	-	50(50)	RHO= .028
X=225	39(39)	11(11)	-	-	-	-	-	-	-	50(50)	RHO= .281
X=240	6(6)	44(44)	-	-	-	-	-	-	-	50(50)	RHO= .484
X=255	1(1)	7(7)	3(3)	-	-	-	10(39)	-	-	21(50)	RHO= .846
X=270	-	-	-	8(13)	-	-	6(12)	20(20)	5(5)	39(50)	RHO= .525
X=285	7(7)	-	5(5)	0(3)	-	-	10(34)	1(1)	-	23(50)	RHO= .065
X=300	15(15)	25(25)	10(10)	-	-	-	-	-	-	50(50)	RHO= -.457
X=315	10(10)	39(39)	1(1)	-	-	-	-	-	-	50(50)	RHO= -.544
X=330	17(17)	27(27)	6(6)	-	-	-	-	-	-	50(50)	RHO= -.198
X=345	15(15)	19(19)	13(13)	3(3)	-	-	-	-	-	50(50)	RHO= -.643
TOTAL	304(304)	484(484)	99(99)	59(67)	48(48)	63(63)	28(87)	43(43)	5(5)	1133(1200)	

TABLE 40 53.5°, 24 M/S, 25 CELLS

25 CELLS

WIND SPEED 24.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

SELECTION BY OBJECTIVE CRITERIA

CLASS	1*	2A*	2B	3	4*	5*	6	7	8*	9	UNIQUE	CORRECT
X= 0	6(8)	3(3)	0(3, 0)	15(15)	24(24)	-	-	-	-	-	35(50)	35(35)
X= 15	39(39)	3(3)	0(3, 0)	8(6)	-	-	-	-	-	-	42(50)	42(42)
X= 30	47(47)	3(3)	0(3, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 45	39(39)	11(11)	0(11, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 60	3(3)	47(47)	0(47, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 75	9(9)	41(41)	0(41, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X= 90	1(1)	2(2)	2(2, -2)	28(26)	19(19)	-	-	-	-	-	22(50)	22(22)
X=105	48(48)	-	-	2(2)	-	-	-	-	-	-	48(50)	48(48)
X=120	41(41)	6(6)	0(8, 0)	1(1)	-	-	-	-	-	-	49(50)	49(49)
X=135	32(32)	16(16)	0(18, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=150	35(35)	15(15)	0(15, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=165	3(3)	-	-	-	-	30(30)	17(17)	-	-	-	33(50)	33(33)
X=180	-	-	-	-	-	11(11)	17(17)	-	22(22)	-	33(50)	33(33)
X=195	-	-	-	-	-	28(28)	22(22)	-	-	-	28(50)	28(28)
X=210	49(49)	1(1)	0(1, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=225	50(50)	-	-	-	-	-	-	-	-	-	50(50)	50(50)
X=240	11(11)	39(39)	0(39, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=255	-	1(1)	0(1, 0)	-	-	-	-	0(49)	-	-	1(50)	1(1)
X=270	-	-	-	-	0(7)	-	-	0(11)	32(32)	-	39(50)	32(39)
X=285	3(3)	-	-	-	-	-	-	0(47)	-	-	3(50)	3(3)
X=300	30(30)	20(20)	0(20, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=315	26(26)	24(24)	0(24, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=330	40(40)	10(10)	0(10, 0)	-	-	-	-	-	-	-	50(50)	50(50)
X=345	29(29)	5(5)	3(5, -3)	15(15)	1(1)	-	-	-	-	-	35(50)	35(35)
TOTAL	543(543)	251(251)	5(251, -5)	69(69)	44(51)	69(69)	56(56)	0(107)	54(54)	-	968(1200)	961(968)

25 CELLS

WIND SPEED 24.0 THETA BEAMS 1,2,3: 53.5 44.3 53.5

MAXIMUM LIKELIHOOD ESTIMATES

CLASS	1	2	3	4	5	6	7	8	9	TOTALS	
X= 0	6(8)	3(3)	15(15)	24(24)	-	-	-	-	-	50(50)	RHO= -.675
X= 15	39(39)	3(3)	8(6)	-	-	-	-	-	-	50(50)	RHO= -.484
X= 30	47(47)	3(3)	-	-	-	-	-	-	-	50(50)	RHO= -.409
X= 45	39(39)	11(11)	-	-	-	-	-	-	-	50(50)	RHO= .204
X= 60	3(3)	47(47)	-	-	-	-	-	-	-	50(50)	RHO= .784
X= 75	9(9)	41(41)	-	-	-	-	-	-	-	50(50)	RHO= .696
X= 90	1(1)	2(2)	26(28)	19(19)	-	-	-	-	-	50(50)	RHO= .595
X=105	48(48)	-	2(2)	-	-	-	-	-	-	50(50)	RHO= -.086
X=120	41(41)	6(6)	1(1)	-	-	-	-	-	-	50(50)	RHO= -.716
X=135	32(32)	16(18)	-	-	-	-	-	-	-	50(50)	RHO= -.173
X=150	35(35)	15(15)	-	-	-	-	-	-	-	50(50)	RHO= -.233
X=165	3(3)	-	-	-	30(30)	17(17)	-	-	-	50(50)	RHO= -.382
X=180	-	-	-	-	11(11)	17(17)	-	22(22)	-	50(50)	RHO= -.713
X=195	-	-	-	-	28(28)	22(22)	-	-	-	50(50)	RHO= -.287
X=210	49(49)	1(1)	-	-	-	-	-	-	-	50(50)	RHO= .055
X=225	50(50)	-	-	-	-	-	-	-	-	50(50)	RHO= .455
X=240	11(11)	39(39)	-	-	-	-	-	-	-	50(50)	RHO= .530
X=255	-	1(1)	-	-	-	-	47(49)	-	-	48(50)	RHO= .838
X=270	-	-	-	7(7)	-	-	11(11)	32(32)	-	50(50)	RHO= .445
X=285	3(3)	-	-	-	-	-	47(47)	-	-	50(50)	RHO= -.119
X=300	30(30)	20(20)	-	-	-	-	-	-	-	50(50)	RHO= -.605
X=315	26(26)	24(24)	-	-	-	-	-	-	-	50(50)	RHO= -.524
X=330	40(40)	10(10)	-	-	-	-	-	-	-	50(50)	RHO= -.358
X=345	29(29)	5(5)	15(15)	1(1)	-	-	-	-	-	50(50)	RHO= -.232
TOTAL	543(543)	251(251)	69(69)	51(51)	69(69)	56(56)	105(107)	54(54)	-	1198(1200)	

Table 16 is the summary of Table 12, which has just been discussed. It shows the various classes into which the 1200 Monte Carlo simulations fell as a function of aspect angle for the objective criteria method and the corresponding number of times that the MLE method made an exactly correct choice. There are other statistics that can be garnered from these tables that will be discussed later.

From Table 16, and those like it, many features of the two methods can be seen. For example, classes 4 through 9 occur near 0^{o}, 90^{o}, 180^{o} and 270^{o}. The patterns for classes 4 through 7 are related to upwind downwind ratios for the backscatter values in the model function. For this example, 471 of the first choices for class 2 out of 496 were correct. If it was not correct the second was correct. There were therefore no "errors" in the class 2 selections. In the objective method for classes 1, 4, 5 and 8 there were 9 incorrect "unique" choices out of a total of 238, or 229 correct unique winds.

These plus 471 correct first choices for class 2 would yield 700 correct winds out of 1200. The second choice for class 2 would usually obviously be correct in a wind field so that there would be 725 correct winds. There would be 45 more two vector solutions, one of which would be correct and 418 four vector solutions.

The MLE selects one vector wind only and it is either correct or incorrect. For this example 1137 selections were correct (95%). The MLE, if it were to work as well on actual data as it does in this simulation is clearly superior. There are differences between actual data and these simulations, such as mesoscale turbulence effects, that may not make the choice between the two methods as obvious.

Figure 10 summarizes the top part of Table 16 in graphical form. The cumulative sum of the various classes in terms of the number of solutions generated by the decisions of the method is graphed.

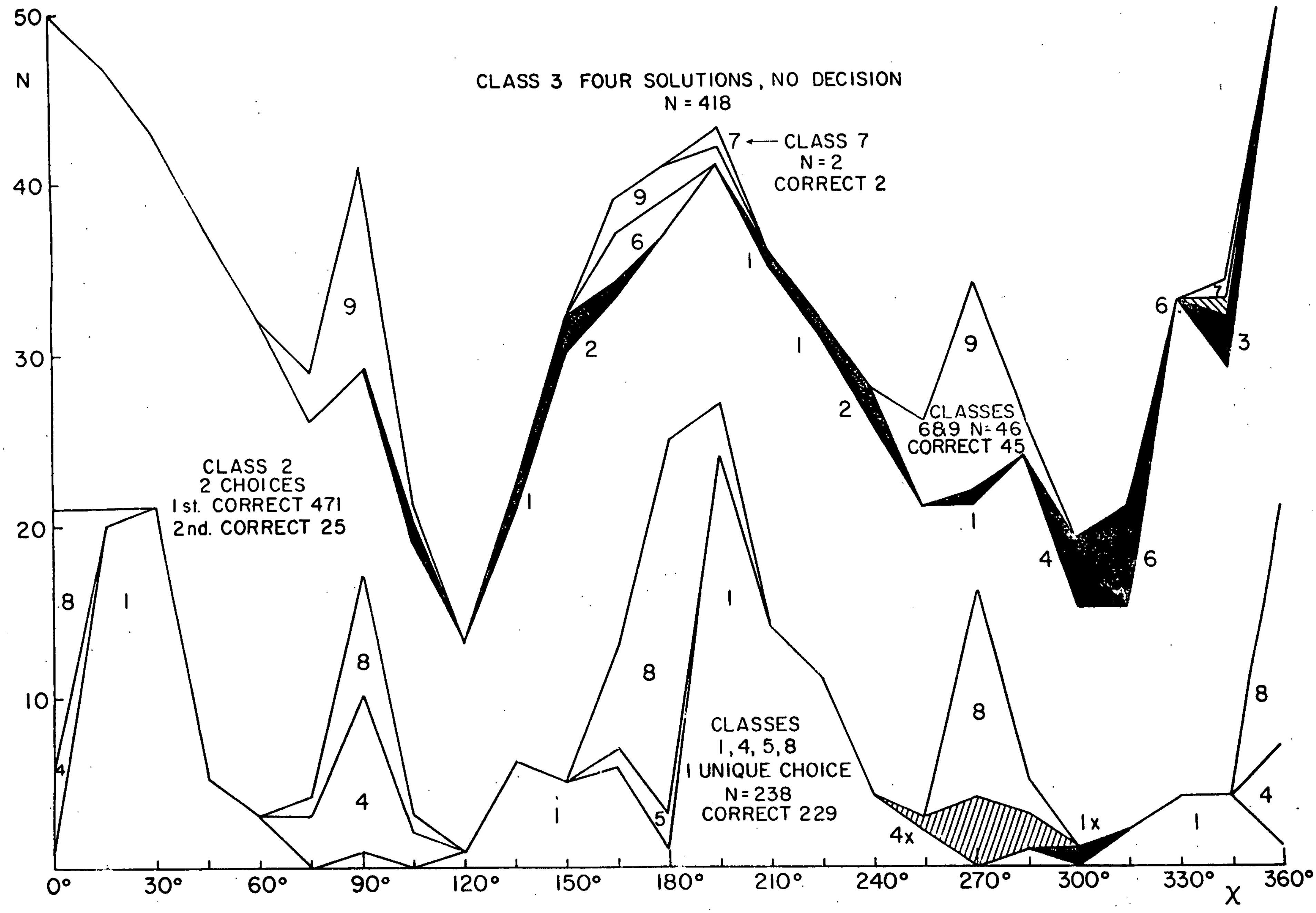


FIG. 10 THE NUMBER OF DIFFERENT CLASSES AS A FUNCTION OF ASPECT ANGLE FOR A 29° INCIDENCE ANGLE AND 12 M/S (1 CELL).

Classes 1, 4, 5 and 8 are unique. The lowest line shows class 1, and the shaded black area shows the one incorrect class 1 choice at 300°. Class 4 is next and a cluster of 7 incorrect class 4 choices is shown near 270°. The curve just above areas marked with 1's, 4's, 5's and 8's represents the total out of 50 unique choices, a few of which are wrong.

The next band marks off the class 2 solutions with the black band marking the number of times the second choice would be correct.

The next group of 9's and 6's show these cases where 2 solutions would be selected. Class 7 is for 3 solutions. Class 3 is "don't know" with four possible solutions.

If this pattern were to exist over a range of speeds, directions and incidence angles so that the objectively determined vectors could be plotted as a modified "chicken plot" as in the various SEASAT workshops, some portions of the field would be well defined near zero, 90, 180 and 270° relative to beam 1. Near 120° and 300°, the fields would not be well defined but elementary meteorology around lows and highs could be used to complete the field. Further illustrations of this point will be given later.

Table 13 illustrates an additional feature of class 2. At $\chi = 0^{\circ}$, under 2B one finds 0(9,0). At $\chi = 120^{\circ}$, one finds 2(12,2) and at $\chi = 270^{\circ}$, one finds 3(3, -1). The 0° entry shows that all 9 first choices are correct. The 120° entry shows that 2 of 12 second choices were correct and that neither of the 2 choices was correct for two simulations. The 270° entry shows that for one pair of class 2 choices both were within $\pm 45^{\circ}$ and could be treated as correct. The closest of the two was used in the statistics. The bottom line with 33(248, 8) should be interpreted in terms of the total column to mean that there were 10 pairs, neither of which was correct, and 2 pairs for which either solution could be considered to be correct. Of the 248 class 2 cases, the first choice was correct 207 times, the second was 31(33-2) times and neither was correct 10 times.

Properties of the Normalized Likelihood Results

Each search of the V-χ plane resulted in the location of the largest maximum of the normalized likelihood function. This was selected as the "true" wind and was correct if the wind direction was within $\pm 45^{o}$ of the input wind.* If the wind was correct, it was used in the table given above to compute the various statistics.

Also if the result was correct, the value of $L'(V, \chi; \hat{\sigma}^{o}_{ij})$ was assigned to its speed and direction and tabulated in a two-way table. Multiple values in the same bin were averaged and their total tabulated. The result was 24 x 28 = 672 such scatter plots. A few examples will be discussed below. They can all be found in the Data Appendix.

Each scatter plot covered the central speed and direction, $\pm$ 0.8 m/s and $\pm 10^{o}$. Correct values outside this range were simply not plotted. Typical examples were selected for discussion.

The conditions for the tables which follow, are summarized below

	SPEED	INC. ANGLE	DIR.	
Table 41	12 m/s,	29^{o},	15^{o},	1 cell
Table 42	12 m/s,	29^{o},	255^{o},	1 cell
Table 43	24 m/s,	29^{o},	45^{o},	1 cell
Table 44	24 m/s,	29^{o},	285^{o},	1 cell
Table 45	8 m/s,	39^{o},	270^{o},	1 cell
Table 46	4 m/s,	47^{o},	285^{o},	1 cell
Table 47	4 m/s,	39^{o},	90^{o},	5 cells
Table 48	4 m/s,	53.5^{o},	60^{o},	5 cells
Table 49	4 m/s,	53.5^{o},	180^{o},	5 cells
Table 50	4 m/s,	53.5^{o},	345^{o},	5 cells
Table 51	4 m/s,	53.5^{o},	60^{o},	25 cells
Table 52	4 m/s,	53.5^{o},	285^{o},	25 cells

*45^{o} is simply a yes-no criterion. The actual error statistics are given in Table 12 as examples, in later analyses and in the appendix.

†The incidence angle is for beam 1.

Table 41 is for 12 m/s, 15° direction, one cell and 29° incidence angle. The MLE winds scatter from 21.5° to 9.5° and from 11.5 m/s to 12.3 m/s. There were 49 out of 50 correct choices. There are 46 values in the table so that 3 are off the page. The normalized likelihood function varies from 0.01 (to two significant figures) to 0.98. The value of the function has no relationship to how close the point is to the correct wind.

For this data, the average wind was 11.95 m/s, the average direction was 14.53°, the standard deviation of the wind speed was 0.2121 and the standard deviation of that direction was 4.30 degrees. The correlation between the wind speed and the wind direction is -0.283. (Standard bivariate statistics have been used.)

Table 42 shows a similar pattern except that there were only 48 correct choices and the correlation between speed and direction is 0.5354. For each table, regression equations that predict either V from χ or χ from V can be derived. Over an area of the ocean, the wind direction might be averaged for a number of cells, used to represent the entire area and to correct part of the error in wind speed. These regression equations will be tabulated later.

Table 43 shows a large scatter in speed. The correlation coefficient is 0.3874. The standard deviation of the speed is 0.536 m/s.

Table 44 shows an even larger scatter in speed and direction for only 37 correct solutions.

The remaining tables can be studied individually. Of interest is Table 46 with only 5 values out of 19 in the range of the variables. For 5 cell averages as in Table 47, the values cluster nicely. The correlation coefficient is 0.6994.

At a 53.5° incidence angle (Table 48) a 5 cell average for 4 m/s does not reduce the scatter. The correlation coefficient is 0.8907.

At 180°, 4 m/s for a 53.5 incidence angle for a 5 cell average, only 5 points out of 19 are in range (Table 49).

For a 25 cell average, 4 m/s at 53.5° and 60°, Table 51) 39 were correct and the correlation was 0.8524. Finally for 53.5°, 4 m/s, 25 cells, and 285°, the scatter is as great as for 1 cell averages and higher winds for lower incidence angles. The correlation coefficient is 0.1103 (Table 52).

1 CELL

V= 12.0 THETA= 29.0 CHI= 15.0

	11.2	11.3	11.4	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5	12.6	12.7	12.8
25.0																	
24.5																	
24.0																	
23.5																	
23.0																	
22.5																	
22.0																	
21.5								.12/ 1	.97/ 1								
21.0																	
20.5																	
20.0											.01/ 1						
19.5									.23/ 1								
19.0						.75/ 1		.94/ 1	.25/ 1								
18.5																	
18.0						.30/ 1											
17.5							.22/ 1	.74/ 1									
17.0								.34/ 1	.50/ 2	.34/ 1							
16.5						.12/ 1	.67/ 1			.67/ 1	.06/ 1						
16.0					.96/ 1		.13/ 1										
15.5									.57/ 2	.30/ 1							
15.0				.56/ 1					.50/ 2								
14.5						.60/ 2		.61/ 1	.22/ 2								
14.0																	
13.5								.54/ 2	.37/ 2								
13.0																	
12.5								.51/ 1									
12.0				.72/ 1		.51/ 1	.55/ 2										
11.5								.08/ 1									
11.0									.30/ 1								
10.5								.54/ 1									
10.0												.22/ 1					
9.5																	
9.0									.58/ 2								
8.5																	
8.0																	
7.5																	
7.0																	
6.5																	
6.0																	
5.5																	
5.0																	

1 CELL

V= 49 VBAR= 11.95 XBAR= 14.53 SDV= .2121 SDX 4.3064 RHO= -.2832

V = 11.95 + -.0139(X - 14.53) X = 14.53 + -5.7507(V - 11.95)

V= 12.0 THETA= 29.0 CHI= 255.0

	11.2	11.3	11.4	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5	12.6	12.7	12.8
265.0										.44/ 1							
264.5																	
264.0																	
263.5																	
263.0													.00/ 1				
262.5																	
262.0													.13/ 1				
261.5																	
261.0																	
260.5											.42/ 1						
260.0																	
259.5																	
259.0																	
258.5																	
258.0																	
257.5												.26/ 1					
257.0						.08/ 1		.56/ 1					.56/ 1				
256.5									.58/ 3								
256.0								.07/ 1	.26/ 1	.13/ 1							
255.5							.06/ 1	.66/ 2	.45/ 1								
255.0							.43/ 1	.35/ 1	.18/ 1					.08/ 1			
254.5							.02/ 1		.26/ 2	.23/ 1							
254.0													.15/ 1				
253.5									.01/ 1								
253.0								.62/ 1	.27/ 1								
252.5						.79/ 1				.52/ 1							
252.0					.08/ 1		.21/ 2		.23/ 3	.24/ 1							
251.5						.65/ 1			.04/ 1	.21/ 1							
251.0																	
250.5																	
250.0										.49/ 1							
249.5										.10/ 1							
249.0																	
248.5							.24/ 1										
248.0																	
247.5																	
247.0						.37/ 1											
246.5																	
246.0																	
245.5																	
245.0																	

N= 48 VBAR= 12.05 XBAR= 255.29 SDV= .2500 SDX 5.8365 RHO= .5354

V = 12.05 + .0229(X - 255.29) X = 255.29 + 12.5026(V - 12.05)

TABLE 45 24 M/S, 29, 45°, 1 CELL

1 CELL DATE 010581 PAGE 263

V= 24.0 THETA= 29.0 CHI= 45.0

	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8
55.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
54.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
54.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
53.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
53.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
52.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
52.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
51.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
51.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
50.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
50.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
49.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
49.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
48.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
48.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
47.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
47.0	-	-	-	-	.0[illegible]/ 1	-	-	-	-	.05/ 1	-	-	-	-	-	.32/ 1	-
46.5	-	-	-	-	-	-	-	-	.03/ 2	.06/ 1	.01/ 1	.06/ 1	.1[illegible]/ 1	-	-	.02/ 1	-
46.0	-	-	-	-	-	-	-	-	.10/ 1	-	-	-	-	-	-	-	.00/ 1
45.5	-	-	-	-	-	-	-	-	.11/ 2	-	-	-	-	-	-	-	.04/ 1
45.0	.11/ 1	-	-	-	.05/ 1	-	-	.03/ 1	-	.04/ 1	.15/ 1	-	-	-	-	-	-
44.5	-	-	-	-	-	-	.26/ 1	.03/ 1	-	-	-	-	-	-	-	-	-
44.0	-	-	-	-	-	-	-	-	-	-	-	.45/ 2	-	-	-	-	-
43.5	-	-	.10/ 1	-	-	.08/ 1	-	-	.09/ 3	-	-	-	-	-	-	.02/ 1	-
43.0	-	-	-	-	-	-	-	-	.07/ 1	-	.12/ 1	-	.57/ 1	-	-	-	-
42.5	-	.08/ 1	.09/ 1	-	-	-	-	-	-	.13/ 1	-	-	-	-	-	-	-
42.0	-	-	-	-	-	-	.06/ 1	-	.09/ 1	-	-	-	-	-	-	-	-
41.5	-	-	.00/ 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
41.0	-	-	-	-	-	-	-	-	-	-	-	.06/ 1	-	-	-	-	-
40.5	-	-	.01/ 1	-	-	-	-	-	-	-	.17/ 1	-	-	-	-	-	-
40.0	-	-	-	-	-	-	-	-	-	.07/ 1	-	-	-	-	-	-	-
39.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
39.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
38.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
38.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
37.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
37.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
35.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
35.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

* See page 175 for data that are missing.

V= 24.0 THETA= 29.0 CHI= 255.0

	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8
295.0																	
294.5																	
294.0																	
293.5								.00/ 1									
293.0																	
292.5																	
292.0																	
291.5																	
291.0										.36/ 1							
290.5																	
290.0							.41/ 1										
289.5				.56/ 1	.73/ 1							.25/ 1				.40/ 2	
289.0													.01/ 1				
288.5				.19/ 1													
288.0																	
287.5							.86/ 1								.13/ 1		
287.0									.15/ 1								
286.5						.10/ 1											
286.0									.18/ 1								
285.5													.15/ 1				.34/ 1
285.0	.77/ 1	.63/ 1					.67/ 1		.05/ 2			.26/ 1					
284.5			.35/ 1		.08/ 1				.00/ 1			.15/ 1					
284.0																	
283.5									.12/ 1								
283.0					.02/ 1		.21/ 1										
282.5											.00/ 1						
282.0								.72/ 1									
281.5																	
281.0																	
280.5																	
280.0														.33/ 1			
279.5												.19/ 1					
279.0																	
278.5																	
278.0																	
277.5																	
277.0							.43/ 1										
276.5																	
276.0																	
275.5																	
275.0																	

N= 37 VBAR= 23.99 XBAR= 285.30 SDV= .4535 SDX 4.0685 RHO= .1121

V = 23.99 + .0125(X - 285.30) X = 285.30 + 1.0057(V - 23.99)

V= 8.0 THETA= 39.0 CHI= 270.0

	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8
280.0	-	-	-	-	-	-	-	-	-	.33/ 1	-	-	-	-	-	-	-
279.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
279.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
278.5	-	-	-	-	-	-	-	-	.00/ 1	-	-	-	-	-	-	-	-
278.0	-	-	-	-	-	-	.00/ 1	-	-	.23/ 2	-	-	-	-	-	-	-
277.5	-	-	-	-	-	-	-	-	-	-	.06/ 1	-	-	-	-	-	-
277.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
276.5	-	-	-	-	-	-	-	-	-	.45/ 1	-	-	-	-	-	-	-
276.0	-	-	-	-	-	-	.12/ 1	-	-	-	-	-	-	-	-	-	-
275.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
275.0	-	-	-	-	-	-	-	-	-	.22/ 1	-	-	-	-	-	-	-
274.5	-	-	-	-	-	-	-	-	-	-	-	.44/ 1	-	-	-	-	-
274.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
273.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
273.0	-	-	-	-	-	-	-	.03/ 1	.21/ 2	.00/ 1	.54/ 1	-	-	-	-	-	-
272.5	-	-	-	-	-	.21/ 1	-	-	-	-	-	-	-	-	-	-	-
272.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
271.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
271.0	-	-	-	-	-	-	-	.26/ 1	-	-	-	-	-	-	-	-	-
270.5	-	-	-	-	-	-	-	-	.15/ 1	-	-	-	-	-	-	-	-
270.0	-	-	-	-	.47/ 1	-	-	.74/ 1	.44/ 1	-	-	-	-	-	-	-	-
269.5	-	-	-	-	-	-	.08/ 1	-	-	.34/ 1	-	-	-	-	-	-	-
269.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
268.5	-	-	-	-	-	-	-	-	-	.09/ 1	-	-	-	-	-	-	-
268.0	-	-	-	-	-	-	.01/ 1	-	-	-	-	-	-	-	-	-	-
267.5	-	-	-	-	-	-	-	-	.31/ 1	-	-	-	-	-	-	-	-
267.0	-	-	-	-	-	-	-	.72/ 1	.28/ 1	-	-	-	-	-	-	-	-
266.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
266.0	-	-	-	-	-	-	-	.61/ 2	-	-	-	-	-	-	-	-	-
265.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
265.0	-	-	-	-	-	-	-	.38/ 1	-	-	-	-	-	-	-	-	-
264.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
264.0	-	-	-	-	-	.15/ 1	-	-	-	-	-	-	-	-	-	-	-
263.5	-	-	-	-	-	-	-	-	.42/ 1	-	-	-	-	-	-	-	-
263.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
262.5	-	-	-	-	-	.01/ 1	-	.48/ 1	-	-	-	-	-	-	-	-	-
262.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
261.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
261.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
260.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
260.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

V= 40 VBAR= 7.96 XBAR= 270.56 SDV= .2586 SDX 6.9793 RHO= .7239

V = 7.96 + .0268(X - 270.55) X = 270.55 + 19.5405(V - 7.96)

1 CELL DATE 010581 PAGE 40

V= 4.0 THETA= 47.0 CHI= 285.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
295.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
294.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
294.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
293.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
293.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
292.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
292.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
291.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
291.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
290.5	.	.	.	.	.	.	.	.	.	.	.	.93/ 1	.	.	.	.	.
290.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
289.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
289.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
288.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
288.0	.	.	.	.	.	.	.	.	.13/ 1	.	.	.	.	.	.	.	.
287.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
287.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
286.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
286.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
285.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
285.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
284.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
284.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
283.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
283.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
282.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
282.0	.	.	.	.	.	.	.	.	.	.	.	.60/ 1	.	.	.	.	.
281.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
281.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
280.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
280.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
279.5	.	.	.	.	.	.	.	.	.	.	.	.20/ 1	.	.	.	.	.
279.0	.	.	.	.40/ 1	.	.	.	.	.	.	.	.	.	.	.	.	.
278.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
278.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
277.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
277.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
276.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
276.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
275.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
275.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

1 CELL DATE 010581 PAGE 41

N= 19 VBAR= 3.61 XBAR= 287.66 SDV= .5776 SDX 21.7783 RHO= -.0882

V = 3.61 + -.0023(X - 287.66) X = 287.66 + -3.3251(V - 3.61)

TABLE 47 4 M/S, 39°, 90°, 5 CELLS

5 CELL DATE 010581 PAGE 62

V= 4.0 THETA= 39.0 CHI= 90.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
100.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
99.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
99.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
98.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
98.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
97.5	.	.	.	.	.	.	.	.	.	.04/ 1	.	.	.	.	.	.	.
97.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
96.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
96.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
95.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
95.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
94.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
94.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
93.5	.	.	.	.	.	.	.	.	.32/ 1	.	.	.	.	.	.	.	.
93.0	.	.	.	.	.	.	.	.19/ 1	.38/ 3	.	.	.	.	.	.	.	.
92.5	.	.	.	.	.	.	.	.	.22/ 1	.	.	.	.	.	.	.	.
92.0	.	.	.	.	.	.	.	.22/ 2	.00/ 1	.	.	.	.	.	.	.	.
91.5	.	.	.	.	.	.	.	.13/ 1	.24/ 1	.	.	.	.	.	.	.	.
91.0	.	.	.	.	.	.	.	.	.47/ 2	.	.	.	.	.	.	.	.
90.5	.	.	.	.	.	.	.	.62/ 1	.	.	.	.	.	.	.	.	.
90.0	.	.	.	.	.	.	.	.68/ 2	.47/ 6	.	.	.	.	.	.	.	.
89.5	.	.	.	.	.	.	.	.46/ 4	.50/ 3	.	.	.	.	.	.	.	.
89.0	.	.	.	.	.	.	.	.76/ 3	.35/ 1	.	.	.	.	.	.	.	.
88.5	.	.	.	.	.	.	.	.	.65/ 1	.	.	.	.	.	.	.	.
88.0	.	.	.	.	.	.	.58/ 3	.56/ 2	.57/ 1	.	.	.	.	.	.	.	.
87.5	.	.	.	.	.	.	.25/ 2	.35/ 2	.	.	.	.	.	.	.	.	.
87.0	.	.	.	.	.	.	.31/ 2	.61/ 1	.	.	.	.	.	.	.	.	.
86.5	.	.	.	.	.	.	.70/ 1	.	.	.	.	.	.	.	.	.	.
86.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
85.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
85.0	.	.	.	.	.	.20/ 1	.	.	.	.	.	.	.	.	.	.	.
84.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
84.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
83.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
83.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
82.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
82.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
81.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
81.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
80.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
80.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

5 CELL DATE 010581 PAGE 63

N= 50 VBAR= 3.94 XBAR= 89.87 SDV= .1020 SDX 2.3726 RHO= .6994

V = 3.94 + .0301(X = 89.87) X = 89.87 + 16.2697(V = 3.94)

V= 4.0 THETA= 53.5 CHI= 60.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
70.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
69.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
69.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
68.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
68.0	.	.	.	.	.	.	.	.	.	.	.	.	.00/ 1	.	.	.	.
67.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
67.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
66.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
66.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
65.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
65.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
64.5	.	.	.	.	.	.	.	.	.	.	.	.	.00/ 1	.	.	.	.
64.0	.	.	.	.	.	.	.	.	.	.	.	.01/ 1	.32/ 1	.	.	.	.
63.5	.	.	.	.	.	.	.	.	.	.	.01/ 1	.	.11/ 1	.	.	.	.
63.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
62.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
62.0	.	.	.	.	.	.	.	.	.	.	.	.12/ 1	.	.	.	.	.
61.5	.	.	.	.	.	.	.	.	.	.03/ 2	.	.	.01/ 1	.	.	.	.
61.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
60.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
60.0	.	.	.	.	.	.	.	.05/ 1	.	.	.39/ 1	.	.	.	.	.	.
59.5	.	.	.	.	.	.	.	.03/ 1	.10/ 1	.	.	.	.	.	.	.	.
59.0	.	.	.	.	.	.	.02/ 2	.	.	.	.	.	.	.	.	.	.
58.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
58.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
57.5	.	.	.	.	.	.	.	.	.11/ 1	.	.	.	.	.	.	.	.
57.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
56.5	.	.	.	.	.	.	.	.	.01/ 2	.	.	.	.	.	.	.	.
56.0	.	.	.	.	.	.02/ 1	.	.	.01/ 1	.	.	.	.	.	.	.	.
55.5	.	.	.	.	.	.	.	.11/ 1	.	.	.10/ 1	.	.	.	.	.	.
55.0	.	.	.	.	.	.20/ 1	.	.02/ 1	.	.	.	.	.	.	.	.	.
54.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
54.0	.	.	.	.	.37/ 1	.	.	.	.	.	.	.	.	.	.	.	.
53.5	.	.	.	.	.01/ 1	.	.	.	.	.	.	.	.	.	.	.	.
53.0	.	.	.	.12/ 1	.	.	.	.	.	.	.	.	.	.	.	.	.
52.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
52.0	.	.	.	.63/ 1	.	.	.	.	.	.	.	.	.	.	.	.	.
51.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
50.5	.	.	.	.	.12/ 1	.	.	.	.	.	.	.	.	.	.	.	.
50.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

N= 33 VBAR= 3.90 XBAR= 57.58 SDV= .3619 SDX 4.8449 RHO= .8907

V = 3.29 + [illegible](X = 57.58)   X = 57.59 + 11.9233(V = 3.98)

V= 4.0 THETA= 53.5 CHI= 180.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
190.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
189.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
189.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
188.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
188.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
187.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
187.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
186.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
186.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
185.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
185.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
184.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
184.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
183.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
183.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
182.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
182.0	.	.	.	.	.	.	.	.79/ 1	.	.	.	.	.	.	.	.	.
181.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
181.0	.	.	.	.	.	.	.	.	.30/ 1	.	.	.	.	.	.	.	.
180.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
180.0	.	.	.	.	.	.	.	.	.31/ 1	.	.	.	.	.	.	.	.
179.5	.	.	.	.	.	.	.	.58/ 1	.	.	.	.	.	.	.	.	.
179.0	.	.	.	.	.	.	.	.	.94/ 1	.	.	.	.	.	.	.	.
178.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
178.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
177.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
177.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
176.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
176.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
175.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
175.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
174.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
174.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
173.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
173.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
172.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
172.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
171.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
171.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
170.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
170.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

5 CELL DATE 010581 PAGE 75

N= 19 VBAR= 3.93 XBAR= 191.39 SDV= .2033 SDX 17.4663 RHO= -.6986

V = 3.94 + -.[illegible](X - 191.39)   X = 191.39 + -60.0182(V - 3.93)

5 CELL

V= 4.0 THETA= 53.5 CHI= 345.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
355.0	.	.	.	.	.47/ 1	.80/ 1	.	.	.	.	.	.	.	.	.	.	.
354.5	.	.	.45/ 1	.62/ 1	.	.19/ 1	.	.	.	.	.	.	.	.	.	.	.
354.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
353.5	.	.	.	.	.55/ 1	.	.	.	.	.	.	.	.	.	.	.	.
353.0	.	.	.	.	.13/ 2	.	.	.	.	.	.	.	.	.	.	.	.
352.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
352.0	.	.	.	.	.	.	.	.	.65/ 1	.	.	.	.	.	.	.	.
351.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
351.0	.	.	.	.	.	.	.	.08/ 1	.	.	.	.	.	.	.	.	.
350.5	.	.	.	.	.	.	.41/ 1	.	.	.	.	.	.	.	.	.	.
350.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
349.5	.	.	.	.	.	.	.	.29/ 1	.	.	.	.	.	.	.	.	.
349.0	.	.	.	.	.	.	.79/ 1	.	.	.	.	.	.	.	.	.	.
348.5	.	.	.	.	.66/ 1	.	.	.	.12/ 2	.	.	.	.	.	.	.	.
348.0	.	.	.	.	.	.	.	.68/ 1	.	.	.	.	.	.	.	.	.
347.5	.	.	.	.	.	.	.	.54/ 1	.	.	.	.	.	.	.	.	.
347.0	.	.	.	.	.	.	.16/ 1	.	.	.	.	.	.	.	.	.	.
346.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
346.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
345.5	.	.	.	.	.	.	.	.	.16/ 1	.	.	.	.	.	.	.	.
345.0	.	.	.	.	.	.	.	.36/ 1	.	.	.	.	.	.	.	.	.
344.5	.	.	.	.	.	.	.	.	.	.02/ 1	.	.	.	.	.	.	.
344.0	.	.	.	.	.	.	.	.	.	.	.	.65/ 1	.	.	.	.	.
343.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
343.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
342.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
342.0	.	.	.	.	.	.	.	.	.	.	.07/ 1	.42/ 1	.	.	.	.	.
341.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
341.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
340.5	.	.	.	.	.	.	.	.	.33/ 1	.	.	.13/ 1	.	.	.	.	.
340.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
339.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
339.0	.	.	.	.	.	.	.	.	.	.33/ 1	.	.	.	.	.	.	.
338.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
338.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
337.5	.	.	.	.	.	.	.	.	.	.39/ 1	.	.	.07/ 1	.	.	.	.
337.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
336.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
336.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.28/ 1	.06/ 1	.	.
335.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
335.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

5 CELL

N= 46 VBAR= 3.87 XBAR= 349.98 SDV= .4162 SDX 10.3585 RHO= -.9047

V = 3.87 + -.0364(X - 349.98) X = 349.98 + -22.5155(V - 3.87)

PFREE 20

25 CELL

V= 4.0 THETA= 53.5 CHI= 60.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
70.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
69.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
69.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
68.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
68.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
67.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
67.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
66.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
66.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
65.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
65.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
64.5	.	.	.	.	.	.	.	.	.	.	.02/ 1	.08/ 1	.	.	.	.	.
64.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
63.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
63.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
62.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
62.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
61.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
61.0	.	.	.	.	.	.	.	.	.29/ 2	.	.	.	.	.	.	.	.
60.5	.	.	.	.	.	.	.	.	.39/ 2	.07/ 1	.	.	.	.	.	.	.
60.0	.	.	.	.	.	.	.	.	.24/ 9	.	.	.	.	.	.	.	.
59.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
59.0	.	.	.	.	.	.	.	.02/ 2	.01/ 3	.	.	.	.	.	.	.	.
58.5	.	.	.	.	.	.	.	.24/ 4	.24/ 2	.	.	.	.	.	.	.	.
58.0	.	.	.	.	.	.	.07/ 2	.	.14/ 1	.	.	.	.	.	.	.	.
57.5	.	.	.	.	.	.	.40/ 1	.04/ 2	.06/ 1	.	.	.	.	.	.	.	.
57.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
56.5	.	.	.	.	.	.	.	.01/ 1	.	.	.	.	.	.	.	.	.
56.0	.	.	.	.	.	.00/ 1	.	.29/ 1	.	.	.	.	.	.	.	.	.
55.5	.	.	.	.	.	.	.03/ 2	.	.	.	.	.	.	.	.	.	.
55.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
54.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
54.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
53.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
53.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
52.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
52.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
50.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
50.0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

25 CELL

N= 39 VBAR= 3.98 XBAR= 59.05 SDV= .1388 SDX 2.0122 RHO= .8524

V = 3.99 + .0538(X = 59.05) X = 59.05 + 12.3609(V = 3.98)

V= 4.0 THETA= 53.5 CHI= 285.0

	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
295.0									.67/ 1								
294.5																	
294.0																	
293.5																	
293.0																	
292.5																	
292.0																	
291.5										.23/ 1							
291.0							.49/ 1		.60/ 2								
290.5																	
290.0										.72/ 1							
289.5									.21/ 2								
289.0																	
288.5																	
288.0									.72/ 2								
287.5																	
287.0											.30/ 1						
286.5										.15/ 1							
286.0								.65/ 1									
285.5									.45/ 2								
285.0																	
284.5								.34/ 1		.64/ 1							
284.0									.14/ 1								
283.5																	
283.0								.41/ 1									
282.5																	
282.0									.22/ 1								
281.5																	
281.0																	
280.5																	
280.0												.25/ 1					
279.5																	
279.0								.31/ 1	.61/ 1								
278.5									.59/ 2								
278.0									.56/ 1								
277.5							.04/ 1										
277.0				.54/ 1													
276.5						.08/ 1			.84/ 1								
276.0																	
275.5										.12/ 1							
275.0																	

V= 34 VBAR= 3.99 XBAR= 285.12 SDV= .2031 SDX 7.4642 RHO= -.1103

V = 3.99 + -.[illegible](X - 285.12) X = 285.12 + -4.0596(V - 3.99)

It was shown in a previous section that $-2\,\ln\,L'(V,\chi;\,\hat{\sigma}^{o}_{ij})$ would be approximately distributed as a Chi-squared variable with six degrees of freedom if it could be computed at the "true" wind speed and direction. The location of the maximum increases the value of the function so that the question arises as to how far the actual distribution departs from the Chi-squared probabilities.

Figure 11 shows the cumulative densities for Chi-squared distributions for 3, 4, 5 and 6 degrees of freedom as smooth curves. The sample comulative density for curve A was evaluated from tabulated values such as those in the preceeding figures for data with an incidence angle of 47^{o}, a speed of 4 m/s and one cell averages Curve B is for 53.5^{o}, 4 m/s and 5 cell averages. Curve A is fairly close to the curve for 3 degrees of freedom. Curve B is mostly between the curves for 5 and 6 degrees of freedom. The coarseness of the step function for large Chi Square is because of the rounding to two significant figures in the data. The figure nevertheless explains most of the variability of the normalized likelihood function as slightly modified Chi-squared variables.

Statistics of the Scatter Plots of the Wind Speeds and Directions Determined from the Maxima of the Normalized Likelihood Function.

Each of the 672 scatter plots yielded the number correct and the five statistics* typically associated with such scatter plots. The five statistics in turn yield two regression equations. The 28 tables that follow give these statistics and the regression equations.

Table 67 shows correlation coefficients at 0^{o} and 180^{o} of - 0.9164 and - 0.9209. Absolute values greater than 0.9 and 0.8 are frequent, which means that 80% to 64% of the variability of either speed or direction can be predicted from the other quantity if the true values are known. The results also imply rather large variations in direction compared to rather small fluctuations in speed about a correct value.

*These are the means and standard deviations for speed and direction and the correlation between speed and direction errors as symbolized by V Bar, X Bar, SDV, SDX, and RHO.

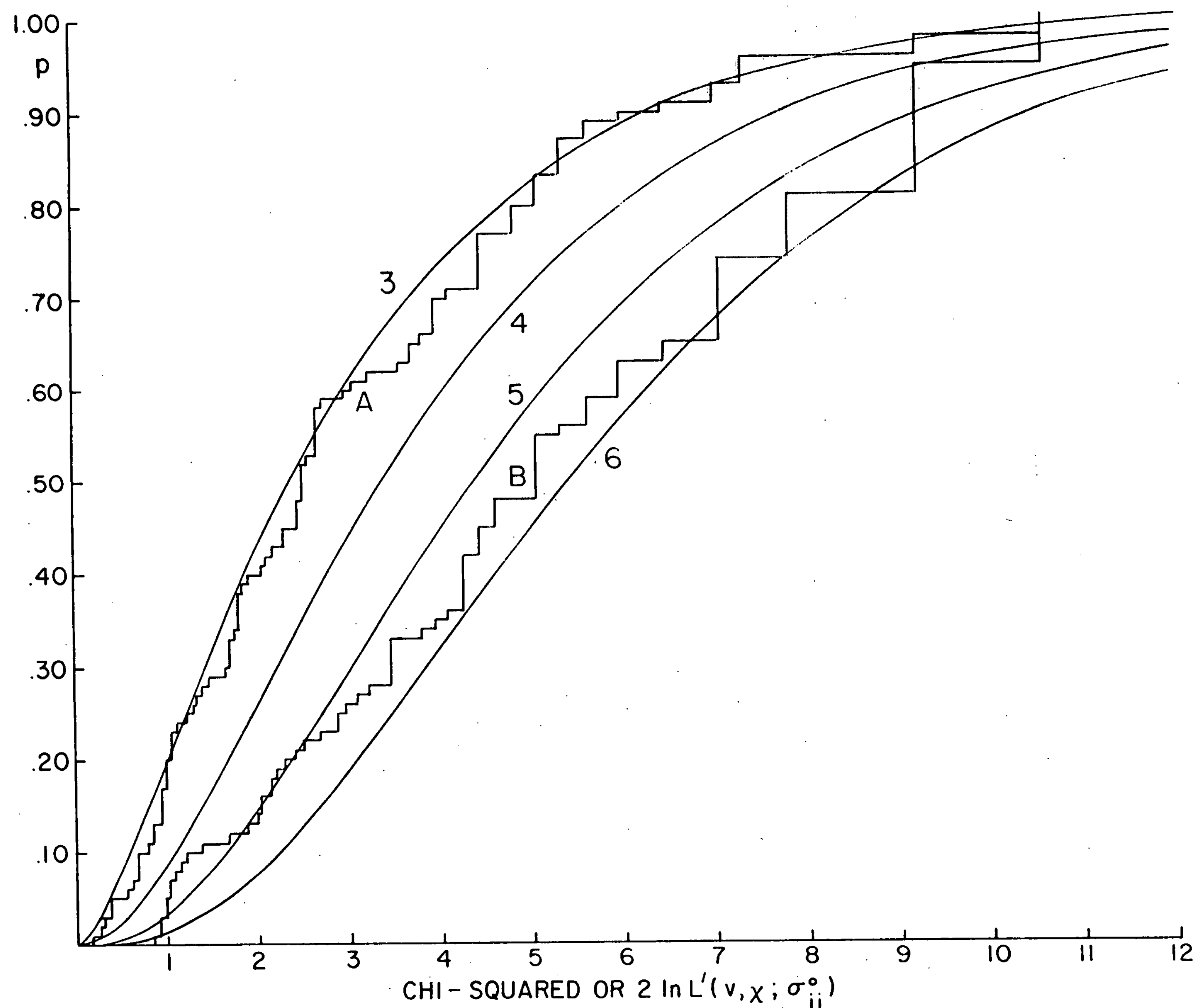


FIG. 11 CUMULATIVE DENSITIES OF CHI-SQUARED DISTRIBUTIONS WITH 3, 4, 5 AND 6 DEGREES OF FREEDOM COMPARED TO MONTE CARLO CUMULATIVE DISTRIBUTIONS FOR $\theta = 47^{\circ}$, ONE CELL AVERAGES AND 4 M/S (CURVE A) AND FOR $\theta = 53.5^{\circ}$, FIVE CELL AVERAGES AND 4 M/S (CURVE B), (VARIOUS ASPECT ANGLES TO TOTAL A SAMPLE OF 100).

TABLE 53 V = 4, θ = 29, 1 CELL

1 CELL

```
V=   4.0 THETA=  29.0 CHI=     .0
N= 49 VBAR=   3.95 XBAR=    .95 SDV=    .1554 SDX   5.9344 RHO=  -.6613
      V =   3.95 +   -.0173( X -    .95)          X =    .95 + -25.2605( V -   3.95)

V=   4.0 THETA=  29.0 CHI=  15.0
N= 48 VBAR=   3.98 XBAR=  14.17 SDV=    .1342 SDX   6.1537 RHO=  -.0521
      V =   3.98 +   -.0011( X -  14.17)          X =  14.17 +  -2.3891( V -   3.98)

V=   4.0 THETA=  29.0 CHI=  30.0
N= 50 VBAR=   3.96 XBAR=  29.03 SDV=    .1229 SDX   3.3637 RHO=   .5130
      V =   3.96 +    .0187( X -  29.03)          X =  29.03 +  14.0388( V -   3.96)

V=   4.0 THETA=  29.0 CHI=  45.0
N= 35 VBAR=   3.97 XBAR=  43.83 SDV=    .1976 SDX   3.5071 RHO=   .4348
      V =   3.97 +    .0245( X -  43.83)          X =  43.83 +   7.7152( V -   3.97)

V=   4.0 THETA=  29.0 CHI=  60.0
N= 38 VBAR=   4.06 XBAR=  60.14 SDV=    .2277 SDX   3.2866 RHO=   .3417
      V =   4.06 +    .0237( X -  60.14)          X =  60.14 +   4.9307( V -   4.06)

V=   4.0 THETA=  29.0 CHI=  75.0
N= 42 VBAR=   4.00 XBAR=  76.36 SDV=    .2126 SDX   9.2937 RHO=   .4431
      V =   4.00 +    .0101( X -  76.36)          X =  76.36 +  19.3663( V -   4.00)

V=   4.0 THETA=  29.0 CHI=  90.0
N= 48 VBAR=   3.90 XBAR=  91.98 SDV=    .1486 SDX   7.6322 RHO=   .4162
      V =   3.90 +    .0081( X -  91.98)          X =  91.98 +  21.3807( V -   3.90)

V=   4.0 THETA=  29.0 CHI= 105.0
N= 50 VBAR=   3.80 XBAR=  98.02 SDV=    .1849 SDX   9.1127 RHO=   .4884
      V =   3.80 +    .0099( X -  98.02)          X =  98.02 +  24.0657( V -   3.80)

V=   4.0 THETA=  29.0 CHI= 120.0
N= 39 VBAR=   3.91 XBAR= 117.85 SDV=    .1856 SDX  10.2852 RHO=   .6706
      V =   3.91 +    .0121( X - 117.85)          X = 117.85 +  37.1585( V -   3.91)

V=   4.0 THETA=  29.0 CHI= 135.0
N= 37 VBAR=   3.87 XBAR= 135.31 SDV=    .1209 SDX   3.1993 RHO=  -.0856
      V =   3.87 +   -.0032( X - 135.31)          X = 135.31 +  -2.2652( V -   3.87)

V=   4.0 THETA=  29.0 CHI= 150.0
N= 36 VBAR=   3.91 XBAR= 151.83 SDV=    .2246 SDX   4.8978 RHO=  -.4579
      V =   3.91 +   -.0210( X - 151.83)          X = 151.83 +  -9.9864( V -   3.91)

V=   4.0 THETA=  29.0 CHI= 165.0
N= 44 VBAR=   4.04 XBAR= 167.82 SDV=    .2336 SDX  10.3029 RHO=  -.6081
      V =   4.04 +   -.0138( X - 167.82)          X = 167.82 + -26.8143( V -   4.04)
```

TABLE 53 (CONT'd.)

```
V=   4.0 THETA=  29.0 CHI= 180.0
N= 47 VBAR=   3.91 XBAR= 182.93 SDV=    .1810 SDX   8.0999 RHO=  -.4455
       V =   3.91 +   -.0100( X - 182.93)          X = 182.93 + -19.9349( V -   3.91)

V=   4.0 THETA=  29.0 CHI= 195.0
N= 48 VBAR=   4.06 XBAR= 189.93 SDV=    .2203 SDX  13.6269 RHO=  -.5424
       V =   4.06 +   -.0088( X - 189.93)          X = 189.93 + -33.5495( V -   4.06)

V=   4.0 THETA=  29.0 CHI= 210.0
N= 45 VBAR=   3.97 XBAR= 209.61 SDV=    .1223 SDX   4.8847 RHO=   .4056
       V =   3.97 +    .0102( X - 209.61)          X = 209.61 +  16.2049( V -   3.97)

V=   4.0 THETA=  29.0 CHI= 225.0
N= 40 VBAR=   3.95 XBAR= 225.02 SDV=    .1717 SDX   5.7612 RHO=   .4182
       V =   3.95 +    .0125( X - 225.02)          X = 225.02 +  14.0273( V -   3.95)

V=   4.0 THETA=  29.0 CHI= 240.0
N= 37 VBAR=   4.01 XBAR= 239.05 SDV=    .1485 SDX   5.9073 RHO=   .3738
       V =   4.01 +    .0094( X - 239.05)          X = 239.05 +  14.8722( V -   4.01)

V=   4.0 THETA=  29.0 CHI= 255.0
N= 35 VBAR=   4.05 XBAR= 258.69 SDV=    .1991 SDX  11.6927 RHO=   .5837
       V =   4.05 +    .0099( X - 258.69)          X = 258.69 +  34.2835( V -   4.05)

V=   4.0 THETA=  29.0 CHI= 270.0
N= 38 VBAR=   4.05 XBAR= 272.86 SDV=    .1788 SDX   9.4531 RHO=   .6037
       V =   4.05 +    .0114( X - 272.86)          X = 272.86 +  31.9138( V -   4.05)

V=   4.0 THETA=  29.0 CHI= 285.0
N= 34 VBAR=   3.97 XBAR= 284.81 SDV=    .1501 SDX  10.6601 RHO=   .5648
       V =   3.97 +    .0080( X - 284.81)          X = 284.81 +  40.1058( V -   3.97)

V=   4.0 THETA=  29.0 CHI= 300.0
N= 31 VBAR=   3.98 XBAR= 295.98 SDV=    .1505 SDX  12.3837 RHO=   .2466
       V =   3.98 +    .0030( X - 295.98)          X = 295.98 +  20.2919( V -   3.98)

V=   4.0 THETA=  29.0 CHI= 315.0
N= 37 VBAR=   3.99 XBAR= 316.27 SDV=    .1436 SDX   4.3202 RHO=  -.4121
       V =   3.99 +   -.0137( X - 316.27)          X = 316.27 + -12.3955( V -   3.99)

V=   4.0 THETA=  29.0 CHI= 330.0
N= 44 VBAR=   4.07 XBAR= 329.05 SDV=    .2020 SDX   4.8670 RHO=  -.5809
       V =   4.07 +   -.0241( X - 329.05)          X = 329.05 + -13.9952( V -   4.07)

V=   4.0 THETA=  29.0 CHI= 345.0
N= 47 VBAR=   3.90 XBAR= 346.82 SDV=    .2104 SDX   5.6467 RHO=  -.6340
       V =   3.90 +   -.0236( X - 346.82)          X = 346.82 + -17.0161( V -   3.90)
```

TABLE 54 V = 4, θ = 29, 5 CELLS

5 CELL

```
V=   4.0 THETA=  29.0 CHI=    .0
N= 50 VBAR=   4.00 XBAR= 359.34 SDV=    .0775 SDX   2.6785 RHO=  -.6167
       V =    4.00 +   -.0178( X - 359.34)          X = 359.34 + -21.3201( V -   4.00)

V=   4.0 THETA=  29.0 CHI=  15.0
N= 50 VBAR=   3.99 XBAR=  14.46 SDV=    .0566 SDX   2.3385 RHO=   .1997
       V =    3.99 +    .0048( X -  14.46)          X =  14.46 +   8.2453( V -   3.99)

V=   4.0 THETA=  29.0 CHI=  30.0
N= 50 VBAR=   3.99 XBAR=  29.82 SDV=    .0765 SDX   1.5581 RHO=   .5773
       V =    3.99 +    .0284( X -  29.82)          X =  29.82 +  11.7507( V -   3.99)

V=   4.0 THETA=  29.0 CHI=  45.0
N= 48 VBAR=   4.01 XBAR=  44.71 SDV=    .1092 SDX   1.5507 RHO=   .4398
       V =    4.01 +    .0310( X -  44.71)          X =  44.71 +   6.2440( V -   4.01)

V=   4.0 THETA=  29.0 CHI=  60.0
N= 47 VBAR=   4.02 XBAR=  59.98 SDV=    .1036 SDX   1.5710 RHO=   .2254
       V =    4.02 +    .0149( X -  59.98)          X =  59.98 +   3.4181( V -   4.02)

V=   4.0 THETA=  29.0 CHI=  75.0
N= 49 VBAR=   3.99 XBAR=  74.92 SDV=    .0843 SDX   1.6610 RHO=   .3099
       V =    3.99 +    .0157( X -  74.92)          X =  74.92 +   6.1064( V -   3.99)

V=   4.0 THETA=  29.0 CHI=  90.0
N= 50 VBAR=   3.95 XBAR=  90.08 SDV=    .0899 SDX   2.7919 RHO=   .5671
       V =    3.95 +    .0183( X -  90.08)          X =  90.08 +  17.6054( V -   3.95)

V=   4.0 THETA=  29.0 CHI= 105.0
N= 50 VBAR=   3.93 XBAR= 103.11 SDV=    .1051 SDX   4.7743 RHO=   .6921
       V =    3.93 +    .0152( X - 103.11)          X = 103.11 +  31.4369( V -   3.93)

V=   4.0 THETA=  29.0 CHI= 120.0
N= 49 VBAR=   3.96 XBAR= 119.84 SDV=    .0633 SDX   2.2052 RHO=   .2327
       V =    3.96 +    .0067( X - 119.84)          X = 119.84 +   8.1031( V -   3.96)

V=   4.0 THETA=  29.0 CHI= 135.0
N= 49 VBAR=   3.96 XBAR= 134.96 SDV=    .0729 SDX   1.4846 RHO=  -.3838
       V =    3.96 +   -.0188( X - 134.96)          X = 134.96 +  -7.8210( V -   3.96)

V=   4.0 THETA=  29.0 CHI= 150.0
N= 50 VBAR=   3.97 XBAR= 150.44 SDV=    .0886 SDX   2.3040 RHO=  -.6320
       V =    3.97 +   -.0243( X - 150.44)          X = 150.44 + -16.4374( V -   3.97)
```

TABLE 54 (CONT'D.)

```
V=   4.0 THETA=  29.0 CHI= 165.0

N= 50 VBAR=   4.00 XBAR= 165.65 SDV=    .1421 SDX   4.4548 RHO=  -.7496

       V =   4.00 +   -.0239( X - 165.65)          X = 165.65 + -23.4959( V -   4.00)

V=   4.0 THETA=  29.0 CHI= 180.0

N= 50 VBAR=   4.01 XBAR= 179.96 SDV=    .1221 SDX   4.7077 RHO=  -.6238

       V =   4.01 +   -.0162( X - 179.96)          X = 179.96 + -24.0553( V -   4.01)

V=   4.0 THETA=  29.0 CHI= 195.0

N= 50 VBAR=   3.97 XBAR= 194.14 SDV=    .0602 SDX   5.0692 RHO=  -.0656

       V =   3.97 +   -.0008( X - 194.14)          X = 194.14 +  -5.5294( V -   3.97)

V=   4.0 THETA=  29.0 CHI= 210.0

N= 50 VBAR=   3.99 XBAR= 209.55 SDV=    .0664 SDX   2.2285 RHO=   .3971

       V =   3.99 +    .0118( X - 209.55)          X = 209.55 +  13.3305( V -   3.99)

V=   4.0 THETA=  29.0 CHI= 225.0

N= 47 VBAR=   3.97 XBAR= 223.90 SDV=    .1086 SDX   2.5793 RHO=   .6025

       V =   3.97 +    .0254( X - 223.90)          X = 223.90 +  14.3114( V -   3.97)

V=   4.0 THETA=  29.0 CHI= 240.0

N= 47 VBAR=   4.04 XBAR= 240.17 SDV=    .0955 SDX   1.8418 RHO=   .3226

       V =   4.04 +    .0167( X - 240.17)          X = 240.17 +   6.2212( V -   4.04)

V=   4.0 THETA=  29.0 CHI= 255.0

N= 47 VBAR=   3.99 XBAR= 255.44 SDV=    .0881 SDX   2.8622 RHO=   .2593

       V =   3.99 +    .0080( X - 255.44)          X = 255.44 +   8.4243( V -   3.99)

V=   4.0 THETA=  29.0 CHI= 270.0

N= 50 VBAR=   4.01 XBAR= 271.12 SDV=    .0956 SDX   5.4632 RHO=   .5920

       V =   4.01 +    .0104( X - 271.12)          X = 271.12 +  33.8313( V -   4.01)

V=   4.0 THETA=  29.0 CHI= 285.0

N= 49 VBAR=   3.99 XBAR= 283.63 SDV=    .0814 SDX   5.3031 RHO=   .5185

       V =   3.99 +    .0080( X - 283.63)          X = 283.63 +  33.7592( V -   3.99)

V=   4.0 THETA=  29.0 CHI= 300.0

N= 41 VBAR=   3.98 XBAR= 299.68 SDV=    .1057 SDX   2.2483 RHO=  -.3148

       V =   3.98 +   -.0148( X - 299.68)          X = 299.68 +  -6.6963( V -   3.98)

V=   4.0 THETA=  29.0 CHI= 315.0

N= 44 VBAR=   3.99 XBAR= 315.24 SDV=    .1104 SDX   2.3723 RHO=  -.7375

       V =   3.99 +   -.0343( X - 315.24)          X = 315.24 + -15.8437( V -   3.99)

V=   4.0 THETA=  29.0 CHI= 330.0

N= 49 VBAR=   4.03 XBAR= 329.37 SDV=    .0977 SDX   2.0560 RHO=  -.6330

       V =   4.03 +   -.0301( X - 329.37)          X = 329.37 + -13.3176( V -   4.03)

V=   4.0 THETA=  29.0 CHI= 345.0

N= 50 VBAR=   3.95 XBAR= 346.59 SDV=    .1252 SDX   3.4538 RHO=  -.8083

       V =   3.95 +   -.0293( X - 346.59)          X = 346.59 + -22.2887( V -   3.95)
```

TABLE 55 $V = 8$, $V = 8$; $\theta = 29$, 1 CELL

```
1 CELL

V=   8.0 THETA=  29.0 CHI=     .0
N= 50 VBAR=    7.91 XBAR=    1.69 SDV=     .1931 SDX    4.9728 RHO=  -.4882
      V =    7.91 +    -.0190( X -    1.69)          X =    1.69 + -12.5688( V -    7.91)

V=   8.0 THETA=  29.0 CHI=   15.0
N= 50 VBAR=    7.99 XBAR=   14.47 SDV=     .1074 SDX    3.2425 RHO=    .0108
      V =    7.99 +     .0004( X -   14.47)          X =   14.47 +     .3258( V -    7.99)

V=   8.0 THETA=  29.0 CHI=   30.0
N= 50 VBAR=    7.93 XBAR=   29.66 SDV=     .1601 SDX    2.8696 RHO=    .4170
      V =    7.93 +     .0233( X -   29.66)          X =   29.66 +    7.4746( V -    7.93)

V=   8.0 THETA=  29.0 CHI=   45.0
N= 41 VBAR=    7.99 XBAR=   44.96 SDV=     .2835 SDX    2.1848 RHO=    .4680
      V =    7.99 +     .0607( X -   44.96)          X =   44.96 +    3.6058( V -    7.99)

V=   8.0 THETA=  29.0 CHI=   60.0
N= 47 VBAR=    8.01 XBAR=   60.09 SDV=     .2083 SDX    2.3414 RHO=    .2439
      V =    8.01 +     .0217( X -   60.09)          X =   60.09 +    2.7411( V -    8.01)

V=   8.0 THETA=  29.0 CHI=   75.0
N= 47 VBAR=    7.91 XBAR=   75.53 SDV=     .2748 SDX    4.2054 RHO=    .2632
      V =    7.91 +     .0172( X -   75.53)          X =   75.53 +    4.0280( V -    7.91)

V=   8.0 THETA=  29.0 CHI=   90.0
N= 49 VBAR=    7.89 XBAR=   90.90 SDV=     .2499 SDX    6.2082 RHO=    .3884
      V =    7.89 +     .0156( X -   90.90)          X =   90.90 +    9.6494( V -    7.89)

V=   8.0 THETA=  29.0 CHI= 105.0
N= 47 VBAR=    7.83 XBAR= 103.79 SDV=     .1672 SDX    5.0528 RHO=    .3180
      V =    7.83 +     .0105( X - 103.79)          X = 103.79 +    9.6089( V -    7.83)

V=   8.0 THETA=  29.0 CHI= 120.0
N= 47 VBAR=    7.85 XBAR= 119.53 SDV=     .1832 SDX    2.8294 RHO=    .1838
      V =    7.85 +     .0119( X - 119.53)          X = 119.53 +    2.8390( V -    7.85)

V=   8.0 THETA=  29.0 CHI= 135.0
N= 43 VBAR=    7.93 XBAR= 134.87 SDV=     .1877 SDX    2.4044 RHO=    .0918
      V =    7.93 +     .0072( X - 134.87)          X = 134.87 +    1.1757( V -    7.93)

V=   8.0 THETA=  29.0 CHI= 150.0
N= 44 VBAR=    7.93 XBAR= 150.76 SDV=     .1949 SDX    2.7379 RHO=  -.4977
      V =    7.93 +    -.0354( X - 150.76)          X = 150.76 +  -6.9898( V -    7.93)

V=   8.0 THETA=  29.0 CHI= 165.0
N= 47 VBAR=    7.88 XBAR= 165.78 SDV=     .2046 SDX    3.9248 RHO=  -.3769
      V =    7.88 +    -.0196( X - 165.78)          X = 165.78 +  -7.2321( V -    7.88)
```

TABLE 55 (CONT'D.)

V= 8.0 THETA= 29.0 CHI= 180.0

N= 47 VBAR= 8.08 XBAR= 180.29 SDV= .2025 SDX 6.5778 RHO= -.4659

V = 8.08 + -.0143(X - 180.29)   X = 180.29 + -15.1344(V - 8.08)

V= 8.0 THETA= 29.0 CHI= 195.0

N= 50 VBAR= 7.97 XBAR= 195.32 SDV= .1877 SDX 6.7620 RHO= -.1354

V = 7.97 + -.0038(X - 195.32)   X = 195.32 + -4.8783(V - 7.97)

V= 8.0 THETA= 29.0 CHI= 210.0

N= 46 VBAR= 8.01 XBAR= 209.65 SDV= .2139 SDX 3.2560 RHO= .1710

V = 8.01 + .0112(X - 209.65)   X = 209.65 + 2.6026(V - 8.01)

V= 8.0 THETA= 29.0 CHI= 225.0

N= 43 VBAR= 8.07 XBAR= 225.56 SDV= .2245 SDX 2.8444 RHO= .4215

V = 8.07 + .0333(X - 225.56)   X = 225.56 + 5.3413(V - 8.07)

V= 8.0 THETA= 29.0 CHI= 240.0

N= 45 VBAR= 8.02 XBAR= 240.71 SDV= .1977 SDX 3.0894 RHO= .4020

V = 8.02 + .0257(X - 240.71)   X = 240.71 + 6.2829(V - 8.02)

V= 8.0 THETA= 29.0 CHI= 255.0

N= 47 VBAR= 8.03 XBAR= 256.21 SDV= .2151 SDX 6.1673 RHO= .5082

V = 8.03 + .0177(X - 256.21)   X = 256.21 + 14.5711(V - 8.03)

V= 8.0 THETA= 29.0 CHI= 270.0

N= 45 VBAR= 7.94 XBAR= 270.03 SDV= .2679 SDX 7.6335 RHO= .6210

V = 7.94 + .0218(X - 270.03)   X = 270.03 + 17.6933(V - 7.94)

V= 8.0 THETA= 29.0 CHI= 285.0

N= 46 VBAR= 7.99 XBAR= 283.54 SDV= .2292 SDX 7.6412 RHO= .5793

V = 7.99 + .0174(X - 283.54)   X = 283.54 + 19.3125(V - 7.99)

V= 8.0 THETA= 29.0 CHI= 300.0

N= 42 VBAR= 7.95 XBAR= 298.96 SDV= .2519 SDX 3.1622 RHO= .1474

V = 7.95 + .0117(X - 298.96)   X = 298.96 + 1.8508(V - 7.95)

V= 8.0 THETA= 29.0 CHI= 315.0

N= 43 VBAR= 7.99 XBAR= 315.14 SDV= .2247 SDX 2.6739 RHO= -.4879

V = 7.99 + -.0410(X - 315.14)   X = 315.14 + -5.8050(V - 7.99)

V= 8.0 THETA= 29.0 CHI= 330.0

N= 43 VBAR= 8.00 XBAR= 330.35 SDV= .2029 SDX 2.5891 RHO= -.3588

V = 8.00 + -.0281(X - 330.35)   X = 330.35 + -4.5785(V - 8.00)

V= 8.0 THETA= 29.0 CHI= 345.0

N= 48 VBAR= 7.94 XBAR= 347.06 SDV= .2429 SDX 7.8555 RHO= -.6289

V = 7.94 + -.0194(X - 347.06)   X = 347.06 + -20.3343(V - 7.94)

TABLE 56   V = 12, θ = 29, 1 CELL

1 CELL

```
V=  12.0 THETA=  29.0 CHI=    .0
N= 50 VBAR=  11.98 XBAR= 359.52 SDV=    .1605 SDX   5.2364 RHO=  -.2706
       V =  11.98 +   -.0093( X - 359.52)         X = 359.52 +  -7.8509( V -  11.98)

V=  12.0 THETA=  29.0 CHI=  15.0
N= 49 VBAR=  11.95 XBAR=  14.53 SDV=    .2121 SDX   4.3064 RHO=  -.2832
       V =  11.95 +   -.0139( X -  14.53)         X =  14.53 +  -5.7507( V -  11.95)

V=  12.0 THETA=  29.0 CHI=  30.0
N= 49 VBAR=  11.99 XBAR=  29.93 SDV=    .2179 SDX   2.2543 RHO=   .1876
       V =  11.99 +    .0181( X -  29.93)         X =  29.93 +   1.9412( V -  11.99)

V=  12.0 THETA=  29.0 CHI=  45.0
N= 43 VBAR=  11.91 XBAR=  44.76 SDV=    .3003 SDX   1.9120 RHO=   .1113
       V =  11.91 +    .0175( X -  44.76)         X =  44.76 +    .7087( V -  11.91)

V=  12.0 THETA=  29.0 CHI=  60.0
N= 49 VBAR=  11.97 XBAR=  60.02 SDV=    .2358 SDX   2.2406 RHO=   .2716
       V =  11.97 +    .0286( X -  60.02)         X =  60.02 +   2.5803( V -  11.97)

V=  12.0 THETA=  29.0 CHI=  75.0
N= 49 VBAR=  11.93 XBAR=  76.28 SDV=    .3127 SDX   6.7257 RHO=   .4622
       V =  11.93 +    .0215( X -  76.28)         X =  76.28 +   9.9407( V -  11.93)

V=  12.0 THETA=  29.0 CHI=  90.0
N= 50 VBAR=  11.77 XBAR=  90.91 SDV=    .2628 SDX   4.8845 RHO=   .3927

       V =  11.77 +    .0211( X -  90.91)         X =  90.91 +   7.2983( V -  11.77)

V=  12.0 THETA=  29.0 CHI= 105.0
N= 48 VBAR=  11.87 XBAR= 104.42 SDV=    .2559 SDX   6.5753 RHO=   .0914
       V =  11.87 +    .0036( X - 104.42)         X = 104.42 +   2.3479( V -  11.87)

V=  12.0 THETA=  29.0 CHI= 120.0
N= 50 VBAR=  11.81 XBAR= 120.05 SDV=    .2050 SDX   2.3714 RHO=   .0377
       V =  11.81 +    .0033( X - 120.05)         X = 120.05 +    .4364( V -  11.81)

V=  12.0 THETA=  29.0 CHI= 135.0
N= 46 VBAR=  11.92 XBAR= 135.11 SDV=    .2236 SDX   2.0957 RHO=   .0664
       V =  11.92 +    .0071( X - 135.11)         X = 135.11 +    .6219( V -  11.92)

V=  12.0 THETA=  29.0 CHI= 150.0
N= 46 VBAR=  11.94 XBAR= 150.16 SDV=    .2515 SDX   2.5221 RHO=  -.2874
       V =  11.94 +   -.0287( X - 150.16)         X = 150.16 +  -2.8821( V -  11.94)
```

```
V=  12.0 THETA=  29.0 CHI= 165.0
N= 47 VBAR=  11.96 XBAR= 166.14 SDV=   .2773 SDX   3.4731 RHO=  -.4380
      V =  11.96 +   -.0350( X - 166.14)          X = 166.14 +  -5.4855( V -  11.96)

V=  12.0 THETA=  29.0 CHI= 180.0
N= 48 VBAR=  12.11 XBAR= 179.18 SDV=   .2760 SDX   6.9976 RHO=  -.3696
      V =  12.11 +   -.0146( X - 179.18)          X = 179.18 +  -9.3685( V -  12.11)

V=  12.0 THETA=  29.0 CHI= 195.0
N= 50 VBAR=  12.00 XBAR= 195.52 SDV=   .2159 SDX   4.8277 RHO=  -.2059
      V =  12.00 +   -.0092( X - 195.52)          X = 195.52 +  -4.6031( V -  12.00)

V=  12.0 THETA=  29.0 CHI= 210.0
N= 48 VBAR=  11.96 XBAR= 210.74 SDV=   .2370 SDX   2.1392 RHO=   .0293
      V =  11.96 +    .0032( X - 210.74)          X = 210.74 +    .2648( V -  11.96)

V=  12.0 THETA=  29.0 CHI= 225.0
N= 49 VBAR=  11.93 XBAR= 225.04 SDV=   .2882 SDX   2.4273 RHO=   .1911
      V =  11.93 +    .0227( X - 225.04)          X = 225.04 +   1.6099( V -  11.93)

V=  12.0 THETA=  29.0 CHI= 240.0
N= 47 VBAR=  11.99 XBAR= 239.85 SDV=   .2532 SDX   2.5422 RHO=   .0305
      V =  11.99 +    .0030( X - 239.85)          X = 239.85 +    .3059( V -  11.99)

V=  12.0 THETA=  29.0 CHI= 255.0
N= 48 VBAR=  12.05 XBAR= 255.29 SDV=   .2500 SDX   5.8385 RHO=   .5354
      V =  12.05 +    .0229( X - 255.29)          X = 255.29 +  12.5026( V -  12.05)

V=  12.0 THETA=  29.0 CHI= 270.0
N= 48 VBAR=  11.89 XBAR= 272.04 SDV=   .2991 SDX   7.3005 RHO=   .3592
      V =  11.89 +    .0147( X - 272.04)          X = 272.04 +   8.7676( V -  11.89)

V=  12.0 THETA=  29.0 CHI= 285.0
N= 47 VBAR=  12.01 XBAR= 283.80 SDV=   .2140 SDX   5.7758 RHO=   .0317
      V =  12.01 +    .0012( X - 283.80)          X = 283.80 +    .8553( V -  12.01)

V=  12.0 THETA=  29.0 CHI= 300.0
N= 42 VBAR=  11.98 XBAR= 299.95 SDV=   .2708 SDX   2.6431 RHO=  -.5098
      V =  11.98 +   -.0522( X - 299.95)          X = 299.95 +  -4.9764( V -  11.98)

V=  12.0 THETA=  29.0 CHI= 315.0
N= 36 VBAR=  12.01 XBAR= 315.00 SDV=   .2743 SDX   1.8234 RHO=  -.4498
      V =  12.01 +   -.0677( X - 315.00)          X = 315.00 +  -2.9905( V -  12.01)

V=  12.0 THETA=  29.0 CHI= 330.0
N= 49 VBAR=  12.12 XBAR= 329.80 SDV=   .2848 SDX   2.6257 RHO=  -.3822
      V =  12.12 +   -.0415( X - 329.80)          X = 329.80 +  -3.5233( V -  12.12)

V=  12.0 THETA=  29.0 CHI= 345.0
N= 49 VBAR=  12.01 XBAR= 344.57 SDV=   .2333 SDX   3.5294 RHO=  -.3599
      V =  12.01 +   -.0238( X - 344.57)          X = 344.57 +  -5.4458( V -  12.01)
```

TABLE 57 V = 24, θ = 29 1 CELL

```
1 CELL

V=  24.0 THETA=  29.0 CHI=     .0
N= 49 VBAR=  23.90 XBAR=    .92 SDV=    .2577 SDX   5.5045 RHO=  -.4383
       V =  23.90 +   -.0205( X -     .92)          X =     .92 +  -9.3625( V -  23.90)

V=  24.0 THETA=  29.0 CHI=  15.0
N= 50 VBAR=  23.89 XBAR=  13.41 SDV=    .3734 SDX   6.6473 RHO=  -.5898
       V =  23.89 +   -.0331( X -  13.41)          X =   13.41 + -10.4995( V -  23.89)

V=  24.0 THETA=  29.0 CHI=  30.0
N= 50 VBAR=  23.89 XBAR=  30.06 SDV=    .3398 SDX   1.7224 RHO=  -.0676
       V =  23.89 +   -.0133( X -  30.06)          X =   30.06 +   -.3429( V -  23.89)

V=  24.0 THETA=  29.0 CHI=  45.0
N= 48 VBAR=  24.10 XBAR=  44.19 SDV=    .5359 SDX   2.1594 RHO=   .3874
       V =  24.10 +    .0961( X -  44.19)          X =   44.19 +   1.5611( V -  24.10)

V=  24.0 THETA=  29.0 CHI=  60.0
N= 49 VBAR=  24.00 XBAR=  59.72 SDV=    .4178 SDX   2.1431 RHO=   .1694
       V =  24.00 +    .0330( X -  59.72)          X =   59.72 +    .8689( V -  24.00)

V=  24.0 THETA=  29.0 CHI=  75.0
N= 50 VBAR=  23.86 XBAR=  75.37 SDV=    .5597 SDX   3.5705 RHO=   .5329
       V =  23.86 +    .0835( X -  75.37)          X =   75.37 +   3.3993( V -  23.86)

V=  24.0 THETA=  29.0 CHI=  90.0
N= 50 VBAR=  23.64 XBAR=  91.23 SDV=    .2707 SDX   4.0166 RHO=   .0542
       V =  23.64 +    .0037( X -  91.23)          X =   91.23 +    .8044( V -  23.64)

V=  24.0 THETA=  29.0 CHI= 105.0
N= 49 VBAR=  23.88 XBAR= 104.50 SDV=    .4750 SDX   3.9631 RHO=  -.0968
       V =  23.88 +   -.0116( X - 104.50)          X =  104.50 +   -.8078( V -  23.88)

V=  24.0 THETA=  29.0 CHI= 120.0
N= 48 VBAR=  23.61 XBAR= 120.27 SDV=    .3090 SDX   2.5189 RHO=  -.1653
       V =  23.61 +   -.0203( X - 120.27)          X =  120.27 +  -1.3472( V -  23.61)

V=  24.0 THETA=  29.0 CHI= 135.0
N= 45 VBAR=  23.91 XBAR= 135.06 SDV=    .3506 SDX   1.7556 RHO=  -.1517
       V =  23.91 +   -.0303( X - 135.06)          X =  135.06 +   -.7598( V -  23.91)

V=  24.0 THETA=  29.0 CHI= 150.0
N= 49 VBAR=  23.82 XBAR= 149.87 SDV=    .3581 SDX   2.1829 RHO=  -.0655
       V =  23.82 +   -.0107( X - 149.87)          X =  149.87 +   -.3994( V -  23.82)
```

TABLE 57 (CONT'D.)

V= 24.0 THETA= 29.0 CHI= 165.0

N= 50 VBAR= 23.92 XBAR= 165.98 SDV= .4291 SDX 7.2382 RHO= -.6508

V = 23.92 + -.0386(X - 165.98)   X = 165.98 + -10.9788(V - 23.92)

V= 24.0 THETA= 29.0 CHI= 180.0

N= 48 VBAR= 24.12 XBAR= 180.84 SDV= .4499 SDX 6.8590 RHO= -.1661

V = 24.12 + -.0109(X - 180.84)   X = 180.84 + -2.5325(V - 24.12)

V= 24.0 THETA= 29.0 CHI= 195.0

N= 49 VBAR= 24.03 XBAR= 194.12 SDV= .3702 SDX 4.6192 RHO= -.1196

V = 24.03 + -.0096(X - 194.12)   X = 194.12 + -1.4925(V - 24.03)

V= 24.0 THETA= 29.0 CHI= 210.0

N= 45 VBAR= 23.93 XBAR= 210.03 SDV= .4408 SDX 2.4912 RHO= .0062

V = 23.93 + .0011(X - 210.03)   X = 210.03 + .0353(V - 23.93)

V= 24.0 THETA= 29.0 CHI= 225.0

N= 49 VBAR= 23.92 XBAR= 225.16 SDV= .3343 SDX 1.5992 RHO= .2104

V = 23.92 + .0440(X - 225.16)   X = 225.16 + 1.0067(V - 23.92)

V= 24.0 THETA= 29.0 CHI= 240.0

N= 49 VBAR= 24.03 XBAR= 240.34 SDV= .4388 SDX 1.6928 RHO= .0507

V = 24.03 + .0131(X - 240.34)   X = 240.34 + .1954(V - 24.03)

V= 24.0 THETA= 29.0 CHI= 255.0

N= 34 VBAR= 24.08 XBAR= 254.07 SDV= .4738 SDX 2.7542 RHO= .5798

V = 24.08 + .0997(X - 254.07)   X = 254.07 + 3.3705(V - 24.08)

V= 24.0 THETA= 29.0 CHI= 270.0

N= 37 VBAR= 23.97 XBAR= 270.95 SDV= .4446 SDX 5.4949 RHO= .2037

V = 23.97 + .0165(X - 270.95)   X = 270.95 + 2.5174(V - 23.97)

V= 24.0 THETA= 29.0 CHI= 285.0

N= 37 VBAR= 23.99 XBAR= 285.30 SDV= .4535 SDX 4.0685 RHO= .1121

V = 23.99 + .0125(X - 285.30)   X = 285.30 + 1.0057(V - 23.99)

V= 24.0 THETA= 29.0 CHI= 300.0

N= 46 VBAR= 24.07 XBAR= 299.75 SDV= .4527 SDX 2.6463 RHO= -.4634

V = 24.07 + -.0793(X - 299.75)   X = 299.75 + -2.7084(V - 24.07)

V= 24.0 THETA= 29.0 CHI= 315.0

N= 45 VBAR= 23.94 XBAR= 314.68 SDV= .4100 SDX 2.4704 RHO= -.2984

V = 23.94 + -.0495(X - 314.68)   X = 314.68 + -1.7977(V - 23.94)

V= 24.0 THETA= 29.0 CHI= 330.0

N= 48 VBAR= 24.08 XBAR= 330.34 SDV= .4078 SDX 2.3117 RHO= -.0748

V = 24.08 + -.0132(X - 330.34)   X = 330.34 + -.4237(V - 24.08)

V= 24.0 THETA= 29.0 CHI= 345.0

N= 50 VBAR= 23.88 XBAR= 345.75 SDV= .5491 SDX 5.9129 RHO= -.4793

V = 23.88 + -.0445(X - 345.75)   X = 345.75 + -5.1609(V - 23.88)

TABLE 58 V = 4, θ = 39, 1 CELL

```
  1 CELL

V=   4.0 THETA=  39.0 CHI=     .0
N= 50 VBAR=    3.97 XBAR=   1.11 SDV=     .1778 SDX   7.1535 RHO=  -.7279
        V =    3.97 +   -.0181( X -    1.11)          X =   1.11 + -29.2838( V -   3.97)

V=   4.0 THETA=  39.0 CHI=   15.0
N= 48 VBAR=    4.00 XBAR=  14.15 SDV=     .1224 SDX   6.3941 RHO=  -.0325
        V =    4.00 +   -.0006( X -   14.15)          X =  14.15 +  -1.6972( V -   4.00)

V=   4.0 THETA=  39.0 CHI=   30.0
N= 47 VBAR=    3.95 XBAR=  28.53 SDV=     .1165 SDX   3.9510 RHO=   .5815
        V =    3.95 +    .0171( X -   28.53)          X =  28.53 +  19.7256( V -   3.95)

V=   4.0 THETA=  39.0 CHI=   45.0
N= 29 VBAR=    3.95 XBAR=  43.16 SDV=     .1850 SDX   4.1504 RHO=   .5577
        V =    3.95 +    .0249( X -   43.16)          X =  43.16 +  12.5117( V -   3.95)

V=   4.0 THETA=  39.0 CHI=   60.0
N= 23 VBAR=    4.01 XBAR=  58.76 SDV=     .2466 SDX   3.3586 RHO=   .6509
        V =    4.01 +    .0478( X -   58.76)          X =  58.76 +   8.8665( V -   4.01)

V=   4.0 THETA=  39.0 CHI=   75.0
N= 39 VBAR=    3.89 XBAR=  73.81 SDV=     .1947 SDX   4.1288 RHO=   .4644
        V =    3.89 +    .0219( X -   73.81)          X =  73.81 +   9.8458( V -   3.89)

V=   4.0 THETA=  39.0 CHI=   90.0
N= 50 VBAR=    3.87 XBAR=  90.76 SDV=     .1816 SDX   6.4532 RHO=   .5533
        V =    3.87 +    .0156( X -   90.76)          X =  90.76 +  19.6598( V -   3.87)

V=   4.0 THETA=  39.0 CHI=  105.0
N= 50 VBAR=    3.79 XBAR=  99.29 SDV=     .2067 SDX   8.4310 RHO=   .4988
        V =    3.79 +    .0122( X -   99.29)          X =  99.29 +  20.3417( V -   3.79)

V=   4.0 THETA=  39.0 CHI=  120.0
N= 39 VBAR=    3.79 XBAR= 109.86 SDV=     .2757 SDX  16.1753 RHO=   .7975
        V =    3.79 +    .0136( X -  109.86)          X = 109.86 +  46.7935( V -   3.79)

V=   4.0 THETA=  39.0 CHI=  135.0
N= 36 VBAR=    3.84 XBAR= 135.50 SDV=     .1439 SDX   4.3591 RHO=  -.0818
        V =    3.84 +   -.0027( X -  135.50)          X = 135.50 +  -2.4787( V -   3.84)

V=   4.0 THETA=  39.0 CHI=  150.0
N= 35 VBAR=    3.89 XBAR= 152.40 SDV=     .2412 SDX   5.9748 RHO=  -.5877
        V =    3.89 +   -.0237( X -  152.40)          X = 152.40 + -14.5607( V -   3.89)
```

```
V=   4.0 THETA=  39.0 CHI= 165.0
N= 31 VBAR=   4.11 XBAR= 165.40 SDV=    .3031 SDX  10.5131 RHO=  -.8638
       V =   4.11 +   -.0249( X - 165.40)          X = 165.40 + -29.9666( V -   4.11)

V=   4.0 THETA=  39.0 CHI= 180.0
N= 38 VBAR=   3.89 XBAR= 186.16 SDV=    .2018 SDX  10.6539 RHO=  -.5495
       V =   3.89 +   -.0104( X - 186.16)          X = 186.16 + -29.0106( V -   3.89)

V=   4.0 THETA=  39.0 CHI= 195.0
N= 41 VBAR=   4.10 XBAR= 191.46 SDV=    .1999 SDX  14.7362 RHO=  -.5545
       V =   4.10 +   -.0075( X - 191.46)          X = 191.46 + -40.8679( V -   4.10)

V=   4.0 THETA=  39.0 CHI= 210.0
N= 45 VBAR=   3.98 XBAR= 208.31 SDV=    .1247 SDX   9.2919 RHO=  -.0351
       V =   3.98 +   -.0005( X - 208.31)          X = 208.31 +  -2.6166( V -   3.98)

V=   4.0 THETA=  39.0 CHI= 225.0
N= 34 VBAR=   3.96 XBAR= 223.75 SDV=    .2075 SDX   9.2502 RHO=   .6333
       V =   3.96 +    .0142( X - 223.75)          X = 223.75 +  28.2309( V -   3.96)

V=   4.0 THETA=  39.0 CHI= 240.0
N= 32 VBAR=   3.99 XBAR= 238.39 SDV=    .1798 SDX   7.2984 RHO=   .7168
       V =   3.99 +    .0177( X - 238.39)          X = 238.39 +  29.0889( V -   3.99)

V=   4.0 THETA=  39.0 CHI= 255.0
N= 27 VBAR=   4.07 XBAR= 258.80 SDV=    .2494 SDX  13.4077 RHO=   .6586
       V =   4.07 +    .0123( X - 258.80)          X = 258.80 +  35.3970( V -   4.07)

V=   4.0 THETA=  39.0 CHI= 270.0
N= 23 VBAR=   4.11 XBAR= 275.65 SDV=    .2185 SDX  10.6246 RHO=   .5463
       V =   4.11 +    .0112( X - 275.65)          X = 275.65 +  26.5602( V -   4.11)

V=   4.0 THETA=  39.0 CHI= 285.0
N= 29 VBAR=   3.98 XBAR= 286.16 SDV=    .1454 SDX   9.8721 RHO=   .1828
       V =   3.98 +    .0027( X - 286.16)          X = 286.16 +  12.4134( V -   3.98)

V=   4.0 THETA=  39.0 CHI= 300.0
N= 36 VBAR=   3.98 XBAR= 297.25 SDV=    .2323 SDX  11.5916 RHO=   .2750
       V =   3.98 +    .0055( X - 297.25)          X = 297.25 +  13.7226( V -   3.98)

V=   4.0 THETA=  39.0 CHI= 315.0
N= 36 VBAR=   3.94 XBAR= 316.68 SDV=    .2033 SDX   4.7952 RHO=  -.6431
       V =   3.94 +   -.0273( X - 316.68)          X = 316.68 + -15.1705( V -   3.94)

V=   4.0 THETA=  39.0 CHI= 330.0
N= 44 VBAR=   4.11 XBAR= 328.35 SDV=    .2735 SDX   5.0082 RHO=  -.7566
       V =   4.11 +   -.0413( X - 328.35)          X = 328.35 + -13.8537( V -   4.11)

V=   4.0 THETA=  39.0 CHI= 345.0
N= 46 VBAR=   3.88 XBAR= 347.82 SDV=    .2664 SDX   7.4373 RHO=  -.7549
       V =   3.88 +   -.0270( X - 347.82)          X = 347.82 + -21.0787( V -   3.88)
```

TABLE 59 V = 4, θ = 39, 5 CELLS

5 CELL

```
V=   4.0 THETA=  39.0 CHI=     .0
N= 49 VBAR=    4.00 XBAR= 359.59 SDV=     .0756 SDX   2.9410 RHO=  -.7984
        V =    4.00 +   -.0205( X - 359.59)           X = 359.59 + -31.0513( V -   4.00)

V=   4.0 THETA=  39.0 CHI=  15.0
N= 50 VBAR=    3.99 XBAR=  14.25 SDV=     .0440 SDX   3.0826 RHO=   .0737
        V =    3.99 +    .0011( X -  14.25)           X =  14.25 +   5.1592( V -   3.99)

V=   4.0 THETA=  39.0 CHI=  30.0
N= 50 VBAR=    3.99 XBAR=  29.76 SDV=     .0700 SDX   1.7896 RHO=   .6193
        V =    3.99 +    .0242( X -  29.76)           X =  29.76 +  15.8277( V -   3.99)

V=   4.0 THETA=  39.0 CHI=  45.0
N= 46 VBAR=    4.00 XBAR=  44.40 SDV=     .0999 SDX   1.6472 RHO=   .5853
        V =    4.00 +    .0355( X -  44.40)           X =  44.40 +   9.6485( V -   4.00)

V=   4.0 THETA=  39.0 CHI=  60.0
N= 40 VBAR=    4.00 XBAR=  59.69 SDV=     .0908 SDX   1.4349 RHO=   .4377
        V =    4.00 +    .0277( X -  59.69)           X =  59.69 +   6.9171( V -   4.00)

V=   4.0 THETA=  39.0 CHI=  75.0
N= 46 VBAR=    3.98 XBAR=  74.79 SDV=     .0955 SDX   1.5798 RHO=   .5053
        V =    3.98 +    .0305( X -  74.79)           X =  74.79 +   8.3591( V -   3.98)

V=   4.0 THETA=  39.0 CHI=  90.0
N= 50 VBAR=    3.94 XBAR=  89.87 SDV=     .1020 SDX   2.3726 RHO=   .6994
        V =    3.94 +    .0301( X -  89.87)           X =  89.87 +  16.2697( V -   3.94)

V=   4.0 THETA=  39.0 CHI= 105.0
N= 50 VBAR=    3.93 XBAR= 102.95 SDV=     .1139 SDX   5.0124 RHO=   .7559
        V =    3.93 +    .0172( X - 102.95)           X = 102.95 +  33.2569( V -   3.93)

V=   4.0 THETA=  39.0 CHI= 120.0
N= 50 VBAR=    3.96 XBAR= 119.89 SDV=     .0633 SDX   2.4155 RHO=   .2398
        V =    3.96 +    .0063( X - 119.89)           X = 119.89 +   9.1558( V -   3.96)

V=   4.0 THETA=  39.0 CHI= 135.0
N= 48 VBAR=    3.95 XBAR= 135.39 SDV=     .0913 SDX   1.7244 RHO=  -.3950
        V =    3.95 +   -.0209( X - 135.39)           X = 135.39 +  -7.4632( V -   3.95)

V=   4.0 THETA=  39.0 CHI= 150.0
N= 50 VBAR=    3.95 XBAR= 150.88 SDV=     .0899 SDX   2.2063 RHO=  -.5819
        V =    3.95 +   -.0237( X - 150.88)           X = 150.88 + -14.2774( V -   3.95)

V=   4.0 THETA=  39.0 CHI= 165.0
N= 48 VBAR=    3.99 XBAR= 165.90 SDV=     .1728 SDX   5.7074 RHO=  -.8411
        V =    3.99 +   -.0255( X - 165.90)           X = 165.90 + -27.7879( V -   3.99)
```

```
V=   4.0 THETA=  39.0 CHI= 180.0
N= 47 VBAR=   4.01 XBAR= 181.14 SDV=    .1499 SDX   7.3846 RHO=  -.7754
      V =   4.01 +   -.0157( X - 181.14)           X = 181.14 + -38.1844( V -   4.01)

V=   4.0 THETA=  39.0 CHI= 195.0
N= 49 VBAR=   3.99 XBAR= 194.07 SDV=    .0763 SDX   6.4575 RHO=  -.3547
      V =   3.99 +   -.0042( X - 194.07)           X = 194.07 + -30.0268( V -   3.99)

V=   4.0 THETA=  39.0 CHI= 210.0
N= 50 VBAR=   3.98 XBAR= 208.86 SDV=    .0713 SDX   3.2503 RHO=   .5031
      V =   3.98 +    .0110( X - 208.86)           X = 208.86 +  22.9424( V -   3.98)

V=   4.0 THETA=  39.0 CHI= 225.0
N= 47 VBAR=   3.97 XBAR= 223.46 SDV=    .1222 SDX   3.4122 RHO=   .7264
      V =   3.97 +    .0260( X - 223.46)           X = 223.46 +  20.2774( V -   3.97)

V=   4.0 THETA=  39.0 CHI= 240.0
N= 40 VBAR=   4.04 XBAR= 240.39 SDV=    .1259 SDX   2.7366 RHO=   .6803
      V =   4.04 +    .0313( X - 240.39)           X = 240.39 +  14.7887( V -   4.04)

V=   4.0 THETA=  39.0 CHI= 255.0
N= 41 VBAR=   4.01 XBAR= 255.41 SDV=    .0993 SDX   3.2523 RHO=   .4984
      V =   4.01 +    .0152( X - 255.41)           X = 255.41 +  16.3306( V -   4.01)

V=   4.0 THETA=  39.0 CHI= 270.0
N= 33 VBAR=   4.01 XBAR= 271.47 SDV=    .1148 SDX   5.0349 RHO=   .6166
      V =   4.01 +    .0141( X - 271.47)           X = 271.47 +  27.0329( V -   4.01)

V=   4.0 THETA=  39.0 CHI= 285.0
N= 46 VBAR=   3.97 XBAR= 282.82 SDV=    .1140 SDX   5.7789 RHO=   .5891
      V =   3.97 +    .0116( X - 282.82)           X = 282.82 +  29.8708( V -   3.97)

V=   4.0 THETA=  39.0 CHI= 300.0
N= 30 VBAR=   3.97 XBAR= 300.40 SDV=    .1215 SDX   2.7502 RHO=  -.3778
      V =   3.97 +   -.0167( X - 300.40)           X = 300.40 +  -8.5502( V -   3.97)

V=   4.0 THETA=  39.0 CHI= 315.0
N= 47 VBAR=   3.99 XBAR= 315.36 SDV=    .1505 SDX   2.5855 RHO=  -.8318
      V =   3.99 +   -.0484( X - 315.36)           X = 315.36 + -14.2877( V -   3.99)

V=   4.0 THETA=  39.0 CHI= 330.0
N= 48 VBAR=   4.02 XBAR= 329.53 SDV=    .1233 SDX   2.1259 RHO=  -.8012
      V =   4.02 +   -.0465( X - 329.53)           X = 329.53 + -13.8095( V -   4.02)

V=   4.0 THETA=  39.0 CHI= 345.0
N= 50 VBAR=   3.94 XBAR= 346.90 SDV=    .1411 SDX   3.5925 RHO=  -.8744
      V =   3.94 +   -.0343( X - 346.90)           X = 346.90 + -22.2636( V -   3.94)
```

TABLE 60   V = 8, θ = 39, 1 CELL

1 CELL

```
V=   8.0 THETA=  39.0 CHI=    .0
N= 50 VBAR=   7.91 XBAR=   1.88 SDV=    .1851 SDX   5.2503 RHO=  -.5974
      V =   7.91 +   -.0211( X -   1.88)            X =   1.88 + -16.9443( V -   7.91)

V=   8.0 THETA=  39.0 CHI=  15.0
N= 50 VBAR=   8.01 XBAR=  13.78 SDV=    .1121 SDX   3.7056 RHO=  -.3891
      V =   8.01 +   -.0118( X -  13.78)            X =  13.78 + -12.8609( V -   8.01)

V=   8.0 THETA=  39.0 CHI=  30.0
N= 48 VBAR=   7.94 XBAR=  29.69 SDV=    .1186 SDX   2.5650 RHO=   .3009
      V =   7.94 +    .0139( X -  29.69)            X =  29.69 +   6.5081( V -   7.94)

V=   8.0 THETA=  39.0 CHI=  45.0
N= 41 VBAR=   7.98 XBAR=  44.87 SDV=    .2575 SDX   2.1070 RHO=   .5443
      V =   7.98 +    .0665( X -  44.87)            X =  44.87 +   4.4538( V -   7.98)

V=   8.0 THETA=  39.0 CHI=  60.0
N= 46 VBAR=   8.02 XBAR=  60.01 SDV=    .1876 SDX   2.1981 RHO=   .4266
      V =   8.02 +    .0364( X -  60.01)            X =  60.01 +   4.9968( V -   8.02)

V=   8.0 THETA=  39.0 CHI=  75.0
N= 47 VBAR=   7.89 XBAR=  74.89 SDV=    .2661 SDX   3.1504 RHO=   .5094
      V =   7.89 +    .0430( X -  74.89)            X =  74.89 +   6.0300( V -   7.89)

V=   8.0 THETA=  39.0 CHI=  90.0
N= 50 VBAR=   7.87 XBAR=  89.72 SDV=    .2291 SDX   3.5259 RHO=   .4464
      V =   7.87 +    .0290( X -  89.72)            X =  89.72 +   6.8684( V -   7.87)

V=   8.0 THETA=  39.0 CHI= 105.0
N= 50 VBAR=   7.85 XBAR= 104.15 SDV=    .1616 SDX   4.6664 RHO=   .1835
      V =   7.85 +    .0064( X - 104.15)            X = 104.15 +   5.2987( V -   7.85)

V=   8.0 THETA=  39.0 CHI= 120.0
N= 47 VBAR=   7.87 XBAR= 119.60 SDV=    .1692 SDX   3.1382 RHO=   .0983
      V =   7.87 +    .0053( X - 119.60)            X = 119.60 +   1.8235( V -   7.87)

V=   8.0 THETA=  39.0 CHI= 135.0
N= 47 VBAR=   7.93 XBAR= 135.01 SDV=    .1705 SDX   2.1748 RHO=   .0392
      V =   7.93 +    .0031( X - 135.01)            X = 135.01 +    .5002( V -   7.93)

V=   8.0 THETA=  39.0 CHI= 150.0
N= 49 VBAR=   7.93 XBAR= 150.69 SDV=    .1715 SDX   2.4723 RHO=  -.5690
      V =   7.93 +   -.0395( X - 150.69)            X = 150.69 +  -8.2046( V -   7.93)
```

TABLE 60 (CONT'D.)

```
V=   8.0 THETA=  39.0 CHI= 165.0
N= 47 VBAR=   7.89 XBAR= 165.82 SDV=    .1761 SDX   3.3079 RHO=  -.3916
      V =   7.89 +   -.0208( X - 165.82)           X = 165.82 +  -7.3565( V -   7.89)

V=   8.0 THETA=  39.0 CHI= 180.0
N= 45 VBAR=   8.08 XBAR= 179.86 SDV=    .1905 SDX   6.4730 RHO=  -.5718
      V =   8.08 +   -.0168( X - 179.86)           X = 179.86 + -19.4326( V -   8.08)

V=   8.0 THETA=  39.0 CHI= 195.0
N= 47 VBAR=   7.96 XBAR= 194.55 SDV=    .1538 SDX   7.2382 RHO=  -.4252
      V =   7.96 +   -.0090( X - 194.55)           X = 194.55 + -20.0165( V -   7.96)

V=   8.0 THETA=  39.0 CHI= 210.0
N= 45 VBAR=   8.01 XBAR= 209.51 SDV=    .1953 SDX   3.6980 RHO=   .1738
      V =   8.01 +    .0092( X - 209.51)           X = 209.51 +   3.2900( V -   8.01)

V=   8.0 THETA=  39.0 CHI= 225.0
N= 45 VBAR=   8.05 XBAR= 225.47 SDV=    .2157 SDX   3.2332 RHO=   .5205
      V =   8.05 +    .0347( X - 225.47)           X = 225.47 +   7.8041( V -   8.05)

V=   8.0 THETA=  39.0 CHI= 240.0
N= 40 VBAR=   8.02 XBAR= 240.65 SDV=    .1935 SDX   3.3049 RHO=   .5806
      V =   8.02 +    .0340( X - 240.65)           X = 240.65 +   9.9151( V -   8.02)

V=   8.0 THETA=  39.0 CHI= 255.0
N= 39 VBAR=   8.02 XBAR= 256.27 SDV=    .2300 SDX   6.4154 RHO=   .6236
      V =   8.02 +    .0224( X - 256.27)           X = 256.27 +  17.3906( V -   8.02)

V=   8.0 THETA=  39.0 CHI= 270.0
N= 40 VBAR=   7.96 XBAR= 270.56 SDV=    .2586 SDX   6.9793 RHO=   .7239
      V =   7.96 +    .0268( X - 270.56)           X = 270.56 +  19.5405( V -   7.96)

V=   8.0 THETA=  39.0 CHI= 285.0
N= 45 VBAR=   7.99 XBAR= 284.30 SDV=    .1841 SDX   6.1133 RHO=   .5097
      V =   7.99 +    .0153( X - 284.30)           X = 284.30 +  16.9287( V -   7.99)

V=   8.0 THETA=  39.0 CHI= 300.0
N= 41 VBAR=   7.98 XBAR= 299.22 SDV=    .2178 SDX   2.9058 RHO=   .0088
      V =   7.98 +    .0007( X - 299.22)           X = 299.22 +    .1170( V -   7.98)

V=   8.0 THETA=  39.0 CHI= 315.0
N= 47 VBAR=   8.01 XBAR= 314.90 SDV=    .2078 SDX   2.5747 RHO=  -.6250
      V =   8.01 +   -.0504( X - 314.90)           X = 314.90 +  -7.7462( V -   8.01)

V=   8.0 THETA=  39.0 CHI= 330.0
N= 48 VBAR=   8.01 XBAR= 329.99 SDV=    .1696 SDX   2.1442 RHO=  -.4085
      V =   8.01 +   -.0323( X - 329.99)           X = 329.99 +  -5.1661( V -   8.01)

V=   8.0 THETA=  39.0 CHI= 345.0
N= 49 VBAR=   7.94 XBAR= 346.77 SDV=    .2278 SDX   7.5620 RHO=  -.6662
      V =   7.94 +   -.0201( X - 346.77)           X = 346.77 + -22.1197( V -   7.94)
```

TABLE 61 V = 8 , θ = 39, 1 CELL ± 0.7 DB

```
1 CELL WITH 7/10'S DB ERROR

V=   8.0 THETA=  39.0 CHI=     .0
N= 50 VBAR=    7.81 XBAR=   3.15 SDV=    .3454 SDX   8.2609 RHO=  -.4554
      V =    7.81 +   -.0190( X -    3.15)          X =    3.15 + -10.8920( V -    7.81)

V=   8.0 THETA=  39.0 CHI=  15.0
N= 44 VBAR=    8.01 XBAR=  11.05 SDV=    .2338 SDX   7.6067 RHO=  -.5447
      V =    8.01 +   -.0167( X -   11.05)          X =   11.05 + -17.7244( V -    8.01)

V=   8.0 THETA=  39.0 CHI=  30.0
N= 37 VBAR=    7.89 XBAR=  28.82 SDV=    .2287 SDX   5.2636 RHO=  -.0532
      V =    7.89 +   -.0023( X -   28.82)          X =   28.82 +  -1.2243( V -    7.89)

V=   8.0 THETA=  39.0 CHI=  45.0
N= 30 VBAR=    8.03 XBAR=  45.12 SDV=    .4777 SDX   3.6939 RHO=   .3529
      V =    8.03 +    .0456( X -   45.12)          X =   45.12 +   2.7288( V -    8.03)

V=   8.0 THETA=  39.0 CHI=  60.0
N= 40 VBAR=    8.13 XBAR=  60.65 SDV=    .3387 SDX   4.4934 RHO=   .5463
      V =    8.13 +    .0412( X -   60.65)          X =   60.65 +   7.2483( V -    8.13)

V=   8.0 THETA=  39.0 CHI=  75.0
N= 45 VBAR=    7.82 XBAR=  76.58 SDV=    .5437 SDX   8.6929 RHO=   .4371
      V =    7.82 +    .0273( X -   76.58)          X =   76.58 +   6.9882( V -    7.82)

V=   8.0 THETA=  39.0 CHI=  90.0
N= 48 VBAR=    7.75 XBAR=  90.28 SDV=    .4218 SDX   7.7982 RHO=   .3740
      V =    7.75 +    .0202( X -   90.28)          X =   90.28 +   6.9143( V -    7.75)

V=   8.0 THETA=  39.0 CHI= 105.0
N= 47 VBAR=    7.72 XBAR= 104.03 SDV=    .2595 SDX   7.4766 RHO=  -.0535
      V =    7.72 +   -.0019( X - 104.03)          X = 104.03 +  -1.5421( V -    7.72)

V=   8.0 THETA=  39.0 CHI= 120.0
N= 47 VBAR=    7.76 XBAR= 119.26 SDV=    .3003 SDX   5.9841 RHO=  -.0617
      V =    7.76 +   -.0032( X - 119.26)          X = 119.26 +  -1.2046( V -    7.76)

V=   8.0 THETA=  39.0 CHI= 135.0
N= 36 VBAR=    7.91 XBAR= 134.96 SDV=    .2951 SDX   4.4961 RHO=   .2034
      V =    7.91 +    .0134( X - 134.96)          X = 134.96 +   3.0989( V -    7.91)

V=   8.0 THETA=  39.0 CHI= 150.0
N= 42 VBAR=    7.90 XBAR= 151.48 SDV=    .3024 SDX   4.5358 RHO=  -.3419
      V =    7.90 +   -.0228( X - 151.48)          X = 151.48 +  -5.1281( V -    7.90)

V=   8.0 THETA=  39.0 CHI= 165.0
N= 35 VBAR=    7.76 XBAR= 167.80 SDV=    .3053 SDX  10.1047 RHO=  -.2451
      V =    7.76 +   -.0074( X - 167.80)          X = 167.80 +  -8.1115( V -    7.76)
```

```
V=   8.0 THETA=  39.0 CHI= 180.0

N= 32 VBAR=   8.08 XBAR= 181.94 SDV=    .3321 SDX  11.0728 RHO=  -.3951

      V =    8.08 +   -.0118( X - 181.94)          X = 181.94 + -13.1730( V -   8.08)

V=   8.0 THETA=  39.0 CHI= 195.0

N= 35 VBAR=   7.96 XBAR= 193.81 SDV=    .2553 SDX  11.0404 RHO=  -.4033

      V =    7.96 +   -.0093( X - 193.81)          X = 193.81 + -17.4360( V -   7.96)

V=   8.0 THETA=  39.0 CHI= 210.0

N= 35 VBAR=   8.09 XBAR= 208.59 SDV=    .3872 SDX   9.3142 RHO=  -.1863

      V =    8.09 +   -.0077( X - 208.59)          X = 208.59 +  -4.4821( V -   8.09)

V=   8.0 THETA=  39.0 CHI= 225.0

N= 32 VBAR=   8.09 XBAR= 225.36 SDV=    .3544 SDX   6.2028 RHO=   .4674

      V =    8.09 +    .0267( X - 225.36)          X = 225.36 +   8.1801( V -   8.09)

V=   8.0 THETA=  39.0 CHI= 240.0

N= 32 VBAR=   8.03 XBAR= 242.05 SDV=    .3167 SDX   4.5148 RHO=   .1969

      V =    8.03 +    .0138( X - 242.05)          X = 242.05 +   2.8067( V -   8.03)

V=   8.0 THETA=  39.0 CHI= 255.0

N= 32 VBAR=   8.05 XBAR= 257.95 SDV=    .4153 SDX  11.0783 RHO=   .5710

      V =    8.05 +    .0214( X - 257.95)          X = 257.95 +  15.2315( V -   8.05)

V=   8.0 THETA=  39.0 CHI= 270.0

N= 25 VBAR=   7.87 XBAR= 271.24 SDV=    .4268 SDX  11.4755 RHO=   .5381

      V =    7.87 +    .0200( X - 271.24)          X = 271.24 +  14.4684( V -   7.87)

V=   8.0 THETA=  39.0 CHI= 285.0

N= 32 VBAR=   7.94 XBAR= 283.69 SDV=    .2657 SDX   9.0970 RHO=   .4796

      V =    7.94 +    .0140( X - 283.69)          X = 283.69 +  16.4207( V -   7.94)

V=   8.0 THETA=  39.0 CHI= 300.0

N= 33 VBAR=   7.93 XBAR= 297.20 SDV=    .4269 SDX   9.3722 RHO=   .0162

      V =    7.93 +    .0007( X - 297.20)          X = 297.20 +    .3552( V -   7.93)

V=   8.0 THETA=  39.0 CHI= 315.0

N= 42 VBAR=   7.96 XBAR= 315.51 SDV=    .3697 SDX   4.8171 RHO=  -.5799

      V =    7.96 +   -.0445( X - 315.51)          X = 315.51 +  -7.5571( V -   7.96)

V=   8.0 THETA=  39.0 CHI= 330.0

N= 38 VBAR=   8.02 XBAR= 330.43 SDV=    .3313 SDX   4.0554 RHO=  -.0960

      V =    8.02 +   -.0078( X - 330.43)          X = 330.43 +  -1.1754( V -   8.02)

V=   8.0 THETA=  39.0 CHI= 345.0

N= 45 VBAR=   7.84 XBAR= 351.13 SDV=    .4337 SDX  14.1363 RHO=  -.5495

      V =    7.84 +   -.0169( X - 351.13)          X = 351.13 + -17.9089( V -   7.84)
```

TABLE 62   V = 12, θ = 39, 1 CELL

```
   1 CELL

 V=  12.0 THETA=  39.0 CHI=     .0
 N= 50 VBAR=  11.98 XBAR= 359.67 SDV=    .1608 SDX   5.5078 RHO=  -.3974
         V =  11.98 +   -.0116( X - 359.67)          X = 359.67 + -13.6154( V -  11.98)

 V=  12.0 THETA=  39.0 CHI=  15.0
 N= 49 VBAR=  11.97 XBAR=  14.06 SDV=    .2115 SDX   5.1869 RHO=  -.5682
         V =  11.97 +   -.0232( X -  14.06)          X =  14.06 + -13.9321( V -  11.97)

 V=  12.0 THETA=  39.0 CHI=  30.0
 N= 49 VBAR=  11.99 XBAR=  29.92 SDV=    .1765 SDX   1.9096 RHO=  -.0327
         V =  11.99 +   -.0030( X -  29.92)          X =  29.92 +   -.3542( V -  11.99)

 V=  12.0 THETA=  39.0 CHI=  45.0
 N= 45 VBAR=  11.94 XBAR=  44.79 SDV=    .2525 SDX   1.5899 RHO=   .1151
         V =  11.94 +    .0183( X -  44.79)          X =  44.79 +    .7250( V -  11.94)

 V=  12.0 THETA=  39.0 CHI=  60.0
 N= 50 VBAR=  11.97 XBAR=  59.93 SDV=    .2155 SDX   1.9901 RHO=   .4223
         V =  11.97 +    .0457( X -  59.93)          X =  59.93 +   3.8992( V -  11.97)

 V=  12.0 THETA=  39.0 CHI=  75.0
 N= 50 VBAR=  11.94 XBAR=  75.23 SDV=    .2497 SDX   2.8866 RHO=   .4757
         V =  11.94 +    .0411( X -  75.23)          X =  75.23 +   5.5008( V -  11.94)

 V=  12.0 THETA=  39.0 CHI=  90.0
 N= 48 VBAR=  11.82 XBAR=  90.77 SDV=    .2131 SDX   3.0466 RHO=   .3845
         V =  11.82 +    .0269( X -  90.77)          X =  90.77 +   5.4962( V -  11.82)

 V=  12.0 THETA=  39.0 CHI= 105.0
 N= 47 VBAR=  11.89 XBAR= 105.06 SDV=    .2152 SDX   4.7263 RHO=  -.2273
         V =  11.89 +   -.0104( X - 105.06)          X = 105.06 +  -4.9919( V -  11.89)

 V=  12.0 THETA=  39.0 CHI= 120.0
 N= 50 VBAR=  11.86 XBAR= 120.07 SDV=    .1711 SDX   2.4333 RHO=  -.1370
         V =  11.86 +   -.0096( X - 120.07)          X = 120.07 +  -1.9484( V -  11.86)

 V=  12.0 THETA=  39.0 CHI= 135.0
 N= 49 VBAR=  11.93 XBAR= 135.21 SDV=    .1880 SDX   1.8045 RHO=   .0716
         V =  11.93 +    .0075( X - 135.21)          X = 135.21 +    .6873( V -  11.93)

 V=  12.0 THETA=  39.0 CHI= 150.0
 N= 50 VBAR=  11.93 XBAR= 150.24 SDV=    .1840 SDX   2.0628 RHO=  -.1661
         V =  11.93 +   -.0148( X - 150.24)          X = 150.24 +  -1.8615( V -  11.93)

 V=  12.0 THETA=  39.0 CHI= 165.0
 N= 47 VBAR=  11.98 XBAR= 165.69 SDV=    .2180 SDX   2.7380 RHO=  -.3910
         V =  11.98 +   -.0311( X - 165.69)          X = 165.69 +  -4.9099( V -  11.98)
```

TABLE 62 (CONT'D.)

```
V=  12.0 THETA=  39.0 CHI= 180.0

N= 46 VBAR=  12.08 XBAR= 178.99 SDV=    .2461 SDX   6.6739 RHO=  -.3864

      V =  12.08 +   -.0143( X - 178.99)          X = 178.99 + -10.4765( V -  12.08)

V=  12.0 THETA=  39.0 CHI= 195.0

N= 49 VBAR=  11.99 XBAR= 195.55 SDV=    .1833 SDX   4.7520 RHO=  -.3012

      V =  11.99 +   -.0116( X - 195.55)          X = 195.55 +  -7.8081( V -  11.99)

V=  12.0 THETA=  39.0 CHI= 210.0

N= 48 VBAR=  11.95 XBAR= 210.78 SDV=    .1871 SDX   2.1608 RHO=  -.0642

      V =  11.95 +   -.0056( X - 210.78)          X = 210.78 +   -.7413( V -  11.95)

V=  12.0 THETA=  39.0 CHI= 225.0

N= 50 VBAR=  11.96 XBAR= 225.03 SDV=    .2434 SDX   2.5333 RHO=   .2797

      V =  11.96 +    .0269( X - 225.03)          X = 225.03 +   2.9102( V -  11.96)

V=  12.0 THETA=  39.0 CHI= 240.0

N= 47 VBAR=  11.99 XBAR= 239.84 SDV=    .2066 SDX   2.4835 RHO=   .1341

      V =  11.99 +    .0112( X - 239.84)          X = 239.84 +   1.6124( V -  11.99)

V=  12.0 THETA=  39.0 CHI= 255.0

N= 47 VBAR=  12.04 XBAR= 254.84 SDV=    .2017 SDX   3.3938 RHO=   .5083

      V =  12.04 +    .0302( X - 254.84)          X = 254.84 +   8.5529( V -  12.04)

V=  12.0 THETA=  39.0 CHI= 270.0

N= 46 VBAR=  11.92 XBAR= 271.07 SDV=    .2607 SDX   5.8803 RHO=   .4397

      V =  11.92 +    .0195( X - 271.07)          X = 271.07 +   9.9175( V -  11.92)

V=  12.0 THETA=  39.0 CHI= 285.0

N= 49 VBAR=  12.01 XBAR= 283.99 SDV=    .1730 SDX   4.1732 RHO=  -.0743

      V =  12.01 +   -.0031( X - 283.99)          X = 283.99 +  -1.7917( V -  12.01)

V=  12.0 THETA=  39.0 CHI= 300.0

N= 43 VBAR=  11.99 XBAR= 299.93 SDV=    .2319 SDX   2.2849 RHO=  -.6073

      V =  11.99 +   -.0616( X - 299.93)          X = 299.93 +  -5.9834( V -  11.99)

V=  12.0 THETA=  39.0 CHI= 315.0

N= 48 VBAR=  12.01 XBAR= 314.89 SDV=    .2244 SDX   1.7078 RHO=  -.4641

      V =  12.01 +   -.0610( X - 314.89)          X = 314.89 +  -3.5315( V -  12.01)

V=  12.0 THETA=  39.0 CHI= 330.0

N= 49 VBAR=  12.10 XBAR= 329.78 SDV=    .2313 SDX   2.3114 RHO=  -.3668

      V =  12.10 +   -.0362( X - 329.78)          X = 329.78 +  -3.7133( V -  12.10)

V=  12.0 THETA=  39.0 CHI= 345.0

N= 49 VBAR=  11.99 XBAR= 345.11 SDV=    .2049 SDX   5.4147 RHO=  -.4686

      V =  11.99 +   -.0177( X - 345.11)          X = 345.11 + -12.3811( V -  11.99)
```

TABLE 63 V = 12, θ = 39, 1 CELL ± 0.7 DB

1 CELL WITH 7/10'S DB ERROR

```
V=  12.0 THETA=  39.0 CHI=     .0
N= 47 VBAR=  11.87 XBAR=    .24 SDV=    .3537 SDX  10.0746 RHO=  -.4674
      V =  11.87 +   -.0165( X -    .24)            X =    .24 + -13.3708( V -  11.87)

V=  12.0 THETA=  39.0 CHI=   15.0
N= 46 VBAR=  11.97 XBAR=  10.33 SDV=    .4753 SDX  11.5402 RHO=  -.5567
      V =  11.97 +   -.0229( X -  10.33)            X =  10.33 + -13.5186( V -  11.97)

V=  12.0 THETA=  39.0 CHI=   30.0
N= 34 VBAR=  11.97 XBAR=  28.16 SDV=    .4756 SDX   7.0936 RHO=  -.3748
      V =  11.97 +   -.0251( X -  28.16)            X =  28.16 +  -5.5894( V -  11.97)

V=  12.0 THETA=  39.0 CHI=   45.0
N= 38 VBAR=  11.85 XBAR=  44.32 SDV=    .5990 SDX   3.9760 RHO=  -.0773
      V =  11.85 +   -.0117( X -  44.32)            X =  44.32 +   -.5133( V -  11.85)

V=  12.0 THETA=  39.0 CHI=   60.0
N= 40 VBAR=  12.00 XBAR=  60.21 SDV=    .4853 SDX   4.2307 RHO=   .5152
      V =  12.00 +    .0591( X -  60.21)            X =  60.21 +   4.4916( V -  12.00)

V=  12.0 THETA=  39.0 CHI=   75.0
N= 44 VBAR=  11.87 XBAR=  76.59 SDV=    .5906 SDX   8.0109 RHO=   .4545
      V =  11.87 +    .0335( X -  76.59)            X =  76.59 +   6.1645( V -  11.87)

V=  12.0 THETA=  39.0 CHI=   90.0
N= 42 VBAR=  11.62 XBAR=  92.96 SDV=    .4264 SDX   6.3382 RHO=   .0779
      V =  11.62 +    .0052( X -  92.96)            X =  92.96 +   1.1575( V -  11.62)

V=  12.0 THETA=  39.0 CHI= 105.0
N= 44 VBAR=  11.67 XBAR= 104.67 SDV=    .4738 SDX  10.3951 RHO=  -.1117
      V =  11.67 +   -.0051( X - 104.67)            X = 104.67 +  -2.4510( V -  11.67)

V=  12.0 THETA=  39.0 CHI= 120.0
N= 47 VBAR=  11.67 XBAR= 118.81 SDV=    .4382 SDX   8.5444 RHO=   .1421
      V =  11.67 +    .0073( X - 118.81)            X = 118.81 +   2.7697( V -  11.67)

V=  12.0 THETA=  39.0 CHI= 135.0
N= 38 VBAR=  11.92 XBAR= 135.50 SDV=    .4338 SDX   4.5756 RHO=   .0365
      V =  11.92 +    .0035( X - 135.50)            X = 135.50 +    .3852( V -  11.92)

V=  12.0 THETA=  39.0 CHI= 150.0
N= 36 VBAR=  11.91 XBAR= 150.26 SDV=    .4088 SDX   5.1148 RHO=  -.1342
      V =  11.91 +   -.0107( X - 150.26)            X = 150.26 +  -1.6786( V -  11.91)

V=  12.0 THETA=  39.0 CHI= 165.0
N= 36 VBAR=  11.79 XBAR= 172.94 SDV=    .5334 SDX  14.3720 RHO=  -.5691
      V =  11.79 +   -.0211( X - 172.94)            X = 172.94 + -15.3332( V -  11.79)
```

```
V=  12.0 THETA=  39.0 CHI= 180.0
N= 32 VBAR=  12.13 XBAR= 180.28 SDV=    .4977 SDX  10.3569 RHO=  -.2710
       V =  12.13 +   -.0130( X - 180.28)          X = 180.28 +  -5.6401( V -  12.13)

V=  12.0 THETA=  39.0 CHI= 195.0
N= 39 VBAR=  11.99 XBAR= 194.63 SDV=    .3950 SDX  11.1281 RHO=  -.4115
       V =  11.99 +   -.0146( X - 194.63)          X = 194.63 + -11.5923( V -  11.99)

V=  12.0 THETA=  39.0 CHI= 210.0
N= 37 VBAR=  11.92 XBAR= 211.05 SDV=    .4957 SDX   9.1290 RHO=  -.3579
       V =  11.92 +   -.0194( X - 211.05)          X = 211.05 +  -6.5919( V -  11.92)

V=  12.0 THETA=  39.0 CHI= 225.0
N= 37 VBAR=  11.93 XBAR= 225.55 SDV=    .5608 SDX   5.0911 RHO=   .0392
       V =  11.93 +    .0043( X - 225.55)          X = 225.55 +    .3561( V -  11.93)

V=  12.0 THETA=  39.0 CHI= 240.0
N= 36 VBAR=  12.06 XBAR= 239.78 SDV=    .4609 SDX   6.1877 RHO=   .1826
       V =  12.06 +    .0136( X - 239.78)          X = 239.78 +   2.4516( V -  12.06)

V=  12.0 THETA=  39.0 CHI= 255.0
N= 36 VBAR=  12.07 XBAR= 258.18 SDV=    .4435 SDX  12.7303 RHO=   .5450
       V =  12.07 +    .0190( X - 258.18)          X = 258.18 +  15.6453( V -  12.07)

V=  12.0 THETA=  39.0 CHI= 270.0
N= 28 VBAR=  11.81 XBAR= 271.21 SDV=    .5561 SDX  11.3797 RHO=   .2921
       V =  11.81 +    .0143( X - 271.21)          X = 271.21 +   5.9780( V -  11.81)

V=  12.0 THETA=  39.0 CHI= 285.0
N= 37 VBAR=  12.01 XBAR= 282.97 SDV=    .3740 SDX   9.0176 RHO=  -.1352
       V =  12.01 +   -.0056( X - 282.97)          X = 282.97 +  -3.2605( V -  12.01)

V=  12.0 THETA=  39.0 CHI= 300.0
N= 35 VBAR=  11.98 XBAR= 298.44 SDV=    .5403 SDX   9.1560 RHO=  -.1532
       V =  11.98 +   -.0090( X - 298.44)          X = 298.44 +  -2.5971( V -  11.98)

V=  12.0 THETA=  39.0 CHI= 315.0
N= 38 VBAR=  11.98 XBAR= 315.47 SDV=    .5081 SDX   3.8485 RHO=  -.4798
       V =  11.98 +   -.0634( X - 315.47)          X = 315.47 +  -3.6343( V -  11.98)

V=  12.0 THETA=  39.0 CHI= 330.0
N= 41 VBAR=  12.18 XBAR= 329.66 SDV=    .5100 SDX   5.2411 RHO=  -.2788
       V =  12.18 +   -.0271( X - 329.66)          X = 329.66 +  -2.8649( V -  12.18)

V=  12.0 THETA=  39.0 CHI= 345.0
N= 37 VBAR=  11.94 XBAR= 347.20 SDV=    .4345 SDX  11.7624 RHO=  -.3303
       V =  11.94 +   -.0122( X - 347.20)          X = 347.20 +  -8.9422( V -  11.94)
```

TABLE 64 V = 24, θ = 39, 1 CELL

1 CELL

```
V=  24.0 THETA=  39.0 CHI=     .0
N= 50 VBAR=  23.86 XBAR=   1.41 SDV=    .2089 SDX   6.5450 RHO=  -.5093
      V =  23.86 +   -.0163( X -    1.41)          X =    1.41 + -15.9602( V -  23.86)

V=  24.0 THETA=  39.0 CHI=  15.0
N= 50 VBAR=  23.93 XBAR=  12.62 SDV=    .3565 SDX   8.0458 RHO=  -.7356
      V =  23.93 +   -.0326( X -   12.62)          X =   12.62 + -16.6030( V -  23.93)

V=  24.0 THETA=  39.0 CHI=  30.0
N= 50 VBAR=  23.91 XBAR=  30.10 SDV=    .2825 SDX   1.5812 RHO=  -.3268
      V =  23.91 +   -.0584( X -   30.10)          X =   30.10 +  -1.8289( V -  23.91)

V=  24.0 THETA=  39.0 CHI=  45.0
N= 50 VBAR=  24.10 XBAR=  44.32 SDV=    .3988 SDX   2.1137 RHO=   .3270
      V =  24.10 +    .0617( X -   44.32)          X =   44.32 +   1.7330( V -  24.10)

V=  24.0 THETA=  39.0 CHI=  60.0
N= 49 VBAR=  24.00 XBAR=  59.68 SDV=    .3500 SDX   2.1062 RHO=   .3503
      V =  24.00 +    .0582( X -   59.68)          X =   59.68 +   2.1080( V -  24.00)

V=  24.0 THETA=  39.0 CHI=  75.0
N= 50 VBAR=  23.91 XBAR=  75.31 SDV=    .4988 SDX   3.3925 RHO=   .6912
      V =  23.91 +    .1016( X -   75.31)          X =   75.31 +   4.7006( V -  23.91)

V=  24.0 THETA=  39.0 CHI=  90.0
N= 50 VBAR=  23.73 XBAR=  90.71 SDV=    .2207 SDX   2.9584 RHO=   .1342
      V =  23.73 +    .0100( X -   90.71)          X =   90.71 +   1.7986( V -  23.73)

V=  24.0 THETA=  39.0 CHI= 105.0
N= 49 VBAR=  23.90 XBAR= 104.89 SDV=    .3702 SDX   3.6048 RHO=  -.2110
      V =  23.90 +   -.0217( X -  104.89)          X =  104.89 +  -2.0547( V -  23.90)

V=  24.0 THETA=  39.0 CHI= 120.0
N= 50 VBAR=  23.72 XBAR= 120.02 SDV=    .2521 SDX   2.8984 RHO=  -.3671
      V =  23.72 +   -.0319( X -  120.02)          X =  120.02 +  -4.2200( V -  23.72)

V=  24.0 THETA=  39.0 CHI= 135.0
N= 50 VBAR=  23.90 XBAR= 135.09 SDV=    .2951 SDX   1.8944 RHO=  -.1742
      V =  23.90 +   -.0271( X -  135.09)          X =  135.09 +  -1.1184( V -  23.90)

V=  24.0 THETA=  39.0 CHI= 150.0
N= 50 VBAR=  23.86 XBAR= 149.87 SDV=    .2724 SDX   1.8788 RHO=   .0362
      V =  23.86 +    .0052( X -  149.87)          X =  149.87 +    .2496( V -  23.86)
```

V= 24.0 THETA= 39.0 CHI= 165.0

N= 50 VBAR= 23.93 XBAR= 166.08 SDV= .3198 SDX 7.2908 RHO= -.6182

V = 23.93 + -.0271(X - 166.08) X = 166.08 + -14.0946(V - 23.93)

V= 24.0 THETA= 39.0 CHI= 180.0

N= 50 VBAR= 24.06 XBAR= 180.96 SDV= .3488 SDX 6.6552 RHO= -.2688

V = 24.06 + -.0141(X - 180.96) X = 180.96 + -5.1286(V - 24.06)

V= 24.0 THETA= 39.0 CHI= 195.0

N= 50 VBAR= 24.03 XBAR= 193.96 SDV= .2838 SDX 4.5238 RHO= -.3351

V = 24.03 + -.0210(X - 193.96) X = 193.96 + -5.3424(V - 24.03)

V= 24.0 THETA= 39.0 CHI= 210.0

N= 50 VBAR= 23.94 XBAR= 209.96 SDV= .3317 SDX 2.4207 RHO= -.0739

V = 23.94 + -.0101(X - 209.96) X = 209.96 + -.5392(V - 23.94)

V= 24.0 THETA= 39.0 CHI= 225.0

N= 50 VBAR= 23.94 XBAR= 225.21 SDV= .2547 SDX 1.5977 RHO= .1304

V = 23.94 + .0208(X - 225.21) X = 225.21 + .8179(V - 23.94)

V= 24.0 THETA= 39.0 CHI= 240.0

N= 49 VBAR= 24.03 XBAR= 240.34 SDV= .3329 SDX 1.6835 RHO= .0403

V = 24.03 + .0080(X - 240.34) X = 240.34 + .2036(V - 24.03)

V= 24.0 THETA= 39.0 CHI= 255.0

N= 30 VBAR= 24.00 XBAR= 253.78 SDV= .3536 SDX 2.5171 RHO= .5616

V = 24.00 + .0789(X - 253.78) X = 253.78 + 3.9974(V - 24.00)

V= 24.0 THETA= 39.0 CHI= 270.0

N= 36 VBAR= 23.96 XBAR= 270.21 SDV= .3369 SDX 4.2344 RHO= .1665

V = 23.96 + .0132(X - 270.21) X = 270.21 + 2.0925(V - 23.96)

V= 24.0 THETA= 39.0 CHI= 285.0

N= 35 VBAR= 23.98 XBAR= 285.69 SDV= .3386 SDX 3.0270 RHO= .0929

V = 23.98 + .0104(X - 285.69) X = 285.69 + .8309(V - 23.98)

V= 24.0 THETA= 39.0 CHI= 300.0

N= 48 VBAR= 24.04 XBAR= 299.83 SDV= .3667 SDX 2.3338 RHO= -.5242

V = 24.04 + -.0824(X - 299.83) X = 299.83 + -3.3357(V - 24.04)

V= 24.0 THETA= 39.0 CHI= 315.0

N= 50 VBAR= 23.95 XBAR= 314.66 SDV= .3252 SDX 2.3008 RHO= -.3537

V = 23.95 + -.0500(X - 314.66) X = 314.66 + -2.5024(V - 23.95)

V= 24.0 THETA= 39.0 CHI= 330.0

N= 49 VBAR= 24.06 XBAR= 330.32 SDV= .3151 SDX 2.1153 RHO= .0104

V = 24.06 + .0016(X - 330.32) X = 330.32 + .0698(V - 24.06)

V= 24.0 THETA= 39.0 CHI= 345.0

N= 50 VBAR= 23.90 XBAR= 345.66 SDV= .4046 SDX 5.8946 RHO= -.4410

V = 23.90 + -.0303(X - 345.66) X = 345.66 + -6.4250(V - 23.90)

TABLE 65 V = 4, θ = 47, 1 CELL

1 CELL

```
V=   4.0 THETA=  47.0 CHI=    .0
N= 46 VBAR=   4.00 XBAR=   1.52 SDV=    .2695 SDX  10.7990 RHO=  -.7048
      V =   4.00 +   -.0176( X -   1.52)           X =   1.52 + -28.2474( V -   4.00)

V=   4.0 THETA=  47.0 CHI=  15.0
N= 42 VBAR=   4.01 XBAR=  11.33 SDV=    .1767 SDX   8.4689 RHO=  -.2721
      V =   4.01 +   -.0057( X -  11.33)           X =  11.33 + -13.0379( V -   4.01)

V=   4.0 THETA=  47.0 CHI=  30.0
N= 38 VBAR=   4.00 XBAR=  25.17 SDV=    .1547 SDX  11.6908 RHO=   .0628
      V =   4.00 +    .0008( X -  25.17)           X =  25.17 +   4.7451( V -   4.00)

V=   4.0 THETA=  47.0 CHI=  45.0
N= 14 VBAR=   4.04 XBAR=  45.00 SDV=    .2638 SDX   6.3133 RHO=   .4417
      V =   4.04 +    .0185( X -  45.00)           X =  45.00 +  10.5717( V -   4.04)

V=   4.0 THETA=  47.0 CHI=  60.0
N= 20 VBAR=   4.09 XBAR=  58.83 SDV=    .3948 SDX   5.5414 RHO=   .8150
      V =   4.09 +    .0581( X -  58.83)           X =  58.83 +  11.4384( V -   4.09)

V=   4.0 THETA=  47.0 CHI=  75.0
N= 32 VBAR=   3.76 XBAR=  72.03 SDV=    .2945 SDX   5.9843 RHO=   .5956
      V =   3.76 +    .0293( X -  72.03)           X =  72.03 +  12.1036( V -   3.76)

V=   4.0 THETA=  47.0 CHI=  90.0
N= 48 VBAR=   3.83 XBAR=  93.34 SDV=    .3037 SDX  10.8408 RHO=   .6050
      V =   3.83 +    .0169( X -  93.34)           X =  93.34 +  21.5960( V -   3.83)

V=   4.0 THETA=  47.0 CHI= 105.0
N= 50 VBAR=   3.70 XBAR=  98.87 SDV=    .3352 SDX   9.6731 RHO=   .3529
      V =   3.70 +    .0122( X -  98.87)           X =  98.87 +  10.1838( V -   3.70)

V=   4.0 THETA=  47.0 CHI= 120.0
N= 38 VBAR=   3.65 XBAR= 105.74 SDV=    .3697 SDX  17.1755 RHO=   .5840
      V =   3.65 +    .0126( X - 105.74)           X = 105.74 +  27.1280( V -   3.65)

V=   4.0 THETA=  47.0 CHI= 135.0
N= 25 VBAR=   3.68 XBAR= 135.32 SDV=    .2550 SDX  13.6198 RHO=   .0967
      V =   3.68 +    .0018( X - 135.32)           X = 135.32 +   5.1629( V -   3.68)

V=   4.0 THETA=  47.0 CHI= 150.0
N= 23 VBAR=   3.94 XBAR= 153.24 SDV=    .3987 SDX  11.6973 RHO=  -.7018
      V =   3.94 +   -.0239( X - 153.24)           X = 153.24 + -20.5898( V -   3.94)
```

```
V=   4.0 THETA=  47.0 CHI= 165.0
N= 20 VBAR=   4.29 XBAR= 159.70 SDV=    .3640 SDX  10.0380 RHO=  -.8277
      V =    4.29 +   -.0300( X - 159.70)         X = 159.70 + -22.8264( V -   4.29)

V=   4.0 THETA=  47.0 CHI= 180.0
N= 19 VBAR=   3.93 XBAR= 191.26 SDV=    .2792 SDX  18.0762 RHO=  -.6649
      V =    3.93 +   -.0103( X - 191.26)         X = 191.26 + -43.0482( V -   3.93)

V=   4.0 THETA=  47.0 CHI= 195.0
N= 30 VBAR=   4.19 XBAR= 190.17 SDV=    .2620 SDX  21.5467 RHO=  -.4347
      V =    4.19 +   -.0053( X - 190.17)         X = 190.17 + -35.7572( V -   4.19)

V=   4.0 THETA=  47.0 CHI= 210.0
N= 28 VBAR=   4.09 XBAR= 207.61 SDV=    .1811 SDX  17.5807 RHO=   .1741
      V =    4.09 +    .0018( X - 207.61)         X = 207.61 +  16.8985( V -   4.09)

V=   4.0 THETA=  47.0 CHI= 225.0
N= 22 VBAR=   4.10 XBAR= 226.11 SDV=    .2992 SDX  16.2732 RHO=   .6550
      V =    4.10 +    .0120( X - 226.11)         X = 226.11 +  35.6231( V -   4.10)

V=   4.0 THETA=  47.0 CHI= 240.0
N= 18 VBAR=   4.11 XBAR= 241.44 SDV=    .4156 SDX  11.9966 RHO=   .7756
      V =    4.11 +    .0269( X - 241.44)         X = 241.44 +  22.3854( V -   4.11)

V=   4.0 THETA=  47.0 CHI= 255.0
N= 20 VBAR=   4.07 XBAR= 257.97 SDV=    .3551 SDX  14.2000 RHO=   .5194
      V =    4.07 +    .0130( X - 257.97)         X = 257.97 +  20.7711( V -   4.07)

V=   4.0 THETA=  47.0 CHI= 270.0
N= 22 VBAR=   3.87 XBAR= 271.25 SDV=    .2753 SDX  17.1642 RHO=   .1503
      V =    3.87 +    .0024( X - 271.25)         X = 271.25 +   9.3693( V -   3.87)

V=   4.0 THETA=  47.0 CHI= 285.0
N= 19 VBAR=   3.61 XBAR= 287.66 SDV=    .5776 SDX  21.7783 RHO=  -.0882
      V =    3.61 +   -.0023( X - 287.66)         X = 287.66 +  -3.3261( V -   3.61)

V=   4.0 THETA=  47.0 CHI= 300.0
N= 28 VBAR=   3.91 XBAR= 296.71 SDV=    .5167 SDX  16.7666 RHO=  -.2487
      V =    3.91 +   -.0077( X - 296.71)         X = 296.71 +  -8.0722( V -   3.91)

V=   4.0 THETA=  47.0 CHI= 315.0
N= 42 VBAR=   4.03 XBAR= 315.08 SDV=    .3713 SDX   8.5929 RHO=  -.5959
      V =    4.03 +   -.0257( X - 315.08)         X = 315.08 + -13.7920( V -   4.03)

V=   4.0 THETA=  47.0 CHI= 330.0
N= 45 VBAR=   4.24 XBAR= 327.21 SDV=    .5343 SDX  10.4429 RHO=  -.8792
      V =    4.24 +   -.0450( X - 327.21)         X = 327.21 + -17.1848( V -   4.24)

V=   4.0 THETA=  47.0 CHI= 345.0
N= 46 VBAR=   3.89 XBAR= 348.76 SDV=    .4599 SDX  10.1103 RHO=  -.8966
      V =    3.89 +   -.0408( X - 348.76)         X = 348.76 + -19.7127( V -   3.89)
```

TABLE 66 $V = 4, \theta = 47$, 5 CELLS

5 CELL

```
V=   4.0 THETA=  47.0 CHI=     .0
N= 49 VBAR=   4.03 XBAR= 358.64 SDV=    .1318 SDX   5.3690 RHO=  -.8679
      V =   4.03 +   -.0213( X - 358.64)          X = 358.64 + -35.3533( V -   4.03)

V=   4.0 THETA=  47.0 CHI=  15.0
N= 48 VBAR=   4.02 XBAR=  12.29 SDV=    .0697 SDX   5.9903 RHO=  -.5394
      V =   4.02 +   -.0063( X -  12.29)          X =  12.29 + -46.3441( V -   4.02)

V=   4.0 THETA=  47.0 CHI=  30.0
N= 36 VBAR=   3.98 XBAR=  29.29 SDV=    .0898 SDX   3.3611 RHO=   .8079
      V =   3.98 +    .0216( X -  29.29)          X =  29.29 +  30.2523( V -   3.98)

V=   4.0 THETA=  47.0 CHI=  45.0
N= 30 VBAR=   4.02 XBAR=  44.55 SDV=    .1400 SDX   2.6844 RHO=   .7823
      V =   4.02 +    .0408( X -  44.55)          X =  44.55 +  14.9988( V -   4.02)

V=   4.0 THETA=  47.0 CHI=  60.0
N= 23 VBAR=   3.99 XBAR=  59.33 SDV=    .1349 SDX   1.9427 RHO=   .7660
      V =   3.99 +    .0532( X -  59.33)          X =  59.33 +  11.0340( V -   3.99)

V=   4.0 THETA=  47.0 CHI=  75.0
N= 45 VBAR=   3.94 XBAR=  74.02 SDV=    .1649 SDX   2.3781 RHO=   .6838
      V =   3.94 +    .0474( X -  74.02)          X =  74.02 +   9.8638( V -   3.94)

V=   4.0 THETA=  47.0 CHI=  90.0
N= 50 VBAR=   3.89 XBAR=  89.33 SDV=    .1633 SDX   2.2443 RHO=   .7040
      V =   3.89 +    .0512( X -  89.33)          X =  89.33 +   9.6772( V -   3.89)

V=   4.0 THETA=  47.0 CHI= 105.0
N= 50 VBAR=   3.92 XBAR= 104.12 SDV=    .1739 SDX   6.3709 RHO=   .6971
      V =   3.92 +    .0190( X - 104.12)          X = 104.12 +  25.5462( V -   3.92)

V=   4.0 THETA=  47.0 CHI= 120.0
N= 50 VBAR=   3.86 XBAR= 114.32 SDV=    .2124 SDX  13.2435 RHO=   .8268
      V =   3.86 +    .0133( X - 114.32)          X = 114.32 +  51.5573( V -   3.86)

V=   4.0 THETA=  47.0 CHI= 135.0
N= 34 VBAR=   3.89 XBAR= 136.47 SDV=    .1417 SDX   3.3956 RHO=  -.4378
      V =   3.89 +   -.0183( X - 136.47)          X = 136.47 + -10.4914( V -   3.89)

V=   4.0 THETA=  47.0 CHI= 150.0
N= 43 VBAR=   3.92 XBAR= 151.87 SDV=    .1311 SDX   3.8885 RHO=  -.6392
      V =   3.92 +   -.0215( X - 151.87)          X = 151.87 + -18.9626( V -   3.92)
```

```
V=   4.0 THETA=  47.0 CHI= 165.0
N= 30 VBAR=   4.10 XBAR= 164.25 SDV=    .2273 SDX  11.3313 RHO=  -.7031
       V =   4.10 +   -.0141( X - 164.25)          X = 164.25 + -35.0535( V -   4.10)

V=   4.0 THETA=  47.0 CHI= 180.0
N= 34 VBAR=   4.03 XBAR= 183.74 SDV=    .2272 SDX  14.7142 RHO=  -.6569
       V =   4.03 +   -.0101( X - 183.74)          X = 183.74 + -42.5517( V -   4.03)

V=   4.0 THETA=  47.0 CHI= 195.0
N= 25 VBAR=   4.04 XBAR= 193.46 SDV=    .1292 SDX  11.3763 RHO=  -.5553
       V =   4.04 +   -.0063( X - 193.46)          X = 193.46 + -48.8764( V -   4.04)

V=   4.0 THETA=  47.0 CHI= 210.0
N= 30 VBAR=   4.01 XBAR= 205.52 SDV=    .1076 SDX   9.8155 RHO=   .1183
       V =   4.01 +    .0013( X - 205.52)          X = 205.52 +  10.7949( V -   4.01)

V=   4.0 THETA=  47.0 CHI= 225.0
N= 34 VBAR=   3.99 XBAR= 222.62 SDV=    .1900 SDX   7.2966 RHO=   .7741
       V =   3.99 +    .0202( X - 222.62)          X = 222.62 +  29.7269( V -   3.99)

V=   4.0 THETA=  47.0 CHI= 240.0
N= 32 VBAR=   4.06 XBAR= 241.05 SDV=    .2150 SDX   4.6664 RHO=   .7887
       V =   4.06 +    .0363( X - 241.05)          X = 241.05 +  17.1200( V -   4.06)

V=   4.0 THETA=  47.0 CHI= 255.0
N= 24 VBAR=   4.02 XBAR= 256.52 SDV=    .2126 SDX   6.6812 RHO=   .7124
       V =   4.02 +    .0227( X - 256.52)          X = 256.52 +  22.3858( V -   4.02)

V=   4.0 THETA=  47.0 CHI= 270.0
N= 31 VBAR=   4.01 XBAR= 270.37 SDV=    .2355 SDX   6.7768 RHO=   .6283
       V =   4.01 +    .0218( X - 270.37)          X = 270.37 +  18.0821( V -   4.01)

V=   4.0 THETA=  47.0 CHI= 285.0
N= 32 VBAR=   3.99 XBAR= 284.31 SDV=    .1756 SDX   7.6801 RHO=   .4285
       V =   3.99 +    .0098( X - 284.31)          X = 284.31 +  18.7349( V -   3.99)

V=   4.0 THETA=  47.0 CHI= 300.0
N= 30 VBAR=   3.94 XBAR= 297.12 SDV=    .2483 SDX  11.2974 RHO=   .2118
       V =   3.94 +    .0047( X - 297.12)          X = 297.12 +   9.6357( V -   3.94)

V=   4.0 THETA=  47.0 CHI= 315.0
N= 47 VBAR=   3.95 XBAR= 316.14 SDV=    .2767 SDX   4.5142 RHO=  -.8889
       V =   3.95 +   -.0545( X - 316.14)          X = 316.14 + -14.5031( V -   3.95)

V=   4.0 THETA=  47.0 CHI= 330.0
N= 44 VBAR=   4.07 XBAR= 329.01 SDV=    .2153 SDX   3.4503 RHO=  -.8760
       V =   4.07 +   -.0547( X - 329.01)          X = 329.01 + -14.0400( V -   4.07)

V=   4.0 THETA=  47.0 CHI= 345.0
N= 49 VBAR=   3.96 XBAR= 347.04 SDV=    .2356 SDX   5.9734 RHO=  -.9006
       V =   3.96 +   -.0355( X - 347.04)          X = 347.04 + -22.8324( V -   3.96)
```

TABLE 67 V = 4, θ = 47, 25 CELLS

25 CELL

```
V=   4.0 THETA=  47.0 CHI=     .0
N= 50 VBAR=   3.98 XBAR=   1.09 SDV=    .0737 SDX   3.2859 RHO=  -.9164
      V =   3.98 +   -.0205( X -    1.09)          X =   1.09 + -40.8728( V -   3.98)

V=   4.0 THETA=  47.0 CHI=   15.0
N= 50 VBAR=   4.01 XBAR=  13.89 SDV=    .0382 SDX   2.8466 RHO=  -.6866
      V =   4.01 +   -.0092( X -   13.89)          X =  13.89 + -51.1447( V -   4.01)

V=   4.0 THETA=  47.0 CHI=   30.0
N= 36 VBAR=   3.99 XBAR=  30.04 SDV=    .0393 SDX   1.4161 RHO=   .6321
      V =   3.99 +    .0176( X -   30.04)          X =  30.04 +  22.7672( V -   3.99)

V=   4.0 THETA=  47.0 CHI=   45.0
N= 39 VBAR=   4.00 XBAR=  44.81 SDV=    .0751 SDX   1.2332 RHO=   .7751
      V =   4.00 +    .0472( X -   44.81)          X =  44.81 +  12.7242( V -   4.00)

V=   4.0 THETA=  47.0 CHI=   60.0
N= 42 VBAR=   3.99 XBAR=  59.75 SDV=    .0684 SDX   1.0016 RHO=   .6957
      V =   3.99 +    .0475( X -   59.75)          X =  59.75 +  10.1918( V -   3.99)

V=   4.0 THETA=  47.0 CHI=   75.0
N= 47 VBAR=   3.96 XBAR=  74.49 SDV=    .0941 SDX   1.3310 RHO=   .7437
      V =   3.96 +    .0526( X -   74.49)          X =  74.49 +  10.5184( V -   3.96)

V=   4.0 THETA=  47.0 CHI=   90.0
N= 50 VBAR=   3.96 XBAR=  89.58 SDV=    .0829 SDX   1.5311 RHO=   .7137
      V =   3.96 +    .0386( X -   89.58)          X =  89.58 +  13.1863( V -   3.96)

V=   4.0 THETA=  47.0 CHI= 105.0
N= 50 VBAR=   3.93 XBAR= 103.21 SDV=    .0815 SDX   3.8147 RHO=   .5848
      V =   3.93 +    .0125( X - 103.21)          X = 103.21 +  27.3647( V -   3.93)

V=   4.0 THETA=  47.0 CHI= 120.0
N= 50 VBAR=   3.95 XBAR= 120.03 SDV=    .0608 SDX   1.9277 RHO=   .0477
      V =   3.95 +    .0015( X - 120.03)          X = 120.03 +   1.5116( V -   3.95)

V=   4.0 THETA=  47.0 CHI= 135.0
N= 45 VBAR=   3.98 XBAR= 135.42 SDV=    .0629 SDX   1.2563 RHO=  -.0075
      V =   3.98 +   -.0004( X - 135.42)          X = 135.42 +   -.1496( V -   3.98)

V=   4.0 THETA=  47.0 CHI= 150.0
N= 50 VBAR=   3.96 XBAR= 150.96 SDV=    .0802 SDX   2.0223 RHO=  -.8296
      V =   3.96 +   -.0329( X - 150.96)          X = 150.96 + -20.9064( V -   3.96)

V=   4.0 THETA=  47.0 CHI= 165.0
N= 49 VBAR=   3.92 XBAR= 167.43 SDV=    .1016 SDX   3.8270 RHO=  -.9041
      V =   3.92 +   -.0240( X - 167.43)          X = 167.43 + -34.0654( V -   3.92)
```

TABLE 67 (CONT'D.)

```
V=   4.0 THETA=  47.0 CHI= 180.0
N= 37 VBAR=   4.03 XBAR= 180.18 SDV=    .1266 SDX   6.9831 RHO=  -.9209
       V =   4.03 +   -.0167( X - 180.18)          X = 180.18 + -50.7934( V -   4.03)

V=   4.0 THETA=  47.0 CHI= 195.0
N= 32 VBAR=   4.02 XBAR= 195.45 SDV=    .0363 SDX   5.3202 RHO=  -.5454
       V =   4.02 +   -.0037( X - 195.45)          X = 195.45 + -79.8296( V -   4.02)

V=   4.0 THETA=  47.0 CHI= 210.0
N= 36 VBAR=   4.01 XBAR= 209.78 SDV=    .0631 SDX   4.0770 RHO=   .3419
       V =   4.01 +    .0053( X - 209.78)          X = 209.78 +  22.1077( V -   4.01)

V=   4.0 THETA=  47.0 CHI= 225.0
N= 39 VBAR=   4.01 XBAR= 225.27 SDV=    .1067 SDX   3.6134 RHO=   .8529
       V =   4.01 +    .0252( X - 225.27)          X = 225.27 +  28.8918( V -   4.01)

V=   4.0 THETA=  47.0 CHI= 240.0
N= 39 VBAR=   4.02 XBAR= 240.68 SDV=    .1042 SDX   3.1624 RHO=   .8604
       V =   4.02 +    .0284( X - 240.68)          X = 240.68 +  26.1041( V -   4.02)

V=   4.0 THETA=  47.0 CHI= 255.0
N= 37 VBAR=   4.03 XBAR= 255.54 SDV=    .1062 SDX   2.4798 RHO=   .8477
       V =   4.03 +    .0363( X - 255.54)          X = 255.54 +  19.7908( V -   4.03)

V=   4.0 THETA=  47.0 CHI= 270.0
N= 44 VBAR=   3.97 XBAR= 269.53 SDV=    .1095 SDX   3.6872 RHO=   .7796
       V =   3.97 +    .0231( X - 269.53)          X = 269.53 +  26.2562( V -   3.97)

V=   4.0 THETA=  47.0 CHI= 285.0
N= 42 VBAR=   3.97 XBAR= 283.73 SDV=    .0859 SDX   5.6307 RHO=   .7212
       V =   3.97 +    .0110( X - 283.73)          X = 283.73 +  47.2802( V -   3.97)

V=   4.0 THETA=  47.0 CHI= 300.0
N= 36 VBAR=   4.01 XBAR= 299.94 SDV=    .0911 SDX   2.0425 RHO=  -.2589
       V =   4.01 +   -.0116( X - 299.94)          X = 299.94 +  -5.8026( V -   4.01)

V=   4.0 THETA=  47.0 CHI= 315.0
N= 50 VBAR=   4.00 XBAR= 314.98 SDV=    .1140 SDX   1.9554 RHO=  -.8381
       V =   4.00 +   -.0489( X - 314.98)          X = 314.98 + -14.3734( V -   4.00)

V=   4.0 THETA=  47.0 CHI= 330.0
N= 50 VBAR=   4.00 XBAR= 330.10 SDV=    .0883 SDX   1.5124 RHO=  -.8736
       V =   4.00 +   -.0510( X - 330.10)          X = 330.10 + -14.9604( V -   4.00)

V=   4.0 THETA=  47.0 CHI= 345.0
N= 50 VBAR=   3.94 XBAR= 346.27 SDV=    .1061 SDX   2.3433 RHO=  -.9036
       V =   3.94 +   -.0409( X - 346.27)          X = 346.27 + -19.9494( V -   3.94)
```

TABLE 68 V = 8, θ = 47, 1 CELL

1 CELL

V= 8.0 THETA= 47.0 CHI= .0

N= 50 VBAR= 8.01 XBAR= 359.76 SDV= .2111 SDX 8.2977 RHO= -.5323

V = 8.01 + -.0135(X - 359.76) X = 359.76 + -20.9219(V - 8.01)

V= 8.0 THETA= 47.0 CHI= 15.0

N= 48 VBAR= 8.03 XBAR= 11.23 SDV= .2695 SDX 9.7104 RHO= -.7086

V = 8.03 + -.0197(X - 11.23) X = 11.23 + -25.5272(V - 8.03)

V= 8.0 THETA= 47.0 CHI= 30.0

N= 35 VBAR= 7.97 XBAR= 29.13 SDV= .1918 SDX 3.2101 RHO= .1562

V = 7.97 + .0093(X - 29.13) X = 29.13 + 2.6151(V - 7.97)

V= 8.0 THETA= 47.0 CHI= 45.0

N= 37 VBAR= 7.82 XBAR= 43.45 SDV= .2670 SDX 2.7773 RHO= .2219

V = 7.82 + .0213(X - 43.45) X = 43.45 + 2.3085(V - 7.82)

V= 8.0 THETA= 47.0 CHI= 60.0

N= 34 VBAR= 7.95 XBAR= 59.40 SDV= .2253 SDX 2.6060 RHO= .4698

V = 7.95 + .0406(X - 59.40) X = 59.40 + 5.4349(V - 7.95)

V= 8.0 THETA= 47.0 CHI= 75.0

N= 42 VBAR= 7.89 XBAR= 74.71 SDV= .3289 SDX 3.6680 RHO= .6297

V = 7.89 + .0565(X - 74.71) X = 74.71 + 7.0217(V - 7.89)

V= 8.0 THETA= 47.0 CHI= 90.0

N= 46 VBAR= 7.76 XBAR= 90.15 SDV= .2983 SDX 4.1728 RHO= .6769

V = 7.76 + .0484(X - 90.15) X = 90.15 + 9.4681(V - 7.76)

V= 8.0 THETA= 47.0 CHI= 105.0

N= 50 VBAR= 7.78 XBAR= 104.07 SDV= .3005 SDX 8.9723 RHO= .3084

V = 7.78 + .0103(X - 104.07) X = 104.07 + 9.2082(V - 7.78)

V= 8.0 THETA= 47.0 CHI= 120.0

N= 43 VBAR= 7.81 XBAR= 119.30 SDV= .2073 SDX 6.9368 RHO= -.0176

V = 7.81 + -.0005(X - 119.30) X = 119.30 + -.5889(V - 7.81)

V= 8.0 THETA= 47.0 CHI= 135.0

N= 39 VBAR= 7.90 XBAR= 135.69 SDV= .2259 SDX 3.1780 RHO= -.1795

V = 7.90 + -.0128(X - 135.69) X = 135.69 + -2.5259(V - 7.90)

V= 8.0 THETA= 47.0 CHI= 150.0

N= 46 VBAR= 7.90 XBAR= 150.80 SDV= .2327 SDX 3.5793 RHO= -.5306

V = 7.90 + -.0345(X - 150.80) X = 150.80 + -8.1620(V - 7.90)

V= 8.0 THETA= 47.0 CHI= 165.0

N= 37 VBAR= 7.95 XBAR= 167.05 SDV= .2928 SDX 8.5905 RHO= -.7253

V = 7.95 + -.0247(X - 167.05) X = 167.05 + -21.2795(V - 7.95)

```
V=   8.0 THETA=  47.0 CHI= 180.0

N= 35 VBAR=   8.12 XBAR= 178.73 SDV=    .3121 SDX  10.4043 RHO=  -.4560

        V =   8.12 +   -.0137( X - 178.73)           X = 178.73 + -15.2004( V -   8.12)

V=   8.0 THETA=  47.0 CHI= 195.0

N= 37 VBAR=   8.05 XBAR= 194.61 SDV=    .1954 SDX  10.6969 RHO=  -.5815

        V =   8.05 +   -.0106( X - 194.61)           X = 194.61 + -31.8368( V -   8.05)

V=   8.0 THETA=  47.0 CHI= 210.0

N= 35 VBAR=   7.98 XBAR= 209.99 SDV=    .2606 SDX   8.9289 RHO=  -.3403

        V =   7.98 +   -.0099( X - 209.99)           X = 209.99 + -11.6608( V -   7.98)

V=   8.0 THETA=  47.0 CHI= 225.0

N= 41 VBAR=   7.95 XBAR= 224.34 SDV=    .3029 SDX   6.0565 RHO=   .6036

        V =   7.95 +    .0302( X - 224.34)           X = 224.34 +  12.0683( V -   7.95)

V=   8.0 THETA=  47.0 CHI= 240.0

N= 38 VBAR=   7.99 XBAR= 239.42 SDV=    .2728 SDX   5.2253 RHO=   .5277

        V =   7.99 +    .0275( X - 239.42)           X = 239.42 +  10.1070( V -   7.99)

V=   8.0 THETA=  47.0 CHI= 255.0

N= 31 VBAR=   8.09 XBAR= 257.16 SDV=    .4086 SDX  10.1962 RHO=   .8198

        V =   8.09 +    .0328( X - 257.16)           X = 257.16 +  20.4576( V -   8.09)

V=   8.0 THETA=  47.0 CHI= 270.0

N= 27 VBAR=   7.89 XBAR= 271.35 SDV=    .4018 SDX   9.1074 RHO=   .7042

        V =   7.89 +    .0311( X - 271.35)           X = 271.35 +  15.9623( V -   7.89)

V=   8.0 THETA=  47.0 CHI= 285.0

N= 42 VBAR=   7.94 XBAR= 281.80 SDV=    .2422 SDX   7.1912 RHO=   .3490

        V =   7.94 +    .0118( X - 281.80)           X = 281.80 +  10.3620( V -   7.94)

V=   8.0 THETA=  47.0 CHI= 300.0

N= 35 VBAR=   7.93 XBAR= 300.04 SDV=    .3324 SDX   5.9171 RHO=  -.4901

        V =   7.93 +   -.0275( X - 300.04)           X = 300.04 +  -8.7238( V -   7.93)

V=   8.0 THETA=  47.0 CHI= 315.0

N= 46 VBAR=   7.98 XBAR= 315.04 SDV=    .3226 SDX   3.0632 RHO=  -.6998

        V =   7.98 +   -.0737( X - 315.04)           X = 315.04 +  -6.6454( V -   7.98)

V=   8.0 THETA=  47.0 CHI= 330.0

N= 48 VBAR=   8.15 XBAR= 329.02 SDV=    .3354 SDX   3.9091 RHO=  -.7243

        V =   8.15 +   -.0622( X - 329.02)           X = 329.02 +  -8.4418( V -   8.15)

V=   8.0 THETA=  47.0 CHI= 345.0

N= 45 VBAR=   7.98 XBAR= 346.89 SDV=    .3328 SDX  10.3958 RHO=  -.7266

        V =   7.98 +   -.0233( X - 346.89)           X = 346.89 + -22.6972( V -   7.98)
```

TABLE 69   V = 8, θ = 47, 5 CELLS

```
5 CELL

V=   8.0 THETA=  47.0 CHI=     .0
N= 50 VBAR=   7.98 XBAR=    .52 SDV=    .1014 SDX   3.9585 RHO=  -.6667
       V =   7.98 +   -.0171( X -    .52)            X =    .52 + -26.0253( V -   7.98)

V=   8.0 THETA=  47.0 CHI=  15.0
N= 50 VBAR=   7.98 XBAR=  14.46 SDV=    .0849 SDX   3.3909 RHO=  -.5482
       V =   7.98 +   -.0137( X -  14.46)            X =  14.46 + -21.8955( V -   7.98)

V=   8.0 THETA=  47.0 CHI=  30.0
N= 45 VBAR=   7.98 XBAR=  30.04 SDV=    .0821 SDX   1.0792 RHO=   .0249
       V =   7.98 +    .0019( X -  30.04)            X =  30.04 +    .3266( V -   7.98)

V=   8.0 THETA=  47.0 CHI=  45.0
N= 46 VBAR=   8.00 XBAR=  44.35 SDV=    .1311 SDX   1.3017 RHO=   .5563
       V =   8.00 +    .0560( X -  44.35)            X =  44.35 +   5.5250( V -   8.00)

V=   8.0 THETA=  47.0 CHI=  60.0
N= 48 VBAR=   7.97 XBAR=  59.65 SDV=    .1248 SDX   1.2247 RHO=   .4070
       V =   7.97 +    .0415( X -  59.65)            X =  59.65 +   3.9953( V -   7.97)

V=   8.0 THETA=  47.0 CHI=  75.0
N= 48 VBAR=   7.94 XBAR=  74.65 SDV=    .1655 SDX   1.6801 RHO=   .8072
       V =   7.94 +    .0795( X -  74.65)            X =  74.65 +   8.1931( V -   7.94)

V=   8.0 THETA=  47.0 CHI=  90.0
N= 50 VBAR=   7.90 XBAR=  89.64 SDV=    .0990 SDX   1.3309 RHO=   .5565
       V =   7.90 +    .0414( X -  89.64)            X =  89.64 +   7.4795( V -   7.90)

V=   8.0 THETA=  47.0 CHI= 105.0
N= 50 VBAR=   7.96 XBAR= 104.68 SDV=    .1413 SDX   3.3286 RHO=   .4422
       V =   7.96 +    .0188( X - 104.68)            X = 104.68 +  10.4169( V -   7.96)

V=   8.0 THETA=  47.0 CHI= 120.0
N= 50 VBAR=   7.90 XBAR= 120.22 SDV=    .0894 SDX   2.1054 RHO=  -.2131
       V =   7.90 +   -.0090( X - 120.22)            X = 120.22 +  -5.0180( V -   7.90)

V=   8.0 THETA=  47.0 CHI= 135.0
N= 49 VBAR=   7.96 XBAR= 135.28 SDV=    .0922 SDX   1.4365 RHO=  -.3886
       V =   7.96 +   -.0250( X - 135.28)            X = 135.28 +  -6.0530( V -   7.96)

V=   8.0 THETA=  47.0 CHI= 150.0
N= 50 VBAR=   7.94 XBAR= 150.43 SDV=    .0917 SDX   1.4939 RHO=  -.2640
       V =   7.94 +   -.0162( X - 150.43)            X = 150.43 +  -4.3017( V -   7.94)
```

TABLE 69 (CONT'D.)

```
V=   8.0 THETA=  47.0 CHI= 165.0
N= 50 VBAR=   7.99 XBAR= 165.05 SDV=    .1260 SDX   2.5426 RHO=  -.7504
        V =   7.99 +   -.0372( X - 165.05)             X = 165.05 + -15.1474( V -   7.99)

V=   8.0 THETA=  47.0 CHI= 180.0
N= 48 VBAR=   8.04 XBAR= 180.28 SDV=    .1790 SDX   6.8542 RHO=  -.7324
        V =   8.04 +   -.0191( X - 180.28)             X = 180.28 + -28.0392( V -   8.04)

V=   8.0 THETA=  47.0 CHI= 195.0
N= 46 VBAR=   8.04 XBAR= 193.72 SDV=    .1186 SDX   5.1023 RHO=  -.6271
        V =   8.04 +   -.0146( X - 193.72)             X = 193.72 + -26.9660( V -   8.04)

V=   8.0 THETA=  47.0 CHI= 210.0
N= 42 VBAR=   8.01 XBAR= 210.00 SDV=    .1056 SDX   2.5501 RHO=   .0711
        V =   8.01 +    .0029( X - 210.00)             X = 210.00 +   1.7167( V -   8.01)

V=   8.0 THETA=  47.0 CHI= 225.0
N= 49 VBAR=   8.00 XBAR= 225.23 SDV=    .0903 SDX   1.5606 RHO=   .4567
        V =   8.00 +    .0264( X - 225.23)             X = 225.23 +   7.8925( V -   8.00)

V=   8.0 THETA=  47.0 CHI= 240.0
N= 48 VBAR=   8.02 XBAR= 240.41 SDV=    .1195 SDX   1.5117 RHO=   .3421
        V =   8.02 +    .0270( X - 240.41)             X = 240.41 +   4.3279( V -   8.02)

V=   8.0 THETA=  47.0 CHI= 255.0
N= 47 VBAR=   8.01 XBAR= 254.81 SDV=    .1593 SDX   2.5057 RHO=   .7235
        V =   8.01 +    .0460( X - 254.81)             X = 254.81 +  11.3810( V -   8.01)

V=   8.0 THETA=  47.0 CHI= 270.0
N= 45 VBAR=   8.00 XBAR= 269.73 SDV=    .1453 SDX   2.9691 RHO=   .5558
        V =   8.00 +    .0272( X - 269.73)             X = 269.73 +  11.3560( V -   8.00)

V=   8.0 THETA=  47.0 CHI= 285.0
N= 46 VBAR=   7.99 XBAR= 284.75 SDV=    .1290 SDX   3.2534 RHO=   .4294
        V =   7.99 +    .0170( X - 284.75)             X = 284.75 +  10.8326( V -   7.99)

V=   8.0 THETA=  47.0 CHI= 300.0
N= 47 VBAR=   8.01 XBAR= 300.00 SDV=    .1479 SDX   2.0875 RHO=  -.7025
        V =   8.01 +   -.0498( X - 300.00)             X = 300.00 +  -9.9134( V -   8.01)

V=   8.0 THETA=  47.0 CHI= 315.0
N= 50 VBAR=   8.01 XBAR= 314.68 SDV=    .1348 SDX   1.7371 RHO=  -.7257
        V =   8.01 +   -.0563( X - 314.68)             X = 314.68 +  -9.3511( V -   8.01)

V=   8.0 THETA=  47.0 CHI= 330.0
N= 50 VBAR=   8.01 XBAR= 330.16 SDV=    .1237 SDX   1.4849 RHO=  -.6498
        V =   8.01 +   -.0541( X - 330.16)             X = 330.16 +  -7.7990( V -   8.01)

V=   8.0 THETA=  47.0 CHI= 345.0
N= 50 VBAR=   7.96 XBAR= 345.26 SDV=    .1683 SDX   2.7203 RHO=  -.8243
        V =   7.96 +   -.0510( X - 345.26)             X = 345.26 + -13.3248( V -   7.96)
```

TABLE 70 V = 12, θ = 47, 1 CELL

```
   1 CELL

V=  12.0 THETA=  47.0 CHI=     .0
N= 50 VBAR=  11.84 XBAR=   2.17 SDV=    .2368 SDX   7.0891 RHO=  -.5489
        V =  11.84 +   -.0183( X -   2.17)          X =   2.17 + -16.4292( V -  11.84)

V=  12.0 THETA=  47.0 CHI=  15.0
N= 49 VBAR=  11.94 XBAR=  14.41 SDV=    .2186 SDX   5.6520 RHO=  -.6336
        V =  11.94 +   -.0245( X -  14.41)          X =  14.41 + -16.3840( V -  11.94)

V=  12.0 THETA=  47.0 CHI=  30.0
N= 50 VBAR=  11.92 XBAR=  29.59 SDV=    .1729 SDX   2.1206 RHO=  -.2171
        V =  11.92 +   -.0177( X -  29.59)          X =  29.59 +  -2.6628( V -  11.92)

V=  12.0 THETA=  47.0 CHI=  45.0
N= 48 VBAR=  11.96 XBAR=  44.74 SDV=    .2521 SDX   2.1771 RHO=   .1977
        V =  11.96 +    .0229( X -  44.74)          X =  44.74 +   1.7070( V -  11.96)

V=  12.0 THETA=  47.0 CHI=  60.0
N= 47 VBAR=  12.00 XBAR=  60.02 SDV=    .3753 SDX   3.3118 RHO=   .5770
        V =  12.00 +    .0654( X -  60.02)          X =  60.02 +   5.0917( V -  12.00)

V=  12.0 THETA=  47.0 CHI=  75.0
N= 41 VBAR=  11.94 XBAR=  74.89 SDV=    .3964 SDX   3.1595 RHO=   .7313
        V =  11.94 +    .0918( X -  74.89)          X =  74.89 +   5.8293( V -  11.94)

V=  12.0 THETA=  47.0 CHI=  90.0
N= 49 VBAR=  11.79 XBAR=  89.77 SDV=    .2978 SDX   3.5617 RHO=   .6443
        V =  11.79 +    .0539( X -  89.77)          X =  89.77 +   7.7051( V -  11.79)

V=  12.0 THETA=  47.0 CHI= 105.0
N= 49 VBAR=  11.75 XBAR= 104.47 SDV=    .2149 SDX   4.7030 RHO=   .1873
        V =  11.75 +    .0086( X - 104.47)          X = 104.47 +   4.0994( V -  11.75)

V=  12.0 THETA=  47.0 CHI= 120.0
N= 47 VBAR=  11.91 XBAR= 119.45 SDV=    .2607 SDX   3.4074 RHO=  -.3656
        V =  11.91 +   -.0280( X - 119.45)          X = 119.45 +  -4.7787( V -  11.91)

V=  12.0 THETA=  47.0 CHI= 135.0
N= 47 VBAR=  11.78 XBAR= 135.81 SDV=    .2388 SDX   2.5944 RHO=  -.3258
        V =  11.78 +   -.0300( X - 135.81)          X = 135.81 +  -3.5400( V -  11.78)

V=  12.0 THETA=  47.0 CHI= 150.0
N= 47 VBAR=  11.86 XBAR= 150.77 SDV=    .2276 SDX   2.5601 RHO=  -.2788
        V =  11.86 +   -.0248( X - 150.77)          X = 150.77 +  -3.1369( V -  11.86)

V=  12.0 THETA=  47.0 CHI= 165.0
N= 44 VBAR=  12.00 XBAR= 165.06 SDV=    .3584 SDX   6.8436 RHO=  -.4651
        V =  12.00 +   -.0244( X - 165.06)          X = 165.06 +  -8.8819( V -  12.00)
```

TABLE 70 (CONT'D.)

```
V=  12.0 THETA=  47.0 CHI= 180.0
N= 39 VBAR=  11.90 XBAR= 180.29 SDV=    .3317 SDX   8.2430 RHO=  -.5240
       V =  11.90 +   -.0211( X - 180.29)           X = 180.29 + -13.0229( V -  11.90)

V=  12.0 THETA=  47.0 CHI= 195.0
N= 47 VBAR=  12.02 XBAR= 190.32 SDV=    .3140 SDX   9.1320 RHO=  -.5103
       V =  12.02 +   -.0175( X - 190.32)           X = 190.32 + -14.8415( V -  12.02)

V=  12.0 THETA=  47.0 CHI= 210.0
N= 37 VBAR=  11.99 XBAR= 209.35 SDV=    .2186 SDX   7.0903 RHO=  -.2971
       V =  11.99 +   -.0092( X - 209.35)           X = 209.35 +  -9.6371( V -  11.99)

V=  12.0 THETA=  47.0 CHI= 225.0
N= 44 VBAR=  11.92 XBAR= 224.59 SDV=    .2575 SDX   3.8268 RHO=   .4628
       V =  11.92 +    .0311( X - 224.59)           X = 224.59 +   6.8769( V -  11.92)

V=  12.0 THETA=  47.0 CHI= 240.0
N= 42 VBAR=  11.95 XBAR= 239.44 SDV=    .2914 SDX   3.4842 RHO=   .6460
       V =  11.95 +    .0540( X - 239.44)           X = 239.44 +   7.7247( V -  11.95)

V=  12.0 THETA=  47.0 CHI= 255.0
N= 40 VBAR=  11.97 XBAR= 255.62 SDV=    .4046 SDX   7.5301 RHO=   .8154
       V =  11.97 +    .0438( X - 255.62)           X = 255.62 +  15.1756( V -  11.97
V=  12.0 THETA=  47.0 CHI= 270.0
N= 38 VBAR=  12.03 XBAR= 271.05 SDV=    .2937 SDX   5.6189 RHO=   .7287
       V =  12.03 +    .0381( X - 271.05)           X = 271.05 +  13.9395( V -  12.03)

V=  12.0 THETA=  47.0 CHI= 285.0
N= 42 VBAR=  11.94 XBAR= 284.89 SDV=    .2451 SDX   4.9142 RHO=  -.0097
       V =  11.94 +   -.0005( X - 284.89)           X = 284.89 +   -.1935( V -  11.94)

V=  12.0 THETA=  47.0 CHI= 300.0
N= 43 VBAR=  12.01 XBAR= 300.23 SDV=    .2398 SDX   2.6346 RHO=  -.4743
       V =  12.01 +   -.0432( X - 300.23)           X = 300.23 +  -5.2114( V -  12.01)

V=  12.0 THETA=  47.0 CHI= 315.0
N= 47 VBAR=  12.03 XBAR= 314.79 SDV=    .3626 SDX   3.0746 RHO=  -.6659
       V =  12.03 +   -.0785( X - 314.79)           X = 314.79 +  -5.6468( V -  12.03)

V=  12.0 THETA=  47.0 CHI= 330.0
N= 48 VBAR=  11.95 XBAR= 330.00 SDV=    .2558 SDX   2.8148 RHO=  -.6073
       V =  11.95 +   -.0552( X - 330.00)           X = 330.00 +  -6.6822( V -  11.95)

V=  12.0 THETA=  47.0 CHI= 345.0
N= 47 VBAR=  11.96 XBAR= 347.05 SDV=    .3481 SDX   8.3048 RHO=  -.7369
       V =  11.96 +   -.0309( X - 347.05)           X = 347.05 + -17.5787( V -  11.96)
```

TABLE 71   V = 12, θ = 47, 5 CELLS

5 CELL

```
V=  12.0 THETA=  47.0 CHI=     .0
N= 50 VBAR=  11.95 XBAR=     .84 SDV=     .1316 SDX   4.8543 RHO=  -.7304
         V =  11.95 +   -.0198( X -     .84)          X =     .84 + -26.9360( V -  11.95)

V=  12.0 THETA=  47.0 CHI=  15.0
N= 50 VBAR=  11.98 XBAR=  15.03 SDV=     .0886 SDX   1.8719 RHO=  -.3454
         V =  11.98 +   -.0163( X -  15.03)           X =  15.03 +  -7.2970( V -  11.98)

V=  12.0 THETA=  47.0 CHI=  30.0
N= 50 VBAR=  11.96 XBAR=  30.07 SDV=     .0854 SDX    .8945 RHO=  -.0640
         V =  11.96 +   -.0061( X -  30.07)           X =  30.07 +   -.6703( V -  11.96)

V=  12.0 THETA=  47.0 CHI=  45.0
N= 49 VBAR=  12.00 XBAR=  44.79 SDV=     .1762 SDX   1.4286 RHO=   .2317
         V =  12.00 +    .0286( X -  44.79)           X =  44.79 +   1.8793( V -  12.00)

V=  12.0 THETA=  47.0 CHI=  60.0
N= 50 VBAR=  11.99 XBAR=  59.77 SDV=     .1514 SDX   1.3124 RHO=   .7130
         V =  11.99 +    .0823( X -  59.77)           X =  59.77 +   6.1794( V -  11.99)

V=  12.0 THETA=  47.0 CHI=  75.0
N= 50 VBAR=  11.98 XBAR=  74.87 SDV=     .1925 SDX   1.5422 RHO=   .6398
         V =  11.98 +    .0799( X -  74.87)           X =  74.87 +   5.1252( V -  11.98)

V=  12.0 THETA=  47.0 CHI=  90.0
N= 50 VBAR=  11.88 XBAR=  89.61 SDV=     .1725 SDX   1.6894 RHO=   .7063
         V =  11.88 +    .0721( X -  89.61)           X =  89.61 +   6.9149( V -  11.88)

V=  12.0 THETA=  47.0 CHI= 105.0
N= 50 VBAR=  11.94 XBAR= 104.76 SDV=     .0958 SDX   2.2703 RHO=   .0702
         V =  11.94 +    .0030( X - 104.76)           X = 104.76 +   1.6627( V -  11.94)

V=  12.0 THETA=  47.0 CHI= 120.0
N= 50 VBAR=  11.91 XBAR= 119.81 SDV=     .0986 SDX   1.9518 RHO=  -.3327
         V =  11.91 +   -.0168( X - 119.81)           X = 119.81 +  -6.5830( V -  11.91)

V=  12.0 THETA=  47.0 CHI= 135.0
N= 50 VBAR=  11.95 XBAR= 135.54 SDV=     .1154 SDX   1.0047 RHO=  -.1725
         V =  11.95 +   -.0198( X - 135.54)           X = 135.54 +  -1.5011( V -  11.95)

V=  12.0 THETA=  47.0 CHI= 150.0
N= 50 VBAR=  11.95 XBAR= 150.38 SDV=     .1006 SDX   1.0238 RHO=  -.3501
         V =  11.95 +   -.0344( X - 150.38)           X = 150.38 +  -3.5618( V -  11.95)

V=  12.0 THETA=  47.0 CHI= 165.0
N= 50 VBAR=  11.97 XBAR= 164.69 SDV=     .1286 SDX   1.2044 RHO=  -.5238
         V =  11.97 +   -.0559( X - 164.69)           X = 164.69 +  -4.9068( V -  11.97)
```

```
V=  12.0 THETA=  47.0 CHI= 180.0
N= 50 VBAR=  12.03 XBAR= 180.29 SDV=    .1576 SDX   4.8676 RHO=  -.7209
       V =  12.03 +   -.0233( X - 180.29)          X = 180.29 + -22.2641( V -  12.03)

V=  12.0 THETA=  47.0 CHI= 195.0
N= 50 VBAR=  11.99 XBAR= 194.56 SDV=    .0986 SDX   2.6689 RHO=  -.2130
       V =  11.99 +   -.0079( X - 194.56)          X = 194.56 +  -5.7643( V -  11.99)

V=  12.0 THETA=  47.0 CHI= 210.0
N= 50 VBAR=  12.00 XBAR= 209.97 SDV=    .1040 SDX   1.6023 RHO=  -.0239
       V =  12.00 +   -.0016( X - 209.97)          X = 209.97 +   -.3685( V -  12.00)

V=  12.0 THETA=  47.0 CHI= 225.0
N= 50 VBAR=  12.00 XBAR= 224.69 SDV=    .1050 SDX   1.8176 RHO=   .4538
       V =  12.00 +    .0262( X - 224.69)          X = 224.69 +   7.8573( V -  12.00)

V=  12.0 THETA=  47.0 CHI= 240.0
N= 50 VBAR=  12.03 XBAR= 240.12 SDV=    .1238 SDX   1.4835 RHO=   .4827
       V =  12.03 +    .0403( X - 240.12)          X = 240.12 +   5.7825( V -  12.03)

V=  12.0 THETA=  47.0 CHI= 255.0
N= 48 VBAR=  11.99 XBAR= 255.42 SDV=    .1330 SDX   2.1651 RHO=   .7808
       V =  11.99 +    .0480( X - 255.42)          X = 255.42 +  12.7089( V -  11.99)

V=  12.0 THETA=  47.0 CHI= 270.0
N= 50 VBAR=  11.99 XBAR= 269.13 SDV=    .1355 SDX   1.8965 RHO=   .4684
       V =  11.99 +    .0335( X - 269.13)          X = 269.13 +   6.5561( V -  11.99)

V=  12.0 THETA=  47.0 CHI= 285.0
N= 50 VBAR=  11.98 XBAR= 285.30 SDV=    .0987 SDX   2.0207 RHO=  -.0560
       V =  11.98 +   -.0027( X - 285.30)          X = 285.30 +  -1.1466( V -  11.98)

V=  12.0 THETA=  47.0 CHI= 300.0
N= 50 VBAR=  12.04 XBAR= 299.92 SDV=    .1311 SDX   1.4357 RHO=  -.5459
       V =  12.04 +   -.0498( X - 299.92)          X = 299.92 +  -5.9798( V -  12.04)

V=  12.0 THETA=  47.0 CHI= 315.0
N= 50 VBAR=  12.00 XBAR= 315.19 SDV=    .1379 SDX   1.3702 RHO=  -.6263
       V =  12.00 +   -.0630( X - 315.19)          X = 315.19 +  -6.2226( V -  12.00)

V=  12.0 THETA=  47.0 CHI= 330.0
N= 50 VBAR=  11.99 XBAR= 329.92 SDV=    .1297 SDX   1.1505 RHO=  -.5687
       V =  11.99 +   -.0641( X - 329.92)          X = 329.92 +  -5.0429( V -  11.99)

V=  12.0 THETA=  47.0 CHI= 345.0
N= 50 VBAR=  11.99 XBAR= 345.24 SDV=    .1097 SDX   1.4937 RHO=  -.5884
       V =  11.99 +   -.0432( X - 345.24)          X = 345.24 +  -8.0138( V -  11.99)
```

TABLE 72 V = 24, θ = 47, 1 CELL

```
1 CELL

V=  24.0 THETA=  47.0 CHI=     .0
N= 50 VBAR=  23.87 XBAR=     .46 SDV=     .3059 SDX   7.4221 RHO=  -.5258
       V =  23.87 +   -.0217( X -     .46)           X =     .46 + -12.7570( V -  23.87)

V=  24.0 THETA=  47.0 CHI=  15.0
N= 50 VBAR=  23.90 XBAR=  14.12 SDV=     .3173 SDX   7.8629 RHO=  -.6271
       V =  23.90 +   -.0253( X -  14.12)           X =  14.12 + -15.5389( V -  23.90)

V=  24.0 THETA=  47.0 CHI=  30.0
N= 49 VBAR=  23.84 XBAR=  30.84 SDV=     .3677 SDX   2.6657 RHO=  -.4852
       V =  23.84 +   -.0669( X -  30.84)           X =  30.84 +  -3.5179( V -  23.84)

V=  24.0 THETA=  47.0 CHI=  45.0
N= 50 VBAR=  24.04 XBAR=  44.91 SDV=     .3595 SDX   2.2994 RHO=  -.0393
       V =  24.04 +   -.0061( X -  44.91)           X =  44.91 +   -.2513( V -  24.04)

V=  24.0 THETA=  47.0 CHI=  60.0
N= 50 VBAR=  24.10 XBAR=  60.24 SDV=     .5449 SDX   3.7806 RHO=   .5715
       V =  24.10 +    .0824( X -  60.24)           X =  60.24 +   3.9651( V -  24.10)

V=  24.0 THETA=  47.0 CHI=  75.0
N= 49 VBAR=  23.94 XBAR=  75.11 SDV=     .5911 SDX   3.4807 RHO=   .6161
       V =  23.94 +    .1046( X -  75.11)           X =  75.11 +   3.6284( V -  23.94)

V=  24.0 THETA=  47.0 CHI=  90.0
N= 49 VBAR=  23.70 XBAR=  89.24 SDV=     .3459 SDX   3.6828 RHO=   .4823
       V =  23.70 +    .0453( X -  89.24)           X =  89.24 +   5.1346( V -  23.70)

V=  24.0 THETA=  47.0 CHI= 105.0
N= 49 VBAR=  23.64 XBAR= 105.33 SDV=     .2384 SDX   4.1029 RHO=  -.3197
       V =  23.64 +   -.0186( X - 105.33)           X = 105.33 +  -5.5028( V -  23.64)

V=  24.0 THETA=  47.0 CHI= 120.0
N= 50 VBAR=  23.89 XBAR= 119.23 SDV=     .3576 SDX   3.6501 RHO=  -.3819
       V =  23.89 +   -.0374( X - 119.23)           X = 119.23 +  -3.8983( V -  23.89)

V=  24.0 THETA=  47.0 CHI= 135.0
N= 50 VBAR=  23.84 XBAR= 134.29 SDV=     .3029 SDX   2.1845 RHO=  -.3108
       V =  23.84 +   -.0431( X - 134.29)           X = 134.29 +  -2.2413( V -  23.84)

V=  24.0 THETA=  47.0 CHI= 150.0
N= 50 VBAR=  23.73 XBAR= 150.63 SDV=     .2944 SDX   1.9030 RHO=   .0271
       V =  23.73 +    .0042( X - 150.63)           X = 150.63 +    .1753( V -  23.73)
```

TABLE 72 (CONT'D.)

V= 24.0 THETA= 47.0 CHI= 165.0

N= 50 VBAR= 24.04 XBAR= 166.49 SDV= .3828 SDX 5.8427 RHO= -.3903

V = 24.04 + -.0256(X - 166.49) X = 166.49 + -5.9574(V - 24.04)

V= 24.0 THETA= 47.0 CHI= 180.0

N= 50 VBAR= 23.86 XBAR= 181.95 SDV= .3551 SDX 7.7024 RHO= -.4605

V = 23.86 + -.0212(X - 181.95) X = 181.95 + -9.9902(V - 23.86)

V= 24.0 THETA= 47.0 CHI= 195.0

N= 50 VBAR= 24.09 XBAR= 192.41 SDV= .3645 SDX 8.8665 RHO= -.4725

V = 24.09 + -.0194(X - 192.41) X = 192.41 + -11.4937(V - 24.09)

V= 24.0 THETA= 47.0 CHI= 210.0

N= 48 VBAR= 23.82 XBAR= 209.92 SDV= .3598 SDX 3.2751 RHO= -.0555

V = 23.82 + -.0061(X - 209.92) X = 209.92 + -.5048(V - 23.82)

V= 24.0 THETA= 47.0 CHI= 225.0

N= 50 VBAR= 23.97 XBAR= 225.03 SDV= .2648 SDX 2.7055 RHO= .2140

V = 23.97 + .0210(X - 225.03) X = 225.03 + 2.1867(V - 23.97)

V= 24.0 THETA= 47.0 CHI= 240.0

N= 49 VBAR= 23.99 XBAR= 240.17 SDV= .3358 SDX 2.8068 RHO= .4666

V = 23.99 + .0558(X - 240.17) X = 240.17 + 3.9000(V - 23.99)

V= 24.0 THETA= 47.0 CHI= 255.0

N= 30 VBAR= 23.99 XBAR= 254.00 SDV= .5201 SDX 4.4052 RHO= .7378

V = 23.99 + .0871(X - 254.00) X = 254.00 + 6.2488(V - 23.99)

V= 24.0 THETA= 47.0 CHI= 270.0

N= 38 VBAR= 23.96 XBAR= 269.25 SDV= .3767 SDX 5.6148 RHO= .3570

V = 23.96 + .0240(X - 269.25) X = 269.25 + 5.3202(V - 23.96)

V= 24.0 THETA= 47.0 CHI= 285.0

N= 33 VBAR= 24.01 XBAR= 284.79 SDV= .4445 SDX 6.2442 RHO= .3064

V = 24.01 + .0218(X - 284.79) X = 284.79 + 4.3034(V - 24.01)

V= 24.0 THETA= 47.0 CHI= 300.0

N= 48 VBAR= 24.00 XBAR= 299.82 SDV= .3193 SDX 2.4330 RHO= -.3060

V = 24.00 + -.0402(X - 299.82) X = 299.82 + -2.3314(V - 24.00)

V= 24.0 THETA= 47.0 CHI= 315.0

N= 49 VBAR= 24.01 XBAR= 315.27 SDV= .4124 SDX 2.5890 RHO= -.4379

V = 24.01 + -.0697(X - 315.27) X = 315.27 + -2.7491(V - 24.01)

V= 24.0 THETA= 47.0 CHI= 330.0

N= 50 VBAR= 24.07 XBAR= 330.56 SDV= .3192 SDX 2.4576 RHO= -.1401

V = 24.07 + -.0182(X - 330.56) X = 330.56 + -1.0789(V - 24.07)

V= 24.0 THETA= 47.0 CHI= 345.0

N= 50 VBAR= 23.98 XBAR= 347.51 SDV= .4401 SDX 10.6413 RHO= -.5056

V = 23.98 + -.0209(X - 347.51) X = 347.51 + -12.2255(V - 23.98)

TABLE 73 V = 4, θ = 53.5, 5 CELLS

```
5 CELL

V=   4.0 THETA=  53.5 CHI=     .0
N= 41 VBAR=    4.04 XBAR=     .12 SDV=     .2438 SDX    9.4828 RHO=   -.7684
        V =    4.04 +    -.0198( X -     .12)          X =     .12 + -29.8835( V -    4.04)

V=   4.0 THETA=  53.5 CHI=   15.0
N= 43 VBAR=    4.03 XBAR=   12.49 SDV=     .1349 SDX    9.3842 RHO=   -.3590
        V =    4.03 +    -.0052( X -   12.49)          X =   12.49 + -24.9833( V -    4.03)

V=   4.0 THETA=  53.5 CHI=   30.0
N= 35 VBAR=    4.01 XBAR=   28.16 SDV=     .1508 SDX    6.2000 RHO=    .5621
        V =    4.01 +     .0137( X -   28.16)          X =   28.16 +  23.1143( V -    4.01)

V=   4.0 THETA=  53.5 CHI=   45.0
N= 43 VBAR=    3.95 XBAR=   40.74 SDV=     .2203 SDX    6.8594 RHO=    .7789
        V =    3.95 +     .0250( X -   40.74)          X =   40.74 +  24.2499( V -    3.95)

V=   4.0 THETA=  53.5 CHI=   60.0
N= 33 VBAR=    3.98 XBAR=   57.58 SDV=     .3619 SDX    4.8448 RHO=    .8907
        V =    3.98 +     .0665( X -   57.58)          X =   57.58 +  11.9233( V -    3.98)

V=   4.0 THETA=  53.5 CHI=   75.0
N= 39 VBAR=    3.75 XBAR=   71.04 SDV=     .3020 SDX    5.3282 RHO=    .7366
        V =    3.75 +     .0418( X -   71.04)          X =   71.04 +  12.9945( V -    3.75)

V=   4.0 THETA=  53.5 CHI=   90.0
N= 50 VBAR=    3.87 XBAR=   93.07 SDV=     .2754 SDX   10.6293 RHO=    .6034
        V =    3.87 +     .0156( X -   93.07)          X =   93.07 +  23.2894( V -    3.87)

V=   4.0 THETA=  53.5 CHI=  105.0
N= 50 VBAR=    3.76 XBAR=  100.74 SDV=     .2979 SDX    9.0788 RHO=    .3446
        V =    3.76 +     .0113( X -  100.74)          X =  100.74 +  10.5023( V -    3.76)

V=   4.0 THETA=  53.5 CHI=  120.0
N= 41 VBAR=    3.69 XBAR=  105.73 SDV=     .3549 SDX   16.5076 RHO=    .6000
        V =    3.69 +     .0129( X -  105.73)          X =  105.73 +  27.9081( V -    3.69)

V=   4.0 THETA=  53.5 CHI=  135.0
N= 26 VBAR=    3.77 XBAR=  136.00 SDV=     .2197 SDX   13.5690 RHO=    .1122
        V =    3.77 +     .0018( X -  136.00)          X =  136.00 +   6.9307( V -    3.77)

V=   4.0 THETA=  53.5 CHI=  150.0
N= 25 VBAR=    4.00 XBAR=  151.34 SDV=     .3066 SDX    9.1987 RHO=   -.6090
        V =    4.00 +    -.0203( X -  151.34)          X =  151.34 + -18.2722( V -    4.00)
```

TABLE 73 (CONT'D.)

```
V=   4.0 THETA=  53.5 CHI= 165.0
N= 19 VBAR=   4.29 XBAR= 159.11 SDV=    .2954 SDX   8.6384 RHO=  -.7885
       V =    4.29 +    -.0270( X - 159.11)          X = 159.11 + -23.0584( V -    4.29)

V=   4.0 THETA=  53.5 CHI= 180.0
N= 19 VBAR=   3.98 XBAR= 191.39 SDV=    .2033 SDX  17.4663 RHO=  -.6986
       V =    3.98 +    -.0081( X - 191.39)          X = 191.39 + -60.0182( V -    3.98)

V=   4.0 THETA=  53.5 CHI= 195.0
N= 22 VBAR=   4.20 XBAR= 182.50 SDV=    .2836 SDX  20.2530 RHO=  -.5211
       V =    4.20 +    -.0073( X - 182.50)          X = 182.50 + -37.2120( V -    4.20)

V=   4.0 THETA=  53.5 CHI= 210.0
N= 21 VBAR=   4.06 XBAR= 205.19 SDV=    .1294 SDX  19.5129 RHO=  -.2759
       V =    4.06 +    -.0018( X - 205.19)          X = 205.19 + -41.6189( V -    4.06)

V=   4.0 THETA=  53.5 CHI= 225.0
N= 25 VBAR=   4.01 XBAR= 221.88 SDV=    .2637 SDX  17.7626 RHO=   .6531
       V =    4.01 +     .0097( X - 221.88)          X = 221.88 +  43.9910( V -    4.01)

V=   4.0 THETA=  53.5 CHI= 240.0
N= 14 VBAR=   4.19 XBAR= 244.07 SDV=    .2865 SDX  11.3104 RHO=   .8245 *W
       V =    4.19 +     .0209( X - 244.07)          X = 244.07 +  32.5483( V -    4.19)

V=   4.0 THETA=  53.5 CHI= 255.0
N= 24 VBAR=   4.09 XBAR= 255.46 SDV=    .3718 SDX  11.0729 RHO=   .7731
       V =    4.09 +     .0260( X - 255.46)          X = 255.46 +  23.0214( V -    4.09)

V=   4.0 THETA=  53.5 CHI= 270.0
N= 19 VBAR=   3.87 XBAR= 274.34 SDV=    .3826 SDX  16.0920 RHO=   .3104
       V =    3.87 +     .0074( X - 274.34)          X = 274.34 +  13.0548( V -    3.87)

V=   4.0 THETA=  53.5 CHI= 285.0
N= 19 VBAR=   3.68 XBAR= 292.16 SDV=    .5277 SDX  17.2560 RHO=  -.4458
       V =    3.68 +    -.0136( X - 292.16)          X = 292.16 + -14.5770( V -    3.68)

V=   4.0 THETA=  53.5 CHI= 300.0
N= 31 VBAR=   3.88 XBAR= 298.58 SDV=    .5274 SDX  14.8923 RHO=  -.3112
       V =    3.88 +    -.0110( X - 298.58)          X = 298.58 +  -8.7865( V -    3.88)

V=   4.0 THETA=  53.5 CHI= 315.0
N= 42 VBAR=   4.09 XBAR= 314.05 SDV=    .3497 SDX   7.0291 RHO=  -.6410
       V =    4.09 +    -.0319( X - 314.05)          X = 314.05 + -12.8843( V -    4.09)

V=   4.0 THETA=  53.5 CHI= 330.0
N= 47 VBAR=   4.29 XBAR= 326.64 SDV=    .4654 SDX   7.5720 RHO=  -.8942
       V =    4.29 +    -.0550( X - 326.64)          X = 326.64 + -14.5480( V -    4.29)

V=   4.0 THETA=  53.5 CHI= 345.0
N= 46 VBAR=   3.87 XBAR= 349.98 SDV=    .4162 SDX  10.3586 RHO=  -.9047
       V =    3.87 +    -.0364( X - 349.98)          X = 349.98 + -22.5155( V -    3.87)
```

TABLE 74 V = 4, θ = 53.5, 25 CELLS

25 CELL

```
V=   4.0 THETA=  53.5 CHI=     .0
N= 50 VBAR=   4.03 XBAR= 359.07 SDV=    .1269 SDX   5.5453 RHO=  -.9111
      V =   4.03 +   -.0208( X - 359.07)          X = 359.07 + -39.8116( V -   4.03)

V=   4.0 THETA=  53.5 CHI=  15.0
N= 49 VBAR=   4.03 XBAR=  12.04 SDV=    .0670 SDX   5.8256 RHO=  -.5282
      V =   4.03 +   -.0061( X -  12.04)          X =  12.04 + -45.8994( V -   4.03)

V=   4.0 THETA=  53.5 CHI=  30.0
N= 46 VBAR=   4.00 XBAR=  29.88 SDV=    .0692 SDX   2.8441 RHO=   .7348
      V =   4.00 +    .0179( X -  29.88)          X =  29.88 +  30.2106( V -   4.00)

V=   4.0 THETA=  53.5 CHI=  45.0
N= 49 VBAR=   4.01 XBAR=  44.14 SDV=    .1158 SDX   2.9864 RHO=   .8348
      V =   4.01 +    .0324( X -  44.14)          X =  44.14 +  21.5306( V -   4.01)

V=   4.0 THETA=  53.5 CHI=  60.0
N= 39 VBAR=   3.98 XBAR=  59.05 SDV=    .1388 SDX   2.0122 RHO=   .8524
      V =   3.98 +    .0588( X -  59.05)          X =  59.05 +  12.3609( V -   3.98)

V=   4.0 THETA=  53.5 CHI=  75.0
N= 45 VBAR=   3.93 XBAR=  73.88 SDV=    .1561 SDX   2.0955 RHO=   .7726
      V =   3.93 +    .0576( X -  73.88)          X =  73.88 +  10.3706( V -   3.93)

V=   4.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=   3.90 XBAR=  89.41 SDV=    .1549 SDX   1.6210 RHO=   .7344
      V =   3.90 +    .0702( X -  89.41)          X =  89.41 +   7.6868( V -   3.90)

V=   4.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=   3.94 XBAR= 104.93 SDV=    .1649 SDX   5.9634 RHO=   .7024
      V =   3.94 +    .0194( X - 104.93)          X = 104.93 +  25.3959( V -   3.94)

V=   4.0 THETA=  53.5 CHI= 120.0
N= 50 VBAR=   3.89 XBAR= 115.37 SDV=    .1868 SDX  11.6159 RHO=   .8049
      V =   3.89 +    .0129( X - 115.37)          X = 115.37 +  50.0459( V -   3.89)

V=   4.0 THETA=  53.5 CHI= 135.0
N= 36 VBAR=   3.89 XBAR= 136.69 SDV=    .1201 SDX   2.9893 RHO=  -.4960
      V =   3.89 +   -.0199( X - 136.69)          X = 136.69 + -12.3504( V -   3.89)

V=   4.0 THETA=  53.5 CHI= 150.0
N= 44 VBAR=   3.93 XBAR= 151.82 SDV=    .1254 SDX   3.6608 RHO=  -.7036
      V =   3.93 +   -.0241( X - 151.82)          X = 151.82 + -20.5448( V -   3.93)
```

```
V=   4.0 THETA=  53.5 CHI= 165.0
N= 33 VBAR=   4.04 XBAR= 164.35 SDV=    .2031 SDX   7.8823 RHO=  -.8470
      V =   4.04 +   -.0218( X - 164.35)          X = 164.35 + -32.8785( V -   4.04)

V=   4.0 THETA=  53.5 CHI= 180.0
N= 34 VBAR=   4.02 XBAR= 181.76 SDV=    .1926 SDX  11.6059 RHO=  -.8067
      V =   4.02 +   -.0134( X - 181.76)          X = 181.76 + -48.6103( V -   4.02)

V=   4.0 THETA=  53.5 CHI= 195.0
N= 43 VBAR=   4.04 XBAR= 189.21 SDV=    .1038 SDX  11.2629 RHO=  -.4746
      V =   4.04 +   -.0044( X - 189.21)          X = 189.21 + -51.5177( V -   4.04)

V=   4.0 THETA=  53.5 CHI= 210.0
N= 39 VBAR=   4.02 XBAR= 204.37 SDV=    .1059 SDX  13.0216 RHO=  -.5941
      V =   4.02 +   -.0048( X - 204.37)          X = 204.37 + -73.0419( V -   4.02)

V=   4.0 THETA=  53.5 CHI= 225.0
N= 26 VBAR=   3.97 XBAR= 222.08 SDV=    .1380 SDX   7.4147 RHO=   .6675
      V =   3.97 +    .0124( X - 222.08)          X = 222.08 +  35.8526( V -   3.97)

V=   4.0 THETA=  53.5 CHI= 240.0
N= 36 VBAR=   4.03 XBAR= 239.68 SDV=    .2068 SDX   6.1777 RHO=   .8139
      V =   4.03 +    .0272( X - 239.68)          X = 239.68 +  24.3088( V -   4.03)

V=   4.0 THETA=  53.5 CHI= 255.0
N= 24 VBAR=   4.07 XBAR= 256.50 SDV=    .1818 SDX   5.7611 RHO=   .7400
      V =   4.07 +    .0234( X - 256.50)          X = 256.50 +  23.4467( V -   4.07)

V=   4.0 THETA=  53.5 CHI= 270.0
N= 30 VBAR=   4.02 XBAR= 270.82 SDV=    .2088 SDX   6.0905 RHO=   .6399
      V =   4.02 +    .0219( X - 270.82)          X = 270.82 +  18.6637( V -   4.02)

V=   4.0 THETA=  53.5 CHI= 285.0
N= 34 VBAR=   3.99 XBAR= 285.12 SDV=    .2031 SDX   7.4642 RHO=  -.1103
      V =   3.99 +   -.0030( X - 285.12)          X = 285.12 +  -4.0546( V -   3.99)

V=   4.0 THETA=  53.5 CHI= 300.0
N= 33 VBAR=   3.99 XBAR= 300.27 SDV=    .2151 SDX   5.2921 RHO=  -.3917
      V =   3.99 +   -.0159( X - 300.27)          X = 300.27 +  -9.6353( V -   3.99)

V=   4.0 THETA=  53.5 CHI= 315.0
N= 47 VBAR=   3.97 XBAR= 315.71 SDV=    .2549 SDX   3.7809 RHO=  -.8867
      V =   3.97 +   -.0598( X - 315.71)          X = 315.71 + -13.1515( V -   3.97)

V=   4.0 THETA=  53.5 CHI= 330.0
N= 45 VBAR=   4.04 XBAR= 329.39 SDV=    .2006 SDX   2.8102 RHO=  -.8956
      V =   4.04 +   -.0639( X - 329.39)          X = 329.39 + -12.5451( V -   4.04)

V=   4.0 THETA=  53.5 CHI= 345.0
N= 49 VBAR=   3.95 XBAR= 346.71 SDV=    .2052 SDX   4.9057 RHO=  -.9292
      V =   3.95 +   -.0389( X - 346.71)          X = 346.71 + -22.2175( V -   3.95)
```

TABLE 75 V = 8, θ = 53.5, 5 CELLS

```
5 CELL

V=   8.0 THETA=  53.5 CHI=    .0
N= 49 VBAR=   8.02 XBAR= 359.90 SDV=    .1867 SDX   7.4772 RHO=  -.7569
       V =   8.02 +   -.0189( X - 359.90)            X = 359.90 + -30.3190( V -   8.02)

V=   8.0 THETA=  53.5 CHI=  15.0
N= 48 VBAR=   8.05 XBAR=  11.53 SDV=    .2217 SDX   9.1187 RHO=  -.8075
       V =   8.05 +   -.0196( X -  11.53)            X =  11.53 + -33.2051( V -   8.05)

V=   8.0 THETA=  53.5 CHI=  30.0
N= 31 VBAR=   7.99 XBAR=  29.37 SDV=    .1279 SDX   2.5494 RHO=   .2138
       V =   7.99 +    .0107( X -  29.37)            X =  29.37 +   4.2605( V -   7.99)

V=   8.0 THETA=  53.5 CHI=  45.0
N= 39 VBAR=   7.87 XBAR=  43.51 SDV=    .1750 SDX   2.3574 RHO=   .3612
       V =   7.87 +    .0268( X -  43.51)            X =  43.51 +   4.8658( V -   7.87)

V=   8.0 THETA=  53.5 CHI=  60.0
N= 44 VBAR=   7.95 XBAR=  59.33 SDV=    .1901 SDX   2.0225 RHO=   .6951
       V =   7.95 +    .0653( X -  59.33)            X =  59.33 +   7.3951( V -   7.95)

V=   8.0 THETA=  53.5 CHI=  75.0
N= 43 VBAR=   7.89 XBAR=  74.30 SDV=    .2595 SDX   2.5108 RHO=   .7377
       V =   7.89 +    .0763( X -  74.30)            X =  74.30 +   7.1371( V -   7.89)

V=   8.0 THETA=  53.5 CHI=  90.0
N= 48 VBAR=   7.83 XBAR=  89.79 SDV=    .2548 SDX   3.1834 RHO=   .7970
       V =   7.83 +    .0638( X -  89.79)            X =  89.79 +   9.9588( V -   7.83)

V=   8.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=   7.84 XBAR= 104.65 SDV=    .2496 SDX   7.1586 RHO=   .3513
       V =   7.84 +    .0122( X - 104.65)            X = 104.65 +  10.0736( V -   7.84)

V=   8.0 THETA=  53.5 CHI= 120.0
N= 46 VBAR=   7.86 XBAR= 120.13 SDV=    .1511 SDX   5.7281 RHO=  -.2213
       V =   7.86 +   -.0058( X - 120.13)            X = 120.13 +  -8.3925( V -   7.86)

V=   8.0 THETA=  53.5 CHI= 135.0
N= 44 VBAR=   7.93 XBAR= 135.83 SDV=    .1657 SDX   2.3455 RHO=  -.3826
       V =   7.93 +   -.0270( X - 135.83)            X = 135.83 +  -5.4170( V -   7.93)

V=   8.0 THETA=  53.5 CHI= 150.0
N= 49 VBAR=   7.93 XBAR= 150.77 SDV=    .1743 SDX   2.7241 RHO=  -.6393
       V =   7.93 +   -.0409( X - 150.77)            X = 150.77 +  -9.9936( V -   7.93)

V=   8.0 THETA=  53.5 CHI= 165.0
N= 41 VBAR=   7.98 XBAR= 166.41 SDV=    .2236 SDX   7.3866 RHO=  -.7644
       V =   7.98 +   -.0231( X - 166.41)            X = 166.41 + -25.2479( V -   7.98)
```

```
V=   8.0 THETA=  53.5 CHI= 180.0
N= 35 VBAR=   8.05 XBAR= 180.23 SDV=    .2359 SDX  10.7654 RHO=  -.6909
        V =   8.05 +   -.0151( X - 180.23)          X = 180.23 + -31.5304( V -   8.05)

V=   8.0 THETA=  53.5 CHI= 195.0
N= 24 VBAR=   8.07 XBAR= 194.46 SDV=    .1639 SDX  10.4553 RHO=  -.7359
        V =   8.07 +   -.0115( X - 194.46)          X = 194.46 + -46.9293( V -   8.07)

V=   8.0 THETA=  53.5 CHI= 210.0
N= 33 VBAR=   8.00 XBAR= 209.89 SDV=    .2153 SDX   9.0520 RHO=  -.3095
        V =   8.00 +   -.0074( X - 209.89)          X = 209.89 + -13.0125( V -   8.00)

V=   8.0 THETA=  53.5 CHI= 225.0
N= 42 VBAR=   7.99 XBAR= 225.04 SDV=    .2240 SDX   5.4345 RHO=   .6848
        V =   7.99 +    .0282( X - 225.04)          X = 225.04 +  16.6096( V -   7.99)

V=   8.0 THETA=  53.5 CHI= 240.0
N= 40 VBAR=   7.99 XBAR= 239.30 SDV=    .2302 SDX   4.7055 RHO=   .7240
        V =   7.99 +    .0354( X - 239.30)          X = 239.30 +  14.8005( V -   7.99)

V=   8.0 THETA=  53.5 CHI= 255.0
N= 33 VBAR=   8.10 XBAR= 256.03 SDV=    .3040 SDX   5.7264 RHO=   .8817
        V =   8.10 +    .0468( X - 256.03)          X = 256.03 +  16.6076( V -   8.10)

V=   8.0 THETA=  53.5 CHI= 270.0
N= 32 VBAR=   7.97 XBAR= 270.70 SDV=    .3341 SDX   6.0806 RHO=   .7788
        V =   7.97 +    .0428( X - 270.70)          X = 270.70 +  14.1729( V -   7.97)

V=   8.0 THETA=  53.5 CHI= 285.0
N= 43 VBAR=   7.97 XBAR= 282.55 SDV=    .1990 SDX   5.3043 RHO=   .4553
        V =   7.97 +    .0171( X - 282.55)          X = 282.55 +  12.1373( V -   7.97)

V=   8.0 THETA=  53.5 CHI= 300.0
N= 41 VBAR=   7.94 XBAR= 300.59 SDV=    .2712 SDX   3.4227 RHO=  -.7415
        V =   7.94 +   -.0588( X - 300.59)          X = 300.59 +  -9.3585( V -   7.94)

V=   8.0 THETA=  53.5 CHI= 315.0
N= 49 VBAR=   8.00 XBAR= 314.95 SDV=    .2524 SDX   2.2350 RHO=  -.7828
        V =   8.00 +   -.0884( X - 314.95)          X = 314.95 +  -6.9355( V -   8.00)

V=   8.0 THETA=  53.5 CHI= 330.0
N= 49 VBAR=   8.13 XBAR= 329.18 SDV=    .2568 SDX   2.7880 RHO=  -.8250
        V =   8.13 +    .0760( X - 329.18)          X = 329.18 +  -8.9565( V -   8.13)

V=   8.0 THETA=  53.5 CHI= 345.0
N= 46 VBAR=   7.98 XBAR= 346.66 SDV=    .2912 SDX   9.1373 RHO=  -.7830
        V =   7.98 +   -.0249( X - 346.66)          X = 346.66 + -24.5709( V -   7.98)
```

TABLE 76   V = 8, θ = 53.5, 25 CELLS

25 CELL

```
V=   8.0 THETA=  53.5 CHI=     .0
N= 50 VBAR=   7.97 XBAR=   1.17 SDV=    .0960 SDX   3.4966 RHO=  -.8225
       V =   7.97 +   -.0226( X -   1.17)          X =   1.17 + -29.9430( V -   7.97)

V=   8.0 THETA=  53.5 CHI=  15.0
N= 50 VBAR=   8.01 XBAR=  14.18 SDV=    .0567 SDX   2.0994 RHO=  -.5671
       V =   8.01 +   -.0153( X -  14.18)          X =  14.18 + -20.9953( V -   8.01)

V=   8.0 THETA=  53.5 CHI=  30.0
N= 48 VBAR=   7.99 XBAR=  29.87 SDV=    .0511 SDX   1.3051 RHO=   .2148
       V =   7.99 +    .0084( X -  29.87)          X =  29.87 +   5.4849( V -   7.99)

V=   8.0 THETA=  53.5 CHI=  45.0
N= 47 VBAR=   7.99 XBAR=  44.72 SDV=    .1092 SDX   1.2369 RHO=   .7182
       V =   7.99 +    .0634( X -  44.72)          X =  44.72 +   8.1386( V -   7.99)

V=   8.0 THETA=  53.5 CHI=  60.0
N= 50 VBAR=   7.99 XBAR=  59.81 SDV=    .0997 SDX   1.0996 RHO=   .7160
       V =   7.99 +    .0649( X -  59.81)          X =  59.81 +   7.8949( V -   7.99)

V=   8.0 THETA=  53.5 CHI=  75.0
N= 46 VBAR=   7.94 XBAR=  74.57 SDV=    .1496 SDX   1.1962 RHO=   .8300
       V =   7.94 +    .1038( X -  74.57)          X =  74.57 +   6.6356( V -   7.94)

V=   8.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=   7.92 XBAR=  89.51 SDV=    .1159 SDX    .9827 RHO=   .8149
       V =   7.92 +    .0961( X -  89.51)          X =  89.51 +   6.9095( V -   7.92)

V=   8.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=   7.95 XBAR= 104.73 SDV=    .0877 SDX   2.4009 RHO=   .3065
       V =   7.95 +    .0112( X - 104.73)          X = 104.73 +   8.3891( V -   7.95)

V=   8.0 THETA=  53.5 CHI= 120.0
N= 50 VBAR=   7.92 XBAR= 120.39 SDV=    .0838 SDX   1.8640 RHO=  -.4370
       V =   7.92 +   -.0197( X - 120.39)          X = 120.39 +  -9.7159( V -   7.92)

V=   8.0 THETA=  53.5 CHI= 135.0
N= 50 VBAR=   7.97 XBAR= 135.36 SDV=    .0863 SDX   1.1007 RHO=  -.3654
       V =   7.97 +   -.0287( X - 135.36)          X = 135.36 +  -4.6586( V -   7.97)

V=   8.0 THETA=  53.5 CHI= 150.0
N= 50 VBAR=   7.95 XBAR= 150.58 SDV=    .0830 SDX   1.2472 RHO=  -.6978
       V =   7.95 +   -.0464( X - 150.58)          X = 150.58 + -10.4839( V -   7.95)

V=   8.0 THETA=  53.5 CHI= 165.0
N= 50 VBAR=   7.95 XBAR= 165.70 SDV=    .0877 SDX   1.7184 RHO=  -.7147
       V =   7.95 +   -.0365( X - 165.70)          X = 165.70 + -14.0030( V -   7.95)
```

TABLE 76 (CONT'D.)

```
V=   8.0 THETA=  53.5 CHI= 180.0
N= 47 VBAR=   8.02 XBAR= 180.29 SDV=    .1271 SDX   4.8816 RHO=  -.8686
       V =   8.02 +   -.0226( X - 180.29)          X = 180.29 + -33.3648( V -   8.02)

V=   8.0 THETA=  53.5 CHI= 195.0
N= 27 VBAR=   7.99 XBAR= 194.85 SDV=    .0685 SDX   3.9953 RHO=  -.7024
       V =   7.99 +   -.0120( X - 194.85)          X = 194.85 + -40.9481( V -   7.99)

V=   8.0 THETA=  53.5 CHI= 210.0
N= 42 VBAR=   8.00 XBAR= 210.04 SDV=    .0899 SDX   2.1623 RHO=   .2774
       V =   8.00 +    .0115( X - 210.04)          X = 210.04 +   6.6728( V -   8.00)

V=   8.0 THETA=  53.5 CHI= 225.0
N= 45 VBAR=   8.03 XBAR= 225.48 SDV=    .1075 SDX   2.2892 RHO=   .7441
       V =   8.03 +    .0350( X - 225.48)          X = 225.48 +  15.8386( V -   8.03)

V=   8.0 THETA=  53.5 CHI= 240.0
N= 48 VBAR=   8.03 XBAR= 240.64 SDV=    .1154 SDX   2.0949 RHO=   .8070
       V =   8.03 +    .0445( X - 240.64)          X = 240.64 +  14.6430( V -   8.03)

V=   8.0 THETA=  53.5 CHI= 255.0
N= 47 VBAR=   8.02 XBAR= 255.24 SDV=    .1129 SDX   1.7094 RHO=   .8174
       V =   8.02 +    .0540( X - 255.24)          X = 255.24 +  12.3753( V -   8.02)

V=   8.0 THETA=  53.5 CHI= 270.0
N= 50 VBAR=   7.98 XBAR= 269.91 SDV=    .1332 SDX   2.4585 RHO=   .7655
       V =   7.98 +    .0415( X - 269.91)          X = 269.91 +  14.1242( V -   7.98)

V=   8.0 THETA=  53.5 CHI= 285.0
N= 50 VBAR=   8.01 XBAR= 285.05 SDV=    .0791 SDX   2.5894 RHO=   .5204
       V =   8.01 +    .0159( X - 285.05)          X = 285.05 +  17.0274( V -   8.01)

V=   8.0 THETA=  53.5 CHI= 300.0
N= 49 VBAR=   8.02 XBAR= 299.64 SDV=    .1137 SDX   1.4936 RHO=  -.4824
       V =   8.02 +   -.0367( X - 299.64)          X = 299.64 +  -6.3341( V -   8.02)

V=   8.0 THETA=  53.5 CHI= 315.0
N= 50 VBAR=   8.01 XBAR= 314.87 SDV=    .1291 SDX   1.2752 RHO=  -.7611
       V =   8.01 +   -.0770( X - 314.87)          X = 314.87 +  -7.5189( V -   8.01)

V=   8.0 THETA=  53.5 CHI= 330.0
N= 50 VBAR=   8.02 XBAR= 329.85 SDV=    .0856 SDX    .8965 RHO=  -.7478
       V =   8.02 +   -.0714( X - 329.85)          X = 329.85 +  -7.8321( V -   8.02)

V=   8.0 THETA=  53.5 CHI= 345.0
N= 50 VBAR=   7.97 XBAR= 345.43 SDV=    .1040 SDX   1.6394 RHO=  -.7661
       V =   7.97 +   -.0486( X - 345.43)          X = 345.43 + -12.0710( V -   7.97)
```

TABLE 77 V = 12, θ = 53.5, 5 CELLS

```
5 CELL

V=  12.0 THETA=  53.5 CHI=    .0
N= 50 VBAR=  11.91 XBAR=   2.15 SDV=    .1611 SDX   5.7282 RHO=  -.6676
      V =  11.91 +   -.0188( X -   2.15)          X =   2.15 + -23.7358( V -  11.91)

V=  12.0 THETA=  53.5 CHI=  15.0
N= 50 VBAR=  11.97 XBAR=  14.86 SDV=    .1325 SDX   3.6511 RHO=  -.5761
      V =  11.97 +   -.0209( X -  14.86)          X =  14.86 + -15.8767( V -  11.97)

V=  12.0 THETA=  53.5 CHI=  30.0
N= 50 VBAR=  11.96 XBAR=  29.68 SDV=    .1097 SDX   1.6055 RHO=  -.2259
      V =  11.96 +   -.0154( X -  29.68)          X =  29.68 +  -3.3075( V -  11.96)

V=  12.0 THETA=  53.5 CHI=  45.0
N= 50 VBAR=  11.97 XBAR=  44.64 SDV=    .1631 SDX   1.7293 RHO=   .2854
      V =  11.97 +    .0269( X -  44.64)          X =  44.64 +   3.0252( V -  11.97)

V=  12.0 THETA=  53.5 CHI=  60.0
N= 49 VBAR=  11.95 XBAR=  59.61 SDV=    .2628 SDX   2.3437 RHO=   .6883
      V =  11.95 +    .0772( X -  59.61)          X =  59.61 +   6.1390( V -  11.95)

V=  12.0 THETA=  53.5 CHI=  75.0
N= 45 VBAR=  11.92 XBAR=  74.52 SDV=    .2854 SDX   2.1448 RHO=   .8216
      V =  11.92 +    .1093( X -  74.52)          X =  74.52 +   6.1753( V -  11.92)

V=  12.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=  11.87 XBAR=  89.69 SDV=    .2333 SDX   2.1818 RHO=   .7945
      V =  11.87 +    .0850( X -  89.69)          X =  89.69 +   7.4289( V -  11.87)

V=  12.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=  11.84 XBAR= 104.49 SDV=    .1526 SDX   3.5393 RHO=   .3567
      V =  11.84 +    .0154( X - 104.49)          X = 104.49 +   8.2730( V -  11.84)

V=  12.0 THETA=  53.5 CHI= 120.0
N= 48 VBAR=  11.95 XBAR= 119.77 SDV=    .1871 SDX   2.5601 RHO=  -.4443
      V =  11.95 +   -.0325( X - 119.77)          X = 119.77 +  -6.0773( V -  11.95)

V=  12.0 THETA=  53.5 CHI= 135.0
N= 50 VBAR=  11.85 XBAR= 135.82 SDV=    .1591 SDX   1.9775 RHO=  -.3966
      V =  11.85 +   -.0319( X - 135.82)          X = 135.82 +  -4.9275( V -  11.85)

V=  12.0 THETA=  53.5 CHI= 150.0
N= 49 VBAR=  11.92 XBAR= 150.53 SDV=    .1552 SDX   1.9130 RHO=  -.4620
      V =  11.92 +   -.0375( X - 150.53)          X = 150.53 +  -5.6936( V -  11.92)

V=  12.0 THETA=  53.5 CHI= 165.0
N= 50 VBAR=  11.97 XBAR= 166.11 SDV=    .2829 SDX   8.0898 RHO=  -.7160
      V =  11.97 +   -.0250( X - 166.11)          X = 166.11 + -20.4738( V -  11.97)
```

TABLE 77 (CONT'D.)

```
V=  12.0 THETA=  53.5 CHI= 180.0
N= 45 VBAR=  11.94 XBAR= 180.36 SDV=    .2408 SDX   6.4021 RHO=  -.6766
        V =  11.94 +   -.0254( X - 180.36)          X = 180.36 + -17.9919( V -  11.94)

V=  12.0 THETA=  53.5 CHI= 195.0
N= 47 VBAR=  12.02 XBAR= 192.31 SDV=    .2282 SDX   6.3220 RHO=  -.6583
        V =  12.02 +   -.0238( X - 192.31)          X = 192.31 + -18.2359( V -  12.02)

V=  12.0 THETA=  53.5 CHI= 210.0
N= 45 VBAR=  11.98 XBAR= 210.03 SDV=    .1365 SDX   2.4286 RHO=   .1568
        V =  11.98 +    .0088( X - 210.03)          X = 210.03 +   2.7895( V -  11.98)

V=  12.0 THETA=  53.5 CHI= 225.0
N= 46 VBAR=  11.96 XBAR= 224.67 SDV=    .1621 SDX   2.9148 RHO=   .6009
        V =  11.96 +    .0334( X - 224.67)          X = 224.67 +  10.8062( V -  11.96)

V=  12.0 THETA=  53.5 CHI= 240.0
N= 49 VBAR=  11.99 XBAR= 239.57 SDV=    .2037 SDX   2.7341 RHO=   .7549
        V =  11.99 +    .0563( X - 239.57)          X = 239.57 +  10.1308( V -  11.99)

V=  12.0 THETA=  53.5 CHI= 255.0
N= 44 VBAR=  11.98 XBAR= 254.77 SDV=    .2439 SDX   3.3010 RHO=   .7843
        V =  11.98 +    .0580( X - 254.77)          X = 254.77 +  10.6137( V -  11.98)

V=  12.0 THETA=  53.5 CHI= 270.0
N= 40 VBAR=  12.02 XBAR= 270.26 SDV=    .2136 SDX   3.4233 RHO=   .7934
        V =  12.02 +    .0495( X - 270.26)          X = 270.26 +  12.7157( V -  12.02)

V=  12.0 THETA=  53.5 CHI= 285.0
N= 46 VBAR=  11.97 XBAR= 285.10 SDV=    .1481 SDX   2.9410 RHO=   .0389
        V =  11.97 +    .0020( X - 285.10)          X = 285.10 +    .7720( V -  11.97)

V=  12.0 THETA=  53.5 CHI= 300.0
N= 48 VBAR=  11.99 XBAR= 300.40 SDV=    .2248 SDX   2.3843 RHO=  -.7418
        V =  11.99 +   -.0699( X - 300.40)          X = 300.40 +  -7.8676( V -  11.99)

V=  12.0 THETA=  53.5 CHI= 315.0
N= 50 VBAR=  12.03 XBAR= 314.78 SDV=    .2665 SDX   2.1654 RHO=  -.7430
        V =  12.03 +   -.0914( X - 314.78)          X = 314.78 +  -6.0375( V -  12.03)

V=  12.0 THETA=  53.5 CHI= 330.0
N= [illegible] VBAR=  11.98 XBAR= 329.96 SDV=    .1826 SDX   1.8[illegible]08 RHO=   [illegible]
        V =  11.98 +    .0[illegible]74( X - [illegible])          X = 329.96 + -[illegible].6712( V -  11.98)

V=  12.0 THETA=  53.5 CHI= [illegible]
N= [illegible] VBAR=  [illegible] XBAR= [illegible] SDV=    .2524 SDX   [illegible] RHO=  [illegible]118
        V =  11.97 +    [illegible]( X - [illegible])          X = [illegible] + [illegible]( V -  [illegible])
```

TABLE 78 V = 12, θ = 53.5, 25 CELLS

25 CELL

```
V=  12.0 THETA=  53.5 CHI=     .0
N= 50 VBAR=  11.96 XBAR=   1.59 SDV=    .0802 SDX   3.5365 RHO=  -.7488
      V =  11.96 +   -.0170( X -   1.59)          X =   1.59 + -33.0163( V -  11.96)

V=  12.0 THETA=  53.5 CHI=  15.0
N= 50 VBAR=  11.99 XBAR=  14.84 SDV=    .0610 SDX   1.6415 RHO=  -.5049
      V =  11.99 +   -.0188( X -  14.84)          X =  14.84 + -13.5762( V -  11.99)

V=  12.0 THETA=  53.5 CHI=  30.0
N= 50 VBAR=  11.99 XBAR=  29.99 SDV=    .0643 SDX    .7176 RHO=  -.2400
      V =  11.99 +   -.0215( X -  29.99)          X =  29.99 +  -2.6765( V -  11.99)

V=  12.0 THETA=  53.5 CHI=  45.0
N= 50 VBAR=  12.00 XBAR=  44.56 SDV=    .0950 SDX    .8981 RHO=   .4677
      V =  12.00 +    .0495( X -  44.56)          X =  44.56 +   4.4208( V -  12.00)

V=  12.0 THETA=  53.5 CHI=  60.0
N= 50 VBAR=  11.99 XBAR=  59.82 SDV=    .1002 SDX    .9208 RHO=   .6347
      V =  11.99 +    .0691( X -  59.82)          X =  59.82 +   5.8341( V -  11.99)

V=  12.0 THETA=  53.5 CHI=  75.0
N= 50 VBAR=  11.95 XBAR=  74.74 SDV=    .1286 SDX    .9963 RHO=   .8475
      V =  11.95 +    .1094( X -  74.74)          X =  74.74 +   6.5669( V -  11.95)

V=  12.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=  11.93 XBAR=  89.73 SDV=    .0679 SDX    .6657 RHO=   .6146
      V =  11.93 +    .0627( X -  89.73)          X =  89.73 +   6.0259( V -  11.93)

V=  12.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=  11.98 XBAR= 105.02 SDV=    .0840 SDX   1.6465 RHO=   .2937
      V =  11.98 +    .0150( X - 105.02)          X = 105.02 +   5.7542( V -  11.98)

V=  12.0 THETA=  53.5 CHI= 120.0
N= 50 VBAR=  11.93 XBAR= 120.18 SDV=    .0649 SDX   1.2405 RHO=  -.5169
      V =  11.93 +   -.0270( X - 120.18)          X = 120.18 +  -9.8772( V -  11.93)

V=  12.0 THETA=  53.5 CHI= 135.0
N= 50 VBAR=  11.98 XBAR= 135.16 SDV=    .0704 SDX    .8158 RHO=  -.5815
      V =  11.98 +   -.0502( X - 135.16)          X = 135.16 +  -6.7416( V -  11.98)

V=  12.0 THETA=  53.5 CHI= 150.0
N= 50 VBAR=  11.96 XBAR= 150.22 SDV=    .0728 SDX    .7502 RHO=  -.2422
      V =  11.96 +   -.0235( X - 150.22)          X = 150.22 +  -2.4964( V -  11.96)

V=  12.0 THETA=  53.5 CHI= 165.0
N= 50 VBAR=  12.00 XBAR= 164.93 SDV=    .0737 SDX   1.1233 RHO=  -.6524
      V =  12.00 +   -.0428( X - 164.93)          X = 164.93 +  -9.9492( V -  12.00)
```

TABLE 78 (CONT'D.)

V= 12.0 THETA= 53.5 CHI= 180.0

N= 50 VBAR=' 12.02 XBAR= 180.17 SDV= .1376 SDX 4.0876 RHO= -.8206

V = 12.02 + -.0276(X - 180.17)   X = 180.17 + -24.3712(V - 12.02)

V= 12.0 THETA= 53.5 CHI= 195.0

N= 50 VBAR= 12.02 XBAR= 194.56 SDV= .0625 SDX 2.2337 RHO= -.5155

V = 12.02 + -.0144(X - 194.56)   X = 194.56 + -18.4258(V - 12.02)

V= 12.0 THETA= 53.5 CHI= 210.0

N= 49 VBAR= 12.00 XBAR= 209.99 SDV= .0783 SDX 1.2924 RHO= .0116

V = 12.00 + .0007(X - 209.99)   X = 209.99 + .1916(V - 12.00)

V= 12.0 THETA= 53.5 CHI= 225.0

N= 50 VBAR= 12.00 XBAR= 225.15 SDV= .0695 SDX .9637 RHO= .5867

V = 12.00 + .0423(X - 225.15)   X = 225.15 + 8.1355(V - 12.00)

V= 12.0 THETA= 53.5 CHI= 240.0

N= 50 VBAR= 12.01 XBAR= 240.15 SDV= .0915 SDX .9592 RHO= .5715

V = 12.01 + .0545(X - 240.15)   X = 240.15 + 5.9904(V - 12.01)

V= 12.0 THETA= 53.5 CHI= 255.0

N= 50 VBAR= 12.02 XBAR= 254.95 SDV= .1103 SDX 1.3081 RHO= .7426

V = 12.02 + .0626(X - 254.95)   X = 254.95 + 8.8028(V - 12.02)

V= 12.0 THETA= 53.5 CHI= 270.0

N= 50 VBAR= 11.99 XBAR= 269.61 SDV= .1115 SDX 1.5401 RHO= .6922

V = 11.99 + .0501(X - 269.61)   X = 269.61 + 9.5607(V - 11.99)

V= 12.0 THETA= 53.5 CHI= 285.0

N= 50 VBAR= 12.00 XBAR= 285.18 SDV= .0826 SDX 1.4169 RHO= .2290

V = 12.00 + .0133(X - 285.18)   X = 285.18 + 3.9291(V - 12.00)

V= 12.0 THETA= 53.5 CHI= 300.0

N= 50 VBAR= 12.00 XBAR= 300.13 SDV= .1150 SDX 1.2191 RHO= -.7933

V = 12.00 + -.0748(X - 300.13)   X = 300.13 + -8.4131(V - 12.00)

V= 12.0 THETA= 53.5 CHI= 315.0

N= 50 VBAR= 12.00 XBAR= 314.85 SDV= .0918 SDX .9110 RHO= -.7430

V = 12.00 + -.0748(X - 314.85)   X = 314.85 + -7.3771(V - 12.00)

V= 12.0 THETA= 53.5 CHI= 330.0

N= 50 VBAR= 12.02 XBAR= 330.03 SDV= .0811 SDX .8216 RHO= -.6308

V = 12.02 + -.0623(X - 330.03)   X = 330.03 + -6.3893(V - 12.02)

V= 12.0 THETA= 53.5 CHI= 345.0

N= 50 VBAR= 11.98 XBAR= 345.09 SDV= .0993 SDX 1.3186 RHO= -.7512

V = 11.98 + -.0566(X - 345.09)   X = 345.09 + -9.9760(V - 11.98)

TABLE 79   V = 24, θ = 53.5, 5 CELLS

5 CELL

```
V=  24.0 THETA=  53.5 CHI=    .0
N= 50 VBAR=  23.95 XBAR=    .19 SDV=    .1630 SDX   5.6541 RHO=  -.5238
      V =  23.95 +   -.0151( X -    .19)          X =    .19 + -18.1656( V -  23.95)

V=  24.0 THETA=  53.5 CHI=  15.0
N= 50 VBAR=  23.99 XBAR=  13.36 SDV=    .2144 SDX   7.8677 RHO=  -.7593
      V =  23.99 +   -.0207( X -  13.36)          X =  13.36 + -27.8635( V -  23.99)

V=  24.0 THETA=  53.5 CHI=  30.0
N= 50 VBAR=  23.92 XBAR=  30.57 SDV=    .2119 SDX   1.9000 RHO=  -.5150
      V =  23.92 +   -.0574( X -  30.57)          X =  30.57 +  -4.6179( V -  23.92)

V=  24.0 THETA=  53.5 CHI=  45.0
N= 50 VBAR=  24.02 XBAR=  45.00 SDV=    .2008 SDX   1.8331 RHO=   .0791
      V =  24.02 +    .0087( X -  45.00)          X =  45.00 +    .7219( V -  24.02)

V=  24.0 THETA=  53.5 CHI=  60.0
N= 50 VBAR=  24.05 XBAR=  60.20 SDV=    .3350 SDX   2.6927 RHO=   .6481
      V =  24.05 +    .0806( X -  60.20)          X =  60.20 +   5.2100( V -  24.05)

V=  24.0 THETA=  53.5 CHI=  75.0
N= 50 VBAR=  23.97 XBAR=  75.11 SDV=    .3572 SDX   1.9605 RHO=   .7069
      V =  23.97 +    .1288( X -  75.11)          X =  75.11 +   3.8800( V -  23.97)

V=  24.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=  23.85 XBAR=  89.46 SDV=    .2025 SDX   1.7462 RHO=   .5498
      V =  23.85 +    .0637( X -  89.46)          X =  89.46 +   4.7413( V -  23.85)

V=  24.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=  23.83 XBAR= 105.01 SDV=    .1331 SDX   2.3614 RHO=  -.1974
      V =  23.83 +   -.0111( X - 105.01)          X = 105.01 +  -3.5019( V -  23.83)

V=  24.0 THETA=  53.5 CHI= 120.0
N= 50 VBAR=  23.95 XBAR= 119.45 SDV=    .2140 SDX   2.4746 RHO=  -.5369
      V =  23.95 +   -.0464( X - 119.45)          X = 119.45 +  -6.2081( V -  23.95)

V=  24.0 THETA=  53.5 CHI= 135.0
N= 50 VBAR=  23.92 XBAR= 134.71 SDV=    .1766 SDX   1.5722 RHO=  -.4994
      V =  23.92 +   -.0561( X - 134.71)          X = 134.71 +  -4.4458( V -  23.92)

V=  24.0 THETA=  53.5 CHI= 150.0
N= 50 VBAR=  23.84 XBAR= 150.54 SDV=    .1678 SDX   1.1493 RHO=  -.1641
      V =  23.84 +   -.0240( X - 150.54)          X = 150.54 +  -1.1240( V -  23.84)
```

TABLE 79 (CONT'D.)

```
V=  24.0 THETA=  53.5 CHI= 165.0
N= 50 VBAR=  24.05 XBAR= 165.35 SDV=    .2044 SDX   1.7818 RHO=  -.3096
      V =  24.05 +   -.0355( X - 165.35)          X = 165.35 + -[illegible].[illegible]994( V -  24.05)

V=  24.0 THETA=  53.5 CHI= 180.0
N= 50 VBAR=  23.92 XBAR= 181.65 SDV=    .2305 SDX   5.6246 RHO=  -.6216
      V =  23.92 +   -.0255( X - 181.65)          X = 181.65 + -15.1672( V -  23.92)

V=  24.0 THETA=  53.5 CHI= 195.0
N= 50 VBAR=  24.03 XBAR= 194.33 SDV=    .2120 SDX   4.8613 RHO=  -.5321
      V =  24.03 +   -.0232( X - 194.33)          X = 194.33 + -12.2025( V -  24.03)

V=  24.0 THETA=  53.5 CHI= 210.0
N= 50 VBAR=  23.91 XBAR= 209.92 SDV=    .1845 SDX   2.1114 RHO=   .0285
      V =  23.91 +    .0025( X - 209.92)          X = 209.92 +    .3259( V -  23.91)

V=  24.0 THETA=  53.5 CHI= 225.0
N= 50 VBAR=  23.99 XBAR= 225.02 SDV=    .1492 SDX   1.7532 RHO=   .2811
      V =  23.99 +    .0239( X - 225.02)          X = 225.02 +   3.3039( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 240.0
N= 50 VBAR=  24.01 XBAR= 240.21 SDV=    .1930 SDX   1.8293 RHO=   .4838
      V =  24.01 +    .0510( X - 240.21)          X = 240.21 +   4.5858( V -  24.01)

V=  24.0 THETA=  53.5 CHI= 255.0
N= 21 VBAR=  23.93 XBAR= 253.50 SDV=    .3669 SDX   3.0006 RHO=   .8457
      V =  23.93 +    .1034( X - 253.50)          X = 253.50 +   6.9161( V -  23.93)

V=  24.0 THETA=  53.5 CHI= 270.0
N= 39 VBAR=  23.98 XBAR= 269.42 SDV=    .2227 SDX   3.3092 RHO=   .5248
      V =  23.98 +    .0353( X - 269.42)          X = 269.42 +   7.8000( V -  23.98)

V=  24.0 THETA=  53.5 CHI= 285.0
N= 23 VBAR=  23.97 XBAR= 285.87 SDV=    .1894 SDX   2.7396 RHO=   .0648
      V =  23.97 +    .0045( X - 285.87)          X = 285.87 +    .9370( V -  23.97)

V=  24.0 THETA=  53.5 CHI= 300.0
N= 50 VBAR=  24.01 XBAR= 299.96 SDV=    .1872 SDX   1.7011 RHO=  -.4566
      V =  24.01 +   -.0502( X - 299.96)          X = 299.96 +  -4.1488( V -  24.01)

V=  24.0 THETA=  53.5 CHI= 315.0
N= 50 VBAR=  24.01 XBAR= 315.18 SDV=    .2409 SDX   1.6394 RHO=  -.5442
      V =  24.01 +   -.0800( X - 315.18)          X = 315.18 +  -3.7032( V -  24.01)

V=  24.0 THETA=  53.5 CHI= 330.0
N= 50 VBAR=  24.04 XBAR= 330.23 SDV=    .1803 SDX   1.4374 RHO=  -.1978
      V =  24.04 +   -.0248( X - 330.23)          X = 330.23 +  -1.5772( V -  24.04)

V=  24.0 THETA=  53.5 CHI= 345.0
N= 50 VBAR=  23.98 XBAR= 347.42 SDV=    .2790 SDX   9.6407 RHO=  -.6426
      V =  23.98 +   -.0186( X - 347.42)          X = 347.42 + -22.2051( V -  23.98)
```

TABLE 80   V = 24, θ = 53.5, 25 CELLS

25 CELL

```
V=  24.0 THETA=  53.5 CHI=     .0
N= 50 VBAR=  23.95 XBAR=     .82 SDV=    .0971 SDX   3.9354 RHO=  -.6751
       V =  23.95 +   -.0167( X -     .82)            X =     .82 + -27.3628( V -  23.95)

V=  24.0 THETA=  53.5 CHI=  15.0
N= 50 VBAR=  23.97 XBAR=  15.14 SDV=    .0735 SDX   1.0911 RHO=  -.4840
       V =  23.97 +   -.0326( X -  15.14)            X =  15.14 +  -7.1891( V -  23.97)

V=  24.0 THETA=  53.5 CHI=  30.0
N= 50 VBAR=  23.96 XBAR=  30.07 SDV=    .0665 SDX    .6404 RHO=  -.4095
       V =  23.96 +   -.0426( X -  30.07)            X =  30.07 +  -3.9405( V -  23.96)

V=  24.0 THETA=  53.5 CHI=  45.0
N= 50 VBAR=  24.00 XBAR=  44.95 SDV=    .1487 SDX   1.1969 RHO=   .2036
       V =  24.00 +    .0253( X -  44.95)            X =  44.95 +   1.6389( V -  24.00)

V=  24.0 THETA=  53.5 CHI=  60.0
N= 50 VBAR=  24.00 XBAR=  59.92 SDV=    .1432 SDX   1.0696 RHO=   .7840
       V =  24.00 +    .1050( X -  59.92)            X =  59.92 +   5.8544( V -  24.00)

V=  24.0 THETA=  53.5 CHI=  75.0
N= 50 VBAR=  24.00 XBAR=  75.09 SDV=    .1565 SDX    .9887 RHO=   .6964
       V =  24.00 +    .1103( X -  75.09)            X =  75.09 +   4.3985( V -  24.00)

V=  24.0 THETA=  53.5 CHI=  90.0
N= 50 VBAR=  23.93 XBAR=  89.89 SDV=    .1120 SDX    .8448 RHO=   .5949
       V =  23.93 +    .0789( X -  89.89)            X =  89.89 +   4.4854( V -  23.93)

V=  24.0 THETA=  53.5 CHI= 105.0
N= 50 VBAR=  23.95 XBAR= 105.02 SDV=    .0790 SDX   1.1620 RHO=  -.0857
       V =  23.95 +   -.0058( X - 105.02)            X = 105.02 +  -1.2604( V -  23.95)

V=  24.0 THETA=  53.5 CHI= 120.0
N= 50 VBAR=  23.94 XBAR= 119.81 SDV=    .1067 SDX   1.5034 RHO=  -.7158
       V =  23.94 +   -.0508( X - 119.81)            X = 119.81 + -10.0867( V -  23.94)

V=  24.0 THETA=  53.5 CHI= 135.0
N= 50 VBAR=  23.96 XBAR= 135.27 SDV=    .0855 SDX    .6431 RHO=  -.1733
       V =  23.96 +   -.0230( X - 135.27)            X = 135.27 +  -1.3031( V -  23.96)

V=  24.0 THETA=  53.5 CHI= 150.0
N= 50 VBAR=  23.97 XBAR= 150.19 SDV=    .0840 SDX    .5662 RHO=  -.2325
       V =  23.97 +   -.0345( X - 150.19)            X = 150.19 +  -1.5679( V -  23.97)

V=  24.0 THETA=  53.5 CHI= 165.0
N= 50 VBAR=  23.98 XBAR= 164.86 SDV=    .0957 SDX    .5772 RHO=  -.3820
       V =  23.98 +   -.0633( X - 164.86)            X = 164.86 +  -2.3040( V -  23.98)
```

```
V=  24.0 THETA=  53.5 CHI= 180.0
N= 50 VBAR=  24.03 XBAR= 180.08 SDV=    .1283 SDX   2.7121 RHO=  -.7130
       V =  24.03 +   -.0337( X - 180.08)          X = 180.08 + -15.0679( V -  24.03)

V=  24.0 THETA=  53.5 CHI= 195.0
N= 50 VBAR=  23.99 XBAR= 194.87 SDV=    .0874 SDX   1.2975 RHO=  -.2868
       V =  23.99 +   -.0193( X - 194.87)          X = 194.87 +  -4.2554( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 210.0
N= 50 VBAR=  23.99 XBAR= 210.00 SDV=    .0888 SDX    .9131 RHO=   .0554
       V =  23.99 +    .0054( X - 210.00)          X = 210.00 +    .5693( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 225.0
N= 50 VBAR=  24.01 XBAR= 224.85 SDV=    .0792 SDX    .9585 RHO=   .4550
       V =  24.01 +    .0376( X - 224.85)          X = 224.85 +   5.5078( V -  24.01)

V=  24.0 THETA=  53.5 CHI= 240.0
N= 50 VBAR=  24.03 XBAR= 240.07 SDV=    .1027 SDX    .8337 RHO=   .5304
       V =  24.03 +    .0653( X - 240.07)          X = 240.07 +   4.3077( V -  24.03)

V=  24.0 THETA=  53.5 CHI= 255.0
N= 48 VBAR=  24.00 XBAR= 255.22 SDV=    .1084 SDX   1.1968 RHO=   .8381
       V =  24.00 +    .0759( X - 255.22)          X = 255.22 +   9.2537( V -  24.00)

V=  24.0 THETA=  53.5 CHI= 270.0
N= 50 VBAR=  24.00 XBAR= 269.54 SDV=    .1045 SDX    .9817 RHO=   .4446
       V =  24.00 +    .0473( X - 269.54)          X = 269.54 +   4.1752( V -  24.00)

V=  24.0 THETA=  53.5 CHI= 285.0
N= 50 VBAR=  23.99 XBAR= 285.23 SDV=    .0800 SDX   1.0280 RHO=  -.1187
       V =  23.99 +   -.0092( X - 285.23)          X = 285.23 +  -1.5256( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 300.0
N= 50 VBAR=  24.03 XBAR= 299.94 SDV=    .1044 SDX    .9785 RHO=  -.6047
       V =  24.03 +   -.0645( X - 299.94)          X = 299.94 +  -5.6652( V -  24.03)

V=  24.0 THETA=  53.5 CHI= 315.0
N= 50 VBAR=  23.99 XBAR= 315.09 SDV=    .1033 SDX    .7961 RHO=  -.5236
       V =  23.99 +   -.0679( X - 315.09)          X = 315.09 +  -4.0366( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 330.0
N= 50 VBAR=  23.99 XBAR= 329.99 SDV=    .0998 SDX    .6837 RHO=  -.3583
       V =  23.99 +   -.0523( X - 329.99)          X = 329.99 +  -2.4537( V -  23.99)

V=  24.0 THETA=  53.5 CHI= 345.0
N= 50 VBAR=  23.99 XBAR= 345.11 SDV=    .0750 SDX    .9394 RHO=  -.2317
       V =  23.99 +   -.0185( X - 345.11)          X = 345.11 +  -2.9028( V -  23.99)
```

Figures 12, 13, and 14 show graphs of the correlation coefficients as a function of aspect angle for 1 cell averages and the four different wind speeds at 29°, 30° and 47° incidence angles. The small sample size of 50 values introduces considerable scatter. The pattern of the correlation coefficients is consistent throughout the entire set of tables and these three figures are representative examples.

An error of 5° in the synoptic scale wind (say the true wind is from 0° and the SCATT gives 5°) is equivalent to a vector speed error in the component at right angles of 2 m/s at 24 m/s and to an error of 1 m/s at 12 m/s. Such vector errors dominate the error structure of the simulations compared to the standard deviations of the wind speeds given in these tables.

An error in wind direction can affect numerical weather prediction models in the same way as errors in speed. The numerical models use the east-west and north-south components of the wind. A 10 m/s wind reported to be from due north that was actually from 355° would have an error of 0.9 m/s in the east-west component. If it were actually from 350° the error would be 1.7 m/s.

The biases in the wind direction and the standard deviations of wind direction errors dominate the corresponding values for speed. They are also strongly aspect angle dependent. A wind direction error propagates via the high correlation coefficients into a systematic wind speed error.

Figures 15 through 19 are graphs as a function of aspect angle* of the bias plus and minus one standard deviation as graphed from the data given in Table 53 through 80. Figure 15 is for an incidence angle of 29°, 16 is for 39°, 17 is for 39° with the 0.7 db added error, 18 is for 47°, and 19 is for 53.5°.

Viewed as a set, all 5 figures showing 16 different conditions, have very similar patterns. Direction errors are largest for light winds, and largest near 0°, 90°, 180° and 270°. The bias changes

---

*i.e. wind direction relative to beam 1.

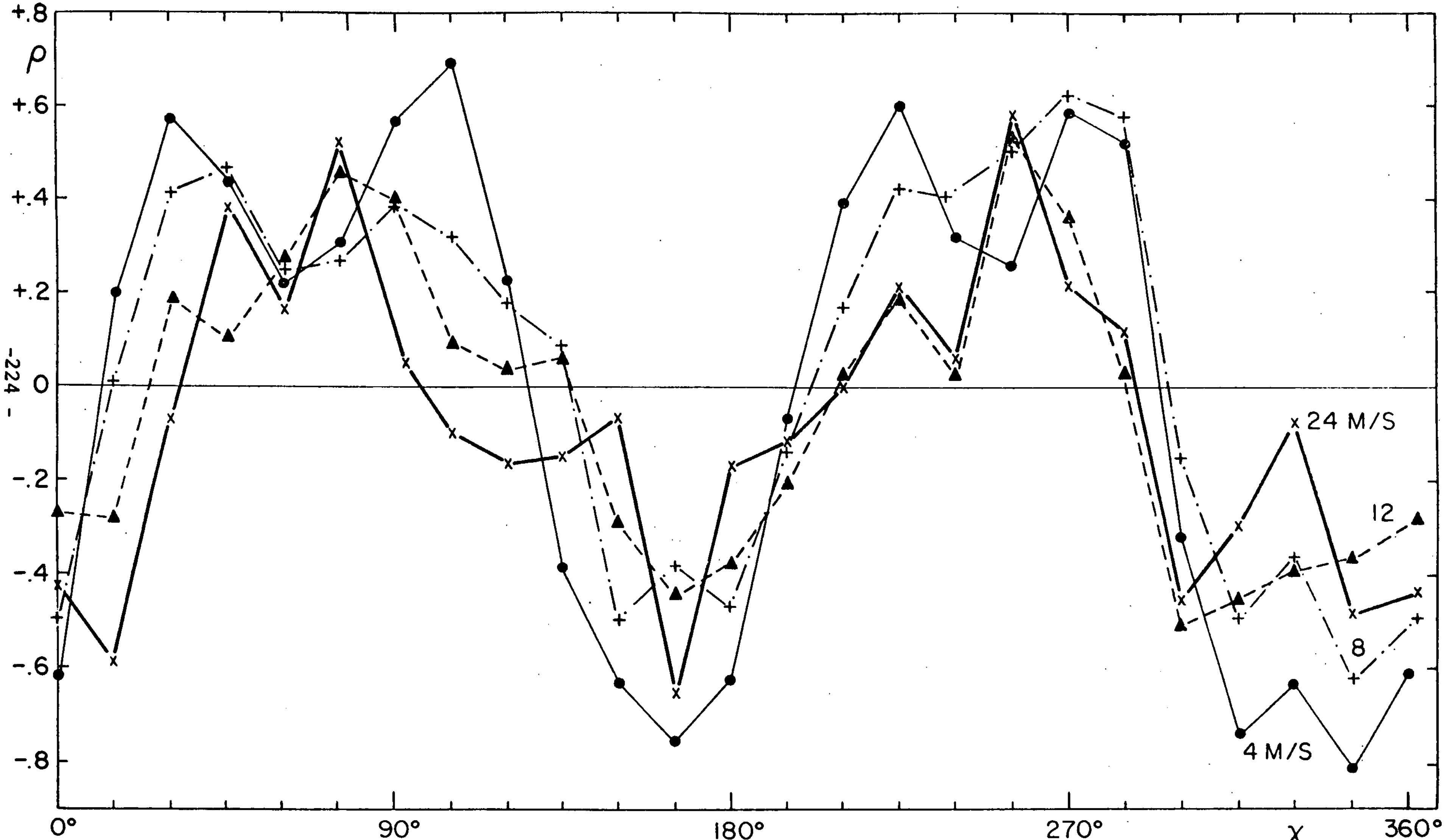


FIG. 12 CORRELATION BETWEEN SPEED AND DIRECTION ERRORS FOR A 29° INCIDENCE ANGLE FOR WINDS OF 4, 8, 12 AND 24 M/S AS A FUNCTION OF ASPECT ANGLE (1 CELL VALUES).

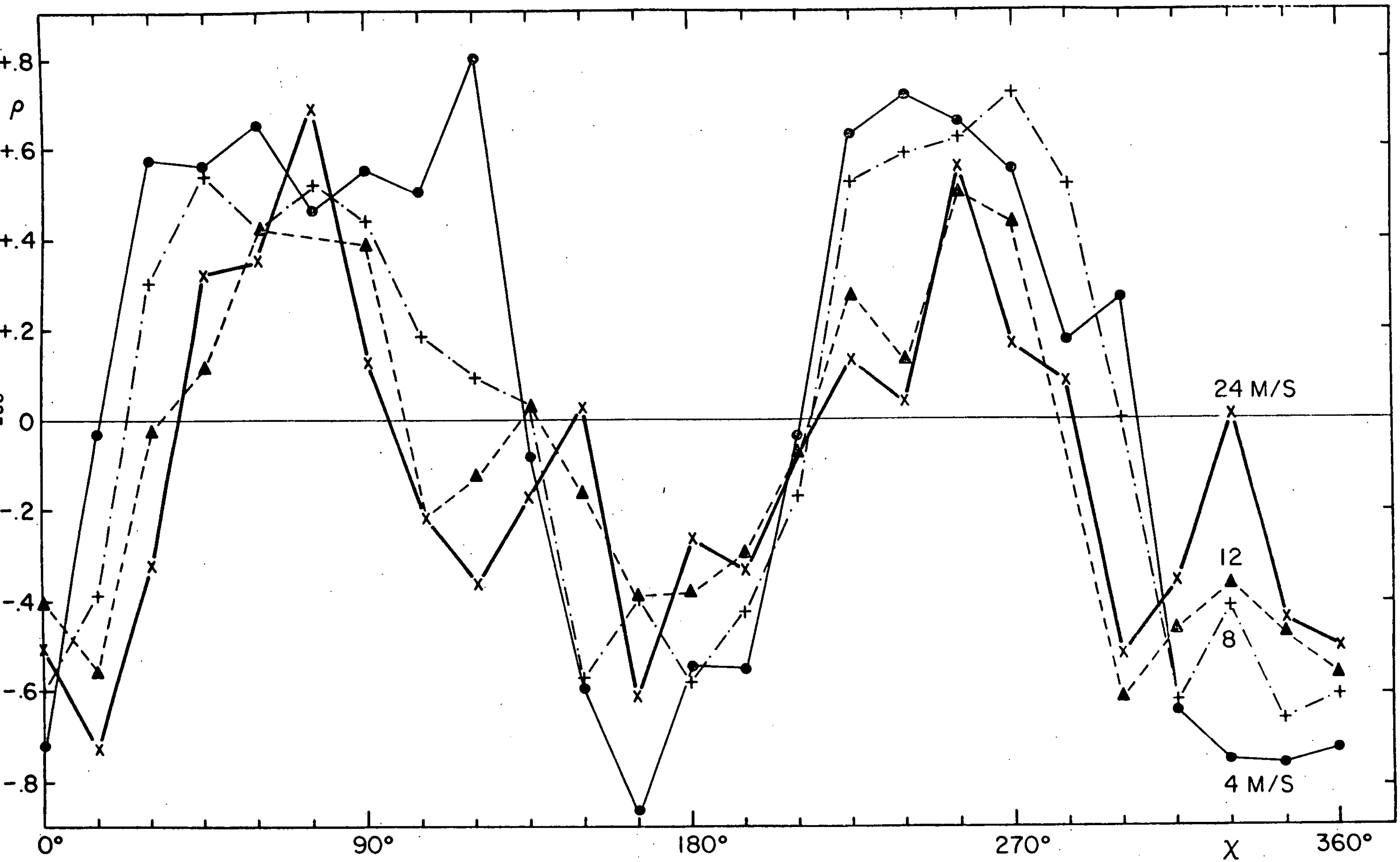


FIG. 13 CORRELATION BETWEEN SPEED AND DIRECTION ERRORS FOR A 39° INCIDENCE ANGLE FOR WINDS OF 4, 8, 12 AND 24 M/S AS A FUNCTION OF ASPECT ANGLE (1 CELL VALUES)

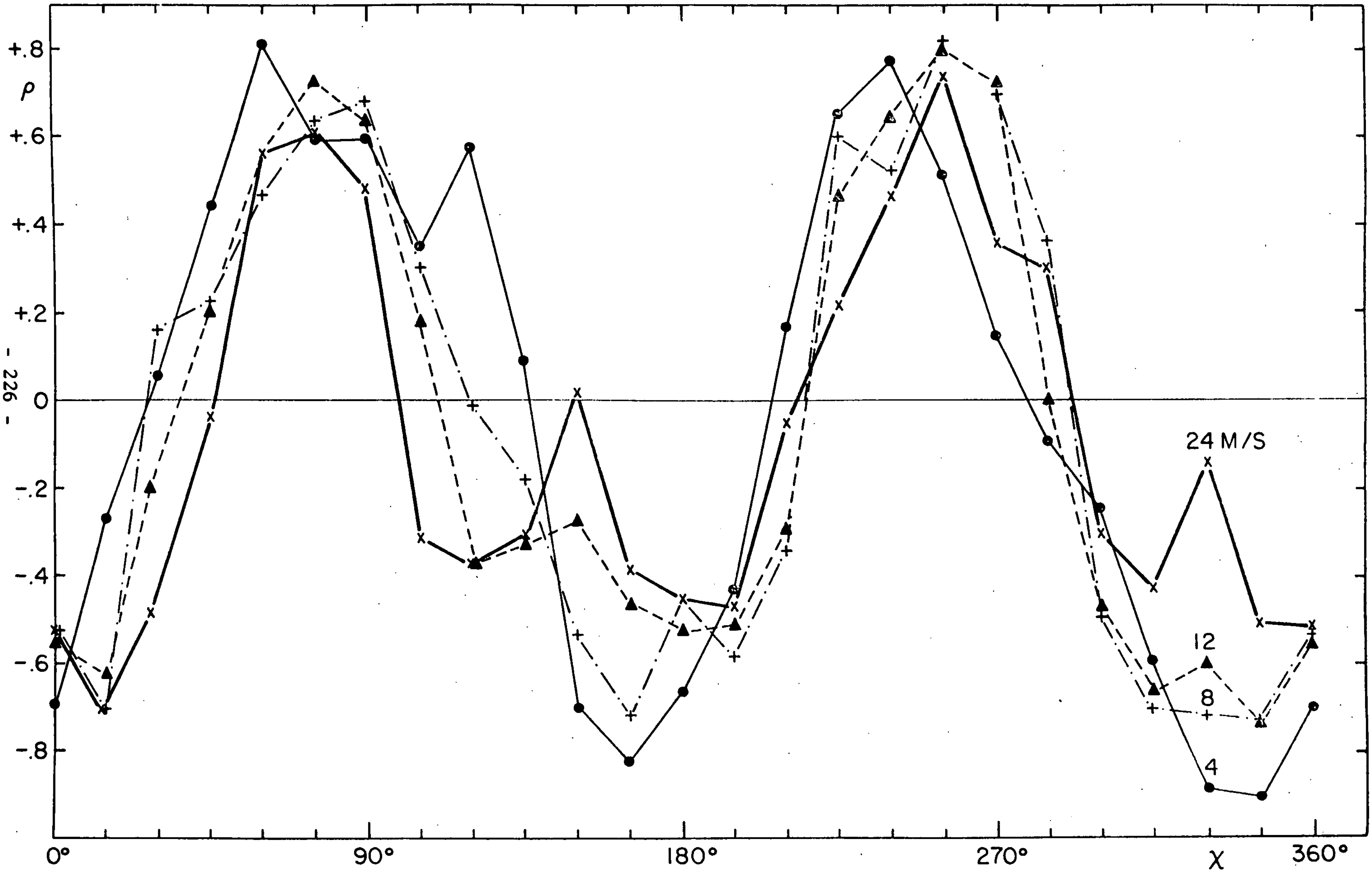


FIG. 14 CORRELATION BETWEEN SPEED AND DIRECTION ERROR FOR A 47° INCIDENCE ANGLE FOR WINDS OF 4, 8, 12 AND 24 M/S AS A FUNCTION OF ASPECT ANGLE (1 CELL AVERAGES)

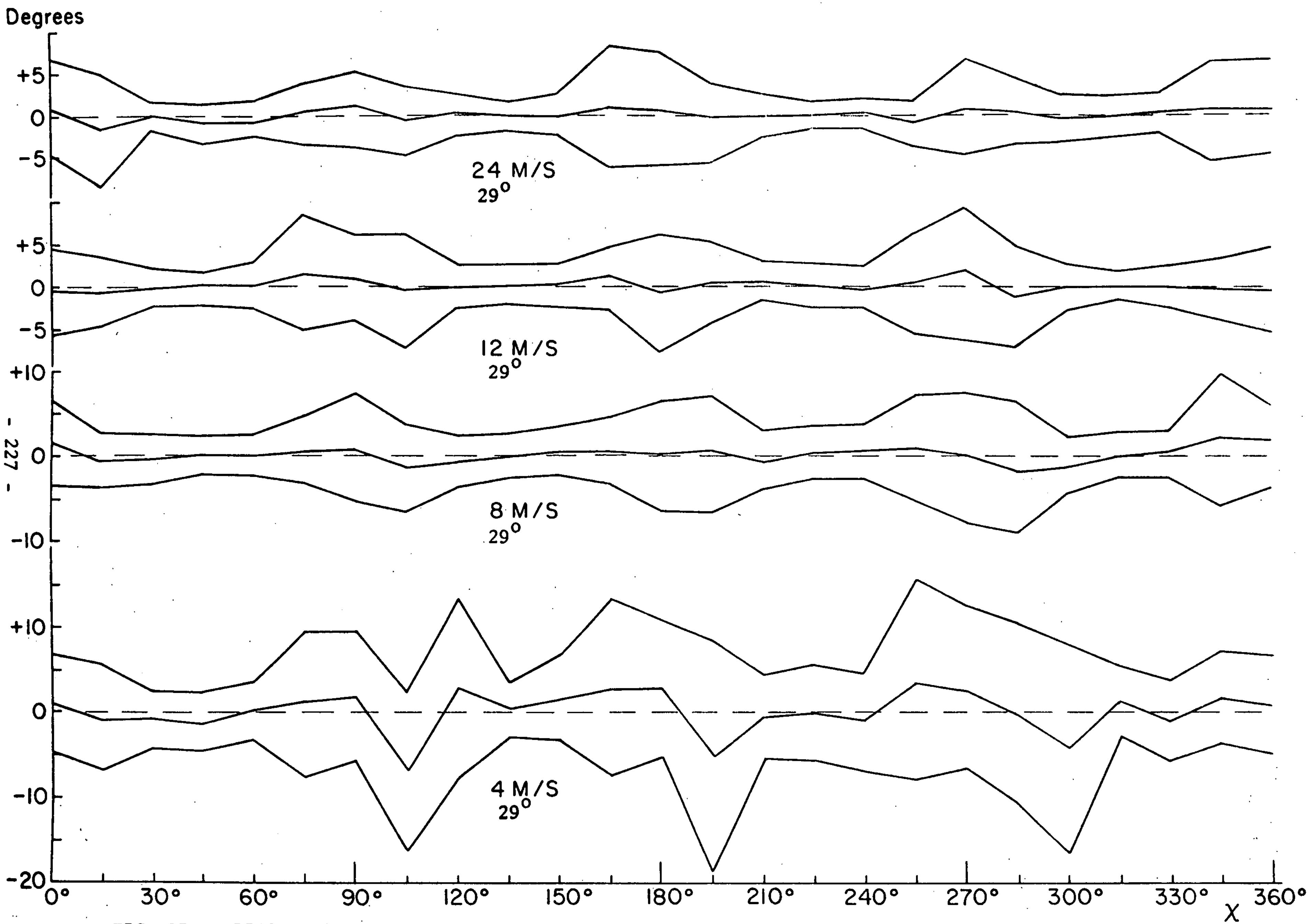


FIG. 15 BIAS PLUS AND MINUS ONE STANDARD DEVIATION (1 CELL)

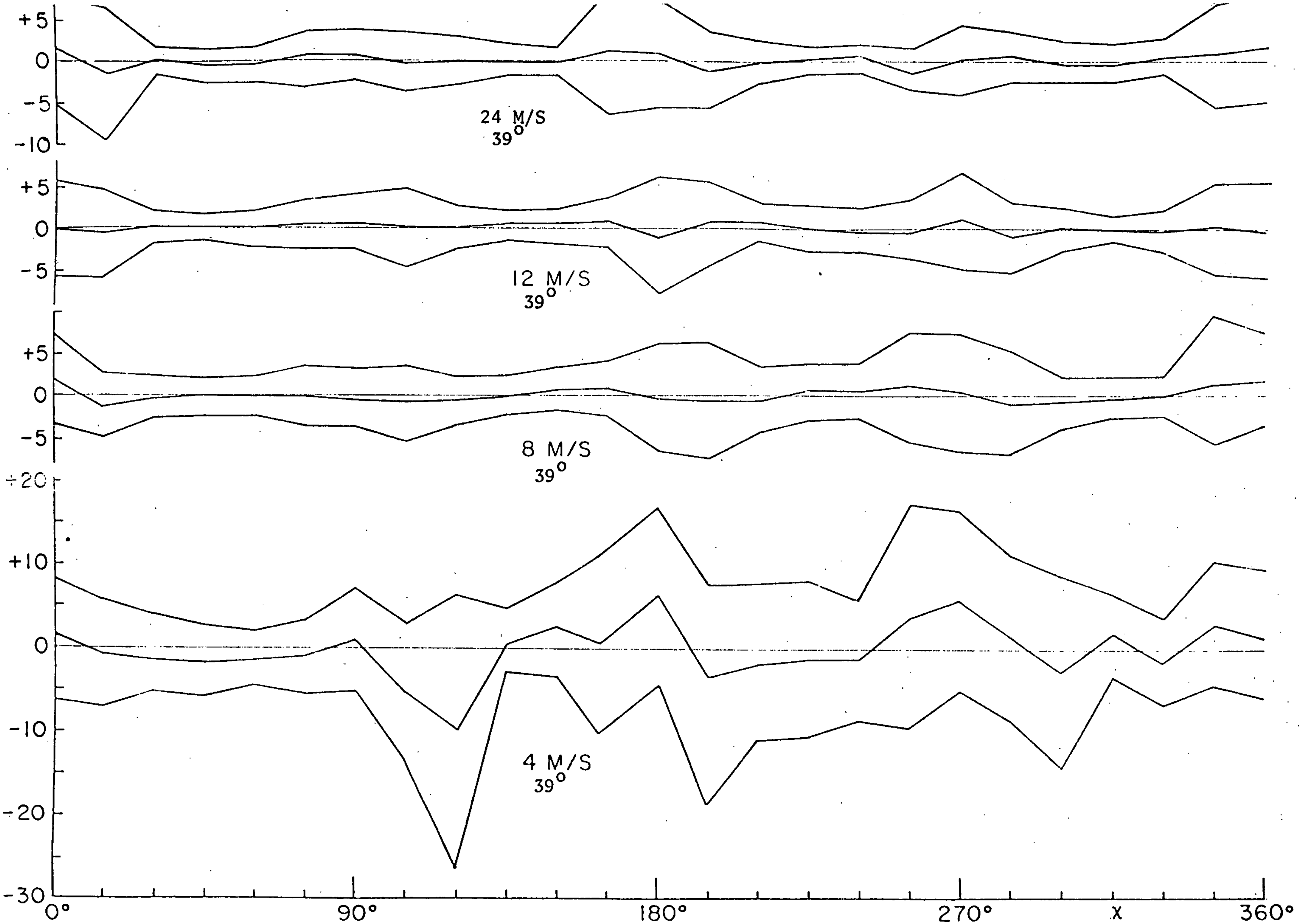


FIG. 16 BIAS PLUS AND MINUS ONE STANDARD DEVIATION

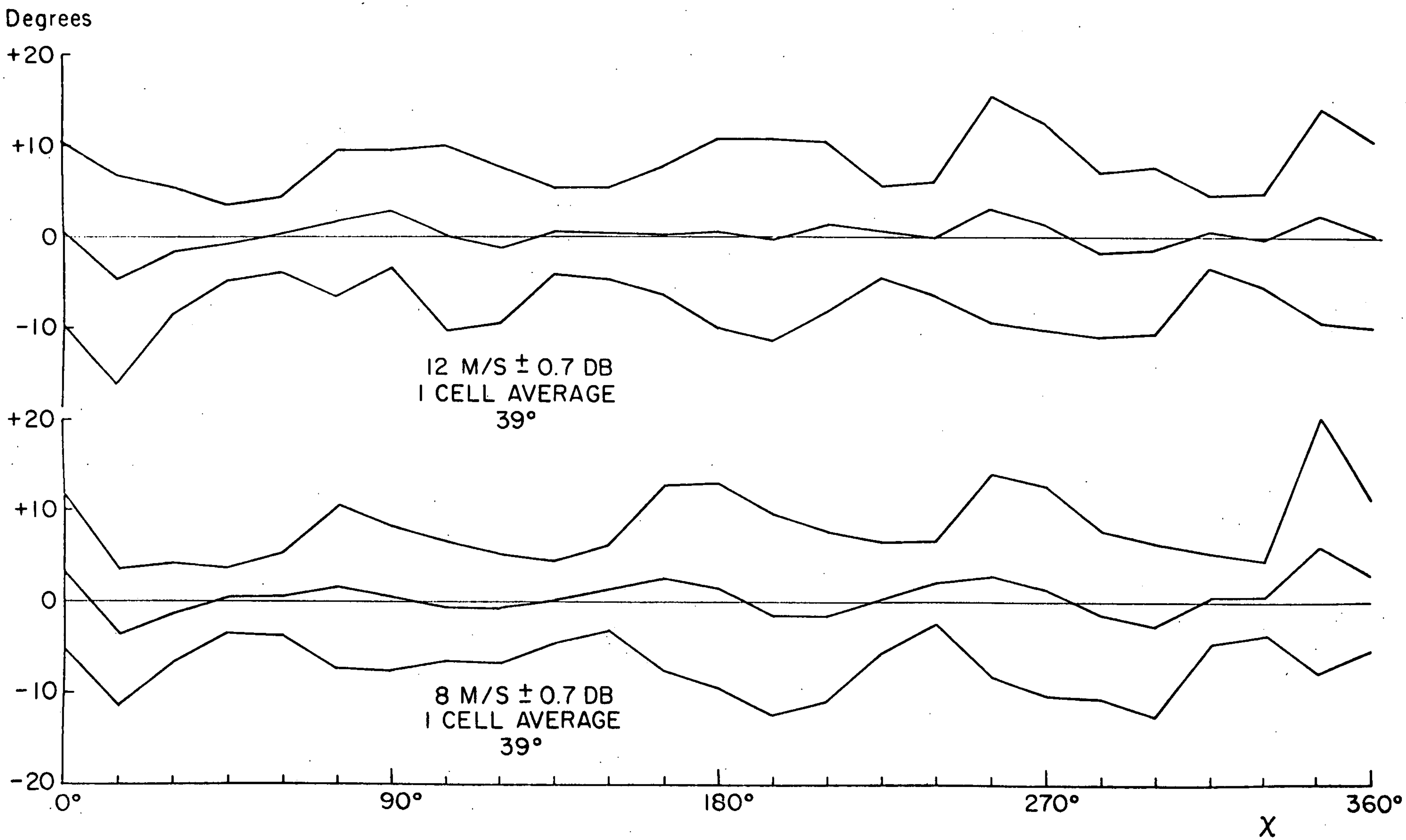


FIG. 17 BIAS PLUS AND MINUS ONE STANDARD DEVIATION

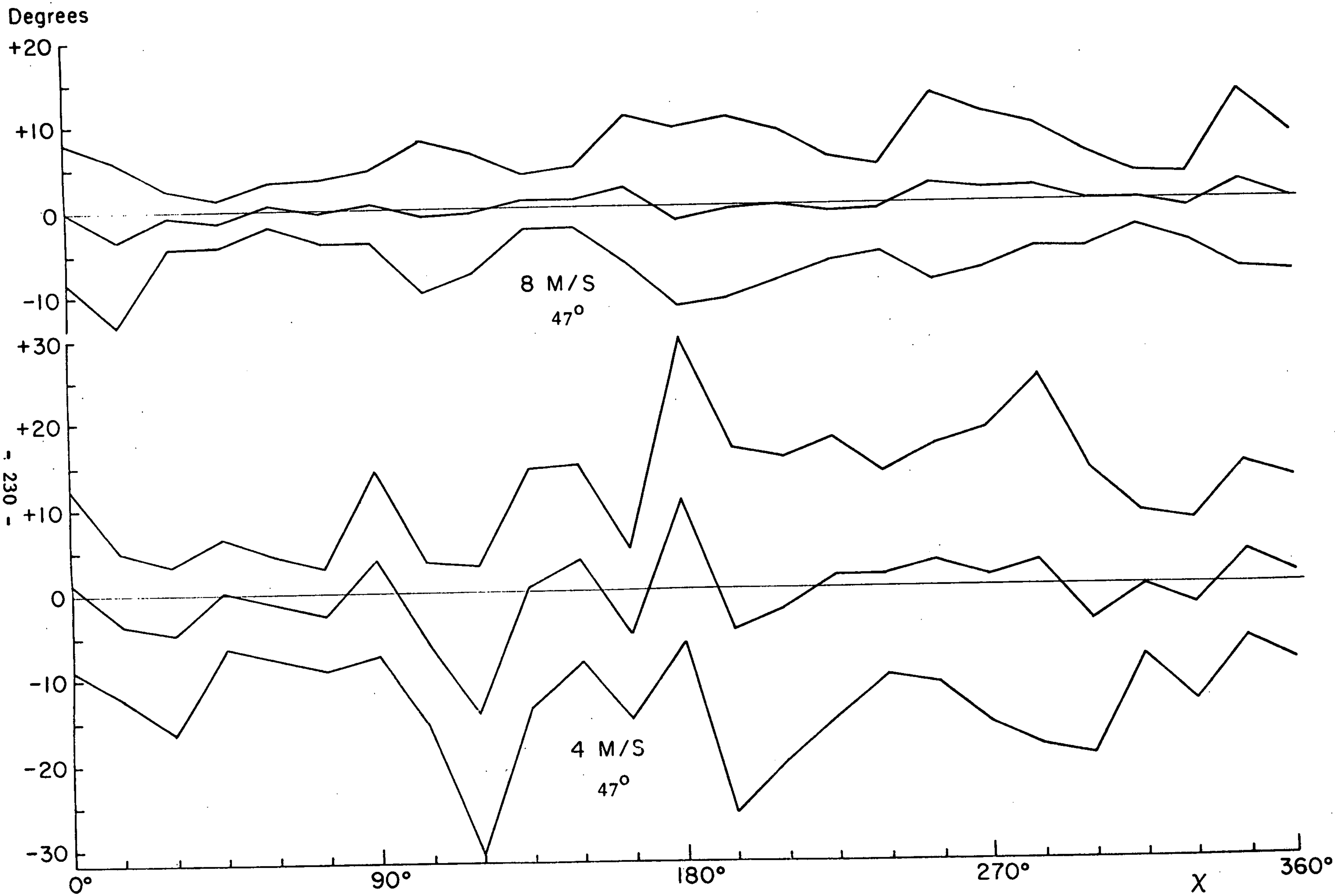


FIG. 18A BIAS PLUS AND MINUS ONE STANDARD DEVIATION (1 CELL)

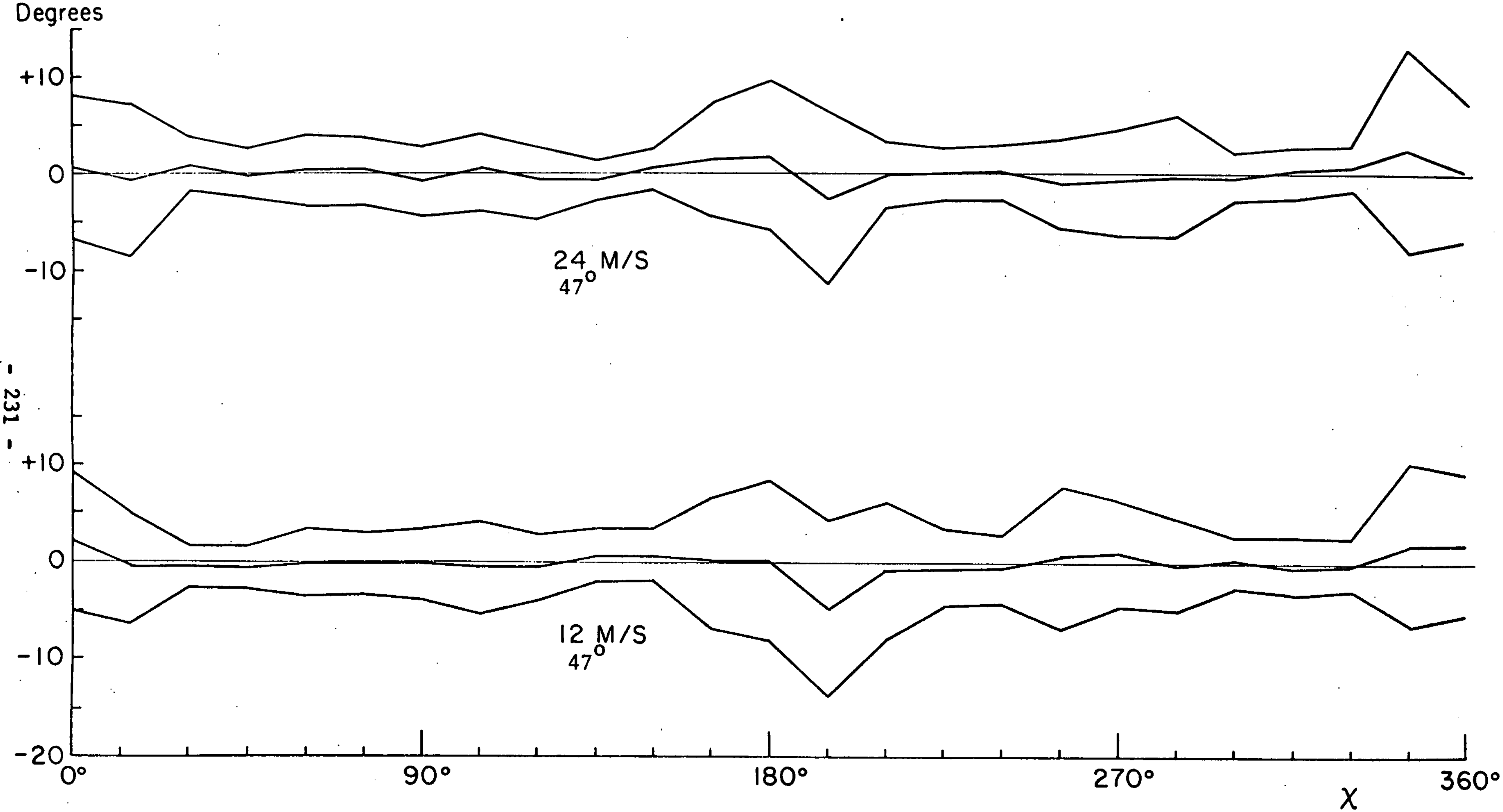


FIG. 18B BIAS PLUS AND MINUS ONE STANDARD DEVIATION (1 CELL)

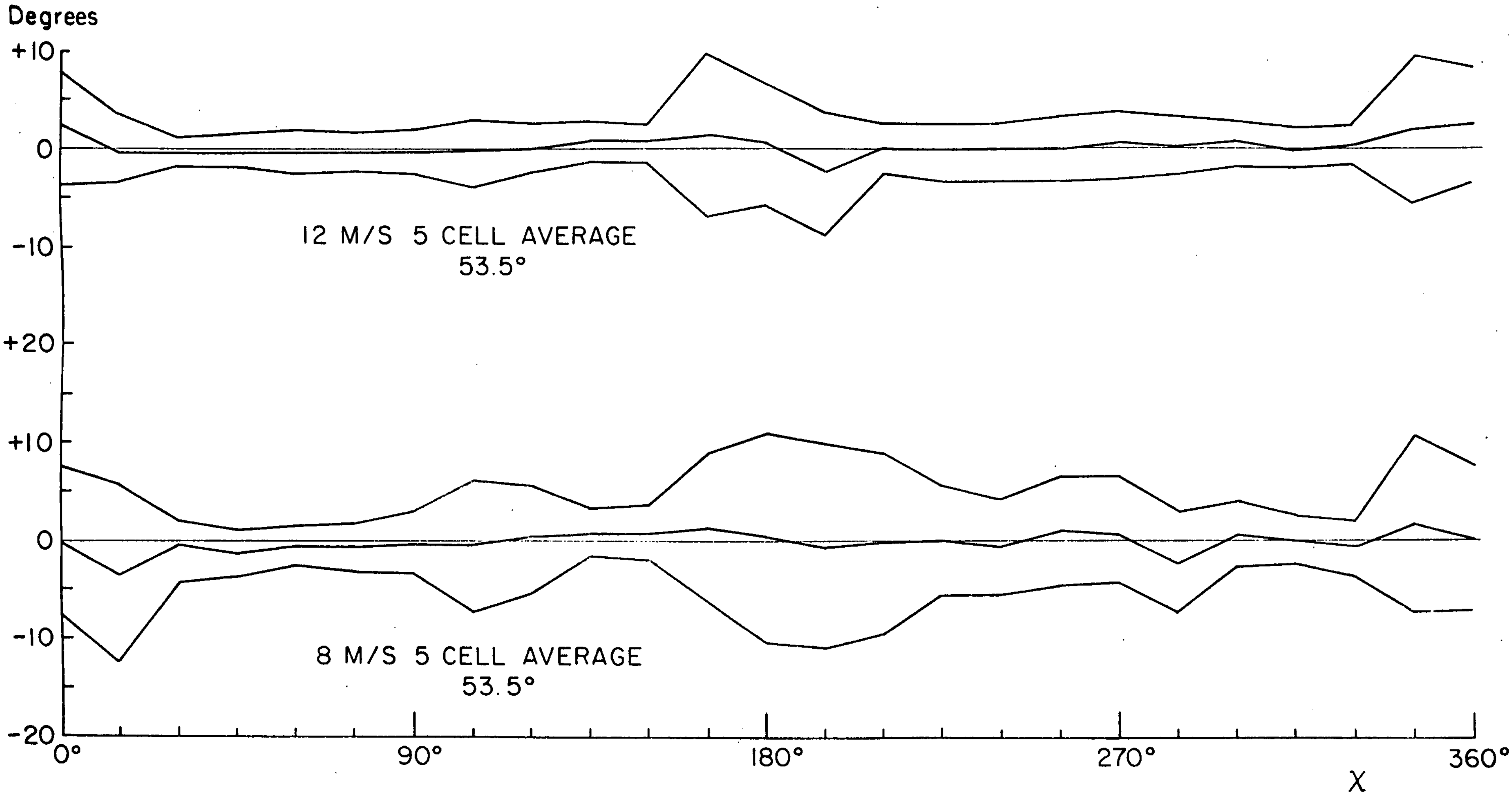


FIG. 19 BIAS PLUS AND MINUS ONE STANDARD DEVIATION

sign as the aspect angle varies through 0^o, 90^o, 180^o and 270^o. The errors are largest near 270^o. The errors increase with increasing incidence angle. Even 5 cell averages at 53.5^o hardly produce acceptable results.

This set of graphs is a representative subset of the entire data set. To show this, Table 81 gives the maximum values for the bias of both speed and direction, for the standard deviations of both speed and direction and for the correlation coefficients as extracted from Tables 53 through 80.

The addition of the new antenna and the measurements at 20^o relative to beam 1 for the right side of the spacecraft looking forward can successfully eliminate aliases by means of either the objective criteria technique or the likelihood function. However, the attempt to obtain higher resolution, at least at a nominally 10 km by 10 km cell, appears to have increased the effects of communication noise and attitude errors to such an extent that the errors in the estimates of the vector winds may be prohibitively large.

The 0.7 db added error was Monte Carloed in the following way. The theoretical backscatter value for a given measurement was found in db. A value of 0.7 db was added to it. Then the antilogs of both the theoretical backscatter and this increased value were found. The differences yielded a standard deviation, SD (0.7). The Monte Carloed value of the backscatter for a given simulation was then computed from

$$\hat{\sigma}^o = \sigma^o + t_1 \left[\mathrm{VAR}\ \sigma^o + (\mathrm{SD}(0.7))^2\right]^{\frac{1}{2}} \tag{68}$$

TABLE 81 SOME SUMMARY STATISTICS FOR THE MAXIMUM LIKELIHOOD ESTIMATES (SD = STANDARD DEVIATION; RHO = CORRELATION). FOR SOME WIND DIRECTION IN TABLES 53 TO 80, THERE IS A MAXIMUM SPEED BIAS. FOR SOME, PERHAPS, OTHER DIRECTION, THERE IS A MAXIMUM DIRECTION BIAS, AND SO ON. THESE VALUES ARE TABULATED. NUMBERS IN PARENTHESES ARE FOR ONE CELL WITH ± 0.7 DB ADDED ERROR.

	SPEED	4			8			12			24		
INCIDENCE ANGLE	NUMBER OF CELLS	1	5	25	1	5	25	1	5	25	1	5	25
29^{o}	BIAS V	- 0.20	- 0.05		- 0.17			- 0.23			- 0.39		
	BIAS χ	- 6.98	- 1.89		+ 2.06			+ 2.04			- 1.59		
	SD V	0.23	0.14		0.28			0.31			0.56		
	SD χ	13.6	5.5		7.9			7.3			7.2		
	RHO +	+ 0.67	+ 0.69		+ 0.58			+ 0.53			+ 0.58		
	RHO -	- 0.66	- 0.81		- 0.63			- 0.51			- 0.65		
39^{o}	BIAS V	- 0.21	- 0.07		- 0.15	(- 0.28)		- 0.18	(- 0.38)		- 0.28		
	BIAS χ	-10.14	+ 2.82		+ 1.88	(+ 6.13)		+ 3.99	(+ 7.94)		- 2.38		
	SD V	0.30	0.17		0.27	(0.54)		0.26	(0.60)		0.50		
	SD χ	16.2	7.4		7.6	(14.1)		6.7	(14.7)		8.1		
	RHO +	+ 0.80	+ 0.73		+ 0.72	(+ 0.57)		+ 0.51	(+ 0.55)		+ 0.69		
	RHO -	- 0.86	- 0.87		- 0.67	(- 0.58)		- 0.61	(- 0.57)		- 0.74		
47^{o}	BIAS V	- 0.39	- 0.14	- 0.08	- 0.24	- 0.06		- 0.25	- 0.25		- 0.36		
	BIAS χ	-14.26	- 5.68	+ 2.43	- 3.77	- 1.28		- 4.68	- 0.87		2.59		
	SDV V	0.58	0.28	0.13	0.41	0.18		0.40	0.16		0.59		
	SD χ	21.8	14.7	7.0	10.7	6.9		9.1	4.9		10.6		
	RHO +	+ 0.82	+ 0.83	+ 0.86	+ 0.82	+ 0.81		+ 0.73	+ 0.78		+ 0.74		
	RHO -	- 0.90	- 0.90	- 0.92	- 0.73	- 0.82		- 0.74	- 0.73		- 0.63		
53.5^{o}	BIAS V		- 0.32	- 0.11		- 0.17	- 0.08		- 0.16	- 0.07		- 0.17	- 0.07
	BIAS χ		-12.50	- 5.79		- 3.47	+ 1.17		- 2.69	+ 1.59		+ 2.42	+ 0.82
	SDV V		0.53	0.25		0.33	0.15		0.29	0.14		0.37	0.16
	SD χ		20.3	13.0		10.8	4.9		8.1	4.1		9.64	3.93
	RHO +		+ 0.85	+ 0.85		+ 0.88	+ 0.83		+ 0.82	+ 0.85		+ 0.85	+ 0.84
	RHO -		- 0.91	- 0.93		- 0.83	- 0.87		- 0.74	- 0.82		- 0.76	- 0.71

in antilog form. This is one way to attempt to account for some of the other unknown effects on the model. There are probably better ways.

The beam directions as described on page 48 make these results applicable to both sides of the spacecraft. Errors seem to be consistently larger near 180° and 270°.

SUMMARY TABLES FOR THE TWO METHODS

Tables 82, 83, 84, 85 and 86 summarize some of the alias removal statistics for the two methods. Each method does what it does quite well, but what is done is different.

Table 82 is for an incidence angle of 29^{o}. The first two rows of numbers are for 4 m/s, one cell, and the objective and MLE method. For these conditions, the objective method selected 126 unique solutions (classes 1, 4, 5, 8) of which 113 (or 90%) were correct. For these same 126 conditions, the MLE selected 117 unique maxima (or 93%) which were correct.

There were 248 class 2 selections. The first choice for the objective method was correct 207 times, and the second choice was correct 33 times. Neither was correct 10 times. The MLE selected 202 correct choices (81%).

The objective method selected 2 wind vectors 120 times without any priority. One or the other was always correct. The MLE method was able to select 105 correct unique winds from the 120 cases.

Class 7 occurred 102 times and one of the three winds in each class 7 case was always correct. The MLE selected 91 correct unique winds from these 102 cases.

Class 3 occurred 604 times, and one of the four winds was always within 45^{o} of the correct wind. The MLE selected 484 unique correct winds from this set. Other parts of these tables show that this does not always occur.

For the 1200 simulations, the objective classifications were correct 1177 times of which 353 specified a nearly unique wind. Two winds were given 121 times, 3 were given 102 times and all four, 604 times. The MLE gave a correct unique wind based on the single greatest maximum in the V-χ plane 999 times.

TABLE 82 SUMMARY OF RESULTS FOR AN INCIDENCE ANGLE OF 29°
(ONE CELL AVERAGES EXCEPT 4* WHICH IS AN AVERAGE OF 5 CELLS)

SPD	TYPE	UNIQUE 1,4,5,8 TOTAL	CORRECT	%	CLASS 2A-2B TOTAL	1ST CORRECT	2ND CORRECT	NEITHER CORRECT	%	CLASSES 6,9 TOTAL	CORRECT	%	CLASS 7 TOTAL	CORRECT	%	CLASS 3 TOTAL	CORRECT	%	TOTAL CLASSIFIED CORRECTLY OR TOTAL MLE.
4	OBJ	126	113	90	248	207	33	10	97	120	120	100	102	102	100	604	604	100	1177
	MLE		117	93		202			81		105	88		91	89		484	80	999
8	OBJ	200	193	97	444	409	35	0	100	67	67	100	52	52	100	437	437	100	1193
	MLE		195	98		407			92		61	91		48	92		396	91	1107
12	OBJ	238	229	96	496	471	25	0	100	46	45	98	2	2	100	418	418	100	1190
	MLE		234	98		474			96		46	100		2	100		381	91	1137
24	OBJ	398	315	79	380	368	10	2	99	54	54	100	27	27	100	341	341	100	1115
	MLE		356	89		370			97		54	100		26	100		318	93	1124
SUMS	OBJ	962	850	88	1568	1455	103	12	99	287	286	99	183	183	100	1800	1800	100	4675
	MLE		902	94		1453			93		266	93		167	81		1579	88	4367
CUM % OF TOTAL		20		17.7 18.7	52.7				50.2 49	58.7		56.1 54.6	62.5		59.9 58.0	100		97 91	
4*	OBJ	281	278	99	368	352	15	1	100	49	49	100	136	136	100	366	366	100	1196
	MLE		279	99		360			98		49	100		135	100		343	94	1166

*5 CELL AVERAGES

The next rows give similar results for 8, 12 and 24 m/s and then sums over all speeds for 1 cell averages. Unique cases increase with increasing wind speed. Class 3 cases decrease with increasing wind speed. The sums are self explanatory.

The row labled "cumulative % of total" shows that the objective method would give about 20% unique wind vectors over a wind field varying from 4 to 24 m/s at a 29° incidence angle. Another 32.7% for a total of 52.7% would be essentially unique. Another 6% would consist of two vectors. One of which might be chosen from other principles. About 37.5% of the vector field would be given by four vectors.

The last row is for 5 cell averages at 4 m/s. Unique and class 2 results increase. Class 3 results decrease. The MLE does much better.

Table 83 is for 39°. Comparisons versus wind speed and with the other tables are what would be expected. The ± 0.7 db additional random error at 8 and 12 m/s more than halves the unique solutions and greatly reduces the class 2 situations. Class 3 soluions are increased about 50%.

Table 84 is for 47°. It provides the opportunity to compare 5 cell averages with 1 cell averages for 4, 8 and 12 m/s. About 85% are correctly classified for 1 cell data and 92% for 5 cell data by the objective methods. The MLE gets 82% correct and 93% correct respectively.

Table 85 is for 53.5° for 5 cell and 25 cell averages and can be interpreted in the same way.

The final table in this section (Table 86) shows the total number of 1st choice MLE decisions that were correct for the conditions of the numerical experiment.

From these tables in terms of the objective method, there are about 23 unique solutions out of each 100, two or three of which are wrong. There are another 27 class 2 solutions, three of which

TABLE 83 SUMMARY OF RESULTS FOR AN INCIDENCE ANGLE OF 39° (ONE CELL AVRRAGES EXCEPT 8* HAS 0.7 DB ADDED RANDOM ERROR, 12* HAS 0.7 DB ADDED RANDOM ERROR AND 4* IS A 5 CELL AVERAGE)

		UNIQUE 1,4,5,8			CLASS 2A-2B					CLASS 6,9			CLASS 7			CLASS 3			TOTAL CLASSIFIED CORRECT OR
						1ST	2ND	NEITHER											
SPD	TYPE	TOTAL	CORRECT	%	TOTAL	CORRECT	CORRECT	CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL MLE
4	OBJ	99	68	69	109	63	40	6	94	140	127	91	247	225	91	605	553	91	1076
	MLE		83	84		75			69		118	84		199	81		433	72	908
8	OBJ	218	206	94	275	240	34	1	100	58	58	100	203	203	100	446	446	100	1187
	MLE		216	99		251			91		58	100		186	92		392	88	1103
12	OBJ	309	305	99	385	365	20	0	100	43	43	100	98	98	100	365	100	100	1195
	MLE		306	99		372			97		43	100		93	95		341	93	1155
24	OBJ	443	358	81	452	444	4	4	99	53	53	100	18	15	83	234	234	100	1108
	MLE		397	90		451			100		53	100		16	89		228	97	1145
SUMS	OBJ	1069	937	88	1221	1112	98	11	99	294	281	96	566	541	96	1650	1598	97	4567
	MLE		992	93		1149			94		272	93		494	87		1394	84	4311
CUM %		22.2		19.5	47.7				44.7	53.8		50.6	65.6		61.8	100		95.1	
OF TOTAL				20.7					44.6			50.2			60.5	100		89.9	
8*	OBJ	98	83	85	225	158	59	8	96	96	96	100	172	172	100	609	609	100	1177
	MLE		89	91		166			74		84	88		138	80		437	72	914
12*	OBJ	127	104	82	256	205	43	8	97	91	90	99	90	90	100	638	636	100	1125
	MLE		77	61		215			84		65	71		75	83		463	73	925
SUMS	OBJ	225	187	83	481	363	102		97	187	186	100	262	262	100	1247	1247	100	2302
	MLE		166	74		381			79		149	80		213	81		900	72	1839
4*	OBJ	201	185	92	149	113	36		100	62	61	98	263	263	100	525			
	MLE		195	97		136			91		62	100		239	91		473		

TABLE 84 SUMMARY OF RESULTS FOR AN INCIDENCE ANGLE OF 47° (ONE CELL AVERAGES, *FOR 5 CELL AVERAGES, **FOR 25 CELL AVERAGES)

		UNIQUE 1,4,5,8			CLASS 2A-2B					CLASSES 6,9			CLASS 7			CLASS 3			TOTAL CLASSIFIED CORRECT OR
SPD	TYPE	TOTAL	CORRECT	%	TOTAL	1ST CORRECT	2ND CORRECT	NEITHER CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL MLE
4	OBJ	46	27	59	38	18	16	4	89	287	130	45	238	128	54	591	304	51	623
	MLE		37	80		21			55		198	69		159	67		320	54	735
8	OBJ	105	82	78	147	116	31	0	100	75	73	97	247	237	96	626	606	97	1145
	MLE		94	89		115			78		67	89		199	81		492	79	967
12	OBJ	213	200	94	313	281	31	1	100	73	70	96	170	170	100	431	431	100	1183
	MLE		201	94		288			92		68	93		149	88		376	87	1082
24	OBJ	385	329	85	505	497	5	3	99	73	72	99	39	16	41	198	198	100	1117
	MLE		355	92		498			99		70	96		26	67		192	97	1141
SUMS	OBJ	749	638	85	1003	912	83	8	99	508	345	68	694	551	79	1846	1539	83	4068
	MLE		687	92		922			92		403	79		533	77		1380	75	3929
CUM %		15.6		13.3	36.5				34.0	47.1		41.2	61.5		52.6	100		84.8	
OF TOTAL				14.3					33.5			41.9			53.0			81.8	
4*	OBJ	106	72	68	69	38	28	2	96	131	98	75	272	207	76	622	492	79	935
	MLE		91	86		46			67		114	87		207	76		442	71	900
8*	OBJ	306	293	96	193	168	25	0	100	49	49	100	220	220	100	432	432	100	1187
	MLE		301	98		186			96		49	100		213	97		405	94	1154
12*	OBJ	526	524	100	296	292	4	0	100	47	47	100	170	170	100	161	161	100	1198
	MLE		525	100		296			100		47	100		170	100		159	99	1197
SUMS	OBJ	938	889	95	558	498	57	2	99	227	194	85	662	597	90	1215	1085	89	3320
	MLE		917	98		528			95		210	93		590	89		1006	83	3251
CUM %		26.0	24.7	24.7	41.6				41.4	47.9		45.5	66.3		62.1	100		92.2	
TOTAL				25.5					40.1			45.9			62.4			90.3	
4**	OBJ	203	190	94	100	54	46	0	100	82	76	93		209	94	593	571	96	1146
	MLE		192	95		77			77		81	93	222	203	91		495	93	1050

TABLE 85 SUMMARY OF RESULTS FOR AN INCIDENCE ANGLE OF 53.5°
5 CELL AVERAGES FIRST FOLLOWED BY 25 CELL AVERAGES*

	UNIQUE 1,4,5,8			CLASS 2A-2B					CLASSES 6, 9			CLASS 7			CLASS 3			TOTAL CLASSIFIED CORRECT OR
SPD TYPE	TOTAL	CORRECT	%	TOTAL	1 ST CORRECT	2 ND CORRECT	NEITHER CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL	CORRECT	%	TOTAL MLE
4 OBJ	29	14	48	41	21	19	1	97	272	105	39	261	113	43	597	250	42	522
MLE		21	72		27			66		198	73		165	63		363	61	774
8 OBJ	95	77	81	130	102	28	0	100	70	66	94	259	240	93	646	593	92	1106
MLE		85	89		111			85		58	83		219	85		526	81	999
12 OBJ	268	262	98	314	292	22	0	100	66	64	97	164	164	100	388	388	100	1192
MLE		263	98		304			97		64	97		155	94		364	94	1150
24 OBJ	462	446	97	484	484	0	0	100	68	68	100	87	2	1	99	99	100	1099
MLE		454	98		484			100		68	100		28	32		99	100	1133
SUMS OBJ	854	799	94	969	899	69	1	100	476	303	64	771	519	67	1730	1330	77	3919
MLE		823	96		926			96		388	82		567	74		1352	78	4056
COM %	17.8		16.6	38.0				36.8	47.9		41.7	64.0		52.5	100		81.6	
OF TOTAL			17.1					36.4			44.5			56.3			84.5	
4* OBJ	75	48	64	58	30	26	2	97	134	75	56	275	161	59	658	448	68	788
MLE		73	97		43			74		117	87		220	80		528	80	981
8* OBJ	271	263	97	149	122	27	0	100	69	67	97	213	213	100	498	497	100	1189
MLE		270	100		138			93		69	100		208	98		461	93	1146
12* OBJ	557	557	100	310	306	4	0	100	70	70	100	142	142	100	121	121	100	1200
MLE		557	100		309			100		70	100		142	100		121	100	1199
24* OBJ	717	710	99	251	251	0	0	100	56	56	100	107	0	0	69	69	100	1086
MLE		717	100		251			100		56	100		105	98		69	100	1199
SUMS OBJ	1620	1578	97	768	709	57		100	329	268	81	737	516	70	1346	1135	84	4263
MLE		1617	100		741			96		312	84		675	92		1179	88	4524
CUM %	32.8		32.9	49.8				48.8	56.6		54.4	71.9		67.3	100		88.8	
OF TOTAL			33.6					49.1			55.6			69.7	100		94.3	

TABLE 86. SUMMARY OF CORRECT MAXIMUM LIKELIHOOD SOLUTIONS† (FIRST VALUE IS TOTAL CORRECT OUT OF 1200, SECOND VALUE IS PERCENT) AND OF CORRECT UNIQUE SOLUTIONS FOR THE OBJECTIVE METHOD (FIRST VALUE IS TOTAL CORRECT OUT OF 1200, SECOND VALUE IS PERCENT). AN UNDERLINED VALUE OF X SUGGESTS THAT IF THESE CONDITIONS HAD BEEN MONTE CARLOED, THE NUMBER CORRECT WOULD HAVE EXCEEDED 90%.

WIND SPEED		4			8			12			24		
NUMBER OF CELLS		1	5	25	1	5	25	1	5	25	1	5	25
INCIDENCE	ANGLE												
29^{o}	MLE	999	1166	x	1107	x	x	1137	x	x	1124	x	x
	%	83.3	97.2	x	92.3	x	x	94.8	x	x	93.7	x	x
	OBJ	113	278	x	193	x	x	229	x	x	315	x	x
	%	9.4	23.2	x	16.1	x	x	19.1	x	x	26.3	x	x
39^{o}	MLE	908	1105	x	1103	x	x	1155	x	x	1145	x	x
	%	75.7	92.1	x	91.9	x	x	96.3	x	x	95.4	x	x
	OBJ	68	185	x	206	x	x	305	x	x	358	x	x
	%	5.7	15.4	x	17.2	x	x	25.4	x	x	29.8	x	x
47^{o}	MLE	735	900	1050	967	1154	x	1082	1197	x	1141	x	x
	%	61.3	75.0	87.5	80.6	96.2	x	90.2	99.8	x	95.1	x	x
	OBJ	27	72	190	82	293	x	200	524	x	329	x	x
	%	2.3	6.0	15.8	6.8	24.4	x	16.7	43.7	x	27.4	x	x
53^{o}	MLE	x	774	981	x	999	1146	x	1150	1199	x	1133	1198
	%	x	64.5	81.8	x	83.3	95.5	x	95.8	99.9	x	94.4	99.9
	OBJ	x	14	48	x	77	270	x	262	557	x	446	710
	%	x	1.2	4.0	x	6.4	22.5	x	21.8	46.4	x	37.2	59.2
39^{o}*	MLE	x	x	x	914	x	x	925	x	x	x	x	x
	%	x	x	x	76.2	x	x	77	x	x	x	x	x
	OBJ	x	x	x	83	x	x	104	x	x	x	x	x
	%	x	x	x	6.9	x	x	8.7	x	x	x	x	x

†For 8 m/s, 1 cell, with χ varied in one degree steps relative to beam 1 from zero to 47^{o}, 2096 correct MLE choices out of 2400, or 87.3%.

*With ± 0.7 db added error.

wrong, and a number of which requires the use of the second choice. There are a scattering of 3 vector solutions and the rest (crudely) are 4 vector solutions. (Two of these could probably we flagged as more probable than the other two).

Figure 20 shows schematically how the objective results might be plotted and analysed. It is not realistic in the sense that the density of the vectors would be far greater around an actual extra-tropical cyclone.

The unique solutions as shown are single heavy wind barbs with a single flag (there is no attempt to show speed). They are in proportion to the percentage that would actually occur. The ones that are incorrect can be easily spotted and crossed out. The class 2 solutions are shown as a heavy wind barb for the first choice and a dashed barb for the second choice. The ones that should be reversed can be easily spotted. The incorrect ones can also easily be crossed out.

The field shows no class 9 cases, but actually there ought to be a few. The remaining class 7 and class 3 sets of 3 and 4 vectors complete the field. The information from the unique vectors and the class 2 vectors allows correct choices to be made for all of the class 7 and class 3 cases by continuity and inspection.

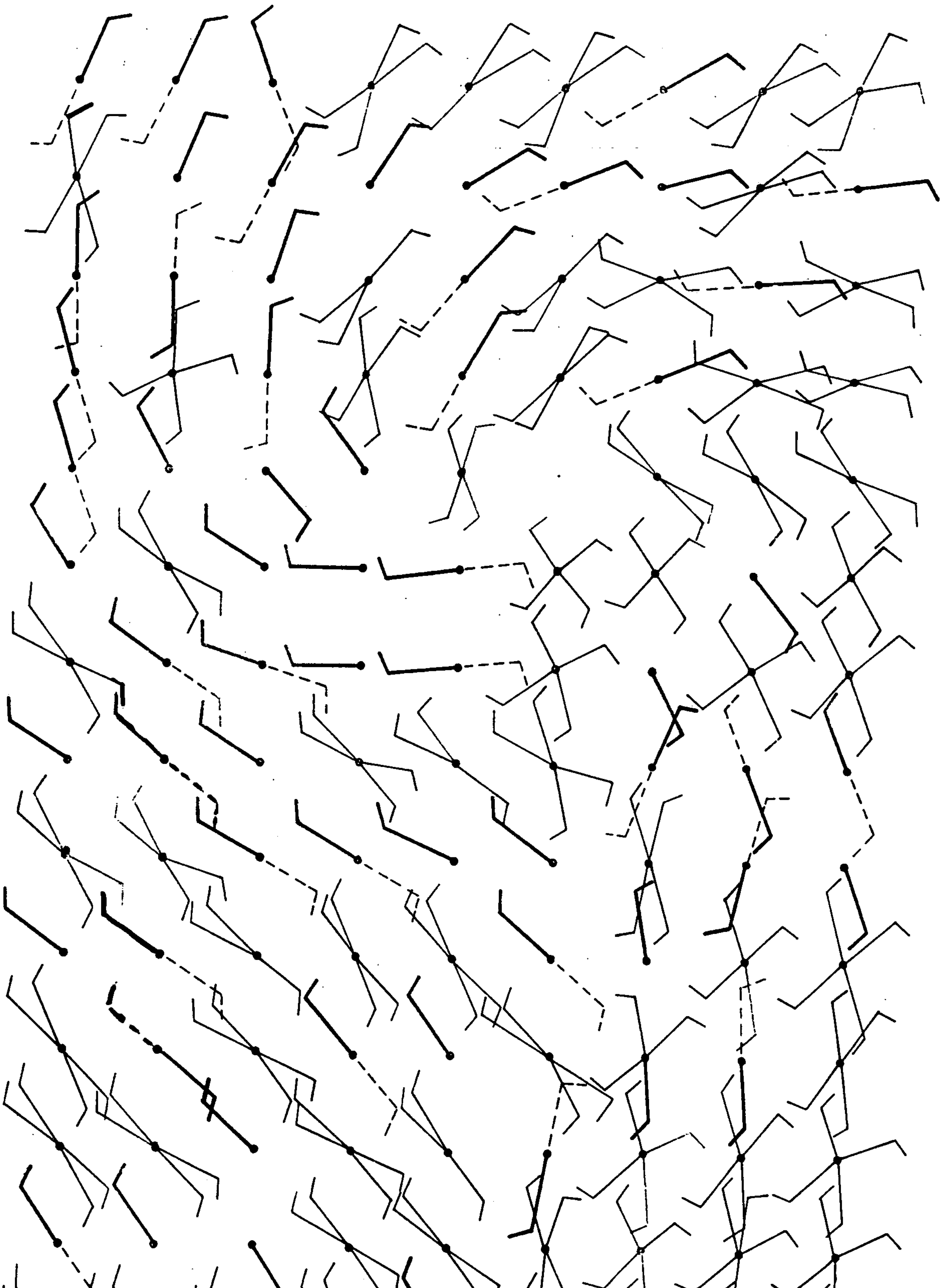


FIG. 20 EXAMPLE OF HOW THE OBJECTIVE TECHNIQUE COULD BE USED TO DETERMINE A UNIQUE WIND FIELD.

COMPARISONS WITH SASS RESULTS FROM SEASAT

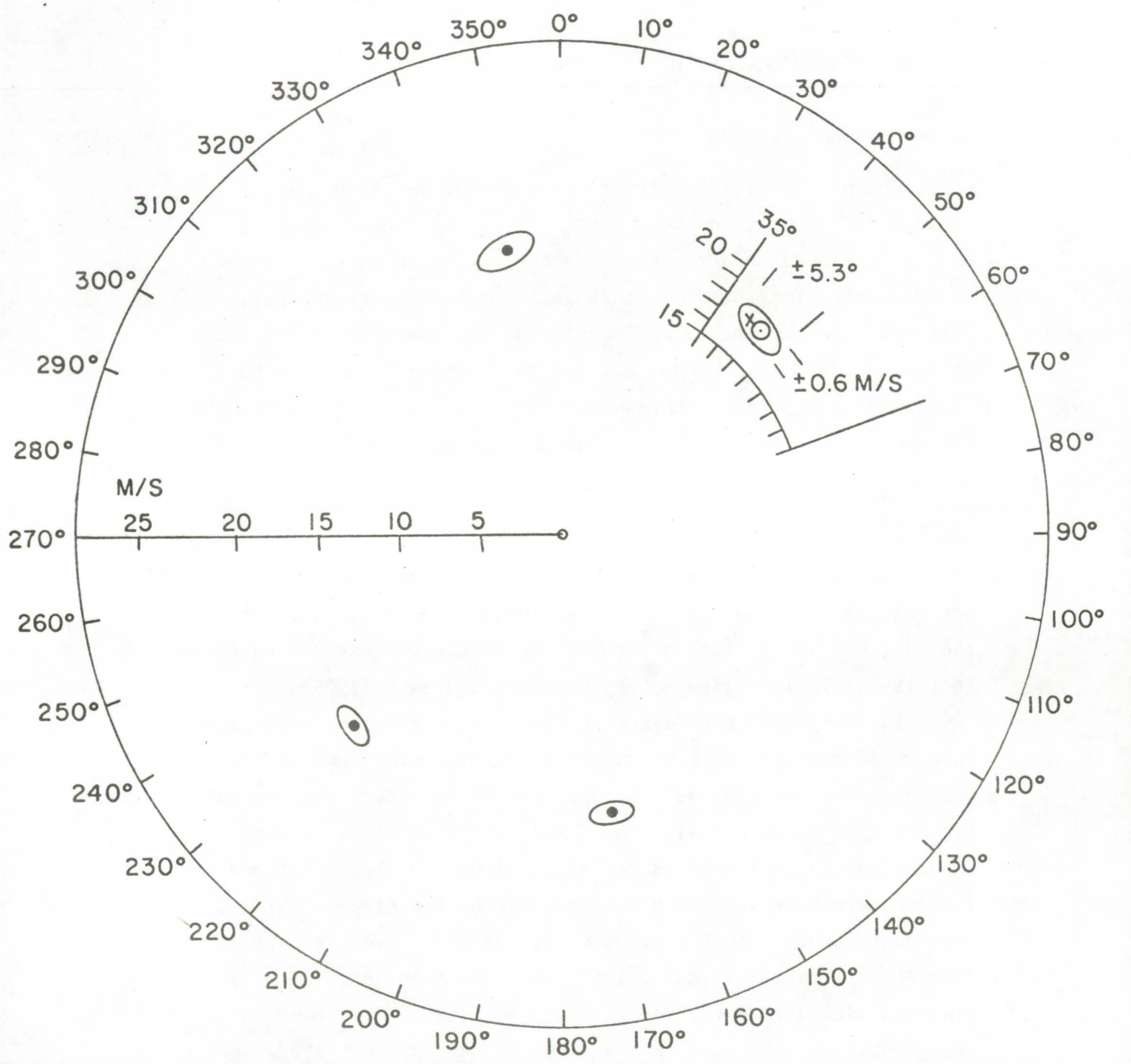
Introduction

During the starting stages of the SASS on SEASAT, similar Monte Carlo simulations were carried out, and predictions were made as to the accuracy with which the vector winds would be measured by that instrument. It is possible to compare these predictions with what actually happened on the basis of the results of the two GOASEX workshops and the forthcoming results of the JASIN workshop. The techniques were not exactly the same for the two comparisons, but there are enough similarities to provide some information.

Early Studies

One early study of this subject, completed in April 1978, was that of Pierson (1978). It was part of a final report to the Jet Propulsion Lab and to the Naval Environmental Prediction Facility jointly caried out by Pierson and Salfi (1978).

In this study the region in the V-χ polar coordinate plane that would contain the true vector wind nine times out of ten was calculated on the basis of appropriate theories, and several figures from these calculations follow. Figure 21 is for an input wind of 17.1 meters per second from 41°. The solutions that resulted are shown by the black dots and by the circled white dot. The Monte Carlo result that came out was 17.4 meters per second from 45°. A 90% confidence interval is much larger than plus or minus one standard deviation in speed and direction. Therefore, the quantities that have been derived in this present study need to be approximately doubled in order to construct the 90% confidence region. Stated another way, the very tiny ellipses around the four solutions for the case under discussion would be half of their radii for a corresponding variability in terms of standard deviations. At an incidence angle of 44°, the simulations, therefore, suggest that for winds near 17 meters per second, the errors would be extremely small in both speed and direction.



TRUE : 17.1 M/S FROM 41°
UNBIASED ESTIMATE : 17.4 M/S FROM 45°

FIGURE 21 EXAMPLE OF A FOUR SOLUTION CASE. THE 90% CONFIDENCE REGION IS ±0.6 M/S AND ±5.3° ABOUT THE ESTIMATE. $\bar{\theta}$ = 44°, VERTICAL POLARIZATION.

Figure 22 illustrates an input of 10.4 meters per second from 227°. The Monte Carlo simulation produced 10.3 meters per second from 229°. The error ellipses that would enclose the true value 90% of the time, if something near the correct direction were known, should again be roughly halved in both dimensions for the results to correspond to the analysis in this study.

The same holds true for lighter winds and other incidence angles as shown by the next few figures. The error ellipses change form and become larger, but, nevertheless, there is every indication that the error structure of the SASS was such that the errors in both wind speed and wind direction would be fairly small. Even the tendency for the direction error to spread out over a large range of angles does not show any evidence of a tilt in these curves that would suggest that an error in direction of one sense would result in a decreasing wind speed and an error in direction in the opposite sense would result in an increasing wind speed.

A Monte Carlo simulation involving a total of 390 values was carried out during the SEASAT program in order to try to separate the effects of synoptic scale gradients in the wind field from noise and communication errors. The SEASAT SASS cells did not fall exactly one on top of the other. The winds at the centers of the cells differed slightly in both speed and direction. These effects were used along with the Monte Carlo errors for communication and attitude and the results are tabulated in Table 87, as taken from Pierson (1978). The pertinent rows in these data are those for noise alone. The effect of gradients on both the SASS and the SCATT will be discussed later.

It is clear from these tables that the predicted errors for the SASS are much smaller than the predicted errors in direction based on the preceeding material for a single cell with an effective area of 100 square kilometers for the SCATT. Even averages of 5 cells at times do not approach the statistical results that were obtained

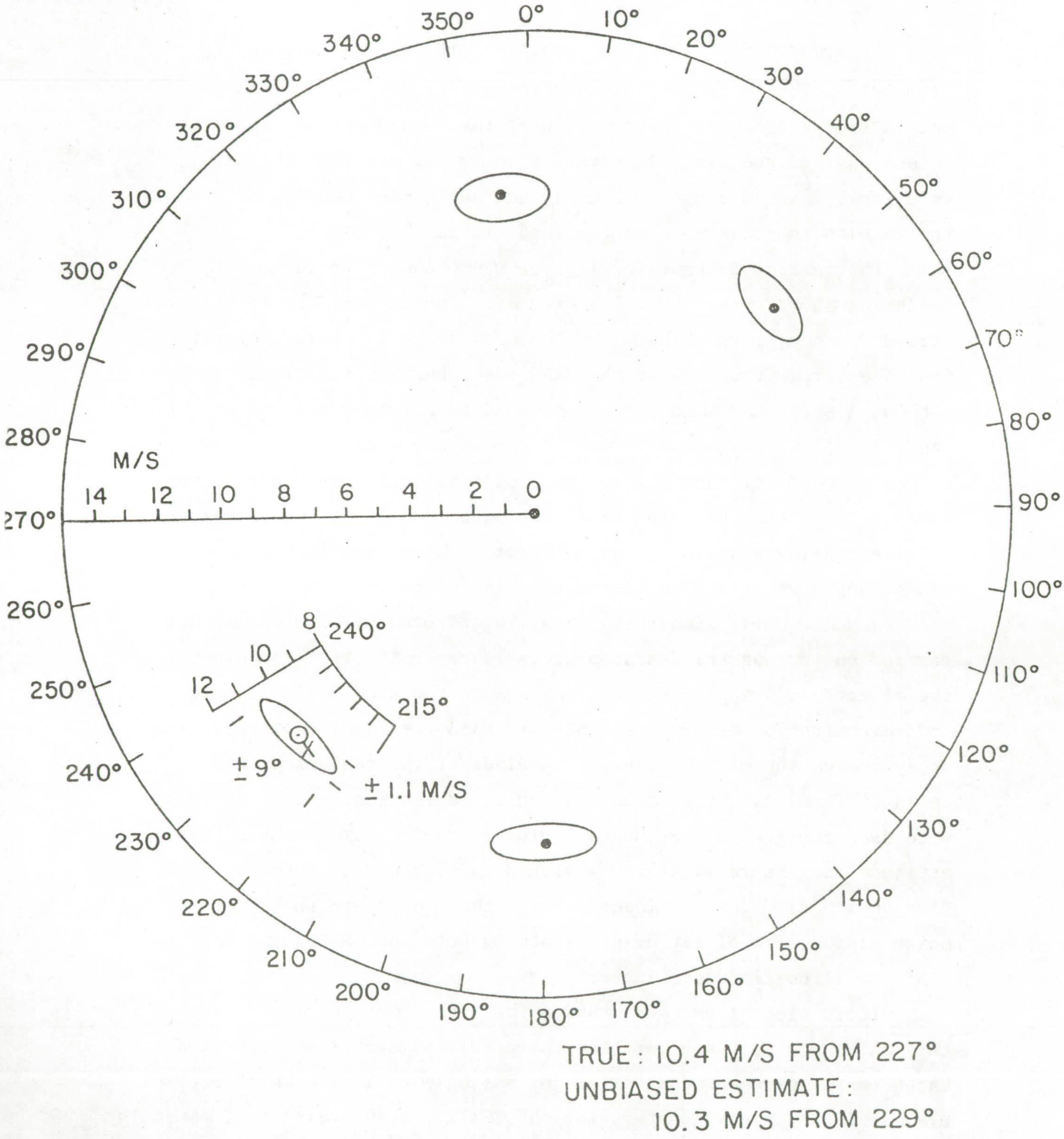


FIGURE 22 EXAMPLE OF THE FOUR SOLUTION CASE. THE 90% FIDUCIAL CONFIDENCE REGION IS SHOWN. $\bar{\theta} = 40°$, VERTICAL POLARIZATION

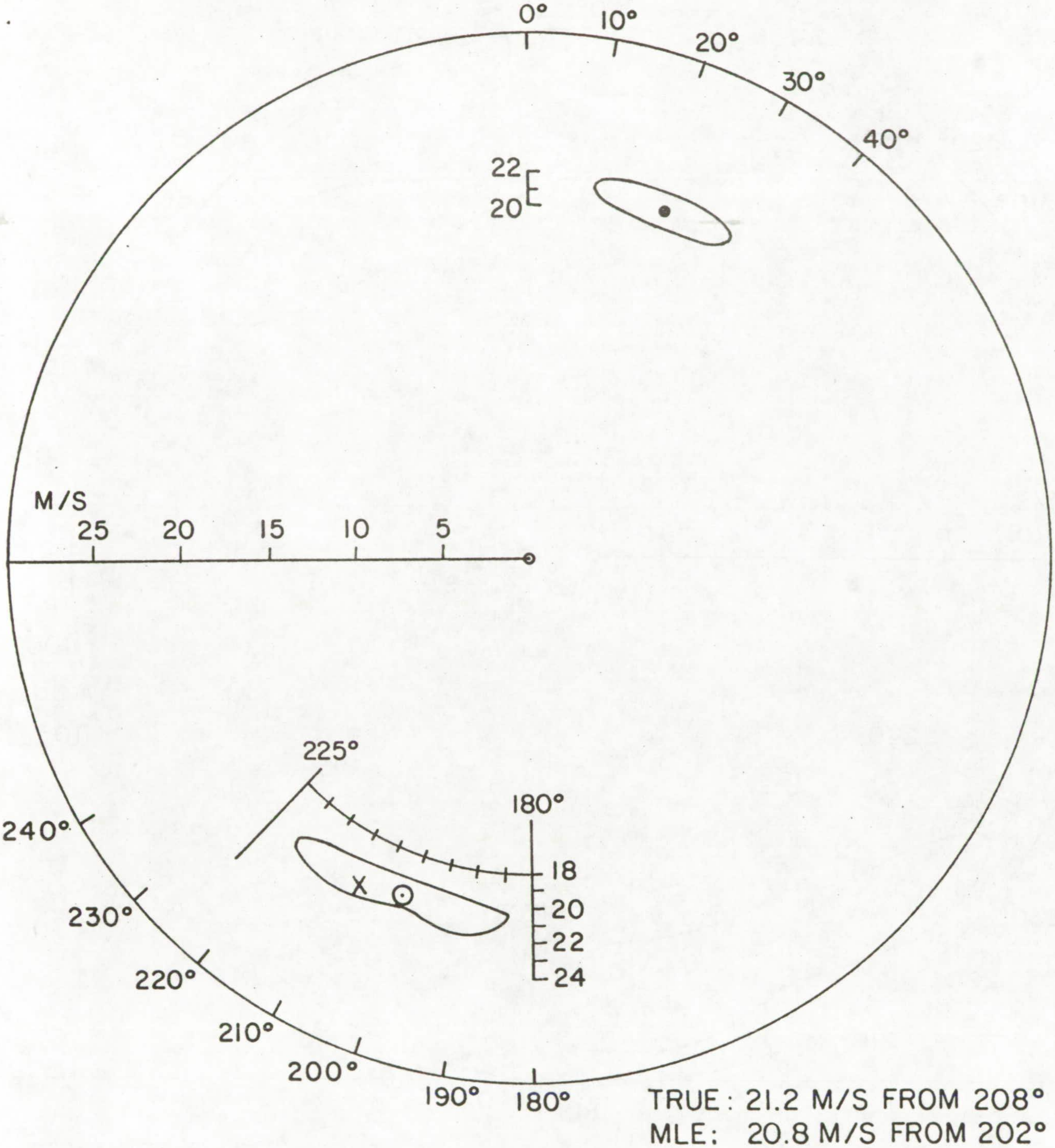
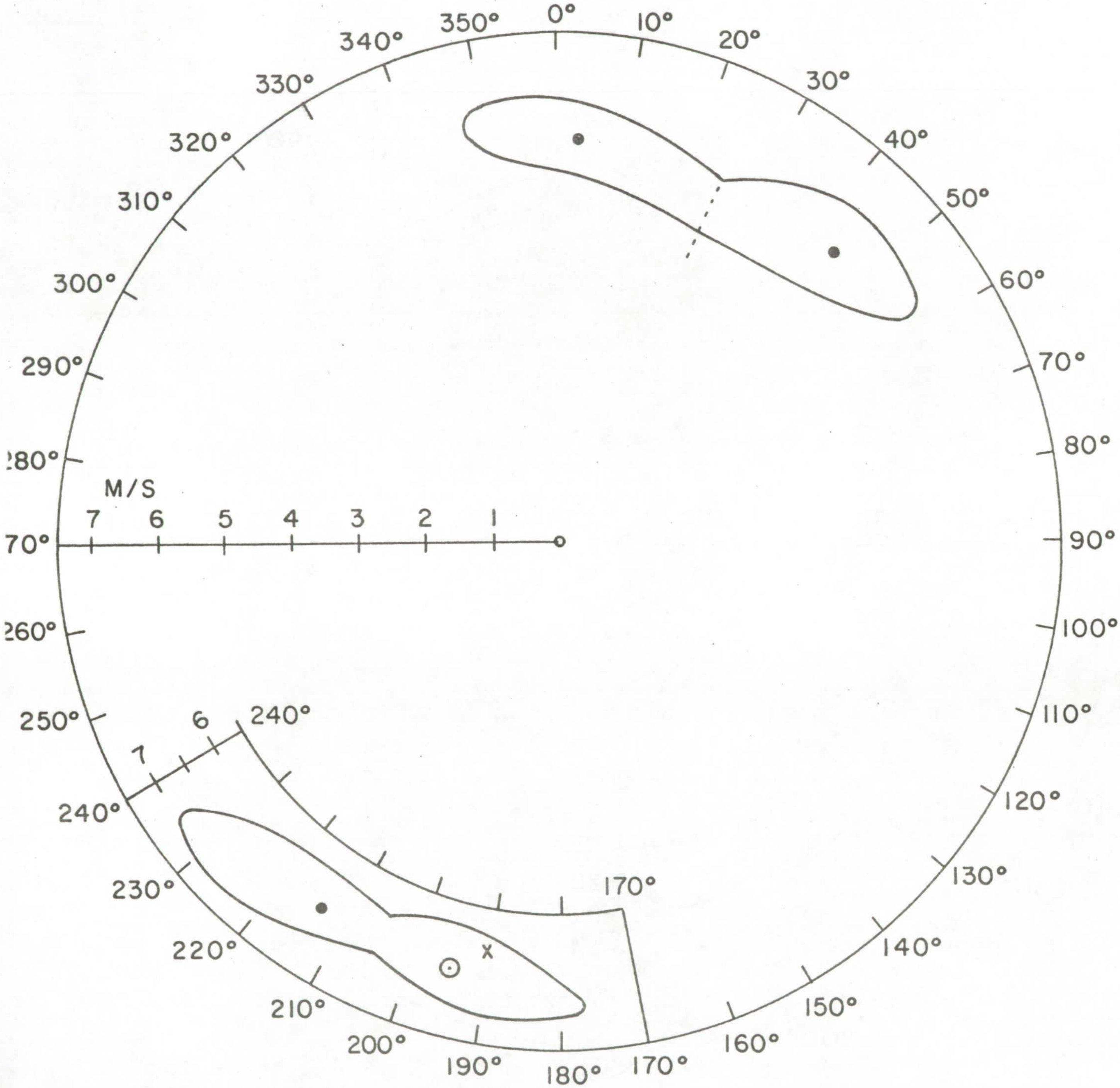
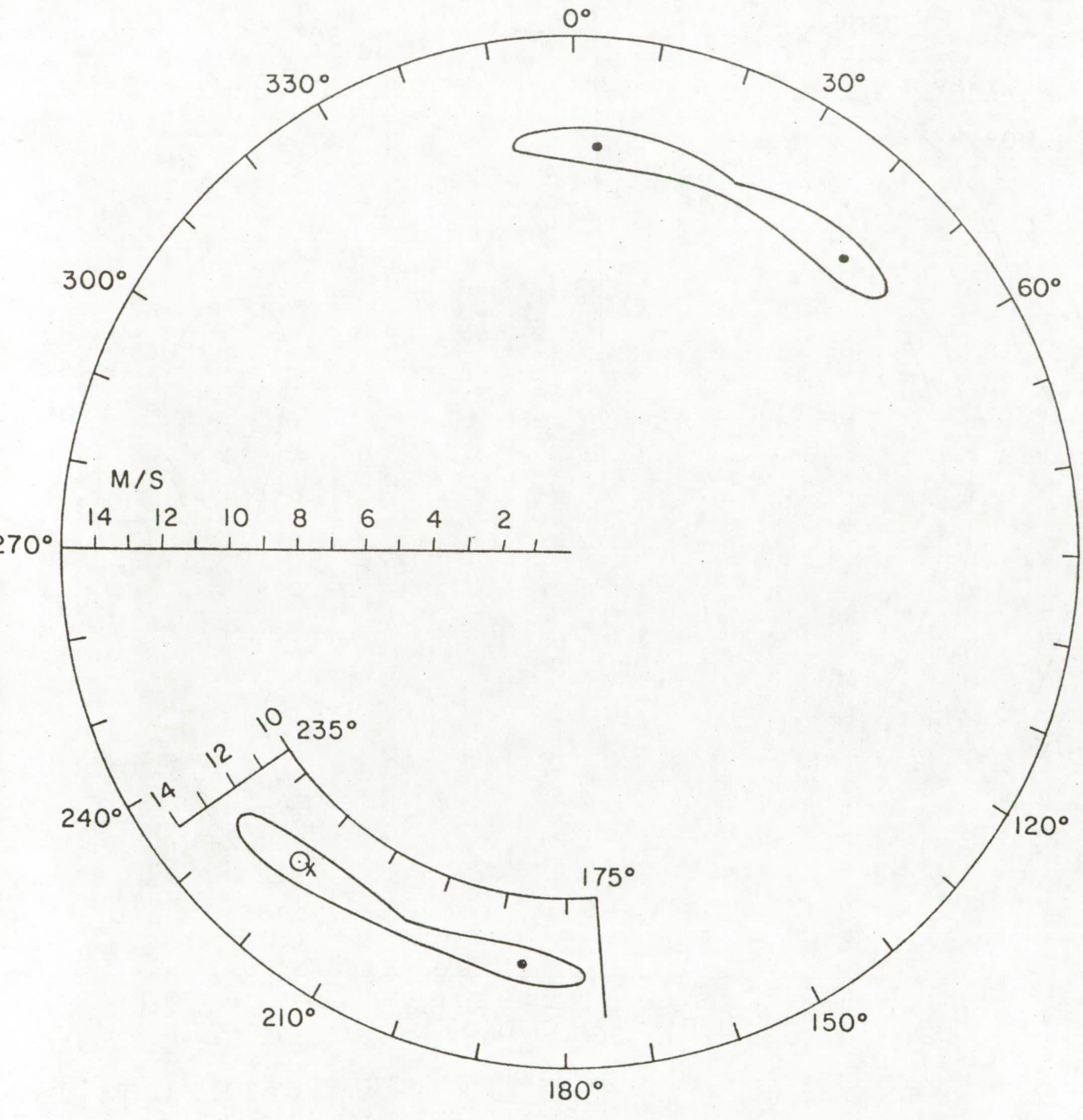


FIGURE 23 EXAMPLE OF A TWO SOLUTION CASE WITH MLE'S AT BOTH UPWIND AND DOWNWIND WITH A NARROW RANGE OF DIRECTIONS AND SPEEDS FOR THE 90% CONFIDENCE REGION. $\bar{\theta} = 52°$, VERTICAL POLARIZATION.



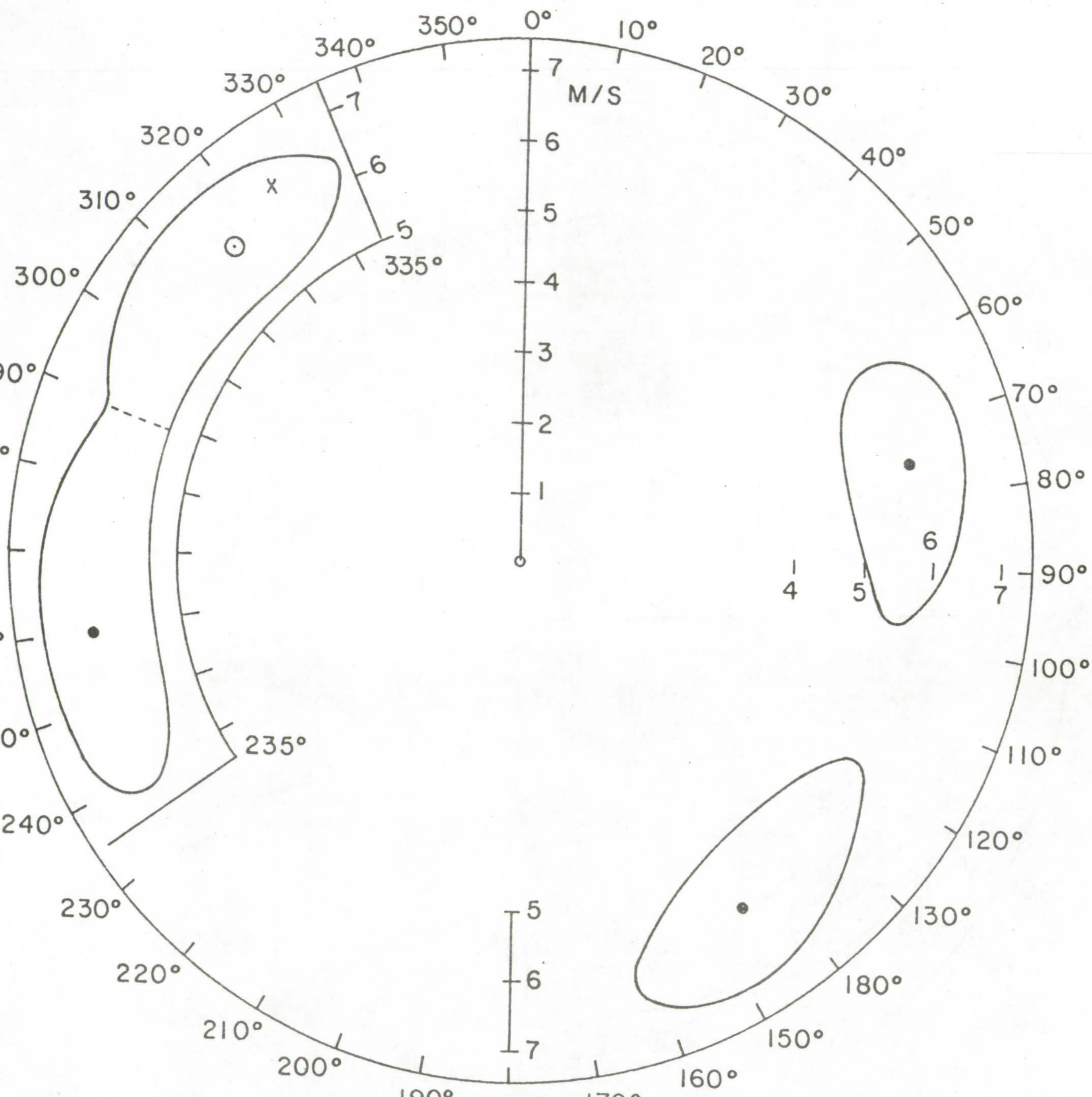
TRUE: 6.2 M/S FROM 191°
UNBIASED ESTIMATE: 6.5 M/S FROM 196°

FIGURE 24 EXAMPLE OF A FOUR SOLUTION CASE FOR WHICH THERE ARE INADMISSABLE POINTS NEAR BOTH UPWIND AND DOWNWIND FOR THE 90% CONFIDENCE REGION.



TRUE: 12.0 M/S FROM 220°
UNBIASED ESTIMATE: 12.0 M/S FROM 221°

FIGURE 25 EXAMPLE OF A FOUR SOLUTION CASE FOR WHICH THERE ARE INADMISSABLE POINTS NEAR BOTH UPWIND AND DOWNWIND FOR THE CONFIDENCE REGION. $\bar{\theta} = 45°$, VERTICAL POLARIZATION.



TRUE: 6.5 M/S FROM 325°
UNBIASED ESTIMATE: 6.2 M/S FROM 316°

FIGURE 26 EXAMPLE OF A FOUR SOLUTION CASE FOR WHICH THE 90% CONFIDENCE REGION CONTAINS INADMISSABLE POINTS FOR UPWIND BUT NOT DOWNWIND. $\bar{\theta} = 30°$, VERTICAL POLARIZATION.

TABLE 87 RESULTS OF A MONTE CARLO SIMULATICN OF 380 VECTOR WIND RECOVERIES WITH THE SASS ON SEASAT-A BY MEANS OF AN EARLIER VERSION OF THE RESULTS DESCRIBED IN THIS PAPER. THE EFFECTS OF GRADIENTS IN THE WIND FIELD ARE ALSO SHOWN. THE WIND SPEED IS IN METERS PER SECOND OR IN PERCENTAGE ERROR. G represents the effects of gradients in the wind field alone; NA represents the effects of noise alone (as computed from G and N+G); and N+G represents the combined effects of gradients and random errors (or noise).

		ALL θ		$20^\circ - 30^\circ$		$30^\circ - 40^\circ$		$40^\circ - 50^\circ$		$50^\circ - 60^\circ$	
		RMS(χ)	RMS(V)	RMS(χ)	RMS(V)	RMS(χ)	RMS(V)	RMS(χ)	RMS(V)	RMS(χ)	RMS(V)
		138		18		43		47		30	
4-10	G	6.1°	0.3	7.3°	0.3	7.9°	0.2	11.1°	0.3	5.5°	0.4
	NA	7.0°	0.9	9.3°	0.6	7.8°	0.2	3.0°	0.4	8.2°	1.8
	N+G	9.3°	1.0	11.8°	0.7	11.1°	0.3	11.5°	0.5	9.9°	1.8
		195		17		35		71		72	
10-20	G	5.4°	0.5	18.8°	0.5	17.1°	0.3	3.4°	0.2	3.6°	0.7
	NA	3.6°	0.6	x	0.7	x	0.6	6.8°	0.3	4.4°	0.7
	N+G	6.5°	0.8	12.8°	0.9	16.8°	0.7	7.6°	0.4	5.7°	1.0
		57		17		27		13		0	
20+	G	14.6°	2%	20.5°	2.5%	22.3°	2.0%	3.2°	1.0%	—	
	NA	4.2°	6.1%	7.7°	9.0%	4.2°	5.2%	x	2.2%	—	
	N+G	15.2°	6.4%	21.9°	9.3%	22.7°	5.6%	3.1°	2.4%	—	

in this simulation of the SASS. When the additional compounding effect of elementry gradients is added to the SASS, the results get very large. Elementary wind gradients will not be as big a problem with the SCATT because of the closer location of the cells that are grouped in groups of six for each wind vector determination.

The main point of this table is that all of the RMS errors in wind direction over the entire range of incidence angles for the spacecraft were predicted to be under 10^{o} and many of them were predicted to be under 5^{o}. The RMS errors in wind speed varied from a maximum of 0.9 meters per second to values such as 0.2 meters per second, or a few percent of the wind when the wind exceeded 20 meters per second.

The comparison of the results in this table, which are typical for the Monte Carlo simulations of the SASS, show that the predicted direction errors for the SCATT on the NOSS solely due to attitude and communication errors will far exceed the direction errors of the SASS.

Compounded with the correlations between the speed and direction errors the SCATT winds for single 10 km by 10 km cells will be extremely difficult to interpret. The 5 cell data, summarized for worst case conditions in Table 81, and the 25 cell data also in Table 81, shows that at some incidence angles even pooled cells have large errors.

The archives of the SASS team meetings also contain minutes where other Monte Carlo simulations were compared for a variety of model functions, although there were differences from one simulation to another and spirited discussions about the applicability of the particular model functions that were used, the overall result was that the wind direction errors would be small and that the wind speed errors would also be quite small.

The Results of Actual Experiments

When actual meteorological data, as measured with an anemometer, were compared with the winds obtained by the SASS on SEASAT, the RMS direction differences and the RMS wind speed differences were much larger. After numerous iterations of the problem, the results of the JASIN workshop yielded what were probably the best results in such comparisons. These results in an early form, which may need correction because of improvements in the JASIN data, indicate that the RMS difference between the wind speeds measured by one instrument and the wind speeds measured by the other instrument was about 1.3 meters per second over a range from 4 meters per second to about 16 meters per second. The RMS direction differences were slightly over 20 degrees on the whole, but there was a large range of incidence angles in the middle of the swath for wind speeds above 8 meters per second, or so, for which the RMS direction differences were under 10 degrees. It is difficult to see on the basis of the figures and data in this present study how the SCATT can even come close to wind direction errors for 10 km resolution as small as those obtained by the SASS on SEASAT. The simulated averages by rows of five over a 50 km by 10 km rectangle or by groups of 25 for a 50 by 50 kilometer square produce acceptable results, using the criterion that the direction error is under 10°, for 5 cell averages for all winds tested at incidence angles of 29° and 35°, for winds above 8 m/s for 47°, 39° and 29°, and for high winds at all incidence angles. Twenty-five cell averages yield acceptable results for all conditions except 4 m/s at a 53.5° incidence angle.

MESOSCALE EDDIES, GRADIENTS AND CELL SIZE

Introduction

The basic problem in deciding upon the resolution of any space borne instrument is whether or not the resolution used will yield the information desired at that resolution. For the SCATT on the NOSS, this reduces to the problem of understanding mesoscale eddies in the winds versus gradients in the wind field at the synoptic scale, and how these two variables respond to variations in the size of the cells that obtain the basic radar backscatter measurements. Attempts to define these mesoscale eddies have been made in one of the GOASEX workshops, and some new results have recently been obtained Pierson (1981). However, even the most elementry inspection of an anemometer record as obtained by a ship at sea, such as those shown in Pierson, Peteherych and Wilkerson (1980), illustrates that the minute-by-minute variations of the wind over a given point on the ocean can be large. These variations are not those associated with cumulus clouds in the trade winds. They are the natural variability in the large scale synoptic flow for the winds around an extra-tropical cyclone. The anemometer record is a manifestation of a very complicated three-dimensional variability in the winds over the ocean. This three-dimensional variability has a vertical scale that extends upward to at least one kilometer, and the lateral scales are many times greater than one kilometer extending throughout the lower planetary boundary layer. The wavelengths involved in the horizontal scale are probably anywhere from a few kilometers to several hundred kilometers, so as to describe the variation of the two components of the horizontal wind.

The mesoscale fluctuations in wind speed can be 10 to 20 percent of the average wind speed as averaged over a half hour or more. The fluctuations in direction can be 5 degrees to 10 degrees from one side to the other of the average wind direction. Such fluctuations in speed and direction as a first approximation can be thought to be advected over the sea surface at the speed of the average wind.

Table 88 shows the time required in minutes for an eddy advected by the mean wind to travel 10 kilometers, 30 kilometers, or 70 kilometers.* For high wind speeds, the 10 kilometer cell of the SCATT is thus the equivalent of a 7 or 14 minute average of the wind. It may even be somewhat less than this for other reasons. For the SASS on SEASAT, depending upon whether the long or the short side of the cell is used the averaging times were 20 minutes to three hours. A seven minute average of the wind when the average wind is twenty-four meters per second could easily be 3 or 4 meters per second stronger or weaker than the average value, and a 14 minute average of the wind for a synoptic scale wind of 12 m/s could easily differ by one or two m/s from a longer average of an hour or so.

Another way to look at this problem would be to take a very long anemometer record, break it up into time pieces with the durations indicated by this table and assign successive pieces in groups of five to a row of simulated SCATT cells on the NOSS. Quite clearly each piece would yield a different average wind speed and a different direction. The communication noise errors would then perturb each of these winds by the amounts shown in these simulations. The Monte Carlo 5 cell average would then attempt to reproduce whatever mean wind speed and direction represent the entire sample.

*The 30 kilometer distance is a compromise between the narrow width of a SASS cell and the long dimension of a SASS cell. The 70 kilometer distance is the long dimension of a SASS cell on the diagonal dimension of a 50 km SCATT cell.

TABLE 88 TIME REQUIRED BY AN EDDY ADVECTED BY THE MEAN WIND SPEED TO TRAVEL THE DISTANCE SHOWN

DISTANCE $\bar{V}$ (m/s)	10 KM	30 KM	70 KM
4	42 m	2 h 6 m	4 h 54 m
8	21 m	1 h 3 m	2 h 27 m
12	14 m	42 m	3 h 38 m
24	7 m	21 m	49 m

Even if these considerations are a gross over simplification of the actual problem because the turbulence in the horizontal is two dimensional, and this aspect needs to be considered, a 10 kilometer SCATT cell effectively covers within 20 or 30%, about 100 squre kilometers, whereas the SASS cell for the SEASAT covered 2100 square kilometers. The area was thus 21 times greater. This larger area both smoothed out the mesoscale turbulence effects in the signal and allowed a longer integration (averaging) time for a larger area which yielded a stronger return signal. The net result was the greatly reduced scatter indicated by the Monte Carlo simulations of the SASS.

The question that must, therefore, be asked is whether or not there is any information on the individual vector winds that might be recovered for the 10 kilometer by 10 kilometer cells of the present design of the SCATT. First of all, the winds for each of these cells will differ by 10 or 20% in speed from the wind that would be computed from a larger area or from a longer time average. Secondly, the directions will fluctuate by 5^{o} or 10^{o} compared to the wind direction that would be gotten from a longer time average, in our opinion, based on our present understanding of the problem.

The six measurements that would be made by the SCATT on a spacecraft for one of these cells would then be perturbed about their true synoptic values for this particular wind speed and direction This speed and direction does not equal any of the other wind speeds and directions in the row of five cells under consideration or in the square of 25 cells under consideration. The additional perturbation due to communication noise and attitude error, would yield a vector wind which even if it could be properly selected would differ from the wind that was actually there by a fairly large amount.

The five winds that would be obtained in this way, or the one wind that would be obtained by averaging the five sets of backscatter values might or might not represent the 10 by 50 kilometer piece of ocean that was sampled (500 square kilometers). If 25 winds obtained in this way were vector averaged, they might or might not represent the wind for the 50 x 50 kilometer square, or the 2500 square kilometer area.

An alternative is to average the backscatter values for cells with the same incidence angle in sets of five, or so, and then to average cross track with a correction for incidence angle. The resulting six numbers would then yield a single vector wind at the center of the 50 km by 50 km area. The average of the backscatter values gives a vector wind that is not the same vector wind as the average of the winds computed from the original 25 ten kilometer cells.

One thing that can be stated as certain though is that the individual winds for each of the 100 square kilometer cells will usually contain very little information about the mesoscale winds for those cells because the errors in their measurement as shown by this study will be as large as their internal, natural fluctuations. The combination of the two will be of such a nature that it will be virtually impossible to separate one effect from the other. (See Pierson (1982)).

A corrolary of this is that it will be difficult to determine most gradients over a distance of 50 kilometers because the sampling variability in the individual 10 x 10 kilometer cells will mask any gradient that could conceiveably occur in nature except possibly near the eye around a hurricane. At sharp wind shift lines such as fronts the 10 km resolution cells may be able to detect the wind shift for moderate incidence angles. If the wind shift exceeds 30°, or so, since the values for the biases and standard deviations are less than this, the wind shift should be detectable.

Ccomplications Concerned with Pooling Backscatter Data

Before drawing conclusions from this Monte Carlo simulation of the SCATT, it is necessary to clarify some of the statements made immediately above. The 10 km resolution of the SCATT was selected with the idea that in the operational mode, very nearly all of the time, the data would be pooled so that the measurement would represent a larger cell that was 50 km by 50 km. This larger cell is clearly sampled very efficiently over nearly its entire area by the SCATT scanning pattern, and the backscatter measurements at a given beam and polarization when averaged over the entire set of 25 10 km by 10 km cells should be fairly stable numbers. The six different backscatter measurements more nearly cover the full 2500 square kilometers uniformly than for alternate designs such as the SASS or the one described in Appendix G.

However, the problem of pooling data is not simple. In this study, it was assumed that the input wind speeds and directions for the 5 cell averages and for the 25 cell averages were all the same.

If this assumption is not made, the model takes on an additional complexity. This additional complexity can be understood only in part at the present stage of knowledge of mesoscale variability, but it exists nevertheless and ought not to be ignored.

Consider 25 cell averages, which would be the operational mode of the SCATT most of the time. For a given 10 km cell one way to determine the wind for a 50 km cell would be to use the six backscatter measurements for each of the 10 km cells and compute a wind speed and direction from them. The 25 resulting values of speed and direction could then be averaged to obtain the average speed and direction for the 50 km cell. Alternatively the 25 measurements of backscatter for a given beam, polarization and incidence angle (with a correction for the slight cross track variations of incidence angle) can be averaged to obtain a "more stable estimate", and the six backscatter values for two polarization and three pointing directions so obtained can be used to compute a wind speed and direction for the center of the 50 km by 50 km cell.

The 25 winds obtained the first way might be averaged either vectorially or by magnitude and direction (with due consideration of directions around zero or three hundred sixty degrees). These two different averages differ, but this aspect of the problem will be ignored to keep the problem simple.

Because of mesoscale turbulence, the actual average winds over each of the 25 cells illuminated by a given beam and polarization in a 50 km by 50 square will differ one from another. These 25 winds can be represented by Equations (69) and (70)

$$V_i = \bar{V} + \Delta V_i \tag{69}$$

$$\chi_i = \bar{\chi} + \Delta \chi_i \tag{70}$$

where i goes from 1 to 25.

Strictly described, the winds are changing with time over each of the cells, and between the measurements made with beam 1 and beam 3 they may have changed in both speed and direction over the cell being scanned compared to what they were earlier. This effect will be neglected for this analysis.

By definition

$$\bar{V} = \frac{1}{N} \Sigma V_i \tag{71}$$

$$\bar{\chi} = \frac{1}{N} \Sigma \chi_i \tag{72}$$

$$\Sigma \Delta V_i = 0 \tag{73}$$

and

$$\Sigma \Delta \chi_i = 0 \tag{74}$$,

where χ is measured relative to beam 1.

For a particular beam, the theoretical value of the backscatter for a constant incidence angle is given by Equation (75)*.

$$\sigma_i^o = A(\chi_i)\ V^{\ B(\chi_i)} \tag{75}$$

The value of the backscatter because of the effects of communication noise and attitude errors is given by Equation (76), (See Appendix D).

$$\hat{\sigma}_i^o = A(\chi_i)\ V^{\ B(\chi_i)} + t_i(\alpha(\sigma_i^o(V_i,\ \chi_i))^2 + \beta\ \sigma_i^o\ (V_i,\ \chi_i) + \gamma)^{\frac{1}{2}} \tag{76}$$

Each of the values of $\hat{\sigma}_i$ can be expanded in a Taylor series around the values of $\bar{V}$ and $\bar{\chi}$ such that all of the partial derivatives in Equation (27) are evaluated at $\bar{V}$ and $\bar{\chi}$ and the functional dependence is understood. Only the first few terms of such an infinite expansion are given. Subscripts in $\bar{V}$ and $\bar{\chi}$ denote partial differentiation. The functional dependence on $\bar{V}$ and $\bar{\chi}$ is understood.

$$\begin{aligned}\hat{\sigma}_i^o &= \sigma^o(\bar{V},\ \bar{\chi}) + \sigma_{\bar{V}}^o\ \Delta V_i + \sigma_{\bar{\chi}}^o\ \Delta\chi_i \\ &+ \sigma_{\bar{V}\bar{V}}^o\ \frac{(\Delta V_i)^2}{2} + \sigma_{\bar{V}\bar{\chi}}^o\ \Delta V_i\Delta\chi_i + \sigma_{\bar{\chi}\bar{\chi}}^o\ \frac{(\Delta\chi_i)^2}{2} \\ &+ t_i\ (\ SD(\bar{V},\bar{\chi}) + SD_{\bar{V}}\ \Delta V_i + SD_{\bar{\chi}}\ \Delta\chi_i + SD_{\bar{V}\bar{V}}\ \frac{(\Delta V_i)^2}{2} \\ &+ SD_{\bar{V}\bar{\chi}}\ \Delta V_i\Delta\chi_i + SD_{\bar{\chi}\bar{\chi}}\ \frac{(\Delta\chi i)^2}{2}\ )\end{aligned} \tag{77}$$

If the expected value of the 25 estimates of the $\hat{\sigma}_i$ is computed, the terms that yield $\Sigma\ \Delta\ V_i$ and $\Sigma\ \Delta\ \chi_i$ will be zero but the quadratic terms will not be zero. The expected value of $\hat{\sigma}_i$ becomes Equation (78).

---

*Where from (2), $A = 10^{G/10}$ and $B = H/10$

$$E(\hat{\sigma}_i) = \sigma^o(\bar{V},\bar{\chi}) + \tfrac{1}{2}\left(\sigma^o_{\bar{V}\bar{V}}\cdot VAR(V_i) + 2\sigma^o_{\bar{V}\bar{\chi}}\cdot COV(V_i\chi_i) + \sigma^o_{\bar{\chi}\bar{\chi}}\cdot VAR(\chi_i) \right) \tag{78}$$

since $E((\Sigma t_i)/N) = 0$ and $E((\Sigma t_i \Delta V_i)/N) = 0$.

The variance (and hence the standard deviation) is found from (79).

$$\begin{aligned} E(\hat{\sigma}^o - E(\hat{\sigma}^o))^2 &= VAR(\hat{\sigma}^o) \\ &= \frac{(SD)^2}{N} + \frac{1}{5}\left((SD_{\bar{V}})^2 + SD(SD_{\bar{V}\bar{V}})\right)\cdot VAR(V_i) \\ &+ \frac{1}{5}\left((SD_{\bar{\chi}})^2 + SD(SD_{\bar{\chi}\bar{\chi}})\right)\cdot VAR(\chi_i) \\ &+ \frac{1}{10} SD_{\bar{V}\bar{V}}\ SD_{\bar{\chi}\bar{\chi}}\ VAR(V_i)\ VAR(\chi_i) \\ &+ \frac{(SD_{\bar{V}\bar{V}})^2}{20} M_4(V_i) + \frac{(SD_{\bar{\chi}\bar{\chi}})^2}{20} M_4(\chi_i) \end{aligned} \tag{79}$$

where M_4 is the fourth moment about the mean. In (79), $E(\Sigma t_i^{\,2}) = 5$, odd mixed moments of ΔV_i and $\Delta\chi_i$ are assumed to be zero, and the correlation between ΔV_i and $\Delta\chi_i$ is assumed to be zero.

In the Monte Carlo studies, this result was simplified to Equation (79) for the 25 cell case since the ΔV_i and $\Delta\chi_i$ were assumed to be zero.

$$\bar{\hat{\sigma}}^o = \sigma^o(\bar{V}, \bar{\chi}) + t(SD(\bar{V}, \bar{\chi})/5) \tag{80}$$

The terms in Equation (78) show contributions from the variance of the mesoscale variability independent of the effects of radar communication noise. The term involving the variance of the V_i, for B greater than 2 will be positive. The term involving the variance of the χ_i can have either sign depending on the value of $\bar{\chi}$ and the incidence angle. It will be particularly important near upwind, downwind and crosswind. The correlation (covariance) between V_i and χ_i will be small for mesoscale turbulance but large across fronts.

The various terms in both (78) and (79) can be computed by means of finite differences and the equations at the beginning of this report without going through the process of formal differentiation of the analytical equations. The average value of the $\hat{\sigma}_i^o$ can be either larger or smaller than the value computed from the average wind over the 25 cells, and the variance will probably be larger because four of the terms in (79) are always positive.

Complications such as this are built into the SASS analysis when attempts are made to pool data and hidden by the large cell size. For the SCATT, they need to be investigated more thoroughly.

This analysis is still oversimplified compared to the actual situation. Because of the effect of sampling variability in (76) each of the 25 wind speeds and directions that would be recovered from the 10 km by 10 km cells would have a "random", or perhaps partially systematic, error in the recovered values of V and χ as in Equations (81) and (82).

$$V_{iR} = V_i + \Delta V_i + \Delta V_{iR} \tag{81}$$

$$\chi_{iR} = \chi_i + \Delta \chi_i + \Delta \chi_{iR} \tag{82}$$

where ΔV_{iR} and $\Delta \chi_{iR}$ are introduced by the communication errors and the attitude errors.

The "correct" value for the 50 km cell is, by definition, $\bar{V}$ and $\bar{\chi}$. The averages of (81) and (82) give (83) and (84).

$$\bar{V}_R = \bar{V} + \frac{1}{N} \Sigma \Delta V_{iR} \tag{83}$$

$$\bar{\chi}_R = \bar{\chi} + \frac{1}{N} \Sigma \Delta \chi_{iR} \tag{84}$$

If the six values of $\bar{\bar{\sigma}}^o$ of the form of (78) are used to find the wind for the center of the cell, the result might be written as

$$\begin{aligned} \bar{V}_{PR} &= V(\bar{\bar{\sigma}}^o_{V1}, \bar{\bar{\sigma}}^o_{V2}, \bar{\bar{\sigma}}^o_{V3}, \bar{\bar{\sigma}}^o_{H1}, \bar{\bar{\sigma}}^o_{H2}, \bar{\bar{\sigma}}^o_{H3}) \\ &= \bar{V} + \Delta V_P \end{aligned} \tag{85}$$

$$\begin{aligned} \bar{\chi}_{PR} &= \chi(\bar{\bar{\sigma}}^o_{V1}, \bar{\bar{\sigma}}^o_{V2}, \bar{\bar{\sigma}}^o_{V3}, \bar{\bar{\sigma}}^o_{H1}, \bar{\bar{\sigma}}^o_{H2}, \bar{\bar{\sigma}}^o_{H3}) \\ &= \bar{\chi} + \Delta \chi_P \end{aligned} \tag{86}$$

where P is for pooled.

Which of these two results, i.e. (83) and (84) versus (85) and (86), is closer to $\bar{V}$ and $\bar{\chi}$ needs to be determined. Both are probably more representative of the wind for the 50 km cell than winds computed from alternative designs.

CONCLUSIONS

Introduction

This investigation has studied the properties of a preliminary design for a scatterometer called the SCATT. It is based on knowledge gained from the SASS on SEASAT. It is already apparent that wind fields obtained from the SASS and meteorological analyses based thereon will be far superior to wind fields and analyses from conventional sources. The errors from the conventional data because of inadequate averaging times, the poor calibration and exposure of most anemometers and the poor coverage of the world ocean by ships and buoys undoubtedly produce very large, and at times systematic, errors in wind fields and meteorological analyses compared to those that would result from the correct use of the SASS data.

The SCATT is an improved instrument compared to the SASS. The SCATT vector wind for a 50 km cell, whether obtained by pooling backscatter values for 25 ten km cells or by averaging 25 vector winds as recovered from each of the 25 sets of six cells, will be a very accurate wind compared to a conventionally measured wind. There are further investigations that need to be made about the error structure of the radar winds for pooled and unpooled data and about the properties of mesoscale turbulence.

The full details of the Monte Carlo simulations are given in Appendix H. Tables similar to Table 12 are contained in this appendix for each check mark shown in Table 11. Tables similar to Tables 41 through 52 are given for every wind speed and direction required by Table 11. There is a mass of basic data that have been summarized and partially interpreted by other graphs and tables in the text.

Overall MLE Ability to Select the Correct Wind

Table 86 is a good place to start in order to draw conclusions from the data. With the ± 0.7 db added error calculations neglected, there would have been a total of 3 x 4 x 4 = 48 possible data sets. Of these, 28 were evaluated. The number, or percentage, of correct MLE choices either increases for the same incidence angle with increasing wind speed or levels off for the same number of cells. For the same number of cells and the same wind speed, the percentage correct decreases with increasing incidence angle. For a fixed wind speed and incidence angle, the percentage correct increases with increasing number of cells. In Table 86, based on the above observations, an underlined X is to indicate that the percentage of correct MLE choices would be expected to be greater than 90% if the data for the tabulated conditions had been computed. The MLE solutions are clearly the best way to proceed if they prove to be as accurate for actual data as they seem to be for this simulation. The starting values for the MLE search can be found from the starting procedure of the objective method.

Winds of 4 m/s and under for all incidence angles and one 10 km cell and winds under 8 or 9 m/s for incidence angles greater than 46°, or so, and one 10 km cell would not be recovered uniquely by the MLE, 90% of the time. For 5 cell averages, most simply at a constant incidence angle, or perhaps for 2 by 2 clusters, ($\sqrt{5}$ is not that much larger than 2), only 3 of the 16 values in the complete table, as extrapolated above, would be under 90% for high incidence angles and low winds (4 m/s at 47° and 53° and 8 m/s at 53°). The 25 cell averages yield 90% correct MLE solutions for all cases except 4 m/s for 47° and 53°. The MLE solutions, if found routinely for 10 km cells, would yield winds that could be easily corrected for the incorrect vectors over large areas of the ocean. These could then be vector averaged to obtain more stable results and eliminate mesoscale effects. For areas where this procedure gave too many errors, the backscatter data could be pooled and the procedure repeated.

Even if the $\pm$ 0.7 db added error were a real effect in the data, the results would be perhaps 75% correct instead of 90% correct. A consideration of this added error is probably an incorrect way to account for mesoscale variability from one 10 km cell to another. Some of the variability at 10 km resolution is the result of mesoscale wind variability as discussed above.

If the SCATT were used to recover 10 km resolution vector wind fields, the fields would not be very useful because the errors in the wind vectors introduced by communication noise and attitude errors would usually mask the mesoscale features of the wind field as shown by Pierson (1982), except for strong wind shear lines, near hurricanes and in areas where 10 km cells would not touch land but larger cells would. It would be necessary to average these winds so as to smooth the vector wind field and obtain a representative synoptic scale wind field for most mid ocean conditions.

An investigation of the complications concerned with pooling backscatter data might show that pooling sets of 25 ten km cells yields useful results with small speed and direction errors, and this would reduce the amount of computations. The choice here is engineering terminology is still T B D, (to be determined) because the effects of turbulence has not been modeled adequately. Pooled data so as to provide winds routinely at a 50 km resolution, either way will be more accurate than winds from the SASS.

The computation of the MLE estimate of speed and direction requires considerably more effort than the computation of the winds by means of the objective method. By keeping track of the wind direction relative to the pointing direction of beam 1 around a full circle, information about the model function can be inferred that may permit its refinement. The behavior of the curves plotted in each of the four quadrants for Figures 4, 5 and 8 is not the same. Should the maximum likelihood method not work as well on actual data as it does on simulated data, the objective method can serve as both a stand-by procedure, that is quickly computed, and as a way to understand how the model function could be improved.

Bias and Correlation of MLE Errors and their Removal

As shown by Tables 41 to 52 and Tables 53 to 80 and by Figures 12 to 19, and as summarized in Table 81, the present design with six antennas introduces systematic correlations between the errors in wind speed and direction and systematically larger errors for certain wind directions relative to beam 1 once the "correct" wind, is the solution closest to the input time wind, has been selected. For the missing entries in Table 81, extrapolation can provide an estimate of what the values might be. For example, for one cell, the bias and standard deviations for any wind speed would be expected to be as large as several degrees for the bias and greater than 10^{o} for the standard deviation. For actual data, the variability could be much worse. These values are larger than those found in similar studies of the SASS.

The reason for these correlated errors in wind speed and wind direction was explained on page 85. Once the reason is known, there is a possible way to eliminate this effect and, at the same time, to make the MLE estimates more accurate.

Instead of three measurements for vertical polarization, suppose that there are four on each side at $\pm 45^{o}$, $\pm 65^{o}$, $\pm 115^{o}$ and $\pm 135^{o}$ and that the horizontal polarization measurements are made at $\pm 45^{o}$ and $\pm 135^{o}$. There would still be a total of six measurements on each side. Recomputed curves such as those in Figure 8 for this modification for wind directions near 45^{o}, 135^{o}, 225^{o} and 315^{o} would then show three ascending curves and three descending curves going through the correct speed and direction. There would be no tendency for four such curves to overpower the two remaining curves and produce the correlation between speed and direction errors as in the present design. The new vertical polarization measurement would reinforce the one at 20^{o} or -115^{o} in the present design and produce a large number of two solution cases for nearly all wind directions relative to beam 1 even without any horizontal polarization information. The two remaining horizontal polarization measurements would still eliminate the incorrect solution for wind directions near 0°, 90°, 180° and 270° relative to beam 1 as shown in Figures 8 and 9.

Alternative Designs

There is nothing yet cast in concrete for the 50 km larger cell and the 10 km cells for finer resolution within the cell. The results obtained about the 10 km cell show rather large sampling variability errors and biases over the full range of wind speeds, directions and incidence angles.

The design advantage of more uniform coverage by means of the 10 km cells and the use of the antenna beam width more efficiently would not be lost if the fundamental cells were enlarged slightly and if the area for routine large scale data was also enlarged.

In statistical analyses, for large samples, improvements of a factor of 10 in the reduction of a standard deviation require a sample size increased by a factor of 100, but when small samples are involved, a 50% increase in the sample size can provide a dramatic improvement. The 10 km resolution seems to be pushing the state of the art a bit too hard even with all of the design improvements of the SCATT compared to the SASS. (As an aside, the antenna design is unchanged. The antenna pattern is symmetric about the peak gain, and some percentage of the 100 watts that are transmitted never reaches the sea surface. An improvement might be possible here.)

If the standard deviation of the estimate of the received power could be decreased by enlarging the dimensions of the cells and lengthening the averaging time for the single cell by a modest amount and if the four vertically and two horizontally polarized antennas were used, the wind data from a single cell would be much more useful.

Basic cells on a 15 km grid with doppler band widths 50% wider and averaging times 50% longer pooled in 4 by 4 arrays so as to provide a 60 km synoptic scale resolution (about 2 values per degree of longitude) might change the values of λ and NR in Equation D7 in such a way as to improve the estimates of the winds substantially for these 15 km by 15 km cells. (The reciprocal of λ is effectively the sample size.) There is time to study numerous options for the antennas to be used on the SCATT and for the cell sizes and scanning patterns to be used in the final design.

ACKNOWLEDGEMENTS

This research was sponsored by the National Aeronautics and Space Administration as administered by the NASA Langley Research Center. The close collaboration and constructive suggestions of W. L. Jones, L. C. Schroeder, W. L. Grantham and E. M. Bracalente during the course of this program are acknowledged with appreciation. Mrs. Jean Berman, our secretary for many years, typed this lengthy text. She will retire at the end of September and we wish her well.

REFERENCES

Born, G., J. C. Wilkerson, D. Lame et al. (1979): Seasat Gulf of Alaska Workshop I Report, NASA 622-101, Vol. 1, Jet Propulsion Laboratory, Pasadena, Calif.

Barrick, D. E., J. C. Wilkerson, P. Woiceshyn et al. (1980): SEASAT Gulf of Alaska Workshop II Report, NASA 622-107, Jet Propulsion Laboratory, Pasadena, Calif.

Businger, J., R. H. Stewart, T. Guymer et al. (1981): SEASAT JASIN Workshop Report. Volume 1 Findings and Conclusions. JPL Pub. 80-62.

Doviak, R. J. and M. Berger (1980): Turbulence and Waves in the Optically Clear Planetary Boundary Layer Resolved by Dual-Doppler Radar. Radio Science, Vol. 15, No. 2 March-April pp. 297-319.

Grantham, W. (1980): Performance Definition Study Results June 1980. Material transmitted to GSFC by NASA LaRC.

James, P. K. (1980): A Review of Radar Observations of the Troposphere in Clear Air Conditions. Radio Science, Vol. 15, No. 2 March-April pp. 151-177.

Jones, W. L., L. C. Schroeder and J. L. Mitchell (1977): Aircraft Measurements of the Microwave Scattering Signature of the Ocean IEEE Trans Antennas and Propagate./IEEE J. Oceanog. Eng. Special Issue of Radio Oceanography January.

Kropfli, R. A. and P. H. Hildebrand (1980): Three-Dimensional Wind Measurements in the Optically Clear Planetary Boundary Layer with Dual-Doppler Radar. Radio Science, Vol. 15, No. 2 March-April - pp. 283-296.

Mood, A. M., F. A. Graybill and D. C. Boes (1963): Introduction to the Theory of Statistics (Third Edition) Mc Graw Hill.

Pierson, W. J., V. Cardone and J. A. Greenwood (1974): The Application of SEASAT-A to Meteorology (The University Institute of Oceanography, now CUNY Institute of Marine and Atmospheric Sciences) Prepared for the SPOC Group of NOAA/NESS.

Pierson, W. J. and R. E. Salfi (1978): The Theory and Data Base for the CUNY SASS Wind Vector Algorithm Part 1 of a Final Report to JPL and NEPRF, Contracts 954411 and Nooo14-77-G-0206 (Including Appendices A through H.), CUNY Institute of Marine and Atmospheric Sciences.

Pierson, W. J. W. Marlatt et al. (1978): Oceans and Atmosphere in SKYLAB EREP Investigations Summary, NASA-SP-399 U.S. Govt. Printing Office Stock No. - 033-000-00741-8

Pierson, W. J. (1979): The Error Structure of the Backscatter Measurements as a Function of Range Gate Effects, of the Properties of the Signal and the Noise and of Errors in the Measurement of Roll Pitch and Yaw. Appendix A to Pierson and Salfi (1978)

Pierson, W. J. (1978b): Verification Procedures for the SEASAT Measurements of the Vector Winds with the SASS. CUNY Institute of Marine and Atmospheric Sciences. Appendix E to Pierson, W. J. and R. E. Salfi (1978) The Theory and Data Base for the CUNY SASS Wind Vector Algorithm. Part 1 of a Final Report submitted to JPL and NEPRF.

Pierson, W. J. (1982): The Measurement of the Synoptic Scale Wind Over the Ocean. CUNY IMAS Technical Report prepared for NASA (Submitted to J. Geophys. Res.)

Schroeder, L. C., D. H. Boggs, G. Dome et. al. (1982): The Relationship Between Wind Vector and Normalized Radar Cross Section Used to Derive Seasat-A Satellite Scatterometer Winds (submitted to J. Geophys. Res.)

Polcyn, F. C., K. Piech et. al. (1978): Data Analysis Techniques in SKYLAB EREP Investigations Summary NASA SP-399, U. S. Govt. Printing Office, Stock No. -033-000-00741-8.

Wentz, F. J. (1978): Documentation for Program WINVEC: Computation of the Sea Surface Friction Velocity and Wind Direction. Tech Report 78-001 under contract JPL-954942 Jet Propulsion Laboratory.

LIST OF FIGURES

LIST OF FIGURES (CONT'D.)

LIST OF FIGURES (CONT'D.)

LIST OF TABLES

LIST OF TABLES (CONT'D.)

LIST OF TABLES (CONT'D)

LIST OF TABLES (CONT'D.)

LIST OF TABLES (CONT'D.)

APPENDIX A

AN ANALYTICAL SOLUTION FOR VERTICAL POLARIZATION PAIRS FOR BEAMS APPROXIMATELY 90^{o} APART

INTRODUCTION

The methods developed in this study require as a starting point the determination of the four vector winds for vertically polarized pairs of backscatter measurements for beams approximately 90^{o} apart as illustrated in Figure 4 of the main text. This is accomplished by solving analytically for the four wind directions, for the four solution case, or the two, that correspond to the places in the V-χ plane where the two curves shown in Figure 4 cross. Given a wind direction, the speed is known immediately from Equation (5).

These wind speeds and directions are then used in the selection by objective criteria method. They also serve as starting points in the V-χ plane for the search for the maximum likelihood estimate.

The equations that are used are equations (5) through (18f). Let beam 1 make a backscatter measurement at an incidence angle of θ_i, so that the two quantities associated with it are σ_1^o and θ_1. (The circumflex will be omitted). The unknown wind direction relative to beam 1 will be χ_1. Let beam 2 make a second backscatter measurement, and the values associated with it will be σ_2^o and θ_2, and let beam 2 be $90^{o} + \varepsilon$ clockwise from beam 1 as in (A1).

$$\varepsilon = \chi_{BEAM1} - \chi_{BEAM2} - \pi/2 \tag{A1}$$

If the second measurement is substituted into Equation (5), the appropriate wind direction relative to beam 1 that will yield the same wind speed as σ_1^o, θ_1 and χ_1 for beam 1 would be $\chi_1 - \pi/2 - \varepsilon$ for beam 2. (A2)

From Equation (13), the result is Equation (A3) for beam 1.

$$Y_1 = Y_{c1} - F(\sigma_1^o, \theta_1)\ F(\chi_1) - G^*(\sigma_1, \theta_1)\ G^{**}(\chi_1, \theta_1) \tag{A3}$$

For beam 2, the result is Equation (A4).

$$Y_2 = Y_{c2} - F(\sigma_2^o, \theta_2)\ F(\chi_1 - \pi/2 - \varepsilon) - G^*(\sigma_2, \theta_2)\ G^{**}(\chi_1 - \pi/2 - \varepsilon, \theta_2) \tag{A4}$$

As χ_1 is varied in these two equations, curves like those in Figure 4 are produced. If they cross at 4 points, for each point where they cross

$$Y_1 = Y_2 \tag{A5}$$

so that the right hand sides of A3 and A4 can be set equal.

The only unknowns in the resulting equation are the four (or perhaps 2) values for χ_1 that satisfy the equation.

Let

$$p = \cos \chi_1 \tag{A6}$$

$$p^* = \cos\ (\chi_1 - \pi/2 - \varepsilon) \tag{A7}$$

From the definitions of the various terms involved, the quantity p^2 can be factored out of part of the left hand side and p^{*2} can be factored out of part of the right hand side with the result

$$Y_{c1} + F^*(p, \sigma_1^o, \theta_1)(\cos \chi_1)^2 = Y_{c2} + M^*(p^*, \sigma_2^o, \theta_2)(\cos\ (\chi_1 - \pi/2 - \varepsilon))^2 \tag{A8}$$

Trigonometric identities and algebraic manipulations then yield Equation (A9).

$$(\text{Cos}\ \chi_1)^2 = \frac{Y_{c1}-Y_{c2} + M^* \ ((\cos\ \varepsilon)^2 + (\sin\ 2\chi,\sin\ 2\varepsilon)/2)}{F^* + M^* \ (\cos\ 2\varepsilon)} \tag{A9}$$

where

$$M^* = F_2(\sigma_2,\theta_2)p^* + G_2^*(\sigma_2,\theta_2)(1 + D(\theta_2)(1 - p^{*2}) + K(\theta_2)\ p^*(1-p^{*2})$$
$$+ E(\theta_2)(1-3p^{*2}+2p^{*4}) + K^*(\theta_2)\ p^*(1-3p^{*2} + 2p^{*4})) \tag{A10}$$

$$F^* = F_1(\sigma_1,\theta_1)\ p + G_1^*\ (\sigma_1,\theta_1)(1+D(\theta_1)(1-p^2) + K(\theta_1)\ p(1-p^2)$$
$$+ E(\theta_1)\ (1-3p^2+2p^4) + K^*(\theta_1)\ p(1-3p^2 + 2p^4)) \tag{A11}$$

The right hand side of (A9) is also a function of χ_1. However it varies only slightly, but in a nontrivial way, as a function of χ_1. For χ_1 in different quadrants, the values of p and p* assume like and opposite signs depending on the quadrant and the value of ε. Equation A9 is thus slightly different for comparable angles in each quadrant (say, χ^*, $-\chi^*$, $\pi + \chi^*$, $\pi - \chi^*$).

A first approximation to χ_1 is found by setting $\chi_1 = \pi/4$ on the right hand side of A9. This yields a value of χ_1 in the first quadrant on the left side which is then substituted into the right side to obtain an improved value. The iteration stops when the left side is equal to the right side to the desired accuracy. The other three values of χ are found by starting at $3\ \pi/4$, $5\ \pi/4$, and $7\ \pi/4$. Given the wind direction relative to beam 1, the use of (5) then gives the wind speed. Sample calculations rapidly converge, and the solution is typically found to $\pm\ 0.01^{\circ}$ and $\pm\ 0.01$ m/s. Examples of the results are given in the main text.

The Monte Carlo calculations in this study were done for the special case of $\varepsilon = 0$. The random fluctuations in the backscatter can result in no formal solutions for a particular pair of values. When two actual crossing solutions are found, a third is found as the base of a "Y" pattern at 0°, 90°, 180° or 270° relative to beam 1 by means of a maximum likelihood estimate based on the two vertical polarization measurements for the speed at that particular direction. For no actual crossing, two solutions are found for the V polarization data that are 180° apart in terms of maximum likelihood estimates. The generalization for conditions where $\varepsilon \neq 0$ is not difficult.

Before solving for the values of χ that satisfy A9, some simple inequalities can be tested by means of (9a), (9b) and (9c) for the two backscatter measurements. These tests determine whether or not there are four solutions. If there are not four solutions, then either there are two crossing solutions and one maximum likelihood solution or there are two maximum likelihood solutions.

For example, if for beam 1, at 0°

$$Y_{u1} < Y_{D1} < Y_{c1} \tag{A12}$$

and if for beam 3

$$Y_{u3} < Y_{D3} < Y_{c3} \tag{A13}$$

and if

$$Y_{u1} < Y_{D1} < Y_{c3} \tag{A14}$$

and

$$Y_{u3} < Y_{D3} < Y_{c1} \tag{A15}$$

then there are four solutions.

If as a second example, (A12) and (A13) hold and if

$$Y_{c3} < Y_{u1} \tag{A16}$$

then there are two maximum likelihood solutions one at 0^{o} and the other at 180^{o}. Other inequalities yield the other possibilities, and simple tests quickly determine the various possibilities.

APPENDIX B A COMPARISON OF PROBABILITY DENSITY FUNCTIONS AND LIKELIHOOD FUNCTIONS FOR VARIOUS ALGORITHMS USED FOR THE SASS AND THE SCATT.

Introduction In the main body of this report, a theory for recovering vector winds from the SCATT has been correctly derived. Every once in a while some incorrect ways to develop a theory and corresponding algorithms have been mentioned in passing. The purpose of this appendix is to summarize the two probability density functions that are involved and the various correct and incorrect likelihood functions that can result from these probability density functions. Some graphical results that compare correct and incorrect pdf's and correct and incorrect likelihood functions will then be presented based on SCATT parameters that will provide guidance on those conditions for which the incorrect theories will lead to incorrect results.

The following material defines all of the functions, constants and variables that are involved. There is one correct pdf and one correct likelihood function. There are three possible ways to define incorrect likelihood functions.

PROBABILITY DENSITY FUNCTIONS AND LIKELIHOOD FUNCTIONS RELATED TO BACKSCATTER MEASUREMENTS (SIMPLIFIED NOTATION).

DEFINITIONS

$\hat{\sigma}$; the randomly varying measurement of the backscatter

σ ; the "true" but (usually) unknown value from the model function.

$$\mathrm{VAR}\ \hat{\sigma} = (\alpha\ \sigma^2 + \beta\ \sigma + \gamma); \tag{B1}$$

A function of σ (not $\hat{\sigma}$). The values of α, β and γ are constants for a given cell.

S/N; a parameter called the signal to noise ratio given by

$$\sigma/N\mathcal{R}$$

where, N and $\mathcal{R}$ are known. In db,

$$S/N\ db = 10\ \log_{10}\ S/N$$

$$\widehat{\mathrm{VAR}}\ \hat{\sigma} = (\alpha\ \hat{\sigma}^2 + \beta\ \hat{\sigma} + \gamma); \tag{B2}$$

an incorrect value for the variance based on the randomly varying measurement of the backscatter.

$\hat{\sigma}_{db} = 10\ \log_{10}\ \hat{\sigma}$; $\hat{\sigma}$ converted to decibels.

$\sigma_{db} = 10\ \log_{10}\ \sigma$; σ converted to decibels.

$$(S\ D\ \hat{\sigma})_{db} = \tfrac{1}{2}\left\{10\ \log_{10}\left(1 + \frac{(\mathrm{VAR}\ \hat{\sigma})^{\frac{1}{2}}}{\sigma}\right) - 10\ \log_{10}\left(1 - \frac{(\mathrm{VAR}\ \hat{\sigma})^{\frac{1}{2}}}{\sigma}\right)\right\} \tag{B3}$$

A quantity used in a theory that assumes an incorrect probability structure for $\hat{\sigma}$; it is a function of σ, not $\hat{\sigma}$.

$$(\hat{SD}\ \hat{\sigma})_{db} = \tfrac{1}{2}\left[10\ \log_{10}\left(1 + \frac{(\hat{VAR}\ \hat{\sigma})^{\frac{1}{2}}}{\hat{\sigma}}\right) - 10\ \log_{10}\left(1 - \frac{(\hat{VAR}\ \hat{\sigma})^{\frac{1}{2}}}{\hat{\sigma}}\right)\right] \quad \text{(B4)}$$

A quantity used in a theory that assumes an incorrect probability structure for $\hat{\sigma}$, and that also uses an incorrect value for the variance based on a randomly varying value for the backscatter. It is a function of $\hat{\sigma}$.

PROBABILITY DENSITY FUNCTIONS

$$f(\hat{\sigma};\ \sigma)\ d\hat{\sigma} = (2\pi)^{-\frac{1}{2}}(\mathrm{VAR}\ \hat{\sigma})^{-\frac{1}{2}}\ \exp\left[-\tfrac{1}{2}\left(\frac{(\hat{\sigma} - \sigma)^2}{\mathrm{VAR}\ \hat{\sigma}}\right)\right]\ d\hat{\sigma} \tag{B5}$$

The correct pdf, $\hat{\sigma}$ is the random variable, σ is assumed known, σ and VAR $\hat{\sigma}$ (which is a function of σ) are known constants. If σ is known, this pdf gives the probability that a value of $\hat{\sigma}$ between $\hat{\sigma} + \frac{d\hat{\sigma}}{2}$ and $\hat{\sigma} - \frac{d\hat{\sigma}}{2}$ will be drawn at random from the assumed population.

$$f(\hat{\sigma}_{db};\ \sigma_{db})\ d\hat{\sigma}_{db} = (2\pi)^{-\frac{1}{2}}(\mathrm{SD}\ \hat{\sigma})_{db}^{-\frac{1}{2}}\ \exp\left[-\tfrac{1}{2}\left(\frac{\hat{\sigma}_{db}-\sigma_{db}}{(\mathrm{SD}\ \hat{\sigma})_{db}}\right)^2\right]\ d\hat{\sigma}_{db} \tag{B6}$$

An incorrect pdf that yields probabilities more or less equal to the correct probabilities. $\hat{\sigma}_{db}$ is the random variable, $(\mathrm{SD}\ \hat{\sigma})_{db}$ and σ_{db} are known constants.

$$f_1(\hat{\sigma},\sigma)d\hat{\sigma} = \frac{4.329}{(2\pi)^{\frac{1}{2}}\hat{\sigma}(\mathrm{SD}\ \hat{\sigma})_{db}}\quad \exp\left[-\tfrac{1}{2}\left(\frac{10\ \log_{10}\hat{\sigma} - 10\ \log_{10}\sigma}{(\mathrm{SD}\ \hat{\sigma})db}\right)^2\right]d\hat{\sigma} \tag{B7}$$

A transformation on $f(\hat{\sigma}_{db},\ \sigma_{db})$ so that probabilities generated by it can be compared to the correct probabilities from $f\ (\hat{\sigma};\sigma)db$, $\hat{\sigma}$ is the variable and σ and $(\mathrm{SD}\ \hat{\sigma})$ db are known constants. The transformation is $\hat{\sigma}_{db} = 10\ \log_{10}\hat{\sigma}$. Equation (B7) is the log-normal distribution (Mood, Graybill and Boes (1963), Pg. 117). Had (B6) been the correct pdf for the backscatter, then (B7) with this transformation would be a possible distribution. However, (B6) is not the correct distribution. This transformation allows (B6) to be compared with (B5) on linear $\hat{\sigma}$ scales.

LIKELIHOOD FUNCTIONS

$$L(\sigma;\hat{\sigma})d\hat{\sigma} = (2\pi)^{-1}(\mathrm{VAR}\ \hat{\sigma})^{-\frac{1}{2}} \exp\left[-\tfrac{1}{2}\left(\frac{(\hat{\sigma} - \sigma)^2}{\mathrm{VAR}\ \hat{\sigma}}\right)\right] d\hat{\sigma} \tag{B8}$$

The correct likelihood function; $\hat{\sigma}$ is known as a sampled value, σ is the variable and VAR $\hat{\sigma}$ is a function of this variable. The likelihood function (with the $d\hat{\sigma}$) gives the probability that a value of $\hat{\sigma}$ between $\hat{\sigma} + d\,\hat{\sigma}/2$ and $\hat{\sigma} - d\,\hat{\sigma}/2$ will be sampled as a function of the values of the variable σ.

$$L_1\ (\sigma;\hat{\sigma})d\hat{\sigma} = (2\pi)^{-1}(\hat{\mathrm{VAR}}\ \hat{\sigma})^{-\frac{1}{2}} \exp\left[-\tfrac{1}{2}\left[\frac{(\hat{\sigma} - \sigma)^2}{\hat{\mathrm{VAR}}\ \hat{\sigma}}\right]\right] d\hat{\sigma} \tag{B9}$$

An incorrect likelihood function, σ is the variable, $\hat{\sigma}$ is known, but $\hat{\mathrm{VAR}}\ \hat{\sigma}$ is treated as a constant, which is calculated from $\hat{\sigma}$ instead of as a variable. It does not yield the correct probability defined by (B8), and, in fact, the roles of the random variable and the population parameter have been completely reversed.

$$L_2\ (\sigma_{db};\hat{\sigma}_{db})\ d\,\hat{\sigma}_{db} = (2\pi)^{-\frac{1}{2}}\left(\left(\mathrm{SD}\ \hat{\sigma}\right)_{db}\right)^{-1} \exp\left(-\tfrac{1}{2}\left(\frac{\hat{\sigma}_{db} - \sigma_{db}}{(\mathrm{SD}\ \hat{\sigma})_{db}}\right)^2\right) d\hat{\sigma}_{db} \tag{B10}$$

A correctly defined likelihood function based on (B6) which is not the correct pdf for backscatter as measured by either the SCATT or the SASS. $\hat{\sigma}_{db}$ is a known constant, σ_{db} is the variable and $(\mathrm{SD}\ \hat{\sigma})_{db}$ is a function of σ_{db}.

$$L_3\ (\sigma_{db},\hat{\sigma}_{db})\ d\,\hat{\sigma}_{db} = (2\pi)^{-\frac{1}{2}}\left(\left(\hat{\mathrm{SD}}\ \hat{\sigma}\right)_{db}\right)^{-1} \exp\left(-\tfrac{1}{2}\left(\frac{\hat{\sigma}_{db} - \sigma_{db}}{(\hat{\mathrm{SD}}\ \hat{\sigma})_{db}}\right)^2\right) d\hat{\sigma}_{db} \tag{B11}$$

An incorrectly defined likelihood function based on B6 $\hat{\sigma}_{db}$ and $(\hat{SD}\ \hat{\sigma})_{db}$ are both treated as constants which is incorrect. σ_{db} is the only variable. This is the likelihood function that is used in the present SASS-1 wind vector algorithm.

$$L_4\ (\sigma;\hat{\sigma})\ d\hat{\sigma} = \frac{4.329}{(2\pi)^{\frac{1}{2}}\ \hat{\sigma}(SD\ \hat{\sigma})_{db}}\ \exp\left[-\tfrac{1}{2}\left(\frac{10\ \log_{10}\hat{\sigma} - 10\ \log_{10}\sigma}{(SD\ \hat{\sigma})_{db}}\right)^2\right]\ d\hat{\sigma} \qquad \text{(B12)}$$

A correctly defined likelihood function based on the transformation of (B6) into (B7) for comparison purposes. σ is the variable and $(SD\ \hat{\sigma})_{db}$ is a function of σ. $\hat{\sigma}$ is known. The pdf from which the likelihood function is obtained is incorrect from a physical point of view.

$$L_5\ (\sigma;\hat{\sigma})\ d\hat{\sigma} = \frac{4.329}{(2\pi)^{-\frac{1}{2}}\ \hat{\sigma}(\hat{SD}\ \hat{\sigma})_{db}}\ \exp\left[-\tfrac{1}{2}\left(\frac{10\ \log_{10}\hat{\sigma} - 10\ \log_{10}\sigma}{(\hat{SD}\ \hat{\sigma})_{db}}\right)^2\right]\ d\hat{\sigma} \qquad \text{(B13)}$$

An incorrectly defined likelihood function, $(\hat{SD}\ \hat{\sigma})_{db}$ has been computed from the known $\hat{\sigma}$ and is a constant. The only variable is the σ in $10\ \log_{10}\sigma$. It is based on an incorrect pdf. This likelihood function can be used to compare (B8) with (B11) because σ can be plotted on a linear scale.

General Discussion The clearest demonstration that $\hat{\sigma}$ in linear form and not $\hat{\sigma}_{db}$ is the correct random variable and that (B5) is the correct pdf lies in the facts that (1) values of $\hat{\sigma}$ (= $R\,\hat{P}_R$) were frequently negative in the SASS data and (2) that Monte Carlo studies generate values of $\hat{\sigma}$ by means of Equation (B5) as described by Equation (29) of the main text. In Monte Carlo studies, especially for low signal-to-noise ratios, it is clearly inconsistent to use the correct pdf to generate the Monte Carlo data (which clearly corresponds to reality) and then to use an incorrect pdf and an incorrect likelihood function obtained from it to recover the winds that generated the perfect theoretical backscatter values and the variances that were then used to generate the Monte Carlo values of $\hat{\sigma}$.

Among other objections such a procedure involves ignoring all of the negative values of $\hat{\sigma}$ that are generated when the signal to noise ratio is low. The negative values of the estimates (measurements) of the received power by the SASS were the source of considerable consternation to some workers with the SASS data. These negative values are a real phenomonon, a value of $\hat{\sigma}^o$ that is negative can be calculated from this negative estimate of the received power and this value can be used to recover winds from the data. The true values of σ, i.e. those computed from a perfect model function are never negative, and this fact is, of course, consistent with the physical model being used.

SOME NUMERICAL AND GRAPHICAL EXAMPLES

For one of the cells of the NOSS, the various parameters involved yield the result that

$$\text{VAR } \hat{\sigma} = 3.36210^{-3}(\sigma)^2 + 8.409 \times 10^{-5}\,\sigma + 7.4675 \times 10^{-7} \qquad \text{(B14)}$$

$$\text{Also NR} = 0.0148 \qquad \text{(B15)}$$

With this information, it is possible to compute the numerical value for the various quantities used in the preceeding equations. They are given in Table B1.

A range of S/N from + 10 db to - 12 db is covered in Table B1. As the values of σ decrease by a factor of 158, the standard deviation decreases by a factor of 10.2, and $SD\sigma/\sigma$ increases from 0.06 to 0.97. The reciprocal of $SD\sigma/\sigma$, when assigned a negative sign, is that value of t from a zero mean unit variance normal pdf such that $\hat{\sigma}$ will be zero, and values of t more negative than this yield negative values of $\hat{\sigma}$. For the last three entries, these values are - 1.57, - 1.28 and - 1.02, and values less than these have an appreciable probability of occurring.

To use the incorrect pdf given by (B6) requires some sort of a standard deviation expressed in db. For this pdf to have approximately the same shape as the correct one, the exponent should equal minus one half at the db values that correspond to $\sigma + SD\hat{\sigma}$ and $\sigma - SD\hat{\sigma}$ for the correct pdf. These values are $\sigma_{db} + SD\hat{\sigma}_{db}$. and $\sigma_{db} - SD\hat{\sigma}_{db}$.

Table B1 Numerical Values for some Graphs of the Preceeding Functions.

S/N	S/N db	σ	σ db	VAR $\hat{\sigma}$	SD $\hat{\sigma}$	$\frac{SD\,\hat{\sigma}}{\sigma}$	SD $\hat{\sigma}$(+)	SD $\hat{\sigma}$(-)	(SD $\hat{\sigma}$) db
10.0	10.0	0.148	- 8.297	$8.6833 \cdot 10^{-5}$	$9.318 \cdot 10^{-3}$	0.06296	.2652	- .2824	0.2738
3.16	5.0	0.0468	- 13.297	$1.20457 \cdot 10^{-5}$	$3.471 \cdot 10^{-3}$	0.07416	.3107	- .3346	0.3227
1.0	.0	0.0148	- 18.297	$2.7277 \cdot 10^{-6}$	$1.6511 \cdot 10^{-3}$	0.1116	.4595	- .5139	0.4867
0.7	- 1.549	0.01036	- 18.665	$1.97876 \cdot 10^{-6}$	$1.4067 \cdot 10^{-3}$	0.1358	.5529	- .6338	0.5934
0.5	- 3.01	0.0074	- 21.308	$1.553 \cdot 10^{-6}$	$1.246 \cdot 10^{-3}$	0.1684	.6760	- .8009	0.7384
0.316	- 5.0	0.00468	- 23.297	$1.2138 \cdot 10^{-6}$	$1.1018 \cdot 10^{-3}$	0.2354	.9181	- 1.1657	1.0419
0.1	-10.0	0.00148	- 28.297	$8.7857 \cdot 10^{-7}$	$9.373 \cdot 10^{-4}$	0.633	2.130	- 4.353	3.024
0.08	-10.97	0.001184	- 29.266	$8.510 \cdot 10^{-7}$	$9.225 \cdot 10^{-4}$	0.7791	2.502	- 6.559	4.5305
0.063	-12.0	0.0009324	- 30.304	$8.284 \cdot 10^{-7}$	$9.102 \cdot 10^{-4}$	0.9762	2.958	-16.227	9.5926

Where $(SD\ \sigma)\ (+) = 10 \log_{10} \left(1 + \frac{SD\ \sigma}{\sigma}\right)$

$(SD\ \sigma)\ (-) = 10 \log_{10} \left(1 - \frac{SD\ \sigma}{\sigma}\right)$

Both sides of the curve cannot be fitted, and so the usual assumption is to use (B3), which compromises part way between, with one value slightly too large and the other slightly too small for high S/N values. The approximation is fairly good for S/N greater than 0 db; it is questionable for S/N less than 0 db but greater than - 10 db. It becomes very poor for S/N less than or equal to - 10 db. For a decrease of a few more db, as soon as $SD\hat{\sigma}/\sigma$ equals one, or more, the approximation falls completely. If $SD\,\sigma(+)$ alone is used, other kinds of inconsistencies arise.

Figures B1 through B6 are graphs of some of the functions defined above for values of S/N of 10, 5, - 3.01, - 5, - 10, and - 10.97 db. The first four figures show the correct pdf (Eq. B5) and LF (Eq. B8) at the top, the incorrect pdf (Eq. B7) transformed from Eq. B6) and the incorrect LF (B13 transformed from B11) in the middle, and the correct LF and incorrect LF at the bottom. The last two figures show only those graphs corresponding to the top and middle of the preceeding four figures.

To obtain graphs that could be easily compared, the pdf's and LF's have been multiplied by one tenth the value of the standard deviation given in Table B1. For a pdf, a point on the curve represents (approximately) the probability that $\hat{\sigma}$ will lie in the interval

$$\hat{\sigma} - SD\hat{\sigma}/20 < \hat{\sigma} < \hat{\sigma} + SD\hat{\sigma}/20.$$

The correct pdf is simply the normal curve, since σ is known as in Figure B1. The likelihood function is a function of σ on the horizontal axis. For Fig. B1, the likelihood function is computed for the observed (estimated, Monte Carloed) value of $\hat{\sigma}$ equal to 0.148. It gives the probability that a value of $\hat{\sigma}$

between $\hat{\sigma} - SD\hat{\sigma}/20$ and $\hat{\sigma} + SD\hat{\sigma}/20$ would have been sampled for any value of σ as given on the horizontal scale.

From the top pdf in B1, for example, the probability of obtaining a value of $\hat{\sigma}$ of $0.14 \pm 4.659 \times 10^{-4}$ is about 0.0272. From the top LF in B1, the probability of obtaining a value of $\hat{\sigma}$ of $1.148 \pm 4.659 \times 10^{-4}$ is 0.004 if σ equals 0.13 (and not 0.006).

The correct LF (compared to the pdf) is lower for low values of σ, higher near the peak for values of σ less than $\hat{\sigma}$, lower past the peak and finally higher for high values of σ. This skewness is produced by simply using the correct definition of the likelihood function for this special radar case where the variance is a function of the mean.

The correct pdf is used to Monte Carlo SASS and SCATT simulations, but the correct LF is not used to recover the winds in the SASS 1 algorithm. The incorrect LF that is used is obtained from an incorrect pdf and both are shown in the middle of Fig. B1.

Since the correct and incorrect likelihood functions are the source of the difficulty, both are graphed as a function of σ on the bottom of B1. For S/N = 10 db, there is very little difference between the correct and incorrect likelihood functions. The incorrect pdf plus the incorrect use of a likelihood function, in which the variance is kept constant, combine to produce a likelihood function that is very close to the correct one.

Fig. B2 shows similar graphs for S/N = 5 db. The correct graphs have properties similar to those of Fig. B1, and the two likelihood functions are very similar. A similar result would be obtained for S/N = 0 db.

Fig. B3 shows the same set of graphs for S/N = - 3.01 db. The correct curves still have a shape similar to those in Figures B1 and B2. However, the two likelihood functions are now discernably different with the incorrect LF being too low for values of σ less than 0.0074 and too high for values of σ greater than 0.0074.

For S/N = - 5 db as in Fig. D4, both the incorrect pdf and the incorrect LF show a definite skewness compared to the correct functions. The errors inherent in using an incorrect LF could be appreciable.

Disaster strikes the incorrect theory for S/N = - 10 and S/N = - 10.97 as shown in Figures B5 and B6. The correct pdf and LF are still perfectly well behaved. Negative values of σ (not $\hat{\sigma}$) are not admissable for theoretical reasons because of the theory of the model function. In B5, for example the probability of observing a value of $\hat{\sigma}$ between $0.0148 - 4.686 \times 10^{-5}$ and $0.0148 + 4.686 \times 10^{-5}$ is 0.01 if, in fact, σ is zero.

Summary

A strange situation has arisen. A correct theory for the pdf of the randomly varying backscatter measurements that were made by the SASS on SEASAT and that will be made by the SCATT on the NOSS was used in the design of these instruments. This theory is used to generate Monte Carlo values so as to study the effect of sampling variability for both the SASS and the SCATT. This theory corresponds to the actual measurements (estimates) obtained with the scatterometer. It is based on the fact that backscatter in antilog form is a normally distributed random variable with an expected value (mean) predicted by a model function and with a variance that is a function of this expected value.

An inconsistency arises in that the present SASS 1 algorithm, which is used to compute the winds from the backscatter measurements, does not use a correct likelihood function in order to obtain maximum likelihood estimates of the vector winds. The discrepancies between the correct and incorrect likelihood functions can be substantial for low signal to noise ratios and can cause difficulties even for moderate signal to noise ratios.

These difficulties do not arise in the final data set for the SASS winds based on the SASS 1 model function. Only pairs of backscatter values 90^{o} apart in aspect angle were used along with a variant of Equation 41b of the text. Curves such as those in Figure 4 usually cross at uniquely defined values of V and χ and Q becomes identically zero. The theories described in this report would yield different results only for the vertical part of the "Y" type of solution in the three solution cases and for the two solution cases.

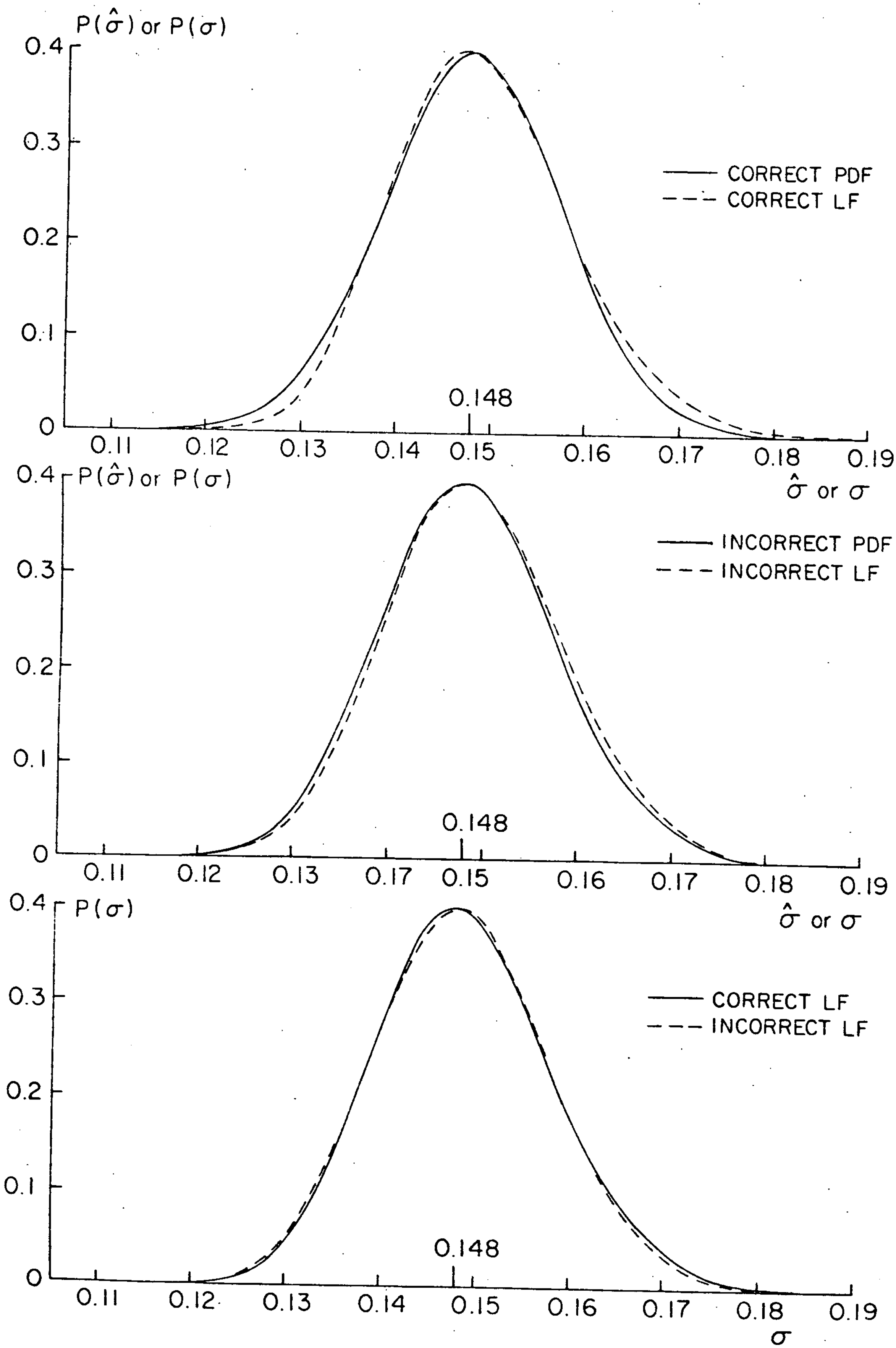


FIG. B1 GRAPHS OF CORRECT AND INCORRECT PROBABILITIES ASSOCIATED WITH BACKSCATTER MEASUREMENTS FOR A PARTICULAR SCATT CELL AND S/N = 10 DB. σ = 0.148 FOR PDF'S. $\hat{\sigma}$ = 0.148 FOR LF'S.

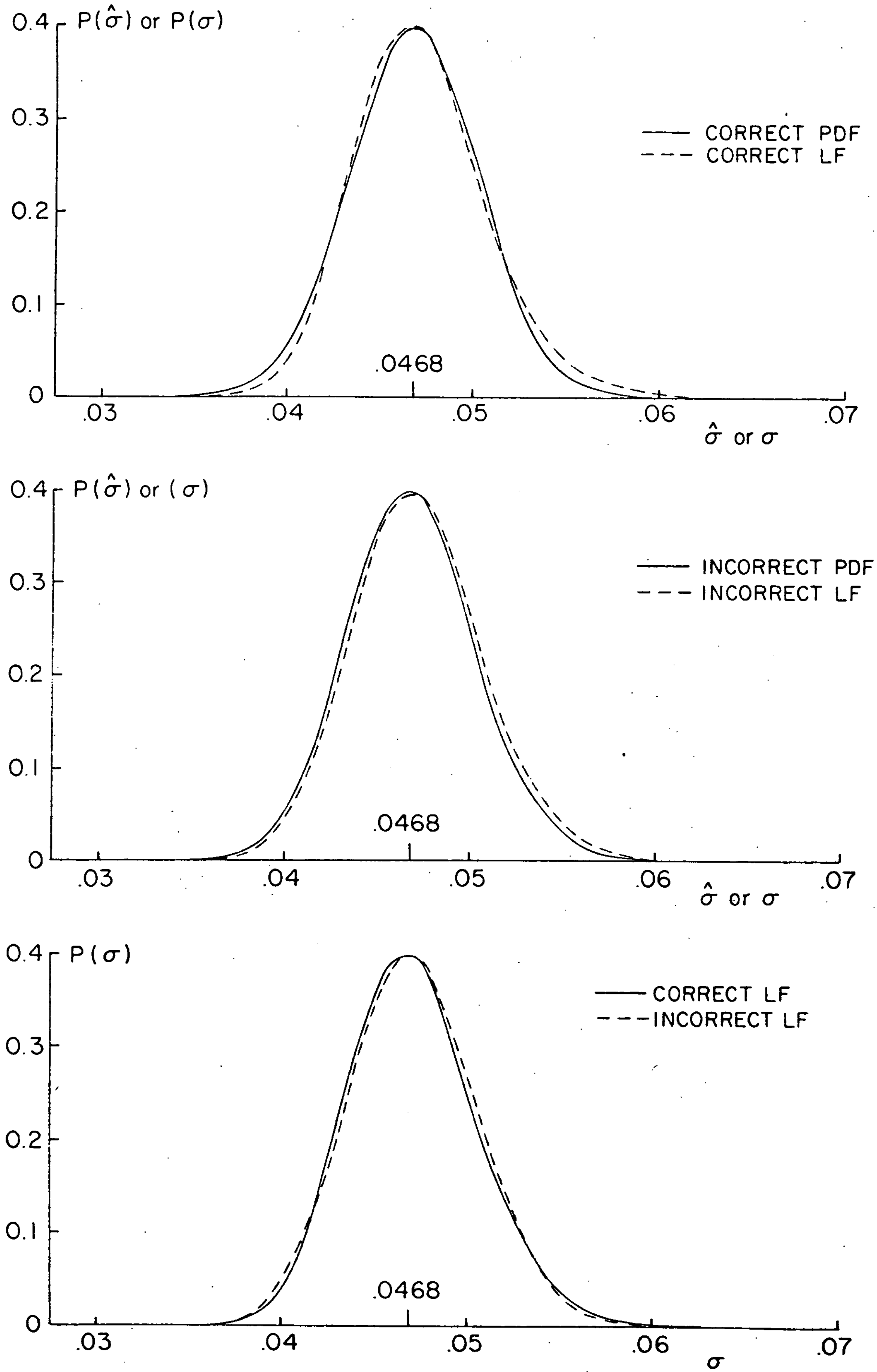


FIG. B2 SAME AS FIG. B1 EXCEPT S/N = 5 DB AND σ = .0468 FOR PDF'S AND σ = 0.0468 FOR LF'S.

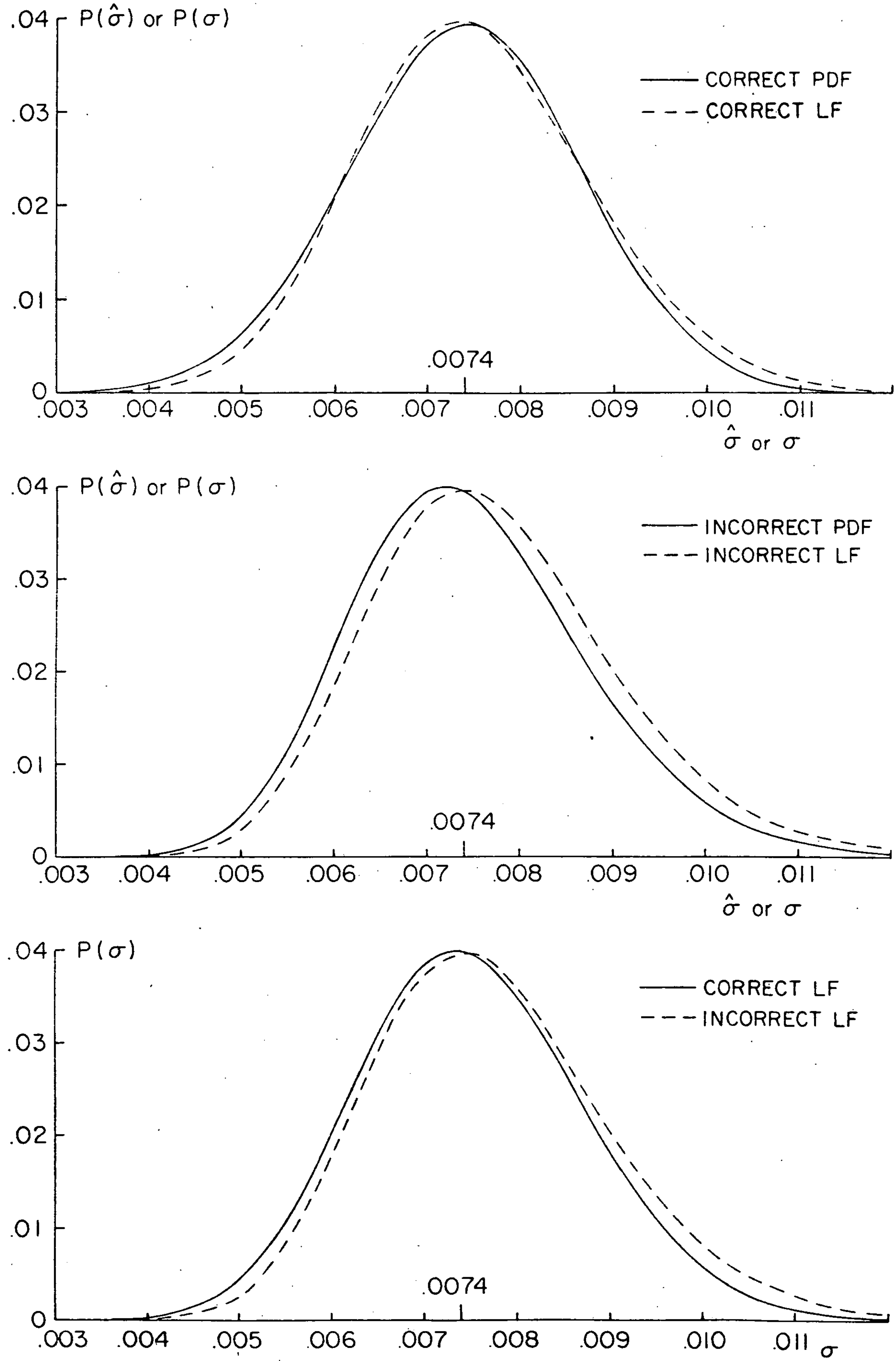


FIG. B3 SAME AS B1 EXCEPT S/N = -3.01 DB AND σ = 0.0074 FOR PDF'S AND $\hat{\sigma}$ = 0.0074 FOR LF'S.

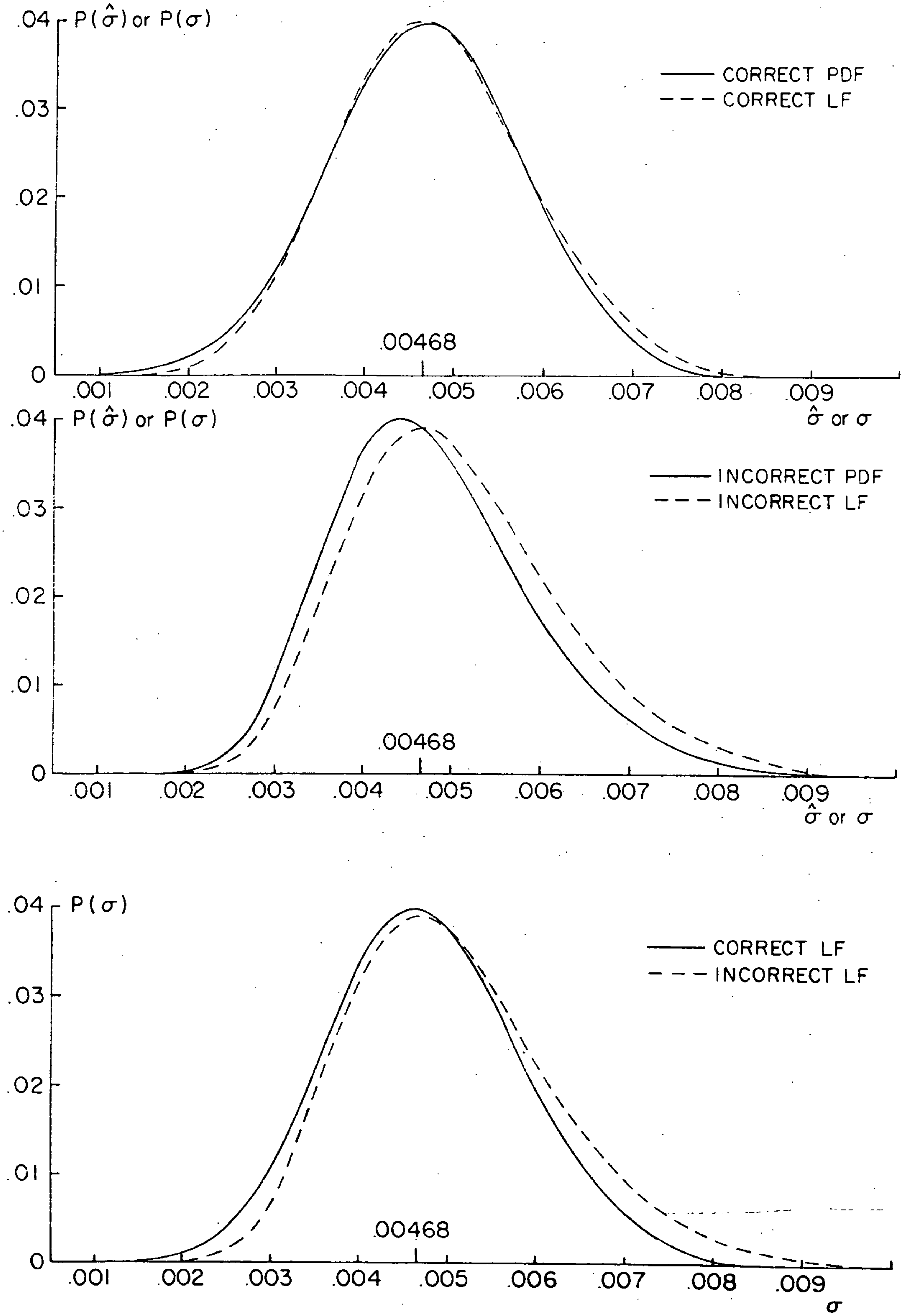


FIG. B4 SAME AS B1 EXCEPT S/N = -5 DB AND $\sigma = 0.00468$ FOR PDF'S AND $\hat{\sigma} = 0.00468$.

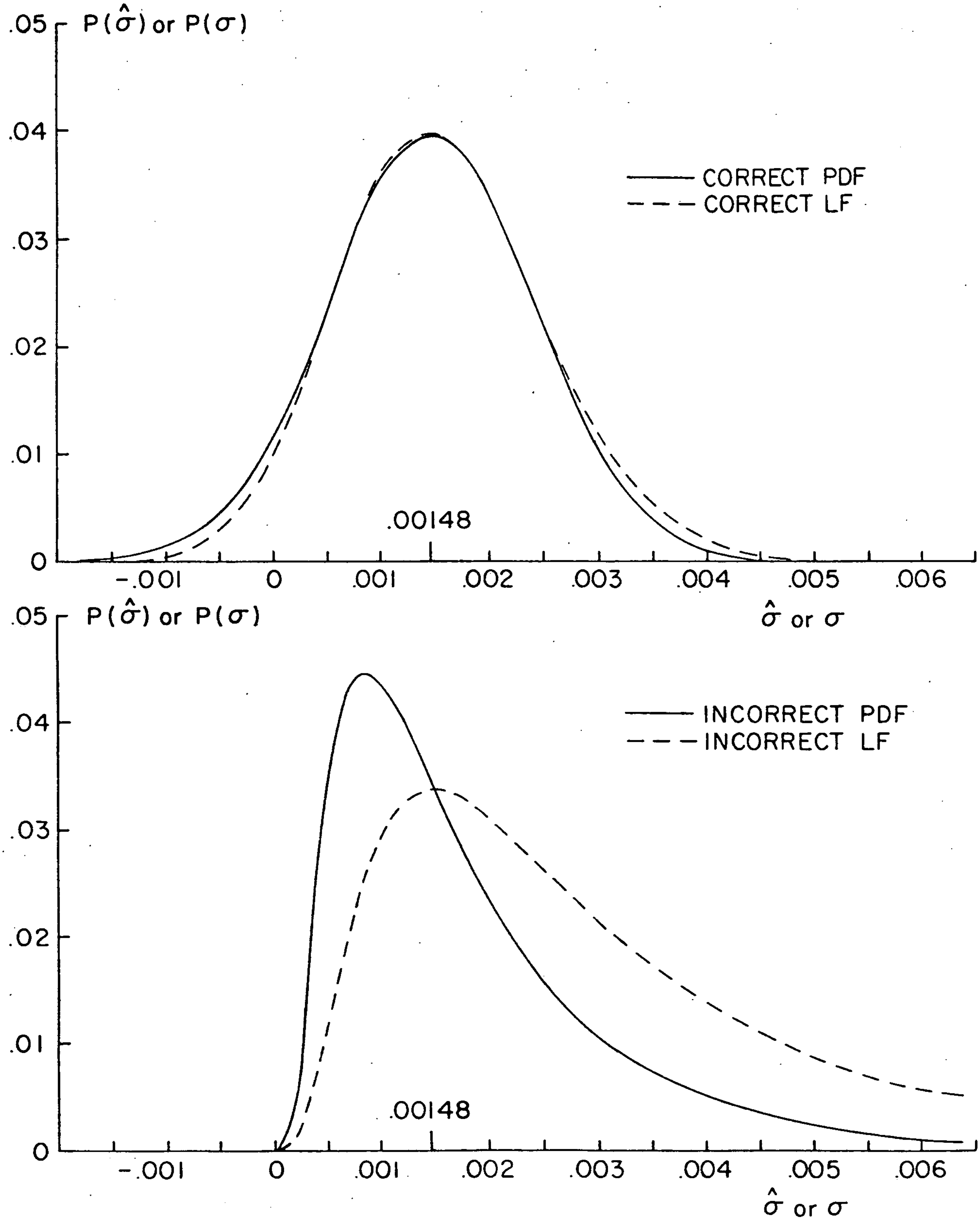
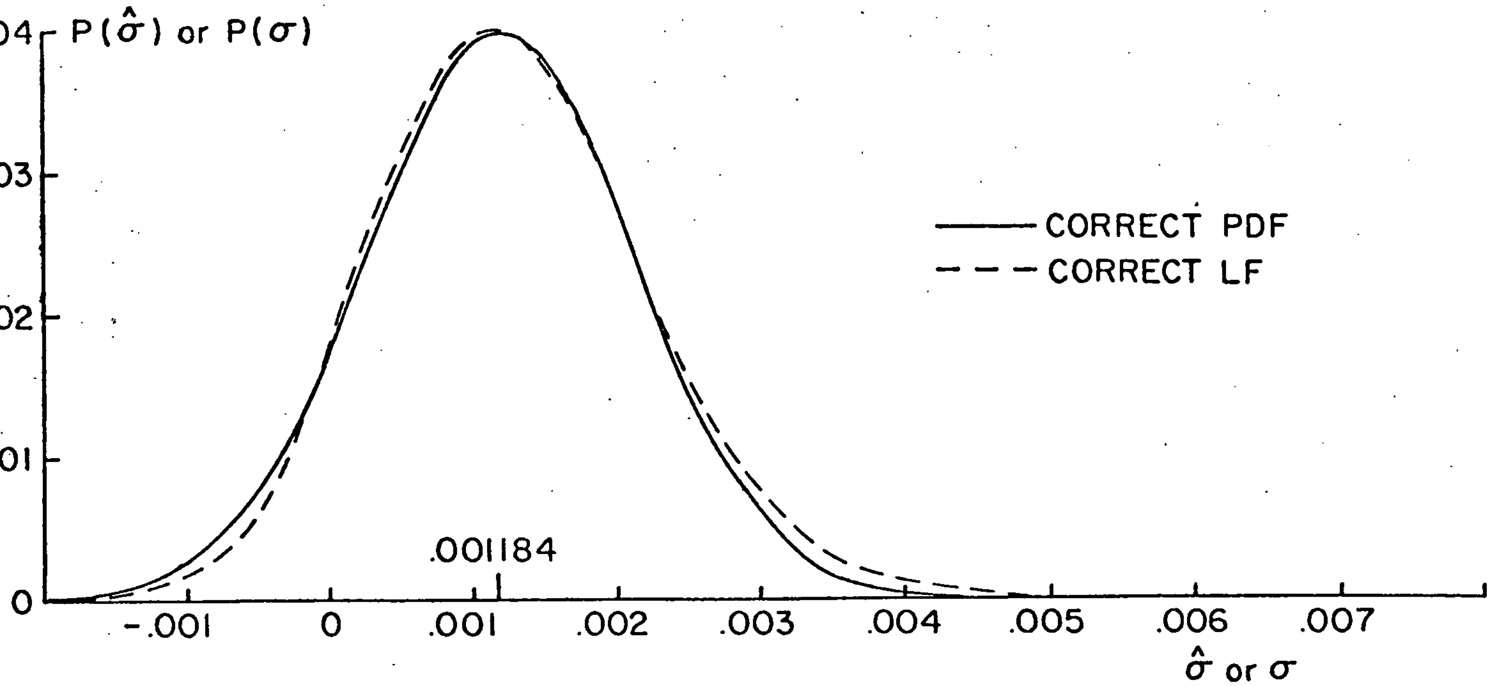


FIG. B5 SAME AS FIG. B1 EXCEPT S/N = -10 DB AND σ = 0.00148 FOR PDF'S AND $\hat{\sigma}$ = 0.00148 FOR LF'S.



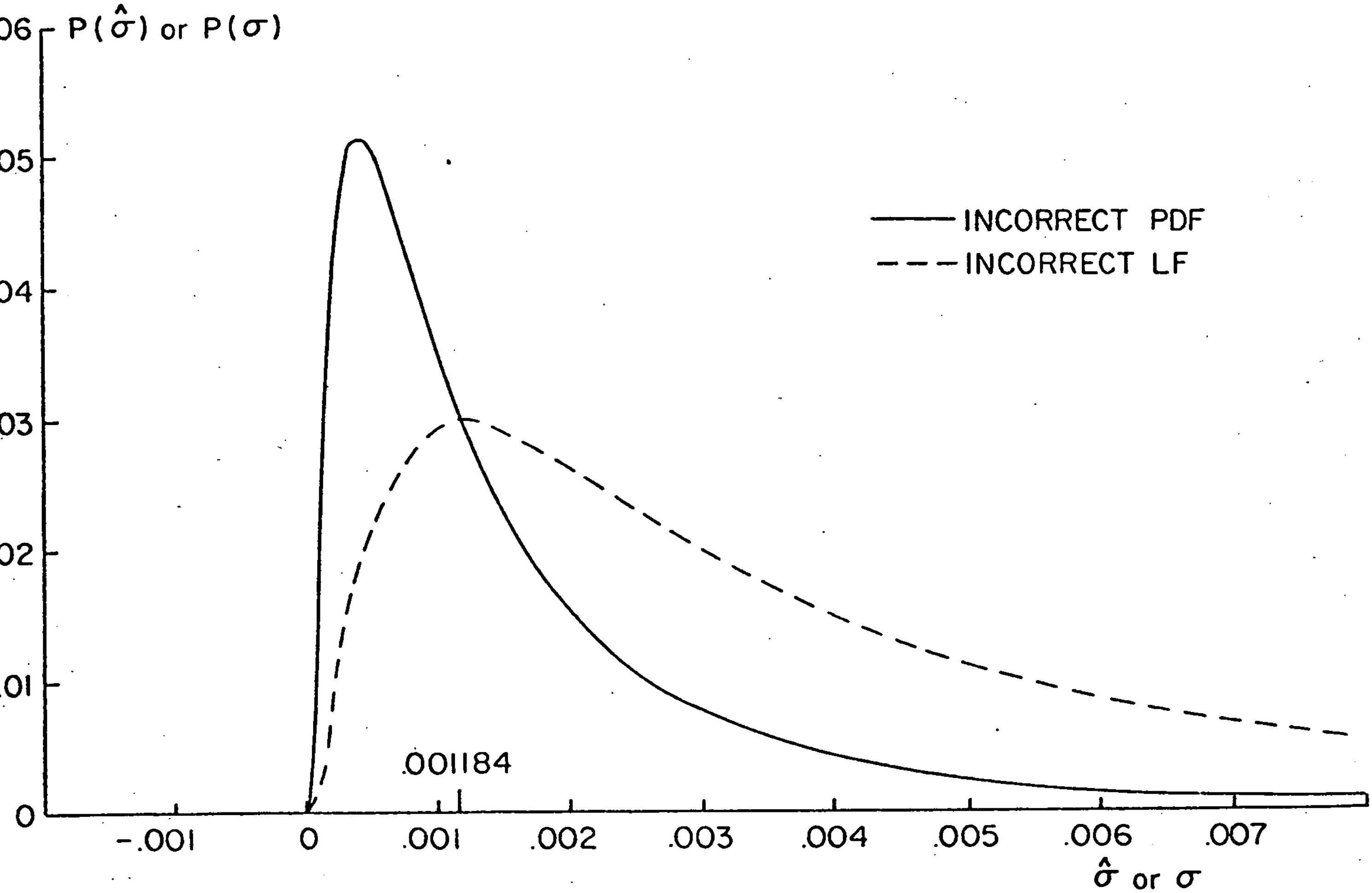


FIG. B6 SAME AS FIG. B1 EXCEPT S/N = -10.97 DB AND σ = 0.001184 FOR PDF'S AND σ = 0.001184 FOR LF'S.

APPENDIX C THE MLE SEARCH ROUTINE AND EXAMPLES OF THE VALUES OF THE NORMALIZED LIKELIHOOD FUNCTION, $L'(V, \chi, \hat{\sigma}^{o}_{ij})$

Search Technique for the Maximum of the Normalized Likelihood Function in the V-χ Plane

A search technique was developed to find the maximum value of the likelihood function that could ensure that the maximum was found to the specified resolution and that would not require a prohibitively expensive exhaustive search of the V-χ plane.

The basic method employs the use of rectangles in the V-χ plane. For a given V-χ pair the value of the function is computed. Then for some ΔV and $\Delta\chi$, the eight points that would form a rectangle surrounding the V-χ pair are computed and the maximum found. Then the value of the maximum is compared to the value for the original V-χ pair. If the maximum on the rectangle is smaller than the value for the original pair, then the search is complete and the maximum has been found. However, if the maximum is larger than the value at the center pair, the maximum has not been found and the search is continued by moving the rectangle, such that the new maximum becomes the center of a new rectangle, and the procedure is continued until a maximum is found.

The advantage of the technique described above is implicit in the move of the rectangle to successive maxima in that the path taken follows the gradient of the likelihood function, thus taking the shortest path through the grid at the resolution specified without ever having to compute the partial derivatives of the function.

In order to reduce the number of steps to find the maximum, rectangles with successively smaller values of ΔV and $\Delta\chi$ are used. The first search is done using $\Delta V = 0.5\ ms^{-1}$ and $\Delta\chi = 2.5^{o}$. When the maximum to that resolution is found, a second search is done with $\Delta V = 0.2\ ms^{-1}$ and $\Delta\chi = 1.5^o$ and finally a third search is done with $\Delta V = 0.1\ ms^{-1}$ and $\Delta\chi = 0.5^o$. Figure C1 illustrates the relative sizes of the rectangles used to search for the maximum of the likelihood function.

<u>Examples of the Function, $L'(V;\ \chi\ \hat{\sigma}^o_{ij})$</u>

The normalized likelihood function varies from a maximum slightly greater than one to zero. When computed over a rather small range of wind speeds and directions near the maximum values, the function varies to values as small as 10^{-23} before the limits of the computer are reached.

The normalized likelihood function was evaluated to a resolution of 0.5 degrees and 0.1 m/s in the search for the maxima. Some of the values of this function (1) for vertical polarization pairs, ninety degrees apart, (2) for vertical polarization pairs, ninety degrees apart plus a new vertical polarization beam 20^o clockwise from beam one and (3) for all six measurements are illustrated by means of computer printout tables in this appendix.

Table C1 summarizes the results for six examples. For the first three examples the input wind speed was 12 m/s and 0^o aspect angle relative to beam 1. For the pair of vertical polarization measurements 90^o apart the maxima are almost equal but not one. The random fluctuations have shifted the locations of the maxima.

When the vertical polarization measurement 20^o away from beam 1 is added to the terms in the calculation of the normalized likelihood function, the maxima decrease slightly and shift to different locations The new value does not produce a marked difference in the values of the maxima.

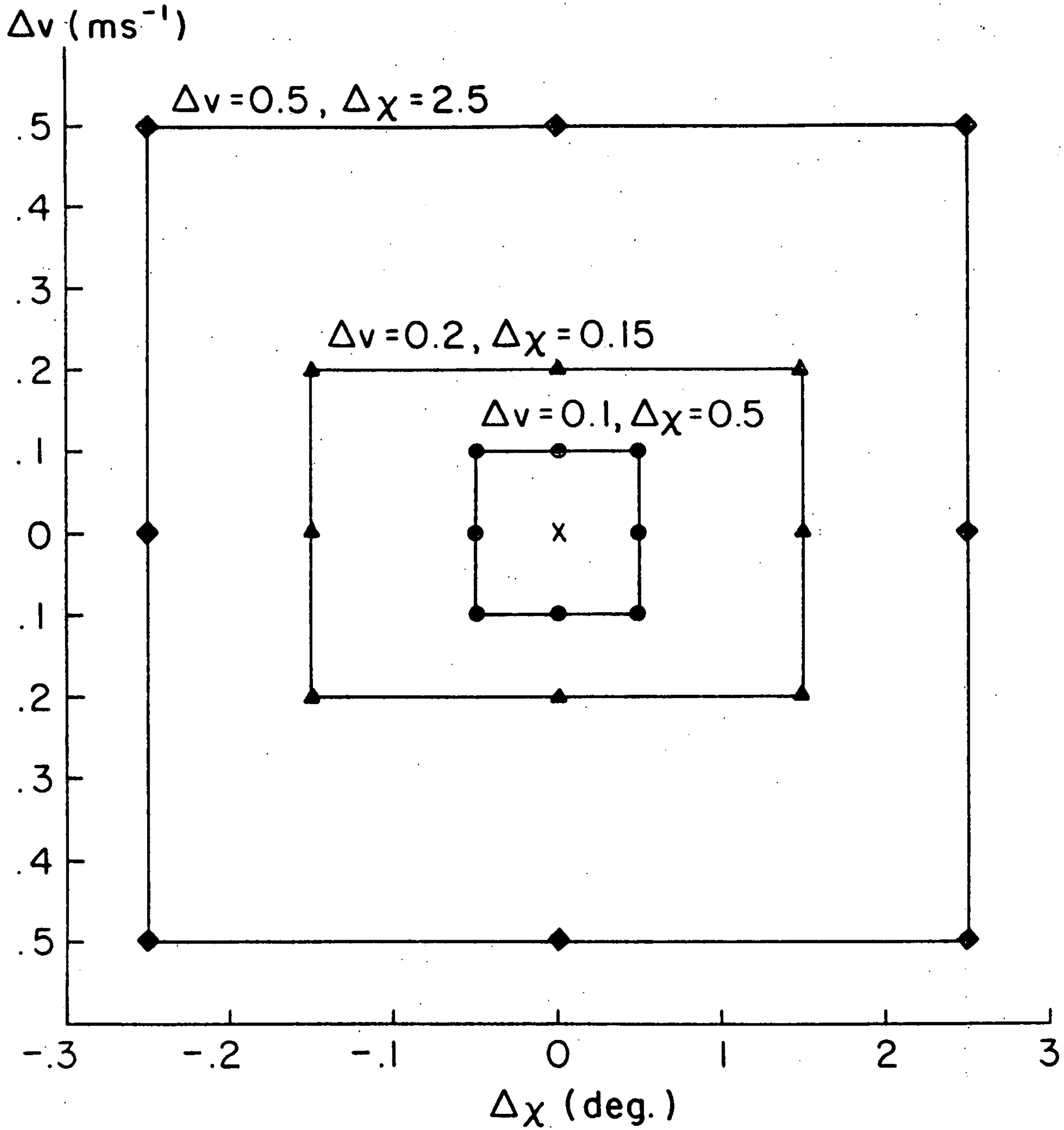


FIG. C3. The Three Resolutions Used in the Search Routine to Locate the Maxima of the Normalized Likelihood Function.

When the three horizontal polarization values are added, the result is dramatic. One maximum is 0.8487. The other is 3.092×10^{-4}. The example properly selects a wind speed and direction near the input speed and direction.

The next three examples are for an input speed and direction of 12 m/s and 30^{o}. For the vertical polarization pair alone, all four maxima ought to be equal. They are not quite equal because of the 0.1 m/s and 0.5 degree resolution. The maxima differ by only 0.0053 and could be made equal by going to an even higher resolution. The values of $L'(V, \chi; \hat{\sigma}^{o}_{ij})$ in the vicinity of these four maxima are shown in Tables C2, C3, C4 and C5.

When the vertical polarization value at 20^{o} is also used, three of the maxima are dramatically reduced. The one 180^{o} away is reduced by more than a factor of 10 compared to the value at the "true" wind. Those not 180^{o} away from the "true" value are reduced by about a factor of 100. At 30^{o}, the new antenna vertical polarization measurement is often enough to permit the selection of the correct wind. The values of the normalized likelihood function in the vicinity of three of these maxima are shown in Table C6, C7 and C8.

The use of all six measurements further reduces the maxima for those speeds and directions that are not the correct values. The maximum 180^{o} away from the true value is smaller by a factor of more than 1000. The values of the function in the vicinity of three of these maxima are shown in Table C9, C10 and C11.

TABLE C1 EXAMPLES OF MLE VALUES FROM V-χ FIELDS

(The output values are speed, direction and the MLE value.)

INPUT 12.0 M/S, 0^{o} ASPECT ANGLE
INCIDENCE ANGLES 39^{o}, 30.5^{o} and 39^{o}

V PAIR	12.1 m/s	8.5^{o}	0.9959
	12.6 m/s	180.0^{o}	0.8165
V PAIR + 20^{o}V	12.1 m/s	354^{o}	0.9630
	12.5 m/s	174.5^{o}	0.6615
ALL SIX	12.2 m/s	353.5^{o}	0.8487
	13.1 m/s	179.5^{o}	3.092×10^{-4}

INPUT 12.0 m/s, 30^{o} ASPECT ANGLE
INCIDENCE ANGLES 39^{o}, 30.5^{o} and 39^{o}

V PAIR	12.1 m/s	31.5^{o}	1.002
	12.3 m/s	326.0^{o}	1.002
	12.7 m/s	153.0^{o}	0.9983
	12.9 m/s	208.5^{o}	0.9967
V PAIR + 20^{o}V	12.0	32.0^{o}	0.8933
	13.3	339.0^{o}	2.875×10^{-4}
	13.4	161.0^{o}	9.434×10^{-3}
	13.0	210.5^{o}	7.772×10^{-2}
ALL SIX	12.0	31.5^{o}	0.7856
	13.3	340.0^{o}	2.97×10^{-7}
	13.3	206.0^{o}	3.159×10^{-4}
	> 13.6	near 166^{o}	out of range (small)

TABLE C2 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.1 M/S, 31.5^{o}, OF 1.002, VERTICAL POLARIZATION PAIR. INPUT 12 M/S, 30^{o}.

	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5
19.5	.4803-04	.1153-03	.2467-03	.4735-03	.8180-03	.1278-02	.1813-02	.2345-02
20.0	.9361-04	.2171-03	.4494-03	.8345-03	.1396-02	.2114-02	.2908-02	.3650-02
20.5	.1799-03	.4031-03	.8072-03	.1451-02	.2351-02	.3449-02	.4603-02	.5608-02
21.0	.3402-03	.7370-03	.1428-02	.2484-02	.3899-02	.5546-02	.7180-02	.8493-02
21.5	.6323-03	.1325-02	.2483-02	.4183-02	.6363-02	.8777-02	.1103-01	.1267-01
22.0	.1154-02	.2338-02	.4241-02	.6921-02	.1020-01	.1366-01	.1665-01	.1858-01
22.5	.2063-02	.4045-02	.7105-02	.1124-01	.1607-01	.2086-01	.2471-01	.2680-01
23.0	.3610-02	.6852-02	.1166-01	.1788-01	.2481-01	.3127-01	.3599-01	.3795-01
23.5	.6175-02	.1136-01	.1873-01	.2786-01	.3752-01	.4595-01	.5140-01	.5272-01
24.0	.1031-01	.1838-01	.2942-01	.4246-01	.5554-01	.6611-01	.7193-01	.7182-01
24.5	.1680-01	.2905-01	.4511-01	.6324-01	.8040-01	.9308-01	.9857-01	.9585-01
25.0	.2666-01	.4474-01	.6749-01	.9197-01	.1137+00	.1281+00	.1321+00	.1252+00
25.5	.4118-01	.6713-01	.9843-01	.1305+00	.1570+00	.1723+00	.1732+00	.1601+00
26.0	.6184-01	.9801-01	.1398+00	.1804+00	.2115+00	.2263+00	.2219+00	.2002+00
26.5	.9021-01	.1391+00	.1933+00	.2430+00	.2778+00	.2900+00	.2777+00	.2447+00
27.0	.1277+00	.1919+00	.2599+00	.3187+00	.3556+00	.3625+00	.3391+00	.2923+00
27.5	.1754+00	.2570+00	.3396+00	.4066+00	.4431+00	.4416+00	.4041+00	.3409+00
28.0	.2334+00	.3339+00	.4309+00	.5043+00	.5376+00	.5243+00	.4697+00	.3882+00
28.5	.3009+00	.4206+00	.5309+00	.6080+00	.6345+00	.6062+00	.5323+00	.4314+00
29.0	.3753+00	.5135+00	.6347+00	.7120+00	.7284+00	.6824+00	.5880+00	.4678+00
29.5	.4529+00	.6072+00	.7358+00	.8097+00	.8128+00	.7477+00	.6328+00	.4949+00
30.0	.5284+00	.6952+00	.8269+00	.8937+00	.8815+00	.7971+00	.6636+00	.5106+00
30.5	.5958+00	.7701+00	.9005+00	.9571+00	.9288+00	.8267+00	.6777+00	.5137+00
31.0	.6489+00	.8253+00	.9499+00	.9943+00	.9506+00	.8339+00	.6740+00	.5039+00
31.5	.6823+00	.8552+00	.9704+00	.1002+01	.9447+00	.8179+00	.6526+00	.4819+00
32.0	.6924+00	.8566+00	.9596+00	.9782+00	.9115+00	.7799+00	.6152+00	.4493+00
32.5	.6779+00	.8290+00	.9184+00	.9260+00	.8537+00	.7229+00	.5645+00	.4082+00
33.0	.6401+00	.7751+00	.8504+00	.8494+00	.7760+00	.6512+00	.5041+00	.3615+00
33.5	.5828+00	.6999+00	.7617+00	.7549+00	.6844+00	.5701+00	.4381+00	.3120+00
34.0	.5114+00	.6101+00	.6598+00	.6499+00	.5856+00	.4849+00	.3705+00	.2624+00
34.5	.4323+00	.5134+00	.5526+00	.5418+00	.4860+00	.4007+00	.3049+00	.2150+00
35.0	.3520+00	.4168+00	.4474+00	.4373+00	.3912+00	.3216+00	.2441+00	.1717+00
35.5	.2761+00	.3265+00	.3500+00	.3417+00	.3053+00	.2507+00	.1901+00	.1335+00
36.0	.2084+00	.2466+00	.2645+00	.2584+00	.2310+00	.1898+00	.1439+00	.1012+00
36.5	.1514+00	.1796+00	.1931+00	.1891+00	.1694+00	.1395+00	.1060+00	.7467-01
37.0	.1058+00	.1261+00	.1361+00	.1339+00	.1204+00	.9954-01	.7593-01	.5367-01
37.5	.7106-01	.8525-01	.9265-01	.9165-01	.8293-01	.6893-01	.5287-01	.3757-01
38.0	.4590-01	.5553-01	.6084-01	.6067-01	.5531-01	.4632-01	.3578-01	.2560-01
38.5	.2849-01	.3483-01	.3855-01	.3882-01	.3572-01	.3019-01	.2353-01	.1698-01
39.0	.1698-01	.2103-01	.2355-01	.2399-01	.2233-01	.1908-01	.1503-01	.1096-01
39.5	.9720-02	.1221-01	.1387-01	.1433-01	.1351-01	.1169-01	.9323-02	.6879-02
40.0	.5339-02	.6919-02	.7874-02	.8259-02	.7908-02	.6944-02	.5616-02	.4200-02
40.5	.2812-02	.3660-02	.4304-02	.4595-02	.4475-02	.3995-02	.3283-02	.2493-02
41.0	.1420-02	.1887-02	.2265-02	.2466-02	.2448-02	.2225-02	.1861-02	.1438-02
41.5	.6870-03	.9345-03	.1147-02	.1276-02	.1293-02	.1200-02	.1024-02	.8061-03
42.0	.3182-03	.4440-03	.5583-03	.6360-03	.6597-03	.6258-03	.5454-03	.4386-03
42.5	.1410-03	.2023-03	.2612-03	.3054-03	.3247-03	.3156-03	.2816-03	.2316-03
43.0	.5978-04	.8831-04	.1174-03	.1411-03	.1542-03	.1538-03	.1407-03	.1186-03
43.5	.2421-04	.3693-04	.5063-04	.6271-04	.7053-04	7237-04	.6805-04	.5890-04

TABLE C3 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.3 M/S 326^{o}, OF 1.002. VERTICAL POLARIZATION PAIR. INPUT 12 M/S, 30^{o}.

	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5
310.0	.6321-09	.2196-08	.6621-08	.1743-07	.4029-07	.8233-07	.1495-06	.2426-06
310.5	.2573-08	.8450-08	.2411-07	.6019-07	.1322-06	.2570-06	.4449-06	.6892-06
311.0	.9850-08	.3063-07	.8294-07	.1967-06	.4114-06	.7627-06	.1261-05	.1868-05
311.5	.3548-07	.1047-06	.2697-06	.6094-06	.1216-05	.2154-05	.3407-05	.4838-05
312.0	.1204-06	.3383-06	.8300-06	.1790-05	.3414-05	.5792-05	.8785-05	.1198-04
312.5	.3856-06	.1033-05	.2421-05	.4995-05	.9126-05	.1485-04	.2165-04	.2840-04
313.0	.1166-05	.2985-05	.6697-05	.1325-04	.2324-04	.3636-04	.5101-04	.6451-04
313.5	.3334-05	.8177-05	.1760-04	.3344-04	.5643-04	.8506-04	.1151-03	.1406-03
314.0	.9027-05	.2125-04	.4397-04	.8044-04	.1309-03	.1904-03	.2490-03	.2942-03
314.5	.2316-04	.5247-04	.1046-03	.1846-03	.2900-03	.4081-03	.5167-03	.5920-03
315.0	.5640-04	.1232-03	.2371-03	.4044-03	.6151-03	.8386-03	.1030-02	.1146-02
315.5	.1305-03	.2754-03	.5127-03	.8473-03	.1250-02	.1654-02	.1975-02	.2138-02
316.0	.2871-03	.5868-03	.1059-02	.1699-02	.2435-02	.3135-02	.3644-02	.3845-02
316.5	.6017-03	.1193-02	.2093-02	.3264-02	.4554-02	.5713-02	.6479-02	.6674-02
317.0	.1202-02	.2318-02	.3957-02	.6014-02	.8184-02	.1002-01	.1111-01	.1119-01
317.5	.2292-02	.4307-02	.7171-02	.1064-01	.1415-01	.1695-01	.1839-01	.1815-01
318.0	.4177-02	.7662-02	.1247-01	.1809-01	.2355-01	.2765-01	.2941-01	.2849-01
318.5	.7280-02	.1306-01	.2081-01	.2960-01	.3779-01	.4353-01	.4549-01	.4331-01
319.0	.1215-01	.2137-01	.3339-01	.4662-01	.5848-01	.6625-01	.6811-01	.6385-01
319.5	.1944-01	.3357-01	.5155-01	.7078-01	.8739-01	.9749-01	.9878-01	.9132-01
320.0	.2983-01	.5069-01	.7663-01	.1037+00	.1262+00	.1389+00	.1389+00	.1268+00
320.5	.4398-01	.7365-01	.1098+00	.1466+00	.1763+00	.1917+00	.1895+00	.1712+00
321.0	.6232-01	.1031+00	.1518+00	.2004+00	.2383+00	.2565+00	.2511+00	.2247+00
321.5	.8499-01	.1390+00	.2027+00	.2649+00	.3121+00	.3330+00	.3233+00	.2871+00
322.0	.1116+00	.1809+00	.2615+00	.3390+00	.3963+00	.4198+00	.4048+00	.3571+00
322.5	.1413+00	.2272+00	.3262+00	.4202+00	.4882+00	.5142+00	.4932+00	.4329+00
323.0	.1725+00	.2759+00	.3939+00	.5049+00	.5839+00	.6123+00	.5850+00	.5116+00
323.5	.2034+00	.3239+00	.4607+00	.5885+00	.6784+00	.7095+00	.6761+00	.5900+00
324.0	.2318+00	.3681+00	.5224+00	.6659+00	.7664+00	.8003+00	.7618+00	.6640+00
324.5	.2554+00	.4051+00	.5744+00	.7319+00	.8421+00	.8793+00	.8372+00	.7300+00
325.0	.2723+00	.4321+00	.6132+00	.7819+00	.9007+00	.9417+00	.8979+00	.7842+00
325.5	.2811+00	.4470+00	.6357+00	.8125+00	.9382+00	.9835+00	.9403+00	.8236+00
326.0	.2813+00	.4489+00	.6405+00	.8216+00	.9524+00	.1002+01	.9620+00	.8460+00
326.5	.2730+00	.4377+00	.6276+00	.8091+00	.9426+00	.9970+00	.9620+00	.8504+00
327.0	.2572+00	.4148+00	.5984+00	.7763+00	.9101+00	.9688+00	.9407+00	.8369+00
327.5	.2352+00	.3822+00	.5555+00	.7261+00	.8577+00	.9199+00	.8999+00	.8066+00
328.0	.2091+00	.3427+00	.5024+00	.6625+00	.7893+00	.8539+00	.8426+00	.7617+00
328.5	.1807+00	.2991+00	.4430+00	.5899+00	.7098+00	.7754+00	.7725+00	.7051+00
329.0	.1520+00	.2544+00	.3809+00	.5128+00	.6239+00	.6890+00	.6938+00	.6400+00
329.5	.1244+00	.2109+00	.3196+00	.4356+00	.5363+00	.5993+00	.6107+00	.5699+00
330.0	.9928-01	.1706+00	.2619+00	.3617+00	.4511+00	.5107+00	.5270+00	.4980+00
330.5	.7722-01	.1345+00	.2097+00	.2937+00	.3715+00	.4264+00	.4461+00	.4272+00
331.0	.5859-01	.1037+00	.1641+00	.2333+00	.2996+00	.3490+00	.3705+00	.3599+00
331.5	.4340-01	.7808-01	.1256+00	.1815+00	.2368+00	.2802+00	.3020+00	.2979+00
332.0	.3140-01	.5748-01	.9407-01	.1383+00	.1835+00	.2207+00	.2418+00	.2424+00
332.5	.2220-01	.4141-01	.6900-01	.1033+00	.1394+00	.1707+00	.1962+00	.1939+00
333.0	.1536-01	.2920-01	.4959-01	.7561-01	.1040+00	.1296+00	.1471+00	.1526+00
333.5	.1040-01	.2017-01	.3494-01	.5433-01	.7618-01	.9677-01	.1119+00	.1181+00
334.0	.6896-02	.1366-01	.2415-01	.3832-01	.5482-01	.7101-01	.8368-01	.9002-01
334.5	.4483-02	.9073-02	.1639-01	.2655-01	.3877-01	.5126-01	.6162-01	.6764-01

TABLE C4 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.7 M/S, 153°, OF 0.9983. VERTICAL POLARIZATION PAIR. INPUT 12 M/S, 30°.

	12.2	12.3	12.4	12.5	12.6	12.7	12.8	12.9	13.0
140.0	.5870-05	.1117-04	.1885-04	.2835-04	.3820-04	.4637-04	.5097-04	.5098-04	.4661-04
140.5	.1387-04	.2578-04	.4249-04	.6246-04	.8232-04	.9780-04	.1053-03	.1031-03	.9240-04
141.0	.3160-04	.5746-04	.9269-04	.1334-03	.1722-03	.2005-03	.2116-03	.2033-03	.1788-03
141.5	.6946-04	.1237-03	.1956-03	.2761-03	.3497-03	.3995-03	.4139-03	.3907-03	.3376-03
142.0	.1473-03	.2574-03	.3996-03	.5538-03	.6890-03	.7738-03	.7881-03	.7317-03	.6220-03
142.5	.3011-03	.5174-03	.7896-03	.1076-02	.1318-02	.1456-02	.1460-02	.1335-02	.1118-02
143.0	.5938-03	.1004-02	.1509-02	.2027-02	.2445-02	.2663-02	.2633-02	.2374-02	.1962-02
143.5	.1129-02	.1883-02	.2791-02	.3697-02	.4400-02	.4732-02	.4619-02	.4113-02	.3357-02
144.0	.2071-02	.3410-02	.4992-02	.6533-02	.7685-02	.8167-02	.7882-02	.6941-02	.5602-02
144.5	.3661-02	.5963-02	.8636-02	.1118-01	.1302-01	.1369-01	.1308-01	.1141-01	.9119-02
145.0	.6240-02	.1007-01	.1445-01	.1854-01	.2139-01	.2230-01	.2112-01	.1826-01	.1447-01
145.5	.1026-01	.1642-01	.2338-01	.2977-01	.3408-01	.3527-01	.3317-01	.2846-01	.2240-01
146.0	.1625-01	.2585-01	.3657-01	.4628-01	.5267-01	.5418-01	.5064-01	.4320-01	.3380-01
146.5	.2482-01	.3929-01	.5533-01	.6968-01	.7892-01	.8080-01	.7516-01	.6383-01	.4971-01
147.0	.3655-01	.5767-01	.8093-01	.1016+00	.1147+00	.1170+00	.1085+00	.9180-01	.7125-01
147.5	.5188-01	.8171-01	.1145+00	.1434+00	.1615+00	.1645+00	.1522+00	.1285+00	.9954-01
148.0	.7102-01	.1118+00	.1565+00	.1960+00	.2206+00	.2245+00	.2075+00	.1751+00	.1355+00
148.5	.9373-01	.1477+00	.2070+00	.2593+00	.2922+00	.2975+00	.2751+00	.2322+00	.1797+00
149.0	.1193+00	.1885+00	.2647+00	.3323+00	.3751+00	.3827+00	.3545+00	.2997+00	.2323+00
149.5	.1465+00	.2322+00	.3273+00	.4124+00	.4670+00	.4779+00	.4440+00	.3764+00	.2925+00
150.0	.1734+00	.2764+00	.3915+00	.4956+00	.5638+00	.5794+00	.5406+00	.4601+00	.3588+00
150.5	.1982+00	.3179+00	.4531+00	.5770+00	.6602+00	.6822+00	.6398+00	.5473+00	.4289+00
151.0	.2186+00	.3533+00	.5073+00	.6508+00	.7498+00	.7800+00	.7362+00	.6337+00	.4996+00
151.5	.2328+00	.3797+00	.5499+00	.7113+00	.8261+00	.8661+00	.8238+00	.7142+00	.5670+00
152.0	.2395+00	.3945+00	.5770+00	.7535+00	.8834+00	.9344+00	.8965+00	.7837+00	.6272+00
152.5	.2380+00	.3966+00	.5865+00	.7741+00	.9169+00	.9796+00	.9490+00	.8374+00	.6763+00
153.0	.2287+00	.3859+00	.5776+00	.7713+00	.9241+00	.9983+00	.9775+00	.8715+00	.7109+00
153.5	.2126+00	.3636+00	.5514+00	.7459+00	.9047+00	.9893+00	.9800+00	.8837+00	.7288+00
154.0	.1913+00	.3319+00	.5106+00	.7003+00	.8609+00	.9536+00	.9567+00	.8733+00	.7288+00
154.5	.1667+00	.2938+00	.4589+00	.6387+00	.7965+00	.8947+00	.9098+00	.8415+00	.7113+00
155.0	.1408+00	.2523+00	.4005+00	.5662+00	.7169+00	.8173+00	.8432+00	.7909+00	.6776+00
155.5	.1154+00	.2104+00	.3396+00	.4882+00	.6282+00	.7275+00	.7620+00	.7254+00	.6305+00
156.0	.9183-01	.1705+00	.2802+00	.4097+00	.5362+00	.6313+00	.6719+00	.6496+00	.5732+00
156.5	.7101-01	.1344+00	.2249+00	.3350+00	.4462+00	.5344+00	.5784+00	.5684+00	.5096+00
157.0	.5342-01	.1031+00	.1760+00	.2670+00	.3623+00	.4417+00	.4865+00	.4862+00	.4431+00
157.5	.3914-01	.7710-01	.1342+00	.2077+00	.2872+00	.3567+00	.4000+00	.4069+00	.3773+00
158.0	.2796-01	.5624-01	.9995-01	.1578+00	.2225+00	.2818+00	.3219+00	.3335+00	.3147+00
158.5	.1949-01	.4007-01	.7274-01	.1172+00	.1687+00	.2178+00	.2537+00	.2678+00	.2573+00
159.0	.1328-01	.2792-01	.5179-01	.8525-01	.1252+00	.1650+00	.1960+00	.2109+00	.2065+00
159.5	.8860-02	.1905-01	.3612-01	.6076-01	.9115-01	.1226+00	.1486+00	.1630+00	.1627+00
160.0	.5791-02	.1274-01	.2470-01	.4248-01	.6512-01	.8945-01	.1106+00	.1239+00	.1260+00
160.5	.3715-02	.8364-02	.1659-01	.2918-01	.4572-01	.6415-01	.8102-01	.9256-01	.9608-01
161.0	.2342-02	.5399-02	.1096-01	.1972-01	.3158-01	.4528-01	.5841-01	.6812-01	.7215-01
161.5	.1454-02	.3431-02	.7131-02	.1312-01	.2149-01	.3150-01	.4151-01	.4942-01	.5342-01
162.0	.8894-03	.2150-02	.4575-02	.8614-02	.1443-01	.2162-01	.2911-01	.3539-01	.3905-01
162.5	.5374-03	.1331-02	.2899-02	.5585-02	.9570-02	.1466-01	.2017-01	.2505-01	.2821-01
163.0	.3212-03	.8147-03	.1817-02	.3583-02	.6279-02	.9833-02	.1383-01	.1754-01	.2017-01
163.5	.1902-03	.4941-03	.1128-02	.2277-02	.4082-02	.6535-02	.9390-02	.1217-01	.1428-01
164.0	.1118-03	.2974-03	.6952-03	.1436-02	.2632-02	.4309-02	.6327-02	.8374-02	.1004-01
164.5	.6529-04	.1779-03	.4257-03	.8995-03	.1687-02	.2823-02	.4235-02	.5724-02	.7004-02
165.0	.3799-04	.1060-03	.2596-03	.5610-03	.1076-02	.1840-02	.2820-02	.3893-02	.4861-02
165.5	.2205-04	.6297-04	.1578-03	.3488-03	.6837-03	.1195-02	.1871-02	.2637-02	.3360-02
166.0	.1279-04	.3737-04	.9579-04	.2165-03	.4338-03	.7745-03	.1238-02	.1781-02	.2316-02
166.5	.7426-05	.2220-04	.5818-04	.1344-03	.2751-03	.5017-03	.8188-03	.1202-02	.1594-02
167.0	.4324-05	.1322-04	.3540-04	.8356-04	.1747-03	.3252-03	.5417-03	.8112-03	.1097-02
167.5	.2529-05	.7902-05	.2162-04	.5212-04	.1113-03	.2114-03	.3591-03	.5484-03	.7560-03
168.0	.1489-05	.4751-05	.1328-04	.3267-04	.7115-04	.1379-03	.2389-03	.3718-03	.5222-03
168.5	.8832-06	.2877-05	.8206-05	.2060-04	.4576-04	.9041-04	.1596-03	.2532-03	.3621-03
169.0	.5290-06	.1758-05	.5115-05	.1309-04	.2965-04	.5969-04	.1074-03	.1733-03	.2524-03
169.5	.3204-06	.1086-05	.3220-05	.8399-05	.1937-04	.3972-04	.7274-04	.1195-03	.1771-03
170.0	.1965-06	.6786-06	.2050-05	.5445-05	.1279-04	.2669-04	.4972-04	.8310-04	.1252-03
170.5	.1222-06	.4298-06	.1322-05	.3573-05	.8538-05	.1812-04	.3433-04	.5832-04	.8927-04
171.0	.7723-07	.2763-06	.8643-06	.2376-05	.5773-05	.1245-04	.2397-04	.4137-04	.6431-04
171.5	.4963-07	.1805-06	.5741-06	.1604-05	.3958-05	.8673-05	.1695-04	.2970-04	.4685-04
172.0	.3249-07	.1201-06	.3877-06	.1100-05	.2756-05	.6128-05	.1215-04	.2160-04	.3455-04
172.5	.2169-07	.8134-07	.2666-06	.7673-06	.1950-05	.4397-05	.8842-05	.1593-04	.2582-04
173.0	.1478-07	.5622-07	.1868-06	.5451-06	.1404-05	.3208-05	.6536-05	.1192-04	.1958-04

TABLE C5 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.9 M/S, 208.5 OF 0.9967. VERTICAL POLARIZATION PAIR. INPUT 12 M/S, 30°.

	12.4	12.5	12.6	12.7	12.8	12.9	13.0	13.1	13.2	13.3
195.0	.1157-03	.2651-03	.5378-03	.9714-03	.1570-02	.2280-02	.2992-02	.3562-02	.3864-02	.3834-02
195.5	.1805-03	.4063-03	.8099-03	.1438-02	.2284-02	.3263-02	.4212-02	.4934-02	.5268-02	.5147-02
196.0	.2808-03	.6207-03	.1216-02	.2121-02	.3313-02	.4654-02	.5910-02	.6812-02	.7158-02	.6887-02
196.5	.4348-03	.9440-03	.1817-02	.3116-02	.4784-02	.6610-02	.8256-02	.9364-02	.9686-02	.9175-02
197.0	.6695-03	.1428-02	.2701-02	.4552-02	.6872-02	.9338-02	.1147-01	.1281-01	.1304-01	.1216-01
197.5	.1024-02	.2146-02	.3988-02	.6608-02	.9809-02	.1311-01	.1585-01	.1741-01	.1745-01	.1603-01
198.0	.1554-02	.3200-02	.5846-02	.9522-02	.1390-01	.1827-01	.2174-01	.2350-01	.2320-01	.2099-01
198.5	.2338-02	.4731-02	.8495-02	.1361-01	.1954-01	.2527-01	.2958-01	.3149-01	.3061-01	.2728-01
199.0	.3482-02	.6926-02	.1223-01	.1926-01	.2721-01	.3464-01	.3992-01	.4183-01	.4005-01	.3517-01
199.5	.5129-02	.1003-01	.1742-01	.2700-01	.3753-01	.4702-01	.5335-01	.5507-01	.5194-01	.4494-01
200.0	.7465-02	.1436-01	.2454-01	.3742-01	.5119-01	.6315-01	.7057-01	.7177-01	.6670-01	.5690-01
200.5	.1073-01	.2030-01	.3413-01	.5124-01	.6903-01	.8388-01	.9234-01	.9254-01	.8478-01	.7131-01
201.0	.1520-01	.2832-01	.4687-01	.6929-01	.9194-01	.1101+00	.1194+00	.1180+00	.1066+00	.8842-01
201.5	.2122-01	.3893-01	.6347-01	.9242-01	.1209+00	.1426+00	.1525+00	.1486+00	.1324+00	.1084+00
202.0	.2917-01	.5272-01	.8468-01	.1215+00	.1567+00	.1823+00	.1923+00	.1848+00	.1625+00	.1314+00
202.5	.3944-01	.7025-01	.1112+00	.1574+00	.2001+00	.2297+00	.2391+00	.2269+00	.1970+00	.1572+00
203.0	.5241-01	.9206-01	.1438+00	.2007+00	.2518+00	.2852+00	.2932+00	.2746+00	.2355+00	.1857+00
203.5	.6840-01	.1185+00	.1827+00	.2518+00	.3118+00	.3488+00	.3541+00	.3277+00	.2777+00	.2164+00
204.0	.8760-01	.1499+00	.2281+00	.3105+00	.3798+00	.4198+00	.4211+00	.3853+00	.3228+00	.2488+00
204.5	.1100+00	.1860+00	.2796+00	.3761+00	.4548+00	.4969+00	.4929+00	.4460+00	.3697+00	.2819+00
205.0	.1354+00	.2262+00	.3363+00	.4473+00	.5350+00	.5783+00	.5676+00	.5082+00	.4169+00	.3147+00
205.5	.1632+00	.2697+00	.3968+00	.5222+00	.6181+00	.6612+00	.6425+00	.5697+00	.4629+00	.3461+00
206.0	.1925+00	.3150+00	.4587+00	.5978+00	.7007+00	.7426+00	.7148+00	.6280+00	.5057+00	.3748+00
206.5	.2221+00	.3601+00	.5195+00	.6709+00	.7794+00	.8187+00	.7813+00	.6805+00	.5434+00	.3995+00
207.0	.2505+00	.4026+00	.5761+00	.7377+00	.8500+00	.8856+00	.8384+00	.7246+00	.5741+00	.4189+00
207.5	.2761+00	.4403+00	.6251+00	.7944+00	.9085+00	.9397+00	.8832+00	.7578+00	.5963+00	.4321+00
208.0	.2971+00	.4705+00	.6634+00	.8375+00	.9514+00	.9775+00	.9128+00	.7782+00	.6085+00	.4383+00
208.5	.3120+00	.4912+00	.6885+00	.8639+00	.9757+00	.9967+00	.9254+00	.7845+00	.6101+00	.4370+00
209.0	.3197+00	.5006+00	.6982+00	.8717+00	.9795+00	.9956+00	.9198+00	.7760+00	.6006+00	.4282+00
209.5	.3193+00	.4980+00	.6916+00	.8599+00	.9622+00	.9740+00	.8962+00	.7531+00	.5805+00	.4122+00
210.0	.3109+00	.4834+00	.6690+00	.8291+00	.9247+00	.9330+00	.8557+00	.7167+00	.5507+00	.3898+00
210.5	.2949+00	.4575+00	.6317+00	.7810+00	.8690+00	.8747+00	.8003+00	.6687+00	.5126+00	.3620+00
211.0	.2725+00	.4220+00	.5820+00	.7186+00	.7984+00	.8024+00	.7331+00	.6116+00	.4681+00	.3301+00
211.5	.2450+00	.3794+00	.5231+00	.6455+00	.7168+00	.7200+00	.6574+00	.5481+00	.4192+00	.2954+00
212.0	.2144+00	.3322+00	.4583+00	.5659+00	.6288+00	.6319+00	.5771+00	.4813+00	.3682+00	.2595+00
212.5	.1825+00	.2833+00	.3915+00	.4842+00	.5387+00	.5421+00	.4957+00	.4139+00	.3170+00	.2236+00
213.0	.1510+00	.2352+00	.3259+00	.4040+00	.4507+00	.4546+00	.4166+00	.3486+00	.2675+00	.1891+00
213.5	.1215+00	.1899+00	.2642+00	.3288+00	.3680+00	.3724+00	.3424+00	.2874+00	.2212+00	.1568+00
214.0	.9495-01	.1492+00	.2086+00	.2608+00	.2933+00	.2981+00	.2752+00	.2319+00	.1792+00	.1275+00
214.5	.7210-01	.1140+00	.1604+00	.2017+00	.2280+00	.2330+00	.2163+00	.1832+00	.1422+00	.1016+00
215.0	.5315-01	.8469-01	.1200+00	.1519+00	.1729+00	.1779+00	.1661+00	.1415+00	.1105+00	.7943-01
215.5	.3804-01	.6114-01	.8733-01	.1115+00	.1279+00	.1325+00	.1247+00	.1069+00	.8408-01	.6083-01

TABLE C6 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.0 M/S, 32° OF 0.8933 VERTICAL POLARIZATION PLUS 20° BEAM. INPUT 12 M/S, 30°.

	11.7	11.8	11.9	12.0	12.1	12.2	12.3
18.0	.1962-05	.5547-05	.1348-04	.2835-04	.5192-04	.8330-04	.1178-03
18.5	.4088-05	.1115-04	.2617-04	.5318-04	.9417-04	.1462-03	.2002-03
19.0	.8442-05	.2222-04	.5036-04	.9891-04	.1694-03	.2546-03	.3376-03
19.5	.1725-04	.4383-04	.9594-04	.1821-03	.3018-03	.4370-03	.5639-03
20.0	.3480-04	.8539-04	.1806-03	.3316-03	.5316-03	.7487-03	.9321-03
20.5	.6925-04	.1641-03	.3356-03	.5959-03	.9246-03	.1262-02	.1522-02
21.0	.1357-03	.3107-03	.6143-03	.1056-02	.1586-02	.2097-02	.2454-02
21.5	.2612-03	.5784-03	.1106-02	.1841-02	.2680-02	.3435-02	.3899-02
22.0	.4936-03	.1057-02	.1958-02	.3156-02	.4453-02	.5537-02	.6102-02
22.5	.9140-03	.1896-02	.3401-02	.5312-02	.7271-02	.8775-02	.9392-02
23.0	.1656-02	.3328-02	.5787-02	.8769-02	.1165-01	.1366-01	.1421-01
23.5	.2933-02	.5714-02	.9640-02	.1418-01	.1830-01	.2085-01	.2110-01
24.0	.5070-02	.9584-02	.1570-01	.2243-01	.2815-01	.3120-01	.3072-01
24.5	.8542-02	.1568-01	.2497-01	.3470-01	.4236-01	.4571-01	.4385-01
25.0	.1401-01	.2501-01	.3874-01	.5240-01	.6230-01	.6551-01	.6129-01
25.5	.[illegible]235-01	.3884-01	.5857-01	.7720-01	.8948-01	.9179-01	.8381-01
26.0	.3464-01	.5864-01	.8623-01	.1109+00	.1254+00	.1256+00	.1121+00
26.5	.5208-01	.8604-01	.1235+00	.1550+00	.1714+00	.1678+00	.1464+00
27.0	.7593-01	.1225+00	.1719+00	.2110+00	.2282+00	.2186+00	.1868+00
27.5	.1072+00	.1692+00	.2323+00	.2792+00	.2957+00	.2777+00	.2326+00
28.0	.1465+00	.2265+00	.3048+00	.3591+00	.3730+00	.3437+00	.2826+00
28.5	.1936+00	.2937+00	.3877+00	.4485+00	.4575+00	.4142+00	.3347+00
29.0	.2471+00	.3683+00	.4780+00	.5436+00	.5454+00	.4858+00	.3863+00
29.5	.3045+00	.4467+00	.5707+00	.6391+00	.6316+00	.5542+00	.4344+00
30.0	.3620+00	.5236+00	.6596+00	.7285+00	.7102+00	.6149+00	.4758+00
30.5	.4150+00	.5928+00	.7375+00	.8046+00	.7751+00	.6632+00	.5072+00
31.0	.4584+00	.6477+00	.7972+00	.8608+00	.8205+00	.6950+00	.5262+00
31.5	.4874+00	.6827+00	.8329+00	.8914+00	.8424+00	.7074+00	.5311+00
32.0	.4988+00	.6937+00	.8405+00	.8933+00	.8383+00	.6992+00	.5213+00
32.5	.4910+00	.6794+00	.8189+00	.8658+00	.8084+00	.6707+00	.4975+00
33.0	.4645+00	.6408+00	.7699+00	.8114+00	.7551+00	.6244+00	.4616+00
33.5	.4222+00	.5818+00	.6983+00	.7349+00	.6829+00	.5638+00	.4161+00
34.0	.3684+00	.5083+00	.6105+00	.6430+00	.5978+00	.4937+00	.3644+00
34.5	.3085+00	.4270+00	.5145+00	.5433+00	.5063+00	.4191+00	.3100+00
35.0	.2478+00	.3448+00	.4176+00	.4431+00	.4148+00	.3447+00	.2560+00
35.5	.1907+00	.2675+00	.3263+00	.3487+00	.3285+00	.2747+00	.2051+00
36.0	.1406+00	.1993+00	.2454+00	.2646+00	.2514+00	.2120+00	.1595+00
36.5	.9929-01	.1424+00	.1775+00	.1935+00	.1859+00	.1583+00	.1203+00
37.0	.6706-01	.9764-01	.1234+00	.1364+00	.1327+00	.1144+00	.8799-01
37.5	.4331-01	.6415-01	.8242-01	.9254-01	.9140-01	.7997-01	.6235-01
38.0	.2672-01	.4037-01	.5286-01	.6043-01	.6073-01	.5402-01	.4280-01
38.5	.1574-01	.2432-01	.3253-01	.3795-01	.3890-01	.3526-01	.2845-01
39.0	.8848-02	.1401-01	.1919-01	.2291-01	.2400-01	.2222-01	.1829-01
39.5	.4740-02	.7715-02	.1085-01	.1328-01	.1426-01	.1351-01	.1138-01
40.0	.2419-02	.4057-02	.5873-02	.7393-02	.8152-02	.7926-02	.6840-02
40.5	.1175-02	.2036-02	.3041-02	.3946-02	.4479-02	.4479-02	.3971-02
41.0	.5423-03	.9737-03	.1505-02	.2018-02	.2365-02	.2438-02	.2225-02
41.5	.2378-03	.4436-03	.7114-03	.9883-03	.1198-02	.1276-02	.1202-02

TABLE C7 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 13.3 M/S 339° OF 2.875×10^{-4} VERTICAL POLARIZATION PLUS 20° BEAM. INPUT 12 M/S, 30°.

	13.0	13.1	13.2	13.3	13.4	13.5	13.6
330.0	.1958-06	.3986-06	.7049-06	.1090-05	.1484-05	.1789-05	.1921-05
330.5	.4209-06	.8238-06	.1403-05	.2094-05	.2754-05	.3212-05	.3342-05
331.0	.8593-06	.1621-05	.2664-05	.3842-05	.4892-05	.5531-05	.5587-05
331.5	.1668-05	.3037-05	.4828-05	.6743-05	.8325-05	.9142-05	.8978-05
332.0	.3082-05	.5429-05	.8360-05	.1133-04	.1359-04	.1451-04	.1388-04
332.5	.5426-05	.9263-05	.1384-04	.1823-04	.2128-04	.2215-04	.2067-04
333.0	.9112-05	.1510-04	.2195-04	.2814-04	.3203-04	.3254-04	.2967-04
333.5	.1461-04	.2356-04	.3335-04	.4170-04	.4634-04	.4602-04	.4107-04
334.0	.2240-04	.3520-04	.4861-04	.5939-04	.6454-04	.6276-04	.5488-04
334.5	.3287-04	.5041-04	.6805-04	.8135-04	.8660-04	.8257-04	.7088-04
335.0	.4621-04	.6930-04	.9159-04	.1073-03	.1121-03	.1049-03	.8854-04
335.5	.6233-04	.9156-04	.1186-03	.1364-03	.1400-03	.1289-03	.1071-03
336.0	.8076-04	.1164-03	.1481-03	.1674-03	.1690-03	.1533-03	.1255-03
336.5	.1006-03	.1425-03	.1783-03	.1984-03	.1974-03	.1765-03	.1426-03
337.0	.1208-03	.1682-03	.2073-03	.2274-03	.2232-03	.1971-03	.1574-03
337.5	.1397-03	.1918-03	.2331-03	.2524-03	.2447-03	.2136-03	.1688-03
338.0	.1561-03	.2114-03	.2537-03	.2715-03	.2604-03	.2250-03	.1761-03
338.5	.1686-03	.2255-03	.2677-03	.2835-03	.2692-03	.2306-03	.1789-03
339.0	.1763-03	.2332-03	.2741-03	.2875-03	.2708-03	.2300-03	.1772-03
339.5	.1786-03	.2341-03	.2726-03	.2837-03	.2651-03	.2237-03	.1712-03
340.0	.1757-03	.2284-03	.2639-03	.2726-03	.2531-03	.2123-03	.1616-03
340.5	.1680-03	.2168-03	.2488-03	.2555-03	.2359-03	.1969-03	.1492-03
341.0	.1564-03	.2005-03	.2288-03	.2337-03	.2148-03	.1786-03	.1349-03
341.5	.1419-03	.1809-03	.2055-03	.2090-03	.1914-03	.1586-03	.1194-03
342.0	.1257-03	.1595-03	.1805-03	.1829-03	.1671-03	.1381-03	.1038-03
342.5	.1089-03	.1376-03	.1552-03	.1569-03	.1430-03	.1180-03	.8858-04
343.0	.9228-04	.1163-03	.1309-03	.1321-03	.1202-03	.9909-04	.7433-04
343.5	.7670-04	.9648-04	.1084-03	.1092-03	.9934-04	.8186-04	.6141-04
344.0	.6259-04	.7862-04	.8822-04	.8889-04	.8081-04	.6661-04	.5000-04
344.5	.5023-04	.6303-04	.7070-04	.7124-04	.6480-04	.5345-04	.4016-04
345.0	.3969-04	.4980-04	.5586-04	.5632-04	.5127-04	.4234-04	.3187-04
345.5	.3094-04	.3882-04	.4358-04	.4398-04	.4009-04	.3316-04	.2501-04
346.0	.2381-04	.2990-04	.3360-04	.3396-04	.3101-04	.2571-04	.1943-04
346.5	.1813-04	.2279-04	.2565-04	.2597-04	.2376-04	.1974-04	.1496-04
347.0	.1368-04	.1722-04	.1941-04	.1969-04	.1806-04	.1505-04	.1144-04
347.5	.1024-04	.1291-04	.1458-04	.1482-04	.1363-04	.1139-04	.8683-05
348.0	.7611-05	.9614-05	.1088-04	.1109-04	.1023-04	.8574-05	.6558-05
348.5	.5631-05	.7127-05	.8086-05	.8265-05	.7644-05	.6427-05	.4932-05
349.0	.4151-05	.5265-05	.5988-05	.6137-05	.5694-05	.4802-05	.3698-05
349.5	.3054-05	.3881-05	.4426-05	.4548-05	.4232-05	.3581-05	.2767-05
350.0	.2245-05	.2859-05	.3269-05	.3368-05	.3144-05	.2669-05	.2069-05
350.5	.1651-05	.2108-05	.2415-05	.2496-05	.2336-05	.1990-05	.1548-05
351.0	.1217-05	.1557-05	.1789-05	.1852-05	.1739-05	.1485-05	.1159-05
351.5	.9000-06	.1153-05	.1328-05	.1379-05	.1298-05	.1112-05	.8704-06
352.0	.6687-06	.8585-06	.9902-06	.1031-05	.9725-06	.8353-06	.6558-06
352.5	.4998-06	.6427-06	.7425-06	.7744-06	.7323-06	.6305-06	.4963-06
353.0	.3763-06	.4844-06	.5605-06	.5856-06	.5548-06	.4787-06	.3777-06
353.5	.2857-06	.3681-06	.4264-06	.4461-06	.4233-06	.3659-06	.2893-06
354.0	.2190-06	.2823-06	.3272-06	.3427-06	.3256-06	.2819-06	.2232-06
354.5	.1697-06	.2187-06	.2536-06	.2658-06	.2527-06	.2190-06	.1736-06
355.0	.1330-06	.1714-06	.1987-06	.2083-06	.1981-06	.1718-06	.1363-06
355.5	.1056-06	.1360-06	.1576-06	.1651-06	.1570-06	.1362-06	.1081-06
356.0	.8500-07	.1093-06	.1265-06	.1324-06	.1259-06	.1091-06	.8661-07
356.5	.6743-07	.8909-07	.1030-06	.1076-06	.1022-06	.8851-07	.7020-07

TABLE C8 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 13.4 M/S, 161° OF 9.434×10^{-3} VERTICAL POLARIZATION PLUS 20 DEGREE BEAM. INPUT 12 M/S, 30°.

	13.1	13.2	13.3	13.4	13.5	13.6
	.4862-07	.1277-06	.2853-06	.5165-06	[illegible]	.1301-05
	.1435-06	.3596-06	.7534-06	.1307-05	[illegible]	.2662-05
	.3975-06	[illegible]	.1976-05	.3433-05	[illegible]	.6753-05
	.1050-05	.2480-05	[illegible]	[illegible]	.1163-04	.1495-04
	.2604-05	.5753-05	.1060-04	.1756-04	.2139-04	.3073-04
	.6102-05	.1292-04	.2345-04	.3626-04	.5021-04	.5765-04
	.1352-04	.2753-04	.4830-04	.7312-04	.9648-04	.1116-03
	.2831-04	.5582-04	.9350-04	.1385-03	.1779-03	.1992-03
	.5627-04	.1072-03	.1737-03	.2472-03	.3098-03	.3327-03
	.1059-03	.1954-03	.3107-03	.4279-03	.5177-03	.5421-03
	.1890-03	.3385-03	.5252-03	.7047-03	.8258-03	.8517-03
	.3203-03	.5586-03	.8375-03	.1102-02	.1266-02	.1273-02
	.5162-03	.8758-03	.1285-02	.1661-02	.1858-02	.1817-02
	.7918-03	.1310-02	.1877-02	.2386-02	.2576-02	.2486-02
	.1157-02	.1871-02	.2642-02	.3207-02	.3441-02	.3271-02
	.1614-02	.2554-02	.3506-02	.4203-02	.4432-02	.4132-02
	.2150-02	.3335-02	.4490-02	.5206-02	.5425-02	.5022-02
	.2740-02	.4171-02	.5518-02	.6386-02	.6507-02	.5879-02
	.3346-02	.5606-02	.6511-02	.7432-02	.7353-02	.6677-02
	.3921-02	.5773-02	.7390-02	.8308-02	.8222-02	.7231-02
	.4415-02	.6405-02	.8073-02	.8987-02	.8774-02	.7621-02
	.4785-02	.6848-02	.8543-02	.9353-02	.9045-02	.7772-02
161°	.5061-02	.7067-02	.8713-02	.9134-02	.9029-02	.7603-02
	.5017-02	.7051-02	.8640-02	.9219-02	.8779-02	.7371-02
	.4927-02	.6812-02	.8228-02	.8740-02	.8219-02	.6873-02
	.4662-02	.6434-02	.7643-02	.8052-02	.7519-02	.6240-02
	.4283-02	.5814-02	.6205-02	.7220-02	.6673-02	.5522-02
	.3827-02	.5154-02	.5077-02	.6312-02	.5821-02	.4721-02
	.3372-02	.4436-02	.5317-02	.5331-02	.4741-02	.3911-02
	.2833-02	.3746-02	.4402-02	.4504-02	.4104-02	.3335-02
	.2355-02	.3111-02	.3643-02	.3603-02	.3344-02	[illegible]
	.1919-02	.2521-02	.2905-02	.2924-02	.2671-02	.2159-02
	.1535-02	.2007-02	.2304-02	.2336-02	.2106-02	.1694-02
	.1208-02	.1573-02	.1793-02	.1817-02	.1632-02	.1311-02
	.9370-03	.1215-02	.1387-02	.1373-02	.1219-02	.1001-02
	.7175-03	.9265-03	.1052-02	.1077-02	.9450-03	.7553-03
	.5436-03	.6996-03	.7912-03	.7932-03	.7080-03	.5653-03
	.4081-03	.5256-03	.5912-03	.5910-03	.5261-03	.4194-03
	.3042-03	.3901-03	.4381-03	.4370-03	.3883-03	.3091-03
	.2256-03	.2876-03	.3230-03	.3215-03	.2852-03	.2262-03
	.1666-03	.2118-03	.2373-03	.2357-03	.2087-03	.1652-03
	.1227-03	.1557-03	.1740-03	.1725-03	.1527-03	.1207-03
	.9058-04	.1134-03	.1276-03	.1261-03	.1113-03	.8811-04
	.6687-04	.8412-04	.9361-04	.7236-04	.8134-04	.6429-04
	.4950-04	.6214-04	.6887-04	.6779-04	.5966-04	.4701-04
	.3681-04	.4605-04	.5108-04	.4995-04	.4390-04	.3450-04
	.2753-04	.3431-04	.3779-04	.3700-04	.3236-04	.2515-04
	.2074-04	.2575-04	.2826-04	.2756-04	.2406-04	.1865-04
	.1576-04	.1930-04	.2139-04	.2072-04	.1802-04	.1409-04
	.1209-04	.1483-04	.1620-04	.1570-04	.1361-04	.1061-04
	.9379-05	.1149-04	.1245-04	.1202-04	.1038-04	.8065-05
	.7363-05	.8970-05	.9677-05	.9293-05	.7975-05	.6172-05
	.5876-05	.7143-05	.7613-05	[illegible]	.6234-05	.4997-05
	.4772-05	.5835-05	.6060-05	.5722-05	.4917-05	.3776-05
	.3861-05	.4623-05	.4901-05	.4632-05	.3930-05	.3054-05
	.3212-05	.3816-05	.4021-05	.3780-05	.3187-05	.2421-05
	.2713-05	.3199-05	.3348-05	.3121-05	.2620-05	.1972-05
	.2331-05	.2727-05	.2832-05	.2627-05	.2186-05	.1631-05
	.2037-05	.2363-05	.2435-05	.2241-05	.1852-05	.1371-05
	.1812-05	.2083-05	.2129-05	.1941-05	.1594-05	.1180-05
	.1641-05	.1867-05	.1895-05	.1714-05	.1394-05	.1024-05
	.1514-05	.1708-05	.1713-05	.1537-05	.1240-05	.9031-06
	.1422-05	.1588-05	.1577-05	.1402-05	.1120-05	.8092-06
	.1362-05	.1504-05	.1478-05	.1301-05	.1030-05	.7367-06
	.1328-05	.1450-05	.1410-05	.1228-05	.9619-06	.6817-06
	.1319-05	.1423-05	.1368-05	.1178-05	.9138-06	.6410-06
	.1335-05	.1423-05	.1352-05	.1151-05	.8835-06	.6125-06

TABLE C9 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 12.0 M/S 31.5^{o} OF 0.7856 ALL SIX MEASUREMENTS. INPUT 12 M/S, 30^{o}.

12.0

	.2103-10	.1231-09	.5586-09	.1985-08	.5577-08	.1251-07	.2261-07
	.6134-10	.3424-09	.1483-08	.5031-08	.1351-07	.2899-07	.5014-07
	.1791-09	.9532-09	.3939-08	.1276-07	.3275-07	.6719-07	.1112-06
	.5220-09	.2650-08	.1045-07	.3233-07	.7927-07	.1555-06	.2464-06
	.1516-08	.7338-08	.2761-07	.8159-07	.1912-06	.3587-06	.5439-06
	.4373-08	.2019-07	.7253-07	.2047-06	.4584-06	.8226-06	.1194-05
	.1250-07	.5509-07	.1889-06	.5092-06	.1090-05	.1872-05	.2600-05
	.3535-07	.1486-06	.4867-06	.1254-05	.2567-05	.4216-05	.5609-05
	.9854-07	.3956-06	.1237-05	.3048-05	.5968-05	.9384-05	.1196-04
	.2702-06	.1036-05	.3098-05	.7298-05	.1368-04	.2060-04	.2515-04
	.7272-06	.2666-05	.7622-05	.1718-04	.3083-04	.4449-04	.5209-04
	.1916-05	.6717-05	.1838-04	.3967-04	.6822-04	.9438-04	.1060-03
	.4929-05	.1654-04	.4336-04	.8968-04	.1479-03	.1963-03	.2117-03
	.1235-04	.3973-04	.9982-04	.1980-03	.3134-03	.3995-03	.4140-03
	.3011-04	.9284-04	.2238-03	.4263-03	.6481-03	.7940-03	.7914-03
	.7116-04	.2107-03	.4879-03	.8931-03	.1305-02	.1539-02	.1476-02
	.1628-03	.4633-03	.1032-02	.1817-02	.2557-02	.2902-02	.2684-02
	.3596-03	.9850-03	.2112-02	.3584-02	.4860-02	.5320-02	.4746-02
	.7656-03	.2021-02	.4179-02	.6840-02	.8951-02	.9461-02	.8153-02
	.1568-02	.3995-02	.7977-02	.1261-01	.1595-01	.1630-01	.1359-01
	.3082-02	.7593-02	.1466-01	.2242-01	.2745-01	.2716-01	.2193-01
	.5804-02	.1385-01	.2591-01	.3840-01	.4556-01	.4371-01	.3424-01
	.1045-01	.2420-01	.4394-01	.6321-01	.7282-01	.6787-01	.5166-01
	.1798-01	.4046-01	.7140-01	.9989-01	.1119+00	.1015+00	.7521-01
	.2946-01	.6457-01	.1110+00	.1513+00	.1653+00	.1461+00	.1055+00
	.4593-01	.9827-01	.1649+00	.2195+00	.2341+00	.2021+00	.1426+00
	.6802-01	.1424+00	.2337+00	.3043+00	.3175+00	.2683+00	.1853+00
	.9554-01	.1961+00	.3156+00	.4029+00	.4122+00	.3415+00	.2313+00
	.1271+00	.2563+00	.4055+00	.5086+00	.5114+00	.4164+00	.2772+00
	.1598+00	.3176+00	.4950+00	.6117+00	.6058+00	.4858+00	.3186+00
	.1898+00	.3727+00	.5736+00	.6999+00	.6844+00	.5419+00	.3508+00
	.2126+00	.4134+00	.6300+00	.7611+00	.7366+00	.5772+00	.3698+00
31.5^{o} —	.2242+00	.4330+00	.6552+00	.7856+00	.7546+00	.5866+00	.3728+00
	.2224+00	.4277+00	.6444+00	.7691+00	.7350+00	.5684+00	.3592+00
	.2071+00	.3980+00	.5988+00	.7132+00	.6801+00	.5245+00	.3305+00
	.1809+00	.3484+00	.5250+00	.6260+00	.5972+00	.4606+00	.2901+00
	.1481+00	.2866+00	.4338+00	.5194+00	.4971+00	.3845+00	.2428+00
	.1134+00	.2213+00	.3376+00	.4069+00	.3920+00	.3050+00	.1936+00
	.8112-01	.1602+00	.2470+00	.3008+00	.2925+00	.2296+00	.1469+00
	.5417-01	.1086+00	.1699+00	.2096+00	.2064+00	.1639+00	.1060+00
	.3371-01	.6885-01	.1096+00	.1375+00	.1375+00	.1108+00	.7272-01
	.1953-01	.4078-01	.6627-01	.8483-01	.8647-01	.7095-01	.4736-01
	.1052-01	.2253-01	.3753-01	.4917-01	.5124-01	.4295-01	.2925-01
	.5258-02	.1160-01	.1987-01	.2674-01	.2860-01	.2456-01	.1713-01
	.2437-02	.5559-02	.9829-02	.1364-01	.1501-01	.1326-01	.9494-02
	.1046-02	.2476-02	.4535-02	.6509-02	.7403-02	.6747-02	.4979-02
	.4151-03	.1023-02	.1950-02	.2906-02	.3427-02	.3233-02	.2467-02
	.1521-03	.3921-03	.7799-03	.1212-02	.1497-02	.1458-02	.1154-02
	.5136-04	.1391-03	.2899-03	.4712-03	.6040-03	.6174-03	.5089-03
	.1597-04	.4558-04	.9999-04	.1707-03	.2294-03	.2454-03	.2114-03
	.4561-05	.1379-04	.3195-04	.5752-04	.8135-04	.9144-04	.8257-04
	.1196-05	.3842-05	.9447-05	.1800-04	.2690-04	.3188-04	.3030-04
	.2870-06	.9848-06	.2580-05	.5226-05	.8283-05	.1039-04	.1043-04
	.6300-07	.2318-06	.6498-06	.1405-05	.2371-05	.3161-05	.3365-05
	.1262-07	.5003-07	.1507-06	.3491-06	.6300-06	.8959-06	.1015-05
	.2304-08	.9882-08	.3211-07	.8007-07	.1551-06	.2363-06	.2860-06
	.3825-09	.1783-08	.6278-08	.1692-07	.3534-07	.5788-07	.7516-07
	.5763-10	.2933-09	.1124-08	.3289-08	.7436-08	.1315-07	.1839-07
	.7866-11	.4390-10	.1840-09	.5869-09	.1442-08	.2765-08	.4180-08
	.9705-12	.5968-11	.2748-10	.9596-10	.2575-09	.5372-09	.8817-09
	.1080-12	.7354-12	.3736-11	.1435-10	.4221-10	.9628-10	.1722-09
	.1083-13	.8197-13	.4615-12	.1959-11	.6345-11	.1588-10	.3110-10
	.9747-15	.8247-14	.5171-13	.2435-12	.8725-12	.2408-11	.5181-11
	.7867-16	.7475-15	.5243-14	.2752-13	.1095-12	.3347-12	.7948-12
	.5680-17	.6089-16	.4801-15	.2822-14	.1253-13	.4258-13	.1120-12
	.3660-18	.4449-17	.3961-16	.2620-15	.1304-14	.4948-14	.1449-13
	.2101-19	.2909-18	.2939-17	.2197-16	.1231-15	.5239-15	.1714-14
	.1072-20	.1699-19	.1957-18	.1661-17	.1053-16	.5046-16	.1853-15
	.4848-22	.8843-21	.1167-19	.1130-18	.8132-18	.4412-17	[illegible]
	.1941-23	.4093-22	.6216-21	.6896-20	.5666-19	.3493-18	.1637-17
	.6860-25	.1681-23	.2952-22	.3771-21	.3552-20	.2500-19	.1332-18
	.2137-26	.6113-25	.1248-23	.1844-22	.2000-21	.1614-20	.9819-20
	.5852-28	.1964-26	.4682-25	.8041-24	.1009-22	.9381-22	.6544-21
	.1407-29	.5568-28	.1557-26	.3123-25	.4555-24	.4899-23	.3936-22
	.2962-31	.1389-29	.4581-28	.1078-26	.1835-25	.2294-24	.2132-23
	.5452-33	.3046-31	.1190-29	.3299-28	.6589-27	.9612-26	.1038-24
	.8761-35	.5856-33	.2723-31	.8941-30	.2104-28	.3598-27	.4534-26
	.1227-36	.9859-35	.5482-33	.2142-31	.5965-30	.1201-28	.1774-27

TABLE C10 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 13.3 M/S 340° OF 2.970 x 10^{-7} ALL SIX MEASUREMENTS INPUT 12 M/S, 30°

						13.3			
	.5482-21	.1045-19	.1456-18	.1504-17	.1166-16	.6870-16	.3115-15	.1099-14	.3051-14
	.5552-20	.9410-19	.1171-17	.1085-16	.7579-16	.4040-15	.1663-14	.5346-14	.1357-13
	.5074-19	.7677-18	.8568-17	.7146-16	.4512-15	.2182-14	.8182-14	.2405-13	.5599-13
	.4190-18	.5681-17	.5705-16	.4299-15	.2462-14	.1084-13	.3715-13	.1001-12	.2144-12
	.3131-17	.3818-16	.3462-15	.2366-14	.1233-13	.4960-13	.1558-12	.3861-12	.7633-12
	.2120-16	.2333-15	.1917-14	.1192-13	.5673-13	.2092-12	.6041-12	.1382-11	.2528-11
	.1302-15	.1298-14	.9705-14	.5508-13	.2402-12	.8143-12	.2169-11	.4591-11	.7798-11
	.7269-15	.6589-14	.4496-13	.2337-12	.9370-12	.2930-11	.7223-11	.1419-10	.2244-10
	.3693-14	.3055-13	.1909-12	.9121-12	.3372-11	.9757-11	.2233-10	.4084-10	.6030-10
	.1710-13	.1296-12	.7440-12	.3279-11	.1122-10	.3012-10	.6417-10	.1096-09	.1515-09
	.7235-13	.5036-12	.2667-11	.1087-10	.3452-10	.8632-10	.1717-09	.2746-09	.3565-09
	.2800-12	.1797-11	.8803-11	.3331-10	.9849-10	.2300-09	.4285-09	.6436-09	.7866-09
	.9933-12	.5897-11	.2681-10	.9449-10	.2609-09	.5706-09	.9986-09	.1412-08	.1630-08
	.3236-11	.1783-10	.7550-10	.2485-09	.6427-09	.1320-08	.2177-08	.2907-08	.3176-08
	.9699-11	.4977-10	.1968-09	.6070-09	.1475-08	.2855-08	.4445-08	.5622-08	.5829-08
	.2680-10	.1285-09	.4761-09	.1380-08	.3159-08	.5776-08	.8518-08	.1023-07	.1009-07
	.6842-10	.3073-09	.1070-08	.2923-08	.6325-08	.1096-07	.1534-07	.1754-07	.1650-07
	.1617-09	.6827-09	.2241-08	.5785-08	.1186-07	.1952-07	.2602-07	.2838-07	.2554-07
	.3546-09	.1411-08	.4379-08	.1071-07	.2087-07	.3270-07	.4162-07	.4342-07	.3746-07
	.7227-09	.2720-08	.8001-08	.1860-07	.3452-07	.5165-07	.6290-07	.6291-07	.5214-07
	.1373-08	.4897-08	.1369-07	.3033-07	.5376-07	.7700-07	.8995-07	.8648-07	.6902-07
	.2434-08	.8256-08	.2200-07	.4656-07	.7902-07	.1086-06	.1220-06	.1130-06	.8701-07
	.4039-08	.1306-07	.3326-07	.6742-07	.1098-06	.1451-06	.1571-06	.1404-06	.1046-06
	.6287-08	.1943-07	.4741-07	.9224-07	.1445-06	.1841-06	.1925-06	.1665-06	.1202-06
	.9201-08	.2724-07	.6383-07	.1195-06	.1806-06	.2222-06	.2248-06	.1886-06	.1322-06
	.1269-07	.3608-07	.8136-07	.1469-06	.2145-06	.2556-06	.2508-06	.2043-06	.1394-06
	.1652-07	.4523-07	.9839-07	.1717-06	.2428-06	.2807-06	.2676-06	.2122-06	.1411-06
	.2036-07	.5380-07	.1131-06	.1913-06	.2625-06	.2949-06	.2738-06	.2117-06	.1374-06
340°	.2382-07	.6085-07	.1240-06	.2035-06	.2714-06	.2970-06	.2689-06	.2031-06	.1290-06
	.2650-07	.6559-07	.1297-06	.2070-06	.2691-06	.2873-06	.2541-06	.1878-06	.1169-06
	.2810-07	.6754-07	.1299-06	.2020-06	.2562-06	.2673-06	.2315-06	.1676-06	.1024-06
	.2848-07	.6659-07	.1248-06	.1894-06	.2348-06	.2399-06	.2036-06	.1447-06	.8687-07
	.2766-07	.6300-07	.1153-06	.1710-06	.2076-06	.2079-06	.1733-06	.1211-06	.7153-07
	.2578-07	.5733-07	.1026-06	.1490-06	.1774-06	.1745-06	.1429-06	.9834-07	.5725-07
	.2313-07	.5030-07	.8813-07	.1256-06	.1468-06	.1420-06	.1145-06	.7766-07	.4461-07
	.2003-07	.4264-07	.7328-07	.1025-06	.1179-06	.1123-06	.8928-07	.5975-07	.3391-07
	.1677-07	.3502-07	.5910-07	.8133-07	.9207-07	.8644-07	.6785-07	.4487-07	.2519-07
	.1361-07	.2791-07	.4633-07	.6278-07	.7007-07	.6494-07	.5037-07	.3295-07	.1831-07
	.1074-07	.2165-07	.3538-07	.4727-07	.5207-07	.4769-07	.3659-07	.2370-07	.1305-07
	.8252-08	.1638-07	.2638-07	.3479-07	.3787-07	.3430-07	.2606-07	.1673-07	.9139-08
	.6192-08	.1211-07	.1925-07	.2508-07	.2700-07	.2421-07	.1823-07	.1161-07	.6297-08
	.4547-08	.8776-08	.1378-07	.1775-07	.1891-07	.1681-07	.1255-07	.7934-08	.4276-08
	.3277-08	.6244-08	.9691-08	.1236-07	.1304-07	.1150-07	.8520-08	.5350-08	.2867-08
	.2322-08	.4372-08	.6714-08	.8478-08	.8875-08	.7761-08	.5713-08	.3566-08	.1901-08
	.1622-08	.3020-08	.4592-08	.5747-08	.5968-08	.5182-08	.3791-08	.2354-08	.1249-08
	.1119-08	.2063-08	.3107-08	.3856-08	.3975-08	.3429-08	.2494-08	.1541-08	.8141-09
	.7645-09	.1395-08	.2084-08	.2566-08	.2627-08	.2253-08	.1630-08	.1002-08	.5276-09
	.5184-09	.9375-09	.1388-08	.1697-08	.1726-08	.1472-08	.1060-08	.6491-09	.3404-09
	.3496-09	.6267-09	.9208-09	.1118-08	.1130-08	.9583-09	.6869-09	.4190-09	.2190-09
	.2351-09	.4178-09	.6092-09	.7345-09	.7381-09	.6229-09	.4445-09	.2702-09	.1408-09
	.1579-09	.2783-09	.4028-09	.4825-09	.4822-09	.4049-09	.2877-09	.1743-09	.9054-10
	.1062-09	.1856-09	.2667-09	.3175-09	.3155-09	.2637-09	.1866-09	.1126-09	.5834-10
	.7160-10	.1242-09	.1772-09	.2096-09	.2072-09	.1723-09	.1214-09	.7303-10	.3773-10
	.4854-10	.8354-10	.1184-09	.1391-09	.1367-09	.1132-09	.7943-10	.4760-10	.2452-10
	.3314-10	.5659-10	.7961-10	.9300-10	.9088-10	.7485-10	.5231-10	.3123-10	.1604-10
	.2284-10	.3868-10	.5402-10	.6270-10	.6092-10	.4992-10	.3473-10	.2066-10	.1057-10
	.1591-10	.2672-10	.3704-10	.4271-10	.4125-10	.3362-10	.2329-10	.1379-10	.7033-11
	.1122-10	.1869-10	.2571-10	.2944-10	.2826-10	.2291-10	.1579-10	.9309-11	.4728-11
	.8029-11	.1326-10	.1810-10	.2057-10	.1962-10	.1581-10	.1083-10	.6359-11	.3217-11
	.5838-11	.9553-11	.1293-10	.1459-10	.1382-10	.1106-10	.7540-11	.4403-11	.2217-11
	.4320-11	.7003-11	.9397-11	.1052-10	.9887-11	.7863-11	.5326-11	.3093-11	.1549-11
	.3258-11	.5230-11	.6954-11	.7716-11	.7198-11	.5683-11	.3824-11	.2207-11	.1099-11
	.2508-11	.3984-11	.5246-11	.5769-11	.5337-11	.4181-11	.2793-11	.1601-11	.7927-12
	.1973-11	.3100-11	.4041-11	.4401-11	.4035-11	.3135-11	.2078-11	.1183-11	.5815-12
	.1589-11	.2467-11	.3181-11	.3429-11	.3114-11	.2398-11	.1576-11	.8898-12	.4343-12
	.1310-11	.2010-11	.2562-11	.2732-11	.2455-11	.1872-11	.1220-11	.6828-12	.3306-12

TABLE C11 NORMALIZED LIKELIHOOD FUNCTION FOR A MAXIMUM AT 13.3 M/S, 206, OF 3.159 x 10^{-4}. ALL SIX MEASUREMENTS INPUT 12 M/S, 30°.

13.3

	.7168-05	.1605-04	.2854-04	.4070-04	.4699-04	.4433-04	.3446-04	.2227-04
	.9541-05	.2108-04	.3700-04	.5210-04	.5939-04	.5533-04	.4249-04	.2713-04
	.1250-04	.2729-04	.4732-04	.6583-04	.7418-04	.6831-04	.5186-04	.3274-04
	.1611-04	.3477-04	.5963-04	.8206-04	.9146-04	.8332-04	.6259-04	.3910-04
	.2040-04	.4358-04	.7399-04	.1008-03	.1112-03	.1003-03	.7466-04	.4619-04
	.2535-04	.5367-04	.9030-04	.1219-03	.1333-03	.1192-03	.8793-04	.5393-04
	.3090-04	.6490-04	.1083-03	.1451-03	.1574-03	.1397-03	.1022-03	.6220-04
	.3691-04	.7699-04	.1276-03	.1698-03	.1829-03	.1612-03	.1171-03	.7081-04
	.4316-04	.8952-04	.1475-03	.1951-03	.2090-03	.1831-03	.1323-03	.7952-04
	.4937-04	.1019-03	.1672-03	.2202-03	.2348-03	.2047-03	.1472-03	.8804-04
	.5521-04	.1136-03	.1858-03	.2437-03	.2590-03	.2249-03	.1612-03	.9604-04
	.6030-04	.1238-03	.2021-03	.2644-03	.2803-03	.2429-03	.1736-03	.1032-03
	.6428-04	.1319-03	.2150-03	.2811-03	.2977-03	.2576-03	.1838-03	.1091-03
	.6683-04	.1372-03	.2238-03	.2926-03	.3098-03	.2680-03	.1912-03	.1134-03
206°	6772-04	.1393-03	.2276-03	.2980-03	.3159-03	.2736-03	.1953-03	.1159-03
	.6683-04	.1380-03	.2260-03	.2968-03	.3154-03	.2738-03	.1958-03	.1164-03
	.6419-04	.1332-03	.2191-03	.2889-03	.3082-03	.2684-03	.1926-03	.1149-03
	.5998-04	.1252-03	.2072-03	.2746-03	.2945-03	.2577-03	.1858-03	.1113-03
	.5447-04	.1146-03	.1910-03	.2549-03	.2750-03	.2422-03	.1756-03	.1058-03
	.4807-04	.1020-03	.1715-03	.2308-03	.2510-03	.2227-03	.1626-03	.9857-04
	.4118-04	.8831-04	.1500-03	.2038-03	.2237-03	.2002-03	.1474-03	.9008-04
	.3423-04	.7431-04	.1277-03	.1754-03	.1946-03	.1759-03	.1308-03	.8066-04
	.2760-04	.6073-04	.1057-03	.1471-03	.1651-03	.1510-03	.1135-03	.7074-04
	.2156-04	.4818-04	.8510-04	.1201-03	.1366-03	.1265-03	.9631-04	.6075-04
	.1632-04	.3708-04	.6657-04	.9538-04	.1101-03	.1035-03	.7987-04	.5105-04
	.1196-04	.2768-04	.5057-04	.7370-04	.8653-04	.8260-04	.6471-04	.4197-04
	.8479-05	.2002-04	.3730-04	.5537-04	.6618-04	.6428-04	.5120-04	.3374-04
	.5814-05	.1403-04	.2669-04	.4043-04	.4926-04	.4875-04	.3953-04	.2651-04
	.3853-05	.9517-05	.1852-04	.2866-04	.3567-04	.3602-04	.2978-04	.2035-04
	.2466-05	.6247-05	.1245-04	.1973-04	.2511-04	.2591-04	.2188-04	.1525-04
	.1524-05	.3966-05	.8111-05	.1318-04	.1718-04	.1814-04	.1567-04	.1116-04
	.9090-06	.2433-05	.5115-05	.8534-05	.1141-04	.1236-04	.1093-04	.7969-05
	.5227-06	.1442-05	.3122-05	.5357-05	.7363-05	.8184-05	.7425-05	.5548-05
	.2897-06	.8252-06	.1842-05	.3258-05	.4609-05	.5267-05	.4909-05	.3765-05
	.1546-06	.4556-06	.1051-05	.1918-05	.2797-05	.3293-05	.3158-05	.2489-05
	.7944-07	.2426-06	.5792-06	.1093-05	.1646-05	.1998-05	.1975-05	.1603-05
	.3927-07	.1245-06	.3081-06	.6020-06	.9379-06	.1177-05	.1200-05	.1004-05
	.1866-07	.6152-07	.1582-06	.3206-06	.5175-06	.6720-06	.7086-06	.6124-06
	.8518-08	.2926-07	.7829-07	.1649-06	.2763-06	.3719-06	.4061-06	.3630-06
	.3733-08	.1339-07	.3734-07	.8189-07	.1426-06	.1994-06	.2258-06	.2090-06
	.1570-08	.5888-08	.1715-07	.3923-07	.7117-07	.1035-06	.1217-06	.1169-06
	.6329-09	.2488-08	.7582-08	.1812-07	.3429-07	.5195-07	.6358-07	.6348-07
	.2445-09	.1009-08	.3223-08	.8061-08	.1595-07	.2521-07	.3217-07	.3342-07
	.9039-10	.3924-09	.1317-08	.3453-08	.7154-08	.1182-07	.1575-07	.1706-07
	.3197-10	.1463-09	.5166-09	.1423-08	.3093-08	.5354-08	.7457-08	.8437-08
	.1081-10	.5224-10	.1945-09	.5640-09	.1288-08	.2339-08	.3413-08	.4039-08
	.3491-11	.1785-10	.7020-10	.2147-09	.5160-09	.9851-09	.1508-08	.1871-08
	.1076-11	.5833-11	.2428-10	.7844-10	.1989-09	.3997-09	.6434-09	.8375-09
	.3161-12	.1821-11	.8038-11	.2749-10	.7364-10	.1561-09	.2647-09	.3622-09
	.8850-13	.5428-12	.2546-11	.9234-11	.2619-10	.5867-10	.1049-09	.1512-09
	.2358-13	.1543-12	.7705-12	.2970-11	.8933-11	.2119-10	.4005-10	.6090-10

APPENDIX D THE PARAMETERS FOR THE VARIANCE AND STANDARD DEVIATION OF THE BACKSCATTER MEASUREMENTS.

The parameters used in the Monte Carlo simulations account for attitude error, the slightly longer gate (which adds a slight amount of added noise) for the signal than the duration of the signal and reciever noise. It is possible to Monte Carlo either received power or backscatter since the one differs from the other only by a constant. It is quite possible for the estimate of the received power to be negative and thus negative values of $\hat{\sigma}^o$ in antilog form are also possible. All of the simulations used these negative numbers in the likelihood function whenever they happened. Only if both vertical polarization measurements 90^o apart are negative, will the procedures used fail for that particular set of values.

The variance of the backscatter can be expressed in the general form

$$\mathrm{VAR}\ \sigma^o = C_1(\sigma^o + C_2)^2 + C_3 \tag{D1}$$

in the absense of attitude error. Attitude error (small for the SCATT) adds another term of the form.

$$\mathrm{VAR}\ \sigma^o = C_4(\sigma^o)^2 + C_1(\sigma^o + C_2)^2 + C_3 \tag{D2}$$

Both forms show that the variance can never be negative, and in fact VAR σ^o has a zero lower bound. Equation (D2) can be put in the form

$$\mathrm{VAR}\ \sigma^o = \alpha\, {\sigma^o}^2 + \beta\, \sigma^o + \gamma \tag{D3}$$

where

$$\alpha = C_4 + C_1 \tag{D4}$$

$$\beta = 2C_1 C_2 \tag{D5}$$

$$\gamma = C_1 C_2^2 + C_3 \tag{D6}$$

The parameters used in this study for Beams 1, 2, and 3 are given in the following tables. They represent only a selected subset of the continuous row of cells in the patterns covered by the SCATT.

In terms of the values given in Tables D1 and D2, the variance of σ^o can be computed from Equation (D7).

$$\mathrm{VAR}\ \sigma^o = (A + \lambda)\ {\sigma^o}^2 + 2\lambda\ NR\,\sigma^o + 1.2\ \lambda(NR)^2 \tag{D7}$$

TABLE D1 PARAMETERS FOR BEAMS 1 AND 3

DISTANCE CROSS TRACK (KM)	INCIDENCE ANGLE	λ $(\times 10^{-3})$	$N\mathcal{R}$	A1 BEAM 1 $(\times 10^{-4})$	A2 BEAM 3 $(\times 10^{-4})$	$\mathcal{R}$ $(\times 10^{15})$
180	23^{o}	1.94	0.3905	16.53	29.70	5.083
230	29^{o}	2.184	0.06411	11.95	17.56	1.1748
280	34^{o}	2.501	0.02826	7.95	10.89	0.47435
330	39^{o}	2.841	0.01480	5.21	6.55	0.28794
380	43^{o}	3.390	0.01167	5.12	6.45	0.26589
430	47^{o}	3.979	0.01163	5.54	7.29	0.31038
480	50^{o}	4.682	0.01242	5.96	7.91	0.3900
530	53.5^{o}	5.510	0.01545	6.37	8.53	0.6209
580	56^{o}	6.477	0.02853	6.74	9.55	0.8919
630	59^{o}	7.600	0.03143	7.22	10.50	1.602

TABLE D2 PARAMETERS FOR BEAM 2

DISTANCE CROSS TRACK (KM)	INCIDENCE ANGLE	λ ($\times 10^{-3}$)	NR	A2 ($\times 10^{-4}$)	R ($\times 10^{15}$)
180	17.5^{o}	2.722	0.4834	9.29	8.829
230	22^{o}	2.937	0.1147	9.29	2.261
280	26.5^{o}	3.212	0.0377	9.29	0.8125
330	30.5^{o}	3.552	0.01728	9.29	0.4119
380	34.4^{o}	3.961	0.01047	9.29	0.2782
430	37.9^{o}	4.448	0.007916	9.29	0.2363
480	41.2^{o}	5.018	0.007140	9.29	0.2404
530	44.3^{o}	5.678	0.007347	9.29	0.2799
580	47^{o}	6.437	0.008348	9.29	0.3606
630	49.7^{o}	7.301	0.01023	9.29	0.5010

APPENDIX E DATA ON ONE DEGREE VARIATIONS IN ASPECT ANGLE FOR AN 8 M/S WIND AND AN INCIDENCE ANGLE OF 39°.

The Data Appendix contains the results of a run for one degree changes in aspect angle for a wind of 8 m/s at an incidence angle of 39°. These data provide an indication of the scatter (or coarsness) of the 15° changes in aspect angle.

Figure E1 is a graph of the correlation coefficients in 1 degree steps. Also shown is a 5 point running average plus and minus the standard deviation of this 5 point running average. The scatter of the individual values is large. A trend is clear and would show up for similar calculations at other wind speeds and incidence angles. The scatter is particularly large near zero degrees and just past 20°.

Figure E2 shows the standard deviations of the errors in wind direction plus a five point running average plus and minus one standard deviation. For the smoothed data direction errors exceed 7.5° for aspect angles between 10° and 25°. Above 29°, the errors are one third of those between 10° and 25°. If this effect were to occur for other wind speeds and over other values of aspect and incidence angle, it could be used to locate areas of the wind field for which the wind direction would be very accurate. These areas would correspond to places where the wind direction was more than 30° away from the direction of Beam 1 and less than some value greater than 47°, which could be found by continuing the calculations that gave Figure E2 for a full 360°. There would probably be other direction ranges for which the direction errors would be about $\pm 2.5^{\circ}$.

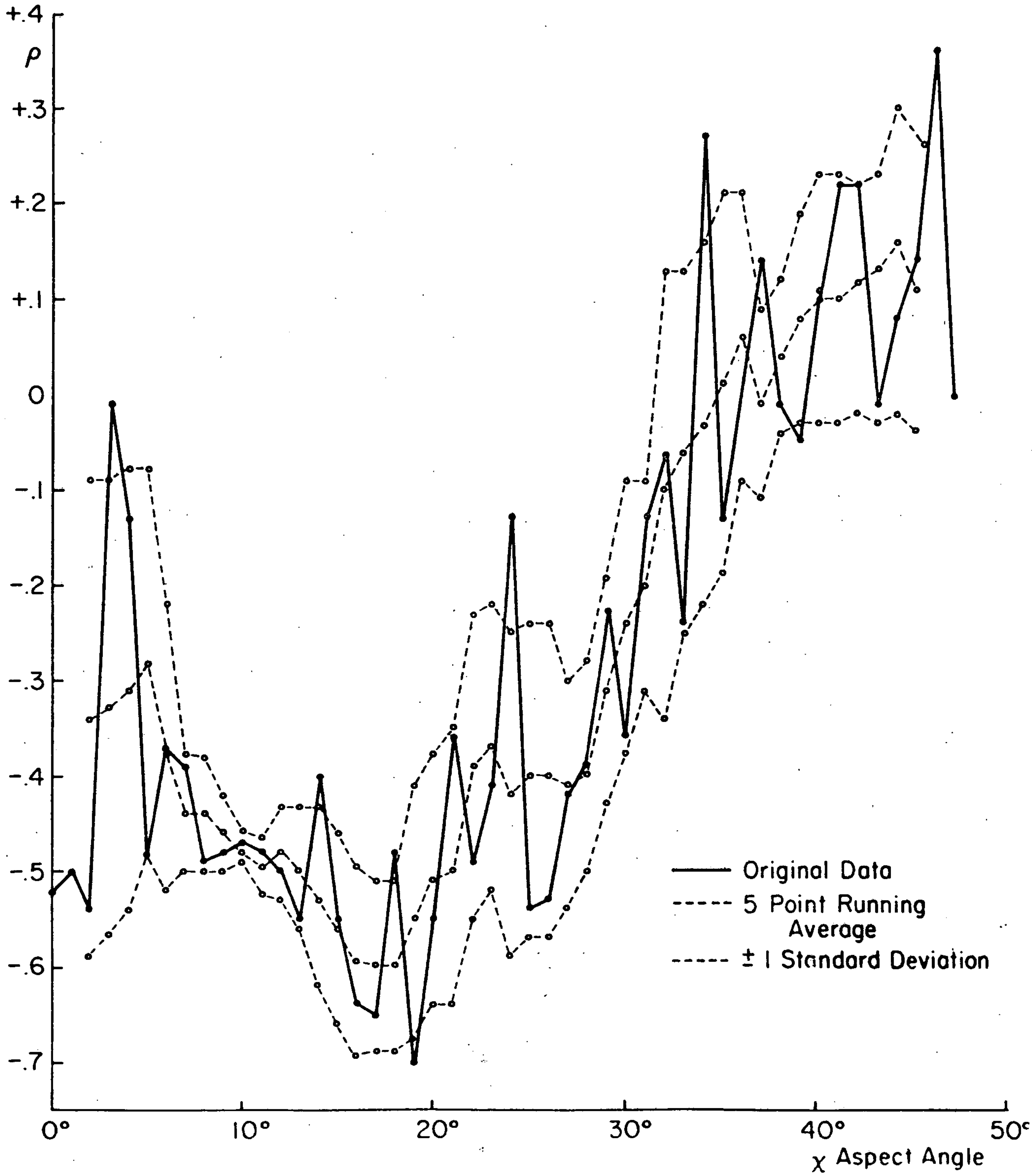


FIGURE E1 CORRELATION BETWEEN WIND SPEED AND WIND DIRECTION ERRORS IN ONE DEGREE AZIMUTH STEPS FOR 8 M/S AND AN INCIDENCE ANGLE OF 39°.

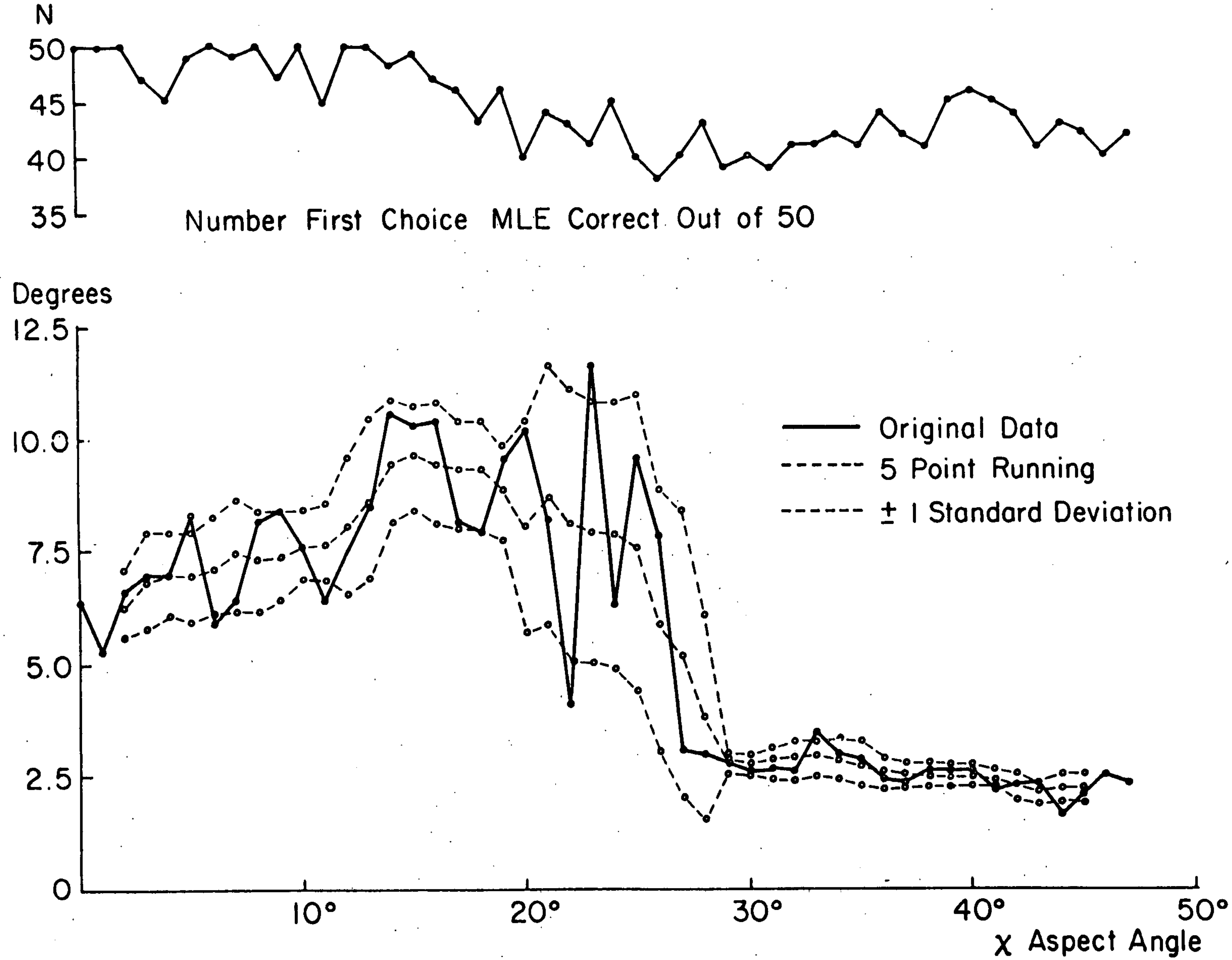


FIGURE E2 STANDARD DEVIATION OF DIRECTION ERRORS FOR FIRST CHOICE MLE AS A FUNCTION OF ASPECT ANGLE FOR A WIND SPEED OF 8 M/S AND AN INCIDENCE ANGLE OF 39°.

APPENDIX F A NOTE ON CELL AVERAGES

The values of the normalized likelihood function will be different depending on how it is calculated for multiple cell combinations. As discussed on Page 88 and following, consider the terms that contributed to the logarithm of the likelihood function from just one antenna and one polarization for a 5 cell average.

The 5 measurements could be averaged to yield

$$\bar{\sigma}^o = \frac{1}{5} \sum_1^5 \hat{\sigma}^o_p \tag{F1}$$

The term in the likelihood function for this one value would be (F2)

$$ln\ L_2' = -\tfrac{1}{2} \frac{(\bar{\sigma}^o - \sigma^o(V,\chi))^2}{((\mathrm{VAR}\ \sigma^o(V,\chi))/5)} \tag{F2}$$

$$+ \text{ other terms.}$$

The other way to use the data would be to compute (F3).

$$ln\ L_2' = -\tfrac{1}{2} \sum_1^5 \frac{(\hat{\sigma}^o_p - \sigma^o(V,\chi))^2}{\mathrm{VAR}\ \sigma^o(V,\chi)} \tag{F3}$$

$$+ \text{ other terms}$$

In (F2), the term is a maximum if $\sigma^o(V,\chi) = \bar{\sigma}^o$. The sum of terms in (F3) will also be a maximum if $\sigma^o(V,\chi) = \bar{\sigma}^o$ since the second moment about the mean is the smallest second moment possible.

Pooled data for the SASS on SEASAT if evaluated for the vertical polarization mode with two beams 90^{o} apart for paired cells with nearly the same incidence angle would yield curves similar to Figure 4 as the loci of the maxima of the two sets of terms one from each beam. The recovered winds would be virtually identical except for the added complications of the small contributions from the terms involving the natural log of the variance.

In a Monte Carlo simulation both (F2) and (F3) could be evaluated at the input wind speed and direction. For this point in the V-χ plane, (F2) becomes

$$ln\, L_1' = -\tfrac{1}{2} \frac{(\sum_1^5 t_p)^2}{5} \; ++ \qquad \text{(F4)}$$

and (F3) becomes

$$ln\, L_2' = -\tfrac{1}{2} \sum_1^5 (t_p^2) \; ++ \qquad \text{(F5)}$$

The quantity in (F4) is essentially a Chi Square variable with one degree of freedom, which has a high probability of being near zero. The quantity in (F5) is essentially a Chi Square variable with 5 degrees of freedom with a small probability of being near zero.

Vertical polarization pair measurements of backscatter 90^{o} apart should yield the same winds whether averaged first or pooled in the likelihood function. For the SCATT design the situation is different. The calculation of the normalized likelihood function from averaged backscatter values based on all 6 SCATT measurements will not yield the same result as calculating the function from individual values since the function responds differently to the scatter of the data for a particular antenna and polarization. In one case, data with large scatter can produce the same mean as data with small scatter. In the second case, the large scatter will greatly reduce the maximum of the function compared to the smaller scatter.

APPENDIX G

A COMPARISON OF THE SCATT CELL PATTERN WITH A COARSER RESOLUTION CELL PATTERN

Introduction

As a spacecraft moves along in a near earth orbit, a radar system has only a small amount of time to make a measurement for a given area of the ocean.With the addition of two more measurements on each side of the spacecraft for a total of six measurements, the time available for each measurement must decrease. The advantages of designing for a nominal 10 km resolution and then pooling the data to obtain larger resolution cells, compared to using a larger nominal 50 km cell and not pooling the data, are many. The first of the two procedures ought to provide more accurate vector wind values. In this appendix, the various features of two alternative designs will be discussed so as to demonstrate the advantages of an initial 10 km resolution. These results are based on information provided by Mr. E. M. Bracalente in a review of an earlier version of this report.

A 50 km Resolution Pattern

For a given range abeam of the subsatellite track of a spacecraft with a 50 km scanning pattern for a scatterometer, the centroids of the six cells scanned by the spacecraft can be made to fall along a line paralleling the subsatellite track at a constant distance abeam. This requires tunable doppler filters to account for the effect of earth rotation.

As shown schematically in Figure 1G, the six centroids for a set of six measurements at a nominal 50 km resolution are shown by circles, squares and triangles labled for the two polarizations, (H, for horizontal and V, for vertical). The outlined measurements are for horizontal polarization. The first measurement would be centered on the vertical polarization circle for beam 1. The second would be centered on the horizontal polarization circle for beam 1, and the illuminated area is outlined by straight lines and dashed lines. The area illuminated by the V pol measurement could be found by shifting the H pol area upward to center it on the V pol circle.

Some time later the V and H polarization measurements for beam 2 would be made, and the illuminated area for H polarization is again outlined. Still later, beam 3 would make two measurements.

The antenna pattern for a given antenna of a scatterometer can be described by a function, $\psi(r, \theta, \phi)$. This function has value on a surface in three dimensional space such that, for beam 1 for r measured along a line at 45^{o} to the spacecraft, the half power points, $(\sqrt{2/2})^2$, relative to the maximum at the center of the line as given by $(\psi(r_1, \theta_1, \pm \epsilon))^2 / (\psi(r_1, \theta_1, \phi_1))^2$, are very close to the line. A long thin band of the sea surface is illuminated each time a radar pluse is transmitted. The motion of the spacecraft plus earth rotation effects introduce different doppler shifts as a function of range to the illuminated area, and by filtering the return signal for various variable doppler frequency bands, a number of cells can be isolated along this long thin band. The dashed portions of the outlined cells are determined by the doppler filters.

The outlined cells also have lines parallel to the sub-track direction. The instantaneously illuminated sea surface area for beam 1, H-pol would be found by drawing a straight line paralled to the long side starting at the lower left corner and connecting it to a different doppler line near the upper right. This instantaneous area is outlined along the top by a dash-dot line.* A succession of pulses is transmitted for a particular beam before switching to a different polarization, or a new beam, and the motion of the spacecraft causes the resulting six sided polygon. The backscatter measurements from all of the pulses for a given beam and instantaneous cell, as it moves, are averaged to obtain the value of σ^{o} to be used in the determination of the wind.

---

*Actually a few milliseconds.

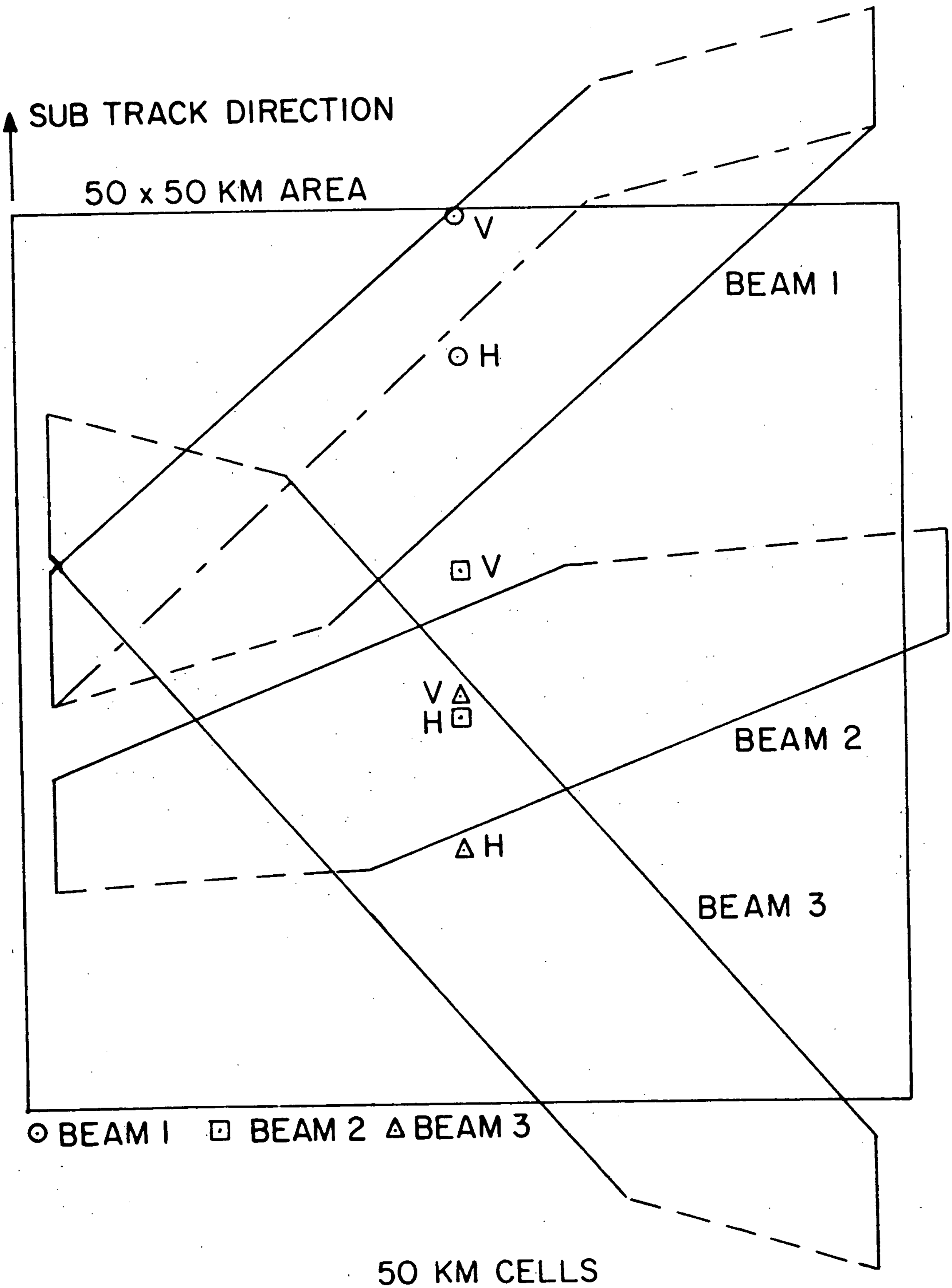


FIG. 1G 50 x 50 CELL GEOMETRY

The full scanning pattern can be approximated, except for earth curvature and other refinements, by laying out this 50 by 50 km square in a repeated pattern on a square grid and by allowing the six cells to shift up and down inside the square and relative to each other.

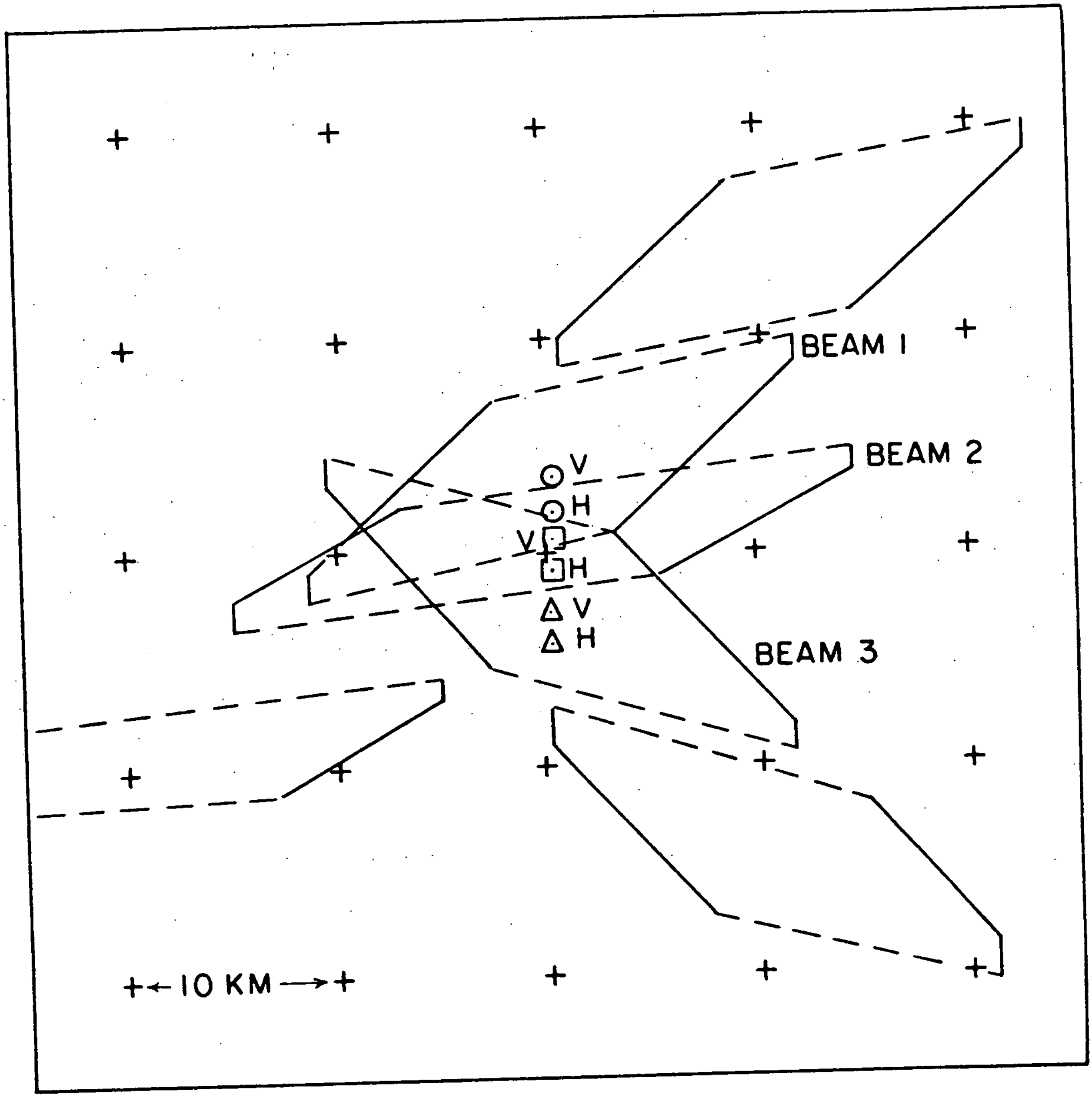
A 10 Kilometer Resolution Pattern

A 10 km resolution pattern is illustrated in Figure 2G. The centroids of six resolution cells are coded as in the previous figure. The three vertical polarization cells associated with the "+" sign at the center of the 50 km by 50 km square are shown. The slanted solid lines are determined by the beam width (ϕ), the dashed lines are the doppler boundaries, and the vertical solid lines are the effect of spacecraft motion. Cells farther out in the pattern for beams one and three are illustrated also as well as one closer in for beam 2.

This new pattern is generated by using five times the number of tunable doppler filters along each beam and by using only one fitth the number of radar pulses per measurement to obtain the data from which the value of σ^o is computed. Note that the long sides of the cells for beam 2 are determined by doppler filters. The beam is acutally pointing at an angle of 20^o to beam 1.

All six cells at the center plus sign could be found by shifting the three cells shown downward so as to be centered on the triangle, square and circle labled with an H. The very small distance moved by the spacecraft during a measurement is shown both by the vertical lines bounding each polygon and by the separation of the pairs of triangles, squares and circles.

The full pattern over the 50 km by 50 km square would be generated by translating each of the six cells defined at the center (three of which are shown), to 25 different locations having the same relative position with reference to each of the 25 plus signs inside the square.



⊙ BEAM 1 ⊡ BEAM 2 △ BEAM 3

10 KM CELLS

FIG. 2G 10 x 10 CELL GEOMETRY

The large square is almost completely covered by beam 1 and beam 3 cells when this is done, and the beam 2 cells cover more than half of the 2500 square kilometer area. This pattern is then repeated by placing 50 km squares side by side and vertically to fill out the swath. Because of earth curvature effects and other complications, the details of the pattern vary with increasing distance abeam of the spacecraft. However, the general properties of the pattern are essentially as illustrated.

Comparisons

From Figure 1G, the area covered by a single cell is roughly 13 by 70 km, or about 910 square kilometers. The twenty-five corresponding cells from Figure 2G nearly completely cover the 2500 square kilometer area (about 2437 square kilometers). Pooling the 25 cells into one measurement of backscatter as tested in the Monte Carlo studies increases the sea surface area sampled by more than a factor of three compared to the 50 km scanning pattern. Moreover, the total ares sampled is obviously more representative, from the meteorological point of view, of the center of the 50 km by 50 km square than the illumination pattern shown in Figure 1G.

The 10 km resolution pattern really does not buy something for nothing. The beam width in the ϕ direction is used more effectively in the 10 km pattern than in the 50 km pattern. It is the same for both designs, but the movement of the spacecraft for the 50 km design nearly doubles the cell areas for beams 1 and 3 and increases them by only 25 percent, or so, for the 10 km design. The pulse repetition frequencies and the procedure for cycling through polarizations and beams make more effective use of the beam width, ϕ, in the 10 km design and make it possible to cover the sea surface more uniformly.

Other Advantages

The data reduction procedures for the 10 km resolution design are not firmly established. In areas of high winds and large wind speed gradients, the higher resolution may prove useful. The data can perhaps be pooled in other patterns such as 2 by 2, (400 sq. km.), 3 by 3 (900 sq. km.), 4 x 4 (1600 sq. km.), 1 by 5 (500 sq. km.), 2 by 5 (1000 sq. km.) and so on. These patterns will depend upon additional knowledge of mesoscale wind features over the ocean. Some aspects of these techniques are discussed in the main text of this report.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-165837-1		
4. Title and Subtitle Monte Carlo Studies of Ocean Wind Vector Measurements by SCATT: Objective Criteria and Maximum Likelihood Estimates for Removal of Aliases, and Effects of Cell Size on Accuracy of Vector Winds		5. Report Date February 1982
		6. Performing Organization Code
7. Author(s) Willard J. Pierson and Robert E. Salfi		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address CUNY Institute of Marine and Atmospheric Sciences The City College of the City University of New York Convent Avenue at 138th Street New York, NY 10031		11. Contract or Grant No. NAS1-15669
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Contractor Report
		14. Sponsoring Agency Code

15. Supplementary Notes

Langley Technical Monitor: Lyle C. Schroeder
Final Report - 1980-1981

16. Abstract

The Scatterometer on the NOSS* is studied by means of Monte Carlo techniques so as to determine the effect of two additional antennas for alias (or ambiguity) removal by means of an objective criteria technique and a normalized maximum likelihood estimator. Cells nominally 10 km by 10 km, 10 km by 50 km and 50 km by 50 km are simulated for winds of 4, 8, 12 and 24 m/s and incidence angles of 29°, 39°, 47°, and 53.5° for 15° changes in direction.

The normalized maximum likelihood estimate (MLE) is correct a large part of the time, but the objective criterion technique is recommended as a reserve, and more quickly computed, procedure. Both methods for alias removal depend on the differences in the present model function at upwind and downwind.

For 10 km by 10 km cells, it is found that the MLE method introduces a correlation between wind speed errors and aspect angle (wind direction) errors that can be as high as 0.8 or 0.9 and that the wind direction errors are unacceptably large, compared to those obtained for the SASS for similar assumptions. These large errors will obscure any information about typical mesoscale wind fields at 10 km resolution.

Pooling the 10 km cells into 50 km cells (5 by 5) effectively samples the 2500 square kilometer area more uniformly and over a larger total area than using a single large cell. Also in areas of high wind gradients, pooling cells in various patterns can provide the required resolution. Thus, the 10 km resolution is still needed for these applications. Other scanning patterns might be an improvement.

*Acronym for National Oceanic Satellite System

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Satellite Scatterometer Ocean Vector Winds Wind Alias Removal Maximum Likelihood Estimates	Unclassified-Unlimited Subject Category 48

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	343	A15

*For sale by the National Technical Information Service, Springfield, Virginia 22161